13. Emmanuel B. Hoja Y Z

- Notemos que

$$\frac{1}{\sqrt{1+x^2}-x} = \frac{\sqrt{1+x^2}+x}{(\sqrt{1-x^2}-x)(\sqrt{1-x^2}+x)} = \frac{\sqrt{1+x^2}+x}{1+x^2-x^2}$$

$$= \boxed{\sqrt{1+x^2}+x}$$

Entonces LHS es

$$\boxed{\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{1+z^2}+x+y+z}$$

- Ahora

$$xy+yz+zx=1$$

$$x^2+xy+yz+zx=1+x^2$$

$$\boxed{(x+y)(x+z)=1+x^2}$$

- Por lo que

$$\sqrt{1+x^2}=\sqrt{(x+y)(x+z)} \leq \frac{(x+y)+(x+z)}{2}$$

- Por lo que

$$\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{1+z^2}+x+y+z \leq \frac{(x+y)+(x+z)+(y+x)+(y+z)+(z+x)+(z+y)}{2}+x+y+z$$

$$= \boxed{3(x+y+z)}$$

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Entonces demostrar que

$$3(x+y+z) \leq \frac{1}{xyz}$$ es suficiente

$$\Leftrightarrow (x+y+z)(xyz) \leq \frac{1}{3}$$

$$\Leftrightarrow \frac{x^2yz + xy^2z + xy^2z + xyz^2 + x^2yz + xyz^2}{2} \leq \frac{1}{3}$$

$$\Leftrightarrow \frac{xy(xz+yz) + yz(xy+xz) + zx(xy+zy)}{2} \leq \frac{1}{3}$$

$$\Leftrightarrow \frac{xy(1-xy) + yz(1-yz) + zx(1-zx)}{2} \leq \frac{1}{3}$$

Sea $f(a) = a(1-a) = a-a^2$

entonces $f'(a) = 1-2a \Rightarrow f''(a) = -2 < 0$

Entonces por Jensen

$$\frac{f(xy)+f(yz)+f(zx)}{3} \leq f\left(\frac{xy+yz+zx}{3}\right) = f\left(\frac{1}{3}\right) = \frac{1}{3}\left(\frac{2}{3}\right) = \frac{2}{9}$$

$$\frac{f(xy)+f(yz)+f(zx)}{2} \leq \frac{\frac{2}{9}\cdot 3}{2} = \frac{1}{3}$$ ∎

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$$\frac{1}{\sqrt{1+x^2}-x} = \frac{\sqrt{1+x^2}+x}{1+x^2-x^2} = \sqrt{1+x^2}+x$$

$$\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{1+z^2}+x+y+z \leq \frac{1}{xyz} \; ?$$

$x=y=z=\sqrt{\frac{1}{3}}$

$$3\cdot\frac{2}{\sqrt{3}}+3\cdot\frac{1}{\sqrt{3}}=\frac{9}{\sqrt{3}}=3\sqrt{3}$$

$$\frac{1}{\cancel{3\sqrt{3}}}$$

$$\frac{1}{3}\cdot\sqrt{\frac{1}{3}}$$

$= 3\sqrt{3}$ Igualdad

$$xy+xz+yx+yz+zx+zy=2$$

$$\cancel{(x+y+z)^2} = 2$$

$$x(y+z)+y(x+z)+z(x+y)=2$$

$$1=xy+yz+zx \geq 3\sqrt[3]{x^2y^2z^2}$$

$$\Rightarrow \left(\frac{1}{3}\right)^{\frac{3}{2}} \geq xyz$$

$$3^{\frac{3}{2}} \leq \frac{1}{xyz}$$

$\cancel{xyz}$

xyz

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$$xy \le \frac{(x+y)^2}{2}$$

$$x^2 + xy + yz + zx$$

$$\sqrt{x(x+y+z)+yz}$$

$$x(x+y+z) \le \left(\frac{2x+y+z}{2}\right)^2 = \frac{(S+x)^2}{4}$$

$xy + yz + zx +$

$$x(y+z) + y(z+x) + z(x+y) = 2$$

$$(x+y+z)^2 = \sqrt{2+x^2+y^2+z^2}$$

$$\sqrt{1+x^2} + \sqrt{1+y^2} + \sqrt{1+z^2} + \sqrt{2+x^2+y^2+z^2} \le \frac{\cancel{3\sqrt{3}}}{xyz} \; ?$$

$$(x+y)(x+z) = 1+x^2$$

$$\sqrt{1+x^2} = \sqrt{(x+y)(x+z)} \le \frac{2x+y+z}{2}$$

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$$\sqrt{1+x^2}+\sqrt{1+y^2}+\sqrt{1+z^2}+x+y+z$$

