Trabajo en Clase 8

1) $\int \frac{e^t}{e^{2t}+3e^t+2}\, dt$

$u = e^t \qquad du = e^t\, dt$

$$\int \frac{du}{u^2+3u+2}$$

$u^2+3u+2 = (u+1)(u+2)$

$$\frac{1}{(u+1)(u+2)} = \frac{A}{u+1} + \frac{B}{u+2} \Rightarrow 1 = A(u+2) + B(u+1)$$

Si $u = -2$ | Si $u = -1$

$1 = -B$ | $1 = A$

$B = -1$

$$\int \frac{du}{u+1} - \int \frac{du}{u+2} = \ln|u+1| - \ln|u+2| + C = \ln|e^t+1| - \ln|e^t+2| + C$$

2) $\int \frac{x^3}{x^2+2x+1}\,dx = \int \left(x + \frac{x^3 - x^3 - 2x^2 - x}{x^2+2x+1}\right)dx = \int\left(x + \frac{-2x^2-x}{x^2+2x+1}\right)dx$

$= \int\left(x-2 + \frac{-2x^2-x+2x^2+4x+2}{x^2+2x+1}\right)dx = \int\left(x-2 + \frac{3x-2}{(x+1)^2}\right)dx$

$= \int (x-2)\,dx + \int \frac{3x-2}{(x+1)^2}\,dx$

$\frac{3x-2}{(x+1)^2} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} \Rightarrow 3x-2 = A_1(x+1) + A_2$

Si $x=-1$	Derivas
$-5 = A_2$	$3 = A_1$

$\int(x-2)dx + \int\frac{3x-2}{(x+1)^2}dx = \int(x-2)dx + 3\int\frac{dx}{x+1} - 5\int\frac{dx}{(x+1)^2} = \frac{(x-2)^2}{2} + 3\ln|x+1| + 5(x+1)^{-1} + C$

3) $\int \frac{dx}{(x^2-1)^2} = \int\frac{dx}{(x-1)^2(x+1)^2}$

$\frac{1}{(x-1)^2(x+1)^2} = \frac{A_1}{x-1} + \frac{A_2}{(x-1)^2} + \frac{B_1}{x+1} + \frac{B_2}{(x+1)^2}$

$\Rightarrow 1 = A_1(x-1)(x+1)^2 + A_2(x+1)^2 + B_1(x-1)^2(x+1) + B_2(x-1)^2$

Si $x=1$	Si $x=-1$	Derivas
$1 = 4A_2$	$1 = 4B_2$	$0 = A_1\left((x+1)^2 + 2(x+1)(x-1)\right) + 2A_2(x+1) + B_1\left(2(x-1)(x+1) + (x-1)^2\right) + 2B_2(x-$
$A_2 = 1/4$	$B_2 = 1/4$	$= A_1(x+1)(3x-1) + \frac{1}{2}(x+1) + B_1(x-1)(3x+1) + \frac{1}{2}(x-1)$

Si $x=1$	Si $x=-1$
$0 = 4A_1 + 1$	$0 = 4B_1 - 1$
$A_1 = -1/4$	$B_1 = +1/4$

$\frac{1}{4}\left(-\int\frac{dx}{x-1} + \int\frac{dx}{(x-1)^2} + \int\frac{dx}{x+1} - \int\frac{dx}{(x+1)^2}\right) = \frac{1}{4}\left(-\ln|x-1| - (x-1)^{-1} + \ln|x+1| - (x+1)^{-1}\right) + C$

