CS:4980 Homework 1

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1 Late day used.

1 SIR Model

1.1 Everyone Loves CS:4980

Here, we assume that N=1. So S(0)=0.98, R_{∞}=0.6 and R(0)=0. So, substituting the values in the equation:

$$R_{\infty} = 1 - S(0)e^{-R_0(R_{\infty}-R(0))} => 0.6 = 1 - 0.98e^{-0.6R_0} => R_0 = 1.493$$

So, the expected fraction of fellow students an interested student spreads their interest to is 1.493.

1.2 ODE $\iff$ Networks

1.2.1

P(t)=$\frac{Y(t)}{N}\beta$

1.2.2

E[X(t+1)]=E[$(1-\beta\frac{Y(t)}{N})X(t)$]
E[Y(t+1)]=E[$(\beta\frac{X(t)}{N}-\gamma+1)Y(t)$]
E[Z(t+1)]=E[$(\gamma+1)Y(t)$]

1.2.3

In this problem, by the assumption, $E[X(t+1)] \approx X(t+1)$, $E[Y(t+1)] \approx Y(t+1)$ and $E[Z(t+1)] \approx Z(t+1)$. Using this assumption,

$$X(t+1) - X(t) = (1-\beta\frac{Y(t)}{N})X(t) - X(t) = -\beta\frac{Y(t)}{N}X(t)$$

Similarly,

$$Y(t+1) - Y(t) = (\beta\frac{X(t)}{N} - \gamma + 1)Y(t) - Y(t) = (\beta\frac{X(t)}{N} - \gamma)Y(t)$$

and

$$Z(t+1) - Z(t) = (\gamma + 1)Y(t) - Y(t) = \gamma Y(t)$$

1.2.4

$$X(t + \Delta t) - X(t) = (1 - \beta \Delta t \frac{Y(t)}{N})X(t) - X(t) = -\beta \Delta t \frac{Y(t)}{N} X(t)$$

$$Y(t + \Delta t) - Y(t) = (\beta \Delta t \frac{X(t)}{N} - \gamma \Delta t + 1)Y(t) - Y(t) = (\beta \Delta t \frac{X(t)}{N} - \gamma \Delta t)Y(t)$$

$$Z(t + \Delta t) - Z(t) = (\gamma \Delta t + 1)Y(t) - Y(t) = \gamma \Delta t Y(t)$$

1.2.5

From the previous section, we can easily construct:

$$X(t + \Delta t) - X(t) = -\beta \Delta t \frac{Y(t)}{N} X(t)$$

$$Y(t + \Delta t) - Y(t) = (\beta \Delta t \frac{X(t)}{N} - \gamma \Delta t)Y(t)$$

$$Z(t + \Delta t) - Z(t) = \gamma \Delta t Y(t)$$

Now, dividing both sides by Δt, we get,

$$\frac{X(t + \Delta t) - X(t)}{\Delta t} = \frac{-\beta \Delta t \frac{Y(t)}{N} X(t)}{\Delta t}$$

$$\frac{Y(t + \Delta t) - Y(t)}{\Delta t} = \frac{(\beta \Delta t \frac{X(t)}{N} - \gamma \Delta t)Y(t)}{\Delta t}$$

$$\frac{Z(t + \Delta t) - Z(t)}{\Delta t} = \frac{\gamma \Delta t Y(t)}{\Delta t}$$

That pretty much evaluates to:

$$X'(t) = -\beta \frac{Y(t)}{N} X(t)$$

$$Y'(t) = (\beta \frac{X(t)}{N} - \gamma)Y(t)$$

$$Z'(t) = \gamma Y(t)$$

This gets us to the standard ODE equations we have seen in class.

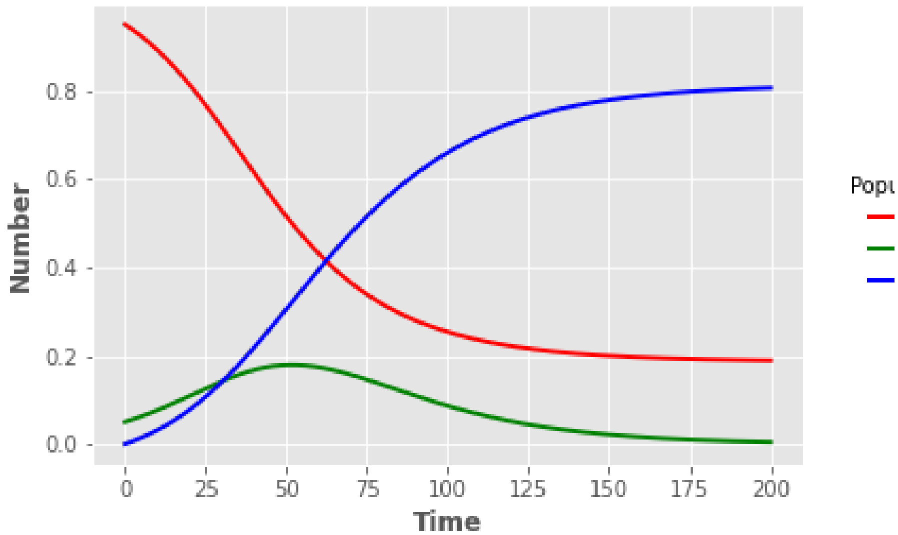

Figure 1: Base SIR model

1.3 Implementation and Calibration

1.3.1

The standard SIR ODE plot with $\beta = 0.1$ and $\gamma = 0.05$ is given by Figure 1. The standard SIR ODE plot with $\beta = 0.05$ and $\gamma = 0.1$ given by Figure 2. We notice that the disease does not take off, which means that the fraction infections fall off very soon after the simulation starts.