$$\le 3(x+y+z) \overset{?}{\le} \frac{1}{xyz}$$

$$\frac{x+y+z}{\sqrt{3}} \le \sqrt{\frac{x^2+y^2+z^2}{2}}$$

$$\frac{(x+y+z)^2}{9} \le \frac{x^2+y^2+z^2}{2}$$

$$\frac{2+x^2+y^2+z^2}{4} \le \frac{x^2+y^2+z^2}{2}$$

$$\frac{xy+yz}{2} \ge \sqrt{xy^2z} = \sqrt{xyz}\sqrt{y}$$

$$1 \ge \sqrt{xyz}\left(\sqrt{x}+\sqrt{y}+\sqrt{z}\right)$$

$$\frac{1}{\sqrt{x}+\sqrt{y}+\sqrt{z}} \le \frac{1}{\sqrt{xyz}}$$

$$\frac{1}{\sqrt{xyz}} \ge \left(\sqrt{x}+\sqrt{y}+\sqrt{z}\right)$$

$$\frac{10}{4} \quad \frac{5}{2} \quad \left(\frac{3}{2}\right)^2 \quad \frac{9}{4}$$

$$\frac{x+y+z}{3} \ge \frac{\left(\sqrt{x}+\sqrt{y}+\sqrt{z}\right)^2}{9}$$

$$xy+yz+zx \le x^2+y^2+z^2$$

$$\frac{(x+y+z)^2}{3} \le x^2+y^2+z^2$$

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P. D

$$3(x+y+z) \le \frac{1}{xyz}$$

$$(x+y+z)(xyz) \le \frac{1}{3}$$

$x=1, y=1$ $z=?$

$$x^2yz + xy^2z + xyz^2 \le \frac{1}{3}$$

$1 + 2z = 1$

$xy(xz+yz+z^2)$

$z=0$?

$xy(1-xy+z^2) \le \frac{1}{3}$?

$x=1$ $y=z=?$

$(\sqrt{2}-1)^2$

$z^2+2z=1$

$= 3-2\sqrt{2}$

$z^2+2z-1=0$

$$\le \left(\frac{xy+yz+z^2+xz}{2}\right)^2$$

$$z = \frac{-2 \pm \sqrt{8}}{2} = \sqrt{2}-1$$

SPG

$x \ge y \ge z$

$$\le \left(\frac{1+z^2}{2}\right)^2$$

$3(2\sqrt{2}-1)(3-2\sqrt{2}) \le 1$

$3(6\sqrt{2}-8-3+2\sqrt{2})$

$\left(\frac{1+z^2}{2}\right) \le \frac{1}{\sqrt{3}}$

$24\sqrt{2}-33 \le 1$

$24\sqrt{2} \le 34$

$z^2 \le \frac{2-\sqrt{3}}{\sqrt{3}}$

$\sqrt{2} \le \frac{34}{24} = \frac{17}{12}$

$2 \le \frac{289}{144}$ ✓

$$\begin{array}{r} 17 \\ 17 \\ \hline 119 \\ 17 \\ \hline 289 \end{array}$$

1} En mand Sucio 3 | S P.D

$$x^2yz + xy^2z + xyz^2 \le \frac{1}{3}$$

$$\sum xy\left(\frac{xz+yz+z^2}{3}\right) \qquad xy(1-xy)$$

$$\cancel{xy(1-xyz)+x}$$

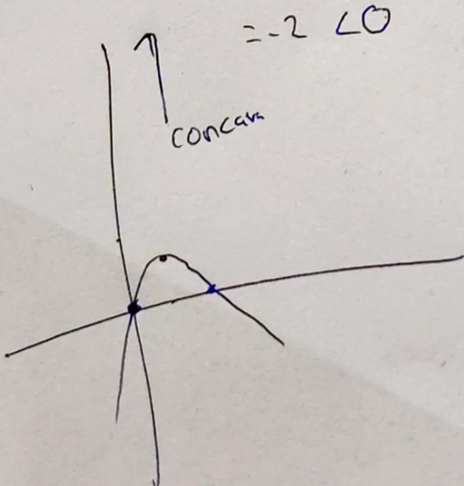
$$\frac{xy(1-xy)+xz(1-xz)+yz(1-yz)}{2} \le \frac{1}{3} \; ?$$

$$f(a) = a(1-a) = a-a^2 \qquad xy(1-xy) \le \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$f'(a) = 1-2a$$

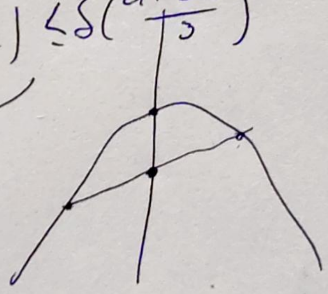
$$f''(a) = 0-2 = -2 < 0 \qquad \le \cancel{\frac{3/4}{2}}$$

concava



Jensen

$$\frac{f(a)+f(b)+f(c)}{3} \le f\left(\frac{a+b+c}{3}\right)$$



$$= f\left(\frac{1}{3}\right) = \frac{1}{3}\left(\frac{2}{3}\right) = \frac{2}{9}$$

$$\frac{f(a)+f(b)+f(c)}{2} \le \frac{7}{9}$$