Trabajo en clase 1²

i) $y' = 2x\sqrt{1-y^2}$, explícita

$$\frac{dy}{dx} = 2x\sqrt{1-y^2}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int 2x\,dx$$

$$\operatorname{arcsen}(y) = x^2 + C$$

$$\boxed{y = \operatorname{sen}(x^2 + c)}$$ General

ii) $y^2 \frac{dy}{dx} = 3x^2y^3 - 6x^2$, $y(0) = 2$, explícita

$$y^2\frac{dy}{dx} = 3x^2(y^3 - 2)$$

$$\int \frac{y^2}{y^3-2}\,dy = \int 3x^2\,dx$$

$u = y^3 - 2$
$du = 3y^2\,dy$

$$\frac{1}{3}\int \frac{du}{u} = \int 3x^2\,dx$$

$$\frac{1}{3}\ln|u| = x^3 + C$$

$$\ln(y^3 - 2) = 3x^3 + C$$

$$y^3 - 2 = e^{3x^3+c} \rightarrow e^{3x^3}\cdot e^c = ce^{3x^3}$$

$$y = \sqrt[3]{ce^{3x^3} + 2}$$ ← Solución general

Particular

$y(0)=2$ $\quad 2 = \sqrt[3]{ce^0 + 2} = \sqrt[3]{c+2}$

$\Rightarrow 8 = c + 2 \Rightarrow c = 6$ $\quad y = \sqrt[3]{6e^{3x^3} + 2}$

3) $\frac{dy}{dx} = e^{x-y}$, $y(\ln 2) = \ln 3$, implícita

$\frac{dy}{dx} = \frac{e^x}{e^y}$

$\int e^y\, dy = \int e^x\, dx$

$e^y = e^x + c$

$e^y - e^x = c$ ← General

$y(\ln 2) = \ln 3$

$e^{\ln 3} - e^{\ln 2} = c$

$3 - 2 = 1 = c$

$\boxed{e^y - e^x = 1}$ ← Particular

4) $y' = \frac{2}{x} - \frac{y}{x}$, explícita

$y' = \frac{1}{x}(2-y)$

$\frac{dy}{dx} = \frac{1}{x}(2-y)$

$\int \frac{dy}{2-y} = \int \frac{dx}{x}$

$u = 2-y$
$du = -dy$

$-\int \frac{du}{u} = \int \frac{dx}{x}$

$-\ln u = \ln x + c$

$\ln(2-y) = -\ln x + c$

$2-y = e^{-\ln x + c} = e^{\ln x^{-1}} e^c = c e^{\ln x^{-1}} = cx^{-1}$

$\boxed{y = 2 - cx^{-1}}$ ← General

$y = 2 - \frac{c}{x}$

5) $\sqrt{x}\,dy = e^{y+\sqrt{x}}\,dx$, implícita.

$$\sqrt{x}\,dy = e^{y}\cdot e^{\sqrt{x}}\,dx$$

$$\int e^{-y}dy = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = e^{\sqrt{x}}\cdot x^{-\frac{1}{2}}$$

$$u=\sqrt{x} \quad du=\frac{1}{2\sqrt{x}}\,dx$$

$$-e^{-y} = 2\int e^{u}du = 2e^{u}+C = 2e^{\sqrt{x}}+C$$

$$\Rightarrow \boxed{C = 2e^{\sqrt{x}} + e^{y}}$$ ← solución general

Trabajo en Clase V3.