1.3.2

The implementation of this is based on the instructions provided at the beginning of Q1.2 The discrete time SIR Network model on the graph given by example.txt with $\beta = 0.1$ and $\gamma = 0.05$ is given in Figure 3.
The discrete time SIR Network model on the graph given by example.txt with $\beta = 0.05$ and $\gamma = 0.1$ is given in Figure 4. In this setting, 5 nodes are initially selected to be infected and no recovered nodes and 95 susceptible nodes at t=0. For a complete graph of 1000 nodes, the result of the simulation using $\beta = 0.1$ and $\gamma = 0.05$ is given in Figure 5. Also, for a complete graph of 1000 nodes, the result of the simulation using $\beta = 0.05$ and $\gamma = 0.1$ is given in Figure 6.

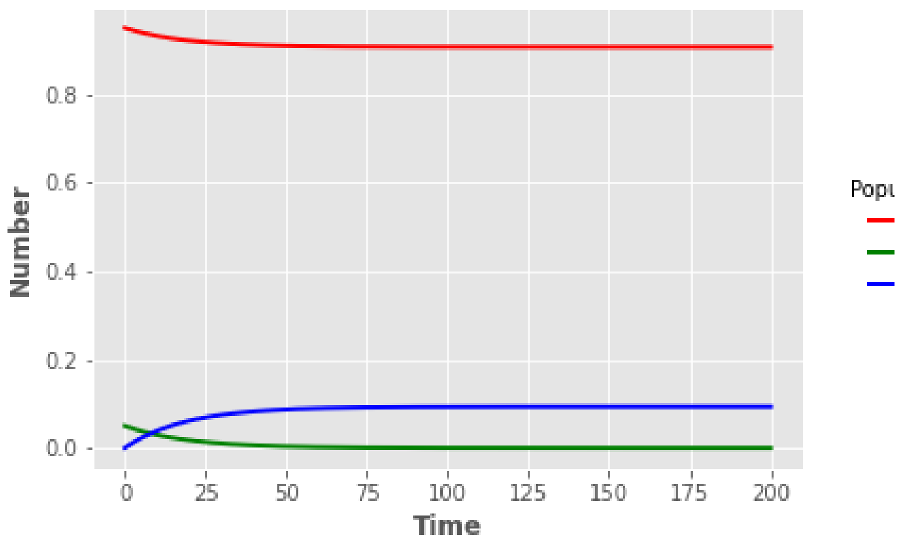

Figure 2: Base SIR model with different parameter values

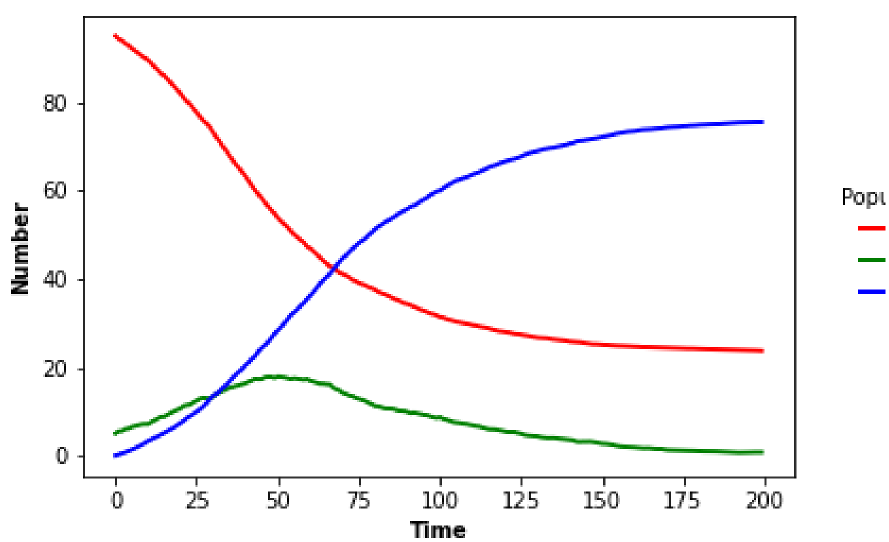

Figure 3: Network SIR Model

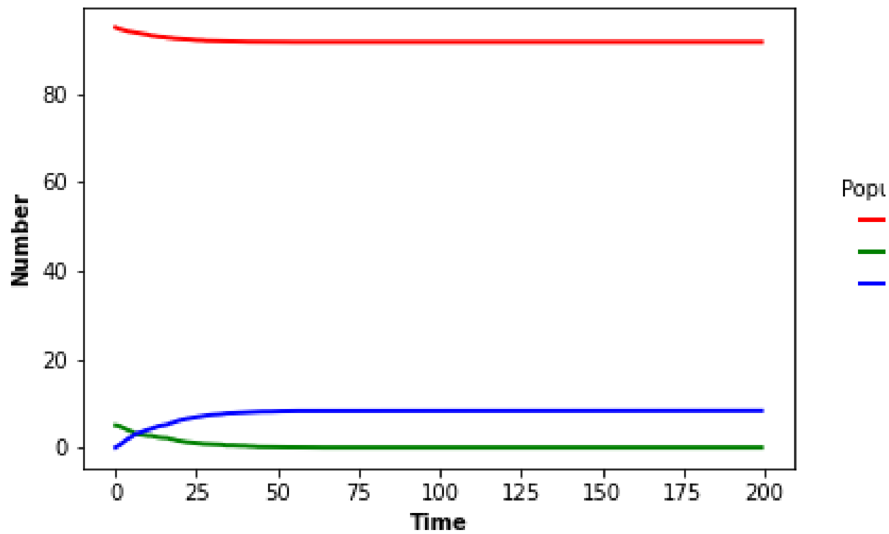

Figure 4: Network SIR model with different parameters

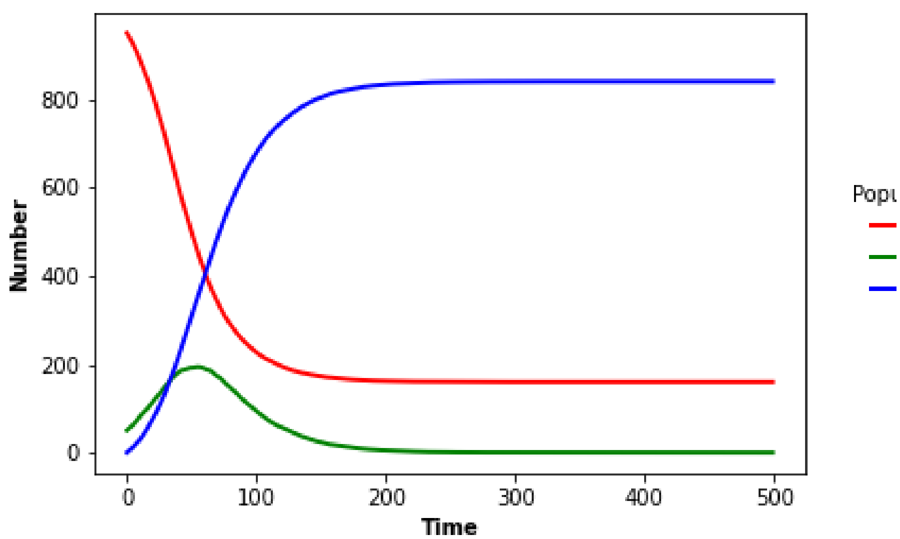

