

# Final Project: Distributed Leader-Follower Affine Formation Maneuver Control for High-Order Multiagent Systems

Soowhan Yi  
*Mechanical Engineering*  
*University of Washington*  
 syi94@uw.edu

## I. INTRODUCTION

Due to emerging demand for its potential applications in robotics, aerospace, and automobile industry, multiagent formation control and maneuver has been widely studied. In applications such as unmanned ground vehicles, with satellites or with drones, multiagent formation control and maneuver algorithm can be designed to maneuver through a unknown map with different designed formation control methods. To design the algorithm behind those controlled formation, it traditionally used agents' relative displacements, distances, or bearing. Although there were different previous works utilizing these traditional methods to solve formation maneuver control problem, they were not able to realize translational, rotational and scaling maneuver at the same time.

To solve this formation maneuver control problem, the paper [1] utilizes stress matrices among other solutions that realizes those maneuvers at the same time, such as berycentric coordinates-based approach and complex Laplacians-based approach, because the berycentric coordinate-based approach requires relative rotation matrix and complex Laplacians-based approach is only applicable in some special dimension, and therefore stress matrix-based approach is more flexible and realizable. In "Affine Formation Maneuver Control of Multiagent Systems" [2], the formation maneuver control problem is solved with stress matrix-based approach and achieves translational, rotational, scaling, and shearing maneuver concurrently. However, it does not consider the case where the leaders of the formation have time varying acceleration. The followers following leaders with time varying acceleration would have coupled inputs as those followers have to use global information of acceleration of the leaders to calculate control input.

Therefore the paper suggests two layered leader-follower control strategy with stress matrix to fully distribute the control strategy for followers in affine formation maneuver. Also, for the leaders, it achieves autonomous maneuver control algorithm with information of their desired trajectory and finite time derivatives.

## II. MATHEMATICAL FORMULATION

In order to properly formulate this multiagent formation maneuver control problem, the paper utilizes two lay-

ered leader follower strategy and stress matrix in undirected graph. First, in graph theory, this stress vector  $\omega = [\omega_1, \omega_2, \dots, \omega_m] \in R^m$  is said to be in equilibrium, when it satisfies,

$$\sum_{j \in N_i} \omega_{ij}(p_j - p_i) = 0,$$

where this  $p_i$  and  $p_j$  are the position of  $i$  and  $j$  agents, and this vector  $\omega_{ij}(p_j - p_i)$  represents the tension force between the two agents. In a matrix form, above equation can be expressed as

$$(\Omega \otimes I_d)p = 0$$

where  $p = [p_1^T, p_2^T, \dots, p_n^T]^T \in R^{dn}$ , and  $\Omega \in R^{n \times n}$  satisfying

$$[\Omega]_{ij} = \begin{cases} 0, & i \neq j, (i, j) \notin \varepsilon \\ -\omega_{ij}, & i \neq j, (i, j) \in \varepsilon \\ \sum_{j \in N_i} \omega_{ij}, & i = j. \end{cases}$$

By letting these tension force in each agents with its neighbors to sumed to be 0, then their formations would be structurally rigid. This structural rigidity of the formation is proved to be universally rigid if and only if there exists a stress matrix  $\Omega$  such that  $\Omega$  is positive semidefinite and  $rank(\Omega) = n - d - 1$ . [3] Also in [2] it shows the desired trajectories of each leaders can be expressed as

$$p_i^*(t) = A(t)r_i + b(t)$$

where  $r_i$  is the nominal configuration of each agents. With this time varying desired trajectory information of each agents, trajectory tracking control algorithm is designed for the leaders' formation. It also introduces the leader-follower strategy that utilizes above stress matrix and leaders positions to calculate the desired trajectories of followers. It shows that the number of leaders selected in a dimension  $d$  in order to be affinely localizable should be  $d+1$ . Also it shows that  $\lim_{t \rightarrow \infty} p_f = -\bar{\Omega}_{ff}^{-1} \bar{\Omega}_{fl} p_l$  where

$$\Omega = \begin{bmatrix} \Omega_{ll} & \Omega_{lf} \\ \Omega_{fl} & \Omega_{ff} \end{bmatrix},$$

where  $\Omega_{ff}$  has to be non-singular. Therefore it suggests the number of leaders in dimension  $d$  and constraint on stress matrix in order to find convergible route for followers. Then it utilizes this desired positions of the leaders and followers to build the control objectives for leaders and followers, respectively.

$$\lim_{t \rightarrow \infty} p_l - p_l^* = 0$$

$$\lim_{t \rightarrow \infty} p_f - p_f^* = p_f - \bar{\Omega}_{ff}^{-1} \bar{\Omega}_{fl} p_l = 0$$

The formulation of this problem heavily builds upon the “Affine Formation Maneuver Control of Multiagent Systems” [2] paper. However, this paper [1] is novel in a way that it builds upon the original paper with higher order so that it can render fully distributed control strategy for the followers and achieve autonomous maneuver for group of leaders.