4) $\displaystyle\int \frac{x^4-81}{x^5+18x^3+81x}\,dx$

$$x^5+18x^3+81x = x(x^4+18x^2+81) = x(x^2+9)^2$$

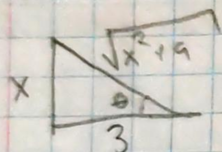
$$\frac{x^4-81}{x(x^2+9)^2} = \frac{(x^2-9)(x^2+9)}{x(x^2+9)^2} = \frac{x^2-9}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$$

$$\Rightarrow x^2-9 = A(x^2+9)+(Bx+C)x = x^2(A+B)+Cx+9A$$

$A+B=1,\quad C=0,\quad A=-1$

$1 = A+B = -1+B \Rightarrow B=2$

$$\int \frac{x^4-81}{x^5+18x^3+81x}\,dx = -\int\frac{dx}{x} + 2\int\frac{x}{x^2+9}\,dx$$



$x = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$\sqrt{x^2+9} = 3\sec\theta \Rightarrow x^2+9 = 9\sec^2\theta$

$$\int\frac{x}{x^2+9}\,dx = \int\frac{3\tan\theta\cdot 3\sec^2\theta\, d\theta}{9\sec^2\theta} = \int\tan\theta\, d\theta = \ln|\sec\theta| + C = \ln\left|\frac{\sqrt{x^2+9}}{3}\right| + C$$

$$= -\ln|x| + 2\ln\left(\frac{\sqrt{x^2+9}}{3}\right) + C$$

5) $\displaystyle\int \frac{dx}{x^2+10x+26}$

$x^2+10x+26 = (x+5)^2+1,\quad u = x+5 \quad du = dx$

$$\int\frac{dx}{(x+5)^2+1} = \int\frac{du}{u^2+1} = \operatorname{Arctan} u + C = \operatorname{Arctan}(x+5) + C$$

b) $\int \cancel{x}\ 1$

c) $\int \frac{1}{(x^{1/3}-1)\,x^{1/2}}\,dx$

$u = \frac{1}{x^{1/3}-1}$ $v = 2x^{1/2}$

$du = -(x^{1/3}-1)^{-2}(x^{-2/3})\,dx$ $dv = x^{-1/2}\,dx$

$\int u\,dv = uv - \int v\,du = \frac{2x^{1/2}}{x^{1/3}-1} + \int 2x^{1/2}(x^{-2/3})(x^{1/3}-1)^{-2}\,dx$

$\frac{2x^{-1/6}}{x^{2/3}-2x^{1/3}+1}\,dx$ $u = x^{1/6}$ $du = \frac{1}{6}x^{-5/6}\,dx$

$\int \frac{2u^4\,du}{u^4-2u^2+1} = \int\left(2 + \frac{2u^4-2u^4+2u^2-1}{u^4-2u^2+1}\right)du$

$= \int\left(2 + \frac{2u^2-1}{u^4-2u^2+1}\right)du$

$\frac{2u^2-1}{(u-1)^2(u+1)^2} = \frac{A_1}{u-1} + \frac{A_2}{(u-1)^2} + \frac{B_1}{(u+1)} + \frac{B_2}{(u+1)^2}$

$2u^2-1 = A_1(u-1)(u+1)^2 + A_2(u+1)^2 + B_1(u+1)(u-1)^2 + B_2(u-1)^2$

Si $u=1$	Si $u=-1$	Derivas
$1 = 4A_2$	$1 = 4B_2$	$4u = A_1\left((u+1)^2 + 2(u-1)(u+1)\right) + 2A_2(u+1) + B_1\left((u-1)^2 + 2(u+1)(u-1)\right) + 2$
$A_2 = 1/4$	$B_2 = 1/4$	$4u = A_1(u+1)(3u-1) + \frac{1}{2}(u+1) + B_1(u-1)(3u+1) + \frac{1}{2}(u-1)$

Si $u=-1$	Si $u=1$
$-4 = 4B_1 - 1$	$4 = 4A_1 + 1$
$B_1 = -3/4$	$A_1 = 3/4$

$$\left(3\int \frac{dw}{w-1} + \int \frac{dw}{(w-1)^2} - 3\int \frac{dw}{w+1} + \int \frac{dw}{(w+1)^2}\right)$$