Figure 5: Network SIR model for complete graph of 1000 nodes for 500 timestamps

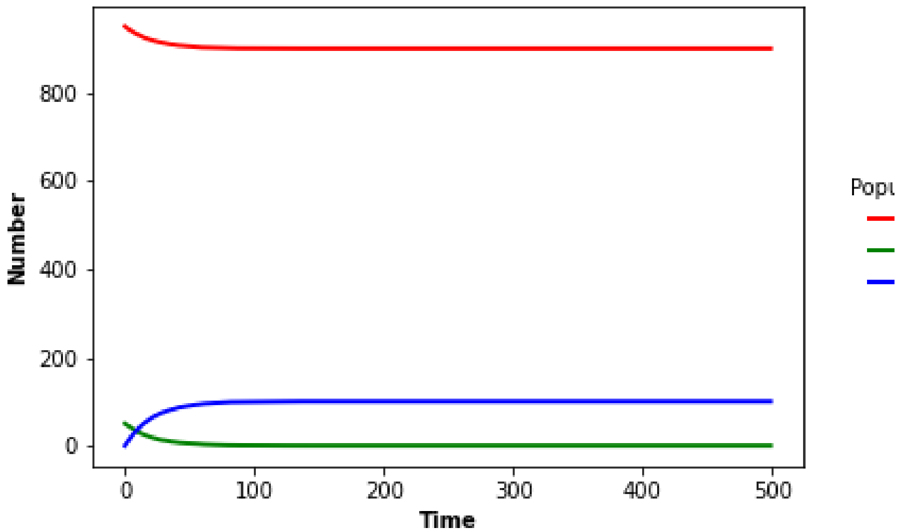

Figure 6: Network SIR model for complete graph of 1000 nodes for 500 timestamps with different parameters

1.3.3

The result of parameter calibration gives the best parameters as $\beta = 4.94614556e-02$ and $\gamma = 9.22423666e-06$. The result of the parameter calibration is summed up in Figure 7. The parameter values put in a SIR model gives us the curve shown in figure 7.

1.3.4

After getting the parameters, we used them on a clique of size 100. The result is shown in Figure 9. After getting the parameters, we used them on a clique of size 1000. The result is shown in Figure 10. After getting the parameters, we used them on a clique of size 1000. The result is shown in Figure 11. Comparing the plots, we see that the nature of spread of infection for the Network SIR model when the graph is a clique is exactly similar to that of the ODE model. This is because in a clique, homogeneous mixing takes place, which is similar to the ODE method.

