PES1201802827
SOORYANATH I.T

DATE 11 4 2020

Assignment - 5

i) wkt $y = A + Bx + Cx^2$

at (1,1)

$1 = A + B + C \rightarrow$ (i)

at (2,-1)

$-1 = A + 2B + 4C \rightarrow$ (ii)

at (3,1)

$1 = A + 3B + 9C \rightarrow$ (iii)

unknowns = A, B, C

$A + B + C = 1$
$A + 2B + 4C = -1$
$A + 3B + 9C = 1$

$Ax = b$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$

$$\approx \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{bmatrix}}_{u} \underbrace{\begin{bmatrix} A \\ B \\ C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}}_{c}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}}_{u} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

Scanned with CamScanner

PES1201801817

DATE

by backward substitution.

$2C = 4 \rightarrow C = 2$

$B + 3C = -2$

$B = -2 - 6 = -8$ //

$A + B + C = 1$

$A - 8 + 2 = 1$

$A - 6 = 1$

$A = 7$ //

sol'n

$\therefore A = 7, B = -8, C = 2$

$y = 7 + (-8)x + 2x^2$.

ii) LU decomposition of a matrix

$A = LU$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

sol'n.

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - (-5)R_1$

$R_4 \rightarrow R_4 - 5R_1$

$$\approx \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 \rightarrow R_3 - (-2)R_2$

$R_4 \rightarrow R_4 - (-2)R_2$

$$\approx \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$R_4 \rightarrow R_4 - 3R_3$

$$\approx \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

DATE

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ -5 & -2 & 3 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 5 & 2 & 5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

3) $T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$.

i) find T wrt standard basis of R^3

basis for $R^3 = \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(1,0,0) = (1,0,1)$
$T(0,1,0) = (2,1,1)$
$T(0,0,1) = (-1,1,-2)$

therefore columnwise transform gives us

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

ii) finding 4 fundamental subspaces of T

columnspace(T)

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_1 \approx \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \Rightarrow R_3 \to R_3 + R_2 \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

DATE

column 1, 2 in T produce pivots, hence they are linear independent

$C(A) = \{(1,0,1),(2,1,1)\}$

$\dim(C(A)) = 2$ ✓

row space

$C(A^T) = \{(1,2,-1),(0,1,1)\}$

$\dim(C(A^T)) = 2$ ✓

finding $N(A)$ & $N(A^T)$.

convert T to row-reduced form.

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xRightarrow[R_1 \to R_1 - 2R_1]{} \begin{array}{ccc} x & y & z \\ \hline 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

z is free variable

$x - 3z = 0$ $\qquad x = 3z$

$y + z = 0$ $\qquad y = -z$

$0.x + 0.y + 0.z = 0$ $\qquad$ let $z = 1$

$$N(A) = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$N(A) = \{(3,-1,1)\}$

$\dim(N(A)) = 1$ //

PES1201802827

DATE

finding N(AT).

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & -2 & b_3 \end{array}\right] \quad R_3 \to R_3 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & -1 & -1 & b_3 - b_1 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_3 - b_1 + b_2 \end{array}\right]$$

for consistency

$(-b_1 + b_2 + b_3 = 0)$ ✓

therefore

$N(AT) = \{(-1, 1, 1)\}$

$\dim(N(AT)) = 1$ //

iii) eigen value & eigenvectors

$$\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix} = (A - \lambda I)$$

$|A - \lambda I| = 0$

$(1-\lambda)\left([1-\lambda][-2-\lambda] - 1\right) = 0$
$+ 1(2 + (1-\lambda))$

$-(1-\lambda)^2(2+\lambda) - \cancel{(1-\lambda)} + 2 + \cancel{(1-\lambda)} = 0$

$(1-\lambda)^2(2+\lambda) = 2$

$(1-\lambda)(1-\lambda)(2+\lambda) - 2 = 0.$

$\lambda^3 - 0(\lambda^2) + (-3 - 1 + 1)\lambda -$

$\lambda^3 = 3\lambda$

PES UNIVERSITY

DATE

PES1201802827

$$\boxed{\lambda = \sqrt{3}, -\sqrt{3}, 0} \checkmark$$

eigen-vector for $\lambda = \sqrt{3}$.

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{1} - \frac{1}{1-\sqrt{3}} R_1$$

$$\approx \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1-\sqrt{3}} & -(2+\sqrt{3})+\frac{1}{1-\sqrt{3}} \end{bmatrix} = \begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 0 & 3.732 & -5.098 \end{bmatrix}$$

R

$$\approx \begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Ux = 0$$

$$-0.732x + 2y - z = 0$$

$$-0.732y + z = 0$$

$$y = \frac{-z}{-0.732} = 1.366z$$

$$-0.732x = -1.7322z$$

$$\boxed{x = 2.3664}$$

eigen vector

$$= z \begin{pmatrix} 2.3664 \\ 1.366 \\ 1 \end{pmatrix}$$

PES UNIVERSITY

PES1201802827

DATE

for $\lambda = \sqrt{3}$

$$\begin{pmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{pmatrix} = \begin{pmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 1 & 1 & -0.2679 \end{pmatrix}$$

$$R_3 \to R_3 - \frac{1}{2.732} R_1 = \begin{pmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 0 & 0.2679 & 0.0981 \end{pmatrix}$$

$$= \begin{pmatrix} \cancel{0.27}\ 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 0 & 0 & \cancel{-0.2679}\ 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

