sasoptpy Documentation

Release 0.2.0

SAS Institute Inc.

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CONTENTS

PDF Version

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sasoptpy is a Python package providing a modeling interface for SAS Viya and SAS/OR Optimization solvers. It provides a quick way for users to deploy optimization models and solve them using SAS Viya Optimization Action Set.

sasoptpy can handle linear optimization, mixed integer linear optimization, and nonlinear optimization problems. Users can benefit from native Python structures like dictionaries, tuples, and list to define an optimization problem. *sasoptpy* supports Pandas objects extensively.

Under the hood, *sasoptpy* uses swat package to communicate SAS Viya, and uses saspy package to communicate SAS 9.4 installations.

sasoptpy is an interface to SAS Optimization solvers. Check SAS/OR and PROC OPTMODEL for more details about optimization tools provided by SAS and an interface to model optimization problems inside SAS.

See our SAS Global Forum paper: Optimization Modeling with Python and SAS Viya

CHAPTER

ONE

OVERVIEW

1.1 What's New

This page outlines changes from each release.

1.1.1 v0.2.0 (July 30, 2018)

New Features

- Support for the new *runOptmodel* CAS action is added
- Nonlinear optimization model building support is added for both SAS 9.4 and SAS Viya solvers
- Abstract model building support is added when using SAS Viya solvers
- New object types, `Set`, `SetIterator`, `Parameter`, `ParameterValue`, `ImplicitVar`, `ExpressionDict`, and `Statement` are added for abstract model building
- `Model.to_optmodel()` method is added for exporting model objects into PROC OPTMODEL codes as a string
- Wrapper functions `read_table()` and `read_data()` are added to read CASTable and DataFrame objects into the models
- Math function wrappers are added
- `_expr` and `_defn` methods are added to all object types for producing OPTMODEL expression and definitions
- Multiple solutions are now being returned when using *solveMilp* action and can be grabbed using `Model.get_solution()` method
- `Model.get_variable_value()` is added to get solution values of abstract variables

Changes

- Variable and constraint naming schemes are replaced with OPTMODEL equivalent versions
- Variables and constraints now preserve the order they are inserted to the problem
- `Model.to_frame()` method is updated to reflect changes to VG and CG orderings
- Two solve methods, `Model.solve_on_cas()` and `Model.solve_on_viya()` are merged into `Model.solve()`
- `Model.solve()` method checks the available CAS actions and uses *runOptmodel* whenever possible

- As part of the merging process, `lp` and `milp` arguments are replaced with `options` argument in *Model.solve()* and *Model.to_optmodel()*
- An optional argument `frame` is added to *Model.solve()* for forcing to use MPS mode and *solveLp-solveMilp* actions
- Minor changes are applied to `__str__` and `__repr__` methods
- Creation indices for objects are being kept using the return of the *register_name()* function
- Objective constant values are now being passed using new CAS action arguments when posssible
- A linearity check is added for models
- Test folder is added to the repository

Bug Fixes

- Nondeterministic behavior when generating MPS files is fixed.

Notes

- Abstract and nonlinear models can be solved on Viya only if *runOptmodel* action is available on the CAS server.
- Three new examples are added which demonstrate abstract model building.
- Some minor changes are applied to the existing examples.

1.1.2 v0.1.2 (April 24, 2018)

New Features

- As an experimental feature, *sasoptpy* supports *saspy* connections now
- `Model.solve_local()` method is added for solving optimization problems using SAS 9.4 installations
- *Model.drop_variable()*, *Model.drop_variables()*, *Model.drop_constraint()*, *Model.drop_constraints()* methods are added
- *Model.get_constraint()* and *Model.get_constraints()* methods are added to grab *Constraint* objects in a model
- *Model.get_variables()* method is added
- `_dual` attribute is added to the *Expression* objects
- *Variable.get_dual()* and `Constraint.get_dual()` methods are added
- *Expression.set_name()* method is added

Changes

- Session argument accepts `saspy.SASsession` objects
- *VariableGroup.mult()* method now supports `pandas.DataFrame`
- Type check for the *Model.set_session()* is removed to support new session types
- Problem and solution summaries are not being printed by default anymore, see *Model.get_problem_summary()* and *Model.get_solution_summary()*

- The default behavior of dropping the table after each solve is changed, but can be controlled with the `drop` argument of the `Model.solve()` method

Bug Fixes

- Fixed: Variables do not appear in MPS files if they are not used in the model
- Fixed: `Model.solve()` primalin argument does not pass into options

Notes

- A .gitignore file is added to the repository.
- A new example is added: Decentralization.
- Both *CAS/Viya* and *SAS* versions of the new example are available.
- There is a known issue with the nondeterministic behavior when creating MPS tables. This will be fixed with a hotfix after the release.
- A new option (no-ex) is added to makedocs script for skipping examples when building docs.

1.1.3 v0.1.1 (February 26, 2018)

New Features

- Initial value argument 'init' is added for `Variable` objects
- `Variable.set_init()` method is added for variables
- Initial value option 'primalin' is added to `Model.solve()` method
- Table name argument 'name', table drop option 'drop' and replace option 'replace' are added to `Model.solve()` method
- Decomposition block implementation is rewritten, block numbers does not need to be consecutive and ordered `Model.upload_user_blocks()`
- `VariableGroup.get_name()` and `ConstraintGroup.get_name()` methods are added
- `Model.test_session()` method is added for checking if session is defined for models
- `quick_sum()` function is added for faster summation of `Expression` objects

Changes

- methods.py is renamed to utils.py

Bug Fixes

- Fixed: Crash in VG and CG when a key not in the list is called
- Fixed: get_value of pandas is depreceated
- Fixed: Variables can be set as temporary expressions
- Fixed: Ordering in `get_solution_table()` is incorrect for multiple entries

1.1.4 v0.1.0 (December 22, 2017)

- Initial release

1.2 Installation

1.2.1 Python version support and dependencies

sasoptpy is developed and tested for Python version 3.5+.

It depends on the following packages:

- numpy
- saspy (Optional)
- swat
- pandas

1.2.2 Getting swat

swat package is a requirement to use SAS Viya solvers.

swat releases are listed at https://github.com/sassoftware/python-swat/releases. After downloading the platform-specific release file, it can be installed using pip:

```
pip install python-swat-X.X.X-platform.tar.gz
```

1.2.3 Getting saspy

saspy package is a requirement to use SAS 9.4 solvers.

saspy releases are listed at https://github.com/sassoftware/saspy/releases. The easiest way to download the latest stable version of saspy is to use:

```
pip install saspy
```

1.2.4 Getting sasoptpy

The latest available version of *sasoptpy* can be obtained from the online repository. Call:

```
git clone https://github.com/sassoftware/sasoptpy.git
```

Then inside the sasoptpy folder, call:

```
pip install .
```

Alternatively, you can use:

```
python setup.py install
```

1.2.5 Step-by-step installation

1. **Installing pandas and numpy**

   First, download and install numpy and pandas using pip:

   ```
   pip install numpy
   pip install pandas
   ```

2. **Installing the swat package**

   First, check the swat release page to find the latest release of the SAS-SWAT package for your environment.

   Then install it using

   ```
   pip install python-swat-X.X.X.platform.tar.gz
   ```

   As an example, run

   ```
   wget https://github.com/sassoftware/python-swat/releases/download/v1.2.1/python-
   ↪swat-1.2.1-linux64.tar.gz
   pip install python-swat-1.2.1-linux64.tar.gz
   ```

   to install the version 1.2.1 of the swat package for 64-bit Linux environments.

3. **Installing sasoptpy**

   Finally you can install *sasoptpy* by downloading the latest archive file and install via pip.

   ```
   wget *url-to-sasoptpy.tar.gz*
   pip install sasoptpy.tar.gz
   ```

   Latest release file is available at Github releases page.

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CHAPTER

TWO

GETTING STARTED

Solving an optimization problem via *sasoptpy* starts with having a running CAS Server or having a SAS 9.4 installation. It is possible to model a problem without a server but solving a problem requires access to SAS/OR solvers.

2.1 Creating a session

2.1.1 Creating a SAS Viya session

sasoptpy uses the CAS connection provided by the swat package. After installation simply use

```
In [1]: from swat import CAS

In [2]: s = CAS(hostname, port, userid, password)
```

The last two parameters are optional for some use cases. See swat Documentation for more details.

2.1.2 Creating a SAS 9.4 session

To create a SAS 9.4 session, see saspy Documentation. After customizing the configurations for your setup, a session can be created as follows:

```
import saspy
s = saspy.SASsession(cfgname='winlocal')
```

2.2 Initializing a model

After having an active CAS/SAS session, an empty model can be defined as follows:

```
In [3]: import sasoptpy as so

In [4]: m = so.Model(name='my_first_model', session=s)
NOTE: Initialized model my_first_model.
```

This command creates an empty model.

2.3 Processing input data

The easiest way to work with *sasoptpy* is to define problem inputs as Pandas DataFrames. Objective and cost coefficients, and lower and upper bounds can be defined using the DataFrame and Series objects. See Pandas Documentation to learn more.

```
In [5]: import pandas as pd

In [6]: prob_data = pd.DataFrame([
   ...:     ['Period1', 30, 5],
   ...:     ['Period2', 15, 5],
   ...:     ['Period3', 25, 0]
   ...:     ], columns=['period', 'demand', 'min_prod']).set_index(['period'])
   ...:

In [7]: price_per_product = 10

In [8]: capacity_cost = 10
```

Set `PERIODS` and other fields `demand`, `min_production` can be extracted as follows

```
In [9]: PERIODS = prob_data.index.tolist()

In [10]: demand = prob_data['demand']

In [11]: min_production = prob_data['min_prod']
```

2.4 Adding variables

You can add a single variables or a set of variables to *`Model`* objects.

- *`Model.add_variable()`* method is used to add a single variable.

  ```
  In [12]: production_cap = m.add_variable(vartype=so.INT, name='production_cap',␣
  ↪lb=0)
  ```

  When working with multiple models, you can create a variable independent of the model, such as

  ```
  >>> production_cap = so.Variable(name='production_cap', vartype=so.INT, lb=0)
  ```

  and add it to an existing model using

  ```
  >>> m.include(production_cap)
  ```

- *`Model.add_variables()`* method is used to add a set of variables.

  ```
  In [13]: production = m.add_variables(PERIODS, vartype=so.INT, name='production',
     ....:                              lb=min_production)
     ....:
  ```

  When passed as a set of variables, individual variables can be obtained by using individual keys, such as `production['Period1']`. To create multi-dimensional variables, simply list all the keys as

  ```
  >>> multivar = m.add_variables(KEYS1, KEYS2, KEYS3, name='multivar')
  ```

2.5 Creating expressions

Expression objects keep mathematical expressions. Although these objects are mostly used under the hood when defining a model, it is possible to define a custom *Expression* to use later.

```
In [14]: totalRevenue = production.sum('*')*price_per_product

In [15]: totalCost = production_cap * capacity_cost
```

The first thing to notice is the use of the *VariableGroup.sum()* method over a variable group. This method returns the sum of variables inside the group as an *Expression* object. Its multiplication with a scalar `profit_per_product` gives the final expression.

Similarly, `totalCost` is simply multiplication of a *Variable* object with a scalar.

2.6 Setting an objective function

Objective functions can be written in terms of expressions. In this problem, the objective is to maximize the profit, so *Model.set_objective()* method is used as follows:

```
In [16]: m.set_objective(totalRevenue-totalCost, sense=so.MAX, name='totalProfit')
Out[16]: sasoptpy.Expression(exp = 10 * production[Period3] + 10 *␣
↪production[Period1] + 10 * production[Period2] - 10 * production_cap, name='obj_1')
```

Notice that you can define the same objective using

```
>>> m.set_objective(production.sum('*')*price_per_product - production_cap*capacity_
↪cost, sense=so.MAX, name='totalProfit')
```

The mandatory argument `sense` should be assigned the value of either `so.MIN` or `so.MAX` for minimization or maximization problems, respectively.

2.7 Adding constraints

In *sasoptpy*, constraints are simply expressions with a direction. It is possible to define an expression and add it to a model by defining which direction the linear relation should have.

There are two methods to add constraints. The first one is *Model.add_constraint()* where a single constraint can be inserted into a model.

The second one is *Model.add_constraints()* where multiple constraints can be added to a model.

```
In [17]: m.add_constraints((production[i] <= production_cap for i in PERIODS),
   ....:                   name='capacity')
   ....:
Out[17]: sasoptpy.ConstraintGroup([production[Period1] - production_cap <=  0,␣
↪production[Period2] - production_cap <=  0, - production_cap + production[Period3]
↪<=  0, ], name='capacity')
```

```
In [18]: m.add_constraints((production[i] <= demand[i] for i in PERIODS),
   ....:                   name='demand')
   ....:
Out[18]: sasoptpy.ConstraintGroup([production[Period1] <=  30, production[Period2] <=␣
↪ 15, production[Period3] <=  25, ], name='demand')
```

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Here, the first term provides a Python generator, which then gets translated into constraints in the problem. The symbols <=, >=, and == are used for less than or equal to, greater than or equal to, and equal to constraints, respectively. Range constraints can be inserted using == and a list of 2 values representing lower and upper bounds.

2.8 Solving a problem

Defined problems can be simply sent to CAS server or SAS sesion by calling the `Model.solve()` method.

See the solution output to the problem.

```
In [19]: m.solve()
NOTE: Added action set 'optimization'.
NOTE: Converting model my_first_model to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 4 variables (0 free, 0 fixed).
NOTE: The problem has 0 binary and 4 integer variables.
NOTE: The problem has 6 linear constraints (6 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 9 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed all variables and constraints.
NOTE: Optimal.
NOTE: Objective = 400.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 6 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 4 rows and 6␣
↪columns.
Out[19]:
                   var   lb            ub  value  rc
0       production_cap -0.0  1.797693e+308   25.0 NaN
1  production[Period1]  5.0  1.797693e+308   25.0 NaN
2  production[Period2]  5.0  1.797693e+308   15.0 NaN
3  production[Period3] -0.0  1.797693e+308   25.0 NaN
```

At the end of the solve operation, the solver returns both Problem Summary and Solution Summary tables. These tables can be later accessed using `m.get_problem_summary()` and `m.get_solution_summary`.

```
In [20]: print(m.get_solution_summary())
Solution Summary

                                Value
Label
Solver                           MILP
Algorithm              Branch and Cut
Objective Function              obj_1
Solution Status               Optimal
Objective Value                   400
```

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```
Relative Gap                    0
Absolute Gap                    0
Primal Infeasibility            0
Bound Infeasibility             0
Integer Infeasibility           0

Best Bound                    400
Nodes                           0
Solutions Found                 1
Iterations                      0
Presolve Time                0.01
Solution Time                0.01
```

The *Model.solve()* method returns the primal solution when available, and `None` otherwise.

2.9 Printing solutions

Solutions provided by the solver can be obtained using *sasoptpy.get_solution_table()* method. It is strongly suggested to group variables and expressions that share the same keys in a call.

```
In [21]: print(so.get_solution_table(demand, production))
         demand  production
1
Period1      30        25.0
Period2      15        15.0
Period3      25        25.0
```

As seen, a Pandas Series and a Variable object that has the same index keys are printed in this example.

2.10 Next steps

You can browse *Examples* to see various uses of aforementioned functionality.

If you have a good understanding of the flow, then check *API Reference* to access API details.

CHAPTER

THREE

HANDLING DATA

sasoptpy can work with native Python types and *pandas* objects for all data operations. Among *pandas* object types, *sasoptpy* works with `pandas.DataFrame` and `pandas.Series` objects to construct and manipulate model components.

3.1 Indices

Methods like *`Model.add_variables()`* can utilize native Python object types like list and range as variable and constraint indices. `pandas.Index` can be used as index as well.

3.1.1 List

```
In [1]: m = so.Model(name='demo')
NOTE: Initialized model demo.

In [2]: SEASONS = ['Fall', 'Winter', 'Spring', 'Summer']

In [3]: prod_lb = {'Fall': 100, 'Winter': 200, 'Spring': 100, 'Summer': 400}

In [4]: production = m.add_variables(SEASONS, lb=prod_lb, name='production')

In [5]: print(production)
Variable Group (production) [
  [Fall: production[Fall]]
  [Spring: production[Spring]]
  [Summer: production[Summer]]
  [Winter: production[Winter]]
]
```

```
In [6]: print(repr(production['Summer']))
sasoptpy.Variable(name='production[Summer]', lb=400,  vartype='CONT')
```

Note that if a list is being used as the index set, associated fields like *lb*, *ub* should be accesible using the index keys. Accepted types are dict and pandas.Series.

3.1.2 Range

```
In [7]: link = m.add_variables(range(3), range(2), vartype=so.BIN, name='link')

In [8]: print(link)
Variable Group (link) [
  [(0, 0): link[0, 0]]
  [(0, 1): link[0, 1]]
  [(1, 0): link[1, 0]]
  [(1, 1): link[1, 1]]
  [(2, 0): link[2, 0]]
  [(2, 1): link[2, 1]]
]
```

```
In [9]: print(repr(link[2, 1]))
sasoptpy.Variable(name='link[2,1]', ub=1,  vartype='BIN')
```

3.1.3 pandas.Index

```
In [10]: import pandas as pd

In [11]: p_data = [[3, 5, 9],
   ....:            [0, -1, 14],
   ....:            [5, 6, 20]]
   ....:

In [12]: df = pd.DataFrame(p_data, columns=['c1', 'col_lb', 'col_ub'])

In [13]: x = m.add_variables(df.index, lb=df['c1'], vartype=so.INT, name='x')
```

```
In [14]: print(x)
Variable Group (x) [
  [0: x[0]]
  [1: x[1]]
  [2: x[2]]
]
```

```
In [15]: df2 = df.set_index([['r1', 'r2', 'r3']])

In [16]: y = m.add_variables(df2.index, lb=df2['col_lb'], ub=df2['col_ub'], name='y')
```

```
In [17]: print(y)
Variable Group (y) [
  [r1: y[r1]]
  [r2: y[r2]]
  [r3: y[r3]]
]
```

```
In [18]: print(repr(y['r1']))
sasoptpy.Variable(name='y[r1]', lb=5, ub=9,  vartype='CONT')
```

3.1.4 Set

sasoptpy can work with data on the server and generate abstract expressions. For this purpose, you can use `Set` objects to represent PROC OPTMODEL sets.

```
In [19]: m2 = so.Model(name='m2')
NOTE: Initialized model m2.

In [20]: I = m2.add_set(name='I')

In [21]: u = m2.add_variables(I, name='u')

In [22]: print(I, u)
I Variable Group (u) [
  [I: u[I]]
]
```

See *Workflows* for more information on working with server-side models.

3.2 Data

sasoptpy can work with both client-side and server-side data. Here are some options to load data into the optimization models.

3.2.1 pandas DataFrame

`pandas.DataFrame` is the preferred object types when passing data into *sasoptpy* models.

```
In [23]: data = [
   ....:     ['clock', 8, 4, 3],
   ....:     ['mug', 10, 6, 5],
   ....:     ['headphone', 15, 7, 2],
   ....:     ['book', 20, 12, 10],
   ....:     ['pen', 1, 1, 15]
   ....:     ]
   ....:

In [24]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit']).set_
↪index(['item'])

In [25]: get = so.VariableGroup(df.index, ub=df['limit'], name='get')

In [26]: print(get)
Variable Group (get) [
  [book: get[book]]
  [clock: get[clock]]
  [headphone: get[headphone]]
  [mug: get[mug]]
  [pen: get[pen]]
]
```

3.2.2 Dictionaries

Lists and dictionaries can be used in expressions and when creating variables.

```
In [27]: items = ['clock', 'mug', 'headphone', 'book', 'pen']
```

(continues on next page)

(continued from previous page)

```
In [28]: limits = {'clock': 3, 'mug': 5, 'headphone': 2, 'book': 10, 'pen': 15}

In [29]: get2 = so.VariableGroup(items, ub=limits, name='get2')

In [30]: print(get2)
Variable Group (get2) [
  [book: get2[book]]
  [clock: get2[clock]]
  [headphone: get2[headphone]]
  [mug: get2[mug]]
  [pen: get2[pen]]
]
```

3.2.3 CASTable

When a data is available on the server-side, a reference to the object can be passed. Note that, using CASTable and Abstract Data requires SAS Viya version 3.4.

```
In [31]: m2 = so.Model(name='m2', session=session)
NOTE: Initialized model m2.
```

```
In [32]: table = session.upload_frame(df)
NOTE: Cloud Analytic Services made the uploaded file available as table TMP_JV5PTTU␣
↪in caslib CASUSERHDFS(casuser).
NOTE: The table TMP_JV5PTTU has been created in caslib CASUSERHDFS(casuser) from␣
↪binary data uploaded to Cloud Analytic Services.
```

```
In [33]: print(type(table), table)
<class 'swat.cas.table.CASTable'> CASTable('TMP_JV5PTTU', caslib='CASUSERHDFS(casuser)
↪')
```

```
In [34]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit'])

In [35]: ITEMS, (value, weight, limit) = m2.read_table(df, key=['item'],
   ....:     key_type='str', columns=['value', 'weight', 'limit'])
   ....:

In [36]: get3 = m2.add_variables(ITEMS, name='get3')

In [37]: print(get3)
Variable Group (get3) [
  [book: get3[book]]
  [clock: get3[clock]]
  [headphone: get3[headphone]]
  [mug: get3[mug]]
  [pen: get3[pen]]
]
```

3.2.4 Abstract Data

If you would like to model your problem first and then load data, you can pass a string for the data sets that will be available later. See following:

```
In [38]: m3 = so.Model(name='m3', session=session)
NOTE: Initialized model m3.

In [39]: ITEMS, (limit) = m3.read_table('DF', key=['item'], key_type=['str'],
   ....:     columns=['limit'])
   ....:

In [40]: print(type(ITEMS), ITEMS)
<class 'sasoptpy.data.Set'> set_item
```

Notice that the key set is created as a reference. We can later solve the problem after having the data available with the same name, e.g. using the *upload_frame* function.

```
In [41]: session.upload_frame(df, casout='DF')
NOTE: Cloud Analytic Services made the uploaded file available as table DF in caslib␣
↪CASUSERHDFS(casuser).
NOTE: The table DF has been created in caslib CASUSERHDFS(casuser) from binary data␣
↪uploaded to Cloud Analytic Services.
Out[41]: CASTable('DF', caslib='CASUSERHDFS(casuser)')
```

3.3 Operations

Lists, `pandas.Series`, and `pandas.DataFrame` objects can be used for mathematical operations like *`VariableGroup.mult()`*.

```
In [42]: sd = [3, 5, 6]

In [43]: z = m.add_variables(3, name='z')
```

```
In [44]: print(z)
Variable Group (z) [
  [0: z[0]]
  [1: z[1]]
  [2: z[2]]
]
```

```
In [45]: print(repr(z))
sasoptpy.VariableGroup([0, 1, 2], name='z')
```

```
In [46]: e1 = z.mult(sd)

In [47]: print(e1)
5 * z[1] + 6 * z[2] + 3 * z[0]
```

```
In [48]: ps = pd.Series(sd)

In [49]: e2 = z.mult(ps)

In [50]: print(e2)
5 * z[1] + 6 * z[2] + 3 * z[0]
```

CHAPTER

FOUR

SESSIONS AND MODELS

4.1 Sessions

4.1.1 CAS Sessions

A swat.cas.connection.CAS session is needed to solve optimization problems with *sasoptpy* using SAS Viya OR solvers. See SAS documentation to learn more about CAS sessions and SAS Viya.

A sample CAS Session can be created using the following commands.

```
>>> import sasoptpy as so
>>> from swat import CAS
>>> s = CAS(hostname=cas_host, username=cas_username, password=cas_password, port=cas_
↪port)
>>> m = so.Model(name='demo', session=s)
>>> print(repr(m))
sasoptpy.Model(name='demo', session=CAS(hostname, port, username, protocol='cas',␣
↪name='py-session-1', session=session-no))
```

4.1.2 SAS Sessions

A saspy.SASsession session is needed to solve optimization problems with *sasoptpy* using SAS/OR solvers on SAS 9.4 clients.

A sample SAS session can be created using the following commands.

```
>>> import sasoptpy as so
>>> import saspy
>>> sas_session = saspy.SASsession(cfgname='winlocal')
>>> m = so.Model(name='demo', session=sas_session)
>>> print(repr(m))
sasoptpy.Model(name='demo', session=saspy.SASsession(cfgname='winlocal'))
```

4.2 Models

4.2.1 Creating a model

An empty model can be created using the *Model* constructor:

```
In [1]: import sasoptpy as so

In [2]: m = so.Model(name='model1')
NOTE: Initialized model model1.
```

4.2.2 Adding new components to a model

Adding a variable:

```
In [3]: x = m.add_variable(name='x', vartype=so.BIN)

In [4]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (1): [
    x
  ]
  Constraints (0): [
  ]
]

In [5]: y = m.add_variable(name='y', lb=1, ub=10)

In [6]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (0): [
  ]
]
```

Adding a constraint:

```
In [7]: c1 = m.add_constraint(x + 2 * y <= 10, name='c1')

In [8]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (1): [
    x + 2 * y <=  10
  ]
]
```

4.2.3 Adding existing components to a model

A new model can use existing variables. The typical way to include a variable is to use the `Model.include()` method:

```
In [9]: new_model = so.Model(name='new_model')
NOTE: Initialized model new_model.

In [10]: new_model.include(x, y)

In [11]: print(new_model)
Model: [
  Name: new_model
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (0): [
  ]
]

In [12]: new_model.include(c1)

In [13]: print(new_model)
Model: [
  Name: new_model
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (1): [
    x + 2 * y <=  10
  ]
]

In [14]: z = so.Variable(name='z', vartype=so.INT, lb=3)

In [15]: new_model.include(z)

In [16]: print(new_model)
Model: [
  Name: new_model
  Objective: MIN [0]
  Variables (3): [
    x
    y
    z
  ]
  Constraints (1): [
    x + 2 * y <=  10
  ]
]
```

Note that variables are added to `Model` objects by reference. Therefore, after `Model.solve()` is called, values of variables will be replaced with optimal values.

4.2.4 Accessing components

You can get a list of model variables using *Model.get_variables()* method.

```
In [17]: print(m.get_variables())
[sasoptpy.Variable(name='x', ub=1,  vartype='BIN'), sasoptpy.Variable(name='y', lb=1,
↪ub=10,  vartype='CONT')]
```

Similarly, you can access a list of constraints using *Model.get_constraints()* method.

```
In [18]: c2 = m.add_constraint(2 * x - y >= 1, name='c2')

In [19]: print(m.get_constraints())
[sasoptpy.Constraint(x + 2 * y <=  10, name='c1'), sasoptpy.Constraint(2 * x - y >= 
↪1, name='c2')]
```

To access a certain constraint using its name, you can use *Model.get_constraint()* method:

```
In [20]: print(m.get_constraint('c2'))
2 * x - y >=  1
```

4.2.5 Dropping components

A variable inside a model can simply be dropped using *Model.drop_variable()*. Similarly, a set of variables can be dropped using *Model.drop_variables()*.

```
In [21]: m.drop_variable(y)

In [22]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (1): [
    x
  ]
  Constraints (2): [
    x + 2 * y <=  10
    2 * x - y >=  1
  ]
]
```

```
In [23]: m.include(y)

In [24]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (2): [
    x + 2 * y <=  10
    2 * x - y >=  1
  ]
]
```

A constraint can be dropped using *Model.drop_constraint()* method. Similarly, a set of constraints can be dropped using *Model.drop_constraints()*.