### III. ANALYSIS

#### A. Tracking Control Algorithm for First Leader

Assuming that the trajectory and its finite time derivative information is already given, the paper designs tracking control algorithm for leaders. First it uses a finite time backstepping approach to achieve  $\lim_{t \rightarrow \infty} p_1 - p_1^* = 0$  for the first leader. Using the existing control objectives as auxiliary variable and a virtual variable that is to be determined later, auxiliary variable can be designed as such,

$$Z_{11} = p_1 - p_1^*$$

$$Z_{1i} = p_1^{i-1} - \alpha_{1(i-1)}(t).$$

Then, if we can determine  $\alpha_{1(i-1)}(t)$  and know all the time derivative information of itself, then we would be able to design the tracking control algorithm for the first leader. By using above equations, the paper achieves the relation between auxiliary variables in higher order.

$$\dot{Z}_{11} = \dot{p}_1 - \dot{p}_1^* = Z_{12} - \alpha_{11} - \dot{p}_1^*$$

$$\alpha_{11}(t) = \dot{p}_1^* - k_{11} \text{sig}^\beta(Z_{11}) - k_{12} Z_{11},$$

where

$$\text{sig}^r(x) = [\text{sig}^r(x_1), \text{sig}^r(x_2), \dots, \text{sig}^r(x_n)]$$

$$\text{sig}^r(x_i) = \text{sgn}(x_i) |x_i|^r$$

and  $\text{sgn}(x_i) = \begin{cases} -1 & x_i < 0 \\ 0 & x_i = 0. \text{ Also } k_{11} \text{ and } k_{12} \text{ are positive} \\ 1 & x_i > 0 \end{cases}$   
control gains, and  $0 \leq \beta \leq 1$ . Therefore, substituting above  $\alpha_{11}$  equation to the  $\dot{Z}_{11}$  equation, yields

$$\dot{Z}_{11} = Z_{12} - k_{11} \text{sig}^\beta(Z_{11}) - k_{12} Z_{11}.$$

Repeating the same procedure for  $Z_{12}$  and continuing on  $Z_{1i}$ ,

$$Z_{12} = p_2^{(1)} - \alpha_{11}(t)$$

$$\dot{Z}_{12} = p_2^{(2)} - \dot{\alpha}_{11} = Z_{13} + \alpha_{12} - \dot{\alpha}_{11}$$

$$\alpha_{12} = \dot{\alpha}_{11} \dot{Z}_{11} + \dot{p}_1^* - k_{11} \text{sig}^\beta(Z_{12}) - k_{12} Z_{12} - Z_{11}$$

$$Z_{1i} = p_1^{i-1} - \alpha_{1(i-1)}(t)$$

$$\dot{Z}_{1i} = Z_{1(i+1)} - k_{11} \text{sig}^\beta(Z_{1i}) - k_{12} Z_{1i} - Z_{1(i-1)}$$

$$\alpha_{1i} = \dot{\alpha}_{1i} \dot{Z}_{1i} - k_{11} \text{sig}^\beta(Z_{1i}) - k_{12} Z_{1i} - Z_{1(i-1)}$$

$$\dot{Z}_{1n} = -k_{11} \text{sig}^\beta(Z_{1n}) - k_{12} Z_{1n} - Z_{1(n-1)} u_i = \alpha_{in}.$$

Finally, it utilizes the  $n$ th order integrator  $\alpha_{1n}$  for the control inputs and its stability is verified with sum of quadratic form of Lyapunov functions.

$$V_{1n} = \frac{1}{2} (Z_{11}^T Z_{11} + Z_{12}^T Z_{12} + \dots + Z_{1n}^T Z_{1n})$$

$$\dot{V}_{1n} = -k_{11} Z_{11}^T \text{sig}^\beta(Z_{11}) - \dots - k_{1n} Z_{1n}^T \text{sig}^\beta(Z_{1n}) - 2k_{12} V_{1n}.$$