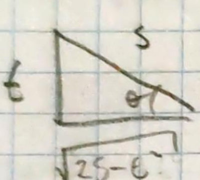
$$\frac{1}{4}\left(3\ln|w-1| - (w-1)^{-1} - 3\ln|w+1| - (w+1)^{-1}\right) + C$$

$$\frac{1}{4}\left(3\ln|x^{1/6}-1| - (x^{1/6}-1)^{-1} - 3\ln|x^{1/6}+1| - (x^{1/6}+1)^{-1}\right) + C$$

$$\frac{2x^{1/2}}{x^{1/3}-1} + \int \frac{2x^{-1/6}}{x^{2/3}-2x^{1/3}+1}\,dx = \frac{2x^{1/2}}{x^{1/3}-1} + \frac{1}{4}\left(3\ln|x^{1/6}-1| - (x^{1/6}-1)^{-1} - 3\ln|x^{1/6}+1| - (x^{1/6}+1)^{-1}\right)$$

Trabajo en Clase 7.

1) $\int \sqrt{25-t^2}\,dt$



$t = 5\,\text{sen}\,\theta$

$dt = 5\cos\theta\, d\theta$

$\sqrt{25-t^2} = 5\cos\theta$

$$\int 25\cos^2\theta\, d\theta = 25\int \cos^2\theta\, d\theta$$

$$= 25\int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= 25\left(\int \frac{1}{2}d\theta + \frac{1}{2}\int \cos 2\theta\, d\theta\right)$$

$$= 25\left(\int \frac{1}{2}d\theta + \frac{1}{4}\int \cos 2\theta\, 2d\theta\right)$$

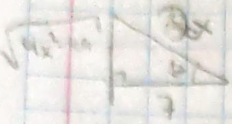
$u = 2\theta \qquad du = 2d\theta$

$$25\left(\int \frac{1}{2}d\theta + \frac{1}{4}\int \cos u\, du\right) = 25\left(\int \frac{1}{2}d\theta + \frac{1}{4}(\text{sen}\, u + C)\right) = 25\left(\frac{1}{2}\theta + C + \frac{1}{4}\text{sen}\,2\theta + \right.$$

$$= \frac{25}{2}\theta + \frac{25}{4}\text{sen}\,2\theta + C = \frac{25}{2}\theta + \frac{25}{2}\text{sen}\,\theta\cos\theta + C = \frac{25}{2}\theta + \frac{t\sqrt{25-t^2}}{2} + C$$

$$= \frac{25}{2}\text{Arcsin}\,\frac{t}{5} + \frac{t\sqrt{25-t^2}}{2} + C$$

2) $\int \frac{1}{\sqrt{4x^2-49}}\,dx$



$x = \frac{7}{2}\sec\theta$

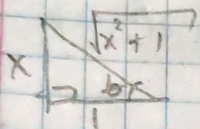
$dx = \frac{7}{2}\sec\theta\tan\theta\,d\theta$

$\sqrt{4x^2-49} = 7\tan\theta$

$$\int \frac{1}{\sqrt{4x^2-49}}\,dx = \int \frac{\frac{7}{2}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{7}\,\cancel{\tan\theta}} = \frac{1}{2}\int \sec\theta\,d\theta = \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$\frac{1}{2}\ln\left|\frac{2}{7}x + \frac{\sqrt{4x^2-49}}{7}\right| + C$$

3) $\int \frac{8dx}{(x^2+1)^2} = 8\int \frac{dx}{(x^2+1)^2}$



$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\sqrt{x^2+1} = \sec\theta \Rightarrow (x^2+1)^2 = \sec^4\theta$

$$8\int \frac{dx}{(x^2+1)^2} = 8\int \frac{\sec^2\theta\,d\theta}{\sec^4\theta} = 8\int \frac{d\theta}{\sec^2\theta} = 8\int \cos^2\theta\,d\theta = 8\int \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= 4\int (1+\cos 2\theta)\,d\theta$$

