P4 Emmanuel Hoja 1/8

Tomamos $m=2$

$\Rightarrow \quad a_n + a_2 - 1 \mid na_n - 2a_2 + 2a_2 - 1$

$\Rightarrow \quad a_n + a_2 - 1 \mid na_n - 1$

Sea $\quad c = a_2 - 1$

$\Rightarrow \quad a_n + c \mid na_n - 1 - n(a_n + c)$

$\Rightarrow \quad a_n + c \mid -1 - nc$

$\Rightarrow \quad a_n + c \mid nc + 1$

$\boxed{\cdot}$ Vamos a probar que $a_n = \cancel{bc+} \; c(n-1)+1$

⊙ Vamos a hacer inducción para abajo.

Si $\quad a_{n+1} = cn + 1 \quad$ para $\quad n \geq 3$

entonces $\quad a_n = c(n-1) + 1 \quad$ se cumple

P4 General Hoja 2/8

Como

$$a_n + c \mid nc+1 = a_{n+1}$$

$$\Rightarrow a_n = \frac{a_{n+1}}{d} - c \quad \text{con } d \text{ un divisor de } a_{n+1}$$

Tenemos que tomando $m = n+1$

$$a_n + a_{n+1} - 1 \mid na_n - (n+1)a_{n+1} + 2a_{n+1} - 1$$

Pero

$$na_n - (n+1)a_{n+1} + 2a_{n+1} - 1$$

$$= n \cdot \frac{a_{n+1}}{d} - nc + a_{n+1}(1-n) - 1$$

$$= a_{n+1}\left(\frac{n}{d} - 1 + n\right) - (nc+1)$$

$$= a_{n+1}\left(\frac{n}{d} - 1 + n\right) - a_{n+1}$$

$$= a_{n+1}\left(\frac{n}{d} + n\right)$$

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Entonces

$$a_n + a_{n+1} - 1 \mid a_{n+1}\left(\frac{n}{d} \cancel{+} n\right)$$

y $a_n + a_{n+1} - 1 = \frac{a_{n+1}}{d} + a_{n+1} \underbrace{-c-1}_{\text{quitamos eso}} \geq \frac{a_{n+1}(d+1)}{d}$

entonces

$$\frac{a_{n+1}}{d} + a_{n+1} - c - 1 \;\Big|\; a_{n+1}\left(n - \frac{n}{d}\right)$$

Si $d > 1$ entonces $n - \frac{n}{d} > 0$ y

$a_{n+1}\left(n - \frac{n}{d}\right) > 0$, ~~además~~

Entonces

$$\frac{a_{n+1}\left(n - \frac{n}{d}\right)}{\frac{a_{n+1}}{d} + a_{n+1} - c - 1} \geq \frac{\cancel{a_{n+1}}\left(\frac{n(d-1)}{\cancel{d}}\right)}{\frac{\cancel{a_{n+1}}(d+1)}{\cancel{d}}} = \frac{n(d-1)}{d+1}$$

pero $\frac{n(d-1)}{d+1} = n\left(1-\frac{2}{d+1}\right) = n - \frac{2}{d+1}$

y $n - \frac{2}{d+1} > n-1$

Entonces

porque es entero

$$\frac{a_{n+1}\left(n - \frac{n}{d}\right)}{\frac{a_{n+1}}{d} + a_{n+1} - c - 1} \geq n$$

esto es por. porque $a_{n+1} \geq cn+1 > c+1$

$$\Leftrightarrow a_{n+1} \cdot \cancel{n} \cdot \frac{d-1}{d} \geq \cancel{n}\left(\frac{a_{n+1}}{d} + a_{n+1} - c - 1\right)$$

$$\Leftrightarrow a_{n+1} \cdot \frac{d-1}{d} \geq a_{n+1}\left(\frac{d+1}{d}\right) - c - 1$$

$$\Leftrightarrow a_{n+1} \cdot \frac{-2}{d} \geq -c-1$$

$$\Leftrightarrow 2 \cdot \frac{a_{n+1}}{d} \leq c+1$$

P4 Funerall Hoja 5/8

$\iff 2\cdot(a_n+c) \le c+1$

$\iff 2a_n+2c \le c+1$

$\iff 2a_n \le 1-c$!

