1 Vehicle Model

Input: δ, F_{xij}, F_{yij}
Output: V_x, V_y, γ
1) longitudinal motion

$$m(\dot{V}_x - V_y \cdot \gamma) = F_{xfl} + F_{xfr} + F_{xr} \cdot \cos\delta + F_{yr} \cdot \sin\delta \tag{1}$$

2) lateral motion

$$m(\dot{V}_y + V_x \cdot \gamma) = F_{yfl} + F_{yfr} + F_{yr} \cdot \cos\delta - F_{xr} \cdot \sin\delta \tag{2}$$

3) yaw motion

$$I_z \cdot \dot{\gamma} = \frac{d}{2}(F_{xfr} - F_{xfl}) + l_f(F_{yfr} + F_{yfl}) - l_r(F_{xr} \cdot \sin\delta + F_{yr} \cdot \cos\delta) \tag{3}$$

2 Wheel Model

Input: $\tau_{ij}, F_{xij}, F_{zij}$
Output: ω_{ij}

$$I_\omega \cdot \dot{\omega}_{ij} = \tau_{ij} - F_{xij} \cdot R_{\text{wheel}} - (F_{zij} \cdot f \cdot R_{\text{wheel}}) \tag{4}$$

Rolling resistance can be chosen to be ignored.

3 Kinematic Model

Input: $\omega_{ij}, V_x, V_y, \gamma$
Output: $\beta, s_{ij}, \alpha_{ij}, F_{zij}$
1) vehicle side slip angle

$$\beta = \arctan\left(\frac{V_y}{V_x}\right) \tag{5}$$

2) tire slip ratio

During Acceleration:

$$s_{ij} = \frac{R_{\text{eff}} \cdot \omega_{ij} - V_x}{R_{\text{eff}} \cdot \omega_{ij}}, \quad \text{where } s > 0, F_x > 0 \tag{6}$$

During Deceleration:

$$s_{ij} = \frac{R_{\text{eff}} \cdot \omega_{ij} - V_x}{V_x}, \quad \text{where } s < 0, F_x < 0 \tag{7}$$

3) tire slip angle

$$\alpha_{\text{fl}} = \arctan\left(\frac{V_y + \gamma \cdot l_f}{V_x - \frac{d}{2} \cdot \gamma}\right) \tag{8}$$

$$\alpha_{\mathrm{fr}} = \arctan\left(\frac{V_y + \gamma \cdot l_f}{V_x + \frac{d}{2} \cdot \gamma}\right) \tag{9}$$

$$\alpha_{\mathrm{r}} = \delta - \arctan\left(\frac{V_y - \gamma \cdot l_r}{V_x}\right) \tag{10}$$

4) vertical load force

$$F_{zfl} = \frac{m}{L} \cdot \left[\frac{1}{2} g \cdot l_r - \frac{1}{2} a_x \cdot h_g - \left(\frac{a_y \cdot h_g \cdot l_r}{d}\right)\right] \tag{11}$$

$$F_{zfr} = \frac{m}{L} \cdot \left[\frac{1}{2} g \cdot l_r - \frac{1}{2} a_x \cdot h_g + \left(\frac{a_y \cdot h_g \cdot l_r}{d}\right)\right] \tag{12}$$

$$F_{zr} = \frac{m}{L} \cdot (g \cdot l_f + a_x \cdot h_g) \tag{13}$$

4 Tire Model

Input: $\beta, s_{ij}, \alpha_{ij}$
Output: F_{xij}, F_{yij}
Pacejka magic formula

$$F_{xij} = D_x \sin\left(C_x \arctan\left(B_x s_{ij} - E_x \left(B_x s_{ij} - \arctan(B_x s_{ij})\right)\right)\right) \tag{14}$$

$$F_{yij} = D_y \sin\left(C_y \arctan\left(B_y \alpha_{ij} - E_y \left(B_y \alpha_{ij} - \arctan(B_y \alpha_{ij})\right)\right)\right) \tag{15}$$