$u = 2\theta \quad du = 2d\theta$

$$= 2\int (1+\cos u)\,du = 2u + 2\operatorname{sen} u + C = 4\theta + 2\operatorname{sen} 2\theta + C = 4\theta + 4\operatorname{sen}\theta\cos\theta$$

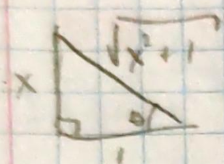
$$4\operatorname{Arctan} x + \frac{4x}{x^2+1} + C$$

4) $\int \frac{e^t\,dt}{\sqrt{e^{2t}+9}}$

$u = e^t \qquad du = e^t\,dt$

$$\int \frac{du}{\sqrt{u^2+9}} = \ln\left(u + \sqrt{u^2+9}\right) + C = \ln\left(e^t + \sqrt{e^{2t}+9}\right) + C$$

5) $\int \frac{2x\,dx}{1+x^2}$



$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$

$\sqrt{x^2+1} = \sec\theta \Rightarrow x^2+1 = \sec^2\theta$

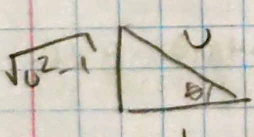
$$\int \frac{2x\,dx}{1+x^2} = \int \frac{2\tan\theta\,\cancel{\sec^2\theta}\,d\theta}{\cancel{\sec^2\theta}} = 2\int \tan\theta\,d\theta = 2\ln|\sec\theta| + C$$

$$= 2\ln\left|\sqrt{x^2+1}\right| + C$$

Opcional

6) $\int \frac{\sqrt{x-2}}{\sqrt{x-1}}\,dx \qquad u^2 = x-1 \quad dx = 2u\,du$

$$\int \frac{\sqrt{u^2-1}\,2\cancel{u}\,du}{\cancel{u}} = 2\int \sqrt{u^2-1}\,du$$



$u = \sec\theta$
$du = \sec\theta\tan\theta\,d\theta$

$\sqrt{u^2-1} = \tan\theta$

$2\int \sec^2\theta\tan^2\theta\,d\theta =$

$= \int \sec\theta \tan^2\theta \, d\theta$

$u_1 = \tan\theta \quad v_1 = \sec\theta$
$du_1 = \sec^2\theta \, d\theta \quad dv_1 = \sec\theta\tan\theta \, d\theta$

$\int u_1 dv_1 = u_1 v_1 - \int v_1 du_1 = \sec\theta\tan\theta - \int \sec^3\theta \, d\theta$

$\int \sec\theta\tan^2\theta \, d\theta = \sec\theta\tan\theta - \int \sec\theta(1+\tan^2\theta) \, d\theta = \sec\theta\tan\theta - \int \sec\theta \, d\theta - \int \sec\theta\tan^2\theta \, d\theta$

$2\int \sec\theta\tan^2\theta \, d\theta = \sec\theta\tan\theta - \ln|\sec\theta + \tan\theta| + C$

$= u\sqrt{u^2-1} - \ln|u + \sqrt{u^2+1}| + C$

$= \sqrt{x-1}\sqrt{x-2} - \ln|\sqrt{x-1} + \sqrt{x}| + C$

$$\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

$g = x^3+1 \quad g' = 3x^2$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx = \int_{g(-1)}^{g(1)} \sqrt{u}\,du = \int_0^2 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_0^2 = \frac{4\sqrt{2}}{3}$$

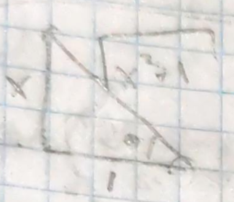
Trabajo en clase 10

1) $\int_0^1 \arctan x \, dx$

$u = \arctan x \qquad v = x$

$du = \frac{1}{1+x^2}dx \qquad dv = dx$

$$\int u\,dv = uv - \int v\,du = x\arctan x - \int \frac{x}{x^2+1}dx$$



$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\sqrt{x^2+1} = \sec\theta$

$$\int \frac{x\,dx}{x^2+1} = \int \tan\theta\, d\theta = \ln|\sec\theta| + C$$

$$= \ln|\sqrt{x^2+1}| + C$$

$$= x\arctan x - \ln|\sqrt{x^2+1}| + C$$

$$\int_0^1 \arctan x\,dx = x\arctan x - \ln|\sqrt{x^2+1}|\Big|_0^1 = [\arctan 1 - \ln|\sqrt{2}|] - [-\ln|1|]$$