Y eso es imposible porque

$1-c \le 1$ y $2a_n \ge 2$

Entonces $d=1$ y ~~$a_n=1+c(n-1$~~

$a_n = \frac{a_{n+1}}{1} - c = 1+cn-c = 1+c(n-1)$ $\square$

~~Además~~

Ahora para todo x existe un y con $a_y = 1+c(y-1)$ porque por Dirichlet existen infinitos primos de la forma $cn+1$

te tomas un primo $p=cr+1$ ~~[illegible]~~ con r más grande que x con $p=cr+1$

↑ son infinitos

P4 Emmanuel B. Hoja 6/8

Entonces ~~P=>~~

$$a_r + c \mid cr+1 = p$$

$\Rightarrow a_r + c = 1$ ó $a_r + c = p$

Si $c=0$

$\Rightarrow a_r = 1 = 1 + c(r-1)$ ✓

$a_r = p - c$

$= cr + 1 - c$

$= c(r-1)+1$ ✓

En ambos casos

$a_r = 1 + c(r-1)$ con $r > x$

entonces y existe.

∴ Inductivamente todo x con $x \geq 3$ es de la forma

$a_x = 1 + c(x-1)$ para alguna constante c

P4 General Hoja 7/8

(a) $x=2$ es de la forma

$a_2 = c+1 = c(2-1)+1$

porque $c = a_2 - 1$

(b) $x=1$ cumple $a_1 = 1$

porque $a_1 + c \mid c+1$

$\Rightarrow a_1 + \cancel{c} \leq \cancel{c} + 1$

$\Rightarrow a_1 \leq 1 \Rightarrow a_1 = 1$

y $a_1 = 1 + c(1-1) = 1$ ✓

Entonces $a_n = 1 + c(n-1)$ es la única posible y cumple por

$a_n + a_m - 1 = 1 + c(n-1) + c(m-1) + 1 - 1$

$= c(n+m-2) + 1$

p4 Emmanuel Hoja 8/8

Entonces

$na_n - ma_m + 2a_m - 1$

$= c(n^2-n) + n - c(m^2-m) - m + c(2m-2) + 2 - 1$

$= c(n^2 - m^2 - n + 3m - 2) + n - m + 1$

$= c(n+m-2)(n-m+1) + n - m + 1$

$= (n-m+1)(c(n+m-2)+1)$

$= (n-m+1)(a_n + a_m - 1)$

$\Rightarrow \quad a_n + a_m - 1 \mid na_n - ma_m + 2a_m + 1$

Demostrando que

$a_n = 1 + c(n-1)$ con $c \in \mathbb{Z}$

y $c \geq 0$ $\leftarrow$ (porque $c = a_2 - 1 \geq 1 - 1 = 0$)

es la unica solución

Emmanuel Buenrostro P4 Sucio 1/14

$$a_n + a_m - 1 \mid na_n - ma_m + 2a_m - 1$$

$\forall n, m$

$n = m$

$$2a_n - 1 \mid 2a_n - 1$$

me cortan en diciembre

$m = 2$

$$a_n + \underbrace{a_2 - 1}_{c} \mid na_n - 1$$

$$na_n - 1 - na_n = nc$$

$$nc + 1$$

$$a_n + c \mid nc - 1$$

≥ 0

$$a_1 + c \mid c - 1$$

si $c > 1 \Rightarrow c - 1 > 1$

$a_1 + c \leq c - 1$

$a_1 \leq -1$!

