6.1.2. Problemas con Ecuaciones Diferenciales.

Prob.
Act. 10 000 casos

Spor un año se reduzca 20%.

· Tasa de cambio de enfermos es proporcional al tiempo

$$\frac{dy}{dt} = ky$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$y = e^{kt+C} = Ce^{kt}$$

$$y(0) = Ce^{k(0)} = 10\,000 \Rightarrow C = 10000$$

$$y = 10\,000\,e^{kt}$$

$$8000 = y(1) = 10000\,e^{k} \Rightarrow e^{k} = \frac{4}{5} \Rightarrow k = \ln\left(\frac{4}{5}\right)$$

$$\boxed{y = 10000\,e^{t\ln\frac{4}{5}}}$$

·¿# casos en 5 años?

$$y = 10000\,e^{5\ln\frac{4}{5}} = 32\cancel{68}.8 \;(76) \Rightarrow 32\cancel{7}7$$

·¿en cuantos años 1000 casos?

$$10000\,e^{t\ln\frac{4}{5}} = 1000$$

$$e^{t\ln\frac{4}{5}} = \frac{1}{10} \Rightarrow t\ln\frac{4}{5} = \ln\frac{1}{10}$$

$$\Rightarrow t = \frac{\ln\frac{1}{10}}{\ln\frac{4}{5}} = \frac{\ln 10}{\ln\frac{5}{4}} = 10.31 \approx 10$$

Curvas ortogonales: tangente perpendiculares

$x^2+y^2=r^2$

$2x+2yy'=0$

$y'=-\frac{x}{y}$

$m_\perp \Rightarrow y'=\frac{y}{x}$

$\frac{dy}{dx}=\frac{y}{x}$

$\int\frac{dy}{y}=\int\frac{dx}{x}$

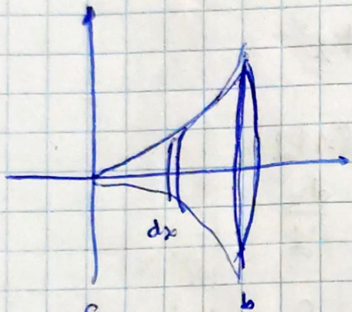
$\ln|y|=\ln|x|+c$

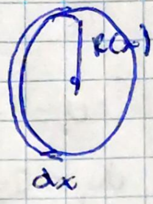
$y=e^{\ln x+c}=ce^{\ln x}=cx$

# 6.2 Aplicaciones de la Integral definida

## 6.2.1 Volumenes de Revolución

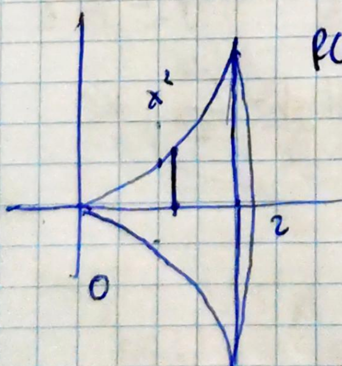
### 6.2.1.1 Metodo de los discos




$$V = \int_a^b \pi \left[R(x)\right]^2 dx$$

$$\begin{cases} y = x^2 \\ y = 0 \\ x = 2 \end{cases}$$

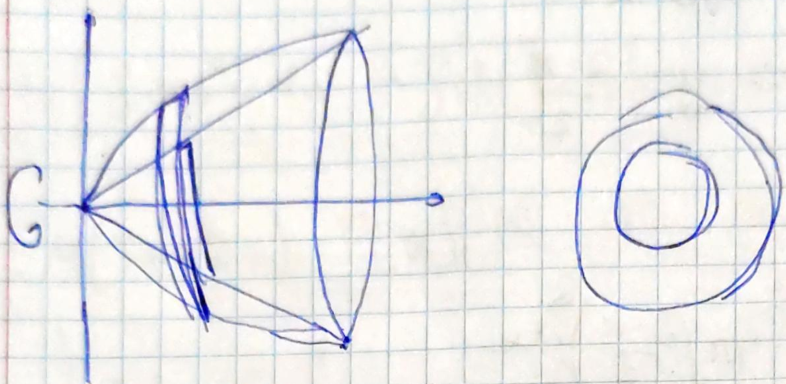



$R(x) = x^2$

$$V = \int_0^2 \pi \left[x^2\right]^2 dx = \pi \int_0^2 x^4 dx = \pi \left( \frac{x^5}{5} \Big|_0^2 \right)$$