```
In [25]: m.drop_constraint(c1)

In [26]: m.drop_constraint(c2)

In [27]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (0): [
  ]
]
```

```
In [28]: m.include(c1)

In [29]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (1): [
    x + 2 * y <=  10
  ]
]
```

4.2.6 Copying a model

An exact copy of the existing model can be obtained by including the *Model* object itself.

```
In [30]: copy_model = so.Model(name='copy_model')
NOTE: Initialized model copy_model.

In [31]: copy_model.include(m)

In [32]: print(copy_model)
Model: [
  Name: copy_model
  Objective: MIN [0]
  Variables (2): [
    x
    y
  ]
  Constraints (1): [
    x + 2 * y <=  10
  ]
]
```

Note that all variables and constraints are included by reference.

4.2.7 Solving a model

A model is solved using the `Model.solve()` method. This method converts Python definitions into an MPS file and uploads to a CAS server for the optimization action. The type of the optimization problem (Linear Optimization or Mixed Integer Linear Optimization) is determined based on variable types.

```
>>> m.solve()
NOTE: Initialized model model_1
NOTE: Converting model model_1 to DataFrame
NOTE: Added action set 'optimization'.
...
NOTE: Optimal.
NOTE: Objective = 124.343.
NOTE: The Dual Simplex solve time is 0.01 seconds.
```

4.2.8 Solve options

Solver Options

Both PROC OPTMODEL solve options and `solveLp`, `solveMilp` action options can be passed using `options` argument of the `Model.solve()` method.

```
>>> m.solve(options={'with': 'milp', 'maxtime': 600})
>>> m.solve(options={'with': 'lp', 'algorithm': 'ipm'})
```

The only special option for the `Model.solve()` method is `with`. If not passed, PROC OPTMODEL chooses a solver that depends on the problem type. Possible `with` options are listed in SAS/OR documentation: http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_optmodel_syntax11.htm&docsetVersion=14.3&locale=en#ormpug.optmodel.npxsolvestmt

See specific solver options at following links:

- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_lpsolver_syntax02.htm&docsetVersion=14.3&locale=en for a list of LP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_milpsolver_syntax02.htm&docsetVersion=14.3&locale=en for a list of MILP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_nlpsolver_syntax02.htm&docsetVersion=14.3&locale=en for a list of NLP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_qpsolver_syntax02.htm&docsetVersion=14.3&locale=en for a list of QP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_clpsolver_syntax01.htm&docsetVersion=14.3&locale=en for a list of CLP solver options.

The `options` argument can also pass `solveLp` and `solveMilp` action options when `frame=True` is used when calling the `Model.solve()` method.

- See http://go.documentation.sas.com/?cdcId=vdmmlcdc&cdcVersion=8.11&docsetId=casactmopt&docsetTarget=casactmopt_solvelp_syntax.htm&locale=en for a list of LP options.
- See http://go.documentation.sas.com/?cdcId=vdmmlcdc&cdcVersion=8.11&docsetId=casactmopt&docsetTarget=casactmopt_solvemilp_syntax.htm&locale=en for a list of MILP options.

Package Options

Besides the `options` argument, there are 7 arguments that can be passed into *`Model.solve()`* method:

- name: Name of the uploaded problem information
- drop: Option for dropping the data from server after solve
- replace: Option for replacing an existing data with the same name
- primalin: Option for using the current values of the variables as an initial solution
- submit: Option for calling the CAS / SAS action
- frame: Option for using frame (MPS) method (if False, it uses OPTMODEL)
- verbose: Option for printing the generated OPTMODEL code before solve

When `primalin` argument is `True`, it grabs *`Variable`* objects `_init` field. This field can be modified with *`Variable.set_init()`* method.

4.2.9 Getting solutions

After the solve is completed, all variable and constraint values are parsed automatically. A summary of the problem can be accessed using the *`Model.get_problem_summary()`* method, and a summary of the solution can be accesed using the *`Model.get_solution_summary()`* method.

To print values of any object, *`get_solution_table()`* can be used:

```
>>> print(so.get_solution_table(x, y))
```

All variables and constraints passed into this method are returned based on their indices. See *Examples* for more details.

CHAPTER

FIVE

MODEL COMPONENTS

In this part, several model components are discussed with examples. See *Examples* to learn more about how these components can be used to define optimization models.

5.1 Expressions

`Expression` objects represent linear and nonlinear mathematical expressions in *sasoptpy*.

5.1.1 Creating expressions

An `Expression` can be created as follows:

```
In [1]: profit = so.Expression(5 * sales - 3 * material, name='profit')

In [2]: print(repr(profit))
sasoptpy.Expression(exp = - 3 * material + 5 * sales, name='profit')
```

5.1.2 Nonlinear expressions

`Expression` objects are linear by default. It is possible to create nonlinear expressions, but there are some limitations.

```
In [3]: nonexp = sales ** 2 + (1 / material) ** 3

In [4]: print(nonexp)
(sales) ** (2) + ((1) / (material)) ** (3)
```

Currently, it is not possible to get or print values of nonlinear expressions. Moreover, if your model includes a nonlinear expression, you need to be using SAS Viya >= 3.4 or any SAS version for solving your problem.

For using mathematical operations, you need to import *sasoptpy.math* functions.

5.1.3 Mathematical expressions

sasoptpy provides mathematical functions for generating mathematical expressions to be used in optimization models.

You need to import *sasoptpy.math* to your code to start using these functions. A list of available mathematical functions are listed at *Math Functions*.

```
In [5]: import sasoptpy.math as sm

In [6]: newexp = sm.max(sales, 10) ** 2

In [7]: print(newexp._expr())
(max(sales , 10)) ^ (2)
```

```
In [8]: import sasoptpy.math as sm

In [9]: angle = so.Variable(name='angle')

In [10]: newexp = sm.sin(angle) ** 2 + sm.cos(angle) ** 2

In [11]: print(newexp._expr())
(sin(angle)) ^ (2) + (cos(angle)) ^ (2)
```

5.1.4 Operations

Getting the current value

After the solve is completed, the current value of an expression can be obtained using the *Expression.get_value()* method:

```
>>> print(profit.get_value())
42.0
```

Getting the dual value

Dual values of *Expression* objects can be obtained using *Variable.get_dual()* and `Constraint.get_dual()` methods.

```
>>> m.solve()
>>> ...
>>> print(x.get_dual())
1.0
```

Addition

There are two ways to add elements to an expression. The first and simpler way creates a new expression at the end:

```
In [12]: tax = 0.5

In [13]: profit_after_tax = profit - tax
```

```
In [14]: print(repr(profit_after_tax))
sasoptpy.Expression(exp = - 3 * material + 5 * sales - 0.5, name=None)
```

The second way, *Expression.add()* method, takes two arguments: the element to be added and the sign (1 or -1):

```
In [15]: profit_after_tax = profit.add(tax, sign=-1)
```

```
In [16]: print(profit_after_tax)
- 3 * material + 5 * sales - 0.5
```

```
In [17]: print(repr(profit_after_tax))
sasoptpy.Expression(exp = - 3 * material + 5 * sales - 0.5, name=None)
```

If the expression is a temporary one, then the addition is performed in place.

Multiplication

You can multiply expressions with scalar values:

```
In [18]: investment = profit.mult(0.2)

In [19]: print(investment)
- 0.6 * material + sales
```

Summation

For faster summations compared to Python's native `sum` function, *sasoptpy* provides `sasoptpy.quick_sum()`.

```
In [20]: import time

In [21]: x = m.add_variables(1000, name='x')
```

```
In [22]: t0 = time.time()

In [23]: e = so.quick_sum(2 * x[i] for i in range(1000))

In [24]: print(time.time()-t0)
0.010161161422729492
```

```
In [25]: t0 = time.time()

In [26]: f = sum(2 * x[i] for i in range(1000))

In [27]: print(time.time()-t0)
0.30232739448547363
```

5.1.5 Renaming an expression

Expressions can be renamed using `Expression.set_name()` method:

```
In [28]: e = so.Expression(x[5] + 2 * x[6], name='e1')

In [29]: print(repr(e))
sasoptpy.Expression(exp = x[5] + 2 * x[6], name='e1')
```

```
In [30]: e.set_name('e2');

In [31]: print(repr(e))
sasoptpy.Expression(exp = x[5] + 2 * x[6], name='e2')
```

5.1.6 Copying an expression

An `Expression` can be copied using `Expression.copy()`.

```
In [32]: copy_profit = profit.copy(name='copy_profit')

In [33]: print(repr(copy_profit))
sasoptpy.Expression(exp = - 3 * material + 5 * sales, name='copy_profit')
```

5.1.7 Temporary expressions

An *Expression* object can be defined as temporary, which enables faster `Expression.sum()` and *Expression.mult()* operations.

```
In [34]: new_profit = so.Expression(10 * sales - 2 * material, temp=True)

In [35]: print(repr(new_profit))
sasoptpy.Expression(exp = - 2 * material + 10 * sales, name=None)
```

The expression can be modified inside a function:

```
In [36]: new_profit + 5
Out[36]: sasoptpy.Expression(exp = - 2 * material + 10 * sales + 5, name=None)
```

```
In [37]: print(repr(new_profit))
sasoptpy.Expression(exp = - 2 * material + 10 * sales + 5, name=None)
```

As you can see, the value of `new_profit` is changed due to an in-place addition. To prevent the change, such expressions can be converted to permanent expressions using the *Expression.set_permanent()* method or constructor:

```
In [38]: new_profit = so.Expression(10 * sales - 2 * material, temp=True)

In [39]: new_profit.set_permanent()
Out[39]: 'expr_11'

In [40]: tmp = new_profit + 5

In [41]: print(repr(new_profit))
sasoptpy.Expression(exp = - 2 * material + 10 * sales, name='expr_11')
```

5.2 Objective Functions

5.2.1 Setting and getting an objective function

Any valid *Expression* can be used as the objective function of a model. An existing expression can be used as an objective function using the *Model.set_objective()* method. The objective function of a model can be obtained using the *Model.get_objective()* method.

```
>>> profit = so.Expression(5 * sales - 2 * material, name='profit')
>>> m.set_objective(profit, so.MAX)
>>> print(m.get_objective())
 -  2.0 * material  +  5.0 * sales
```

5.2.2 Getting the value

After a solve, the objective value can be checked using the *Model.get_objective_value()* method.

```
>>> m.solve()
>>> print(m.get_objective_value())
42.0
```

5.3 Variables

5.3.1 Creating variables

Variables can be created either separately or inside a model.

Creating a variable outside a model

The first way to create a variable uses the default constructor.

```
>>> x = so.Variable(vartype=so.INT, ub=5, name='x')
```

When created separately, a variable needs to be included (or added) inside the model:

```
>>> y = so.Variable(name='y', lb=5)
>>> m.add_variable(y)
```

and

```
>>> y = m.add_variable(name='y', lb=5)
```

are equivalent.

Creating a variable inside a model

The second way is to use *Model.add_variable()*. This method creates a *Variable* object and returns a pointer.

```
>>> x = m.add_variable(vartype=so.INT, ub=5, name='x')
```

5.3.2 Arguments

There are three types of variables: continuous variables, integer variables, and binary variables. Continuous variables are the default type and can be created using the `vartype=so.CONT` argument. Integer variables and binary variables can be created using the `vartype=so.INT` and `vartype=so.BIN` arguments, respectively.

The default lower bound for variables is 0, and the upper bound is infinity. Name is a required argument. If the given name already exists in the namespace, then a different generic name can be used for the variable. The *reset_globals()* function can be used to reset sasoptpy namespace when needed.

5.3.3 Changing bounds

The *Variable.set_bounds()* method changes the bounds of a variable.

```
>>> x = so.Variable(name='x', lb=0, ub=20)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=0, ub=20, vartype='CONT')
>>> x.set_bounds(lb=5, ub=15)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=5, ub=15, vartype='CONT')
```

5.3.4 Setting initial values

Initial values of variables can be passed to the solvers for certain problems. The `Variable.set_init()` method changes the initial value for variables. This value can be set at the creation of the variable as well.

```
>>> x.set_init(5)
>>> print(repr(x))
sasoptpy.Variable(name='x', ub=20, init=5,  vartype='CONT')
```

5.3.5 Working with a set of variables

A set of variables can be added using single or multiple indices. Valid index sets include list, dict, and `pandas.Index` objects. See *Handling Data* for more about allowed index types.

Creating a set of variables outside a model

```
>>> production = VariableGroup(PERIODS, vartype=so.INT, name='production',
                               lb=min_production)
>>> print(repr(production))
sasoptpy.VariableGroup(['Period1', 'Period2', 'Period3'], name='production')
>>> m.include(production)
```

Creating a set of variables inside a model

```
>>> production = m.add_variables(PERIODS, vartype=so.INT,
                                 name='production', lb=min_production)
>>> print(production)
>>> print(repr(production))
Variable Group (production) [
  [Period1: production['Period1',]]
  [Period2: production['Period2',]]
  [Period3: production['Period3',]]
]
sasoptpy.VariableGroup(['Period1', 'Period2', 'Period3'],
name='production')
```

5.4 Constraints

5.4.1 Creating constraints

Similar to `Variable` objects, `Constraint` objects can be created inside or outside optimization models.

Creating a constraint outside a model

```
>>> c1 = so.Constraint(3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( -  5.0 * y  +  3.0 * x  <=  10, name='c1')
```

Creating a constraint inside a model

```
>>> c1 = m.add_constraint(3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( -  5.0 * y  +  3.0 * x  <=  10, name='c1')
```

5.4.2 Modifying variable coefficients

The coefficient of a variable inside a constraint can be updated using the `Constraint.update_var_coef()` method:

```
>>> c1 = so.Constraint(exp=3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( -  5.0 * y  +  3.0 * x  <=  10, name='c1')
>>> c1.update_var_coef(x, -1)
>>> print(repr(c1))
sasoptpy.Constraint( -  5.0 * y  -  x  <=  10, name='c1')
```

5.4.3 Working with a set of constraints

A set of constraints can be added using single or multiple indices. Valid index sets include list, dict, and `pandas.Index` objects. See *Handling Data* for more about allowed index types.

Creating a set of constraints outside a model

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> cg = so.ConstraintGroup((2 * z[i, j] + 3 * z[i-1, j] >= 2 for i in
                            [1] for j in ['a', 'b', 'c']), name='cg')
>>> print(cg)
Constraint Group (cg) [
  [(1, 'a'):  3.0 * z[0, 'a']  +  2.0 * z[1, 'a']  >=  2]
  [(1, 'b'):  3.0 * z[0, 'b']  +  2.0 * z[1, 'b']  >=  2]
  [(1, 'c'):  2.0 * z[1, 'c']  +  3.0 * z[0, 'c']  >=  2]
]
```

Creating a set of constraints inside a model

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> cg2 = m.add_constraints((2 * z[i, j] + 3 * z[i-1, j] >= 2 for i in
                             [1] for j in ['a', 'b', 'c']), name='cg2')
>>> print(cg2)
Constraint Group (cg2) [
  [(1, 'a'):  2.0 * z[1, 'a']  +  3.0 * z[0, 'a']  >=  2]
  [(1, 'b'):  3.0 * z[0, 'b']  +  2.0 * z[1, 'b']  >=  2]
  [(1, 'c'):  2.0 * z[1, 'c']  +  3.0 * z[0, 'c']  >=  2]
]
```

5.4.4 Range constraints

A range for an expression can be given using a list of two value (lower and upper bound) with an == sign:

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> c1 = m.add_constraint(x + 2*y == [2,9], name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( x + 2.0 * y  ==  [2, 9], name='c1')
```

CHAPTER

SIX

WORKFLOWS

sasoptpy can work both with client-side data and server-side data. Some limitations to the functionalities may apply in terms of which workflow is being used. In this part, overall flow of the package is explained.

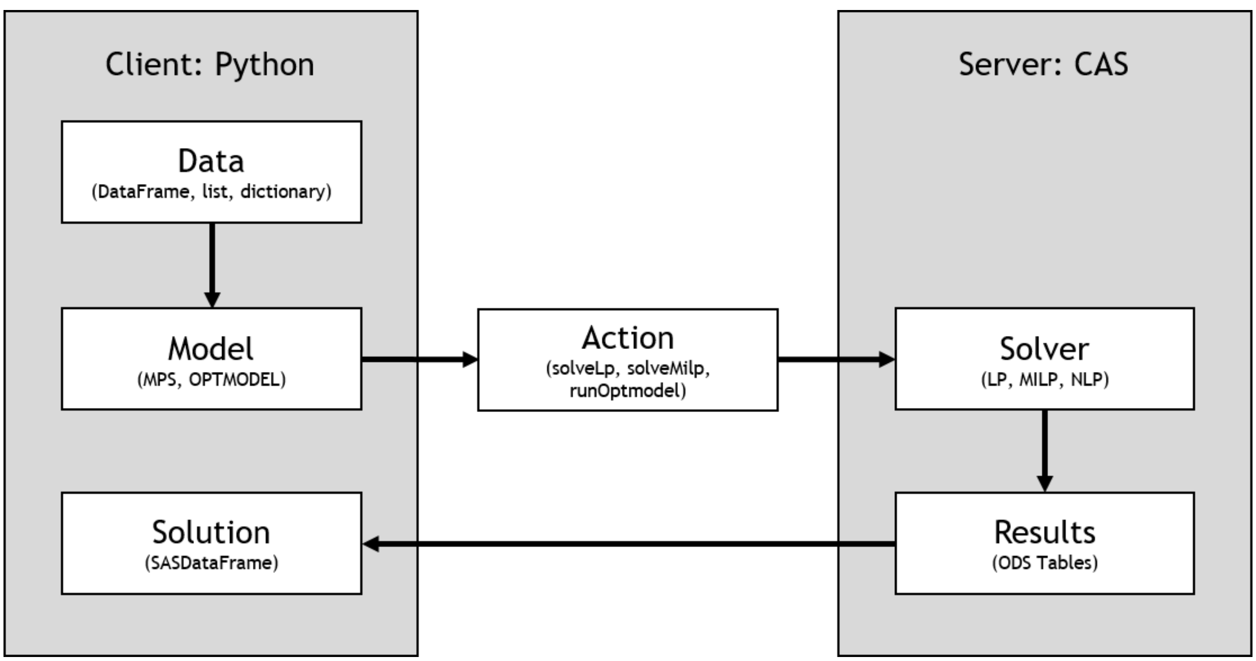
6.1 Client-side models

If the data is on the client-side (Python), then a concrete model is generated on the client-side and uploaded using one of the available CAS actions.

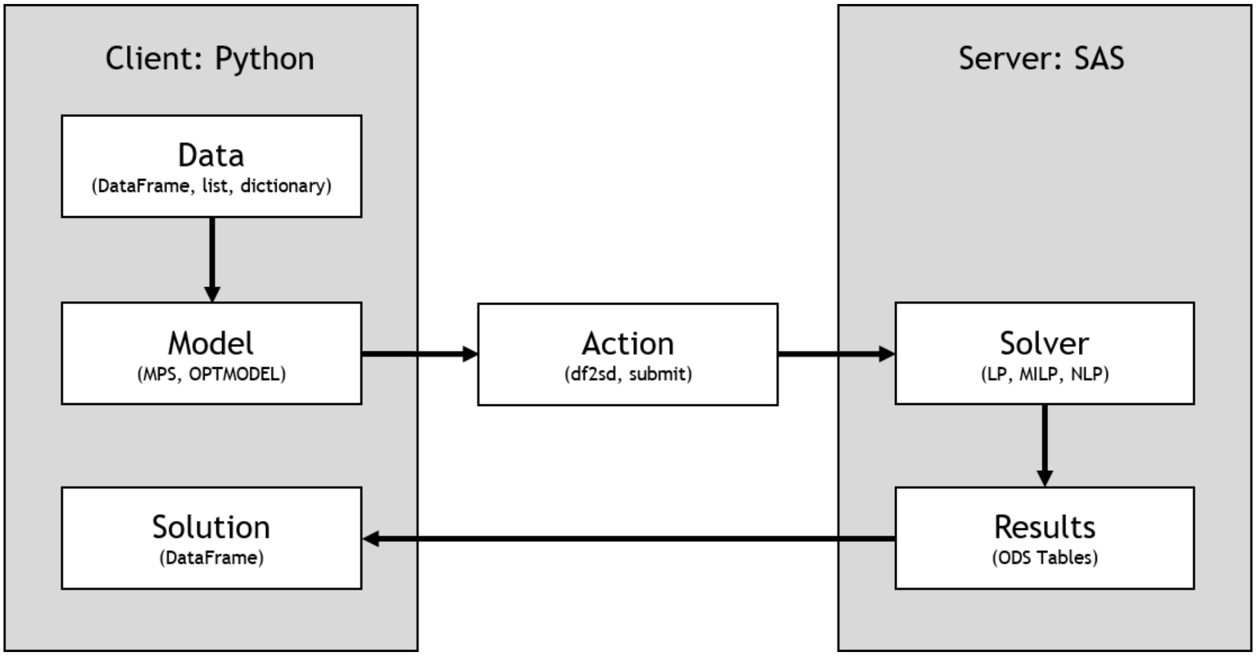
Using client-side models brings several advantages, such as accessing variables, expressions, and constraints directly. You may do more intensive operations like filter data, sort values, changing variable values, and print expressions more easily.

There are two main disadvantages of working with client-side models. First, if your model is relatively big size, then the generated MPS DataFrame or OPTMODEL codes may allocate a large memory on your machine. Second, the information that needs to be passed from client to server might be bigger than using a server-side model.

See the following representation of the client-side model workflow for CAS (Viya) servers:



See the following representation of the client-side model workflow for SAS clients:



Steps of modeling a simple Knapsack problem are shown in the following subsections.

6.1.1 Reading data

```
In [1]: import sasoptpy as so

In [2]: import pandas as pd

In [3]: from swat import CAS

In [4]: session = CAS(hostname, port)

In [5]: m = so.Model(name='client_CAS', session=session)
NOTE: Initialized model client_CAS.

In [6]: data = [
   ...:     ['clock', 8, 4, 3],
   ...:     ['mug', 10, 6, 5],
   ...:     ['headphone', 15, 7, 2],
   ...:     ['book', 20, 12, 10],
   ...:     ['pen', 1, 1, 15]
   ...:     ]
   ...:

In [7]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit'])

In [8]: ITEMS, (value, weight, limit) = m.read_table(
   ...:     df, key=['item'], columns=['value', 'weight', 'limit'])
   ...:

In [9]: total_weight = 55
```

```
In [10]: print(type(ITEMS), ITEMS)
<class 'list'> ['clock', 'mug', 'headphone', 'book', 'pen']
```

```
In [11]: print(type(total_weight), total_weight)
<class 'int'> 55
```

Here, instead of using `Model.read_table()` method, column values can be obtained one by one:

```
>>> df = df.set_index('item')
>>> ITEMS = df.index.tolist()
>>> value = df['value']
>>> weight = df['weight']
>>> limit = df['limit']
```

6.1.2 Model

```
# Variables
In [12]: get = m.add_variables(ITEMS, name='get', vartype=so.INT)

# Constraints
In [13]: m.add_constraints((get[i] <= limit[i] for i in ITEMS), name='limit_con');

In [14]: m.add_constraint(
   ....:     so.quick_sum(weight[i] * get[i] for i in ITEMS) <= total_weight,
   ....:     name='weight_con');
   ....:

# Objective
In [15]: total_value = so.quick_sum(value[i] * get[i] for i in ITEMS)

In [16]: m.set_objective(total_value, name='total_value', sense=so.MAX);

# Solve
In [17]: m.solve(verbose=True)
NOTE: Added action set 'optimization'.
NOTE: Converting model client_CAS to OPTMODEL.
var get {{'clock','mug','headphone','book','pen'}} integer >= 0;
con limit_con_clock : get['clock'] <= 3;
con limit_con_mug : get['mug'] <= 5;
con limit_con_headphone : get['headphone'] <= 2;
con limit_con_book : get['book'] <= 10;
con limit_con_pen : get['pen'] <= 15;

con weight_con : 7 * get['headphone'] + 4 * get['clock'] + get['pen'] + 12 * get['book
↪'] + 6 * get['mug'] <= 55;
max total_value = 15 * get['headphone'] + 8 * get['clock'] + get['pen'] + 20 * get[
↪'book'] + 10 * get['mug'];
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;

NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 5 variables (0 free, 0 fixed).
NOTE: The problem has 0 binary and 5 integer variables.
NOTE: The problem has 6 linear constraints (6 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 10 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
```

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```
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 5 constraints.
NOTE: The MILP presolver removed 5 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 5 variables, 1 constraints, and 5 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      3     99.0000000    199.0000000   50.25%       0
                0        1      3     99.0000000    102.3333333    3.26%       0
                0        1      3     99.0000000    102.3333333    3.26%       0
NOTE: The MILP presolver is applied again.
                0        1      3     99.0000000    102.3333333    3.26%       1
                0        1      3     99.0000000    102.3333333    3.26%       1
NOTE: The MILP solver added 3 cuts with 7 cut coefficients at the root.
NOTE: Optimal.
NOTE: Objective = 99.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 6 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 5 rows and 6␣
↪columns.
Out[17]:
              var   lb             ub  value  rc
0      get[clock] -0.0  1.797693e+308    3.0 NaN
1        get[mug] -0.0  1.797693e+308    4.0 NaN
2  get[headphone] -0.0  1.797693e+308    2.0 NaN
3       get[book] -0.0  1.797693e+308   -0.0 NaN
4        get[pen] -0.0  1.797693e+308    5.0 NaN
```

Using `verbose` option shows the generated OPTMODEL code. Here, we can see the coefficient values of the parameters inside the model.

6.1.3 Parsing results

After the solve, primal and dual solution tables are obtained. We can print the solution tables using the *Model.get_solution()* method.

It is also possible to print the optimal solution using the *get_solution_table()* function.