y

$\Rightarrow \boxed{c = 1}$

y $a_2 = 2$

$3X$

ó $c = 0$

Emmanuel Buenrostro P4 sucio 21/14

$n=2$

$a_m+1 \mid 3 - ma_m + 2a_m$

$= 3 - a_m(m-2)$

$a_m+1 \mid a_m(m-2) - 3 \quad - a_m(m-2) - (m-2)$

$a_m+1 \mid m+1$

$a_m+1 \leq m+1$

$a_m \leq m \Rightarrow a_1 \leq 1$ y $a_1 = 1$

$m=3$

$m=1$

$a_n + a_3 - 1 \mid na_n - a_3 - 1$

$a_n \mid na_n$

$a_n+1 \mid n-1$

$n=1$

$a_m \mid (m-2)a_m$

y $a_n \mid -1$

$a_1 + c \mid c-1$

$c=0$

$\Rightarrow a_1 = 1$ $\quad a_n = +1$

$a_1 \mid -1$

$\Rightarrow$ Unica sol $\{a_i = 1\}$

pn Emmanuel B. Sucio 3/14

$a_n + c \mid nc + 1$

si $c=0$

$a_n \mid 1 \Rightarrow a_n = 1$

$n=1$

$c + a_1 \mid c + 1$

si $c=1 \Rightarrow a_2 = 2$

$a_n + 1 \mid n + 1$

$\Rightarrow a_n \leq n$

$\Rightarrow \cancel{c} + a_1 \leq \cancel{c} + 1$

$\Rightarrow a_1 = 1$

si $c = 2 \Rightarrow a_2 = 3$

$a_n + 2 \mid 2n + 1$

$n=2$

$a_3 + 2 \mid 7$

$2c + 1 \mid 2c + 1$

$a_3 = 5$

$c + a_n \mid nc + 1 - nc - na_n$

$a_4 + 2 \mid 9$

$\Rightarrow$

$a_n + c \mid na_n - 1$

$a_n = (n-1) \cancel{x} + 1$

$na_n = (n^2 - n)x + n$

$-ma_m = -(m^2 - m)x - m$

$+2a_m = (2m - 2)x + 2$

-1

$x(n^2 - n - m^2 + m - 2m + 2) + n - m + 1$

$x(n + m - 2) + 1$

si $x=1$ $\quad n + m - 1$ $\quad n^2 - m^2 - 2m + 3$

P4 Emmanuel B. Sucio 4/14

$$\begin{array}{c|ccc} & n & -m & 1 \\ \hline n & n^2 & -nm & n \\ m & nm & -m^2 & m \\ -1 & -n & m & -1 \end{array}$$

$2\cdot 2 - 3\cdot 3 + 2\cdot 3 - 1 =$

$4 - 9 + 6 - 1 = 0$

$3\cdot 3 - 2\cdot 2 + 2 - 2 - 1 = 8$

$2 + 3 - 1 = 4$

$4\cdot 4 - 3\cdot 3 + 2\cdot 3 - 1$

$16 - 9 + 6 - 1 = 12$

$a_{n+1} \mid n+1$

$a_3 + 1 \mid 4 \Rightarrow a_3 = 1, 3$

$a_4 + 1 \mid 5 \Rightarrow a_4 = 4$

Serviría el caso $c = 1$

$4\cdot 4 - 3\cdot 1 + 2\cdot 1 - 1$

$16 - 3 + 2 - 1 = 14$

pu Emmanuel B, Sucio S) 14

$c = 2$

$a_2 = 3$

$a_3 = 5$

$a_4 + 2 \mid a$ $\qquad a_4 = 1$

$a_4 = 7$

$a_n + c \mid cn + 1$

Si $cn+1$ es primo

$a_n + c = cn + 1$

$a_n = c(n-1) + 1$

Si tenes n, m con $a_n = c(n-1) + 1$

y $a_m = c(m-1) + 1$

$n a_n - m a_m + 2a_m \cancel{+} 1$

$= c(n^2 - n) + n - m(m^2 - m) - m + c(2m-2) + 2 - 1$

$= c(n^2 \cancel{-} m^2 - n + 3m - 2) + n - m + 1$

$a_n + a_m - 1 = c(n + m - 2) + 1$

P4 Emmanuel B, Sucio 6/14

$$\begin{array}{c|ccc} & n & -m & 1 \\ \hline n & n^2 & -nm & n \\ m & nm & -m^2 & m \\ -2 & -2n & +2m & -2 \end{array}$$