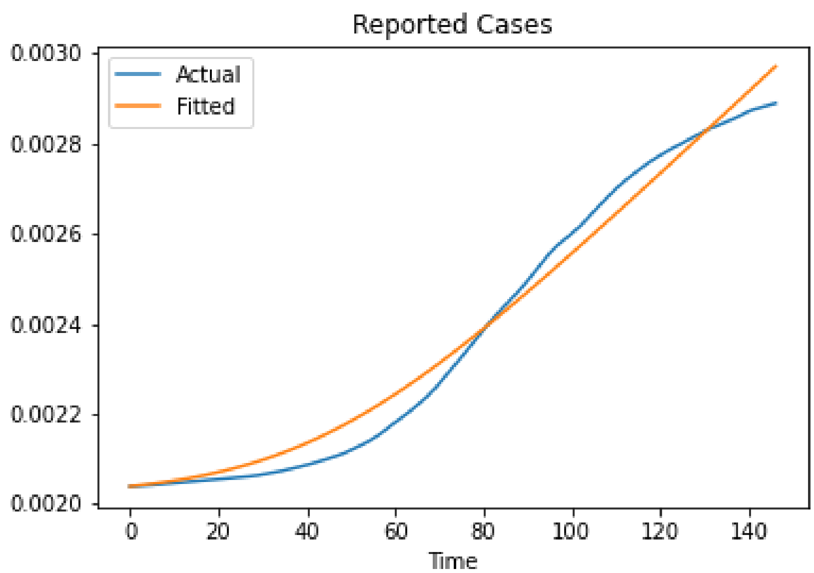

Figure 7: Calibrating Parameters for Georgia deaths

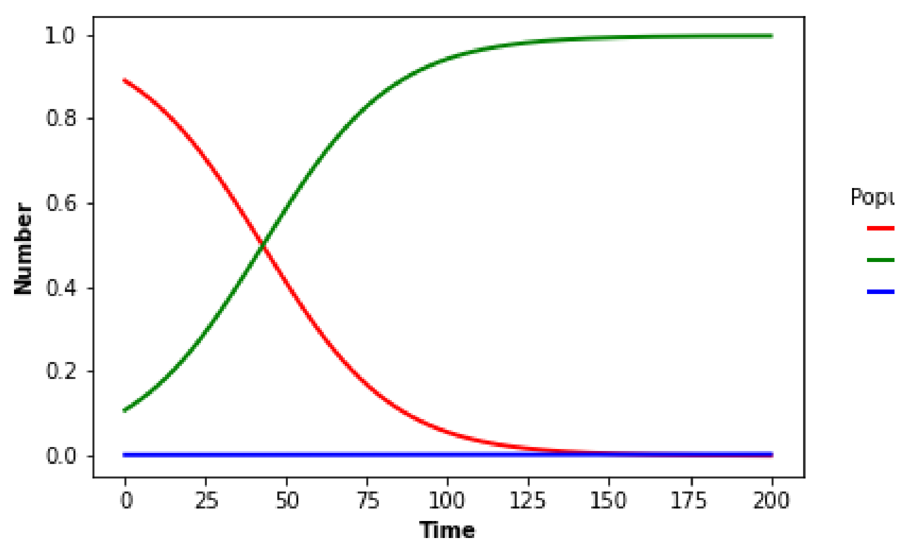

Figure 8: SIR Model with parameters got from calibrating the GA Dataset

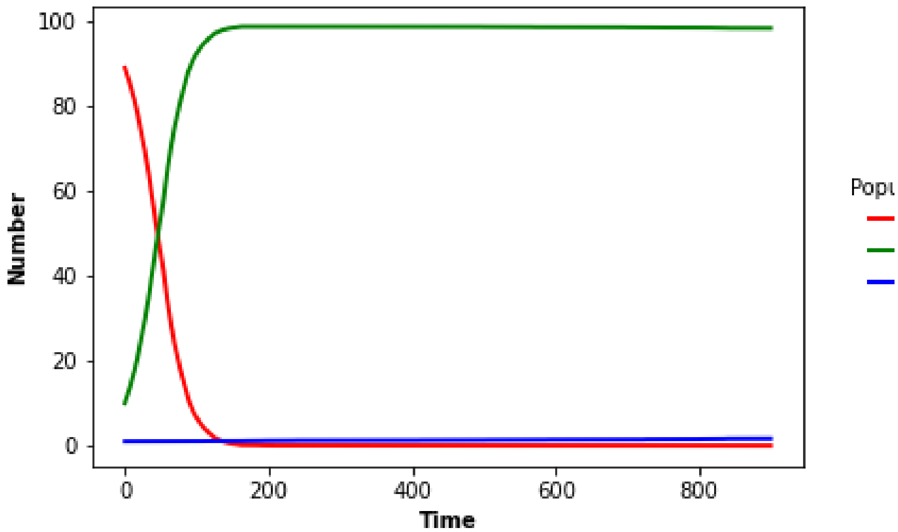

Figure 9: SIR Network Model with parameters got from calibrating the GA Dataset with N=100

2 Cayley Trees

2.1

Let us have a k-Cayley tree. So, by construction, we can see that when d=1, k nodes will be reachable (as these are the immediate neighbours of the root node). When d=2, (k-1)*k nodes will be reachable (as each of the k neighbours of the root node has k-1 immediate leaf nodes). Similarly, when When d=3, $(k-1)^2 * k$ nodes will be reachable. Generalizing the notion, when d=d, $(k-1)^{d-1} * k$ will be reachable from the root node.

2.2

Now, if we simply removed the constraint of only d-reachability, we can obtain the diameter by:

$$\sum_{i=1}^{d}(k-1)^{d-1}k = k + (k-1)k + (k-1)^2k + ... + (k-1)^{d-1}k = k\frac{(k-1)^d - 1}{k-2}$$

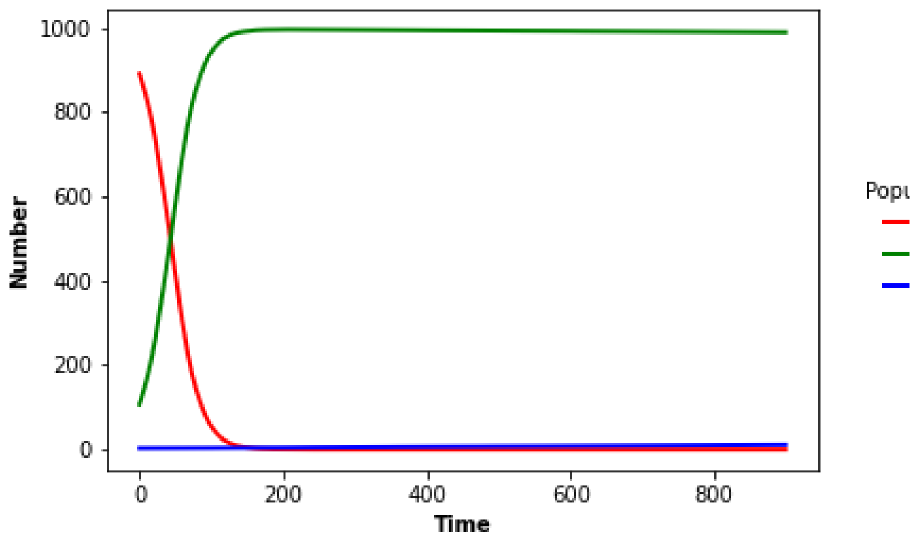

Figure 10: SIR Network Model with parameters got from calibrating the GA Dataset with N=1000