```
In [18]: print(m.get_solution())
              var   lb             ub  value  rc
0      get[clock] -0.0  1.797693e+308    3.0 NaN
1        get[mug] -0.0  1.797693e+308    4.0 NaN
2  get[headphone] -0.0  1.797693e+308    2.0 NaN
3       get[book] -0.0  1.797693e+308   -0.0 NaN
4        get[pen] -0.0  1.797693e+308    5.0 NaN
```

```
In [19]: print(so.get_solution_table(get, key=ITEMS));
```

```
In [20]: print('Total value:', total_value.get_value());
```

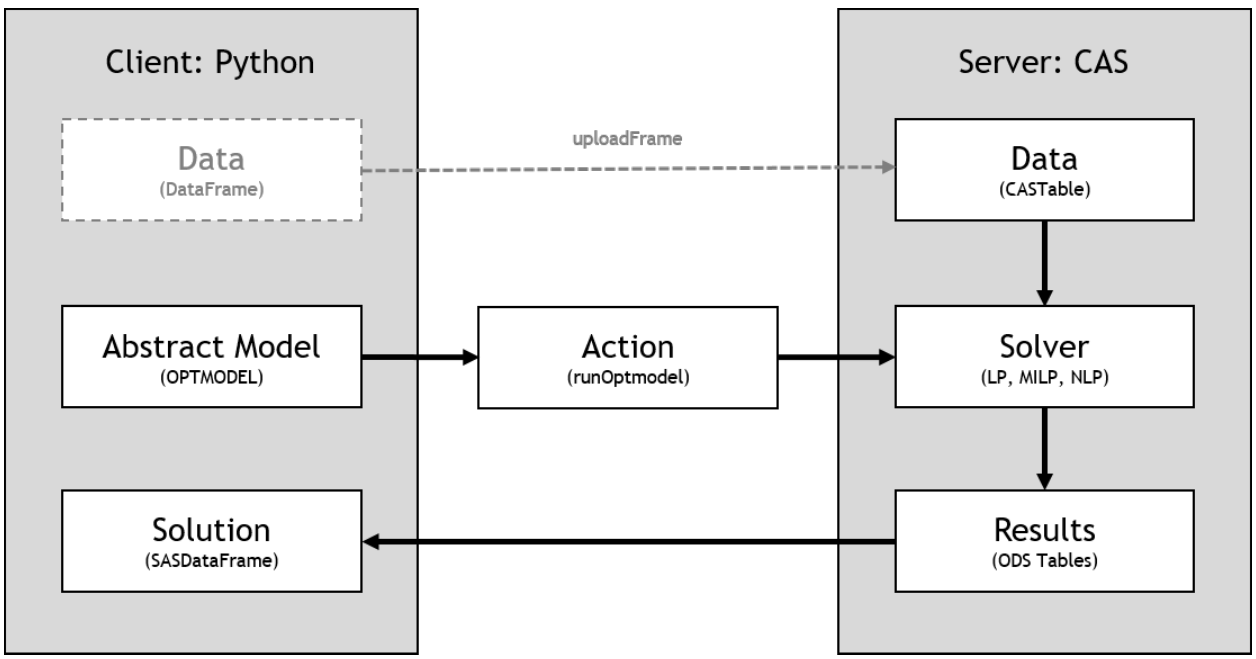
6.2 Server-side models

If the data is on the server-side (CAS or SAS), then an abstract model is generated on the client-side. This abstract model is later converted to PROC OPTMODEL code, which combines the data on the server.

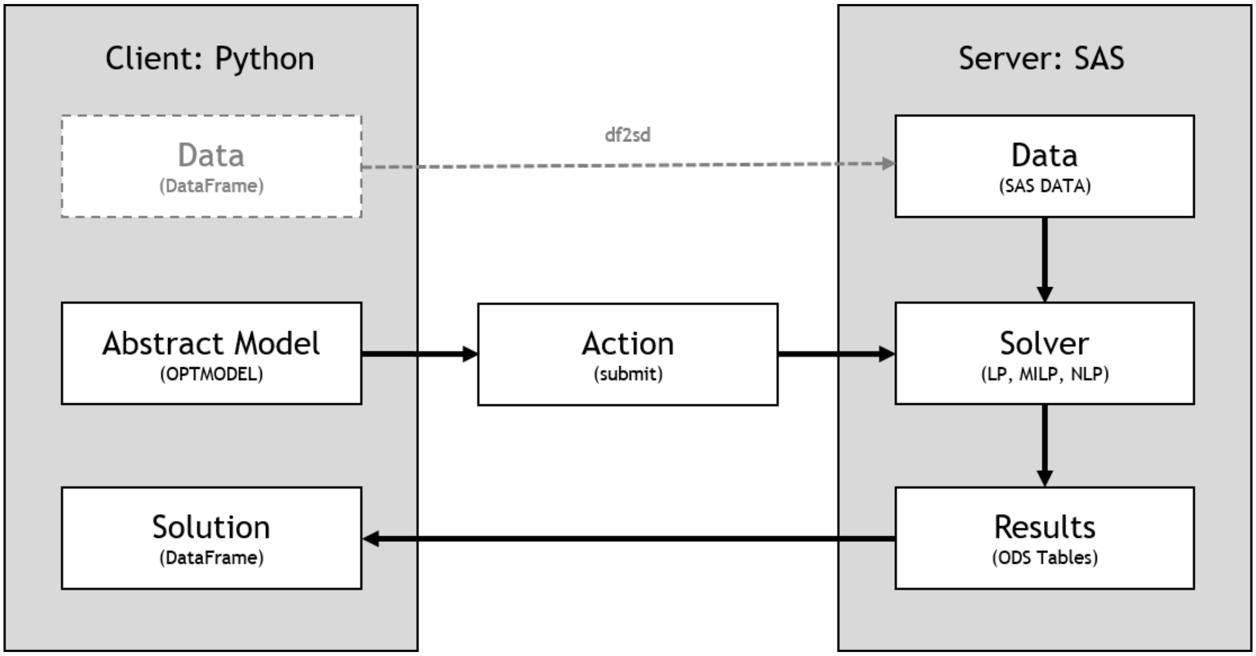
The main advantage of the server-side models is faster upload times compared to client-side. This is especially very noticable when using large chunks of variable and constraint groups.

The only disadvantage of using server-side models is that variables often need to be accessed directly from the resulting SASDataFrame objects. Since components of the models are abstract, accessing objects directly is often not possible.

See the following representation of the server-side model workflow for CAS (Viya) servers:



See the following representation of the server-side model workflow for SAS clients:



In the following subsections, the same example will be solved using server-side data.

6.2.1 Uploading data (Optional)

It is possible to upload client-side data to server-side when working with relatively big models.

sasoptpy supports using `swat.cas.table.CASTable` objects. The `swat.cas.connection.CAS.upload_frame()` method can be used to upload `pandas.DataFrame` objects to the CAS Server. Another way is to use `read_table()` function with `upload=True` option.

```
In [21]: session = CAS(hostname, port)

In [22]: m = so.Model(name='server_CAS', session=session)
NOTE: Initialized model server_CAS.

In [23]: data = [
   ....:     ['clock', 8, 4, 3],
   ....:     ['mug', 10, 6, 5],
   ....:     ['headphone', 15, 7, 2],
   ....:     ['book', 20, 12, 10],
   ....:     ['pen', 1, 1, 15]
   ....:     ]
   ....:

In [24]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit'])

In [25]: ITEMS, (value, weight, limit) = m.read_table(
   ....:     df, key=['item'], key_type='str', columns=['value', 'weight', 'limit'],
   ....:     upload=True, casout='df')
   ....:
NOTE: Cloud Analytic Services made the uploaded file available as table DF in caslib␣
↪CASUSERHDFS(casuser).
NOTE: The table DF has been created in caslib CASUSERHDFS(casuser) from binary data␣
↪uploaded to Cloud Analytic Services.
```

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```
In [26]: total_weight = m.add_parameter(init = 55, name='total_weight')
```

```
In [27]: print(type(ITEMS), ITEMS)
<class 'sasoptpy.data.Set'> set_item
```

```
In [28]: print(type(total_weight), total_weight)
<class 'sasoptpy.data.ParameterValue'> total_weight
```

Since we use `upload=True` option, the data is uploaded to the server and we get a CASTable object. Similarly, `total_weight` is a parameter object here.

6.2.2 Model

```
# Variables
In [29]: get = m.add_variables(ITEMS, name='get', vartype=so.INT)

# Constraints
In [30]: m.add_constraints((get[i] <= limit[i] for i in ITEMS), name='limit_con');

In [31]: m.add_constraint(
   ....:     so.quick_sum(weight[i] * get[i] for i in ITEMS) <= total_weight,
   ....:     name='weight_con');
   ....:

# Objective
In [32]: total_value = so.quick_sum(value[i] * get[i] for i in ITEMS)

In [33]: m.set_objective(total_value, name='total_value', sense=so.MAX);

# Solve
In [34]: m.solve(verbose=True)
NOTE: Added action set 'optimization'.
NOTE: Converting model server_CAS to OPTMODEL.
set <str> set_item;
num value {set_item};
num weight {set_item};
num limit {set_item};
read data DF into set_item=[item] value weight limit;
num total_weight = 55;
var get {set_item} integer >= 0;
con limit_con {i_1 in set_item} : - limit[i_1] + get[i_1] <= 0;

con weight_con : - sum {i_2 in set_item}(weight[i_2] * get[i_2]) + total_weight >= 0;
max total_value = sum {i_3 in set_item}(value[i_3] * get[i_3]);
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;

NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 5 rows read from table 'DF' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 5 variables (0 free, 0 fixed).
NOTE: The problem has 0 binary and 5 integer variables.
```

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```
NOTE: The problem has 6 linear constraints (5 LE, 0 EQ, 1 GE, 0 range).
NOTE: The problem has 10 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 5 constraints.
NOTE: The MILP presolver removed 5 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 5 variables, 1 constraints, and 5 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      3     99.0000000    199.0000000   50.25%       0
                0        1      3     99.0000000    102.3333333    3.26%       0
                0        1      3     99.0000000    102.3333333    3.26%       0
NOTE: The MILP presolver is applied again.
                0        1      3     99.0000000    102.3333333    3.26%       1
NOTE: Optimal.
NOTE: Objective = 99.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 6 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 5 rows and 6␣
↪columns.
Out[34]:
              var   lb            ub  value  rc
0       get[book] -0.0  1.797693e+308    2.0 NaN
1      get[clock] -0.0  1.797693e+308    3.0 NaN
2  get[headphone] -0.0  1.797693e+308    2.0 NaN
3        get[mug] -0.0  1.797693e+308   -0.0 NaN
4        get[pen] -0.0  1.797693e+308    5.0 NaN
```

There is no difference in terms of how client-side and server-side models are written. However, the generated OPTMODEL code is more compact for server-side models.

6.2.3 Parsing results

```
# Print results
In [35]: print(m.get_solution())
              var   lb            ub  value  rc
0       get[book] -0.0  1.797693e+308    2.0 NaN
1      get[clock] -0.0  1.797693e+308    3.0 NaN
2  get[headphone] -0.0  1.797693e+308    2.0 NaN
3        get[mug] -0.0  1.797693e+308   -0.0 NaN
4        get[pen] -0.0  1.797693e+308    5.0 NaN
```

```
In [36]: print('Total value:', m.get_objective_value())
Total value: 99.0
```

Since there is no direct access to expressions and variables, the optimal solution is printed using the server response.

6.3 Limitations

- Nonlinear models can only be solved using runOptmodel action, hence requires SAS Viya version to be greater than or equal to 3.4.
- User defined decomposition blocks are only available in MPS mode, hence only works with client-side data.
- Mixed usage (client-side and server-side data) may not work in some cases. A quick fix would be transferring the data, in either direction.

CHAPTER

SEVEN

EXAMPLES

Examples are provided from SAS/OR documentation.

7.1 Viya Examples / Concrete

7.1.1 Food Manufacture 1

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex1_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex01.html

Model

```
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    # Problem data
    OILS = ['veg1', 'veg2', 'oil1', 'oil2', 'oil3']
    PERIODS = range(1, 7)
    cost_data = [
        [110, 120, 130, 110, 115],
        [130, 130, 110, 90, 115],
        [110, 140, 130, 100,  95],
        [120, 110, 120, 120, 125],
        [100, 120, 150, 110, 105],
        [90, 100, 140,  80, 135]]
    cost = pd.DataFrame(cost_data, columns=OILS, index=PERIODS).transpose()
    hardness_data = [8.8, 6.1, 2.0, 4.2, 5.0]
    hardness = {OILS[i]: hardness_data[i] for i in range(len(OILS))}

    revenue_per_ton = 150
    veg_ub = 200
    nonveg_ub = 250
    store_ub = 1000
    storage_cost_per_ton = 5
```

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```
    hardness_lb = 3
    hardness_ub = 6
    init_storage = 500

    # Problem initialization
    m = so.Model(name='food_manufacture_1', session=cas_conn)

    # Problem definition
    buy = m.add_variables(OILS, PERIODS, lb=0, name='buy')
    use = m.add_variables(OILS, PERIODS, lb=0, name='use')
    manufacture = m.add_implicit_variable((use.sum('*', p) for p in PERIODS),
                                          name='manufacture')
    last_period = len(PERIODS)
    store = m.add_variables(OILS, [0] + list(PERIODS), lb=0, ub=store_ub,
                            name='store')
    for oil in OILS:
        store[oil, 0].set_bounds(lb=init_storage, ub=init_storage)
        store[oil, last_period].set_bounds(lb=init_storage, ub=init_storage)
    VEG = [i for i in OILS if 'veg' in i]
    NONVEG = [i for i in OILS if i not in VEG]
    revenue = so.quick_sum(revenue_per_ton * manufacture[p] for p in PERIODS)
    rawcost = so.quick_sum(cost.at[o, p] * buy[o, p]
                           for o in OILS for p in PERIODS)
    storagecost = so.quick_sum(storage_cost_per_ton * store[o, p]
                               for o in OILS for p in PERIODS)
    m.set_objective(revenue - rawcost - storagecost, sense=so.MAX,
                    name='profit')

    # Constraints
    m.add_constraints((use.sum(VEG, p) <= veg_ub for p in PERIODS),
                      name='veg_ub')
    m.add_constraints((use.sum(NONVEG, p) <= nonveg_ub for p in PERIODS),
                      name='nonveg_ub')
    m.add_constraints((store[o, p-1] + buy[o, p] == use[o, p] + store[o, p]
                      for o in OILS for p in PERIODS),
                      name='flow_balance')
    m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) >=
                      hardness_lb * manufacture[p] for p in PERIODS),
                      name='hardness_ub')
    m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) <=
                      hardness_ub * manufacture[p] for p in PERIODS),
                      name='hardness_lb')

    # Solver call
    res = m.solve()

    # With other solve options
    m.solve(options={'with': 'lp', 'algorithm': 'PS'})
    m.solve(options={'with': 'lp', 'algorithm': 'PS'})
    m.solve(options={'with': 'lp', 'algorithm': 'PS'})

    if res is not None:
        print(so.get_solution_table(buy, use, store))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.food_manufacture_1 import test

In [2]: test(cas_conn)
NOTE: Initialized model food_manufacture_1.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                            Objective
         Phase Iteration        Value         Time
          D 2          1    1.019986E+06         0
          D 2         54    1.233856E+05         0
          P 2         70    1.078426E+05         0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6␣
↪columns.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Primal Simplex algorithm is used.
                            Objective
         Phase Iteration        Value         Time
          P 1          1    2.310290E+03         0
          P 2         47    4.266801E+04         0
          P 2         57    8.634298E+04         0
```

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```
          D 2          71     1.078426E+05         0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Primal Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6␣
↪columns.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Primal Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          P 1           1     2.310290E+03         0
          P 2          47     4.266801E+04         0
          P 2          57     8.634298E+04         0
          D 2          71     1.078426E+05         0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Primal Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6␣
↪columns.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Primal Simplex algorithm is used.
```

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```
                              Objective
         Phase Iteration          Value         Time
          P 1          1    2.310290E+03           0
          P 2         47    4.266801E+04           0
          P 2         57    8.634298E+04           0
          D 2         71    1.078426E+05           0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Primal Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6␣
↪columns.
            buy      use       store
1    2
oil1 0        -        -  500.000000
oil1 1        0        0  500.000000
oil1 2        0        0  500.000000
oil1 3        0        0  500.000000
oil1 4        0        0  500.000000
oil1 5        0        0  500.000000
oil1 6        0        0  500.000000
oil2 0        -        -  500.000000
oil2 1        0        0  500.000000
oil2 2        0        0  500.000000
oil2 3        0        0  500.000000
oil2 4        0      250  250.000000
oil2 5        0      250    0.000000
oil2 6      750      250  500.000000
oil3 0        -        -  500.000000
oil3 1        0      250  250.000000
oil3 2        0      250    0.000000
oil3 3      250      250    0.000000
oil3 4        0        0    0.000000
oil3 5      500        0  500.000000
oil3 6        0        0  500.000000
veg1 0        -        -  500.000000
veg1 1        0  85.1852  414.814815
veg1 2        0  85.1852  329.629630
veg1 3        0  85.1852  244.444444
veg1 4        0  159.259   85.185185
veg1 5        0  85.1852    0.000000
veg1 6  659.259  159.259  500.000000
veg2 0        -        -  500.000000
veg2 1        0  114.815  385.185185
veg2 2        0  114.815  270.370370
veg2 3        0  114.815  155.555556
veg2 4        0  40.7407  114.814815
veg2 5        0  114.815    0.000000
veg2 6  540.741  40.7407  500.000000
Out[2]: 107842.592593
```

7.1.2 Food Manufacture 2

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex2_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex02.html

Model

```
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    # Problem data
    OILS = ['veg1', 'veg2', 'oil1', 'oil2', 'oil3']
    PERIODS = range(1, 7)
    cost_data = [
        [110, 120, 130, 110, 115],
        [130, 130, 110, 90, 115],
        [110, 140, 130, 100,  95],
        [120, 110, 120, 120, 125],
        [100, 120, 150, 110, 105],
        [90, 100, 140,  80, 135]]
    cost = pd.DataFrame(cost_data, columns=OILS, index=PERIODS).transpose()
    hardness_data = [8.8, 6.1, 2.0, 4.2, 5.0]
    hardness = {OILS[i]: hardness_data[i] for i in range(len(OILS))}

    revenue_per_ton = 150
    veg_ub = 200
    nonveg_ub = 250
    store_ub = 1000
    storage_cost_per_ton = 5
    hardness_lb = 3
    hardness_ub = 6
    init_storage = 500
    max_num_oils_used = 3
    min_oil_used_threshold = 20

    # Problem initialization
    m = so.Model(name='food_manufacture_2', session=cas_conn)

    # Problem definition
    buy = m.add_variables(OILS, PERIODS, lb=0, name='buy')
    use = m.add_variables(OILS, PERIODS, lb=0, name='use')
    manufacture = m.add_implicit_variable((use.sum('*', p) for p in PERIODS),
                                          name='manufacture')
    last_period = len(PERIODS)
    store = m.add_variables(OILS, [0] + list(PERIODS), lb=0, ub=store_ub,
                            name='store')
    for oil in OILS:
        store[oil, 0].set_bounds(lb=init_storage, ub=init_storage)
        store[oil, last_period].set_bounds(lb=init_storage, ub=init_storage)
```

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```
    VEG = [i for i in OILS if 'veg' in i]
    NONVEG = [i for i in OILS if i not in VEG]
    revenue = so.quick_sum(revenue_per_ton * manufacture[p] for p in PERIODS)
    rawcost = so.quick_sum(cost.at[o, p] * buy[o, p]
                           for o in OILS for p in PERIODS)
    storagecost = so.quick_sum(storage_cost_per_ton * store[o, p]
                               for o in OILS for p in PERIODS)
    m.set_objective(revenue - rawcost - storagecost, sense=so.MAX,
                    name='profit')

    # Constraints
    m.add_constraints((use.sum(VEG, p) <= veg_ub for p in PERIODS),
                      name='veg_ub')
    m.add_constraints((use.sum(NONVEG, p) <= nonveg_ub for p in PERIODS),
                      name='nonveg_ub')
    m.add_constraints((store[o, p-1] + buy[o, p] == use[o, p] + store[o, p]
                      for o in OILS for p in PERIODS),
                      name='flow_balance')
    m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) >=
                      hardness_lb * manufacture[p] for p in PERIODS),
                      name='hardness_ub')
    m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) <=
                      hardness_ub * manufacture[p] for p in PERIODS),
                      name='hardness_lb')

    # Additions to the first problem
    isUsed = m.add_variables(OILS, PERIODS, vartype=so.BIN, name='is_used')
    for p in PERIODS:
        for o in VEG:
            use[o, p].set_bounds(ub=veg_ub)
        for o in NONVEG:
            use[o, p].set_bounds(ub=nonveg_ub)
    m.add_constraints((use[o, p] <= use[o, p]._ub * isUsed[o, p]
                      for o in OILS for p in PERIODS), name='link')
    m.add_constraints((isUsed.sum('*', p) <= max_num_oils_used
                      for p in PERIODS), name='logical1')
    m.add_constraints((use[o, p] >= min_oil_used_threshold * isUsed[o, p]
                      for o in OILS for p in PERIODS), name='logical2')
    m.add_constraints((isUsed[o, p] <= isUsed['oil3', p]
                      for o in ['veg1', 'veg2'] for p in PERIODS),
                      name='logical3')

    res = m.solve()
    if res is not None:
        print(so.get_solution_table(buy, use, store, isUsed))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.food_manufacture_2 import test

In [2]: test(cas_conn)
NOTE: Initialized model food_manufacture_2.
NOTE: Added action set 'optimization'.
```

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```
NOTE: Converting model food_manufacture_2 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 125 variables (0 free, 10 fixed).
NOTE: The problem has 30 binary and 0 integer variables.
NOTE: The problem has 132 linear constraints (66 LE, 30 EQ, 36 GE, 0 range).
NOTE: The problem has 384 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 50 variables and 10 constraints.
NOTE: The MILP presolver removed 66 constraint coefficients.
NOTE: The MILP presolver modified 6 constraint coefficients.
NOTE: The presolved problem has 75 variables, 122 constraints, and 318 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      3  29000.0000000         343250   91.55%       0
                0        1      3  29000.0000000         107333   72.98%       0
                0        1      3  29000.0000000         105799   72.59%       0
                0        1      3  29000.0000000         105650   72.55%       0
                0        1      3  29000.0000000         105650   72.55%       0
                0        1      3  29000.0000000         105650   72.55%       0
                0        1      3  29000.0000000         105650   72.55%       0
                0        1      3  29000.0000000         105650   72.55%       0
                0        1      4  44000.0000000         105650   58.35%       0
NOTE: The MILP solver added 14 cuts with 69 cut coefficients at the root.
               42       29      5  93416.6666667         104429   10.55%       0
               55       35      6  93416.6666667         104040   10.21%       0
               62       36      7  99008.3333333         104040    4.84%       0
               93       61      8  99683.3333333         104040    4.19%       0
              105       57      9  99872.2222222         103576    3.58%       0
              114       57     10         100214         103576    3.25%       0
              173       52     11         100214         103431    3.11%       1
              249       62     12         100279         103192    2.82%       1
              417        0     12         100279         100279    0.00%       1
NOTE: Optimal.
NOTE: Objective = 100278.7037.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 132 rows and 4␣
↪columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 125 rows and 6␣
↪columns.
           buy      use       store is_used
1    2
oil1 0       -        -  500.000000       -
oil1 1       0        0  500.000000       0
oil1 2       0        0  500.000000       0
oil1 3       0        0  500.000000       0
oil1 4       0        0  500.000000       0
oil1 5       0        0  500.000000       0
```

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```
oil1 6       0        0  500.000000       0
oil2 0       -        -  500.000000       -
oil2 1       0        0  500.000000       0
oil2 2       0        0  500.000000       0
oil2 3       0       40  460.000000       1
oil2 4       0      230  230.000000       1
oil2 5       0      230    0.000000       1
oil2 6     730      230  500.000000       1
oil3 0       -        -  500.000000       -
oil3 1       0      250  250.000000       1
oil3 2       0      250    0.000000       1
oil3 3     770      210  560.000000       1
oil3 4       0       20  540.000000       1
oil3 5      -0       20  520.000000       1
oil3 6       0       20  500.000000       1
veg1 0       -        -  500.000000       -
veg1 1       0  85.1852  414.814815       1
veg1 2       0  85.1852  329.629630       1
veg1 3       0        0  329.629630       0
veg1 4       0      155  174.629630       1
veg1 5      -0      155   19.629630       1
veg1 6  480.37        0  500.000000       0
veg2 0       -        -  500.000000       -
veg2 1       0  114.815  385.185185       1
veg2 2       0  114.815  270.370370       1
veg2 3      -0      200   70.370370       1
veg2 4      -0        0   70.370370       0
veg2 5       0        0   70.370370       0
veg2 6  629.63      200  500.000000       1
Out[2]: 100278.703704
```

7.1.3 Factory Planning 1

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex3_toc.htm&docsetVersion=14.3&locale=en

https://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex03.html

Model

```python
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='factory_planning_1', session=cas_conn)

    # Input data
    product_list = ['prod{}'.format(i) for i in range(1, 8)]
    product_data = pd.DataFrame([10, 6, 8, 4, 11, 9, 3],
```

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```
                                columns=['profit'], index=product_list)
    demand_data = [
        [500, 1000, 300, 300,  800, 200, 100],
        [600,  500, 200,   0,  400, 300, 150],
        [300,  600,   0,   0,  500, 400, 100],
        [200,  300, 400, 500,  200,   0, 100],
        [0,    100, 500, 100, 1000, 300,   0],
        [500,  500, 100, 300, 1100, 500,  60]]
    demand_data = pd.DataFrame(
        demand_data, columns=product_list, index=range(1, 7))
    machine_types_data = [
        ['grinder', 4],
        ['vdrill', 2],
        ['hdrill', 3],
        ['borer', 1],
        ['planer', 1]]
    machine_types_data = pd.DataFrame(machine_types_data, columns=[
        'machine_type', 'num_machines']).set_index(['machine_type'])
    machine_type_period_data = [
        ['grinder', 1, 1],
        ['hdrill',  2, 2],
        ['borer',   3, 1],
        ['vdrill',  4, 1],
        ['grinder', 5, 1],
        ['vdrill',  5, 1],
        ['planer',  6, 1],
        ['hdrill',  6, 1]]
    machine_type_period_data = pd.DataFrame(machine_type_period_data, columns=[
        'machine_type', 'period', 'num_down'])
    machine_type_product_data = [
        ['grinder', 0.5,  0.7,  0,    0,    0.3,  0.2, 0.5],
        ['vdrill',  0.1,  0.2,  0,    0.3,  0,    0.6, 0],
        ['hdrill',  0.2,  0,    0.8,  0,    0,    0,   0.6],
        ['borer',   0.05, 0.03, 0,    0.07, 0.1,  0,   0.08],
        ['planer',  0,    0,    0.01, 0,    0.05, 0,   0.05]]
    machine_type_product_data = \
        pd.DataFrame(machine_type_product_data, columns=['machine_type'] +
                     product_list).set_index(['machine_type'])
    store_ub = 100
    storage_cost_per_unit = 0.5
    final_storage = 50
    num_hours_per_period = 24 * 2 * 8

    # Problem definition
    PRODUCTS = product_list
    PERIODS = range(1, 7)
    MACHINE_TYPES = machine_types_data.index.values

    num_machine_per_period = pd.DataFrame()
    for i in range(1, 7):
        num_machine_per_period[i] = machine_types_data['num_machines']
    for _, row in machine_type_period_data.iterrows():
        num_machine_per_period.at[row['machine_type'],
                                  row['period']] -= row['num_down']

    make = m.add_variables(PRODUCTS, PERIODS, lb=0, name='make')
    sell = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=demand_data.transpose(),
```

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```
                         name='sell')

    store = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=store_ub, name='store')
    for p in PRODUCTS:
        store[p, 6].set_bounds(lb=final_storage, ub=final_storage+1)

    storageCost = storage_cost_per_unit * store.sum('*', '*')
    revenue = so.quick_sum(product_data.at[p, 'profit'] * sell[p, t]
                           for p in PRODUCTS for t in PERIODS)
    m.set_objective(revenue-storageCost, sense=so.MAX, name='total_profit')

    production_time = machine_type_product_data
    m.add_constraints((
        so.quick_sum(production_time.at[mc, p] * make[p, t] for p in PRODUCTS)
        <= num_hours_per_period * num_machine_per_period.at[mc, t]
        for mc in MACHINE_TYPES for t in PERIODS), name='machine_hours')
    m.add_constraints(((store[p, t-1] if t-1 in PERIODS else 0) + make[p, t] ==
                      sell[p, t] + store[p, t] for p in PRODUCTS
                      for t in PERIODS),
                      name='flow_balance')

    res = m.solve()
    if res is not None:
        print(so.get_solution_table(make, sell, store))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.factory_planning_1 import test