$$c(n+m-2)(n-m+1) + n - m + 1$$

$$c(n+m-2) + 1 \quad \uparrow \text{divide}$$

$$c(n+m-2)+1 \mid c(n+m-2)(n-m+1) + n - m + 1 - c(n+m-2)(n-m+1)$$
$$- (n-m+1)$$

$$= \cancel{n-m+1} \quad 0$$

si jala

$$a_n = 1 + c(n-1) \quad \forall c$$

(lo multiplico por ~~n+m-2~~ $n-m+1$

p4 Emanuel Bucarest Sucro t p4

$a_m = c(m-1)+1$ (para $m = p - c$)
primo

$n a_n \cancel{+} m a_m + 2a_m - 1$

$= n a_n - c(m^2 - m) - m + c(2m-2) + 1$

$= c(-m^2 + 3m - 2) - m + 1$ then

$a_n + a_m - 1 = a_n + c(m-1)$

$1 + c(n + m - 2)$

$$\begin{array}{c|cc} & -m & 2 \\ m & -m^2 & 2m \\ -1 & m & -2 \end{array}$$

$+a_n \cdot n$

$a_n + c(m-1) \mid c(-m^2 + 3m - 2) - m + 1 - c(m-1)(-m+2) - a_n(-m+2)$

$= a_n(-m+2) + a_n \cdot n - m + 1$

$= a_n(n + m - 2) - m + 1$

$a_n(n+m-2) - m + 1 - a_n(n+m-2) - c(m-1)(n+m-2)$

$= -m+1 - c(m-1)(n+m-2)$

$= m-1(-1 - c(n+m-2))$

$m=2$ P4 Emanuel B. Suciu 8/14

$a_n + c \mid n a_n - 1$

$a_n + c \mid 1 + cn$

$a_2 = c+1$

$a_n + c \mid 2c+2 - n a_n + 2a_n - 1$

$2c+1 + a_n(2-n)$

$m = n-1$ n prime no or

is forme $1+c(n-1)$

$a_n + c(n-2) \mid (n-2)(-1-c(2n-3))$

$\frac{1+cn}{d} - c$

$\frac{1+c(n-d)}{d}$

$\frac{1+c(n-d)+c(nd-2d)}{d} = \frac{1+c(n(d+1)-3d)}{d}$

P4 Emunciel B Sein 4/14

Si $c=2$

$a_3+2 \mid 7 \Rightarrow a_3=5$

$a_4+2 \mid 9 \quad a_4=1?$

$4\cdot 1-5\cdot 3+2\cdot 5-1$

$4-15+10-1=-2$

$a_3+a_4-1=5$

Si $a_n+c \neq cn+1$

$\Rightarrow a_n+c \leq \frac{cn+1}{2}$

$a_{n-1}=1+c(n-2)$

$a_n \leq \frac{c(n-2)+1}{2} = \frac{a_{n-1}}{2}$

negativo

$n\cdot a_n-(n-1)a_{n-1}+2a_{n-1}-1$

$= n\cdot a_n-a_{n-1}(n-3)-1 \leq a_{n-1}\left(\frac{n}{2}-n+3\right)-1$

$a_{n-1}\left(3-\frac{n}{2}\right)-1$

$(1+c(n-2))\left(3-\frac{n}{2}\right)-1$

$3-\frac{n}{2}+3c(n-2)-\frac{c(n-2)n}{2}$

$\frac{n\cdot a_n}{\cancel{n-(n-1)}}$

$n-a_{n-1}(n-3)-1$

p4 Corrected vs Sheng 10/14

$$a_n + a_{n-1} - 1 \leq \frac{3a_{n-1}}{2} - 1$$

$$a_n + c(n-2) \mid 4(n-2) + c(n-2)(2n-3)$$

$$- c(n-2)(2n-3) - a_n(2n-3)$$

$$\boxed{a_n + c(n-2) \mid n-2 - a_n(2n-3)}$$

$$a_n = cx + r \qquad n-2 - 2ncx + 3cx - 2nr + 3r$$

$$c(x+n-2) + r \mid c(3x - 2nx) + n - 2 - 2nr + 3r$$

$$(c(x+n-2) + r)k = c(3x - 2nx) + n - 2 - 2nr + 3r$$

si $n = 3$

$$a_3 + c \mid \cancel{(-a_n(3))}$$

$$\cancel{a_n(2n-3)}$$

$$3a_n - 1$$

P4 Sucio 1/1/14

