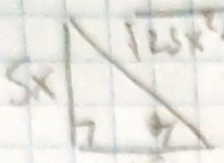
Tarea 7.

1) $\int \frac{100}{25x^2+36}\,dx$



$\tan\theta = \frac{5x}{6} \Rightarrow x = \frac{6}{5}\tan\theta$

$dx = \frac{6}{5}\sec^2\theta\,d\theta$

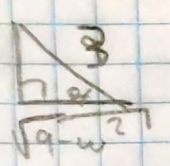
$\sqrt{25x^2+36} = 6\sec\theta$

$25x^2+36 = 36\sec^2\theta$

$\int \frac{100}{36\sec^2\theta}\cdot\frac{6}{5}\sec^2\theta\,d\theta = \int\frac{100}{30}d\theta = \int\frac{10}{3}d\theta = \frac{10}{3}\theta + C$

$= \boxed{\frac{10}{3}\text{Arc tan}\left(\frac{5x}{6}\right) + C}$

2) $\int \frac{\sqrt{9-w^2}}{w^2}\,dw$



$w = 3\operatorname{sen}\theta \Rightarrow w^2 = 9\operatorname{sen}^2\theta$

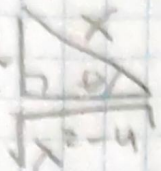
$dw = 3\cos\theta\,d\theta$

$\sqrt{9-w^2} = 3\cos\theta$

$\int \frac{3\cos\theta}{9\operatorname{sen}^2\theta}\cdot 3\cos\theta\,d\theta = \int \frac{\cos^2\theta}{\operatorname{sen}^2\theta}dx = \int(\csc^2\theta - 1)\,d\theta$

$= -\theta - \cot\theta + C = \boxed{-\text{Arcsen}\left(\frac{w}{3}\right) - \frac{\sqrt{9-w^2}}{w} + C}$

3) $\int x\sqrt{x^2-4}^3\,dx$



$x = 2\csc\theta$

$dx = -2\csc\theta\cot\theta\,d\theta$

$\sqrt{x^2-4} = 2\cot\theta$

$\int (2\csc\theta)(2\cot\theta)(-2\csc\theta\cot\theta)\,d\theta$

$= -8\int \csc^2\theta\cot^2\theta\,d\theta =$

$u = \cot\theta \quad du = -\csc^2\theta\,d\theta$

$= 8\int u^2\,du = 8\frac{u^3}{3} + C = 8\frac{\cot^3\theta}{3} + C$

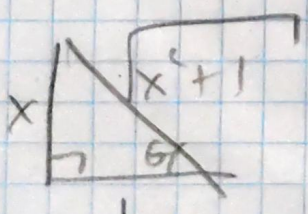
$= \frac{8\cdot\frac{(x^2-4)^{3/2}}{8}}{3} + C = \boxed{\frac{(x^2-4)^{3/2}}{3} + C}$

4) $\int \frac{e^t}{(e^t+1)^{3/2}}\,dt$

$x = e^t \quad dx = e^t\,dt$

$\int \frac{dx}{(x^2+1)^{3/2}} = \int \frac{\sec^2\theta\,d\theta}{\sec^3\theta} = \int \cos\theta\,d\theta = \sin\theta + C$

$= \frac{x}{\sqrt{x^2+1}} + C$



$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\sqrt{x^2+1} = \sec\theta$

$= \boxed{\frac{e^t}{\sqrt{e^t+1}} + C}$

s) $\int \frac{5}{\sqrt{25x^2-9}}\,dx$

$u = 5x \quad du = 5dx$

$\int \frac{du}{\sqrt{u^2-9}} = \ln\left(u+\sqrt{u^2-9}\right)+C$

$= \ln\left(5x+\sqrt{25x^2-9}\right)+C$