Now,

$$k\frac{(k-1)^d-1}{k-2} = n-1 => k[(k-1^d-1] = (n-1)(k-2)$$
$$=> (k-1)^d - 1 = \frac{(n-1)(k-2)}{k}$$
$$=> (k-1)^d = \frac{(n-1)(k-2)}{k} + 1$$

Taking log in both sides, we get:

$$d\log(k-1) = \log(\frac{(n-1)(k-2)}{k}+1) => d = \frac{\log(\frac{(n-1)(k-2)}{k}+1)}{\log(k-1)}$$

So, the diameter is $2\frac{\log(\frac{(n-1)(k-2)}{k}+1)}{\log(k-1)}$

3 Calibrating IC Models

3.1

L= $\binom{A_v}{A_{v2u}} p_{vu}^{A_{v2u}} (1-p_{vu})^{A_v - A_{v2u}}$

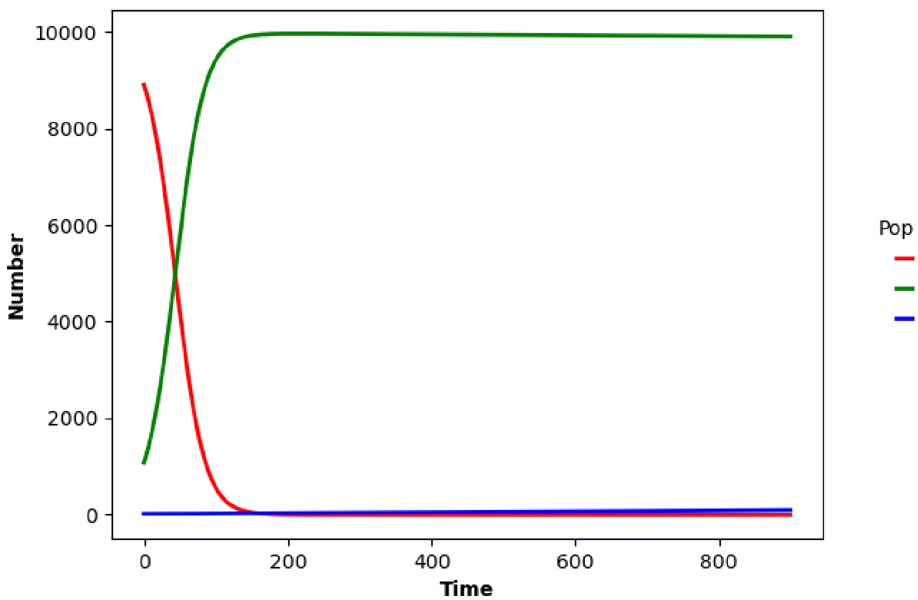

Figure 11: SIR Network Model with parameters got from calibrating the GA Dataset with N=10000

3.2

I get 459 nonzero probability values.

4 Zombie Apocalypse

4.1

For this, we know x+y=2×10^6
Now, if we pose out objective function as:

$$f(x,y) = (40 - \frac{x}{200000})(0.04 - \frac{y}{100000000})$$

So for the first case,

$$f_x(y) = \frac{y}{2 \times 10^{13}} - \frac{2}{10^7}$$

Now equating $f_x(y) = 0$, we get $y == 4 \times 10^5$. Substituting back to f(x,y), we get $f(x) = \frac{144}{100} - \frac{x}{2 \times 10^8}$
Similarly, for the second case,

$$f_y(x) = \frac{x}{2 \times 10^{13}} - \frac{4}{10^7}$$

Now equating $f_y(x) = 0$, we get $x == 8 \times 10^5$. Substituting back to f(x,y), we get $f(y) = \frac{144}{100} - \frac{y}{2 \times 10^8}$ So, amongst x and y, as y has lesser value for f(x,y) to attain maxima, thus the second measure is the better measure to allocate 2 million dollars.

4.2

4.2.1

Setting the values of x(0)=5 and y(0)=2, the plot of x(t) and y(t) vs t and phase diagram is given in Figure 12 and 13 respectively. The values do not oscilate when (x(t),y(t))=(0,0) and (1,1). The figures of which are shown in Figures 14 and 15.

4.2.2

The fixed points of LVM2 model are:

- (0,0,0) which is a saddle point
- $(\frac{\delta}{\gamma}, \frac{\alpha}{\beta}, 0)$
- $(\frac{\epsilon}{\rho}, 0, \frac{\alpha}{\phi})$