~~z = 0~~
~~y = 0~~
~~x = 0~~

let $z = 1$

$2.732y + z = 0$

$2.732y = -z$

$y = -0.3660z$

$2.732x + 2y = z$

$x = \frac{z - 2y}{2.732}$

$x = 0.63396z$

$$x_2 = \begin{pmatrix} 0.63396 \\ -0.3660 \\ 1 \end{pmatrix}$$

if $\lambda = 0$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

PES1201802827

DATE

$$x_3 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$ from N(A) solution obtained previously.

iv) factorize $T = QR$.

$$T = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$q_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$q_2 = \frac{e_2}{\|e_2\|}$$

$$e_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \left(\frac{2}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}}\right) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cancel{\frac{1}{\sqrt{2}}}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3/2 \\ 0 \\ 3/2 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1 \\ -0.5 \end{pmatrix}$$

$$q_2 = \frac{1}{\sqrt{1.5}} \begin{pmatrix} 0.5 \\ 1 \\ -0.5 \end{pmatrix}$$

$$q_3 = \frac{e_3}{\|e_3\|}$$

$$e_3 = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} - \cancel{\frac{1}{1.5}} \begin{pmatrix} 0.5 \\ 1 \\ -0.5 \end{pmatrix} \times (\cancel{-1/2} + 1 + 1) + \frac{3}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

PES1201802419

DATE

$$q_3 = \begin{pmatrix} -2/7 \\ 3/7 \\ -6/7 \end{pmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 0.5/\sqrt{1.5} & -2/7 \\ 0 & 1/\sqrt{1.5} & 3/7 \\ 1/\sqrt{2} & -0.5/\sqrt{1.5} & -6/7 \end{bmatrix}$$

$$A = QR$$

$$Q^T A = R.$$

$$R = \begin{bmatrix} 1.4142 & 2.1213 & -2.121 \\ 0 & 1.2247 & 1.2247 \\ 0 & 0 & 0 \end{bmatrix}$$

(using calculator)

4). find a best fit line.

$$y = mx + c$$

$$\text{let } y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{m \times 1}$$

$$A = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_m \end{bmatrix}_{m \times 2}$$

$$\underset{A}{\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{x}{\begin{bmatrix} c \\ m \end{bmatrix}} = \underset{B.}{\begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}}$$

DATE

PES1201802827

WKT

$$\hat{x} = (A^TA)^{-1} \cdot A^T \cdot B$$

$$\hat{x} = \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot A^T \cdot B$$

$$A^TA = \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \cdot A = \underset{2\times 4}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times 2}{\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}}$$

$$A^TA = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}.$$

$$A^TB = \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$= \underset{2\times 4}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 28 \\ 28+6 \end{bmatrix}}$$

$$= \begin{bmatrix} 28 \\ 34 \end{bmatrix}.$$

$$(A^TA)^{-1} = \begin{bmatrix} \frac{15}{58} & -1/58 \\ -1/58 & 1/29 \end{bmatrix}$$

$$\hat{x} = \begin{pmatrix} 193/29 \\ 20/29 \end{pmatrix}$$

$m = 20/29$.

$x = 193/29$.

PES UNIVERSITY

Scanned with CamScanner

DATE

5) consider the eqn of plane

let $V = \begin{bmatrix} \underset{\text{pivot}}{1} & 1 & 3 & 0 & 4 \end{bmatrix}$

$x_1 = -x_2 + (-3)x_3 - 4x_5$ //

let

$$A x_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = K_1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + K_2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + K_3 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{5 \times 3}$$

WKT $P = A(A^TA)^{-1}A^T$

using calculations

$$P = \begin{bmatrix} 26/27 & -1/27 & -1/9 & 0 & -4/27 \\ -1/27 & 26/27 & -1/9 & 0 & -4/27 \\ -1/9 & -1/9 & 2/3 & 0 & -4/9 \\ 0 & 0 & 0 & 0 & 0 \\ -4/27 & -4/27 & -4/9 & 0 & 11/27 \end{bmatrix}$$

WKT $P + Q = I$.

So $Q = I - P$

$$= \begin{bmatrix} 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/27 & 1/27 & 1/9 & 0 & 4/27 \\ 1/9 & 1/9 & 1/3 & 0 & 4/9 \\ 0 & 0 & 0 & 1 & 0 \\ 4/27 & 4/27 & 4/9 & 0 & 16/27 \end{bmatrix}$$

PES1201802827

6) for what values of a is the matrix +ve definite.

Sol'n: WKT determinant of principle submatrices ≥ 0.