Then,

$$Z_{1i}^T \text{sig}^\beta(Z_{1i}) = \text{sgn}(Z_{1i1}) |Z_{1i1}| \text{sgn}(Z_{1i1}) |Z_{1i1}|^\beta + \dots$$

$$+ \text{sgn}(Z_{1in}) |Z_{1in}| \text{sgn}(Z_{1in}) |Z_{1in}|^\beta$$

$$= \sum_{k \in \dim} |Z_{1ik}|^{1+\beta}$$

$$\|Z_{1i}\|^{\beta+1} = \left( \sum_{k \in \dim} Z_{1ik}^2 \right)^{\frac{\beta+1}{2}} \leq \sum_{k \in \dim} (Z_{1ik}^2)^{\frac{\beta+1}{2}}$$

$$(\because 0 \leq \beta \leq 1)$$

$$\therefore Z_{1i}^T \text{sig}^\beta(Z_{1i}) = \sum_{k \in \dim} |Z_{1ik}|^{1+\beta} \geq \|Z_{1i}\|^{\beta+1}$$

$$\|Z_{1i}\|^{\beta+1} = \|2(V_{1i} - V_{1(i-1)})\|^{\beta+1}$$

$$= 2^{\frac{\beta+1}{2}} (V_{1i} - V_{1(i-1)})^{\frac{\beta+1}{2}}$$

Therefore

$$\dot{V}_{1n} \leq -2^{\frac{\beta+1}{2}} k_{11} (V_{11} + \sum_{i=2}^n (V_{1i} - V_{1(i-1)})^{\frac{\beta+1}{2}}) - 2k_{12} V_{1n}$$

$$\leq -2^{\frac{\beta+1}{2}} k_{11} V_{1n}^{\frac{\beta+1}{2}}.$$

Here we know that  $0 \leq \beta \leq 1$ , and  $k_{11}$  and  $V_{1n}$  are positive. Therefore  $\dot{V}_{1n}$  is negative definite and  $\lim_{t \rightarrow \infty} p_1 - p_1^* = 0$

#### B. Tracking Control Algorithm for Second Leader Group

Before designing control algorithm for second leader group, it needs to have desired trajectory and time derivative information of those second leader group. To estimate those information in a distributed way, the second leader group utilizes the desired trajectory and its  $n$ th order derivatives of the first leader to generate desired trajectory and the time derivatives of second leader group. Since we know the desired trajectory,  $b(t)$ , and time derivatives of the first leader,  $b^{(j)}(t)$ ,

where  $j = 0, 1, \dots, n$ , desired trajectory and time derivatives of second leader group can be estimated with

$$\dot{\hat{b}}_i^{(j)} = -\rho_j \text{sgn} \left[ \sum_{k=1}^M \text{sgn}(\omega_{ik}) (\hat{b}_i^{(j)} - \hat{b}_k^{(j)}) \right],$$

where  $i \in 2, \dots, M$  and  $\rho_j$  is positive constant. Also the time derivative information of  $A(t)$  can be estimated in a distributed way.

$$\dot{\hat{a}}_i^{(j)} = -\rho_j \text{sgn} \left[ \sum_{k=1}^M \text{sgn}(\omega_{ik}) (\hat{a}_i^{(j)} - \hat{a}_k^{(j)}) \right],$$

where  $\hat{a}_i^{(j)} \in R^{d^2}$  is agent  $i$ 's estimation of  $a(t)$ , and  $a(t)$  is column vector form of  $A(t)$ .

Then combining those two estimates generates desired trajectory and time derivative information of each agents.

$$p_i^*(t) = A(t)r_i + b(t).$$

With this generated desired trajectory of each agents, again, finite time backstepping approach is used to generate the control inputs of each agents.

$$\begin{aligned} Z_{i1} &= p_j - p_j^* = Z_{i2} - \alpha_{i1} - \dot{p}_j^* \\ \alpha_{i1}(t) &= \dot{p}_j^* - k_{i1} \text{sig}^\beta(Z_{i1}) - k_j 2Z_{i1} \\ Z_{ik} &= p_1^{k-1} - \alpha_{1(k-1)}(t) \\ \alpha_{ik} &= \dot{\alpha}_{ik} \dot{Z}_{ik} - k_{11} \text{sig}^\beta(Z_{ik}) - k_{12} Z_{ik} - Z_{1(i-1)} \\ u_i &= \alpha_{in} \end{aligned}$$