$$= \frac{32}{5}\pi$$

## 6.2.1 Metodo de las arandelas



$$V = \int_a^b \pi\left[(R(x))^2 - (r(x))^2\right] dx$$

$$\begin{cases} y = \sqrt{x} \\ y = \frac{x}{2} \end{cases}$$

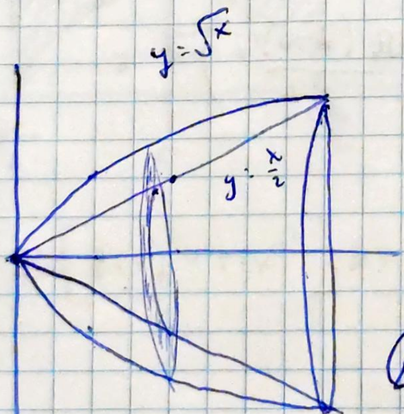
$$\frac{x}{2} = \sqrt{x}$$

$$\frac{x^2}{4} = x$$

$$x^2 = 4x$$

$$x(x-4) = 0$$

$$x = 0 \qquad x = 4$$




$$R(x) = \sqrt{x} \quad \cancel{\frac{x}{2}}$$

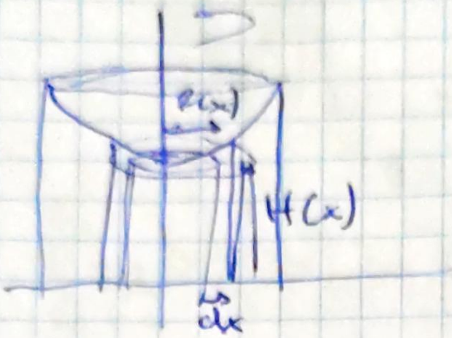
$$r(x) = \frac{x}{2}$$

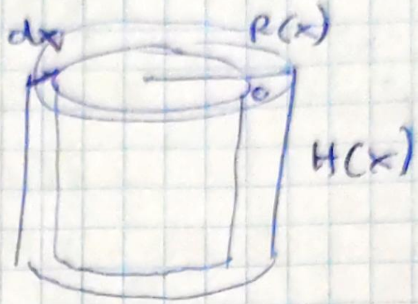
$$\cancel{(R(x))^2 \quad x - x\sqrt{x} + \frac{x^2}{4}}$$

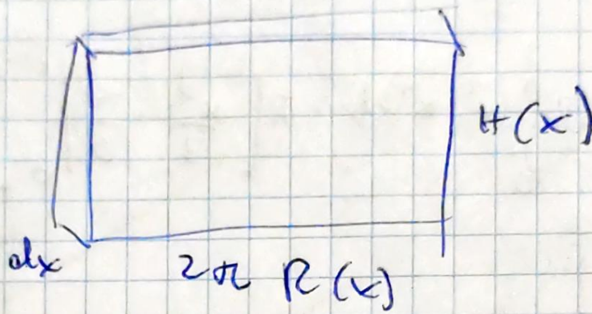
$$V = \int_0^4 \pi\left(x - \frac{x^2}{4}\right)dx = \pi\int_0^4 \left(x - \frac{x^2}{4}\right)dx = \pi\left(\frac{x^2}{2} - \frac{x^3}{12}\Big|_0^4\right)$$

$$= \pi\left(8 - \frac{16}{3}\right) = \pi\left(\frac{24-16}{3}\right) = \frac{8\pi}{3}$$

## 6.2.1.3 Metodo de las capas cilindricas
cascarones cilindricos casquillos, casquetes, anillos

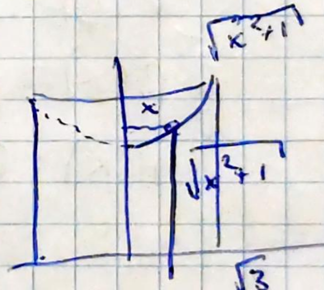



$$\boxed{V = \int_a^b 2\pi R(x)\, H(x)\, dx}$$

Ejemplo
$$\begin{cases} y = \sqrt{x^2+1} \\ x = 0, \quad x = \sqrt{3} \\ y = 0 \end{cases}$$




$R(x) = x \qquad H(x) = \sqrt{x^2+1}$

$$V = \int_0^{\sqrt{3}} 2\pi x \sqrt{x^2+1}\, dx = \int_1^4 \pi u^{1/2}\, du = \pi \cdot \frac{2}{3} u^{3/2} \Big|_1^4 = \frac{16\pi}{3} - \frac{2\pi}{3} = \boxed{\frac{14\pi}{3}}$$

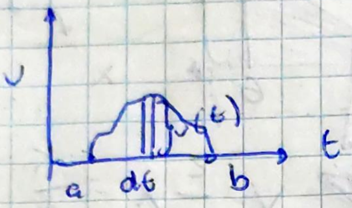
$u = x^2+1$

$du = 2x\,dx$

## Trabajo

- Con velocidad

$$d = vt$$



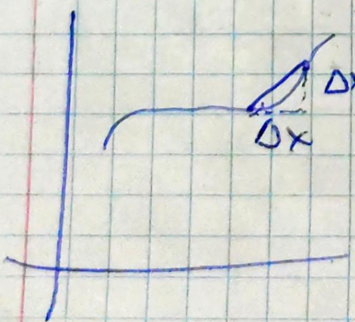

$v(t)\,dt \leftarrow$ distancia

$$d = \int_a^b v(t)\,dt$$

- Ahora $\quad w = Fd$

$$\boxed{w = \int_a^b F(x)\,dx}$$

## Longitud de Curva




$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(\Delta x)^2\left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)}$$

$$= \Delta x \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$$

$$d = \lim_{n \to \infty} \sum \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x = \boxed{\int_a^b \sqrt{1 + (y')^2}\,dx}$$