- ortogonales a $kx^2+y^2=1$

$D(kx^2+y^2=1)$

$2kx+2yy'=0$

$y'=-\frac{kx}{y}$

$$k=\frac{1-y^2}{x^2}$$

$m_2=y'=\frac{y}{kx}=\frac{y}{\frac{1-y^2}{x}}=\frac{xy}{1-y^2}$

$\frac{dy}{dx}=\frac{xy}{1-y^2}$

$\int\frac{1-y^2}{y}dy=\int x\,dx$

$\ln|y|-\frac{y^2}{2}=\frac{x^2}{2}+C$

$$\boxed{C=\frac{x^2}{2}+\frac{y^2}{2}-\ln|y|}$$

$\int\frac{1-y^2}{y}dy$

$u=1-y^2 \quad v=\ln|y|$

$du=-2y\,dy \quad dv=\frac{1}{y}dy$

$\ln|y|-y^2\ln|y|+2\int y\ln|y|\,dy$

$u=\ln|y| \quad v=\frac{y^2}{2}$

$du=\frac{dy}{y} \quad dv=y\,dy$

$+y^2\ln|y|-\int y\,dy=\ln|y|-\frac{y^2}{2}+C$

2. Las curvas $2x^2+3y^2=5$ y $y^2=x^3$ son ortogonales.

$D(2x^2+3y^2=5)$

$4x+6yy'=0$

$y'=-\frac{4x}{6y}=-\frac{2x}{3y}$

$m_1=-\frac{2x}{3y}$

$D(y^2=x^3)$

$2yy'=3x^2$

$y'=\frac{3x^2}{2y}$

$y^2=\frac{5-2x^2}{3}=x^3$

$3x^3+2x^2-5=0$

$x=1$

$x-1 \;\big)\; 3x^3+2x^2+0-5$, cociente $3x^2+5x+5$

$-3x^3+3x^2$

$5x^2-0-5$

$-5x^2+5x$

$5x-5$

2. Las curvas $2x^2+3y^2=5$ y $y^2=x^3$ son ortogonales

$D(2x^2+3y^2=5)$ $D(y^2=x^3)$

$4x+6yy'=0$; $2yy'=3x^2$

$m_1 = y' = -\frac{2x}{3y}$ $y' = \frac{3x^2}{2y}$

ortogonal

$m_2 = -\frac{1}{m_1} = \frac{3y}{2x}$

$y^2=x^3$

$2x^2+3x^3=5$

$x=1$ es una solución $2+3=5$ ✓

$\Rightarrow y=1$

y la pendiente si es $\frac{3}{2}$ y $\frac{3}{2}$. (si son ortogonales)

$$\begin{array}{l} x-1 \,\big|\, 3x^3+2x^2+0-5 \quad = 3x^2+5x+5 \\ -3x^3+3x^2 \\ \quad 5x^2+0-5 \\ \quad -5x^2+5x-5 \\ \quad\quad 5x-5 \end{array}$$

$(x-1)(3x^2+5x+5)=0$ (son ...)

$= \frac{-5 \pm \sqrt{25-60}}{6}$ No soluciones extras.

3. Act. 10000

un año: Reduce 25% : 7500

y = cantidad enfermos t = tiempo

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$y = e^{kt+c} = e^{kt} \cdot e^{c} = ce^{kt}$$

$t=0$

$10000 = y(0) = ce^0 = c \Rightarrow c = 10000$

$t=1$

$7500 = y(1) = 10000e^k \Rightarrow e^k = \frac{3}{4} \Rightarrow k = \ln\left(\frac{3}{4}\right) = -0.287$

$$\boxed{y = 10000e^{t\ln\left(\frac{3}{4}\right)} = 10000e^{-0.287t}}$$

$y = 10000e^{t\ln\left(\frac{3}{4}\right)} = 1000$

$\Rightarrow e^{t\ln\left(\frac{3}{4}\right)} = \frac{1}{10}$

$\Rightarrow t\ln\left(\frac{3}{4}\right) = \ln\left(\frac{1}{10}\right)$

$\Rightarrow t = \frac{\ln\left(\frac{1}{10}\right)}{\ln\left(\frac{3}{4}\right)} = 8.003$

En 8 años (pero el techo es 9)