### C. PDM Control Algorithm for Followers with Constant Gains

The author was inspired from ‘‘Containment Control of Multiagent Systems With Dynamic Leaders Based on a PIn-Type Approach’’ [4] and proposed PDM control algorithm for followers. Using all the desired trajectory information and their time derivative information of neighbors of itself, the agent is able to generate adaptive gains using offline computation. The author proposes control algorithm for followers

$$\begin{aligned} \dot{u}_i &= -\eta_i \sum_{j \in V_i \cup V_f} \omega_{ij} [k_0(p_i - p_j) + k_1(\dot{p}_i - \dot{p}_j) \\ &\quad + \dots + k_m(p_i^{(m)} - p_j^{(m)})]. \end{aligned}$$

Using the dynamics of leaders and followers and their compact form,

$$\begin{aligned} \dot{x}_i &= (B \otimes I_d)x_i, i \in V_l \\ \dot{x}_i &= (B \otimes I_d)x_i + (C \otimes I_d)u_i, i \in V_f \\ \dot{x}_l &= (I_M \otimes B \otimes I_d)x_l \\ \dot{x}_f &= (I_{n_f} \otimes B \otimes I_d)x_f + (\Lambda \Omega_{ff} \otimes C \otimes I_d)x_f \\ &\quad - (\Lambda \Omega_{fl} \otimes CK \otimes I_d)x_l \end{aligned}$$

where  $x_i = [p_i^T, p_i^{(1)T}, m \dots, p_i^{(m)T}]^T \in R^{d(m+1) \times 1}$ , and

$$\begin{aligned} B &= \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ C &= [0 \quad \dots \quad 0 \quad 1]^T \\ \Lambda &= \text{diag}[\eta_{M+1}, \dots, \eta_N] \in R^{n_f \times n_f} \end{aligned}$$

the tracking error can be defined as

$$X_f(t) = x_f(t) - x_f^*(t) = x_f + [\Omega_{ff}^{-1} \Omega_{fl} \otimes I_{(m+1)d}]x_l.$$

From this tracking error equation,  $\eta_i$  will be designed to let this tracking error to converge to zero. By taking the derivative of it,

$$\begin{aligned} \dot{X}_f(t) &= \dot{x}_f + [\Omega_{ff}^{-1} \Omega_{fl} \otimes I_{(m+1)d}]\dot{x}_l \\ &= (I_{n_f} \otimes B \otimes I_d)x_f + (\Lambda \Omega_{ff} \otimes C \otimes I_d)x_f \\ &\quad - (\Lambda \Omega_{fl} \otimes CK \otimes I_d)x_l \\ &\quad + [\Omega_{ff}^{-1} \Omega_{fl} \otimes I_{(m+1)d}](I_M \otimes B \otimes I_d)x_l \\ &= (I_{n_f} \otimes B \otimes I_d - \Lambda \Omega_{ff} \otimes CK \otimes I_d)X_f(t) \end{aligned}$$

With this time derivative information, construct the Lyapunov function and take the derivative.

$$\begin{aligned} V &= X_f^T (\Omega_{ff} \otimes P \otimes I_d) X_f \\ \dot{V} &= \dot{X}_f^T (\Omega_{ff} \otimes P \otimes I_d) X_f + X_f^T (\Omega_{ff} \otimes P \otimes I_d) \dot{X}_f \end{aligned}$$

Using the above  $\dot{X}_f(t)$  equation and Algebraic Riccati Equation,

$$\begin{aligned} B^T P + PB - PCC^T P + I_{m+1} &= 0 \\ \dot{X}_f(t) &= (I_{n_f} \otimes B \otimes I_d - \Lambda \Omega_{ff} \otimes CK \otimes I_d)X_f(t) \end{aligned}$$

$$\begin{aligned} \dot{V} &= X_f^T (\Omega_{ff} \otimes (PB + B^T P) \otimes I_d) X_f \\ &\quad - 2X_f^T (\Omega_{ff} \Lambda \Omega_{ff} \otimes (PCC^T P) \otimes I_d) X_f \\ &= -X_f^T [\Omega_{ff} \otimes I_{m+1} \otimes I_d] X_f + X_f^T [(\Omega_{ff} \\ &\quad - 2\Omega_{ff} \Lambda \Omega_{ff}) \otimes PCC^T P \otimes I_d] X_f \end{aligned}$$

