3.1.2 De funciones trigonométricas

Cuando tenemos funciones trigonométricas, podemos utilizar identidades que nos permitan realizar alguna sustitución, por ello es importante conocer las derivadas de las funciones trigonométricas.

Cuando tenemos potencias de senos y cosenos de la forma:

$$\int \cos^n x \sin^m x \, dx$$

El proceso para realizar se separa en tres casos, que dependen de los exponentes.

Caso 1.n impar

Si n es impar separar

$$\cos^n x = (\cos^2 x)^k \cos x$$

Utilizamos la identidad:

$$\cos^2 x = 1 - \sin^2 x$$

Utilizar

$$u = \sin x$$

$$du = \cos x \, dx$$

Ejemplo:

$$\int \cos^5 x \sin^6 x \, dx = \int (\cos^2 x)^2 \sin^6 x \cos x \, dx = \int (1 - \sin^2 x)^2 \sin^6 x \cos x \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$\int \cos^5 x \sin^6 x \, dx = \int (1 - u^2)^2 u^6 du = \int (u^4 - 2u^2 + 1) u^6 du = \int (u^{10} - 2u^8 + u^6) du$$

$$\int \cos^5 x \sin^6 x \, dx = \frac{u^{11}}{11} - \frac{2u^9}{9} + \frac{u^7}{7} + c = \frac{(\sin x)^{11}}{11} - \frac{2(\sin x)^9}{9} + \frac{(\sin x)^7}{7} + c$$

$$\int \cos^5 x \sin^6 x \, dx = \frac{1}{11} \sin^{11} x - \frac{2}{9} \sin^9 x + \frac{1}{7} \sin^7 x + c$$

Caso 2.m impar

Si m es impar separar

$$\sin^m x = (\sin^2 x)^l \sin x$$

Utilizamos la identidad:

$$\sin^2 x = 1 - \cos^2 x$$

Utilizar

$$u = \cos x$$

$$du = -\sin x\, dx$$

Ejemplo:

$$\int \cos^4 x \sin^3 x\, dx = \int \cos^4 x\, (\sin^2 x) \sin x\, dx = -\int \cos^4 x\, (1 - \cos^2 x)(-\sin x) dx$$

$$u = \cos x$$

$$du = -\sin x\, dx$$

$$\int \cos^4 x \sin^3 x\, dx = -\int u^4(1-u^2)du = -\int (u^4 - u^6)du = -\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + c = \frac{u^7}{7} - \frac{u^5}{5} + c$$

$$\int \cos^4 x \sin^3 x\, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c = \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + c$$

Caso 3. n y m son pares

Si n y m son pares separar

$$\cos^n x = (\cos^2 x)^k$$

$$\sin^m x = (\sin^2 x)^l$$

Luego utilizar las identidades de ángulo doble:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

Ejemplo:

$$\int \cos^2 x \sin^4 x\, dx = \int (\cos^2 x)(\sin^2 x)^2 dx = \int \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$\int \cos^2 x \sin^4 x\, dx = \int \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1 - 2\cos 2x + \cos^2 2x}{4}\right) dx$$

$$\int \cos^2 x \sin^4 x\, dx = \int \left(\frac{1 - 2\cos 2x + \cos^2 2x + \cos 2x - 2\cos^2 2x + \cos^3 2x}{8}\right) dx$$

$$\int \cos^2 x \sin^4 x\, dx = \int \left(\frac{1 - \cos 2x - \cos^2 2x + \cos^3 2x}{8}\right) dx$$

$$\int \cos^2 x \sin^4 x\, dx = \int \frac{1}{8} dx + \int \frac{-\cos 2x}{8} dx + \int \frac{-\cos^2 2x}{8} dx + \int \frac{\cos^3 2x}{8} dx$$

$$\int \frac{1}{8} dx = \frac{1}{8} x$$

$$\int \frac{-\cos 2x}{8} dx = -\frac{1}{8}\frac{1}{2}\int \cos 2x\, 2dx = -\frac{1}{16}\int \cos u\, du = -\frac{1}{16}\sin 2x$$

$$u = 2x$$

$$du = 2dx$$

$$\int \frac{-\cos^2 2x}{8} dx = -\frac{1}{8}\int \cos^2 2x\, dx = -\frac{1}{8}\int \left(\frac{1+\cos 4x}{2}\right) dx = -\frac{1}{16}\int dx - \frac{1}{16}\frac{1}{4}\int \cos 4x\, 4dx$$

$$u = 4x$$

$$du = 4dx$$

$$\int \frac{-\cos^2 2x}{8} dx = -\frac{1}{16}x - \frac{1}{64}\int \cos u\, du = -\frac{1}{16}x - \frac{1}{64}\sin 4x$$

$$\int \frac{\cos^3 2x}{8} dx = \frac{1}{8}\int \cos^3 2x\, dx = \frac{1}{8}\int (\cos^2 2x)\cos 2x\, dx = \frac{1}{8}\int (1-\sin^2 2x)\cos 2x\, dx$$

$$u = \sin 2x$$

$$du = 2\cos 2x\, dx$$

$$\int \frac{\cos^3 2x}{8} dx = \frac{1}{8}\frac{1}{2}\int (1-\sin^2 2x)\, 2\cos 2x\, dx = \frac{1}{16}\int (1-u^2) du = \frac{1}{16}u - \frac{u^3}{48}$$

$$\int \frac{\cos^3 2x}{8} dx = \frac{1}{16}\sin 2x - \frac{1}{48}\sin^3 2x$$

$$\int \cos^2 x \sin^4 x\, dx = \int \frac{\cos^3 2x}{8} dx + \int \frac{-\cos^2 2x}{8} dx + \int \frac{-\cos 2x}{8} dx + \int \frac{1}{8} dx$$

$$\int \cos^2 x \sin^4 x\, dx = \frac{1}{16}\sin 2x - \frac{1}{48}\sin^3 2x - \frac{1}{16}x - \frac{1}{64}\sin 4x - \frac{1}{16}\sin 2x + \frac{1}{8}x + c$$

$$\int \cos^2 x \sin^4 x\, dx = \frac{1}{16}x - \frac{1}{48}\sin^3 2x - \frac{1}{64}\sin 4x + c$$

Trabajo en Clase 6

Integra:

1. $\int \cos^3 7x \sin 7x\, dx =$

2. $\int \sin^3 x \cos^3 x\, dx =$

3. $\int \sec^2 x \tan^2 x \, dx =$

4. $\int \sin^2 u \, du =$

5. $\int \frac{\sec^3 x}{\tan x} dx =$

Opcional

6. $\int \sec^3 x \, dx =$

Tarea 6

Integra:

1. $\int \sin^5 x \, dx =$

2. $\int \cos^2 x \, dx =$

3. $\int \sec^3 x \tan^3 x \, dx =$

4. $\int \frac{\sin^3 x}{\cos^4 x} dx =$

5. $\int \sin^2 2x \cos^3 2x \, dx =$