4. Act: 1000 kg
10 hrs: 800 kg
24 hrs: ?

y: azucar sin refinar $\quad t$ = tiempo

$\frac{dy}{dt} = ky$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + C$

$y = e^{kt+C} = c^{kt} \cdot C$

- $t=0$

$1000 = y(0) = c\,e^{0} = c \Rightarrow c = 1000$

- $t=10$

$800 = y(10) = 1000\,e^{10k} \Rightarrow e^{10k} = \frac{4}{5} \Rightarrow 10k = \ln\left(\frac{4}{5}\right) \Rightarrow k = \frac{\ln\left(\frac{4}{5}\right)}{10} = -0.022$

$y = 1000\,e^{\frac{\ln\left(\frac{4}{5}\right)}{10}t}$

- $t=24$ $\quad y = 1000\,e^{\frac{\ln\left(\frac{4}{5}\right)}{10}\cdot 24} = 1000\,e^{2.4\ln\left(\frac{4}{5}\right)} = 585{,}3\text{kg} \approx 585\text{kg}$

5. Act: 100%
5700 años: 50%
90%? t (se resta el 10%)

y = numero nucleos t = tiempo

$$\frac{dy}{dt} = ky \qquad n = 1\%$$

$$\Rightarrow \int \frac{dy}{y} = \int k\, dt$$

$$\ln|y| = kt + C$$

$$\Rightarrow y = Ce^{kt}$$

• $t = 0$

$$100n = y(0) = Ce^{0} = C \Rightarrow C = 100n$$

• $t = 5700$

$$50n = y(5700) = 100n e^{5700k}$$

$$\Rightarrow e^{5700k} = \frac{50n}{100n} = \frac{1}{2}$$

$$\Rightarrow k = \frac{\ln(\frac{1}{2})}{5700}$$

• 90n

$$90\cancel{n} = 100\cancel{n} e^{\frac{\ln(\frac{1}{2})}{5700}t}$$

$$\Rightarrow e^{\frac{\ln(\frac{1}{2})}{5700}t} = \frac{9}{10} \Rightarrow \frac{\ln(\frac{1}{2})}{5700} t = \ln(\frac{9}{10})$$

$$\Rightarrow t = \frac{5700 \ln(\frac{9}{10})}{\ln(\frac{1}{2})} = 866.41$$

$$\boxed{\text{La edad es aprox 866 años}}$$

1. Ortogonal a $kx^2+y^2=1$

$D(kx^2+y^2=1)$

$2kx+2yy'=0$

$y'=-\frac{kx}{y}$

$m_2=y'=\frac{y}{kx}=\frac{xy}{1-y^2}$

$k=\frac{1-y^2}{x^2}$

$\frac{dy}{dx}=\frac{xy}{1-y^2}$

$\int\frac{1-y^2}{y}dy=\int x\,dx$

$\ln|y|-\frac{y^3}{2}=\frac{x^2}{2}+C$

$\boxed{C=\ln|y|-\frac{y^2}{2}-\frac{x^2}{2}}$

$\int\frac{1-y^2}{y}dy$

$u_1=1-y^2 \quad v_1=\ln|y|$

$du_1=-2y\,dy \quad dv_1=\frac{dy}{y}$

$=(1-y^2)\ln|y|+2\int y\ln|y|dy$

$u_2=\ln|y| \quad v_2=\frac{y^2}{2}$

$du_2=\frac{dy}{y} \quad dv_2=y\,dy$

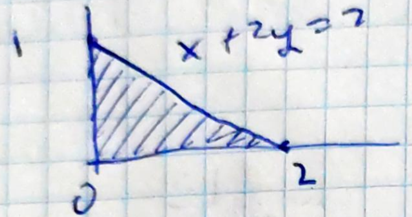
$=\ln|y|-y^2\ln|y|+2\left(\frac{y^2}{2}\ln|y|-\int\frac{y}{2}dy\right)$