$$= \frac{\pi}{4} - \ln\sqrt{2}$$

2) $\int_{\pi/6}^{\pi/4} \operatorname{sen}^5 x \cos^3 x\, dx$

$u = \operatorname{sen} x \quad du = \cos x$

$$\int \operatorname{sen}^5 x\cos^3 x\,dx = \int \operatorname{sen}^5 x(1-\operatorname{sen}^2 x)\cos x\,dx = \int (u^5 - u^7)\,du = \frac{u^6}{6} - \frac{u^8}{8} + C$$

$$= \frac{1}{6}\operatorname{sen}^6 x - \frac{1}{8}\operatorname{sen}^8 x + C$$

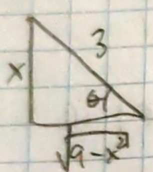
$$\int_{\pi/6}^{\pi/4} \operatorname{sen}^5 x \cos^3 x\,dx = \frac{1}{6}\operatorname{sen}^6 x - \frac{1}{8}\operatorname{sen}^8 x \Big|_{\pi/6}^{\pi/4} = \left[\frac{1}{6}\left(\operatorname{sen}\frac{\pi}{4}\right)^6 - \frac{1}{8}\left(\operatorname{sen}\frac{\pi}{4}\right)^8\right]$$

$$-\left[\frac{1}{6}\left(\operatorname{sen}\frac{\pi}{6}\right)^6 - \frac{1}{8}\left(\operatorname{sen}\frac{\pi}{6}\right)^8\right] = \left[\frac{1}{6}\left(\frac{\sqrt{2}}{2}\right)^6 - \frac{1}{8}\left(\frac{\sqrt{2}}{2}\right)^8\right] - \left[\frac{1}{6}\left(\frac{1}{2}\right)^6 - \frac{1}{8}\left(\frac{1}{2}\right)^8\right]$$

$$= \left[\frac{1}{6}\cdot\frac{1}{8} - \frac{1}{8}\cdot\frac{1}{16}\right] - \left[\frac{1}{6}\cdot\frac{1}{64} - \frac{1}{8}\cdot\frac{1}{256}\right]$$

$$= \frac{256 - 96 - 32 + 6}{6\cdot 8\cdot 256} = \frac{134}{12288} = \boxed{\frac{67}{6144}}$$

3) $\displaystyle\int_0^1 \frac{x^2}{\sqrt{9-x^2}}\,dx$



$x = 3\operatorname{sen}\theta$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-x^2} = 3\cos\theta$

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \int \frac{9\operatorname{sen}^2\theta}{\cancel{3\cos\theta}}\,\cancel{3\cos\theta}\,d\theta$$

$$= 9\int \operatorname{sen}^2\theta\,d\theta = 9\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= 9\left(\int\frac{1}{2}\,d\theta - \frac{1}{2}\int\cos 2\theta\,d\theta\right)$$

$u = 2\theta \qquad du = 2\,d\theta$

$$= 9\left(\int\frac{1}{2}\,d\theta - \frac{1}{4}\int\cos u\,du\right) = 9\left(\frac{1}{2}\theta - \frac{1}{4}\operatorname{sen} 2\theta + C\right) = \frac{9}{2}\operatorname{Arcsen}\frac{x}{3}$$

$$-\frac{9}{4}\cdot 2\operatorname{sen}\theta\cos\theta + C = \frac{9}{2}\operatorname{Arcsen}\left|\frac{x}{3}\right| - \frac{9}{2}\cdot\frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3} + C = \frac{9}{2}\operatorname{Arcsen}\left|\frac{x}{3}\right| - \frac{x\sqrt{9-x^2}}{2} + C$$

$$\int_0^1 \frac{x^2}{\sqrt{9-x^2}}\,dx = \frac{9}{2}\operatorname{Arcsen}\left|\frac{x}{3}\right| - \frac{x\sqrt{9-x^2}}{2}\Big|_0^1 = \frac{9}{2}\operatorname{Arcsen}\left|\frac{1}{3}\right| - \frac{\sqrt{8}}{2}$$

