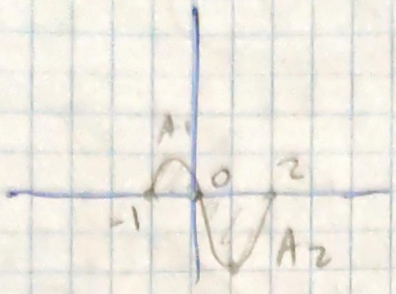
Tarea 11

1.



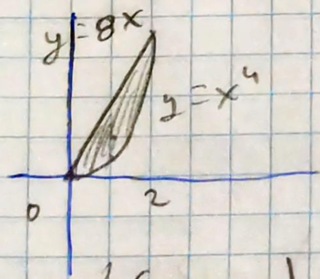
$x^3-x^2-2x = x(x^2-x-2)$
$= x(x+1)(x-2)$

$$A_1=\int_{-1}^{0}(x^3-x^2-2x)\,dx = \frac{x^4}{4}-\frac{x^3}{3}-x^2\Big|_{-1}^{0} = -\left(\frac{1}{4}+\frac{1}{3}-1\right) = -\left(\frac{3+4-12}{12}\right)$$
$$= \frac{5}{12}$$

$$A_2=\int_{0}^{2}(-x^3+x^2+2x)\,dx = -\frac{x^4}{4}+\frac{x^3}{3}+x^2\Big|_{0}^{2} = -\cancel{4}+\frac{8}{3}+\cancel{4}$$

$$A=A_1+A_2=\frac{5}{12}+\frac{8}{3}=\frac{5+32}{12}=\boxed{\frac{37}{12}}$$

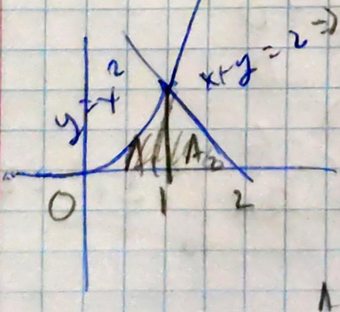
2.



$x^4 = 8x \qquad x=0$
$x^3 = 8 \Rightarrow x=2 \quad -2$

$$A=\int_{0}^{2}(8x-x^4)\,dx = 4x^2-\frac{x^5}{5}\Big|_{0}^{2} = 16-\frac{32}{5}=\frac{48}{5}$$

3. $x+y=2 \Rightarrow y=2-x$



$x^2=0 \Rightarrow x=0$
$x^2=2-x$
$x^2+x-2=0 \Rightarrow (x-1)(x+2)=0 \Rightarrow x=1 \quad -2$
$2-x=0 \Rightarrow x=2$

$$A_1=\int_{0}^{1}x^2\,dx = \frac{x^3}{3}\Big|_{0}^{1} = \frac{1}{3} \qquad A=\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$$

$$A_2=\int_{1}^{2}(2-x)\,dx = 2x-\frac{x^2}{2}\Big|_{1}^{2} = 4-2-2+\frac{1}{2}=\frac{1}{2}$$

4. $12y^2 - 12y^3 = 2y^2 - 2y \Rightarrow y = 0$

$6y - 6y^2 = y - 1$

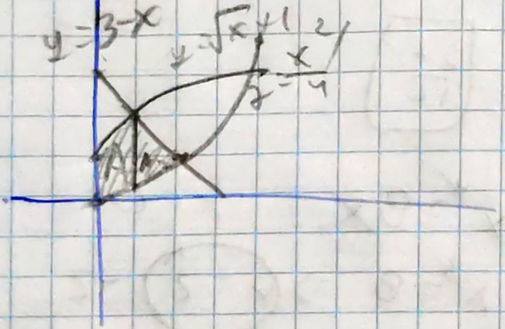
$-6y(y-1) = y-1$ $\quad y=0,\ y=1$

$(y-1)(-6y-1) = 0 \Rightarrow y = 1,\ y = -\frac{1}{6}$

$$\int_0^1 \left(12y^2 - 12y^3 - 2y^2 + 2y\right) dy = \int_0^1 \left(-12y^3 + 10y^2 + 2y\right) dy$$

$$= -3y^4 + \frac{10}{3}y^3 + y^2 \Big|_0^1 = -3 + \frac{10}{3} + 1 = \frac{4}{3}$$

5)



$x = 2\sqrt{y}$

$x^2 = 4y \Rightarrow y = \frac{x^2}{4}$

$\sqrt{x} + 1 = 3 - x$

$-x + \sqrt{x} - 2 = 0 \Rightarrow (\sqrt{x}+2)(\sqrt{x}-1) = 0$

$\Rightarrow \sqrt{x} = 1 \Rightarrow x = 1$

$\frac{x^2}{4} = 3 - x \Rightarrow x^2 = 12 - 4x \Rightarrow x^2 + 4x - 12 = 0$

$(x-2)(x+6) = 0$

$\Rightarrow x = 2,\ -6$

$\cancel{A_1 = \int_0^1 (\sqrt{x} + 1 - 3 + x)\,dx = \int_0^1 (x + \sqrt{x} - 2)\,dx = \frac{x^2}{2} + \frac{2}{3}x^{3/2} - 2x \Big|_0^1} = \frac{1}{2} + \frac{2}{3} - 2$

$$A_1 = \int_0^1 \left(\sqrt{x} + 1 - \frac{x^2}{4}\right) dx = -\frac{x^3}{12} + \frac{2}{3} x^{3/2} + x \Big|_0^1$$

$$= -\frac{1}{12} + \frac{2}{3} + 1 = \frac{-1+8+12}{12} = \frac{19}{12}$$

$$A_2 = \int_1^2 \left(3 - x - \frac{x^2}{4}\right) dx = -\frac{x^3}{12} - \frac{x^2}{2} + 3x \Big|_1^2$$

$$= -\frac{8}{12} - \frac{4}{2} + 6 - \left(-\frac{1}{12} - \frac{1}{2} + 3\right) = -\frac{7}{12} - 2 + 6 + \frac{1}{2} - 3$$

$$= \frac{-7+6+12}{12} = \frac{11}{12}$$

$$A = \frac{11}{12} + \frac{19}{12} = \frac{30}{12} = \frac{15}{6} = \frac{5}{2}$$