In [2]: test(cas_conn)
NOTE: Initialized model factory_planning_1.
NOTE: Added action set 'optimization'.
NOTE: Converting model factory_planning_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 126 variables (0 free, 6 fixed).
NOTE: The problem has 72 linear constraints (30 LE, 42 EQ, 0 GE, 0 range).
NOTE: The problem has 281 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 24 variables and 23 constraints.
NOTE: The LP presolver removed 91 constraint coefficients.
NOTE: The presolved problem has 102 variables, 49 constraints, and 190 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                           Objective
         Phase Iteration        Value         Time
          D 2          1    9.501963E+04         0
          P 2         34    9.371518E+04         0
NOTE: Optimal.
NOTE: Objective = 93715.178571.
```

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```
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 72 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 126 rows and 6␣
↪columns.
                make         sell  store
1      2
prod1 1   500.000000   500.000000    0.0
prod1 2   700.000000   600.000000  100.0
prod1 3     0.000000   100.000000    0.0
prod1 4   200.000000   200.000000    0.0
prod1 5     0.000000     0.000000    0.0
prod1 6   550.000000   500.000000   50.0
prod2 1   888.571429   888.571429    0.0
prod2 2   600.000000   500.000000  100.0
prod2 3     0.000000   100.000000    0.0
prod2 4   300.000000   300.000000    0.0
prod2 5   100.000000   100.000000    0.0
prod2 6   550.000000   500.000000   50.0
prod3 1   382.500000   300.000000   82.5
prod3 2   117.500000   200.000000    0.0
prod3 3     0.000000     0.000000    0.0
prod3 4   400.000000   400.000000    0.0
prod3 5   600.000000   500.000000  100.0
prod3 6     0.000000    50.000000   50.0
prod4 1   300.000000   300.000000    0.0
prod4 2     0.000000     0.000000    0.0
prod4 3     0.000000     0.000000    0.0
prod4 4   500.000000   500.000000    0.0
prod4 5   100.000000   100.000000    0.0
prod4 6   350.000000   300.000000   50.0
prod5 1   800.000000   800.000000    0.0
prod5 2   500.000000   400.000000  100.0
prod5 3     0.000000   100.000000    0.0
prod5 4   200.000000   200.000000    0.0
prod5 5  1100.000000  1000.000000  100.0
prod5 6     0.000000    50.000000   50.0
prod6 1   200.000000   200.000000    0.0
prod6 2   300.000000   300.000000    0.0
prod6 3   400.000000   400.000000    0.0
prod6 4     0.000000     0.000000    0.0
prod6 5   300.000000   300.000000    0.0
prod6 6   550.000000   500.000000   50.0
prod7 1     0.000000     0.000000    0.0
prod7 2   250.000000   150.000000  100.0
prod7 3     0.000000   100.000000    0.0
prod7 4   100.000000   100.000000    0.0
prod7 5   100.000000     0.000000  100.0
prod7 6     0.000000    50.000000   50.0
Out[2]: 93715.178571
```

7.1.4 Factory Planning 2

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex4_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex04.html

Model

```
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='factory_planning_2', session=cas_conn)

    # Input data
    product_list = ['prod{}'.format(i) for i in range(1, 8)]
    product_data = pd.DataFrame([10, 6, 8, 4, 11, 9, 3],
                                columns=['profit'], index=product_list)
    demand_data = [
        [500, 1000, 300, 300,  800, 200, 100],
        [600,  500, 200,   0,  400, 300, 150],
        [300,  600,   0,   0,  500, 400, 100],
        [200,  300, 400, 500,  200,   0, 100],
        [0,    100, 500, 100, 1000, 300,   0],
        [500,  500, 100, 300, 1100, 500,  60]]
    demand_data = pd.DataFrame(
        demand_data, columns=product_list, index=range(1, 7))
    machine_type_product_data = [
        ['grinder', 0.5,  0.7,  0,    0,    0.3,  0.2, 0.5],
        ['vdrill',  0.1,  0.2,  0,    0.3,  0,    0.6, 0],
        ['hdrill',  0.2,  0,    0.8,  0,    0,    0,   0.6],
        ['borer',   0.05, 0.03, 0,    0.07, 0.1,  0,   0.08],
        ['planer',  0,    0,    0.01, 0,    0.05, 0,   0.05]]
    machine_type_product_data = \
        pd.DataFrame(machine_type_product_data, columns=['machine_type'] +
                     product_list).set_index(['machine_type'])
    machine_types_data = [
        ['grinder', 4, 2],
        ['vdrill',  2, 2],
        ['hdrill',  3, 3],
        ['borer',   1, 1],
        ['planer',  1, 1]]
    machine_types_data = pd.DataFrame(machine_types_data, columns=[
        'machine_type', 'num_machines', 'num_machines_needing_maintenance'])\
        .set_index(['machine_type'])

    store_ub = 100
    storage_cost_per_unit = 0.5
    final_storage = 50
    num_hours_per_period = 24 * 2 * 8
```

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```
    # Problem definition
    PRODUCTS = product_list
    profit = product_data['profit']
    PERIODS = range(1, 7)
    MACHINE_TYPES = machine_types_data.index.tolist()

    num_machines = machine_types_data['num_machines']

    make = m.add_variables(PRODUCTS, PERIODS, lb=0, name='make')
    sell = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=demand_data.transpose(),
                           name='sell')

    store = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=store_ub, name='store')
    for p in PRODUCTS:
        store[p, 6].set_bounds(lb=final_storage, ub=final_storage)

    storageCost = so.quick_sum(
        storage_cost_per_unit * store[p, t] for p in PRODUCTS for t in PERIODS)
    revenue = so.quick_sum(profit[p] * sell[p, t]
                           for p in PRODUCTS for t in PERIODS)
    m.set_objective(revenue-storageCost, sense=so.MAX, name='total_profit')

    num_machines_needing_maintenance = \
        machine_types_data['num_machines_needing_maintenance']
    numMachinesDown = m.add_variables(MACHINE_TYPES, PERIODS, vartype=so.INT,
                                      lb=0, name='numMachinesDown')

    production_time = machine_type_product_data
    m.add_constraints((
        so.quick_sum(production_time.at[mc, p] * make[p, t] for p in PRODUCTS)
        <= num_hours_per_period *
        (num_machines[mc] - numMachinesDown[mc, t])
        for mc in MACHINE_TYPES for t in PERIODS), name='machine_hours_con')

    m.add_constraints((so.quick_sum(numMachinesDown[mc, t] for t in PERIODS) ==
                       num_machines_needing_maintenance[mc]
                       for mc in MACHINE_TYPES), name='maintenance_con')

    m.add_constraints(((store[p, t-1] if t-1 in PERIODS else 0) + make[p, t] ==
                      sell[p, t] + store[p, t]
                      for p in PRODUCTS for t in PERIODS),
                      name='flow_balance_con')

    res = m.solve()
    if res is not None:
        print(so.get_solution_table(make, sell, store))
        print(so.get_solution_table(numMachinesDown).unstack(level=-1))

    print(m.get_solution_summary())
    print(m.get_problem_summary())

    return m.get_objective_value()
```

Output

```
In [1]: from examples.factory_planning_2 import test

In [2]: test(cas_conn)
NOTE: Initialized model factory_planning_2.
NOTE: Added action set 'optimization'.
NOTE: Converting model factory_planning_2 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 156 variables (0 free, 13 fixed).
NOTE: The problem has 0 binary and 30 integer variables.
NOTE: The problem has 77 linear constraints (30 LE, 47 EQ, 0 GE, 0 range).
NOTE: The problem has 341 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 27 variables and 15 constraints.
NOTE: The MILP presolver removed 63 constraint coefficients.
NOTE: The MILP presolver modified 16 constraint coefficients.
NOTE: The presolved problem has 129 variables, 62 constraints, and 278 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      2  92755.0000000         116455   20.35%       0
                0        1      2  92755.0000000         116455   20.35%       0
                0        1      2  92755.0000000         116141   20.14%       0
                0        1      2  92755.0000000         115660   19.80%       0
                0        1      2  92755.0000000         114597   19.06%       0
                0        1      2  92755.0000000         113265   18.11%       0
                0        1      2  92755.0000000         111849   17.07%       0
                0        1      2  92755.0000000         110679   16.19%       0
                0        1      2  92755.0000000         109751   15.49%       0
                0        1      2  92755.0000000         109476   15.27%       0
                0        1      2  92755.0000000         109039   14.93%       0
                0        1      2  92755.0000000         108998   14.90%       0
                0        1      2  92755.0000000         108924   14.84%       0
                0        1      2  92755.0000000         108893   14.82%       0
                0        1      2  92755.0000000         108863   14.80%       0
                0        1      3         108855         108863    0.01%       0
NOTE: The MILP solver added 46 cuts with 181 cut coefficients at the root.
NOTE: Optimal within relative gap.
NOTE: Objective = 108855.00931.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 77 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 156 rows and 6␣
↪columns.
                make         sell       store
1     2
prod1 1   500.000000   500.000000    0.000000
prod1 2   600.000665   600.000000    0.000665
```

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```
prod1 3   399.999335   300.000000  100.000000
prod1 4     0.000544   100.000544    0.000000
prod1 5     0.000000     0.000000    0.000000
prod1 6   550.000000   500.000000   50.000000
prod2 1  1000.000000  1000.000000    0.000000
prod2 2   500.000000   500.000000    0.000000
prod2 3   699.998346   599.998891   99.999456
prod2 4     0.003085   100.001089    0.001452
prod2 5   100.000323   100.000000    0.001775
prod2 6   549.998225   500.000000   50.000000
prod3 1   300.000000   300.000000    0.000000
prod3 2   199.999677   199.999677    0.000000
prod3 3    99.999456     0.000000   99.999456
prod3 4     0.002178   100.001633    0.000000
prod3 5   500.000000   500.000000    0.000000
prod3 6   150.000000   100.000000   50.000000
prod4 1   300.000000   300.000000    0.000000
prod4 2     0.000000     0.000000    0.000000
prod4 3    99.999456     0.000000   99.999456
prod4 4     0.002722   100.002178    0.000000
prod4 5   100.001129   100.000000    0.001129
prod4 6   349.998871   300.000000   50.000000
prod5 1   800.000000   800.000000    0.000000
prod5 2   399.999544   399.999322    0.000222
prod5 3   599.998619   499.999113   99.999728
prod5 4     0.000817   100.000544    0.000000
prod5 5  1000.004598  1000.000000    0.004598
prod5 6  1149.995402  1100.000000   50.000000
prod6 1   200.000000   200.000000    0.000000
prod6 2   300.000000   300.000000    0.000000
prod6 3   400.000000   400.000000    0.000000
prod6 4     0.000000     0.000000    0.000000
prod6 5   300.000000   300.000000    0.000000
prod6 6   550.000000   500.000000   50.000000
prod7 1   100.000000   100.000000    0.000000
prod7 2   150.000222   150.000000    0.000222
prod7 3   199.999234   100.000000   99.999456
prod7 4     0.000544   100.000000    0.000000
prod7 5     0.000355     0.000000    0.000355
prod7 6   109.999645    60.000000   50.000000
        numMachinesDown numMachinesDown numMachinesDown numMachinesDown  \
2                     1               2               3               4
1
borer          0.000000        0.000000        0.000002        0.999995
grinder        0.000000        0.000000        0.000000        2.000000
hdrill         0.999999        2.000001        0.000000        0.000000
planer         0.000000        0.000003        0.000002        0.999995
vdrill         0.000000        0.000000        0.000000        1.999996

        numMachinesDown numMachinesDown
2                     5               6
1
borer          0.000000        0.000003
grinder        0.000000        0.000000
hdrill         0.000000        0.000000
planer         0.000000        0.000000
vdrill         0.000001        0.000003
```

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```
Solution Summary

                                               Value
Label
Solver                                          MILP
Algorithm                             Branch and Cut
Objective Function                      total_profit
Solution Status         Optimal within Relative Gap
Objective Value                         108855.00931

Relative Gap                            0.0000746796
Absolute Gap                            8.1298591363
Primal Infeasibility                    2.060574E-13
Bound Infeasibility                                0
Integer Infeasibility                   5.4448817E-6

Best Bound                              108863.13917
Nodes                                              1
Solutions Found                                    3
Iterations                                       328
Presolve Time                                   0.06
Solution Time                                   1.59
Problem Summary

                                  Value
Label
Objective Sense            Maximization
Objective Function         total_profit
Objective Type                   Linear

Number of Variables                 156
Bounded Above                         0
Bounded Below                        72
Bounded Below and Above              71
Free                                  0
Fixed                                13
Binary                                0
Integer                              30

Number of Constraints                77
Linear LE (<=)                       30
Linear EQ (=)                        47
Linear GE (>=)                        0
Linear Range                          0

Constraint Coefficients             341
Out[2]: 108855.009314
```

7.1.5 Manpower Planning

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex5_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex05.html

Model

```
import sasoptpy as so
import pandas as pd
import math


def test(cas_conn):
    # Input data
    demand_data = pd.DataFrame([
        [0, 2000, 1500, 1000],
        [1, 1000, 1400, 1000],
        [2, 500, 2000, 1500],
        [3, 0, 2500, 2000]
        ], columns=['period', 'unskilled', 'semiskilled', 'skilled'])\
        .set_index(['period'])
    worker_data = pd.DataFrame([
        ['unskilled',   0.25, 0.10, 500, 200, 1500, 50, 500],
        ['semiskilled', 0.20, 0.05, 800, 500, 2000, 50, 400],
        ['skilled',     0.10, 0.05, 500, 500, 3000, 50, 400]
        ], columns=['worker', 'waste_new', 'waste_old', 'recruit_ub',
                    'redundancy_cost', 'overmanning_cost', 'shorttime_ub',
                    'shorttime_cost']).set_index(['worker'])
    retrain_data = pd.DataFrame([
        ['unskilled', 'semiskilled', 200, 400],
        ['semiskilled', 'skilled', math.inf, 500],
        ], columns=['worker1', 'worker2', 'retrain_ub', 'retrain_cost']).\
        set_index(['worker1', 'worker2'])
    downgrade_data = pd.DataFrame([
        ['semiskilled', 'unskilled'],
        ['skilled', 'semiskilled'],
        ['skilled', 'unskilled']
        ], columns=['worker1', 'worker2'])

    semiskill_retrain_frac_ub = 0.25
    downgrade_leave_frac = 0.5
    overmanning_ub = 150
    shorttime_frac = 0.5

    # Sets
    WORKERS = worker_data.index.tolist()
    PERIODS0 = demand_data.index.tolist()
    PERIODS = PERIODS0[1:]
    RETRAIN_PAIRS = [i for i, _ in retrain_data.iterrows()]
    DOWNGRADE_PAIRS = [(row['worker1'], row['worker2'])
                       for _, row in downgrade_data.iterrows()]

    waste_old = worker_data['waste_old']
    waste_new = worker_data['waste_new']
    redundancy_cost = worker_data['redundancy_cost']
    overmanning_cost = worker_data['overmanning_cost']
    shorttime_cost = worker_data['shorttime_cost']
    retrain_cost = retrain_data['retrain_cost'].unstack(level=-1)

    # Initialization
    m = so.Model(name='manpower_planning', session=cas_conn)
```

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```
    # Variables
    numWorkers = m.add_variables(WORKERS, PERIODS0, name='numWorkers', lb=0)
    demand0 = demand_data.loc[0]
    for w in WORKERS:
        numWorkers[w, 0].set_bounds(lb=demand0[w], ub=demand0[w])
    numRecruits = m.add_variables(WORKERS, PERIODS, name='numRecruits', lb=0)
    worker_ub = worker_data['recruit_ub']
    for w in WORKERS:
        for p in PERIODS:
            numRecruits[w, p].set_bounds(ub=worker_ub[w])
    numRedundant = m.add_variables(WORKERS, PERIODS, name='numRedundant', lb=0)
    numShortTime = m.add_variables(WORKERS, PERIODS, name='numShortTime', lb=0)
    shorttime_ub = worker_data['shorttime_ub']
    for w in WORKERS:
        for p in PERIODS:
            numShortTime.set_bounds(ub=shorttime_ub[w])
    numExcess = m.add_variables(WORKERS, PERIODS, name='numExcess', lb=0)

    retrain_ub = pd.DataFrame()
    for i in PERIODS:
        retrain_ub[i] = retrain_data['retrain_ub']
    numRetrain = m.add_variables(RETRAIN_PAIRS, PERIODS, name='numRetrain',
                                 lb=0, ub=retrain_ub)

    numDowngrade = m.add_variables(DOWNGRADE_PAIRS, PERIODS,
                                   name='numDowngrade', lb=0)
    # Constraints
    m.add_constraints((numWorkers[w, p]
                      - (1-shorttime_frac) * numShortTime[w, p]
                      - numExcess[w, p] == demand_data.loc[p, w]
                      for w in WORKERS for p in PERIODS), name='demand')
    m.add_constraints((numWorkers[w, p] ==
                      (1 - waste_old[w]) * numWorkers[w, p-1]
                      + (1 - waste_new[w]) * numRecruits[w, p]
                      + (1 - waste_old[w]) * numRetrain.sum('*', w, p)
                      + (1 - downgrade_leave_frac) *
                      numDowngrade.sum('*', w, p)
                      - numRetrain.sum(w, '*', p)
                      - numDowngrade.sum(w, '*', p)
                      - numRedundant[w, p]
                      for w in WORKERS for p in PERIODS),
                      name='flow_balance')
    m.add_constraints((numRetrain['semiskilled', 'skilled', p] <=
                      semiskill_retrain_frac_ub * numWorkers['skilled', p]
                      for p in PERIODS), name='semiskill_retrain')
    m.add_constraints((numExcess.sum('*', p) <= overmanning_ub
                      for p in PERIODS), name='overmanning')
    # Objectives
    redundancy = so.Expression(numRedundant.sum('*', '*'), name='redundancy')
    cost = so.Expression(so.quick_sum(redundancy_cost[w] * numRedundant[w, p] +
                                      shorttime_cost[w] * numShortTime[w, p] +
                                      overmanning_cost[w] * numExcess[w, p]
                                      for w in WORKERS for p in PERIODS)
                         + so.quick_sum(
                             retrain_cost.loc[i, j] * numRetrain[i, j, p]
                             for i, j in RETRAIN_PAIRS for p in PERIODS),
                         name='cost')
```

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```
    m.set_objective(redundancy, sense=so.MIN, name='redundancy_obj')
    res = m.solve()
    if res is not None:
        print('Redundancy:', redundancy.get_value())
        print('Cost:', cost.get_value())
        print(so.get_solution_table(
            numWorkers, numRecruits, numRedundant, numShortTime, numExcess))
        print(so.get_solution_table(numRetrain))
        print(so.get_solution_table(numDowngrade))

    m.set_objective(cost, sense=so.MIN, name='cost_obj')
    res = m.solve()
    if res is not None:
        print('Redundancy:', redundancy.get_value())
        print('Cost:', cost.get_value())
        print(so.get_solution_table(numWorkers, numRecruits, numRedundant,
                                    numShortTime, numExcess))
        print(so.get_solution_table(numRetrain))
        print(so.get_solution_table(numDowngrade))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.manpower_planning import test

In [2]: test(cas_conn)
NOTE: Initialized model manpower_planning.
NOTE: Added action set 'optimization'.
NOTE: Converting model manpower_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 3 fixed).
NOTE: The problem has 24 linear constraints (6 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 108 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 21 variables and 9 constraints.
NOTE: The LP presolver removed 21 constraint coefficients.
NOTE: The presolved problem has 42 variables, 15 constraints, and 87 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    5.223600E+02         0
          P 2         13    8.417969E+02         0
NOTE: Optimal.
NOTE: Objective = 841.796875.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 24 rows and 4 columns.
```

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```
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6␣
↪columns.
Redundancy: 841.796875
Cost: 1462047.697368
               numWorkers numRecruits numRedundant numShortTime numExcess
1           2
semiskilled 0  1500.00000           -            -            -         -
semiskilled 1  1442.96875           0            0           50   17.9688
semiskilled 2  2000.00000     682.198            0            0         0
semiskilled 3  2500.00000     645.724            0            0         0
skilled     0  1000.00000           -            -            -         -
skilled     1  1025.00000           0            0           50         0
skilled     2  1525.00000         500            0           50         0
skilled     3  2000.00000         500            0            0         0
unskilled   0  2000.00000           -            -            -         -
unskilled   1  1157.03125           0      442.969           50   132.031
unskilled   2   675.00000           0      166.328           50       150
unskilled   3   175.00000           0        232.5           50       150
                           numRetrain
1           2           3
semiskilled skilled     1  256.250000
semiskilled skilled     2  106.578947
semiskilled skilled     3  106.578947
unskilled   semiskilled 1  200.000000
unskilled   semiskilled 2  200.000000
unskilled   semiskilled 3  200.000000
                           numDowngrade
1           2           3
semiskilled unskilled   1        0.0000
semiskilled unskilled   2        0.0000
semiskilled unskilled   3        0.0000
skilled     semiskilled 1      168.4375
skilled     semiskilled 2        0.0000
skilled     semiskilled 3        0.0000
skilled     unskilled   1        0.0000
skilled     unskilled   2        0.0000
skilled     unskilled   3        0.0000
NOTE: Added action set 'optimization'.
NOTE: Converting model manpower_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 3 fixed).
NOTE: The problem has 24 linear constraints (6 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 108 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 30 variables and 11 constraints.
NOTE: The LP presolver removed 39 constraint coefficients.
NOTE: The presolved problem has 33 variables, 13 constraints, and 69 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
```

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```
          D 2          1    2.143730E+05         0
          D 2          8    4.986773E+05         0
NOTE: Optimal.
NOTE: Objective = 498677.28532.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 24 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6␣
↪columns.
Redundancy: 1423.718837
Cost: 498677.285319
              numWorkers numRecruits numRedundant numShortTime numExcess
1           2
semiskilled 0      1500.0           -            -            -         -
semiskilled 1      1400.0           0            0            0         0
semiskilled 2      2000.0         800            0            0         0
semiskilled 3      2500.0         800            0            0         0
skilled     0      1000.0           -            -            -         -
skilled     1      1000.0     55.5556            0            0         0
skilled     2      1500.0         500            0            0         0
skilled     3      2000.0         500            0            0         0
unskilled   0      2000.0           -            -            -         -
unskilled   1      1000.0           0        812.5            0         0
unskilled   2       500.0           0      257.618            0         0
unskilled   3         0.0           0      353.601            0         0
                          numRetrain
1           2           3
semiskilled skilled     1    0.000000
semiskilled skilled     2  105.263158
semiskilled skilled     3  131.578947
unskilled   semiskilled 1    0.000000
unskilled   semiskilled 2  142.382271
unskilled   semiskilled 3   96.398892
                          numDowngrade
1           2           3
semiskilled unskilled   1         25.0
semiskilled unskilled   2          0.0
semiskilled unskilled   3          0.0
skilled     semiskilled 1          0.0
skilled     semiskilled 2          0.0
skilled     semiskilled 3          0.0
skilled     unskilled   1          0.0
skilled     unskilled   2          0.0
skilled     unskilled   3          0.0
Out[2]: 498677.285319
```

7.1.6 Refinery Optimization

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex6_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex06.html

Model

```python
import sasoptpy as so
import pandas as pd
import numpy as np


def test(cas_conn):

    m = so.Model(name='refinery_optimization', session=cas_conn)

    crude_data = pd.DataFrame([
        ['crude1', 20000],
        ['crude2', 30000]
        ], columns=['crude', 'crude_ub']).set_index(['crude'])

    arc_data = pd.DataFrame([
        ['source', 'crude1', 6],
        ['source', 'crude2', 6],
        ['crude1', 'light_naphtha', 0.1],
        ['crude1', 'medium_naphtha', 0.2],
        ['crude1', 'heavy_naphtha', 0.2],
        ['crude1', 'light_oil', 0.12],
        ['crude1', 'heavy_oil', 0.2],
        ['crude1', 'residuum', 0.13],
        ['crude2', 'light_naphtha', 0.15],
        ['crude2', 'medium_naphtha', 0.25],
        ['crude2', 'heavy_naphtha', 0.18],
        ['crude2', 'light_oil', 0.08],
        ['crude2', 'heavy_oil', 0.19],
        ['crude2', 'residuum', 0.12],
        ['light_naphtha', 'regular_petrol', np.nan],
        ['light_naphtha', 'premium_petrol', np.nan],
        ['medium_naphtha', 'regular_petrol', np.nan],
        ['medium_naphtha', 'premium_petrol', np.nan],
        ['heavy_naphtha', 'regular_petrol', np.nan],
        ['heavy_naphtha', 'premium_petrol', np.nan],
        ['light_naphtha', 'reformed_gasoline', 0.6],
        ['medium_naphtha', 'reformed_gasoline', 0.52],
        ['heavy_naphtha', 'reformed_gasoline', 0.45],
        ['light_oil', 'jet_fuel', np.nan],
        ['light_oil', 'fuel_oil', np.nan],
        ['heavy_oil', 'jet_fuel', np.nan],
        ['heavy_oil', 'fuel_oil', np.nan],
        ['light_oil', 'light_oil_cracked', 2],
        ['light_oil_cracked', 'cracked_oil', 0.68],
        ['light_oil_cracked', 'cracked_gasoline', 0.28],
        ['heavy_oil', 'heavy_oil_cracked', 2],
        ['heavy_oil_cracked', 'cracked_oil', 0.75],
        ['heavy_oil_cracked', 'cracked_gasoline', 0.2],
        ['cracked_oil', 'jet_fuel', np.nan],
        ['cracked_oil', 'fuel_oil', np.nan],
        ['reformed_gasoline', 'regular_petrol', np.nan],
        ['reformed_gasoline', 'premium_petrol', np.nan],
        ['cracked_gasoline', 'regular_petrol', np.nan],
```

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```
        ['cracked_gasoline', 'premium_petrol', np.nan],
        ['residuum', 'lube_oil', 0.5],
        ['residuum', 'jet_fuel', np.nan],
        ['residuum', 'fuel_oil', np.nan],
        ], columns=['i', 'j', 'multiplier']).set_index(['i', 'j'])

    octane_data = pd.DataFrame([
        ['light_naphtha', 90],
        ['medium_naphtha', 80],
        ['heavy_naphtha', 70],
        ['reformed_gasoline', 115],
        ['cracked_gasoline', 105],
        ], columns=['i', 'octane']).set_index(['i'])

    petrol_data = pd.DataFrame([
        ['regular_petrol', 84],
        ['premium_petrol', 94],
        ], columns=['petrol', 'octane_lb']).set_index(['petrol'])

    vapour_pressure_data = pd.DataFrame([
        ['light_oil', 1.0],
        ['heavy_oil', 0.6],
        ['cracked_oil', 1.5],
        ['residuum', 0.05],
        ], columns=['oil', 'vapour_pressure']).set_index(['oil'])

    fuel_oil_ratio_data = pd.DataFrame([
        ['light_oil', 10],
        ['cracked_oil', 4],
        ['heavy_oil', 3],
        ['residuum', 1],
        ], columns=['oil', 'coefficient']).set_index(['oil'])

    final_product_data = pd.DataFrame([
        ['premium_petrol', 700],
        ['regular_petrol', 600],
        ['jet_fuel', 400],
        ['fuel_oil', 350],
        ['lube_oil', 150],
        ], columns=['product', 'profit']).set_index(['product'])

    vapour_pressure_ub = 1
    crude_total_ub = 45000
    naphtha_ub = 10000
    cracked_oil_ub = 8000
    lube_oil_lb = 500
    lube_oil_ub = 1000
    premium_ratio = 0.40

    ARCS = arc_data.index.tolist()
    arc_mult = arc_data['multiplier'].fillna(1)

    FINAL_PRODUCTS = final_product_data.index.tolist()
    final_product_data['profit'] = final_product_data['profit'] / 100
    profit = final_product_data['profit']

    ARCS = ARCS + [(i, 'sink') for i in FINAL_PRODUCTS]
```

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```
    flow = m.add_variables(ARCS, name='flow', lb=0)
    NODES = np.unique([i for j in ARCS for i in j])

    m.set_objective(so.quick_sum(profit[i] * flow[i, 'sink']
                                 for i in FINAL_PRODUCTS
                                 if (i, 'sink') in ARCS),
                    name='totalProfit', sense=so.MAX)

    m.add_constraints((so.quick_sum(flow[a] for a in ARCS if a[0] == n) ==
                      so.quick_sum(arc_mult[a] * flow[a]
                                   for a in ARCS if a[1] == n)
                      for n in NODES if n not in ['source', 'sink']),
                      name='flow_balance')