To prove stability of this Lyapunov function,  $(\Omega_{ff} - 2\Omega_{ff} \Lambda \Omega_{ff}) \leq 0$ , since  $\Omega_{ff} \otimes I_{m+1} \otimes I_d \geq 0$  and  $PCC^T P \geq 0$ . If we choose  $\eta_i$  as

$$\min_{i \in V_f} \eta_i \geq \frac{1}{2\lambda_{\min}(\Omega_{ff})},$$

then

$$y^T (\Omega_{ff} - 2\Omega_{ff} \Lambda \Omega_{ff}) y = -(\Omega_{ff} y)^T (2\Lambda - \Omega_{ff}^{-1}) \Omega_{ff} y \leq 0,$$

for any  $y \in R^{n_f}$ .

#### D. PDM Control Algorithm for Followers with Adaptive Gains

With constant control gain larger than  $\frac{1}{2\lambda(\Omega_{ff})}$ , the trajectory would be depending on the formation of the graph. So the author suggests using adaptive gains with Algebraic Riccati Equation.

$$\begin{aligned}\dot{u}_i &= -\eta_i \sum_{j \in V_i \cup V_f} \omega_{ij} [k_0(e_{ij}) + k_1(e_{ij}^{(1)}) + \dots + k_m(e_{ij}^{(m)})] \\ \dot{\eta}_i &= \gamma_i s_i^T (PCC^T P \otimes I_d) s_i \\ s_i &= \sum_{j \in V_i \cup V_f} \omega_{ij} (x_i - x_j).\end{aligned}$$

Considering the previous Lyapunov function and building upon it,

$$\begin{aligned}V_1 &= X_f^T (\Omega_{ff} \otimes P \otimes I_d) X_f \\ V_2 &= V_1 + \sum_{i=M+1}^N (\hat{\eta}_i(t) - \bar{\eta})^2 / \gamma_i \\ \dot{V}_1 &= -X_f^T [\Omega_{ff} \otimes I_{m+1} \otimes I_d] X_f + X_f^T [(\Omega_{ff} - 2\Omega_{ff}\Lambda\Omega_{ff}) \otimes PCC^T P \otimes I_d] X_f \\ &= -X_f^T [\Omega_{ff} \otimes I_{m+1} \otimes I_d] X_f \\ &\quad + X_f^T [\Omega_{ff} \otimes PCC^T P \otimes I_d] X_f \\ &\quad - 2[\Omega_{ff} \otimes I_{(m+1)d} X_f]^T [\hat{\Lambda}(t) \otimes PCC^T P \otimes I_d] \\ &\quad \times [(\Omega_{ff} \otimes I_{(m+1)d}) X_f] \\ \dot{V}_2 &= \dot{V}_1 + \sum_{i=M+1}^N 2(\hat{\eta}_i(t) - \bar{\eta}) [s_i^T (PCC^T P \otimes I_d) s_i].\end{aligned}$$

Substituting  $\dot{V}_1$  into  $\dot{V}_2$  equation achieves,

$$\begin{aligned}\therefore \dot{V}_2 &= -X_f^T [\Omega_{ff} \otimes I_{m+1} \otimes I_d] X_f \\ &\quad + X_f^T [\Omega_{ff} \otimes PCC^T P \otimes I_d] X_f \\ &\quad - 2\bar{\eta} \sum_{i=M+1}^N s_i^T (PCC^T P \otimes I_d) s_i \\ &= -X_f^T [\Omega_{ff} \otimes I_{m+1} \otimes I_d] X_f \\ &\quad + X_f^T [\Omega_{ff} \otimes PCC^T P \otimes I_d] X_f \\ &\quad - 2\bar{\eta} X_f^T [\Omega_{ff} \otimes PCC^T P \otimes I_d] X_f.\end{aligned}$$

Applying the same logic that was applied in previous section, to obtain the stability  $\dot{V}_2 \leq 0$ ,

$$\begin{aligned}2\bar{\eta}\Omega_{ff}\Omega_{ff} - \Omega_{ff} &\geq 0 \\ \therefore \bar{\eta} &\geq \frac{1}{2\lambda_{\min}(\Omega_{ff})}.\end{aligned}$$

#### IV. SIMULATION

##### A. Stress matrix Design

The paper designs the stress matrix according to the methods presented in [5] and [2], and the nominal configuration follows,

$$\begin{aligned}r_1 &= [0; 0] & r_2 &= [-1; 1] & r_3 &= [-1; 0] & r_4 &= [-1; -1] \\ r_5 &= [-2; 1] & r_6 &= [-2; -1] & r_7 &= [-3; 1] & r_8 &= [-3; -1]\end{aligned}$$

and figure 1 shows the paper's the nominal configuration and the corresponding weights.