$\ln x$

4) $\displaystyle\int_2^3 \frac{4x^4-5x^2+3}{x^5-x^3}\,dx$ $\qquad \frac{4x^4-5x^2+3}{x^3(x^2-1)} = \frac{A_1}{x}+\frac{A_2}{x^2}+\frac{A_3}{x^3}+\frac{Bx+C}{x^2-1}$

$x^5-x^3 = x^3(x-1)(x+1)$

$$\begin{array}{r} 4x^3+4x^2+x+1 \\ x-1\,\overline{)\,4x^4+0-5x^2+0+3} \\ -4x^4+4x^3 \\ \hline 4x^3-5x^2 \\ -4x^3+4x^2 \\ \hline x^2+0 \\ -x^2+x \\ \hline x+3 \\ -x+1 \\ \hline 4 \end{array} \qquad \begin{array}{r} 4x^3-4x^2-x \\ x+1\,\overline{)\,4x^4+0-5x^2+0+3} \\ -4x^4-4x^3 \\ \hline -4x^3-5x^2 \\ 4x^3+4x^2 \\ \hline -x^2\cdot 0 \\ x^2\ \ x \\ \hline x+3 \end{array}$$

$$\int \frac{4x^5-5x^3+3x}{x^6-x^4}\,dx = \int \frac{(x)(4x^4-5x^2+3)\,dx}{x^6-x^4}$$

$u = x^2 \qquad du = 2x\,dx$

$$= \frac{1}{2}\int \frac{(4u^2-5u+3)}{u^3-u^2}\,du$$

$$\begin{array}{r} \cancel{4u} \\ u-1\,\overline{)\,\cancel{4u^2-5u+3}} \\ \cancel{-4u^2\ \ 4u} \\ \hline \cancel{u\ \ +3} \end{array} \qquad \frac{4u^2-5u+3}{u^2(u-1)} = \frac{A_1}{u}+\frac{A_2}{u^2}+\frac{B_1}{u-1}$$

$4u^2-5u+3 = A_1(u^2-u)+A_2(u-1)+B_1u^2 = u^2(A_1+B_1)+u(-A_1+A_2)-A_2$

$A_2=-3 \qquad A_1=2 \qquad B_1=2$

$$2\int \frac{1}{u}\,du - 3\int\frac{1}{u^2}\,du + 2\int\frac{du}{u-1} = 2\ln|u| + \frac{3}{u} + 2\ln|u-1| + C$$

$$\int_2^3 \frac{4x^4-5x^2+3}{x^5-x^3}\,dx = 2\ln|u| + \frac{3}{u} + 2\ln|u-1|\Big|_2^3 = 2\ln|3| + 1 + 2\ln|2| - 2\ln|2| - \frac{3}{2} + 2\ln|1|$$

$$= 2\ln|3| - \frac{1}{2}$$

$\int_0^{\dots} x^3\,dx =$

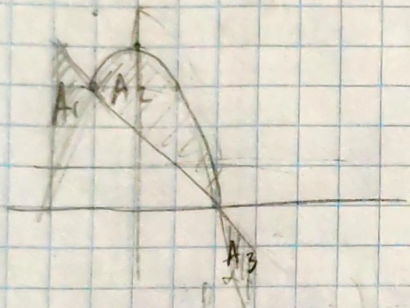
Trabajo en Clase 9

Intervalo $[0,b]$	Área
$[0,1]$	0.3
$[0,2]$	4.84
$[0,3]$	24.5
$[0,4]$	77.44
$[0,5]$	189.06