    CRUDES = crude_data.index.tolist()
    crudeDistilled = m.add_variables(CRUDES, name='crudesDistilled', lb=0)
    crudeDistilled.set_bounds(ub=crude_data['crude_ub'])
    m.add_constraints((flow[i, j] == crudeDistilled[i]
                      for (i, j) in ARCS if i in CRUDES), name='distillation')

    OILS = ['light_oil', 'heavy_oil']
    CRACKED_OILS = [i+'_cracked' for i in OILS]
    oilCracked = m.add_variables(CRACKED_OILS, name='oilCracked', lb=0)
    m.add_constraints((flow[i, j] == oilCracked[i] for (i, j) in ARCS
                      if i in CRACKED_OILS), name='cracking')

    octane = octane_data['octane']
    PETROLS = petrol_data.index.tolist()
    octane_lb = petrol_data['octane_lb']
    vapour_pressure = vapour_pressure_data['vapour_pressure']

    m.add_constraints((so.quick_sum(octane[a[0]] * arc_mult[a] * flow[a]
                                    for a in ARCS if a[1] == p)
                       >= octane_lb[p] *
                      so.quick_sum(arc_mult[a] * flow[a]
                                   for a in ARCS if a[1] == p)
                      for p in PETROLS), name='blending_petrol')

    m.add_constraint(so.quick_sum(vapour_pressure[a[0]] * arc_mult[a] * flow[a]
                                  for a in ARCS if a[1] == 'jet_fuel') <=
                     vapour_pressure_ub *
                     so.quick_sum(arc_mult[a] * flow[a]
                                  for a in ARCS if a[1] == 'jet_fuel'),
                     name='blending_jet_fuel')

    fuel_oil_coefficient = fuel_oil_ratio_data['coefficient']
    sum_fuel_oil_coefficient = sum(fuel_oil_coefficient)
    m.add_constraints((sum_fuel_oil_coefficient * flow[a] ==
                      fuel_oil_coefficient[a[0]] * flow.sum('*', ['fuel_oil'])
                      for a in ARCS if a[1] == 'fuel_oil'),
                      name='blending_fuel_oil')

    m.add_constraint(crudeDistilled.sum('*') <= crude_total_ub,
                     name='crude_total_ub')

    m.add_constraint(so.quick_sum(flow[a] for a in ARCS
                                  if a[0].find('naphtha') > -1 and
```

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```
                                a[1] == 'reformed_gasoline')
                 <= naphtha_ub, name='naphtba_ub')

    m.add_constraint(so.quick_sum(flow[a] for a in ARCS if a[1] ==
                                'cracked_oil') <=
                 cracked_oil_ub, name='cracked_oil_ub')

    m.add_constraint(flow['lube_oil', 'sink'] == [lube_oil_lb, lube_oil_ub],
                 name='lube_oil_range')

    m.add_constraint(flow.sum('premium_petrol', '*') >= premium_ratio *
                 flow.sum('regular_petrol', '*'), name='premium_ratio')

    res = m.solve()
    if res is not None:
        print(so.get_solution_table(crudeDistilled))
        print(so.get_solution_table(oilCracked))
        print(so.get_solution_table(flow))

        octane_sol = []
        for p in PETROLS:
            octane_sol.append(so.quick_sum(octane[a[0]] * arc_mult[a] *
                                           flow[a].get_value() for a in ARCS
                                           if a[1] == p) /
                              sum(arc_mult[a] * flow[a].get_value()
                                  for a in ARCS if a[1] == p))
        octane_sol = pd.Series(octane_sol, name='octane_sol', index=PETROLS)
        print(so.get_solution_table(octane_sol, octane_lb))
        print(so.get_solution_table(vapour_pressure))
        vapour_pressure_sol = sum(vapour_pressure[a[0]] *
                                  arc_mult[a] *
                                  flow[a].get_value() for a in ARCS
                                  if a[1] == 'jet_fuel') /\
            sum(arc_mult[a] * flow[a].get_value() for a in ARCS
                if a[1] == 'jet_fuel')
        print('Vapour_pressure_sol: {:.4f}'.format(vapour_pressure_sol))

        num_fuel_oil_ratio_sol = [arc_mult[a] * flow[a].get_value() /
                                  sum(arc_mult[b] *
                                      flow[b].get_value()
                                      for b in ARCS if b[1] == 'fuel_oil')
                                  for a in ARCS if a[1] == 'fuel_oil']
        num_fuel_oil_ratio_sol = pd.Series(num_fuel_oil_ratio_sol,
                                           name='num_fuel_oil_ratio_sol',
                                           index=[a[0] for a in ARCS
                                                  if a[1] == 'fuel_oil'])
        print(so.get_solution_table(fuel_oil_coefficient,
                                    num_fuel_oil_ratio_sol))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.refinery_optimization import test
```

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```
In [2]: test(cas_conn)
NOTE: Initialized model refinery_optimization.
NOTE: Added action set 'optimization'.
NOTE: Converting model refinery_optimization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 51 variables (0 free, 0 fixed).
NOTE: The problem has 46 linear constraints (4 LE, 38 EQ, 3 GE, 1 range).
NOTE: The problem has 158 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 28 variables and 30 constraints.
NOTE: The LP presolver removed 85 constraint coefficients.
NOTE: The presolved problem has 23 variables, 16 constraints, and 73 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    7.189280E+05         0
          P 2         21    2.113651E+05         0
NOTE: Optimal.
NOTE: Objective = 211365.13477.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 46 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 51 rows and 6␣
↪columns.
        crudesDistilled
1
crude1          15000.0
crude2          30000.0
                   oilCracked
1
heavy_oil_cracked      3800.0
light_oil_cracked      4200.0
                                             flow
1                 2
cracked_gasoline  premium_petrol         0.000000
cracked_gasoline  regular_petrol      1936.000000
cracked_oil       fuel_oil               0.000000
cracked_oil       jet_fuel            5706.000000
crude1            heavy_naphtha      15000.000000
crude1            heavy_oil          15000.000000
crude1            light_naphtha      15000.000000
crude1            light_oil          15000.000000
crude1            medium_naphtha     15000.000000
crude1            residuum           15000.000000
crude2            heavy_naphtha      30000.000000
crude2            heavy_oil          30000.000000
crude2            light_naphtha      30000.000000
crude2            light_oil          30000.000000
crude2            medium_naphtha     30000.000000
```

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```
crude2            residuum           30000.000000
fuel_oil          sink                   0.000000
heavy_naphtha     premium_petrol      1677.804016
heavy_naphtha     reformed_gasoline   5406.861844
heavy_naphtha     regular_petrol      1315.334140
heavy_oil         fuel_oil               0.000000
heavy_oil         heavy_oil_cracked   3800.000000
heavy_oil         jet_fuel            4900.000000
heavy_oil_cracked cracked_gasoline    3800.000000
heavy_oil_cracked cracked_oil         3800.000000
jet_fuel          sink               15156.000000
light_naphtha     premium_petrol      2706.887007
light_naphtha     reformed_gasoline      0.000000
light_naphtha     regular_petrol      3293.112993
light_oil         fuel_oil               0.000000
light_oil         jet_fuel               0.000000
light_oil         light_oil_cracked   4200.000000
light_oil_cracked cracked_gasoline    4200.000000
light_oil_cracked cracked_oil         4200.000000
lube_oil          sink                 500.000000
medium_naphtha    premium_petrol         0.000000
medium_naphtha    reformed_gasoline      0.000000
medium_naphtha    regular_petrol     10500.000000
premium_petrol    sink                6817.778853
reformed_gasoline premium_petrol      2433.087830
reformed_gasoline regular_petrol         0.000000
regular_petrol    sink               17044.447133
residuum          fuel_oil               0.000000
residuum          jet_fuel            4550.000000
residuum          lube_oil            1000.000000
source            crude1             15000.000000
source            crude2             30000.000000
                octane_sol  octane_lb
1
premium_petrol        94.0         94
regular_petrol        84.0         84
             vapour_pressure
1
cracked_oil             1.50
heavy_oil               0.60
light_oil               1.00
residuum                0.05
Vapour_pressure_sol: 0.7737
             coefficient  num_fuel_oil_ratio_sol
1
cracked_oil            4                     NaN
heavy_oil              3                     NaN
light_oil             10                     NaN
residuum               1                     NaN
Out[2]: 211365.134769
```

7.1.7 Mining Optimization

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex7_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex07.html

Model

```
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='mining_optimization', session=cas_conn)

    mine_data = pd.DataFrame([
        ['mine1', 5, 2, 1.0],
        ['mine2', 4, 2.5, 0.7],
        ['mine3', 4, 1.3, 1.5],
        ['mine4', 5, 3, 0.5],
        ], columns=['mine', 'cost', 'extract_ub', 'quality']).\
        set_index(['mine'])

    year_data = pd.DataFrame([
        [1, 0.9],
        [2, 0.8],
        [3, 1.2],
        [4, 0.6],
        [5, 1.0],
        ], columns=['year', 'quality_required']).set_index(['year'])

    max_num_worked_per_year = 3
    revenue_per_ton = 10
    discount_rate = 0.10

    MINES = mine_data.index.tolist()
    cost = mine_data['cost']
    extract_ub = mine_data['extract_ub']
    quality = mine_data['quality']
    YEARS = year_data.index.tolist()
    quality_required = year_data['quality_required']

    isOpen = m.add_variables(MINES, YEARS, vartype=so.BIN, name='isOpen')
    isWorked = m.add_variables(MINES, YEARS, vartype=so.BIN, name='isWorked')
    extract = m.add_variables(MINES, YEARS, lb=0, name='extract')
    [extract[i, j].set_bounds(ub=extract_ub[i]) for i in MINES for j in YEARS]

    extractedPerYear = {j: extract.sum('*', j) for j in YEARS}
    discount = {j: 1 / (1+discount_rate) ** (j-1) for j in YEARS}

    totalRevenue = revenue_per_ton *\
        so.quick_sum(discount[j] * extractedPerYear[j] for j in YEARS)
```

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```
    totalCost = so.quick_sum(discount[j] * cost[i] * isOpen[i, j]
                             for i in MINES for j in YEARS)
    m.set_objective(totalRevenue-totalCost, sense=so.MAX, name='totalProfit')

    m.add_constraints((extract[i, j] <= extract[i, j]._ub * isWorked[i, j]
                      for i in MINES for j in YEARS), name='link')

    m.add_constraints((isWorked.sum('*', j) <= max_num_worked_per_year
                      for j in YEARS), name='cardinality')

    m.add_constraints((isWorked[i, j] <= isOpen[i, j] for i in MINES
                      for j in YEARS), name='worked_implies_open')

    m.add_constraints((isOpen[i, j] <= isOpen[i, j-1] for i in MINES
                      for j in YEARS if j != 1), name='continuity')

    m.add_constraints((so.quick_sum(quality[i] * extract[i, j] for i in MINES)
                      == quality_required[j] * extractedPerYear[j]
                      for j in YEARS), name='quality_con')

    res = m.solve()
    if res is not None:
        print(so.get_solution_table(isOpen, isWorked, extract))
        quality_sol = {j: so.quick_sum(quality[i] * extract[i, j].get_value()
                                       for i in MINES)
                       / extractedPerYear[j].get_value() for j in YEARS}
        qs = so.dict_to_frame(quality_sol, ['quality_sol'])
        epy = so.dict_to_frame(extractedPerYear, ['extracted_per_year'])
        print(so.get_solution_table(epy, qs, quality_required))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.mining_optimization import test

In [2]: test(cas_conn)
NOTE: Initialized model mining_optimization.
NOTE: Added action set 'optimization'.
NOTE: Converting model mining_optimization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 60 variables (0 free, 0 fixed).
NOTE: The problem has 40 binary and 0 integer variables.
NOTE: The problem has 66 linear constraints (61 LE, 5 EQ, 0 GE, 0 range).
NOTE: The problem has 151 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 8 variables and 8 constraints.
NOTE: The MILP presolver removed 16 constraint coefficients.
NOTE: The MILP presolver modified 8 constraint coefficients.
NOTE: The presolved problem has 52 variables, 58 constraints, and 135 constraint␣
↪coefficients.
```

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```
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      7     95.6438817    364.3638322   73.75%       0
                0        1      7     95.6438817    157.7308887   39.36%       0
                0        1      7     95.6438817    153.3061673   37.61%       0
                0        1      7     95.6438817    150.9827514   36.65%       0
                0        1      7     95.6438817    146.8623445   34.88%       0
                0        1      8    146.8619786    146.8623445    0.00%       0
NOTE: The MILP solver added 4 cuts with 19 cut coefficients at the root.
NOTE: Optimal within relative gap.
NOTE: Objective = 146.86197857.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 66 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 60 rows and 6␣
↪columns.
           isOpen  isWorked   extract
1      2
mine1 1  1.000000  1.000000  2.000000
mine1 2  1.000000  0.000000  0.000000
mine1 3  1.000000  1.000000  1.950000
mine1 4  1.000000  1.000000  0.125000
mine1 5  1.000000  1.000000  2.000000
mine2 1  1.000000  0.000000  0.000000
mine2 2  1.000000  1.000000  2.500000
mine2 3  1.000000  0.000000  0.000000
mine2 4  1.000000  1.000000  2.500000
mine2 5  0.999998  0.999998  2.166667
mine3 1  1.000000  1.000000  1.300000
mine3 2  1.000000  1.000000  1.300000
mine3 3  1.000000  1.000000  1.300000
mine3 4  1.000000  0.000000  0.000000
mine3 5  1.000000  1.000000  1.300000
mine4 1  1.000000  1.000000  2.450000
mine4 2  1.000000  1.000000  2.200000
mine4 3  1.000000  0.000000  0.000000
mine4 4  1.000000  1.000000  3.000000
mine4 5  0.000000  0.000000  0.000000
   extracted_per_year  quality_sol  quality_required
1
1            5.750000          0.9               0.9
2            6.000000          0.8               0.8
3            3.250000          1.2               1.2
4            5.625000          0.6               0.6
5            5.466667          1.0               1.0
Out[2]: 146.861979
```

7.1.8 Farm Planning

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex8_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex08.html

Model

```python
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='farm_planning', session=cas_conn)

    # Input Data

    cow_data_raw = []
    for age in range(12):
        if age < 2:
            row = {'age': age,
                   'init_num_cows': 10,
                   'acres_needed': 2/3.0,
                   'annual_loss': 0.05,
                   'bullock_yield': 0,
                   'heifer_yield': 0,
                   'milk_revenue': 0,
                   'grain_req': 0,
                   'sugar_beet_req': 0,
                   'labour_req': 10,
                   'other_costs': 50}
        else:
            row = {'age': age,
                   'init_num_cows': 10,
                   'acres_needed': 1,
                   'annual_loss': 0.02,
                   'bullock_yield': 1.1/2,
                   'heifer_yield': 1.1/2,
                   'milk_revenue': 370,
                   'grain_req': 0.6,
                   'sugar_beet_req': 0.7,
                   'labour_req': 42,
                   'other_costs': 100}
        cow_data_raw.append(row)
    cow_data = pd.DataFrame(cow_data_raw).set_index(['age'])
    grain_data = pd.DataFrame([
        ['group1', 20, 1.1],
        ['group2', 30, 0.9],
        ['group3', 20, 0.8],
        ['group4', 10, 0.65]
        ], columns=['group', 'acres', 'yield']).set_index(['group'])
    num_years = 5
```

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```
    num_acres = 200
    bullock_revenue = 30
    heifer_revenue = 40
    dairy_cow_selling_age = 12
    dairy_cow_selling_revenue = 120
    max_num_cows = 130
    sugar_beet_yield = 1.5
    grain_cost = 90
    grain_revenue = 75
    grain_labour_req = 4
    grain_other_costs = 15
    sugar_beet_cost = 70
    sugar_beet_revenue = 58
    sugar_beet_labour_req = 14
    sugar_beet_other_costs = 10
    nominal_labour_cost = 4000
    nominal_labour_hours = 5500
    excess_labour_cost = 1.2
    capital_outlay_unit = 200
    num_loan_years = 10
    annual_interest_rate = 0.15
    max_decrease_ratio = 0.50
    max_increase_ratio = 0.75

    # Sets

    AGES = cow_data.index.tolist()
    init_num_cows = cow_data['init_num_cows']
    acres_needed = cow_data['acres_needed']
    annual_loss = cow_data['annual_loss']
    bullock_yield = cow_data['bullock_yield']
    heifer_yield = cow_data['heifer_yield']
    milk_revenue = cow_data['milk_revenue']
    grain_req = cow_data['grain_req']
    sugar_beet_req = cow_data['sugar_beet_req']
    cow_labour_req = cow_data['labour_req']
    cow_other_costs = cow_data['other_costs']

    YEARS = list(range(1, num_years+1))
    YEARS0 = [0] + YEARS

    # Variables

    numCows = m.add_variables(AGES + [dairy_cow_selling_age], YEARS0, lb=0,
                              name='numCows')
    for age in AGES:
        numCows[age, 0].set_bounds(lb=init_num_cows[age],
                                   ub=init_num_cows[age])
    numCows[dairy_cow_selling_age, 0].set_bounds(lb=0, ub=0)

    numBullocksSold = m.add_variables(YEARS, lb=0, name='numBullocksSold')
    numHeifersSold = m.add_variables(YEARS, lb=0, name='numHeifersSold')

    GROUPS = grain_data.index.tolist()
    acres = grain_data['acres']
    grain_yield = grain_data['yield']
    grainAcres = m.add_variables(GROUPS, YEARS, lb=0, name='grainAcres')
```

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```
    for group in GROUPS:
        for year in YEARS:
            grainAcres[group, year].set_bounds(ub=acres[group])
    grainBought = m.add_variables(YEARS, lb=0, name='grainBought')
    grainSold = m.add_variables(YEARS, lb=0, name='grainSold')

    sugarBeetAcres = m.add_variables(YEARS, lb=0, name='sugarBeetAcres')
    sugarBeetBought = m.add_variables(YEARS, lb=0, name='sugarBeetBought')
    sugarBeetSold = m.add_variables(YEARS, lb=0, name='sugarBeetSold')

    numExcessLabourHours = m.add_variables(YEARS, lb=0,
                                           name='numExcessLabourHours')
    capitalOutlay = m.add_variables(YEARS, lb=0, name='capitalOutlay')

    yearly_loan_payment = (annual_interest_rate * capital_outlay_unit) /\
                          (1 - (1+annual_interest_rate)**(-num_loan_years))

    # Objective function

    revenue = {year:
               bullock_revenue * numBullocksSold[year] +
               heifer_revenue * numHeifersSold[year] +
               dairy_cow_selling_revenue * numCows[dairy_cow_selling_age,
                                                   year] +
               so.quick_sum(milk_revenue[age] * numCows[age, year]
                            for age in AGES) +
               grain_revenue * grainSold[year] +
               sugar_beet_revenue * sugarBeetSold[year]
               for year in YEARS}

    cost = {year:
            grain_cost * grainBought[year] +
            sugar_beet_cost * sugarBeetBought[year] +
            nominal_labour_cost +
            excess_labour_cost * numExcessLabourHours[year] +
            so.quick_sum(cow_other_costs[age] * numCows[age, year]
                         for age in AGES) +
            so.quick_sum(grain_other_costs * grainAcres[group, year]
                         for group in GROUPS) +
            sugar_beet_other_costs * sugarBeetAcres[year] +
            so.quick_sum(yearly_loan_payment * capitalOutlay[y]
                         for y in YEARS if y <= year)
            for year in YEARS}
    profit = {year: revenue[year] - cost[year] for year in YEARS}

    totalProfit = so.quick_sum(profit[year] -
                               yearly_loan_payment * (num_years - 1 + year) *
                               capitalOutlay[year] for year in YEARS)

    m.set_objective(totalProfit, sense=so.MAX, name='totalProfit')

    # Constraints

    m.add_constraints((
        so.quick_sum(acres_needed[age] * numCows[age, year] for age in AGES) +
        so.quick_sum(grainAcres[group, year] for group in GROUPS) +
        sugarBeetAcres[year] <= num_acres
```

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```
        for year in YEARS), name='num_acres')

    m.add_constraints((
        numCows[age+1, year+1] == (1-annual_loss[age]) * numCows[age, year]
        for age in AGES if age != dairy_cow_selling_age
        for year in YEARS0 if year != num_years), name='aging')

    m.add_constraints((
        numBullocksSold[year] == so.quick_sum(
            bullock_yield[age] * numCows[age, year] for age in AGES)
        for year in YEARS), name='numBullocksSold_def')

    m.add_constraints((
        numCows[0, year] == so.quick_sum(
            heifer_yield[age] * numCows[age, year]
            for age in AGES) - numHeifersSold[year]
        for year in YEARS), name='numHeifersSold_def')

    m.add_constraints((
        so.quick_sum(numCows[age, year] for age in AGES) <= max_num_cows +
        so.quick_sum(capitalOutlay[y] for y in YEARS if y <= year)
        for year in YEARS), name='max_num_cows_def')

    grainGrown = {(group, year): grain_yield[group] * grainAcres[group, year]
                  for group in GROUPS for year in YEARS}
    m.add_constraints((
        so.quick_sum(grain_req[age] * numCows[age, year] for age in AGES) <=
        so.quick_sum(grainGrown[group, year] for group in GROUPS)
        + grainBought[year] - grainSold[year]
        for year in YEARS), name='grain_req_def')

    sugarBeetGrown = {(year): sugar_beet_yield * sugarBeetAcres[year]
                      for year in YEARS}
    m.add_constraints((
        so.quick_sum(sugar_beet_req[age] * numCows[age, year] for age in AGES)
        <=
        sugarBeetGrown[year] + sugarBeetBought[year] - sugarBeetSold[year]
        for year in YEARS), name='sugar_beet_req_def')

    m.add_constraints((
        so.quick_sum(cow_labour_req[age] * numCows[age, year]
                     for age in AGES) +
        so.quick_sum(grain_labour_req * grainAcres[group, year]
                     for group in GROUPS) +
        sugar_beet_labour_req * sugarBeetAcres[year] <=
        nominal_labour_hours + numExcessLabourHours[year]
        for year in YEARS), name='labour_req_def')
    m.add_constraints((profit[year] >= 0 for year in YEARS), name='cash_flow')

    m.add_constraint(so.quick_sum(numCows[age, num_years] for age in AGES
                                  if age >= 2) /
                     sum(init_num_cows[age] for age in AGES if age >= 2) ==
                     [1-max_decrease_ratio, 1+max_increase_ratio],
                     name='final_dairy_cows_range')

    res = m.solve()
```

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```
    if res is not None:
        print(so.get_solution_table(numCows))
        revenue_df = so.dict_to_frame(revenue, cols=['revenue'])
        cost_df = so.dict_to_frame(cost, cols=['cost'])
        profit_df = so.dict_to_frame(profit, cols=['profit'])
        print(so.get_solution_table(numBullocksSold, numHeifersSold,
                                    capitalOutlay, numExcessLabourHours,
                                    revenue_df, cost_df, profit_df))
        gg_df = so.dict_to_frame(grainGrown, cols=['grainGrown'])
        print(so.get_solution_table(grainAcres, gg_df))
        sbg_df = so.dict_to_frame(sugarBeetGrown, cols=['sugerBeetGrown'])
        print(so.get_solution_table(
            grainBought, grainSold, sugarBeetAcres,
            sbg_df, sugarBeetBought, sugarBeetSold))
        num_acres = so.get_obj_by_name('num_acres')
        na_df = num_acres.get_expressions()
        max_num_cows_con = so.get_obj_by_name('max_num_cows_def')
        mnc_df = max_num_cows_con.get_expressions()
        print(so.get_solution_table(na_df, mnc_df))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.farm_planning import test

In [2]: test(cas_conn)
NOTE: Initialized model farm_planning.
NOTE: Added action set 'optimization'.
NOTE: Converting model farm_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 143 variables (0 free, 13 fixed).
NOTE: The problem has 101 linear constraints (25 LE, 70 EQ, 5 GE, 1 range).
NOTE: The problem has 780 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 84 variables and 69 constraints.
NOTE: The LP presolver removed 533 constraint coefficients.
NOTE: The presolved problem has 59 variables, 32 constraints, and 247 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 1          1    4.195000E+02         0
          D 2         37    1.744078E+05         0
          D 2         55    1.217192E+05         0
NOTE: Optimal.
NOTE: Objective = 121719.17286.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 101 rows and 4␣
↪columns.
```

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```
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 143 rows and 6␣
↪columns.
        numCows
1  2
0  0  10.000000
0  1  22.800000
0  2  11.584427
0  3   0.000000
0  4   0.000000
0  5   0.000000
1  0  10.000000
1  1   9.500000
1  2  21.660000
1  3  11.005205
1  4   0.000000
1  5   0.000000
2  0  10.000000
2  1   9.500000
2  2   9.025000
2  3  20.577000
2  4  10.454945
2  5   0.000000
3  0  10.000000
3  1   9.800000
3  2   9.310000
3  3   8.844500
3  4  20.165460
3  5  10.245846
4  0  10.000000
4  1   9.800000
4  2   9.604000
4  3   9.123800
4  4   8.667610
4  5  19.762151
...          ...
8  0  10.000000
8  1   9.800000
8  2   9.604000
8  3   9.411920
8  4   9.223682
8  5   9.039208
9  0  10.000000
9  1   9.800000
9  2   9.604000
9  3   9.411920
9  4   9.223682
9  5   9.039208
10 0  10.000000
10 1   9.800000
10 2   9.604000
10 3   9.411920
10 4   9.223682
10 5   9.039208
11 0  10.000000
11 1   9.800000
```

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```
11 2   9.604000
11 3   9.411920
11 4   9.223682
11 5   9.039208
12 0   0.000000
12 1   9.800000
12 2   9.604000
12 3   9.411920
12 4   9.223682
12 5   9.039208

[78 rows x 1 columns]
   numBullocksSold  numHeifersSold  capitalOutlay  numExcessLabourHours  \
1
1        53.735000       30.935000            0.0                   0.0
2        52.341850       40.757423            0.0                   0.0
3        57.435807       57.435807            0.0                   0.0
4        56.964286       56.964286            0.0                   0.0
5        50.853436       50.853436            0.0                   0.0

        revenue          cost        profit
1
1  41494.530000  19588.466667  21906.063333
2  41153.336497  19264.639818  21888.696679
3  45212.490308  19396.435208  25816.055100
4  45860.056078  19034.285714  26825.770363
5  42716.941438  17434.354053  25282.587385
          grainAcres  grainGrown
1      2
group1 1   20.000000   22.000000
group1 2   20.000000   22.000000
group1 3   20.000000   22.000000
group1 4   20.000000   22.000000
group1 5   20.000000   22.000000
group2 1    0.000000    0.000000
group2 2    0.000000    0.000000
group2 3    3.134152    2.820737
group2 4    0.000000    0.000000
group2 5    0.000000    0.000000
group3 1    0.000000    0.000000
group3 2    0.000000    0.000000
group3 3    0.000000    0.000000
group3 4    0.000000    0.000000
group3 5    0.000000    0.000000
group4 1    0.000000    0.000000
group4 2    0.000000    0.000000
group4 3    0.000000    0.000000
group4 4    0.000000    0.000000
group4 5    0.000000    0.000000
   grainBought  grainSold  sugarBeetAcres  sugerBeetGrown  sugarBeetBought  \
1
1    36.620000        0.0       60.766667       91.150000              0.0
2    35.100200        0.0       62.670049       94.005073              0.0
3    37.836507        0.0       65.100304       97.650456              0.0
4    40.142857        0.0       76.428571      114.642857              0.0
5    33.476475        0.0       87.539208      131.308812              0.0
```