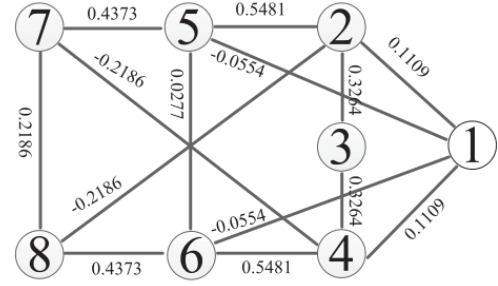


Fig. 1. nominal formation and stresses

##### B. Maneuver Design

Before simulating, the desired trajectory has to be generated according to the given desired position and velocity value at time  $t_k$  and  $t_{k+1}$ . With the previous assumption that desired trajectory,  $p_i = A(t)r_i + b(t)$ , is known and  $n^{th}$  order differentiable, the paper assumes  $3^{rd}$  order time differentiable equation,

$$b(t) = c_3(t - t_k)^3 + c_2(t - t_k)^2 + c_1(t - t_k) + c_0,$$

where  $c_i \in R^2$  are constant vectors at  $t \in [t_k, t_{k+1}]$  for all the leader positions. With known desired position and velocity profile at two different given time, then the desired trajectory can be found with

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \Delta T & \Delta T^2 & \Delta T^3 \\ 0 & 1 & \Delta T & 3\Delta T^2 \end{bmatrix}^{-1} \otimes I_2 \begin{bmatrix} S_{t_k} \\ v_{t_k} \\ S_{t_{k+1}} \\ v_{t_{k+1}} \end{bmatrix}$$

where  $S_{t_k}, v_{t_k}, S_{t_{k+1}}, v_{t_{k+1}}$  are the desired position and velocity profile at  $t_k$  and  $t_{k+1}$ . Also the scaling maneuver is designed with,

$$\begin{aligned}A(t) &= \phi(t)I_2 \\ \phi(t) &= \varsigma_3(t - t_k)^3 + \varsigma_2(t - t_k)^2 + \varsigma_1(t - t_k) + \varsigma_0,\end{aligned}$$

where  $\varsigma_3, \varsigma_2, \varsigma_1, \varsigma_0$  are known desired scaling factor at  $t_k$  and  $t_{k+1}$ . With the same matrix equation used for translational maneuver, scaling maneuver constants,  $\varsigma_i \in R, i = 0, 1, \dots, 3$ , can be also be inferred. Also the rotational and shearing design is described in a same manner, where

$$\begin{aligned}A_{rot}(t) &= R(\theta(t)) = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) \end{bmatrix} \\ A_{she}(t) &= \begin{bmatrix} \psi_1(t) & 0 \\ 0 & \psi_2(t) \end{bmatrix}\end{aligned}$$

are rotational and shearing matrix for each design, and  $\theta(t) \in [0, 2\pi)$  and  $\psi(t) \in (0, 1)$  are rotation angle with respect to the nominal configuration and shearing factors respectively.

### C. Simulation and parameters

The paper also utilizes MATLAB to calculate the unique solution of the Algebraic Riccati Equation to get values of  $K$ .

$$K = [46.72143.817197.9471143.817]$$

Therefore  $\eta_i$  is chosen to be 56 because  $\eta_i \geq \frac{1}{2\lambda_{\min}(\Omega_{ff})}$ . Other constant parameters are chosen abstract

$$\rho_i = 12, \beta = 0.5, k_{i1} = 1, k_{i2} = 16, i = 1, \dots, 4$$

From figure 2 we can see that the paper successfully simulates the designed affine maneuver.

