Tarea 8

1) $\int \frac{x+3}{2x^3-8x}\,dx$

$2x^3-8x = x(2x^2-8) = 2x(x^2-4) = 2x(x-2)(x+2)$

$\frac{x+3}{2x^3-8x} = \frac{A}{2x} + \frac{B}{x-2} + \frac{C}{x+2}$

$A(x^2-4) + B(2x^2+4x) + C(2x^2-4x) = x+3$

$x=2$

$B(16)=5 \Rightarrow B=\frac{5}{16}$

$x=0$

$A(-4)=3 \Rightarrow A=-\frac{3}{4}$

$x=-2$

$C(16)=1 \Rightarrow C=\frac{1}{16}$

$\int \frac{x+3}{2x^3-8x}\,dx = \boxed{\int -\frac{3\,dx}{8x} + \int \frac{5\,dx}{16(x-2)} + \int \frac{dx}{16(x+2)}}$

$= -\frac{3}{8}\ln|x| + \frac{5}{16}\ln|x-2| + \frac{1}{16}\ln|x+2| + C$

d) $\int \frac{y^2+2y+1}{y^4+2y^2+1}\,dy$

$y^4+2y^2+1 = (y^2+1)^2$

$\int \frac{y^2+2y+1}{y^4+2y^2+1} = \frac{A_1y+B_1}{y^2+1} + \frac{A_2y+B_2}{(y^2+1)^2}$

$y^2+2y+1 = (A_1y+B_1)(y^2+1) + A_2y+B_2$

$y=-1$

$0 = -A_2+B_2$

$y=1$

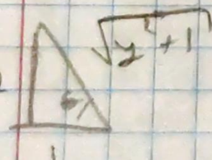
$4 = A_2+B_2$

$2B_2=4 \Rightarrow B_2=2 \Rightarrow A_2=2$

$y^2+2y+1 = A_1y^3+B_1y^2+A_1y+B_1+2y+2$

$\Rightarrow B_1=-1\,,\ A_1=0$

$\int \frac{-dy}{y^2+1} + \int \frac{2y+2}{(y^2+1)^2}\,dy = -\arctan(y)$



$y=\tan\theta$

$dy = \sec^2\theta\,d\theta$

$y^2+1 = \sec^2\theta$

$\int \frac{2\tan\theta+2}{\sec^2\theta}\,d\theta = \int 2\operatorname{sen}\theta\cos\theta\,d\theta + \int 2\cos^2\theta\,d\theta$

$= \int \operatorname{sen}(2\theta)\,d\theta + \int(1+\cos 2\theta)\,d\theta$

$u=2\theta \quad du=2d\theta$

$= \frac{1}{2}\left[\int \operatorname{sen} u\,du + \int du + \int \cos u\,du\right]$

$= \frac{1}{2}\left[-\cos u + u + \operatorname{sen} u\right]$

$= \frac{1}{2}[\sin 2\theta - \cos 2\theta + 2\theta]$

$\frac{1}{2}[2\sin\theta\cos\theta \quad \cos^2\theta + \sin^2\theta + 2\theta]$

$= \frac{1}{2}\left[2 \cdot \frac{y}{\sqrt{y^2+1}} \cdot \frac{1}{\sqrt{y^2+1}} - \frac{1}{y^2+1} + \frac{y^2}{y^2+1} + 2\arctan y\right]$

$= \frac{y^2+2y-1}{2(y^2+1)} + \arctan(y)$

$y^2+2y+1 = A_1y^3 + B_1y^2 + (A_1+A_2)y + B_1 + B_2$

$A_1 = 0, \quad B_1 = 1$

$A_2 = 2, \quad B_2 = 0$

$\int \frac{dy}{y^2+1} + \int \frac{2y}{(y^2+1)^2}dy = \boxed{\arctan y - \frac{1}{(y^2+1)} + C}$

$u = y^2+1 \quad du = 2y\,dy$

$\int \frac{du}{u^2} = \int u^{-2}du = -u^{-1} + C = -\frac{1}{(y^2+1)} + C$

b) $\int \frac{2x^4}{x^3-x^2+x-1}\,dx$

$$\begin{array}{r l}
 & 2x + 2 \\
x^3-x^2+x-1 \,\big) & 2x^4 \\
 & -2x^4 + 2x^3 - 2x^2 + 2x \\
 & \quad 2x^3 - 2x^2 + 2x \\
 & \quad -2x^3 + 2x^2 - 2x + 2 \\
 & \qquad\qquad 2
\end{array}$$

$$\int \left(2x+2 + \frac{2}{x^3-x^2+x-1}\right) dx = x^2 + 2x +$$

$$x^3 - x^2 + x - 1 = (x-1)(x^2+1)$$

$$\frac{2}{x^3-x^2+x-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$2 = A(x^2+1) + (Bx+C)(x-1)$$

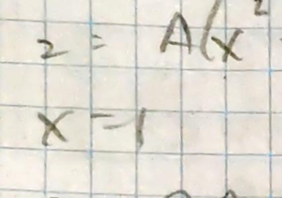
$x = 1$

$2 = 2A \Rightarrow A = 1$

$2 = 1x^2 + \cancel{1} + (Bx^2 + (C-B)x - C)$

$B = -1, \quad C = -1$

$$\int \frac{dx}{x-1} - \int \frac{x+1}{x^2+1}\,dx = \boxed{\ln|x-1| + \ln\sqrt{x^2+1} + \mathrm{Arctan}\,x + C}$$



$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$x^2+1 = \sec^2\theta$

$$\int (\tan\theta + 1)\,d\theta = \ln|\sec\theta| + \theta + C$$

$$= \ln|\sqrt{x^2+1}| + \mathrm{Arctan}(x) + C$$

4) $\int \frac{x^3}{x^2-2x+1}dx$

$$\begin{array}{r|l} x^2-2x+1 & x^3 \\ & -x^3+2x^2-x \\ & \quad 2x^2-x \\ & \quad -2x^2+4x-2 \\ & \qquad 3x-2 \end{array} \quad x+2$$

$\int \left(x+2+\frac{3x-2}{(x-1)^2}\right)dx$

$\frac{3x-2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$A(x-1) + B = 3x-2$

$Ax - A + B = 3x-2$

$A = 3 \ , \ B = 1$

$\int \frac{3\,dx}{x-1} + \int \frac{dx}{(x-1)^2} = 3\ln|x-1| - (x-1)^{-1} + c$

$I = \boxed{\frac{x^2}{2} + 2x + 3\ln|x-1| - (x-1)^{-1} + c}$

5) $\int \frac{\sin\theta}{\cos^2\theta + \cos\theta - 2}\, d\theta$

$u = \cos\theta \quad du = -\sin\theta\, d\theta$

$\int \frac{-du}{(u-1)(u+2)}$

$\frac{1}{(u-1)(u+2)} = \frac{A}{u-1} + \frac{B}{u+2}$

$1 = A(u+2) + B(u-1)$

$u = 1$

$1 = 3A \Rightarrow A = \frac{1}{3}$

$u = -2$

$1 = -3B \Rightarrow B = -\frac{1}{3}$

$-\left[\frac{1}{3}\int \frac{du}{u-1} - \frac{1}{3}\int \frac{du}{u+2}\right] = \frac{1}{3}\ln|u+2| - \frac{1}{3}\ln|u-1| + C$

$= \frac{1}{3}\ln|\cos\theta + 2| - \frac{1}{3}\ln|\cos\theta - 1| + C$