$=\ln|y|-y^2\ln|y|+y^2\ln|y|-\frac{y^2}{2}+C$

$=\ln|y|-\frac{y^2}{2}+C$

Trabajo en Clase 14

1. rotar sobre eje x



$\Rightarrow y = \frac{2-x}{2}$

$$V = \int_0^2 \pi [y]^2 \, dx = \int_0^2 \pi \left(\frac{2-x}{2}\right)^2 dx$$

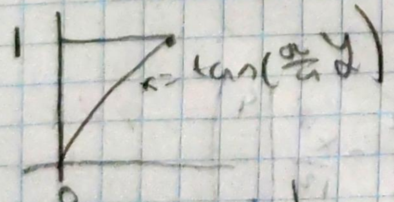
$$\int \pi \frac{(2-x)^2}{4} dx = \frac{\pi}{4} \int (2-x)^2 dx \qquad u = 2-x \quad du = -dx$$

$$= -\frac{\pi}{4} \int u^2 du = -\frac{\pi}{4} \cdot \frac{u^3}{3} + C = -\frac{\pi(2-x)^3}{12} + C$$

$$V = -\frac{\pi(2-x)^3}{12} \Big|_0^2 = \frac{-\pi(2-2)^3}{12} - \left(\frac{-\pi(2-0)^3}{12}\right)$$

$$= \frac{\pi(8)}{12} = \boxed{\frac{2\pi}{3}}$$

2. rotar sobre eje y



$$V = \int_0^1 \pi [x]^2 dy = \int_0^1 \pi \left(\tan\left(\frac{\pi}{4} y\right)\right)^2 dy \qquad u = \frac{\pi}{4} y \quad du = \frac{\pi}{4} dy$$

$$\int 4\tan^2(u)\,du = 4\int \tan^2(u)\,du = 4\int(\sec^2 u - 1)\,du$$

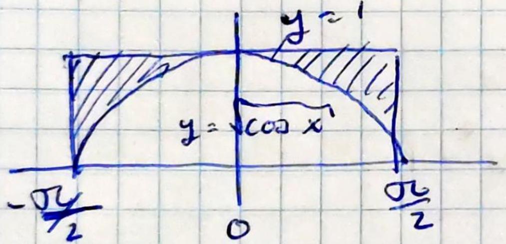
$$= 4\left[\int \sec^2 u\,du - \int du\right] = 4\left[\tan u - u\right] + C$$

$$= 4\left[\tan\left(\tfrac{\pi}{4}y\right) - \tfrac{\pi}{4}y\right] + C = 4\tan\left(\tfrac{\pi}{4}\right)y - \pi y + C$$

$$V = \int_0^1 \pi \tan\left(\tfrac{\pi}{4}y\right)^2 dy = 4\tan\left(\tfrac{\pi}{4}y\right) - \pi y \Big|_0^1 = 4\tan\left(\tfrac{\pi}{4}\right) - \pi - 4\tan(0)$$

$$= \boxed{4 - \pi}$$

3. rotar sobre eje x



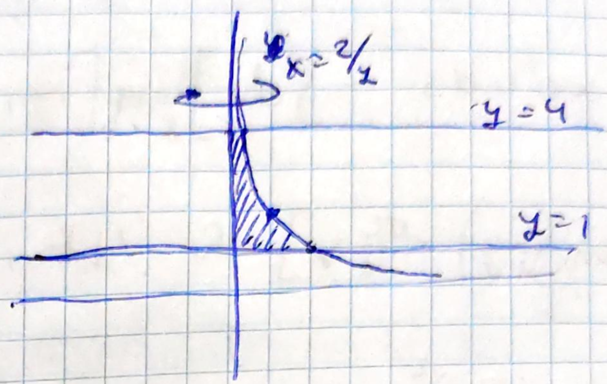
$$V = \int_{-\frac{\pi}{2}}^{\pi/2} \pi\left[R(x)^2 - r(x)^2\right]dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \pi\left[1 - \cos x\right]dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\pi - \pi\cos x)\,dx = \pi x - \pi \operatorname{sen} x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \pi\left(\tfrac{\pi}{2}\right) - \pi\operatorname{sen}\left(\tfrac{\pi}{2}\right) - \left(\pi\left(-\tfrac{\pi}{2}\right) - \pi\operatorname{sen}\left(-\tfrac{\pi}{2}\right)\right)$$

