quiero ser bueno
PONTE A ENTRENAR

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§1 Problemas

Problem 1.1. Let $n > 3$ be a positive integer. Suppose that n children are arranged in a circle, and n coins are distributed between them (some children may have no coins). At every step, a child with at least 2 coins may give 1 coin to each of their immediate neighbors on the right and left. Determine all initial distributions of the coins from which it is possible that, after a finite number of steps, each child has exactly one coin.

Problem 1.2. Show that $r = 2$ is the largest real number r which satisfies the following condition:
If a sequence $a_1, a_2, \ldots$ of positive integers fulfills the inequalities

$$a_n \leq a_{n+2} \leq \sqrt{a_n^2 + ra_{n+1}}$$

for every positive integer n, then there exists a positive integer M such that $a_{n+2} = a_n$ for every $n \geq M$.

Problem 1.3. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(xf(x-y)) + yf(x) = x + y + f(x^2),$$

for all real numbers x and y.

Problem 1.4. We are given an acute triangle ABC. The angle bisector of $\angle BAC$ cuts BC at P. Points D and E lie on segments AB and AC, respectively, so that $BC \parallel DE$. Points K and L lie on segments PD and PE, respectively, so that points A, D, E, K, L are concyclic. Prove that points B, C, K, L are also concyclic.

Problem 1.5. Suppose $a_1 < a_2 < \cdots < a_{2024}$ is an arithmetic sequence of positive integers, and $b_1 < b_2 < \cdots < b_{2024}$ is a geometric sequence of positive integers. Find the maximum possible number of integers that could appear in both sequences, over all possible choices of the two sequences.

§2 Fuentes

- 1.1 IMOSL 2022 C4
- 1.2 APMO 2020/2
- 1.3 Ibero 2020/5