$P(x,y)$

$$\to f\left(x + \frac{f(xy)}{x}\right) = f(xy)\, f\left(y + \frac{1}{y}\right)$$

$x = y = 1$

$P(1,1)$

$$f(f(1)+1) = f(1)\, f(2)$$

$P\left(x, \frac{1}{x}\right)$

$$f(x + x f(1)) = f(1)\, f\left(x + \frac{1}{x}\right)$$

$P(xy, x)$

$$f\left(y + \frac{f(xy)}{y}\right) = f(xy)\, f\left(x + \frac{1}{x}\right)$$

$$\Rightarrow f(xy) \cdot \frac{f\left(y + \frac{f(xy)}{y}\right)}{f\left(x + \frac{1}{x}\right)} = \frac{f\left(x + \frac{f(xy)}{x}\right)}{f\left(y + \frac{1}{y}\right)}$$

$$\boxed{f\left(y + \frac{f(xy)}{y}\right) f\left(y + \frac{1}{y}\right) = f\left(x + \frac{f(xy)}{x}\right) \cdot f\left(x + \frac{1}{x}\right)}$$

$P(2, 1)$

$$f\left(2 + \frac{f(2)}{2}\right) = f(2)^2$$

$P(x, 1)$

$$f\left(x + \frac{f(x)}{\cancel{x}}\right) = f(x)\, f(2)$$

$$\Rightarrow \quad f(2) = \frac{f\left(x + \frac{f(x)}{x}\right)}{f(x)} \qquad \Rightarrow \boxed{f\left(x + \frac{f(x)}{x}\right) = f(x) \cdot c}$$

$$P\left(x + \frac{f(x)}{x}, 1\right)$$

$$p\cancel{f}\left(\begin{array}{c} x + \frac{f(x)}{x} \\ y \quad y \end{array} + \frac{f(x) \cdot c}{x + \frac{f(x)}{x}}\right) = f(x) \cdot c^2$$

$$f(f(y) + 1) = f(y)\, f\left(y + \frac{1}{y}\right)$$

$$a = f(xy)$$

⊛

$$\frac{f(x + \frac{a}{x})}{\cancel{a}} = a\, f(y + \frac{1}{y})$$

Si existe $\cancel{y \neq x}$ con

$f(xy) = x$?

y, con $f(xy) = x$

↳ Eso es si
existe $f(a)$
con $f(a) = x$

entonces

$P(x, y)$

$$f(x + 1) = \cancel{f(x)}\; x\, f(y + \frac{1}{y})$$

$$\left(x + \frac{f(\cancel{0}x)}{x}\right) \neq 2$$

$$x^2 + f(\cancel{0}x) = 2x \qquad \delta \quad \text{Existe?}$$

$$x^2 - 2x + f(x) = 0$$

$$\text{Si} \quad f(1) = 1$$
$$P(1,1)$$
$$f(2) = f(1) f(2) \Rightarrow f(1) = 1 \qquad x$$

- Si existe a
  con $\quad f(a) = 1$

$$P\left(x, \frac{a}{x}\right)$$

$$f\left(x + \frac{1}{x}\right) = \cancel{f(x)} \boxed{f\left(\frac{a}{x} + \frac{x}{a}\right)}$$

$$P(x,y)$$

$$\left[f\left(x + \frac{f(x^2)}{x}\right) = f(x^2) \, f\left(x + \frac{1}{x}\right) \right]$$

$$a = y + \frac{1}{y} \qquad\qquad y^2 - ay + 1 = 0$$

$$\boxed{\frac{a \pm \sqrt{a^2 - 4}}{2}} \qquad\qquad \text{para } a \geq 2$$