$$= \frac{\pi^2}{2} - \pi + \frac{\pi^2}{2} - \pi = \boxed{\pi^2 - 2\pi}$$

4. rotar sobre eje y

$$\begin{cases} x = \frac{2}{y} \\ y = 1 \\ y = 4 \\ \text{eje } y \quad x = 0 \end{cases}$$

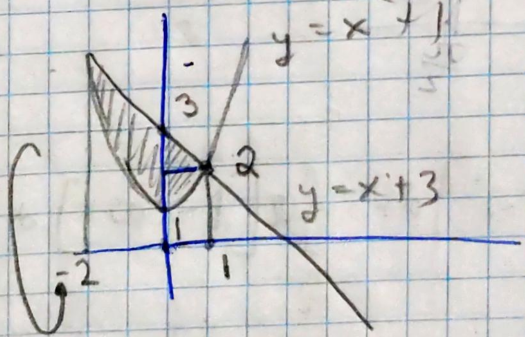


$$V = \int_a^b \pi R^2(y)\,dy = \int_1^4 \pi\left(\frac{2}{y}\right)^2 dy = 4\pi \int_1^4 y^{-2}\,dy = 4\pi\left[-\frac{1}{y}\Big|_1^4\right]$$

$$= 4\pi\left[-\frac{1}{4} - (-1)\right] = 4\pi\left[\frac{3}{4}\right] = \boxed{3\pi}$$

5. rotar sobre eje x

$$\begin{cases} y = x^2 + 1 \\ y = -x + 3 \end{cases}$$



$$x^2 + 1 = -x + 3$$

$$x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2, 1$$

$$V = \int_{-2}^{1} \pi\left((-x+3)^2 - (x^2+1)^2\right)dx = \int_{-2}^{1} \pi\left(x^2 - 6x + 9 - x^4 - 2x^2 - 1\right)dx$$

$$= \pi \int_{-2}^{1} \left(-x^4 - x^2 - 6x + 8\right)dx = \pi\left[-\frac{x^5}{5} - \frac{x^3}{3} - \frac{6x^2}{2} + 8x \Big|_{-2}^{1}\right]$$

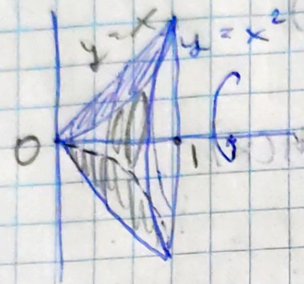
$$= \pi\left[-\frac{1}{3} - \frac{1}{3} + 3 + 8 - \left(\frac{32}{5} + \frac{8}{3} + 12 - 16\right)\right]$$

$$= \pi\left[\frac{-5 - 5 + 45 + 120 - 96 - 40 + 180 + 240}{15}\right]$$

$$= \pi\left[\frac{351}{15}\right] = \pi\left[\frac{117}{5}\right] = \boxed{\frac{117\pi}{5}}$$

