P2 Emmanuel B. Hoja 1/5

(·) Notemos que

$$a_{n+1} = \frac{a_n^2}{a_n-1} = a_n+1 + \frac{1}{a_n-1} > a_n + 1$$

(·) Entonces $a_1 = 850$, $a_2 > a_1+1 = 851$, ...

Inductivamente si $a_k > 850 + (k-1)$

Entonces

$$a_{k+1} > a_k + 1 > 850 + (k-1) + 1 = 850 + k$$
$$= 850 + (k+1) - 1 \quad \square$$

(·) Así que si $\lfloor a_x \rfloor = 2024$

$\Rightarrow \quad 2025 > a_x > 850 + x - 1$

$\Rightarrow \quad 1176 > x$

(·) Entonces para $n < 1176$

tenemos que $a_n < a_1 + n + 1$

Porque

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~~$a_n = a_1 + (n-1)$~~

Notemos primero que $a_n = a_1 + (n-1) + \sum_{i=1}^{n-1} \frac{1}{a_i - 1}$

Inductivamente

$$a_1 = a_1 + 0 + 0 \checkmark$$

$$a_2 = a_1 + 1 + \frac{1}{a_1 - 1} \checkmark$$

Si $a_k = a_1 + (k-1) + \sum_{i=1}^{k-1} \frac{1}{a_i - 1}$

Entonces

$$a_{k+1} = a_k + 1 + \frac{1}{a_k - 1} = a_1 + (k-1) + 1 + \frac{1}{a_k - 1} + \sum_{i=1}^{k-1} \frac{1}{a_i - 1}$$

$$= a_1 + k + \sum_{i=1}^{k} \frac{1}{a_i - 1} \checkmark$$

Se cumple la inducción

Entonces

$$a_n = a_1 + (n-1) + \sum_{i=1}^{n-1} \frac{1}{a_i - 1}$$

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Como $a_i \geq a_1 = 850 \quad \forall i$

$$\sum_{i=1}^{n-1} \frac{1}{a_i - 1} \leq \frac{n-1}{849} \leq \frac{1176}{849} < 2$$

$\Rightarrow a_n < a_1 + (n-1) + 2 = a_1 + n + 1$

~~Ahora~~ Entonces $a_n < 851 + n$

$\boxed{\cdot}$ Por CS ó AM-HM

$$\sum_{i=1}^{n-1} \frac{1}{a_i - 1} \geq \frac{(n-1)^2}{\sum_{i=1}^{n-1} a_i - 1}$$

Pero

$$\sum_{i=1}^{n-1} a_i - 1 = (850+1) + (850+2) + \dots + (850 + (n-1))$$

$$= 850(n-1) + \frac{(n-1)(n)}{2}$$

$$= (n-1)\left(850 + \frac{n}{2}\right) \quad \square$$

Entonces

$$\frac{(n-1)^2}{\sum_{i=1}^{n-1} a_i - 1} \geq \frac{(n-1)^2}{(n-1)(850+\frac{n}{2})} = \frac{n-1}{850+\frac{n}{2}}$$

$\boxed{\cdot}$ Como sabemos que $x < 1176$

Si $x = 1175$

$$a_x = 850 + x - 1 + \sum_{i=1}^{x-1} \frac{1}{a_i - 1} \geq 850 + 1174 + \frac{1174}{850 + \frac{1175}{2}}$$

$$\geq 2024$$

~~Porque~~ Entonces $x = 1175$ funciona que $a_x \geq 2024$

Porque

$a_{1175} \geq 2024$ y

$$a_{1175} = 850 + 1174 + \sum_{i=1}^{x-1} \frac{1}{a_i - 1} \leq 850 + 1174 + \frac{1174}{850}$$

Y si $x \leq 1173 \Rightarrow a_x < 850 + 1173 + 1 = 2024$!

Entonces $x = 1174$ ó 1175

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Si $x = 1175$

Entonces

$a_1, a_2, \dots, a_{1174}$

Son 1174 cosas distintas

cada una tiene $\lfloor a_i \rfloor$ distinto

(porque $a_n > a_{n-1} + 1 \Rightarrow \lfloor a_n \rfloor \geq \lfloor a_{n-1} + 1 \rfloor = \lfloor a_n \rfloor + 1$)

Entonces

$\lceil a_i \rceil$ son valores entre

~~850, 851, ..., 2024~~

849, 850, ..., 2024

así que

$$a_{1174} = 2024 + \sum_{i=1}^{1173} \frac{1}{a_i - 1} > 2024 + \frac{1}{850} + \frac{1}{851} + \dots + \frac{1}{2024}$$

Pero $\sum_{i=850}^{2024} \frac{1}{i} \approx \ln(2024) - \ln(849) = \ln\left(\frac{2024}{849}\right) > 1$

Entonces $a_{1175} > 2025$

y $a_{1174} > 2024$ (Argumento similar a todo eso)