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```
   sugarBeetSold
1
1      22.760000
2      27.388173
3      24.550338
4      42.142857
5      66.586258
   num_acres  max_num_cows_def
1
1      200.0        130.000000
2      200.0        128.411427
3      200.0        115.433945
4      200.0        103.571429
5      200.0         92.460792
Out[2]: 121719.172861
```

7.1.9 Economic Planning

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex9_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex09.html

Model

```python
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='economic_planning', session=cas_conn)

    industry_data = pd.DataFrame([
        ['coal', 150, 300, 60],
        ['steel', 80, 350, 60],
        ['transport', 100, 280, 30]
        ], columns=['industry', 'init_stocks', 'init_productive_capacity',
                    'demand']).set_index(['industry'])

    production_data = pd.DataFrame([
        ['coal', 0.1, 0.5, 0.4],
        ['steel', 0.1, 0.1, 0.2],
        ['transport', 0.2, 0.1, 0.2],
        ['manpower', 0.6, 0.3, 0.2],
        ], columns=['input', 'coal',
                    'steel', 'transport']).set_index(['input'])

    productive_capacity_data = pd.DataFrame([
        ['coal', 0.0, 0.7, 0.9],
        ['steel', 0.1, 0.1, 0.2],
```

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```
        ['transport', 0.2, 0.1, 0.2],
        ['manpower', 0.4, 0.2, 0.1],
        ], columns=['input', 'coal',
                    'steel', 'transport']).set_index(['input'])

    manpower_capacity = 470
    num_years = 5

    YEARS = list(range(1, num_years+1))
    YEARS0 = [0] + list(YEARS)
    INDUSTRIES = industry_data.index.tolist()
    [init_stocks, init_productive_capacity, demand] = so.read_frame(
        industry_data)
    # INPUTS = production_data.index.tolist()
    production_coeff = so.flatten_frame(production_data)
    productive_capacity_coeff = so.flatten_frame(productive_capacity_data)

    static_production = m.add_variables(INDUSTRIES, lb=0,
                                        name='static_production')
    m.set_objective(0, sense=so.MIN, name='Zero')
    m.add_constraints((static_production[i] == demand[i] +
                       so.quick_sum(
                           production_coeff[i, j] * static_production[j]
                           for j in INDUSTRIES) for i in INDUSTRIES),
                      name='static_con')

    m.solve()
    print(so.get_solution_table(static_production, sort=True))

    final_demand = so.get_solution_table(
        static_production, sort=True)['static_production']
    # Alternative way
    # final_demand = {}
    # for i in INDUSTRIES:
    #     final_demand[i] = static_production.get_value()

    production = m.add_variables(INDUSTRIES, range(0, num_years+2), lb=0,
                                 name='production')
    stock = m.add_variables(INDUSTRIES, range(0, num_years+2), lb=0,
                            name='stock')
    extra_capacity = m.add_variables(INDUSTRIES, range(1, num_years+3), lb=0,
                                     name='extra_capacity')
    productive_capacity = {}
    for i in INDUSTRIES:
        for year in range(1, num_years+2):
            productive_capacity[i, year] = init_productive_capacity[i] +\
                so.quick_sum(extra_capacity[i, y] for y in range(2, year+1))
    for i in INDUSTRIES:
        production[i, 0].set_bounds(ub=0)
        stock[i, 0].set_bounds(lb=init_stocks[i], ub=init_stocks[i])

    total_productive_capacity = sum(productive_capacity[i, num_years]
                                    for i in INDUSTRIES)
    total_production = so.quick_sum(production[i, year] for i in INDUSTRIES
                                    for year in [4, 5])
    total_manpower = so.quick_sum(production_coeff['manpower', i] *
                                  production[i, year+1] +
```

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```
                                  productive_capacity_coeff['manpower', i] *
                                  extra_capacity[i, year+2]
                                  for i in INDUSTRIES for year in YEARS)

    continuity_con = m.add_constraints((
        stock[i, year] + production[i, year] ==
        (demand[i] if year in YEARS else 0) +
        so.quick_sum(production_coeff[i, j] * production[j, year+1] +
                     productive_capacity_coeff[i, j] *
                     extra_capacity[j, year+2] for j in INDUSTRIES) +
        stock[i, year+1]
        for i in INDUSTRIES for year in YEARS0), name='continuity_con')

    manpower_con = m.add_constraints((
        so.quick_sum(production_coeff['manpower', j] * production[j, year] +
                     productive_capacity_coeff['manpower', j] *
                     extra_capacity[j, year+1]
                     for j in INDUSTRIES)
        <= manpower_capacity for year in range(1, num_years+2)),
        name='manpower_con')

    capacity_con = m.add_constraints((production[i, year] <=
                                      productive_capacity[i, year]
                                      for i in INDUSTRIES
                                      for year in range(1, num_years+2)),
                                     name='capacity_con')

    for i in INDUSTRIES:
        production[i, num_years+1].set_bounds(lb=final_demand[i])

    for i in INDUSTRIES:
        for year in [num_years+1, num_years+2]:
            extra_capacity[i, year].set_bounds(ub=0)

    problem1 = so.Model(name='Problem1', session=cas_conn)
    problem1.include(production, stock, extra_capacity,
                     continuity_con, manpower_con, capacity_con)
    problem1.set_objective(total_productive_capacity, sense=so.MAX,
                           name='total_productive_capacity')
    problem1.solve()
    productive_capacity_fr = so.dict_to_frame(productive_capacity,
                                              cols=['productive_capacity'])
    print(so.get_solution_table(production, stock, extra_capacity,
                                productive_capacity_fr, sort=True))
    print(so.get_solution_table(manpower_con.get_expressions(), sort=True))

    # Problem 2

    problem2 = so.Model(name='Problem2', session=cas_conn)
    problem2.include(problem1)
    problem2.set_objective(total_production, name='total_production',
                           sense=so.MAX)
    for i in INDUSTRIES:
        for year in YEARS:
            continuity_con[i, year].set_rhs(0)
    problem2.solve()
    print(so.get_solution_table(production, stock, extra_capacity,
```

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```
                                productive_capacity, sort=True))
    print(so.get_solution_table(manpower_con.get_expressions(), sort=True))

    # Problem 3

    problem3 = so.Model(name='Problem3', session=cas_conn)
    problem3.include(production, stock, extra_capacity, continuity_con,
                     capacity_con)
    problem3.set_objective(total_manpower, sense=so.MAX, name='total_manpower')
    for i in INDUSTRIES:
        for year in YEARS:
            continuity_con[i, year].set_rhs(demand[i])
    problem3.solve()
    print(so.get_solution_table(production, stock, extra_capacity,
                                productive_capacity, sort=True))
    print(so.get_solution_table(manpower_con.get_expressions(), sort=True))

    return problem3.get_objective_value()
```

Output

```
In [1]: from examples.economic_planning import test

In [2]: test(cas_conn)
NOTE: Initialized model economic_planning.
NOTE: Added action set 'optimization'.
NOTE: Converting model economic_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 3 variables (0 free, 0 fixed).
NOTE: The problem has 3 linear constraints (0 LE, 3 EQ, 0 GE, 0 range).
NOTE: The problem has 9 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed all variables and constraints.
NOTE: Optimal.
NOTE: Objective = 0.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 3 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 3 rows and 6␣
↪columns.
           static_production
1
coal              166.396761
steel             105.668016
transport          92.307692
NOTE: Initialized model Problem1.
NOTE: Added action set 'optimization'.
NOTE: Converting model Problem1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
```

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```
NOTE: The problem has 63 variables (0 free, 12 fixed).
NOTE: The problem has 42 linear constraints (24 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 255 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 21 variables and 7 constraints.
NOTE: The LP presolver removed 64 constraint coefficients.
NOTE: The presolved problem has 42 variables, 35 constraints, and 191 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    1.360782E+04         0
          P 2         38    2.141875E+03         0
NOTE: Optimal.
NOTE: Objective = 2141.875197.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 42 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6␣
↪columns.
            production    stock extra_capacity productive_capacity
1         2
coal      0          0      150              -                   -
coal      1    260.403        0              0                 300
coal      2    293.406        0              0                 300
coal      3        300        0              0                 300
coal      4    17.9487  148.448        189.203             489.203
coal      5    166.397        0        1022.67             1511.88
coal      6    166.397        0              0             1511.88
coal      7          -        -              0                   -
steel     0          0       80              -                   -
steel     1    135.342  12.2811              0                 350
steel     2     181.66        0              0                 350
steel     3     193.09        0              0                 350
steel     4    105.668        0              0                 350
steel     5    105.668        0              0                 350
steel     6    105.668        0              0                 350
steel     7          -        -              0                   -
transport 0          0      100              -                   -
transport 1    140.722  6.24084              0                 280
transport 2     200.58        0              0                 280
transport 3    267.152        0              0                 280
transport 4    92.3077        0              0                 280
transport 5    92.3077        0              0                 280
transport 6    92.3077        0              0                 280
transport 7          -        -              0                   -
   manpower_con
1
1    224.988515
2    270.657715
3    367.038878
```

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```
4    470.000000
5    150.000000
6    150.000000
NOTE: Initialized model Problem2.
NOTE: Added action set 'optimization'.
NOTE: Converting model Problem2 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 12 fixed).
NOTE: The problem has 42 linear constraints (24 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 255 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 21 variables and 7 constraints.
NOTE: The LP presolver removed 64 constraint coefficients.
NOTE: The presolved problem has 42 variables, 35 constraints, and 191 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    9.413902E+03         0
          P 2         46    2.618579E+03         0
NOTE: Optimal.
NOTE: Objective = 2618.5791147.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 42 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6␣
↪columns.
            production    stock extra_capacity     dict
1         2
coal      0          0      150              -        -
coal      1    184.818  31.6285              0      300
coal      2    430.505  16.3725        130.505  430.505
coal      3    430.505        0              0  430.505
coal      4    430.505        0              0  430.505
coal      5    430.505        0              0  430.505
coal      6    166.397  324.108              0  430.505
coal      7          -        -              0        -
steel     0          0       80              -        -
steel     1    86.7295  11.5323              0      350
steel     2    155.337        0              0      350
steel     3    182.867        0              0      350
steel     4    359.402        0        9.40227  359.402
steel     5    359.402  176.535              0  359.402
steel     6    105.668  490.269              0  359.402
steel     7          -        -              0        -
transport 0          0      100              -        -
transport 1    141.312        0              0      280
transport 2    198.388        0              0      280
transport 3    225.918        0              0      280
transport 4    519.383        0        239.383  519.383
```

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```
transport 5    519.383  293.465              0  519.383
transport 6    92.3077   750.54              0  519.383
transport 7          -        -              0        -
   manpower_con
1
1    217.374162
2    344.581624
3    384.165212
4    470.000000
5    470.000000
6    150.000000
NOTE: Initialized model Problem3.
NOTE: Added action set 'optimization'.
NOTE: Converting model Problem3 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 12 fixed).
NOTE: The problem has 36 linear constraints (18 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 219 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 18 variables and 3 constraints.
NOTE: The LP presolver removed 31 constraint coefficients.
NOTE: The presolved problem has 45 variables, 33 constraints, and 188 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    4.013232E+04         0
          P 2         50    2.450027E+03         0
NOTE: Optimal.
NOTE: Objective = 2450.0266228.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 36 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6␣
↪columns.
            production    stock extra_capacity     dict
1         2
coal      0          0      150              -        -
coal      1    251.793        0              0      300
coal      2    316.015        0        16.0152  316.015
coal      3    319.832        0         3.8168  319.832
coal      4     366.35        0        46.5177   366.35
coal      5     859.36        0         493.01   859.36
coal      6     859.36  460.208              0   859.36
coal      7          -        -              0        -
steel     0          0       80              -        -
steel     1    134.795   11.028              0      350
steel     2    175.041        0              0      350
steel     3    224.064        0              0      350
steel     4    223.136        0              0      350
```

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```
steel     5    220.044        0              0      350
steel     6        350        0              0      350
steel     7          -        -              0        -
transport 0          0      100              -        -
transport 1    143.559  4.24723              0      280
transport 2    181.676        0              0      280
transport 3        280        0              0      280
transport 4    279.072        0              0      280
transport 5     275.98        0              0      280
transport 6    195.539        0              0      280
transport 7          -        -              0        -
   manpower_con
1
1    226.631832
2    279.983537
3    333.725517
4    539.769130
5    636.824849
6    659.723590
Out[2]: 2450.026623
```

7.1.10 Decentralization

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex10_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex10.html

Model

```
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='decentralization', session=cas_conn)

    DEPTS = ['A', 'B', 'C', 'D', 'E']
    CITIES = ['Bristol', 'Brighton', 'London']

    benefit_data = pd.DataFrame([
        ['Bristol', 10, 15, 10, 20, 5],
        ['Brighton', 10, 20, 15, 15, 15]],
        columns=['city'] + DEPTS).set_index('city')

    comm_data = pd.DataFrame([
        ['A', 'B', 0.0],
        ['A', 'C', 1.0],
        ['A', 'D', 1.5],
        ['A', 'E', 0.0],
```

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```
        ['B', 'C', 1.4],
        ['B', 'D', 1.2],
        ['B', 'E', 0.0],
        ['C', 'D', 0.0],
        ['C', 'E', 2.0],
        ['D', 'E', 0.7]], columns=['i', 'j', 'comm']).set_index(['i', 'j'])

    cost_data = pd.DataFrame([
        ['Bristol', 'Bristol', 5],
        ['Bristol', 'Brighton', 14],
        ['Bristol', 'London', 13],
        ['Brighton', 'Brighton', 5],
        ['Brighton', 'London', 9],
        ['London', 'London', 10]], columns=['i', 'j', 'cost']).set_index(
            ['i', 'j'])

    max_num_depts = 3

    benefit = {}
    for city in CITIES:
        for dept in DEPTS:
            try:
                benefit[dept, city] = benefit_data.loc[city, dept]
            except:
                benefit[dept, city] = 0

    comm = {}
    for row in comm_data.iterrows():
        (i, j) = row[0]
        comm[i, j] = row[1]['comm']
        comm[j, i] = comm[i, j]

    cost = {}
    for row in cost_data.iterrows():
        (i, j) = row[0]
        cost[i, j] = row[1]['cost']
        cost[j, i] = cost[i, j]

    assign = m.add_variables(DEPTS, CITIES, vartype=so.BIN, name='assign')
    IJKL = [(i, j, k, l)
            for i in DEPTS for j in CITIES for k in DEPTS for l in CITIES
            if i < k]
    product = m.add_variables(IJKL, vartype=so.BIN, name='product')

    totalBenefit = so.quick_sum(benefit[i, j] * assign[i, j]
                                for i in DEPTS for j in CITIES)

    totalCost = so.quick_sum(comm[i, k] * cost[j, l] * product[i, j, k, l]
                             for (i, j, k, l) in IJKL)

    m.set_objective(totalBenefit-totalCost, name='netBenefit', sense=so.MAX)

    m.add_constraints((so.quick_sum(assign[dept, city] for city in CITIES)
                      == 1 for dept in DEPTS), name='assign_dept')

    m.add_constraints((so.quick_sum(assign[dept, city] for dept in DEPTS)
                      <= max_num_depts for city in CITIES), name='cardinality')
```

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```
    product_def1 = m.add_constraints((assign[i, j] + assign[k, l] - 1
                                     <= product[i, j, k, l]
                                     for (i, j, k, l) in IJKL),
                                     name='product_def1')

    product_def2 = m.add_constraints((product[i, j, k, l] <= assign[i, j]
                                      for (i, j, k, l) in IJKL),
                                     name='product_def2')

    product_def3 = m.add_constraints((product[i, j, k, l] <= assign[k, l]
                                      for (i, j, k, l) in IJKL),
                                     name='product_def3')

    m.solve()
    print(m.get_problem_summary())

    m.drop_constraints(product_def1)
    m.drop_constraints(product_def2)
    m.drop_constraints(product_def3)

    m.add_constraints((
        so.quick_sum(product[i, j, k, l]
                     for j in CITIES if (i, j, k, l) in IJKL) == assign[k, l]
        for i in DEPTS for k in DEPTS for l in CITIES if i < k),
        name='product_def4')

    m.add_constraints((
        so.quick_sum(product[i, j, k, l]
                     for l in CITIES if (i, j, k, l) in IJKL) == assign[i, j]
        for k in DEPTS for i in DEPTS for j in CITIES if i < k),
        name='product_def4')

    m.solve()
    print(m.get_problem_summary())
    totalBenefit.set_name('totalBenefit')
    totalCost.set_name('totalCost')
    print(so.get_solution_table(totalBenefit, totalCost))
    print(so.get_solution_table(assign).unstack(level=-1))

    return m.get_objective_value()
```

Output

```
In [1]: from examples.decentralization import test

In [2]: test(cas_conn)
NOTE: Initialized model decentralization.
NOTE: Added action set 'optimization'.
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 278 linear constraints (183 LE, 5 EQ, 90 GE, 0 range).
```

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```
NOTE: The problem has 660 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 120 constraints.
NOTE: The MILP presolver removed 120 constraint coefficients.
NOTE: The MILP presolver added 120 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 158 constraints, and 540 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      2    -14.9000000    135.0000000  111.04%       0
                0        1      2    -14.9000000     67.5000000  122.07%       0
                0        1      2    -14.9000000     55.0000000  127.09%       0
                0        1      3      8.1000000     55.0000000   85.27%       0
                0        1      3      8.1000000     48.0000000   83.12%       0
                0        1      3      8.1000000     44.8375000   81.93%       0
                0        1      3      8.1000000     42.0000000   80.71%       0
                0        1      3      8.1000000     39.0666667   79.27%       0
                0        1      3      8.1000000     34.7500000   76.69%       0
                0        1      3      8.1000000     33.9000000   76.11%       0
                0        1      3      8.1000000     29.6800000   72.71%       0
                0        1      3      8.1000000     28.5000000   71.58%       0
                0        1      3      8.1000000     28.5000000   71.58%       0
                0        1      3      8.1000000     28.5000000   71.58%       0
                0        1      3      8.1000000     28.5000000   71.58%       0
NOTE: The MILP solver added 31 cuts with 168 cut coefficients at the root.
                2        0      4     14.9000000     14.9000000    0.00%       0
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 278 rows and 4␣
↪columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 105 rows and 6␣
↪columns.
Problem Summary

                                Value
Label
Objective Sense          Maximization
Objective Function         netBenefit
Objective Type                 Linear

Number of Variables               105
Bounded Above                       0
Bounded Below                       0
Bounded Below and Above           105
Free                                0
Fixed                               0
Binary                            105
```

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```
Integer                             0

Number of Constraints             278
Linear LE (<=)                    183
Linear EQ (=)                       5
Linear GE (>=)                     90
Linear Range                        0

Constraint Coefficients           660
NOTE: Added action set 'optimization'.
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 68 linear constraints (3 LE, 65 EQ, 0 GE, 0 range).
NOTE: The problem has 270 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 0 constraints.
NOTE: The MILP presolver removed 0 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 68 constraints, and 270 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      2    -28.1000000    135.0000000  120.81%       0
                0        1      2    -28.1000000     30.0000000  193.67%       0
                0        1      3    -16.3000000     30.0000000  154.33%       0
                0        1      4     14.9000000     14.9000000    0.00%       0
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 68 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 105 rows and 6␣
↪columns.
Problem Summary

                                Value
Label
Objective Sense          Maximization
Objective Function         netBenefit
Objective Type                 Linear

Number of Variables               105
Bounded Above                       0
Bounded Below                       0
Bounded Below and Above           105
Free                                0
Fixed                               0
```

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```
Binary                          105
Integer                           0

Number of Constraints            68
Linear LE (<=)                    3
Linear EQ (=)                    65
Linear GE (>=)                    0
Linear Range                      0

Constraint Coefficients         270
   totalBenefit  totalCost
1
           80.0       65.1
    assign  assign assign
2 Brighton Bristol London
1
A      0.0     1.0    0.0
B      1.0     0.0    0.0
C      1.0     0.0    0.0
D      0.0     1.0    0.0
E      1.0     0.0    0.0
Out[2]: 14.9
```

7.1.11 Optimal Wedding

Reference

SAS Blog: https://blogs.sas.com/content/operations/2014/11/10/do-you-have-an-uncle-louie-optimal-wedding-seat-assignments/

Model

```
import sasoptpy as so
import math


def test(cas_conn, num_guests=20, max_table_size=3, max_tables=None):

    m = so.Model("wedding", session=cas_conn)

    # Check max. tables
    if max_tables is None:
        max_tables = math.ceil(num_guests/max_table_size)

    # Sets
    guests = range(1, num_guests+1)
    tables = range(1, max_tables+1)
    guest_pairs = [[i, j] for i in guests for j in range(i+1, num_guests+1)]

    # Variables
    x = m.add_variables(guests, tables, vartype=so.BIN, name="x")
    unhappy = m.add_variables(tables, name="unhappy", lb=0)

    # Objective
```

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```
    m.set_objective(unhappy.sum('*'), sense=so.MIN, name="obj")

    # Constraints
    m.add_constraints((x.sum(g, '*') == 1 for g in guests), name="assigncon")
    m.add_constraints((x.sum('*', t) <= max_table_size for t in tables),
                      name="tablesizecon")
    m.add_constraints((unhappy[t] >= abs(g-h)*(x[g, t] + x[h, t] - 1)
                       for t in tables for [g, h] in guest_pairs),
                      name="measurecon")

    # Solve
    res = m.solve(options={
        'with': 'milp', 'decomp': {'method': 'set'}, 'presolver': 'none'})

    if res is not None:

        print(so.get_solution_table(x))

        # Print assignments
        for t in tables:
            print('Table {} : [ '.format(t), end='')
            for g in guests:
                if x[g, t].get_value() == 1:
                    print('{} '.format(g), end='')
            print(']')

    return m.get_objective_value()
```

Output

```
In [1]: from examples.sas_optimal_wedding import test

In [2]: test(cas_conn)
NOTE: Initialized model wedding.
NOTE: Added action set 'optimization'.
NOTE: Converting model wedding to DataFrame.
NOTE: Uploading the problem DataFrame to the server.
NOTE: Cloud Analytic Services made the uploaded file available as table TMPNWGUU1QD␣
↪in caslib CASUSERHDFS(casuser).
NOTE: The table TMPNWGUU1QD has been created in caslib CASUSERHDFS(casuser) from␣
↪binary data uploaded to Cloud Analytic Services.
NOTE: The problem wedding has 147 variables (140 binary, 0 integer, 0 free, 0 fixed).
NOTE: The problem has 1357 constraints (7 LE, 20 EQ, 1330 GE, 0 range).
NOTE: The problem has 4270 constraint coefficients.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value NONE is applied.
NOTE: The MILP solver is called.
NOTE: The Decomposition algorithm is used.
NOTE: The Decomposition algorithm is executing in the distributed computing␣
↪environment in single-machine mode.
NOTE: The DECOMP method value SET is applied.
NOTE: All blocks are identical and the master model is set partitioning.
NOTE: The Decomposition algorithm is using an aggregate formulation and Ryan-Foster␣
↪branching.
NOTE: The number of block threads has been reduced to 1 threads.
```

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```
NOTE: The problem has a decomposable structure with 7 blocks. The largest block␣
↪covers 14.08% of the constraints in the problem.
NOTE: The decomposition subproblems cover 147 (100%) variables and 1337 (98.53%)␣
↪constraints.
NOTE: The deterministic parallel mode is enabled.
NOTE: The Decomposition algorithm is using up to 32 threads.
         Iter         Best       Master         Best       LP        IP  CPU Real
                     Bound    Objective      Integer      Gap       Gap Time Time
            .       0.0000      13.0000      13.0000 1.30e+01 1.30e+01    0    0
            1       0.0000      13.0000      13.0000 1.30e+01 1.30e+01    1    1
            .       0.0000      13.0000      13.0000 1.30e+01 1.30e+01    8   12
           10       0.0000      13.0000      13.0000 1.30e+01 1.30e+01    8   13
           18       4.2500      13.0000      13.0000  205.88%  205.88%   21   29
           19       6.0000      13.0000      13.0000  116.67%  116.67%   23   32
            .       6.0000      13.0000      13.0000  116.67%  116.67%   23   32
           20       6.0000      13.0000      13.0000  116.67%  116.67%   24   33
           21       9.5000      13.0000      13.0000   36.84%   36.84%   25   34
           23      13.0000      13.0000      13.0000    0.00%    0.00%   27   37
            Node  Active   Sols         Best         Best      Gap    CPU   Real
                                     Integer        Bound            Time   Time
               0       0      3      13.0000      13.0000    0.00%     27     37
NOTE: The Decomposition algorithm used 32 threads.
NOTE: The Decomposition algorithm time is 37.94 seconds.
NOTE: Optimal.
NOTE: Objective = 13.
        x
1  2
1  1  1.0
1  2  0.0
1  3  0.0
1  4  0.0
1  5  0.0
1  6  0.0
1  7  0.0
2  1  1.0
2  2  0.0
2  3  0.0
2  4  0.0
2  5  0.0
2  6  0.0
2  7  0.0
3  1  1.0
3  2  0.0
3  3  0.0
3  4  0.0
3  5  0.0
3  6  0.0
3  7  0.0
4  1  0.0
4  2  1.0
4  3  0.0
4  4  0.0
4  5  0.0
4  6  0.0
4  7  0.0
5  1  0.0
5  2  1.0
```

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```
...    ...
16 6   1.0
16 7   0.0
17 1   0.0
17 2   0.0
17 3   0.0
17 4   0.0
17 5   0.0
17 6   1.0
17 7   0.0
18 1   0.0
18 2   0.0
18 3   0.0
18 4   0.0
18 5   0.0
18 6   1.0
18 7   0.0
19 1   0.0
19 2   0.0
19 3   0.0
19 4   0.0
19 5   0.0
19 6   0.0
19 7   1.0
20 1   0.0
20 2   0.0
20 3   0.0
20 4   0.0
20 5   0.0
20 6   0.0
20 7   1.0

[140 rows x 1 columns]
Table 1 : [ 1 2 3 ]
Table 2 : [ 4 5 6 ]
Table 3 : [ 7 8 9 ]
Table 4 : [ 10 11 12 ]
Table 5 : [ 13 14 15 ]
Table 6 : [ 16 17 18 ]
Table 7 : [ 19 20 ]
Out[2]: 13.0
```

7.1.12 Kidney Exchange

Reference

SAS Blog: https://blogs.sas.com/content/operations/2015/02/06/the-kidney-exchange-problem/

Model

```
import sasoptpy as so
import random
```

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```
def test(cas_conn):
    # Data generation
    n = 100
    p = 0.02

    random.seed(1)

    ARCS = {}
    for i in range(0, n):
        for j in range(0, n):
            if random.random() < p:
                ARCS[i, j] = random.random()

    max_length = 10

    # Model
    model = so.Model("kidney_exchange", session=cas_conn)

    # Sets
    NODES = set().union(*ARCS.keys())
    MATCHINGS = range(1, int(len(NODES)/2)+1)

    # Variables
    UseNode = model.add_variables(NODES, MATCHINGS, vartype=so.BIN,
                                  name="usenode")
    UseArc = model.add_variables(ARCS, MATCHINGS, vartype=so.BIN,
                                 name="usearc")
    Slack = model.add_variables(NODES, vartype=so.BIN, name="slack")

    print('Setting objective...')