$\frac{1}{4}$ 0.25

Trabajo en Clase 10

1. $y = 4 - x^2$, $y = 2 - x$ $\quad -2 \le x \le 3$



$4 - x^2 = 2 - x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2,\ x = -1$

$A_1 = \int_{-2}^{-1} (2 - x - 4 + x^2)\,dx = \int_{-2}^{-1} (x^2 - x - 2)\,dx$

$A = A_1 + A_2 + A_3$

$= \frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{-2}^{-1} = \frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2(-1) - \left(\frac{(-2)^3}{3} - \frac{(-2)^2}{2} - 2(-2)\right)$

$= -\frac{1}{3} - \frac{1}{2} + 2 - \left(-\frac{8}{3} - 2 + 4\right) = -\frac{1}{3} - \frac{1}{2} + \cancel{2} + \frac{8}{3} + \cancel{2} \cancel{- 4} = \frac{7}{3} - \frac{1}{2} = \frac{11}{6}$

$A_2 = \int_{-1}^{2} (4 - x^2 - 2 + x)\,dx = \int_{-1}^{2} (-x^2 + x + 2)\,dx = -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2} = -\frac{(2)^3}{3} + \frac{(2)^2}{2} + 2(2) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + \dots\right)$

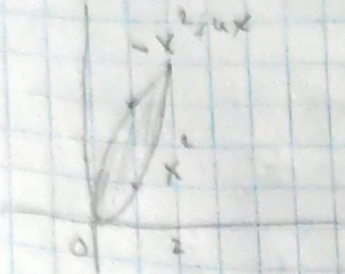
$= -\frac{8}{3} + 2 + 4 - \left(\frac{1}{3} + \frac{1}{2} - 2\right) = -\frac{8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2 = -\frac{9}{3} + 8 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2}$

$A_3 = \int_{2}^{3} (x^2 - x - 2)\,dx = \frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{2}^{3} = \frac{27}{3} - \frac{9}{2} - 6 - \left(\frac{8}{3} - \frac{4}{2} - 4\right) = 9 - \frac{9}{2} - 6 - \frac{8}{3} + 2 + 4$

$= \frac{54 - 27 - 16}{6} = \frac{11}{6}$

$A = \frac{11}{6} + \frac{9}{2} + \frac{11}{6} = \frac{11}{3} + \frac{9}{2} = \boxed{\frac{22 + 27}{6}} = \frac{49}{6}$

2. $y = x^2$, $y = -x^2 + 4x$



$x^2 = -x^2 + 4x$

$2x^2 - 4x = 0$

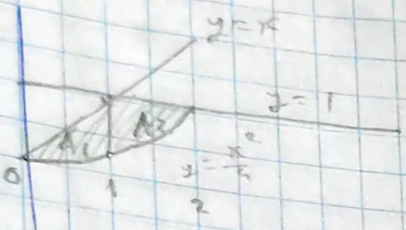
$2x(x-2) = 0$

$x=0$, $x=2$

$$A = \int_0^2 (-x^2+4x-x^2)\,dx = \int_0^2 (-2x^2+4x)\,dx = -\frac{2x^3}{3} + \frac{4x^2}{2}\Big|_0^2 = -\frac{2}{3}x^3 + 2x^2\Big|_0^2$$

$$= -\frac{2}{3}\cdot 8 + 2\cdot 4 = 8 - \frac{16}{3} = \frac{8}{3}$$

3.



