Tarea 12

1. $y' = 2xy^2 e^{x^2}$ $\quad y(0) = 1$

$$\frac{dy}{dx} = 2xy^2 e^{x^2}$$

$$\int \frac{dy}{y^2} = \int 2x e^{x^2} dx$$

$u = x^2 \quad du = 2x\,dx$

$$-\frac{1}{y} + c = \int e^u du = ue^u - \int e^u du = ue^u - e^u + c$$

$$= x^2 e^{x^2} - e^{x^2} + c = e^u + c$$

$$-\frac{1}{y} = x^2 e^{x^2} - e^{x^2} + c$$

$$-\frac{1}{1} = -1 + c \Rightarrow c = 0$$

$$y = -\frac{1}{e^u + c} = -\frac{1}{e^{x^2} + c}$$

Si $x = 0 \quad y = 1$

$$1 = -\frac{1}{1+c} \Rightarrow c = -2$$

$$\boxed{y = -\frac{1}{e^{x^2} - 2}}$$

2. $2x^2\,dx - 3\sqrt{y}\,\csc x\,dy = 0$

$2x^2\,dx = 3\sqrt{y}\,\csc x\,dy$

$\int \frac{2x^2}{\csc x}\,dx = \int 3\sqrt{y}\,dy$

$\int 2x^2 \operatorname{sen} x\,dx = \int 3y^{1/2}\,dy$

$-2x^2\cos x + \int 4x\cos x\,dx$

$4x\operatorname{sen} x - \int 4\operatorname{sen} x\,dx$

$= -2x^2\cos x + 4x\operatorname{sen} x + 4\cos x = 2y^{3/2} + C$

$$C = 2y^{3/2} + 2x^2\cos x - 4x\operatorname{sen} x - 4\cos x$$

$$y = \sqrt[3]{C - x^2\cos x + 2x\operatorname{sen} x + 2\cos x)^2}$$

3. $y' = xe^{x-y}\cos y$ implícita

$$\frac{dy}{dx} = xe^{x-y}\cos y = \frac{xe^x \cdot dx}{e^y \operatorname{sen} y}$$

$$\int e^y \operatorname{sen} y \, dy = \int xe^x \, dx$$

$$y\operatorname{sen} y + \int \cos y \, dy \qquad xe^x - \int e^x \, dx$$

$$= y\operatorname{sen} y + \operatorname{sen} y + C = x e^x - e^x + C$$

$C =$

$$\rightarrow -e^y \cos y + \int e^y \cos y \, dy$$

$$e^y \operatorname{sen} y - \int e^y \operatorname{sen} y \, dy$$

$$\int e^y \operatorname{sen} y \, dy = \frac{e^y \operatorname{sen} y - e^y \cos y}{2} = xe^x - e^x + C$$

$$e^y \operatorname{sen} y - e^y \cos y = 2xe^x - 2e^x + C$$

$$\boxed{C = e^y \operatorname{sen} y - e^y \cos y - 2xe^x + 2e^x}$$

4. $y' = \frac{y^2}{x} - \frac{1}{x}$ $y(1) = 1$ explicar la

$$\frac{dy}{dx} = \frac{y^2 - 1}{x}$$

$$\int \frac{dy}{y^2 - 1} = \int \frac{dx}{x}$$

$$\frac{1}{2} \ln \left| \frac{y-1}{y+1} \right| = \ln|x| + C$$

Si $x = 1$ $y = 1$

$\frac{1}{2}$

$$e^{\ln \left| \frac{y-1}{y+1} \right|} = e^{2\ln|x| + C}$$

$$\left| \frac{y-1}{y+1} \right| = e^{2\ln|x|} \cdot C$$

$x = 1$, $y = 1$

$$0 = e^0 \cdot C \Rightarrow C = 0$$

$$\frac{y-1}{y+1} = 0$$

$$y - 1 = 0$$

$$y = 1$$

5. $y' = xy + x - y - 1$, implicit

$$\frac{dy}{dx} = (x-1)(y+1)$$

$$\int \frac{dy}{y+1} = \int (x-1)\,dx$$

$$\ln|y+1| = \frac{x^2}{2} - x + C$$

$$\boxed{2\ln|y+1| - x^2 + 2x = C}$$