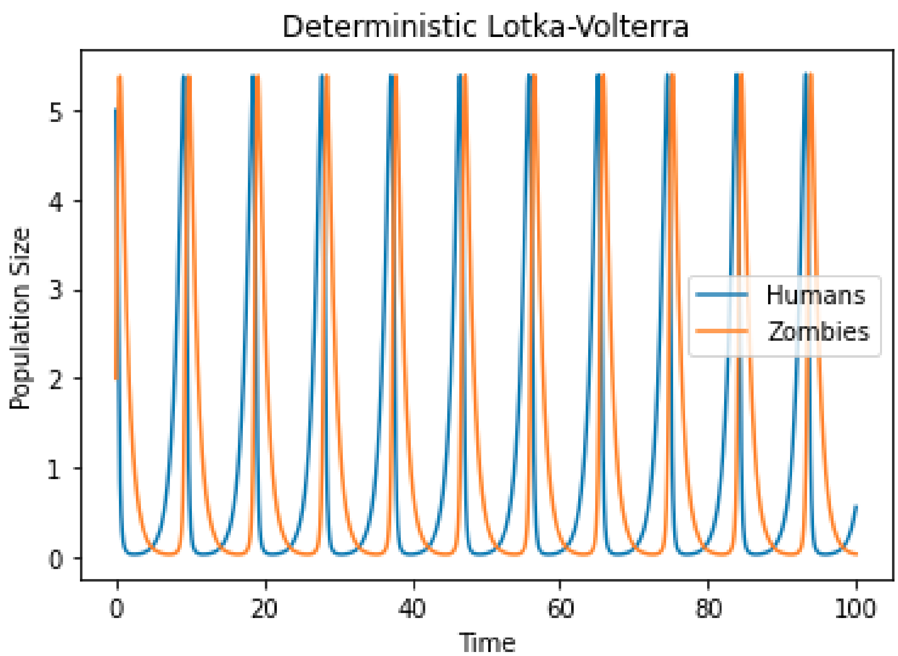

Figure 12: LV model with all parameter values as 1 but x(t)=5 and y(t)=2

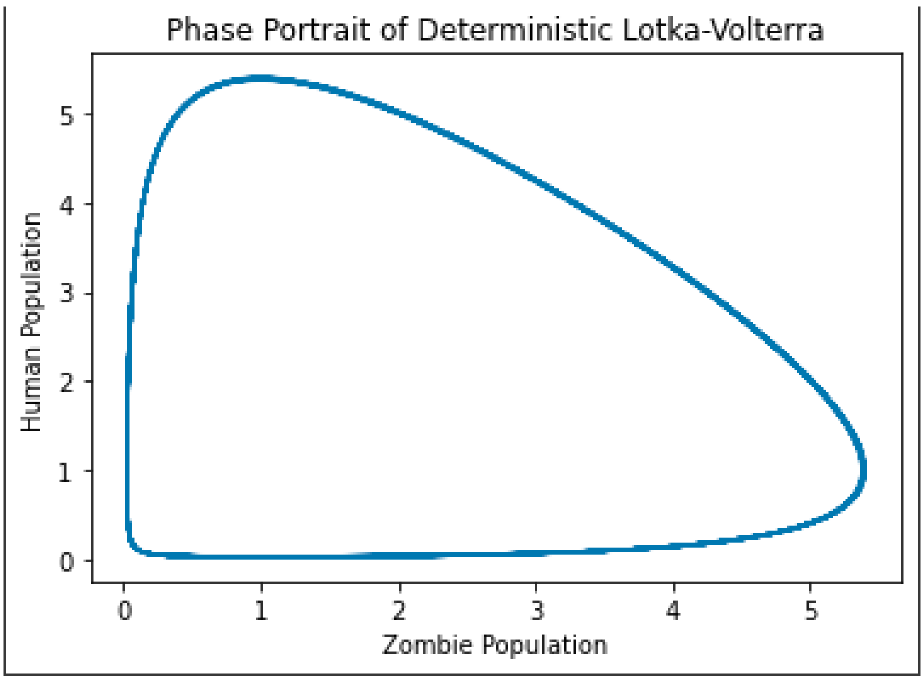

Figure 13: Phase Diagram of LV model with all parameter values as 1 but x(t)=5 and y(t)=2

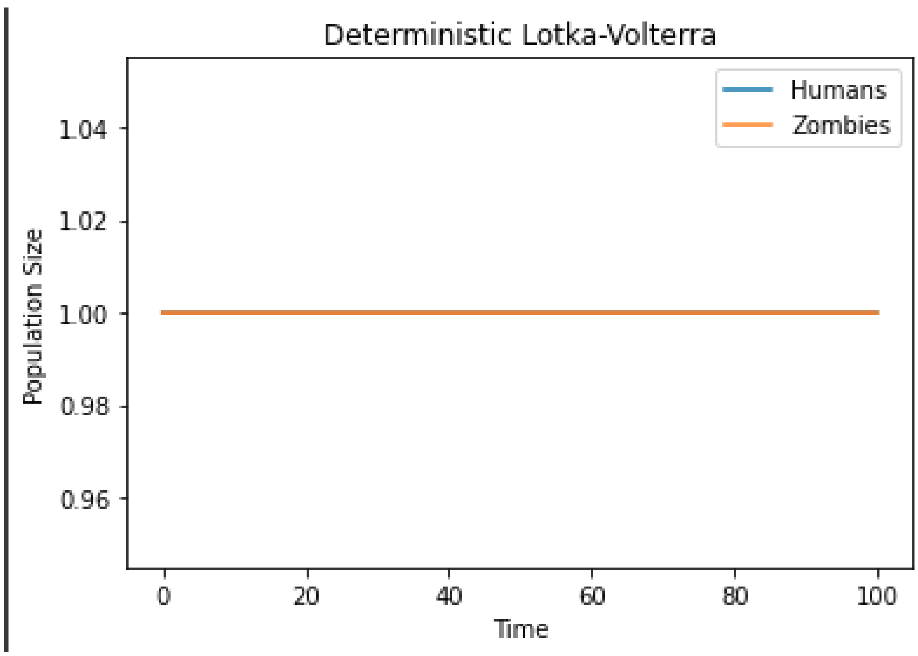

Figure 14: Phase Diagram of LV model with all parameter values as 1 but x(t)=1 and y(t)=1