$a_n k + c(n-2)k = n-2 + c(n-2)(2n-3)$

$c(n-2)(2n-3-k) = a_n k - n + 2$

$(\cancel{a}n-2c)$

$c(-2c)(2n-3-k)$

$n-2 \mid a_n k$

~~si~~ $\cancel{2n-2k}$ ~~por~~ gcd

$c \mid a_n k - n + 2$

$c(2n-3-k) = \frac{a_n k}{n-2} - 1$

~~si $c \mid n+2 \Rightarrow c \mid k$~~

si $c \nmid k$ entonces

$cn \equiv 1 \mod a_n + c$

$\cancel{2c}$

$2n - 3 - k - 4\cancel{\ } + 6c + 3ck \equiv \cancel{a_n k} - n + 2$

$-\cancel{ck}$ mod $a_n + c$

$3n - 9 - k + 6c + 3ck \equiv 0$

$a_n(-6-3k) + 3n - 9 - k \equiv 0$

pu Sheet 12/14 Enumerated

S1 $\quad a_{n+1} = 1 + cn$

$$a_n + c \mid a_{n+1}$$

$$na_n + (1-n)(1+cn) - 1$$

$$na_n + 1 + cn - n - cn^2 - 1$$

$$a_n + cn \mid -cn^2 + cn - n + na_n$$

$$n\underline{(-cn + c - 1 + a_n)}$$

$$c(-n^2 + n) - n(1 - a_n)$$

$$cn(1-n) \cancel{-n(1-a_n)}$$

$$-n + na_n \qquad -cn(1-n) \qquad -a_n(1-n)$$

$$= -n + na_n - a_n + na_n$$

$$= -n + a_n(2n-1)$$

$$\frac{a_{n+1}}{d} - c + cn \mid na_n + (1-n)a_{n+1} - 1$$

$$a_{n+1}\left(\frac{n}{d} + 1 - n\right) - 1 \cancel{+} nc$$

$$\frac{1+cn}{d} - ckn$$

p4 Serio 13/14 Enunciad

$$\cancel{\frac{a_{n+1}}{d} - c + a_{n+1}}$$

a_{n+1}

$$-nc = 1 - a_{n+1}$$

$$a_{n+1}\left(\frac{n}{d} - n\right)$$

$$a_n + a_{n+1} - 1 \mid \cancel{a_{n+1}\left(\frac{n}{d} - n\right)}$$

$$\frac{a_{n+1}}{d} + \boxed{a_{n+1}} - c - 1 \mid \frac{n a_{n+1}}{d} - n a_{n+1}$$

$$-n \frac{a_{n+1}}{d} - n a_{n+1} + (c+1)n$$

$$\left(\frac{a_{n+1}}{d} + a_{n+1} - c - 1\right)k = 2n a_{n+1} + (c+1)n$$

$$a_{n+1} \cdot \boxed{\frac{k}{d}} = a_{n+1}(2n-k) - (c+1)(n-k)$$

$$-\frac{a_{n+1}}{d} \cdot n - a_{n+1} n + (c+1)n$$

$$\frac{a_{n+1}}{d} + a_{n+1} - c - 1 \mid \frac{n(d-1)}{d} a_{n+1}$$

lo cual es positivo si $d > 1$ pero es < a la izq

P4 Sevo 14 lca

$$\frac{cn+1}{d} + cn - c \;\Big|\; \frac{n(d-1)}{d}(cn+1)$$

si $d>1$

$$\frac{\frac{n(d-1)}{d} a_{n+1}}{a_{n+1} \cdot \frac{(d+1)}{d} \boxed{-c-1}} \geq \frac{\frac{n(d-1)\cancel{a_{n+1}}}{\cancel{d}}}{\cancel{a_n} \frac{d+1}{\cancel{d}}} = n \cdot \frac{d-1}{d+1}$$

$$= n - \frac{2}{d+1} > n-1$$

A l veras n

pw

$$\frac{\cancel{n} \cdot (d+1)}{d} a_{n+1} - (c+1)\cancel{n} \overset{?}{>} \frac{\cancel{n}(d-1)}{d} a_{n+1}$$

$$\frac{a_{n+1}}{d} \cdot 2d > c+1$$

$$2a_{n+1} > c+1$$ ✓

$$\Rightarrow d = 1$$

Y ya