Tarea 13.

1. Ortogonales a $y = e^{kx}$

$$D(y = e^{kx})$$

$$y' = ke^{kx} \rightarrow k = \frac{\ln y}{x}$$

$$m_2 = \frac{-1}{y'} = \frac{-1}{ke^{kx}}$$

$$\frac{dy}{dx} = \frac{-1}{ke^{kx}}$$

$$\int dy = \int \frac{-dx}{ke^{kx}} \qquad dy = \frac{-x\,dx}{y \ln y}$$

$$\int y \ln y\, dy = -\int x\, dx$$

$$\frac{y^2}{2}\ln y - \int \frac{y}{2}\, dy$$

$$= \boxed{\frac{y^2}{2}\ln y - \frac{y^2}{4} = -\frac{x^2}{2} + C}$$

$$2y^2(\ln y - 1)$$

$$\frac{y^2}{2}\ln y - \frac{y^2}{4} + \frac{x^2}{2} = C$$

$$\boxed{2y^2 \ln y - y^2 + 2x^2 = C}$$

2. $\frac{dH}{dt} = -k(H - 18)$

$$\int \frac{dH}{H-18} = \int -k\,dt$$

$$\ln|H-18| = -kt + C$$

$$H - 18 = e^{-kt} \cdot e^{C} = Ce^{-kt}$$

Si $t = 0$

$$80 = Ce^{0} \Rightarrow \boxed{C = 80}$$

Si $t = 80$

$$20 = 80e^{-kt}$$

$$\frac{1}{4} = e^{-kt}$$

$$-5k = \ln\left(\frac{1}{4}\right)$$

$$k = -\frac{\ln\left(\frac{1}{4}\right)}{5} = \frac{\ln(4)}{5}$$

$$2 = 80e^{-\frac{\ln 4}{5}t} \qquad \frac{1}{40} = e^{-\frac{\ln 4}{5}t}$$

$$-\ln 40 = -\frac{\ln 4}{5}t$$

$$t = \frac{5\ln 40}{\ln 4} = 13{,}3$$

$$-5$$

$$\boxed{8.3}$$

3. $\frac{dy}{dt} = ky$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + C$

$y = Ce^{kt}$

5: $t = 0 \quad y = 10g$

$10 = C$

5: $t = 24360 \quad y = 5$

$5 = 10e^{24360k}$

$\frac{\ln(\frac{1}{2})}{24360} = k$

$y = 2$

$2 = 10e^{t\frac{\ln(\frac{1}{2})}{24360}}$

$\frac{\ln(0.2)}{\ln(\frac{1}{2})} \cdot 24360 = t = 56562$

4. $\frac{di}{dx} = ki$

$\ln i = kx + c$

$i = ce^{kx}$

$x = 0 \quad i = 100$

$100 = c$

$x = 18 \quad i = 50$

$50 = 100e^{18k}$

$k = \frac{\ln(\frac{1}{2})}{18}$

$i = 10$

$10 = 100e^{\frac{\ln(\frac{1}{2})}{18}t}$

$\frac{\ln(0.1)}{\ln(\frac{1}{2})} \cdot 18 = t = 59$

60

5. $\frac{dy}{dt} = -0,6y$

$\ln y = -0,6t + C$

$y = ce^{-0,6t}$

$t=0 \quad y=100$

$100 = 100$

$t=1$

$y = 100e^{-0.6} = 54.88$ ~~55~~