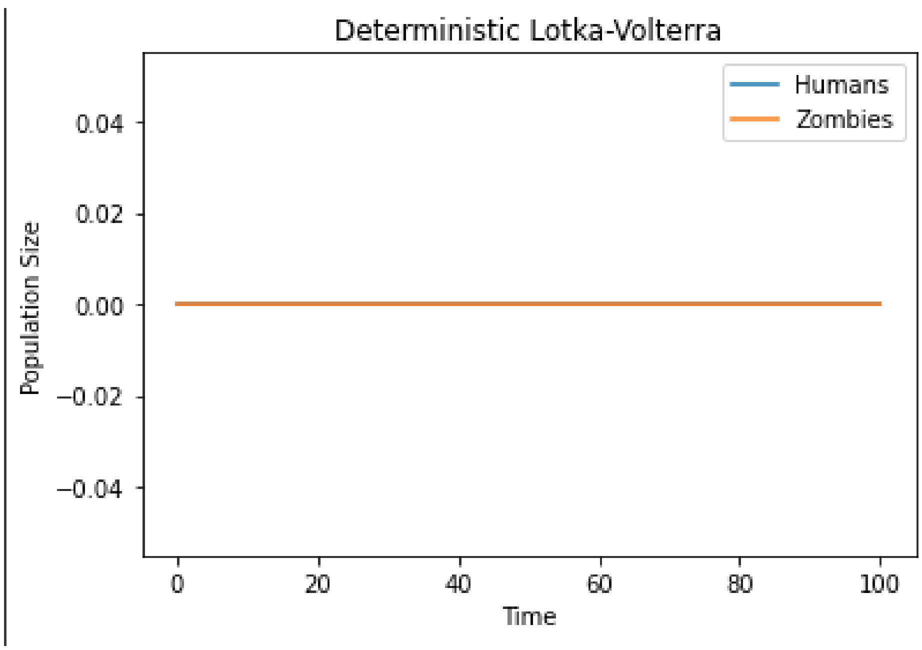

Figure 15: Phase Diagram of LV model with all parameter values as 1 but x(t)=0 and y(t)=0

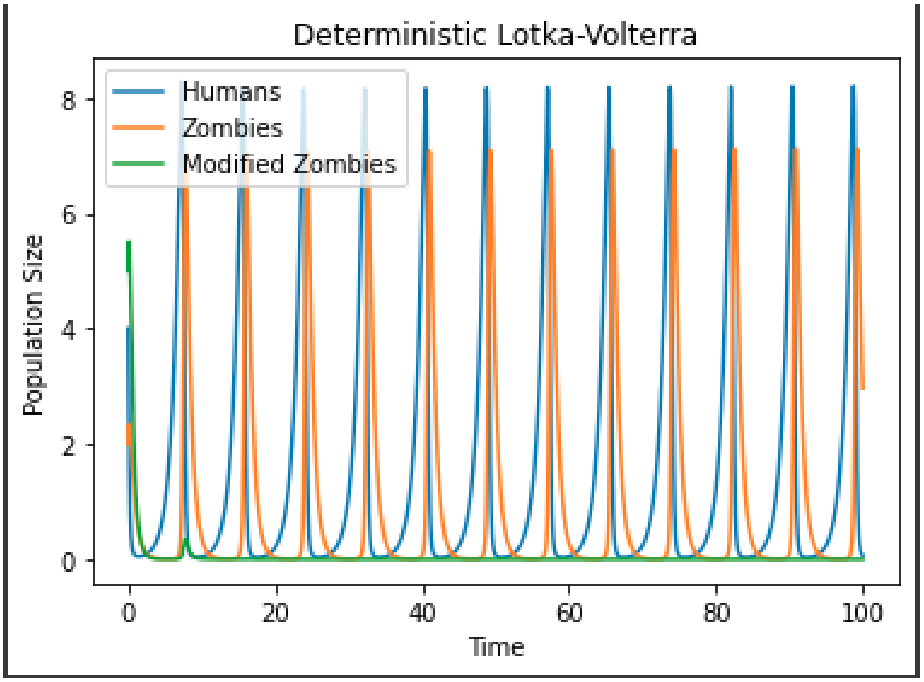

Figure 16: Phase Diagram of LVM2 model with all parameter values and initialization values asked in Q4.2.3

4.2.3

The addition of modified zombies to the LVM2 model plot looks as shown in Figure 16. At the fixed points, the first value is (0,0,0), whose plot is shown in Figure 17. The second fixed point is (1.5,1,0) whose plot is shown in Figure 18. While the third fixed point is (2,0,1) whose plot is shown in Figure 19.

4.2.4

The human and zombie values in LVM2 is periodic while modified zombie is not periodic.

5 Closing Triangles

5.1

The constructed subgraph is given in Figure 20.

5.2

I am already a part of 9 triangles.

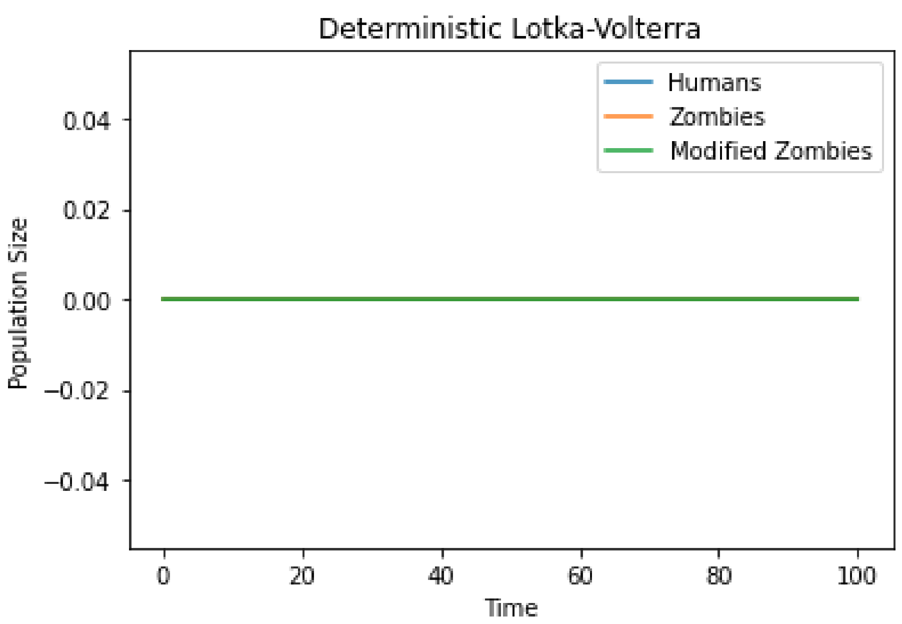

Figure 17: LVM2 model with all parameter values and initialization values of (0,0,0)

5.3

I am a part of 5 incomplete triangles (ie, potential friends).