Trabajo en Clase 15

1. rotar sobre x



$x = x^2$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

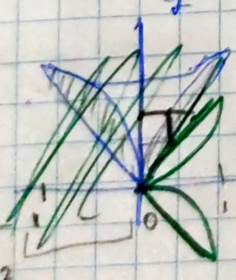
$\Rightarrow x = 0, 1$

a) Discos

$$\int_0^1 \pi\left[x^2 - \left[x^2\right]^2\right]dx = \pi\int_0^1 (x^2 - x^4)dx = \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\Big|_0^1\right]$$

$$= \pi\left[\frac{1}{3} - \frac{1}{5}\right] = \boxed{\frac{2\pi}{15}}$$

b) Casquillos



~~$R(x)$~~

~~$R(y) =$~~

$R(x) = y$

$H(x) = \sqrt{y} - y$

$y = x^2$

$x = \sqrt{y}$

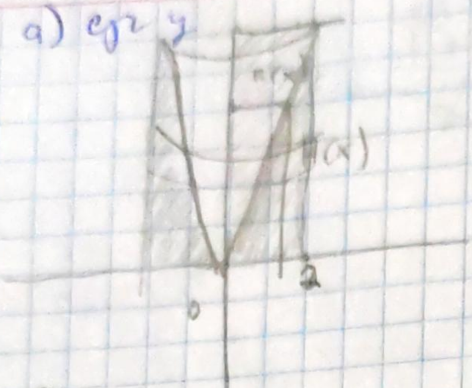
$$V = \int_0^1 2\pi y(\sqrt{y} - y)\,dy = 2\pi\int_0^1 (y^{3/2} - y^2)dy = 2\pi\left[\frac{2}{5}y^{5/2} - \frac{y^3}{3}\Big|_0^1\right]$$

$$= 2\pi\left[\frac{2}{5} - \frac{1}{3}\right] = 2\pi\left[\frac{1}{15}\right] = \boxed{\frac{2\pi}{15}}$$

2. a) eje y

$r(x) = x$

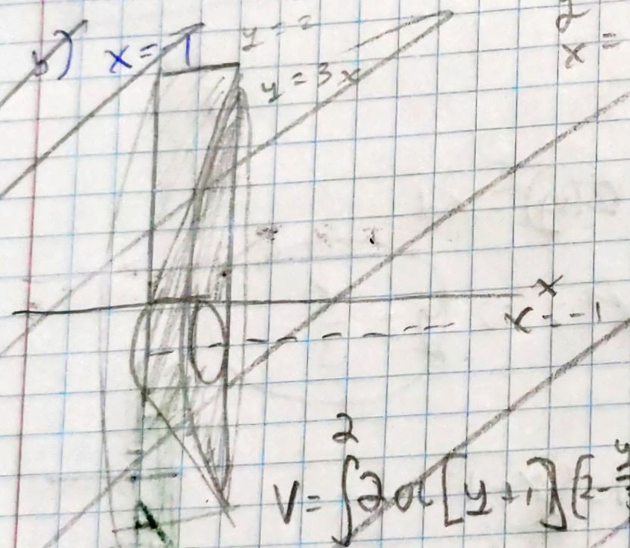
$h(x) = 3x$



$$V = \int_0^2 2\pi(x)(3x)\,dx = 6\pi\int_0^2 x^2\,dx = 6\pi\left[\frac{x^3}{3}\Big|_0^2\right]$$

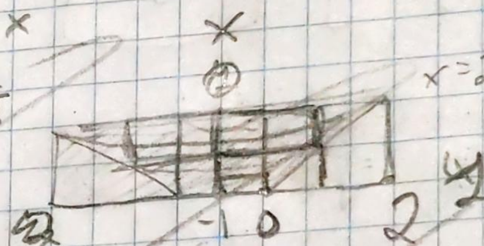
$$= 6\pi\left[\frac{8}{3}\right] = \boxed{16\pi}$$

b) $x = -1$



$y = 3x$

$x = \frac{y}{3}$



$r(y) = y + 1$

$h(y) = 2 - \frac{y}{3}$

$$V = \int_0^2 2\pi[y+1]\left[2 - \frac{y}{3}\right]dy = \frac{2}{3}\pi\int_0^2 (6y - y^2 + 6 - y)\,dy = \int_0^2 (-y^2 + 5y + 6)\,dy$$