    # Objective
    model.set_objective(so.quick_sum((ARCS[i, j] * UseArc[i, j, m]
                                      for [i, j] in ARCS for m in MATCHINGS)),
                        name="total_weight", sense=so.MAX)

    print('Adding constraints...')
    # Constraints
    Node_Packing = model.add_constraints((UseNode.sum(i, '*') + Slack[i] == 1
                                          for i in NODES), name="node_packing")
    Donate = model.add_constraints((UseArc.sum(i, '*', m) == UseNode[i, m]
                                    for i in NODES
                                    for m in MATCHINGS), name="donate")
    Receive = model.add_constraints((UseArc.sum('*', j, m) == UseNode[j, m]
                                     for j in NODES
                                     for m in MATCHINGS), name="receive")
    Cardinality = model.add_constraints((UseArc.sum('*', '*', m) <= max_length
                                         for m in MATCHINGS),
                                        name="cardinality")

    # Solve
    model.solve(options={'with': 'milp', 'maxtime': 300})

    # Define decomposition blocks
    for i in NODES:
        for m in MATCHINGS:
```

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```
            Donate[i, m].set_block(m-1)
            Receive[i, m].set_block(m-1)
    for m in MATCHINGS:
        Cardinality[m].set_block(m-1)

    model.solve(verbose=True, options={
        'with': 'milp', 'maxtime': 300, 'presolver': 'basic',
        'decomp': {'method': 'user'}})

    return model.get_objective_value()
```

Output

```
In [1]: from examples.sas_kidney_exchange import test

In [2]: test(cas_conn)
NOTE: Initialized model kidney_exchange.
Setting objective...
Adding constraints...
NOTE: Added action set 'optimization'.
NOTE: Converting model kidney_exchange to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 15828 variables (0 free, 0 fixed).
NOTE: The problem has 15828 binary and 0 integer variables.
NOTE: The problem has 9850 linear constraints (49 LE, 9801 EQ, 0 GE, 0 range).
NOTE: The problem has 47286 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The remaining solution time after problem generation and solver initialization␣
↪is 299.59 seconds.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 8223 variables and 7096 constraints.
NOTE: The MILP presolver removed 22296 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 7605 variables, 2754 constraints, and 24990␣
↪constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node   Active   Sols    BestInteger      BestBound      Gap    Time
                0        1      2      2.8934453     33.5480359   91.38%      10
NOTE: The MILP solver's symmetry detection found 531 orbits. The largest orbit␣
↪contains 23 variables.
                0        1      2      2.8934453     33.5480359   91.38%      14
                0        1      2      2.8934453     33.5480359   91.38%      15
                0        1      2      2.8934453     33.5480359   91.38%      16
                0        1      2      2.8934453     33.5480359   91.38%      17
NOTE: The MILP solver added 5 cuts with 278 cut coefficients at the root.
                1        2      2      2.8934453     33.5480359   91.38%      19
                3        3      3     13.1640039     33.5480359   60.76%      21
                4        4      3     13.1640039     33.5480359   60.76%      22
               10        9      4     24.0492143     33.5480359   28.31%      24
               14       10      4     24.0492143     33.1877376   27.54%      25
               77       41      4     24.0492143     32.7482323   26.56%      30
```

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```
     126       85       5     26.1164550     32.7482323   20.25%       30
     154       88       6     26.5523364     32.7482323   18.92%       33
     196      106       6     26.5523364     32.7482323   18.92%       35
     273      156       6     26.5523364     32.7482323   18.92%       40
     416      273       7     28.0518710     32.7482323   14.34%       42
     431      242       7     28.0518710     32.7482323   14.34%       46
     615      373       7     28.0518710     32.4428695   13.53%       50
     704      424       7     28.0518710     32.3460905   13.28%       55
     768        1       7     28.0518710     32.2947078   13.14%       62
     771        3       7     28.0518710     32.2947078   13.14%       65
     798       23       7     28.0518710     32.2947078   13.14%       70
     897       85       7     28.0518710     32.2947078   13.14%       75
    1016      176       7     28.0518710     32.2947078   13.14%       81
    1057      183       7     28.0518710     32.2947078   13.14%       85
    1245      313       7     28.0518710     32.2947078   13.14%       90
    1494      524       7     28.0518710     32.2947078   13.14%       96
    1713      690       7     28.0518710     32.2947078   13.14%      100
    1776      709       7     28.0518710     32.2947078   13.14%      105
    2005      880       7     28.0518710     32.2947078   13.14%      110
    2254     1081       7     28.0518710     32.2947078   13.14%      116
    2337     1123       7     28.0518710     32.2947078   13.14%      120
    2529     1271       7     28.0518710     32.2947078   13.14%      125
    2715     1411       7     28.0518710     32.2947078   13.14%      130
    3117     1700       7     28.0518710     32.2947078   13.14%      136
    3186     1718       7     28.0518710     32.0012943   12.34%      140
    3381     1866       7     28.0518710     32.0012943   12.34%      146
    3540     1978       7     28.0518710     31.9947066   12.32%      152
    3724     2089       7     28.0518710     31.9931742   12.32%      155
    4176     2421       7     28.0518710     31.9931742   12.32%      160
    4241     2446       7     28.0518710     31.7554361   11.66%      165
    4492     2629       7     28.0518710     31.7554361   11.66%      170
    4670     2737       7     28.0518710     31.7554361   11.66%      176
    4766     2799       7     28.0518710     31.7554361   11.66%      180
    4862     2849       7     28.0518710     31.7554361   11.66%      185
    5214     3081       7     28.0518710     31.7554361   11.66%      190
    5526     3282       7     28.0518710     31.7554361   11.66%      195
    5867     3520       7     28.0518710     31.7554361   11.66%      200
    6196     3675       7     28.0518710     31.6317022   11.32%      205
    6403     3805       7     28.0518710     31.6317022   11.32%      210
    6544     3884       7     28.0518710     31.6317022   11.32%      215
    6704     3961       7     28.0518710     31.6317022   11.32%      220
    6966     4099       7     28.0518710     31.6234431   11.29%      225
    7242     4267       7     28.0518710     31.6234431   11.29%      230
    7466     4393       7     28.0518710     31.5535230   11.10%      235
    7786     4589       7     28.0518710     31.5534247   11.10%      240
    7869     4643       7     28.0518710     31.5534247   11.10%      245
    8074     4782       7     28.0518710     31.5534247   11.10%      250
    8494     5027       7     28.0518710     31.5534247   11.10%      255
    8855     5244       7     28.0518710     31.5507961   11.09%      260
    9069     5365       7     28.0518710     31.5507961   11.09%      267
    9357     5560       7     28.0518710     31.4998544   10.95%      270
    9408     5565       7     28.0518710     31.4336628   10.76%      275
    9506     5613       7     28.0518710     31.4336628   10.76%      280
    9602     5661       7     28.0518710     31.4336628   10.76%      285
    9920     5841       7     28.0518710     31.4336628   10.76%      290
   10111     5953       7     28.0518710     31.4336628   10.76%      295
   10627     6234       7     28.0518710     31.4336628   10.76%      299
```

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```
NOTE: Real time limit reached.
NOTE: Objective of the best integer solution found = 28.051870979.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 9850 rows and 4␣
↪columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 15828 rows and 6␣
↪columns.
NOTE: Response TIME_LIM_SOL
NOTE: Added action set 'optimization'.
NOTE: Converting model kidney_exchange to DataFrame.
NOTE: Cloud Analytic Services made the uploaded file available as table BLOCKSTABLE␣
↪in caslib CASUSERHDFS(casuser).
NOTE: The table BLOCKSTABLE has been created in caslib CASUSERHDFS(casuser) from␣
↪binary data uploaded to Cloud Analytic Services.
NOTE: Uploading the problem DataFrame to the server.
NOTE: Cloud Analytic Services made the uploaded file available as table TMPYGE321TR␣
↪in caslib CASUSERHDFS(casuser).
NOTE: The table TMPYGE321TR has been created in caslib CASUSERHDFS(casuser) from␣
↪binary data uploaded to Cloud Analytic Services.
NOTE: The problem kidney_exchange has 15828 variables (15828 binary, 0 integer, 0␣
↪free, 0 fixed).
NOTE: The problem has 9850 constraints (49 LE, 9801 EQ, 0 GE, 0 range).
NOTE: The problem has 47286 constraint coefficients.
NOTE: The remaining solution time after solver initialization is 299.78 seconds.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value BASIC is applied.
NOTE: The MILP presolver removed 3156 variables and 2029 constraints.
NOTE: The MILP presolver removed 9526 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 12672 variables, 7821 constraints, and 37760␣
↪constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The Decomposition algorithm is used.
NOTE: The Decomposition algorithm is executing in the distributed computing␣
↪environment in single-machine mode.
NOTE: The DECOMP method value USER is applied.
NOTE: All blocks are identical and the master model is set partitioning.
NOTE: The Decomposition algorithm is using an aggregate formulation and Ryan-Foster␣
↪branching.
NOTE: The number of block threads has been reduced to 1 threads.
NOTE: The problem has a decomposable structure with 49 blocks. The largest block␣
↪covers 2.02% of the constraints in the problem.
NOTE: The decomposition subproblems cover 12593 (99.38%) variables and 7742 (98.99%)␣
↪constraints.
NOTE: The deterministic parallel mode is enabled.
NOTE: The Decomposition algorithm is using up to 32 threads.
         Iter         Best       Master         Best       LP        IP  CPU Real
                     Bound    Objective      Integer      Gap       Gap Time Time
            .     358.5463       8.2725       8.2725   97.69%    97.69%    2    3
            3     350.4519       9.1816       9.1816   97.38%    97.38%    5    7
            4     350.4519      15.5468      15.5468   95.56%    95.56%    7   11
            5     344.6750      15.5468      15.5468   95.49%    95.49%    9   14
            6     316.2089      15.5468      15.5468   95.08%    95.08%   11   16
           10     316.2089      15.5468      22.7247   95.08%    92.81%   15   22
```

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```
          11     260.4562      22.7247      22.7247   91.28%    91.28%    17    24
          14     259.6150      22.7247      22.7247   91.25%    91.25%    20    30
          15     242.7751      22.7554      22.7247   90.63%    90.64%    21    31
          16     219.3485      23.5204      22.7247   89.28%    89.64%    22    33
          17     204.8669      23.7674      22.7247   88.40%    88.91%    23    35
          18     155.3209      24.1564      22.7247   84.45%    85.37%    24    36
           .     155.3209      24.9509      22.7247   83.94%    85.37%    27    40
          20     147.4931      24.9509      22.7247   83.08%    84.59%    27    41
          22     113.1877      25.7096      23.7438   77.29%    79.02%    80    95
          23      79.4927      26.0422      23.7438   67.24%    70.13%    81    97
          30      79.4927      26.8911      23.7438   66.17%    70.13%    90   111
          32      75.0523      26.9424      23.7438   64.10%    68.36%    93   115
          38      74.7758      27.5362      23.7438   63.17%    68.25%   102   130
          39      37.7021      27.6625      23.7438   26.63%    37.02%   104   132
           .      37.7021      27.6678      27.1343   26.61%    28.03%   105   133
          40      37.7021      27.6678      27.1343   26.61%    28.03%   106   135
          43      37.7021      28.0519      28.0519   25.60%    25.60%   110   141
          45      30.7209      28.0519      28.0519    8.69%     8.69%   112   143
          47      30.6065      28.0519      28.0519    8.35%     8.35%   114   146
          50      28.0519      28.0519      28.0519    0.00%     0.00%   116   150
           Node   Active    Sols         Best         Best       Gap      CPU    Real
                                      Integer        Bound                Time    Time
              0        0      12      28.0519      28.0519     0.00%     116     150
NOTE: The Decomposition algorithm used 32 threads.
NOTE: The Decomposition algorithm time is 150.08 seconds.
NOTE: Optimal within relative gap.
NOTE: Objective = 28.051870979.
Out[2]: 28.051871
```

7.2 Viya Examples / Abstract

7.2.1 Curve Fitting

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex11_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex11.html

Model

```python
import sasoptpy as so
import pandas as pd


def test(cas_conn, sols=False):

    # Upload data to server first
    xy_raw = pd.DataFrame([
        [0.0, 1.0],
        [0.5, 0.9],
```

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```
        [1.0, 0.7],
        [1.5, 1.5],
        [1.9, 2.0],
        [2.5, 2.4],
        [3.0, 3.2],
        [3.5, 2.0],
        [4.0, 2.7],
        [4.5, 3.5],
        [5.0, 1.0],
        [5.5, 4.0],
        [6.0, 3.6],
        [6.6, 2.7],
        [7.0, 5.7],
        [7.6, 4.6],
        [8.5, 6.0],
        [9.0, 6.8],
        [10.0, 7.3]
        ], columns=['x', 'y'])
    xy_data = cas_conn.upload_frame(xy_raw, casout={'name': 'xy_data',
                                                    'replace': True})

    # Read observations
    POINTS, (x, y), xy_table_ref = so.read_table(xy_data, columns=['x', 'y'])

    # Parameters and variables
    order = so.Parameter(name='order')
    beta = so.VariableGroup(so.exp_range(0, order), name='beta')
    estimate = so.ImplicitVar(
        (beta[0] + so.quick_sum(beta[k] * x[i] ** k
                                for k in so.exp_range(1, order))
         for i in POINTS), name='estimate')

    surplus = so.VariableGroup(POINTS, name='surplus', lb=0)
    slack = so.VariableGroup(POINTS, name='slack', lb=0)

    objective1 = so.Expression(
        so.quick_sum(surplus[i] + slack[i] for i in POINTS), name='objective1')
    abs_dev_con = so.ConstraintGroup(
        (estimate[i] - surplus[i] + slack[i] == y[i] for i in POINTS),
        name='abs_dev_con')

    minmax = so.Variable(name='minmax')
    objective2 = so.Expression(minmax + 0.0, name='objective2')
    minmax_con = so.ConstraintGroup(
        (minmax >= surplus[i] + slack[i] for i in POINTS), name='minmax_con')

    order.set_init(1)
    L1 = so.Model(name='L1', session=cas_conn)
    L1.set_objective(objective1, sense=so.MIN)
    L1.include(POINTS, x, y, xy_table_ref)
    L1.include(order, beta, estimate, surplus, slack, abs_dev_con)
    L1.add_statement('print x y estimate surplus slack;', after_solve=True)

    L1.solve(verbose=True)
    sol_data1 = L1.response['Print3.PrintTable'].sort_values('x')
    print(so.get_solution_table(beta))
    print(sol_data1.to_string())
```

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```
    Linf = so.Model(name='Linf', session=cas_conn)
    Linf.include(L1, minmax, minmax_con)
    Linf.set_objective(objective2, sense=so.MIN)

    Linf.solve()
    sol_data2 = Linf.response['Print3.PrintTable'].sort_values('x')
    print(so.get_solution_table(beta))
    print(sol_data2.to_string())

    order.set_init(2)

    L1.solve()
    sol_data3 = L1.response['Print3.PrintTable'].sort_values('x')
    print(so.get_solution_table(beta))
    print(sol_data3.to_string())

    Linf.solve()
    sol_data4 = Linf.response['Print3.PrintTable'].sort_values('x')
    print(so.get_solution_table(beta))
    print(sol_data4.to_string())

    if sols:
        return (sol_data1, sol_data2, sol_data3, sol_data4)
    else:
        return Linf.get_objective_value()
```

Output

```
In [1]: from examples.curve_fitting import test

In [2]: (s1, s2, s3, s4) = test(cas_conn, sols=True)
NOTE: Cloud Analytic Services made the uploaded file available as table XY_DATA in␣
↪caslib CASUSERHDFS(casuser).
NOTE: The table XY_DATA has been created in caslib CASUSERHDFS(casuser) from binary␣
↪data uploaded to Cloud Analytic Services.
NOTE: Initialized model L1.
NOTE: Added action set 'optimization'.
NOTE: Converting model L1 to OPTMODEL.
set set_XY_DATA_N;
num x {set_XY_DATA_N};
num y {set_XY_DATA_N};
read data XY_DATA into set_XY_DATA_N=[_N_] x y;
num order = 1;
var beta {0..order};
impvar estimate {i_1 in set_XY_DATA_N} = sum {i_2 in 1..order}(beta[i_2] * (x[i_1]) ^␣
↪(i_2)) + beta[0];
var surplus {set_XY_DATA_N} >= 0;
var slack {set_XY_DATA_N} >= 0;
min objective1 = sum {i_3 in set_XY_DATA_N}(surplus[i_3] + slack[i_3]);
con abs_dev_con {i_4 in set_XY_DATA_N} : surplus[i_4] + y[i_4] - estimate[i_4] -␣
↪slack[i_4] = 0;

solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
```

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```
print _con_.name _con_.body _con_.dual;
print x y estimate surplus slack;

NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 40 variables (2 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 19 linear constraints (0 LE, 19 EQ, 0 GE, 0 range).
NOTE: The problem has 75 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
NOTE: The presolved problem has 40 variables, 19 constraints, and 75 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    0.000000E+00         0
          D 2         23    1.146625E+01         0
NOTE: Optimal.
NOTE: Objective = 11.46625.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 19 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 40 rows and 6␣
↪columns.
      beta
1
0  0.58125
1  0.63750
    COL1     x    y  estimate  surplus    slack
10  11.0   0.0  1.0   0.58125  0.00000  0.41875
18  19.0   0.5  0.9   0.90000  0.00000  0.00000
12  13.0   1.0  0.7   1.21875  0.51875  0.00000
4    5.0   1.5  1.5   1.53750  0.03750  0.00000
0    1.0   1.9  2.0   1.79250  0.00000  0.20750
15  16.0   2.5  2.4   2.17500  0.00000  0.22500
1    2.0   3.0  3.2   2.49375  0.00000  0.70625
11  12.0   3.5  2.0   2.81250  0.81250  0.00000
5    6.0   4.0  2.7   3.13125  0.43125  0.00000
3    4.0   4.5  3.5   3.45000  0.00000  0.05000
8    9.0   5.0  1.0   3.76875  2.76875  0.00000
14  15.0   5.5  4.0   4.08750  0.08750  0.00000
13  14.0   6.0  3.6   4.40625  0.80625  0.00000
7    8.0   6.6  2.7   4.78875  2.08875  0.00000
9   10.0   7.0  5.7   5.04375  0.00000  0.65625
17  18.0   7.6  4.6   5.42625  0.82625  0.00000
2    3.0   8.5  6.0   6.00000  0.00000  0.00000
6    7.0   9.0  6.8   6.31875  0.00000  0.48125
16  17.0  10.0  7.3   6.95625  0.00000  0.34375
```

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```
NOTE: Initialized model Linf.
NOTE: Added action set 'optimization'.
NOTE: Converting model Linf to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 41 variables (3 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 38 linear constraints (0 LE, 19 EQ, 19 GE, 0 range).
NOTE: The problem has 132 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
NOTE: The presolved problem has 41 variables, 38 constraints, and 132 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    -5.000000E-03         0
          P 2         26     1.725000E+00         0
NOTE: Optimal.
NOTE: Objective = 1.725.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 38 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 41 rows and 6␣
↪columns.
    beta
1
0 -0.400
1  0.625
    COL1     x    y  estimate  surplus   slack
10  11.0   0.0  1.0   -0.4000    0.000  1.4000
18  19.0   0.5  0.9   -0.0875    0.000  0.9875
12  13.0   1.0  0.7    0.2250    0.000  0.4750
4    5.0   1.5  1.5    0.5375    0.000  0.9625
0    1.0   1.9  2.0    0.7875    0.000  1.2125
15  16.0   2.5  2.4    1.1625    0.000  1.2375
1    2.0   3.0  3.2    1.4750    0.000  1.7250
11  12.0   3.5  2.0    1.7875    0.000  0.2125
5    6.0   4.0  2.7    2.1000    0.000  0.6000
3    4.0   4.5  3.5    2.4125    0.000  1.0875
8    9.0   5.0  1.0    2.7250    1.725  0.0000
14  15.0   5.5  4.0    3.0375    0.000  0.9625
13  14.0   6.0  3.6    3.3500    0.000  0.2500
7    8.0   6.6  2.7    3.7250    1.025  0.0000
9   10.0   7.0  5.7    3.9750    0.000  1.7250
17  18.0   7.6  4.6    4.3500    0.000  0.2500
2    3.0   8.5  6.0    4.9125    0.000  1.0875
6    7.0   9.0  6.8    5.2250    0.000  1.5750
16  17.0  10.0  7.3    5.8500    0.000  1.4500
```

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```
NOTE: Added action set 'optimization'.
NOTE: Converting model L1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 41 variables (3 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 19 linear constraints (0 LE, 19 EQ, 0 GE, 0 range).
NOTE: The problem has 93 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
NOTE: The presolved problem has 41 variables, 19 constraints, and 93 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    0.000000E+00         0
          D 2         20    1.045896E+01         0
NOTE: Optimal.
NOTE: Objective = 10.458964706.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 19 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 41 rows and 6␣
↪columns.
       beta
1
0  0.982353
1  0.294510
2  0.033725
    COL1     x    y  estimate   surplus     slack
10  11.0   0.0  1.0  0.982353  0.000000  0.017647
18  19.0   0.5  0.9  1.138039  0.238039  0.000000
12  13.0   1.0  0.7  1.310588  0.610588  0.000000
4    5.0   1.5  1.5  1.500000  0.000000  0.000000
0    1.0   1.9  2.0  1.663671  0.000000  0.336329
15  16.0   2.5  2.4  1.929412  0.000000  0.470588
1    2.0   3.0  3.2  2.169412  0.000000  1.030588
11  12.0   3.5  2.0  2.426275  0.426275  0.000000
5    6.0   4.0  2.7  2.700000  0.000000  0.000000
3    4.0   4.5  3.5  2.990588  0.000000  0.509412
8    9.0   5.0  1.0  3.298039  2.298039  0.000000
14  15.0   5.5  4.0  3.622353  0.000000  0.377647
13  14.0   6.0  3.6  3.963529  0.363529  0.000000
7    8.0   6.6  2.7  4.395200  1.695200  0.000000
9   10.0   7.0  5.7  4.696471  0.000000  1.003529
17  18.0   7.6  4.6  5.168612  0.568612  0.000000
2    3.0   8.5  6.0  5.922353  0.000000  0.077647
6    7.0   9.0  6.8  6.364706  0.000000  0.435294
16  17.0  10.0  7.3  7.300000  0.000000  0.000000
```

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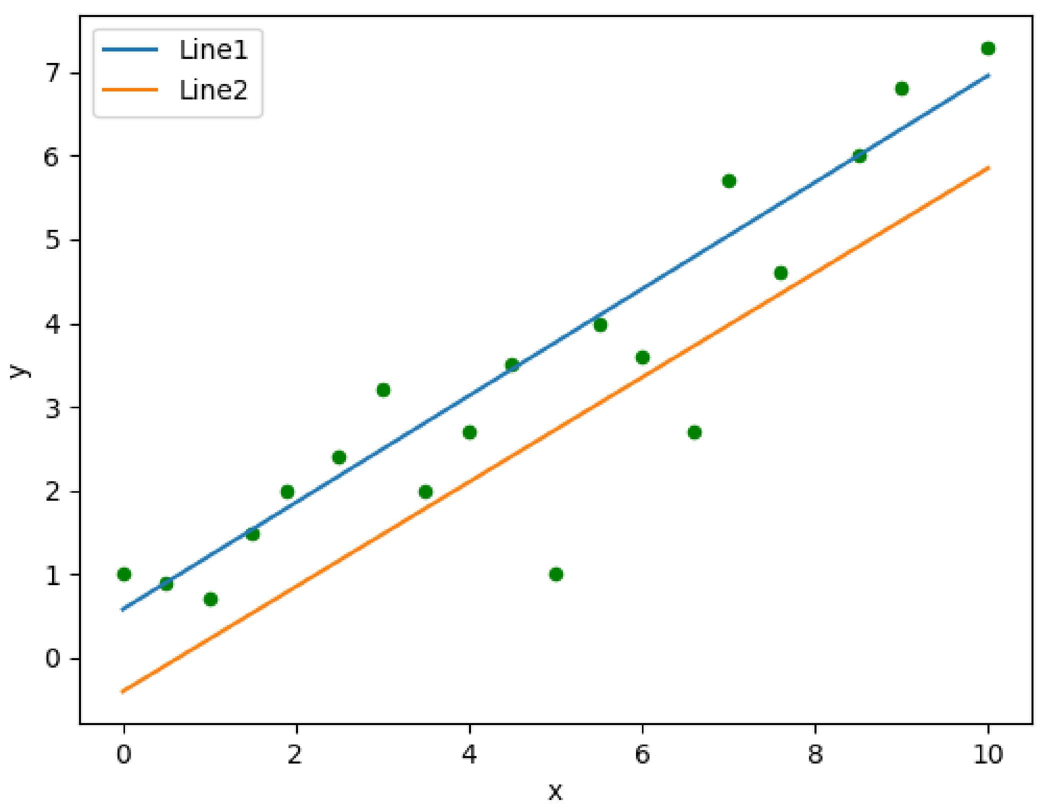
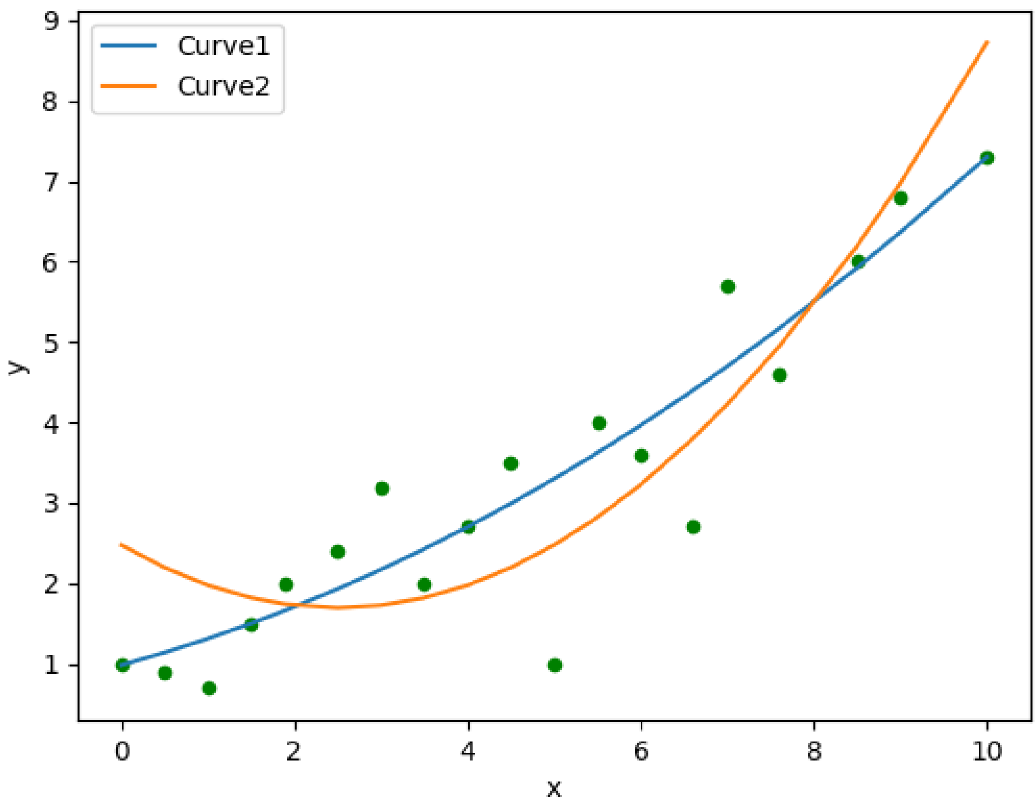
```
NOTE: Added action set 'optimization'.
NOTE: Converting model Linf to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 42 variables (4 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 38 linear constraints (0 LE, 19 EQ, 19 GE, 0 range).
NOTE: The problem has 150 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
NOTE: The presolved problem has 42 variables, 38 constraints, and 150 constraint␣
↪coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration        Value         Time
          D 2          1    -5.000000E-03         0
          P 2         27     1.475000E+00         0
NOTE: Optimal.
NOTE: Objective = 1.475.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 38 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 42 rows and 6␣
↪columns.
    beta
1
0  2.475
1 -0.625
2  0.125
    COL1     x    y  estimate  surplus    slack
10  11.0   0.0  1.0   2.47500  1.47500  0.00000
18  19.0   0.5  0.9   2.19375  1.29375  0.00000
12  13.0   1.0  0.7   1.97500  1.27500  0.00000
4    5.0   1.5  1.5   1.81875  0.31875  0.00000
0    1.0   1.9  2.0   1.73875  0.00000  0.26125
15  16.0   2.5  2.4   1.69375  0.00000  0.70625
1    2.0   3.0  3.2   1.72500  0.00000  1.47500
11  12.0   3.5  2.0   1.81875  0.00000  0.18125
5    6.0   4.0  2.7   1.97500  0.00000  0.72500
3    4.0   4.5  3.5   2.19375  0.00000  1.30625
8    9.0   5.0  1.0   2.47500  1.47500  0.00000
14  15.0   5.5  4.0   2.81875  0.00000  1.18125
13  14.0   6.0  3.6   3.22500  0.00000  0.37500
7    8.0   6.6  2.7   3.79500  1.09500  0.00000
9   10.0   7.0  5.7   4.22500  0.00000  1.47500
17  18.0   7.6  4.6   4.94500  0.34500  0.00000
2    3.0   8.5  6.0   6.19375  0.19375  0.00000
6    7.0   9.0  6.8   6.97500  0.17500  0.00000
16  17.0  10.0  7.3   8.72500  1.42500  0.00000
```

```
# Plots
In [3]: import matplotlib.pyplot as plt

In [4]: p1 = s1.plot.scatter(x='x', y='y', c='g')

In [5]: s1.plot.line(ax=p1, x='x', y='estimate', label='Line1');

In [6]: s2.plot.line(ax=p1, x='x', y='estimate', label='Line2');

In [7]: p1
Out[7]: <matplotlib.axes._subplots.AxesSubplot at 0x7fed5865c6a0>

In [8]: p2 = s3.plot.scatter(x='x', y='y', c='g')

In [9]: s3.plot.line(ax=p2, x='x', y='estimate', label='Curve1');

In [10]: s4.plot.line(ax=p2, x='x', y='estimate', label='Curve2');

In [11]: p2
Out[11]: <matplotlib.axes._subplots.AxesSubplot at 0x7fed586f2fd0>
```





