Naïve Bayes Model and Directed Graphical Model

Akash Choudhuri
Roll: 2019D014

M.Sc (2nd Year), Mathematics with Data Science
Institute of Mathematics & Applications, Bhubaneswar
akashchoudhuri.ima@iomaorissa.ac.in

July, 2021

Outline

- Conditional Independence and Bayes Theorem.
- The Naïve Bayes Model.
- Directed Graphical Models.
- Bayesian Networks.
- Programmed Example (if time permits).

Conditional Independence and Bayes Theorem

Axioms of Probability Theorem:

- For an event A, the probability of occurrence of that event A will be greater than or equal to zero.

$$p(A) \geq 0$$

- If there are disjoint events in a sample space, then the union of all events is the summation of individual probabilities.

$$P\left(\bigcup A_i\right) = \sum_i P(A_i)$$

- In case of an event involving the universal set has the probability of 1.

Important Concepts of Probability Theory

- **Random Variable:** A random variable is a measurable function which maps each outcome of the sample space to a Real value.
- **Joint Probability Distribution:** It finds the probability of many events occurring together by treating each event as a random variable. Eg, for 3 events X_1, X_2, X_3, Joint distribution is denoted by $P(X_1, X_2, X_3)$.
- **Marginal Probability Distribution:** Let X_1, X_2, X_3 be 3 random variables. Then the marginal distribution is:

$$P(x_1) = \sum \Sigma^p(x_1, x_2, x_3)$$

Introduction to Bayes Theorem

- **Conditional Independence:** We say an event X is conditionally independent of event Y given an event Z denoted as:

$$P(X|Y, Z) = P(X|Z).$$

- **Bayes Theorem:** Principled way of calculating a conditional probability without the joint probability.

In simpler terms, the result $P(A|B)$ is referred to as the posterior probability and $P(A)$ is referred to as the prior probability. Sometimes $P(B|A)$ is referred to as the likelihood and $P(B)$ is referred to as the evidence. This allows Bayes Theorem to be restated as:

$$\text{Posterior} = \text{Likelihood} * \text{Prior} / \text{Evidence}$$

The Naïve Bayes Model

Why ‘naïve’?

- This model uses Bayes Theorem with a small assumption that **there is independence among predictors**, ie, the presence of a particular feature in a class is unrelated to the presence of any other feature.

So, our Bayes Theorem formula is re-written by omitting the denominator (a littler bit of maths can show that and it reduces to:

By Bayes Theorem, $P(B|A)=(P(A|B)*P(B))/P(A)$

$$=P(A|B)*P(B)$$

Generalising the Equation,

$$P(c|X)=P(x_1|c)*P(x_2|c)*P(x_3|c)\ldots\ldots P(x_n|c)*P(c)$$

Naïve Bayes Classifier Algorithm for Discrete Data

- **Step 1:** Given a set of features D containing target variable T, calculate $P(X_i \mid Y_i)$ where

$$X_i, Y_i \in D \text{ and } X_i \neq Y_i$$

- **Step 2:** Calculate the Class Probabilities of Y given as P(Y).
- **Step 3:** Train the Model by finding the probabilities.
- **Step 4:** For a new set of features which is a subset of D, find the corresponding T.

Directed Graphical Models

Kinds of Graphical Models

- Undirected Graphical Models also known as Markov Random Fields.
- Directed graphical models also known as Bayesian (belief) networks. The important Characteristics of Bayesian Networks are:
  - Bayesian Networks require that the graph is a DAG (directed acyclic graphs).
  - No directed cycles allowed.

Bayesian Networks

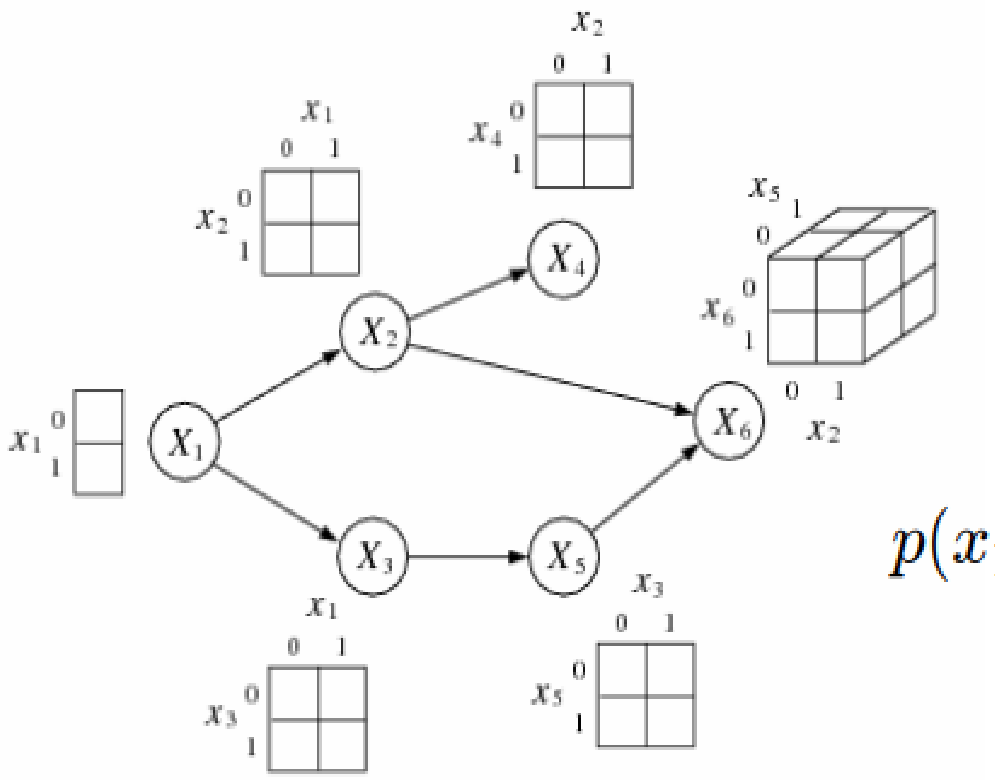
Bayesian Networks

- Judea Paul, who is credited with the invention of Bayesian Networks, won the Turing Award in 2011 for this discovery.
- A probability distribution factorizes according to a DAG if it can be written as:

$$P(x) = \prod_{j=1}^{d} P(x_j | x_{\pi j})$$

Where Π_j are the parents of j , and the nodes are ordered topologically (parents before children).