$$= \frac{2}{3}\pi\left[-\frac{y^3}{3} + \frac{5y^2}{2} + 6y\Big|_0^2\right] = \frac{2}{3}\pi\left[-\frac{8}{3} + 10 + 12\right] = \frac{116\pi}{9}$$

c) $y = 7$

b) $x = -1$

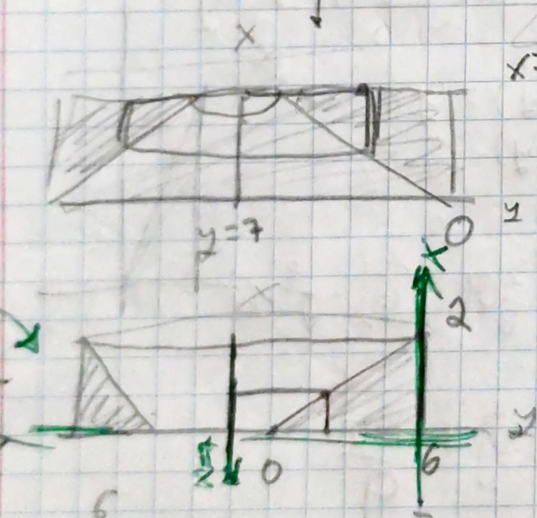
$R(x) = x+1$

$H(x) = 3x$

$$\int_0^2 2\pi[x+1][3x]\,dx = 6\pi\int_0^2 (x^2+x)\,dx = 6\pi\left[\frac{x^3}{3}+\frac{x^2}{2}\Big|_0^2\right]$$

$$= 6\pi\left[\frac{8}{3}+2\right] = \boxed{28\pi}$$

c) $y = 7$

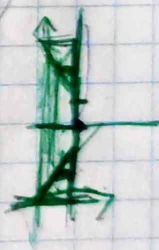


$x = 2$ $\quad$ ~~$R(y) = x-1$~~

$3x = 1 \Rightarrow y = \frac{x}{3}$

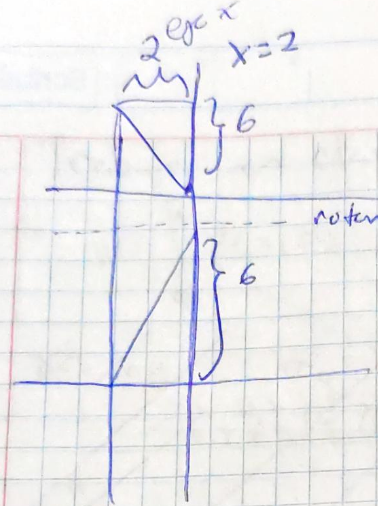
$R(y) = y+1$

$H(y) = \frac{y}{3}$

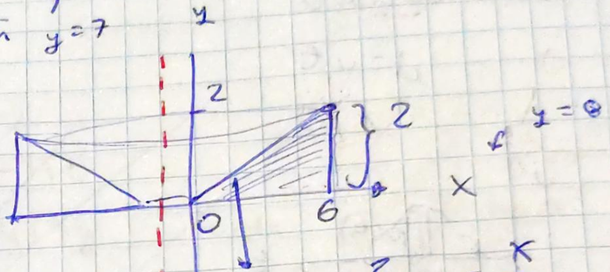


$$\int_0^6 2\pi[y+1]\left[\frac{y}{3}\right]dy = \frac{2}{3}\pi\int_0^6 [y^2+y]\,dy$$

$$\frac{2}{3}\pi\left[\frac{y^3}{3}+\frac{y^2}{2}\Big|_0^6\right] = \frac{2}{3}\pi[72+18] = \frac{2}{3}\pi[90] = \boxed{60\pi}$$



eje y
$y = 8$



$$y = \frac{2}{6}x = \frac{x}{3}$$

$$\boxed{\begin{aligned} R(x) &= \cancel{x}+1 \\ H(x) &= \frac{x}{3} \end{aligned}}$$