$a_{1174} \leq 2023 + \frac{1173}{850} < 2025 \Rightarrow \lfloor a_{1174} \rfloor = 2024$

by

P2 Emmanuel Score 1/10

$a_1 = 850$

$$a_2 = \frac{a_1^2}{a_1 - 1} = a_1 + \frac{a_1^2 - a_1^2 + a_1}{}$$

$$= a_1 + \frac{a_1}{a_1 - 1}$$

$$= a_1 + 1 + \frac{1}{a_1 - 1}$$

$$a_3 = a_2 + \frac{a_2}{a_2 - 1} = a_2 + \frac{a_1 + \frac{a_1}{a_1 - 1}}{a_1 + \frac{a_1}{a_1 - 1} - 1}$$

$$= a_2 + \frac{\frac{a_1^2}{\cancel{a_1 - 1}}}{\frac{a_1^2 - a_1 + 1}{\cancel{a_1 - 1}}}$$

$$a_3 = \frac{a_2^2}{a_2 - 1} = \frac{\frac{a_1^4}{(a_1 - 1)^2}}{\frac{a_1^2 - a_1 + 1}{\cancel{a_1 - 1}}}$$

$$a_{n+1} = a_n \left(\frac{a_n}{a_n - 1} \right) \le \frac{\left(a_n + \frac{a_n}{a_n - 1}\right)^2}{4}$$

$$ab \le \frac{(a+b)^2}{4}$$

$$\frac{a_{n+1}}{a_{n+1} - 1} = \frac{a_n^2}{a_n^2 - a_n + 1}$$

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$$\begin{array}{r} 2026 \\ -\ 850 \\ \hline 1176 \end{array}$$

$$a_3 = a_2 + 1 + \frac{1}{a_2 - 1}$$

$$= a_1 + 1 + 1 + \frac{1}{a_2 - 1} + \frac{1}{a_1 - 1}$$

$$= a_1 + 2 + \frac{1}{a_2 - 1} + \frac{1}{a_1 - 1}$$

$$a_n = \cancel{a_{n-1} +}\ a_1 + (n-1) + \sum_{i=1}^{n-1} \frac{1}{a_i - 1} \leq a_1 + (n-1)\frac{850}{849}$$

$$= 850\left(1 + \frac{n-1}{849}\right)$$

$a_i - 1 \geq 849$

$$\sum_{i=1}^{n-1} \frac{1}{a_i - 1} \leq \frac{n-1}{849}$$

$$2024 \leq a_n \leq 850\left(1 + \frac{n-1}{849}\right)$$

$$a_1 + (n-1) + \sum 2025 > a_n = a_1 + (n-1) + \sum_{i=1}^{n-1} \frac{1}{a_i - 1} > a_1 + (n-1)$$

$$\Rightarrow \quad 2025 > a_1 + (n-1)$$

$$n < 2026 - a_1 = 2026 - 850 = 1176$$

n2 Emmanuel Sect 31 ∠0

$$2024 \le 850\left(1+\frac{n-1}{849}\right) < \cancel{850\left(1+\frac{1176}{849}\right)}$$

$$\frac{1174}{\cancel{2024}\ 850} < \frac{n-1}{849}$$

$$\boxed{1174 \cdot \frac{849}{850} < n-1 < 1175}$$

$$1173 \,\big|\, 850$$
$$2023$$

Si $n-1 = 1173$

$$1174 \cdot \frac{849}{850} < 1173$$

$$\frac{1174}{1173} < \frac{850}{849}$$

$$\cancel{1}+\frac{1}{1173} < \cancel{1}+\frac{1}{849} \checkmark$$

P2 Enmenel Saio 4/20

Si

$n = 1174$

$$a_n = 850 + 1174 + \sum_{i=1}^{1173} \frac{1}{a_i - 1}$$

$$\begin{array}{r} 1173 \\ - 849 \\ \hline \end{array}$$

$= 2024 +$

$a_i > a_1 + i - 1$

Si $n - 1 \le 1173$

$$a_1 + (n-1)\frac{850}{849} \le \underbrace{850 + 1173}_{2024 + \text{algo}} + \frac{1173}{849}$$

$$\sum_{i=1}^{n-1} \frac{1}{a_i - 1} > \frac{n-1}{850+n-2} = 1 - \frac{850+n-2-850+1}{850+n-2}$$

$$= 1 - \frac{849}{850+n-2}$$

Si $n = 1173$

$$\Rightarrow a_n \ge \underbrace{850 + 1173 + 1}_{2024} - \frac{849}{850+n-2}$$

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$$\sum_{i=1}^{n-1} \frac{1}{a_i - 1} \geq \frac{(n-1)^2}{\sum_{i=1}^{n-1} a_i - 1}$$

$$\sum_{i=1}^{n-1} a_i < \cancel{850 + 85} \quad 850 + 851 + 852 + \ldots + (850 + (n-2))$$