5 Appendix

ij is the wheel position, fl - front left, fr - front right, r - rear

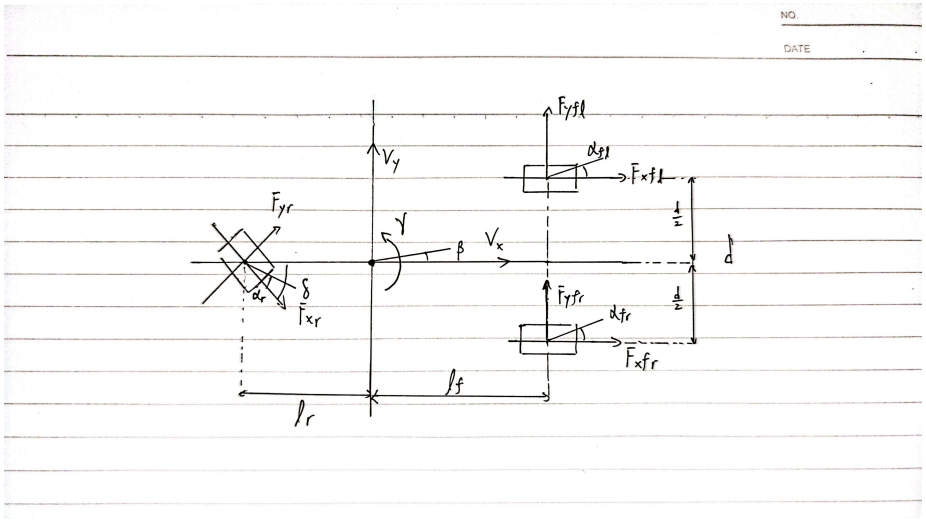

Figure 1: 7 -DOF vehicle model.

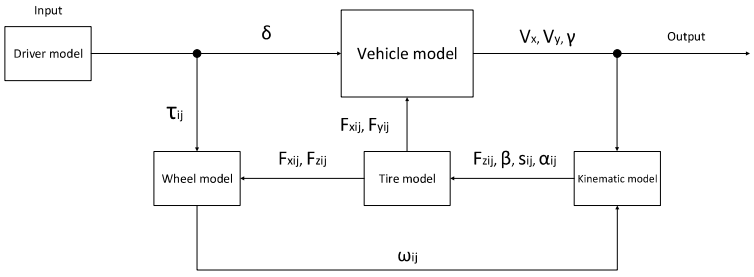

Figure 2: Vehicle dynamics.

Variable	Description
δ	Steering angle
F_{xij}	Longitudinal tire force at tire ij
F_{yij}	Lateral tire force at tire ij
F_{zij}	Vertical load force on tire ij
V_x	Longitudinal velocity of the vehicle
V_y	Lateral velocity of the vehicle
γ	Yaw rate of the vehicle
m	Mass of the vehicle
d	Distance between the left and right wheels (track width)
l_f	Distance from the vehicle's center of gravity to the front axle
l_r	Distance from the vehicle's center of gravity to the rear axle
I_z	Yaw moment of inertia
τ_{ij}	Torque applied to tire ij
ω_{ij}	Rotation speed of tire ij
R_{wheel}	Effective radius of the wheel
f	Coefficient of rolling resistance
R_{eff}	Effective rolling radius of the tire
β	Vehicle side slip angle
s_{ij}	Slip ratio of tire ij
α_{ij}	Slip angle of tire ij
a_x	Longitudinal acceleration
a_y	Lateral acceleration
h_g	Height of the vehicle's center of gravity
L	Distance between front and rear axles
D_x	Peak value factor for longitudinal force
C_x	Shape factor for longitudinal force
B_x	Stiffness factor for longitudinal force
E_x	Curvature factor for longitudinal force
D_y	Peak value factor for lateral force
C_y	Shape factor for lateral force
B_y	Stiffness factor for lateral force
E_y	Curvature factor for lateral force

Table 1: List of Variables and Descriptions