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

$|a| > 0$ so $a > 0$

$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0$ so $a^2 - 4 > 0$ $\quad (a-2)(a+2) > 0$

$a^2 > 4$ $\quad (a-2) > 0$

$a > +2$ $\quad a > 2$ $\quad a+2 > 0$

$a > -2$.

so $a \neq 0, -1, 1, -2, 2$

$a \notin [-2, 2]$.

$|A| > 0$

$a(a^2-4) - 2(2a-4) + 2(4-2a) > 0$

$a(a-2)(a+2) - 4(a-2) + 4(2-a) > 0$

$a(a-2)(a+2) + 4(2-a) + 4(2-a) > 0$

$a(a-2)(a+2) + 8(2-a) > 0$

$(2-a)\left(a(a+2)(-1) + 8\right) > 0$

$a^3 - 12a + 16 > 0$

$2 - a > 0$

$\cancel{a > 2}$ $2 > a$

$a > 2$

$a > -4$

$a > 1$

$\therefore a \in (2, \infty)$ ✓

DATE

$f = x^T A x$

$f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + (-2)(x_2x_3)$

$a_{11} \quad a_{22} \quad a_{33} \quad \overset{-1}{a_{12}} \; \overset{-1}{a_{21}} \quad \overset{-1}{a_{23}} \; \overset{-1}{a_{32}}$

$\therefore a_{31} = a_{13} = 0$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad //$$

7) SVD of $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

find A^TA

$$\underset{(2\times3\times3\times2)}{A^TA} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

λ_1, λ_2 of A^TA ?

$$\begin{bmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{bmatrix} = 0$$

$A - \lambda I$

$|A - \lambda I| = ?$

$(81-\lambda)(9-\lambda) - 27^2 = 0$

$729 - 81\lambda - 9\lambda + \lambda^2 - 27^2 = 0$

$\lambda_1 = 90 \qquad \sqrt{\lambda_1} = 3\sqrt{10}$

$\lambda_2 = 0 \quad //$

$$\Sigma = \begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

PES1201802827

λ_1 vector = x_1

$x_1 =$

$$\begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$+9x + 27y = 0$

$27x + 81y = 0$

$$y \begin{bmatrix} 27/9 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \quad // \qquad y\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

-9 -27

0 0

or

soln

if $\lambda = 0$

$$x_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad //$$

for V = $\lambda_1 = 90$ ✓

$\lambda_2 = 0$ ✓

~~$\lambda_3 = 0$~~ ✓

$$V_1 = \frac{x_1}{\|x_1\|} = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$V_2 = \frac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix} = V^T.$$

$3\times2 = 2\times1 = 3V^T$

PES1201802827

DATE

finding σ

eigen values are 40, 0, 0.

$$u_i = \frac{A \cdot v_i}{\sigma_i}$$

$$u_1 = \frac{A \cdot V_1}{\sigma_1} = \frac{1}{3\sqrt{10}} A \cdot V.$$

$$= \begin{bmatrix} -0.266 \\ 0.5333 \\ 0.5333 \end{bmatrix}$$

$u_2 = ??$ $\sigma_2 = 0$ here.

we do $(AA^T - 0 \cdot I)x = 0$.

so

$$AA^T \cdot x = 0$$

$$\begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

finding null space

$$R_2 \to R_2 + 2R_1 \quad \begin{array}{c} \text{pivot} \\ \begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$10x - 20y - 20z = 0$$

$$x - 2y - 2z = 0$$

$$x = 2y + 2z$$

$$= y\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda = 0$ we get 2 vectors (eigen).

choosing $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$$u_2 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$u = \begin{bmatrix} -0.266 & 0.8944 & a \\ 0.533 & 0.44721 & b \\ 0.533 & 0 & c \end{bmatrix}.$$

u is orthogonal so $u_3 \perp u_2$ & $u_3 \perp u_1$

$$-0.266a + 0.533b + 0.533c = 0$$
$$0.8944a + 0.44721b + 0c = 0.$$

let $c = 1$

$$-0.266a + 0.533b = -0.533 \rightarrow \text{(i)}$$
$$0.8944a + 0.44721b = 0 \rightarrow \text{(ii)}$$

eqn (i) $\times$ 3.3624.

$$\cancel{-0.8944a} + 1.792162b = 1.79216$$
$$\cancel{0.8944a} + 0.44721b = 0$$
$$2.239b = 1.79216$$

$b = 0.8$

$a = 0.4$

$c = 1$

$$u_3 = \frac{1}{\sqrt{1.8}} \begin{bmatrix} 0.4 \\ 0.8 \\ 1 \end{bmatrix},$$

PES1201802827

DATE

$$A = \underbrace{\begin{bmatrix} 1/3 & 2/\sqrt{5} & 0.4/\sqrt{1.8} \\ -2/3 & 1/\sqrt{5} & 0.8/\sqrt{1.8} \\ -2/3 & 0 & 1/\sqrt{1.8} \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}}_{V^T}$$