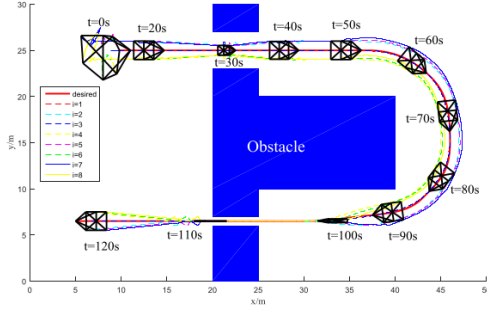


Fig. 2. Formation maneuver trajectories

Here is my attempt, figure 3, to redo the work. It seems like the tracking control algorithm for leaders group is working but it is probably details that I might have missed. But here is my link for the code and future simulation demonstration and effort. <https://github.com/SoowhanYi94/ME597>

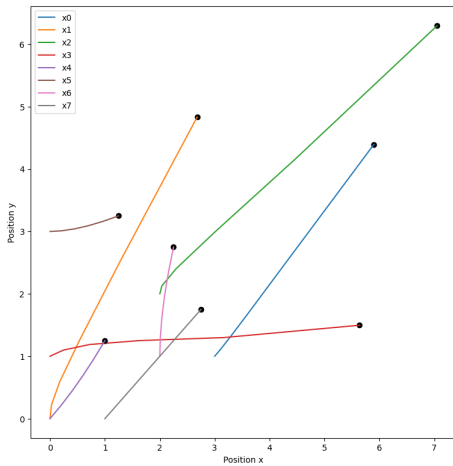


Fig. 3. Formation maneuver trajectories

### V. CONCLUSION

Overall the paper heavily depends on the previous works [2], [4] and [3]. However the contribution of the paper lies

on the fully distributing the followers control strategy for affine formation maneuver and making the leaders maneuver autonomous with given desired trajectory and its finite time derivative information. Also, the PIn type control algorithm does require higher order information than the agents' dynamics, and therefore can not realize the adaptive gain because simply we do not have those information in this setting. Therefore its novelty also lies on usage of different distributed control method for the followers and autonomous tracking control method for leaders.

The assumptions here is that the desired trajectory and its finite time derivative information for each agents are accessible and differentiable at  $n$ th order. This means that the discontinuous maneuver is not available for the leaders and they would be vulnerable sudden change in environment. Also the number of leaders are selected at least more than  $1 + d$  in which  $d$  is the dimension of the dynamics, that affinely spans  $R^d$ , which are realizable with 3 4 agents being in the leader group in 2 or 3 dimensions.

However, another assumption here is that connection between leaders group and the first leader should always exist. Since the continuous maneuver of the first leader and the connection among leaders group are vulnerable to the disturbances, the leaders group should render relatively slow speed for the accurate tracking control algorithm.

Therefore those assumptions are applicable to the situation where agents are maneuvering through unknown map as it needs slowly maneuver to accurately map the environment and avoid any obstacle. Also it has to be applied where the sudden change in environment is not expected. For this reason, I suggest, in the future, the research can be improved with conducting performance measure using variance of the disturbance response on this leader-follower affine formation maneuver. Could it be further researched for disturbance rejection control strategy using [6]?

### REFERENCES

- [1] Liangming Chen, Jie Mei, Chuanjiang Li, and Guangfu Ma. Distributed leader-follower affine formation maneuver control for high-order multi-agent systems. *IEEE Transactions on Automatic Control*, 65(11):4941–4948, 2020.
- [2] Shiyu Zhao. Affine formation maneuver control of multiagent systems. *IEEE Transactions on Automatic Control*, 63(12):4140–4155, 2018.
- [3] A.Y. Alfakih. On the dual rigidity matrix. *Linear Algebra and its Applications*, 428(4):962–972, 2008.
- [4] Long Cheng, Yunpeng Wang, Wei Ren, Zeng-Guang Hou, and Min Tan. Containment control of multiagent systems with dynamic leaders based on a  $pi^n$ -type approach. *IEEE Transactions on Cybernetics*, 46(12):3004–3017, 2016.
- [5] Zhiyun Lin, Lili Wang, Zhiyong Chen, Minyue Fu, and Zhimin Han. Necessary and sufficient graphical conditions for affine formation control. *IEEE Transactions on Automatic Control*, 61(10):2877–2891, 2016.
- [6] Boyang Zhang, Xiuxia Sun, Shuguang Liu, and Jinglin Hu. Active disturbance rejection control approach to tight formation flight of unmanned aerial vehicles. In *2018 37th Chinese Control Conference (CCC)*, pages 6745–6751, 2018.