$$= 850(n-1) + \frac{(n-2)(n-1)}{2}$$

$$= (n-1)\left(850 + \frac{n-2}{2}\right)$$

$$\geq \frac{(n-1)}{850 + \frac{n-2}{2}}$$

So $n-1 \geq 1173$

$$a_n = 850 + n - 1 + \sum_{i=1}^{n-1} \frac{1}{a_i - 1}$$

$$\geq 850 + 1173 + \frac{1173}{850 + \frac{1172}{2}} = 2023 + \frac{1173}{1336} \quad \because C$$

$$2 \overline{)1172} = 586$$

17
12

$$586 + 850 = 1336$$

P2 Seria 6/10

$n-1 \geq 1174$

$a_{n} \geq 850 + 1174 + \frac{1174}{850 + \frac{1173}{2}}$

2024 F

$a_n \geq a_{n-1} + 1 \geq 2024 + 1 = 2025$!

$\Rightarrow$ $n-1 \leq 1173$

$\{n = 1174\}$

$a_n = 850 + 1173 + \sum_{i=1}^{1173} \frac{1}{a_i - 1} \leq 850 + 1173 + \frac{1173}{850}$

$= 2023 + \frac{1173}{850} < 2025$ ✓

P2 Suceo 71 10

$$\sum_{i=1}^{n-1} \frac{1}{a_i-1} \geq \frac{(n-1)^2}{\sum a_i - 1}$$

$$\sum_{i=1}^{n-1} a_i - 1 < \cancel{}$$

A ver pruebo $n \leq 1175$

$$a_n = a_1 + n - 1 + \sum_{i=1}^{n-1} \frac{1}{a_i - 1} \leq a_1 + n + 1 = 851 + n$$

y $\displaystyle\sum_{i=1}^{n-1} \frac{1}{a_i-1} \leq \frac{n-1}{850} \leq \frac{1175}{850} < 2$

$$\Rightarrow a_1 + a_2 + \dots + a_n \leq (851+1)(851+2) + \dots + (851+n)$$

$$= 851n + \frac{n(n+1)}{2} = n\left(851 + \frac{n+1}{2}\right)$$

$$\sum_{i=1}^{n-1} a_i - 1 \leq (n-1)\left(850 + \frac{n}{2}\right)$$

$$\frac{(n-1)}{850 + \frac{n}{2}}$$

$n = 1174$

P2 Emerald Sew 9) 10

-5 ; $a_{1174} < 2024$

$a_{1175} \geq 2025$?

$x = 1175$

$\Rightarrow$ a_{1174} >.

950 + 1174

2024 $\sum_{i=1}$

$\frac{1174}{2023}$

$a_n = a_{n-1}^{+}$

$a_{n+1} = a_n + 1 + \frac{1}{a_n - 1} < a_n + 2$

a_1

$a_{n+1} - 2 \leq a_n \geq a_{n+1} - 1$

$a_{1174} < a_{1174} \leq 2024$

$a_{1175} - 1 \leq 2024$

$a_{1174} \leq a_{1175}$

$a_{n+1} \geq a_n + 1$

$a_{1175} - 1$

S. $x = 1174$

950

1

P 2 Ennunciado Semi 9/10

$a_{1174} \leq 2023 +$

$$\frac{1}{850} + \frac{1}{851} + \dots + \frac{1}{2023}$$

a_{1175}

tomen todos pares diferentes

$$\underbrace{850, 851, \dots, 2024}_{1175}$$

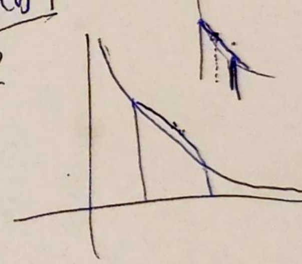
$$\underbrace{\frac{1}{850} + \frac{1}{851} + \dots + \frac{1}{2024}}_{1175 \text{ cosas}} > ?$$

~~Si el~~ ~~expected~~

es

$$\frac{f(a)+f(b)}{2} \geq f\left(\frac{a+b}{2}\right)$$

1 2



$$\frac{1}{a} + \frac{1}{b} =$$

$\cancel{f(850)} + f($

$$f(850+851+\dots+2024) \geq 1175\, f\left(\frac{850+\dots+2024}{1175}\right)$$

P2 Fundamental Serii 10/10

$$\frac{2024 \cdot 2025}{2}$$

$$\begin{array}{r} 2025 \\ 1012 \\ \hline 4050 \\ 2025 \\ 20250 \\ \hline 2049300 \end{array}$$

$$\frac{850 \cdot 851}{2}$$

$$\begin{array}{r} 851 \\ 425 \\ \hline 4255 \\ 1702 \\ 3404 \\ \hline 361675 \end{array}$$

$$\begin{array}{r} 2049300 \\ -\ 361675 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 1175 \\ 1175 \\ \hline 25875 \\ 8225 \\ 1175 \\ 1175 \\ \hline 1380625 \end{array}$$

C

$$\ln\left(\frac{2024}{850}\right) > 1$$