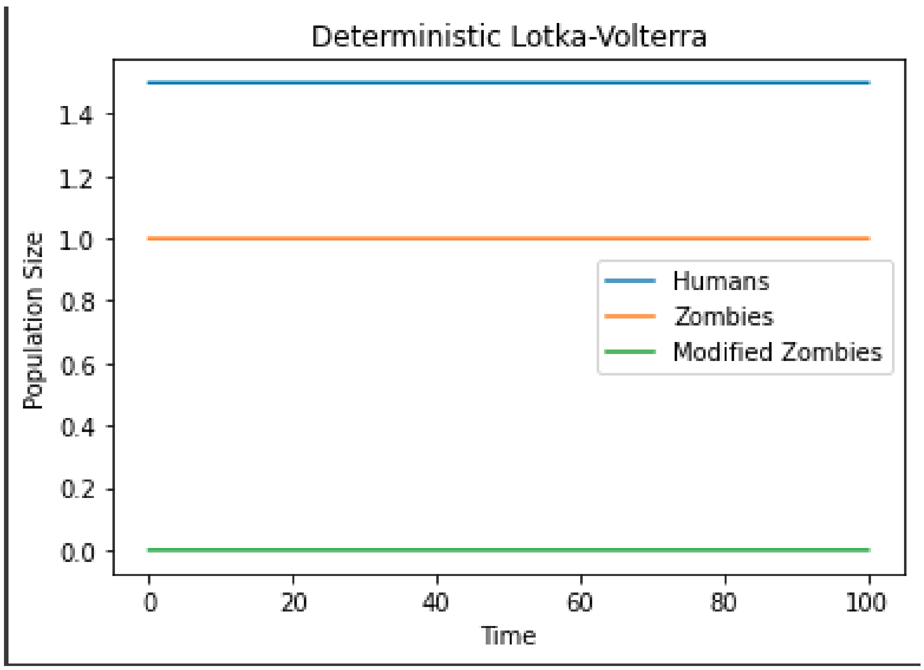

Figure 18: LVM2 model with all parameter values and initialization values of (1.5,1,0)

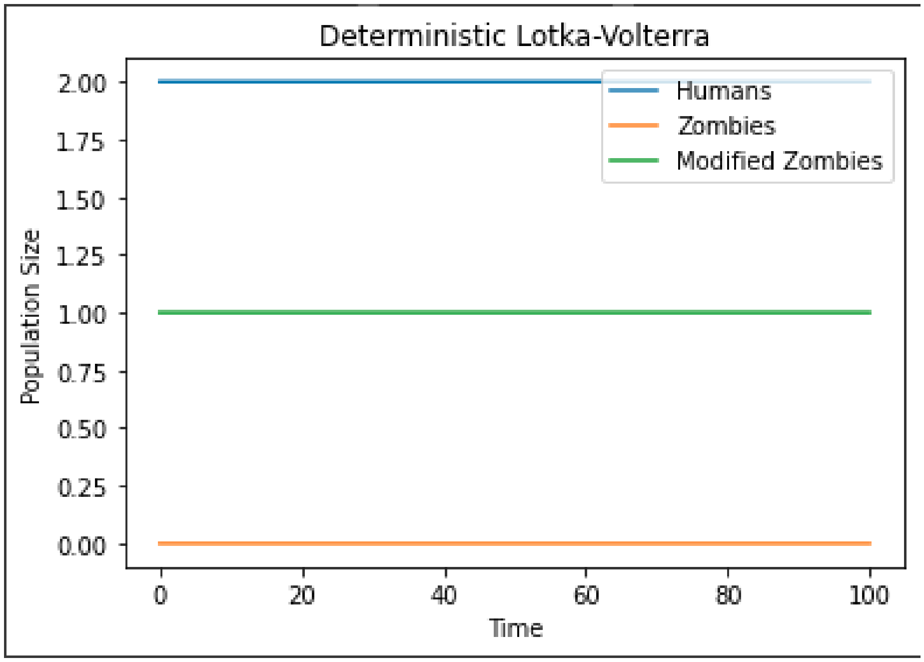

Figure 19: LVM2 model with all parameter values and initialization values of (2,0,1)

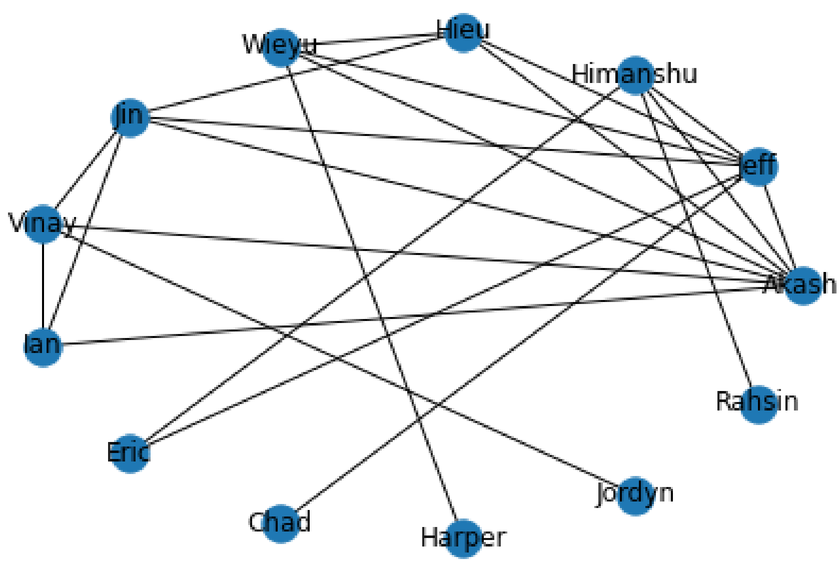

Figure 20: Friends Subgraph