$\frac{x^2}{4} = 1$

$x^2 = 4$, $x = \pm 2 \to x = 2$

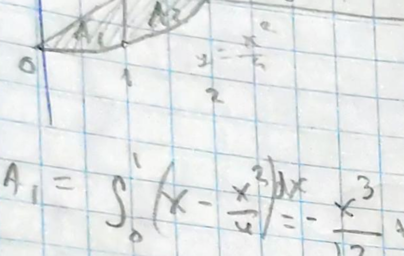
$$A_1 = \int_0^1 \left(x - \frac{x^2}{4}\right)dx = -\frac{x^3}{12} + \frac{x^2}{2}\Big|_0^1 = -\frac{1}{12} + \frac{1}{2} = \frac{5}{12}$$

$$A_2 = \int_1^2 \left(1 - \frac{x^2}{4}\right)dx = -\frac{x^3}{12} + x\Big|_1^2 = -\frac{8}{12} + 2 - \left(-\frac{1}{12}+1\right) = \frac{-8+24+1-12}{12}$$

$= 5/2$

$A = A_1 + A_2 = 5/12 + 5/12 = 10/12 = 5/6$

4.



$= \frac{1}{2}x - \frac{1}{2}\operatorname{sen} x \cos x \Big|_0^{\pi} =$

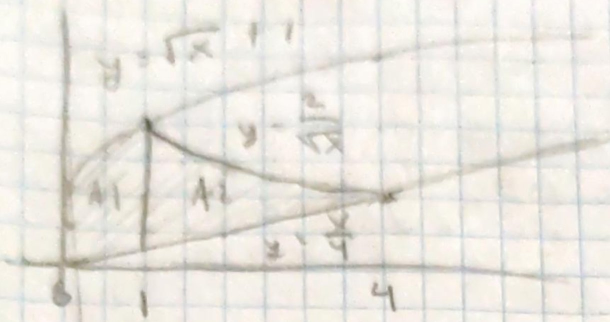
$$A = \int_0^{\pi} (1-\cos^2 x)\,dx = \int_0^{\pi} \operatorname{sen}^2 x\,dx = \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right)dx$$

$u = 2x$ $du = 2$

$$= \frac{1}{4}\int_0^{\pi/2} (1-\cos u)\,du = \frac{1}{4}(u - \operatorname{sen} u)\Big|_0^{\pi/2} = \frac{1}{4}\left(\frac{\pi}{2} - \operatorname{sen}\frac{\pi}{2}\right) = \frac{\pi}{8} - \frac{1}{4}$$

$\frac{1}{4}\,2\pi - \operatorname{sen} 2\pi = \frac{\pi}{2}$

5)



$y = \frac{x}{4} \qquad y = \sqrt{x}+1$

$\frac{x}{4} = \sqrt{x}+1$

$\frac{x^2}{16} - \frac{x}{2} + 1 = 0$

$\frac{x^2}{16} - \frac{3x}{2} + 1 = 0$

$x^2 - 24x + 16 = 0$

$24 \pm \sqrt{576-64}$

$\frac{2}{\sqrt{x}} = \frac{x}{4}$

$8 = x^{3/2}$

$2 = \sqrt{x}, \quad x = 4$

$\sqrt{x}+1 = \frac{2}{\sqrt{x}}$

$x + \sqrt{x} = 2$

$x + \sqrt{x} - 2 = 0$

$(\sqrt{x}+2)(\sqrt{x}-1) = 0$

$\sqrt{x} = -2, \quad \sqrt{x} = 1 \Rightarrow x = 1$

$A_1 = \int_0^1 \left(\sqrt{x} + 1 - \frac{x}{4}\right)dx = \frac{2}{3}x^{3/2} + x - \frac{x^2}{8}\Big|_0^1 = \frac{2}{3} + 1 - \frac{1}{8} = \frac{16+24-3}{24} = \frac{37}{24}$

$A_2 = \int_1^4 \left(\frac{2}{\sqrt{x}} - \frac{x}{4}\right)dx = 4\sqrt{x} - \frac{x^2}{8}\Big|_1^4 = 8 - 2 - \left(4 - \frac{1}{8}\right) = \frac{17}{8} = \frac{51}{24}$

$A = A_1 + A_2 = \frac{37}{24} + \frac{51}{24} = \frac{88}{24} = \frac{11}{3}$