Continued....



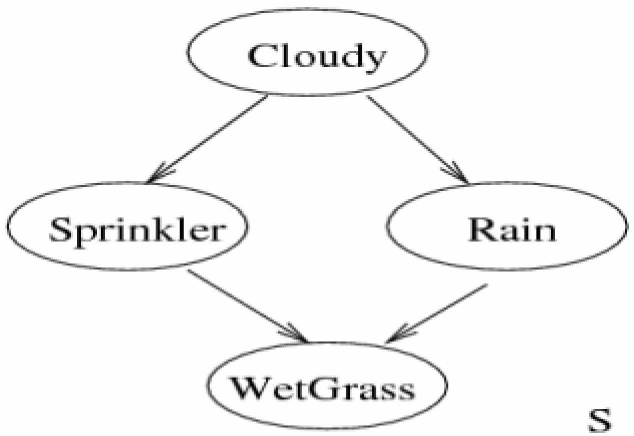
Each row of the conditional probability table (CPT) defines the distribution over the child's values given its parents values. The model is locally normalized.

$$p(x_{1:6}) = p(x_1)p(x_2|x_1)p(x_3|x_1)p(x_4|x_3)$$
$$p(x_5|x_2, x_3)p(x_6|x_2, x_5)$$

Example Bayesian Network

P(C=F)	P(C=T)
0.5	0.5

C	P(S=F)	P(S=T)
F	0.5	0.5
T	0.9	0.1



C	P(R=F)	P(R=T)
F	0.8	0.2
T	0.2	0.8

S	R	P(W=F)	P(W=T)
F	F	1.0	0.0
T	F	0.1	0.9
F	T	0.1	0.9
T	T	0.01	0.99

Continued

c	s	r	w	prob
0	0	0	0	0.200
0	0	0	1	0.000
0	0	1	0	0.005
0	0	1	1	0.045
0	1	0	0	0.020
0	1	0	1	0.180
0	1	1	0	0.001
0	1	1	1	0.050
1	0	0	0	0.090
1	0	0	1	0.000
1	0	1	0	0.036
1	0	1	1	0.324
1	1	0	0	0.001
1	1	0	1	0.009
1	1	1	0	0.000
1	1	1	1	0.040

- The joint distribution is computed using Naïve Bayes Model as:

$$\mathrm{p}\,(\mathrm{C}, \mathrm{S}, \mathrm{R}, \mathrm{W}) = \mathrm{p}\,(\mathrm{C})\,\mathrm{p}\,(\mathrm{S}|\mathrm{C})\,\mathrm{p}\,(\mathrm{R}|\mathrm{C})\,\mathrm{p}\,(\mathrm{W}|\mathrm{S}, \mathrm{R})$$

- Prior that sprinkler is on:

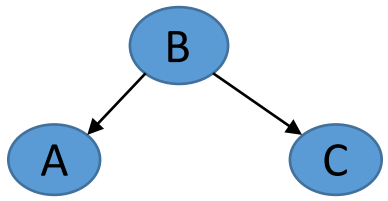
$$p(S=1) = \sum_{c=0}^{1}\sum_{r=0}^{1}\sum_{w=0}^{1} p(C=c, S=1, R=r, W=w) = 0.3$$

- Posterior that sprinkler is on given that grass is wet:

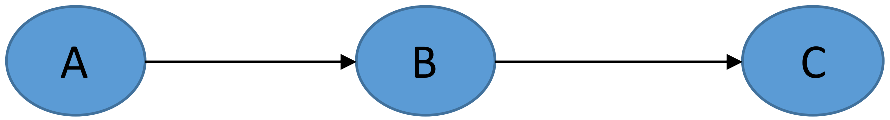
$$p(S=1|W=1) = \frac{p(S=1, W=1)}{p(W=1)} = 0.43$$

Conditional Independencies Implied from Bayesian Networks

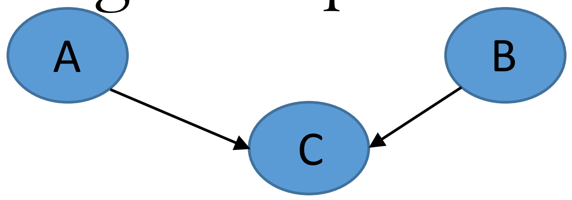
- **Common Parent:** Fixing B, A and C are decoupled in this network ($A \perp C \mid B$).



- **Cascade Structure:** In this network, $A \perp C \mid B$.



- **V- Structure:** Knowing C couples A & B.



D- Separation

Let A,B &C be non-overlapping sets of nodes (vertices) of a graph G. To ascertain $(A \perp B \mid C)$, consider all paths from any node in A to any node in B. Any such path is said to be block if it includes a node such that:

- The arrows on the path meet either head-to-tail or tail-to-tail and the node is in the set C.

OR

- The arrows meet head-to-head at the nodes and neither the node nor any of its descendants is in the set C.

Fact: If A is d-separated from B by C, then $(A \perp B \mid C)$ holds in the graph.

Programmed Example

Thank You

Questions/ Queries? Do reach out to me!