16

Trabajo en Clase

$F = 100 - 4t = 100 - \frac{4}{5}x$ ← esta en lb

1. $x = dis = 5t$

$$W = \int_0^{20} \left(100 - \frac{4}{5}x\right) dx = \underbrace{100x - \frac{2}{5}x^2}_{lb \cdot ft} \Big|_0^{20} = 2000\,lb\cdot ft - 160\,lb\cdot ft$$

$$= \boxed{1840\ lb \cdot ft}$$

2. $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$

$$y' = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} \cdot x^{1/2} = 2\sqrt{2}\sqrt{x}$$

$$d = \int_0^1 \sqrt{1 + (y')^2}\, d\cancel{x} = \int_0^1 \sqrt{8x+1}\, dx$$

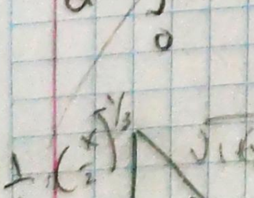
$u = 8x + 1 \qquad du = 8dx \rightarrow \frac{1}{8}\int_1^9 \sqrt{u}\, du = \frac{1}{8}\left[\frac{2}{3} u^{3/2} \Big|_1^9 \right]$

$$= \frac{1}{8}\left[\frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}\right] = \frac{1}{8}\left[18 - \frac{2}{3}\right] = \frac{52}{24} = \frac{26}{12} = \boxed{\frac{13}{6}}$$

3. $y = \left(\frac{x}{2}\right)^{2/3}$

$$y' = \frac{2}{3}\left(\frac{x}{2}\right)^{-1/3} \cdot \frac{1}{2} = \frac{1}{3} \cdot \left(\frac{x}{2}\right)^{-1/3}$$

$$d = \int_0^2 \sqrt{1 + (y')^2}\, dx = \int_0^2 \sqrt{1 + \frac{1}{9}\cdot\left(\frac{x}{2}\right)^{-2/3}}\, dx \qquad \int_0^2 \sqrt{1 + \frac{\sqrt[3]{4}}{9\sqrt[3]{x^2}}}\, dx$$



$$\sqrt{1 + (y')^2} = \cancel{y \sec\theta}\ \frac{y'}{\operatorname{sen}\theta}$$

$$\frac{1}{3}\left(\frac{x}{2}\right)^{-1/3} = \tan\theta$$

$$-\frac{1}{9}\left(\frac{x}{2}\right)^{-4/3} dx = \sec^2\theta\, d\theta$$

3. $y = \left(\frac{x}{2}\right)^{2/3}$ $x=0 \to y=0$ $x=2 \quad y=1$

$x = 2y^{3/2}$

$x' = 2 \cdot \frac{3}{2} y^{1/2} = 3y^{1/2}$

$$l = \int_0^1 \sqrt{1+(x')^2}\, dy = \int_0^1 \sqrt{1+9y}\, dy = \frac{1}{9}\int_1^{10} \sqrt{u}\, du = \frac{1}{9}\left[\frac{2}{3} u^{3/2} \Big|_1^{10}\right]$$

$u = 9y+1$

$du = 9\, dy$

$$= \frac{1}{9}\left[\frac{2}{3}(10)^{3/2} - \frac{2}{3}\right] = \frac{1}{9}\left[\frac{20\sqrt{10} - 2}{3}\right] = \boxed{\frac{20\sqrt{10} - 2}{27}}$$

k N/m.

4. $F = -kx = -16x$ kg

$$W = \int_{0m}^{-0.25m} F(x)\, dx = \int_{0m}^{-0.25} -16x\, dx \text{ kg} = -8x^2 \Big|_{0m}^{-0.25m} = \left[-8\left(-\frac{1}{4}\right)^2 - [0]\right]$$

$$\boxed{= -\frac{1}{2} \text{ kg}\cdot\text{m}}$$

$+\frac{1}{2}$ J