7.2.2 Nonlinear 1

Reference

http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_nlpsolver_examples01.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/nlpse01.html

Model

```
import sasoptpy as so


def test(cas_conn):

    m = so.Model(name='nlpse01', session=cas_conn)
    x = m.add_variables(range(1, 9), lb=0.1, ub=10, name='x')

    f = so.Expression(0.4 * (x[1]/x[7]) ** 0.67 + 0.4 * (x[2]/x[8]) ** 0.67 + 10 -
↪x[1] - x[2], name='f')
    m.set_objective(f, sense=so.MIN)
```

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```
    m.add_constraint(1 - 0.0588*x[5]*x[7] - 0.1*x[1] >= 0, name='c1')
    m.add_constraint(1 - 0.0588*x[6]*x[8] - 0.1*x[1] - 0.1*x[2] >= 0, name='c2')
    m.add_constraint(1 - 4*x[3]/x[5] - 2/(x[3]**0.71 * x[5]) - 0.0588*(x[7]/x[3]**1.
↪3) >= 0, name='c3')
    m.add_constraint(1 - 4*x[4]/x[6] - 2/(x[4]**0.71 * x[6]) - 0.0588*(x[8]/x[4]**1.
↪3) >= 0, name='c4')
    m.add_constraint(f == [0.1, 4.2])

    x[1].set_init(6)
    x[2].set_init(3)
    x[3].set_init(0.4)
    x[4].set_init(0.2)
    x[5].set_init(6)
    x[6].set_init(6)
    x[7].set_init(1)
    x[8].set_init(0.5)

    m.add_statement('print x;', after_solve=True)
    m.solve(verbose=True, options={'with': 'nlp', 'algorithm': 'activeset'})
    print(m.get_problem_summary())
    print(m.get_solution_summary())
    print(m.response['Print3.PrintTable'])

    return m.get_objective_value()
```

Output

```
In [1]: from examples.nonlinear_1 import test

In [2]: test(cas_conn)
NOTE: Initialized model nlpse01.
NOTE: Added action set 'optimization'.
NOTE: Converting model nlpse01 to OPTMODEL.
var x {{1,2,3,4,5,6,7,8}} >= 0.1 <= 10;
x[5] = 6;
x[6] = 6;
x[1] = 6;
x[7] = 1;
x[2] = 3;
x[8] = 0.5;
x[3] = 0.4;
x[4] = 0.2;
min f = - x[2] - x[1] + 0.4 * (((x[2]) / (x[8])) ^ (0.67)) + 0.4 * (((x[1]) / (x[7]))␣
↪^ (0.67)) + 10.0;
con c1 : - 0.0588 * x[5] * x[7] - 0.1 * x[1] >= -1.0;
con c2 : - 0.1 * x[2] - 0.1 * x[1] - 0.0588 * x[6] * x[8] >= -1.0;
con c3 : - 0.0588 * ((x[7]) / ((x[3]) ^ (1.3))) - (4 * x[3]) / (x[5]) - (2) / ((x[3])␣
↪^ (0.71) * x[5]) >= -1.0;
con c4 : - (4 * x[4]) / (x[6]) - 0.0588 * ((x[8]) / ((x[4]) ^ (1.3))) - (2) / ((x[4])␣
↪^ (0.71) * x[6]) >= -1.0;
con con_1 : -9.9 <= - x[2] - x[1] + 0.4 * (((x[2]) / (x[8])) ^ (0.67)) + 0.4 *␣
↪(((x[1]) / (x[7])) ^ (0.67)) <= -5.8;
solve with nlp / algorithm=activeset;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
```

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```
print _con_.name _con_.body _con_.dual;
print x;

NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 8 variables (0 free, 0 fixed).
NOTE: The problem has 0 linear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 5 nonlinear constraints (0 LE, 0 EQ, 4 GE, 1 range).
NOTE: The OPTMODEL presolver removed 0 variables, 0 linear constraints, and 0␣
↪nonlinear constraints.
NOTE: Using analytic derivatives for objective.
NOTE: Using analytic derivatives for nonlinear constraints.
NOTE: The NLP solver is called.
NOTE: The Active Set algorithm is used.
                           Objective                                Optimality
              Iter             Value     Infeasibility                   Error
                 0        3.65736570        0.41664483              0.24247905
                 1        3.65736570        0.41664483              0.24247905
                 2        3.40486061        0.10284726              0.10617988
                 3        3.51178229        0.07506389              0.10593173
                 4        4.23595983        0.03595983              0.33749510
                 5        4.16334906                 0              0.26471063
                 6        4.03168584        0.00791810              0.13742971
                 7        3.88912660        0.11248991              0.06129662
                 8        3.89579714        0.09534670              0.05994916
                 9        3.95046640        0.02649207              0.06776850
                10        3.92833580        0.03517161              0.06442935
                11        3.95179326        0.00494247              0.05837915
                12        3.94741555        0.00651989              0.05477333
                13        3.95209064        0.00058609              0.05265725
                14        3.95058104        0.00122758              0.04772557
                15        3.95055959        0.00099113              0.04613473
                16        3.95141460        0.00000381              0.04497006
                17        3.95132211   0.0000005999371              0.04260039
                18        3.95114031        0.00000941              0.04093117
                19        3.95027690        0.00011307              0.00020755
                20        3.95115797   0.0000007730235              0.00010507
                21        3.95116558                 0              0.00001366
                22        3.95116364   0.0000000153799              0.00000814
                23        3.95116355   0.0000000228326              0.00000595
                24        3.95116352   0.0000000257138              0.00000337
                25        3.95116349   0.0000000200547              0.00000132
                26        3.95116349   0.0000000192412   0.0000002015918
NOTE: Optimal.
NOTE: Objective = 3.9511634887.
NOTE: Objective of the best feasible solution found = 3.9511579677.
NOTE: The best feasible solution found is returned.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 5 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 8 rows and 6␣
↪columns.
NOTE: Response BEST_FEASIBLE
Problem Summary
```

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```
                              Value
Label
Objective Sense        Minimization
Objective Function                f
Objective Type            Nonlinear

Number of Variables               8
Bounded Above                     0
Bounded Below                     0
Bounded Below and Above           8
Free                              0
Fixed                             0

Number of Constraints             5
Linear LE (<=)                    0
Linear EQ (=)                     0
Linear GE (>=)                    0
Linear Range                      0
Nonlinear LE (<=)                 0
Nonlinear EQ (=)                  0
Nonlinear GE (>=)                 4
Nonlinear Range                   1
Solution Summary

                          Value
Label
Solver                      NLP
Algorithm            Active Set
Objective Function            f
Solution Status    Best Feasible
Objective Value    3.9511579677

Optimality Error   0.0001050714
Infeasibility      7.7302351E-7

Iterations                   26
Presolve Time              0.00
Solution Time              0.04
   COL1         x
0   1.0  6.463315
1   2.0  2.234530
2   3.0  0.667455
3   4.0  0.595820
4   5.0  5.932980
5   6.0  5.527231
6   7.0  1.013787
7   8.0  0.400664
Out[2]: 3.951158
```

7.2.3 Nonlinear 2

Reference

http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_nlpsolver_examples02.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/nlpse02.html

Model

```
import sasoptpy as so
import sasoptpy.math as sm


def test(cas_conn):

    m = so.Model(name='nlpse02', session=cas_conn)
    N = m.add_parameter(name='N', init=1000)
    x = m.add_variables(so.exp_range(1, N), name='x', init=1)
    m.set_objective(
        so.quick_sum(-4*x[i]+3 for i in so.exp_range(1, N-1)) +
        so.quick_sum((x[i]**2 + x[N]**2)**2 for i in so.exp_range(1, N-1)),
        name='f', sense=so.MIN)

    m.add_statement('print x;', after_solve=True)
    m.solve(options={'with': 'nlp'}, verbose=True)
    print(m.response['Print3.PrintTable'])

    # Model 2
    so.reset_globals()
    m = so.Model(name='nlpse02_2', session=cas_conn)
    N = m.add_parameter(name='N', init=1000)
    x = m.add_variables(so.exp_range(1, N), name='x', lb=1, ub=2)
    m.set_objective(
        so.quick_sum(sm.cos(-0.5*x[i+1] - x[i]**2) for i in so.exp_range(
            1, N-1)), name='f2', sense=so.MIN)
    m.add_statement('print x;', after_solve=True)
    m.solve(verbose=True, options={'with': 'nlp', 'algorithm': 'activeset'})
    print(m.get_solution_summary())

    return m.get_objective_value()
```

Output

```
In [1]: from examples.nonlinear_2 import test

In [2]: test(cas_conn)
NOTE: Initialized model nlpse02.
NOTE: Added action set 'optimization'.
NOTE: Converting model nlpse02 to OPTMODEL.
num N = 1000;
var x {1..N} init 1;
min f = sum {i_2 in 1..N-1}(((x[i_2]) ^ (2) + (x[N]) ^ (2)) ^ (2)) + sum {i_1 in 1..N-
↪1}(- 4 * x[i_1] + 3);
solve with nlp;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x;

NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
```

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```
NOTE: The problem has 1000 variables (1000 free, 0 fixed).
NOTE: The problem has 0 linear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver removed 0 variables, 0 linear constraints, and 0␣
↪nonlinear constraints.
NOTE: Using analytic derivatives for objective.
NOTE: Using 2 threads for nonlinear evaluation.
NOTE: The NLP solver is called.
NOTE: The Interior Point algorithm is used.
                           Objective                          Optimality
              Iter             Value     Infeasibility             Error
                 0     2997.00000000                 0     3996.00000000
                 1     2903.33927274                 0     3901.96888568
                 2     2720.89298022                 0     3716.59858581
                 3     2375.45256010                 0     3356.96682109
                 4     2050.78067864                 0     3007.33819156
                 5     1479.51953631                 0     2358.96117840
                 6      635.46851927                 0     1297.01852837
                 7       47.88027207                 0      263.90369702
                 8        0.56099667                 0       25.76387053
                 9        0.00010025                 0        0.31770898
                10   0.0000000000556                 0        0.00005787
                11                 0                 0  1.9350622922E-12
NOTE: Optimal.
NOTE: Objective = 0.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 0 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 1000 rows and 6␣
↪columns.
       COL1            x
0       1.0  1.000000e+00
1       2.0  1.000000e+00
2       3.0  1.000000e+00
3       4.0  1.000000e+00
4       5.0  1.000000e+00
5       6.0  1.000000e+00
6       7.0  1.000000e+00
7       8.0  1.000000e+00
8       9.0  1.000000e+00
9      10.0  1.000000e+00
10     11.0  1.000000e+00
11     12.0  1.000000e+00
12     13.0  1.000000e+00
13     14.0  1.000000e+00
14     15.0  1.000000e+00
15     16.0  1.000000e+00
16     17.0  1.000000e+00
17     18.0  1.000000e+00
18     19.0  1.000000e+00
19     20.0  1.000000e+00
20     21.0  1.000000e+00
21     22.0  1.000000e+00
22     23.0  1.000000e+00
23     24.0  1.000000e+00
```

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```
24     25.0  1.000000e+00
25     26.0  1.000000e+00
26     27.0  1.000000e+00
27     28.0  1.000000e+00
28     29.0  1.000000e+00
29     30.0  1.000000e+00
..      ...           ...
970   971.0  1.000000e+00
971   972.0  1.000000e+00
972   973.0  1.000000e+00
973   974.0  1.000000e+00
974   975.0  1.000000e+00
975   976.0  1.000000e+00
976   977.0  1.000000e+00
977   978.0  1.000000e+00
978   979.0  1.000000e+00
979   980.0  1.000000e+00
980   981.0  1.000000e+00
981   982.0  1.000000e+00
982   983.0  1.000000e+00
983   984.0  1.000000e+00
984   985.0  1.000000e+00
985   986.0  1.000000e+00
986   987.0  1.000000e+00
987   988.0  1.000000e+00
988   989.0  1.000000e+00
989   990.0  1.000000e+00
990   991.0  1.000000e+00
991   992.0  1.000000e+00
992   993.0  1.000000e+00
993   994.0  1.000000e+00
994   995.0  1.000000e+00
995   996.0  1.000000e+00
996   997.0  1.000000e+00
997   998.0  1.000000e+00
998   999.0  1.000000e+00
999  1000.0  9.684996e-16

[1000 rows x 2 columns]
NOTE: Initialized model nlpse02_2.
NOTE: Added action set 'optimization'.
NOTE: Converting model nlpse02_2 to OPTMODEL.
num N = 1000;
var x {1..N} >= 1 <= 2;
min f2 = sum {i_1 in 1..N-1}(cos(- (x[i_1]) ^ (2) - 0.5 * x[i_1 + 1]));
solve with nlp / algorithm=activeset;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x;

NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 1000 variables (0 free, 0 fixed).
NOTE: The problem has 0 linear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver removed 0 variables, 0 linear constraints, and 0
↪nonlinear constraints.
```

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```
NOTE: Using analytic derivatives for objective.
NOTE: Using 3 threads for nonlinear evaluation.
NOTE: The NLP solver is called.
NOTE: The Active Set algorithm is used.
NOTE: Initial point was changed to be feasible to bounds.
                           Objective                          Optimality
              Iter             Value     Infeasibility             Error
                 0       70.66646447                 0        1.24686873
                 1       70.66646439                 0        1.24686873
                 2     -996.26893548                 0        0.23815533
                 3     -998.99328004                 0        0.10718277
                 4     -998.99999439                 0        0.00379400
                 5     -999.00000000                 0        0.00000393
                 6     -999.00000000                 0  1.7022658635E-12
NOTE: Optimal.
NOTE: Objective = -999.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows and␣
↪4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 0 rows and 4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows␣
↪and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 1000 rows and 6␣
↪columns.
Solution Summary

                           Value
Label
Solver                       NLP
Algorithm             Active Set
Objective Function            f2
Solution Status          Optimal
Objective Value             -999

Optimality Error    1.702266E-12
Infeasibility                  0

Iterations                     6
Presolve Time               0.00
Solution Time               0.07
Out[2]: -999.0
```

7.3 SAS (saspy) Examples

7.3.1 Decentralization (saspy)

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex10_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex10.html

Model

```
import sasoptpy as so
import pandas as pd


def test(cas_conn):

    m = so.Model(name='decentralization', session=cas_conn)

    DEPTS = ['A', 'B', 'C', 'D', 'E']
    CITIES = ['Bristol', 'Brighton', 'London']

    benefit_data = pd.DataFrame([
        ['Bristol', 10, 15, 10, 20, 5],
        ['Brighton', 10, 20, 15, 15, 15]],
        columns=['city'] + DEPTS).set_index('city')

    comm_data = pd.DataFrame([
        ['A', 'B', 0.0],
        ['A', 'C', 1.0],
        ['A', 'D', 1.5],
        ['A', 'E', 0.0],
        ['B', 'C', 1.4],
        ['B', 'D', 1.2],
        ['B', 'E', 0.0],
        ['C', 'D', 0.0],
        ['C', 'E', 2.0],
        ['D', 'E', 0.7]], columns=['i', 'j', 'comm']).set_index(['i', 'j'])

    cost_data = pd.DataFrame([
        ['Bristol', 'Bristol', 5],
        ['Bristol', 'Brighton', 14],
        ['Bristol', 'London', 13],
        ['Brighton', 'Brighton', 5],
        ['Brighton', 'London', 9],
        ['London', 'London', 10]], columns=['i', 'j', 'cost']).set_index(
            ['i', 'j'])

    max_num_depts = 3

    benefit = {}
    for city in CITIES:
        for dept in DEPTS:
            try:
                benefit[dept, city] = benefit_data.loc[city, dept]
            except:
                benefit[dept, city] = 0

    comm = {}
    for row in comm_data.iterrows():
        (i, j) = row[0]
        comm[i, j] = row[1]['comm']
        comm[j, i] = comm[i, j]

    cost = {}
    for row in cost_data.iterrows():
```

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```
        (i, j) = row[0]
        cost[i, j] = row[1]['cost']
        cost[j, i] = cost[i, j]

    assign = m.add_variables(DEPTS, CITIES, vartype=so.BIN, name='assign')
    IJKL = [(i, j, k, l)
            for i in DEPTS for j in CITIES for k in DEPTS for l in CITIES
            if i < k]
    product = m.add_variables(IJKL, vartype=so.BIN, name='product')

    totalBenefit = so.quick_sum(benefit[i, j] * assign[i, j]
                                for i in DEPTS for j in CITIES)

    totalCost = so.quick_sum(comm[i, k] * cost[j, l] * product[i, j, k, l]
                             for (i, j, k, l) in IJKL)

    m.set_objective(totalBenefit-totalCost, name='netBenefit', sense=so.MAX)

    m.add_constraints((so.quick_sum(assign[dept, city] for city in CITIES)
                       == 1 for dept in DEPTS), name='assign_dept')

    m.add_constraints((so.quick_sum(assign[dept, city] for dept in DEPTS)
                       <= max_num_depts for city in CITIES), name='cardinality')

    product_def1 = m.add_constraints((assign[i, j] + assign[k, l] - 1
                                      <= product[i, j, k, l]
                                      for (i, j, k, l) in IJKL),
                                      name='product_def1')

    product_def2 = m.add_constraints((product[i, j, k, l] <= assign[i, j]
                                       for (i, j, k, l) in IJKL),
                                      name='product_def2')

    product_def3 = m.add_constraints((product[i, j, k, l] <= assign[k, l]
                                       for (i, j, k, l) in IJKL),
                                      name='product_def3')

    m.solve()
    print(m.get_problem_summary())

    m.drop_constraints(product_def1)
    m.drop_constraints(product_def2)
    m.drop_constraints(product_def3)

    m.add_constraints((
        so.quick_sum(product[i, j, k, l]
                     for j in CITIES if (i, j, k, l) in IJKL) == assign[k, l]
        for i in DEPTS for k in DEPTS for l in CITIES if i < k),
        name='product_def4')

    m.add_constraints((
        so.quick_sum(product[i, j, k, l]
                     for l in CITIES if (i, j, k, l) in IJKL) == assign[i, j]
        for k in DEPTS for i in DEPTS for j in CITIES if i < k),
        name='product_def4')

    m.solve()
```

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```
    print(m.get_problem_summary())
    totalBenefit.set_name('totalBenefit')
    totalCost.set_name('totalCost')
    print(so.get_solution_table(totalBenefit, totalCost))
    print(so.get_solution_table(assign).unstack(level=-1))

    return m.get_objective_value()
```

Output

```
>>> from examples.decentralization import test
>>> sas_session = saspy.SASsession(cfgname='winlocal')
>>> test(sas_session)

SAS Connection established. Subprocess id is 18384

NOTE: Initialized model decentralization.
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to SAS server.
NOTE: Writing HTML5(SASPY_INTERNAL) Body file: _TOMODS1
NOTE: Problem generation will use 4 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 278 linear constraints (183 LE, 5 EQ, 90 GE, 0 range).
NOTE: The problem has 660 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 120 constraints.
NOTE: The MILP presolver removed 120 constraint coefficients.
NOTE: The MILP presolver added 120 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 158 constraints, and 540 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 4 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      3    -14.9000000    135.0000000  111.04%       0
             0        1      3    -14.9000000     67.5000000  122.07%       0
             0        1      3    -14.9000000     55.0000000  127.09%       0
             0        1      4      8.1000000     48.0000000   83.12%       0
             0        1      4      8.1000000     44.8375000   81.93%       0
             0        1      4      8.1000000     42.0000000   80.71%       0
             0        1      4      8.1000000     39.0666667   79.27%       0
             0        1      4      8.1000000     34.7500000   76.69%       0
             0        1      4      8.1000000     33.9000000   76.11%       0
             0        1      4      8.1000000     29.6800000   72.71%       0
             0        1      4      8.1000000     28.5000000   71.58%       0
             0        1      4      8.1000000     28.5000000   71.58%       0
             0        1      4      8.1000000     28.5000000   71.58%       0
             0        1      4      8.1000000     28.5000000   71.58%       0
NOTE: The MILP solver added 31 cuts with 168 cut coefficients at the root.
             2        0      5     14.9000000     14.9000000    0.00%       0
```

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```
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The data set WORK.PROB_SUMMARY has 20 observations and 3 variables.
NOTE: The data set WORK.SOL_SUMMARY has 18 observations and 3 variables.
NOTE: The data set WORK.PRIMAL_OUT has 105 observations and 6 variables.
NOTE: The data set WORK.DUAL_OUT has 278 observations and 4 variables.
NOTE: PROCEDURE OPTMODEL used (Total process time):
      real time           0.34 seconds
      cpu time            0.29 seconds

                                Value
Label
Objective Sense           Maximization
Objective Function          netBenefit
Objective Type                  Linear

Number of Variables                105
Bounded Above                        0
Bounded Below                        0
Bounded Below and Above            105
Free                                 0
Fixed                                0
Binary                             105
Integer                              0

Number of Constraints              278
Linear LE (<=)                     183
Linear EQ (=)                        5
Linear GE (>=)                      90
Linear Range                         0

Constraint Coefficients            660
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to SAS server.
NOTE: Writing HTML5(SASPY_INTERNAL) Body file: _TOMODS1
NOTE: Problem generation will use 4 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 68 linear constraints (3 LE, 65 EQ, 0 GE, 0 range).
NOTE: The problem has 270 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 0 constraints.
NOTE: The MILP presolver removed 0 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 68 constraints, and 270 constraint␣
↪coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 4 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      3    -28.1000000    135.0000000  120.81%       0
             0        1      3    -28.1000000     30.0000000  193.67%       0
             0        1      4    -16.3000000     30.0000000  154.33%       0
             0        1      5     14.9000000     14.9000000    0.00%       0
```

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```
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The data set WORK.PROB_SUMMARY has 20 observations and 3 variables.
NOTE: The data set WORK.SOL_SUMMARY has 18 observations and 3 variables.
NOTE: The data set WORK.PRIMAL_OUT has 105 observations and 6 variables.
NOTE: The data set WORK.DUAL_OUT has 68 observations and 4 variables.
NOTE: PROCEDURE OPTMODEL used (Total process time):
      real time           0.19 seconds
      cpu time            0.14 seconds

                                Value
Label
Objective Sense          Maximization
Objective Function         netBenefit
Objective Type                 Linear

Number of Variables               105
Bounded Above                       0
Bounded Below                       0
Bounded Below and Above           105
Free                                0
Fixed                               0
Binary                            105
Integer                             0

Number of Constraints              68
Linear LE (<=)                      3
Linear EQ (=)                      65
Linear GE (>=)                      0
Linear Range                        0

Constraint Coefficients           270
   totalBenefit  totalCost
1
           80.0       65.1
    assign  assign assign
2 Brighton Bristol London
1
A      0.0     1.0    0.0
B      1.0     0.0    0.0
C      1.0     0.0    0.0
D      0.0     1.0    0.0
E      1.0     0.0    0.0
```

CHAPTER

EIGHT

API REFERENCE

8.1 Model

8.1.1 Constructor

Model(name[, session])	Creates an optimization model

sasoptpy.Model

class sasoptpy.**Model**(*name*, *session=None*)

Bases: object

Creates an optimization model

Parameters **name** : string

Name of the model

session : swat.cas.connection.CAS object or saspy.SASsession object, optional

CAS or SAS Session object

Examples

```
>>> from swat import CAS
>>> import sasoptpy as so
>>> s = CAS('cas.server.address', port=12345)
>>> m = so.Model(name='my_model', session=s)
NOTE: Initialized model my_model
```

```
>>> mip = so.Model(name='mip')
NOTE: Initialized model mip
```

8.1.2 Components

Model.set_session(session)	Sets the CAS session for model
Model.add_constraint(c[, name])	Adds a single constraint to the model
Model.add_constraints(argv[, cg, name])	Adds a set of constraints to the model

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Table 2 – continued from previous page

`Model.add_variable`([var, vartype, name, lb, ...])	Adds a new variable to the model
`Model.add_variables`(*argv[, vg, name, ...])	Adds a group of variables to the model
`Model.add_implicit_variable`([argv, name])	Adds an implicit variable to the model
`Model.add_set`(name[, init, settype])	Adds a set to the model
`Model.add_parameter`(*argv[, name, init, p_type])	Adds a parameter to the model
`Model.add_statement`(statement[, after_solve])	Adds a PROC OPTMODEL statement to the model
`Model.set_objective`(expression[, sense, name])	Sets the objective function for the model
`Model.set_coef`(var, con, value)	Updates the coefficient of a variable inside constraints
`Model.drop_constraint`(constraint)	Drops a constraint from the model
`Model.drop_constraints`(constraints)	Drops a constraint group from the model
`Model.drop_variable`(variable)	Drops a variable from the model
`Model.drop_variables`(variables)	Drops a variable group from the model
`Model.get_constraint`(name)	Returns the reference to a constraint in the model
`Model.get_constraints`()	Returns a list of constraints in the model
`Model.get_variable`(name)	Returns the reference to a variable in the model
`Model.get_variables`()	Returns a list of variables
`Model.get_objective`()	Returns the objective function as an `Expression` object
`Model.get_variable_coef`(var)	Returns the objective value coefficient of a variable
`Model.read_data`(table, key_set[, key_cols, ...])	Reads a CASTable into PROC OPTMODEL and adds it to the model
`Model.read_table`(table[, key, columns, ...])	Reads a CAS Table or pandas DataFrame into the model
`Model.include`(*argv)	Adds existing variables and constraints to a model

sasoptpy.Model.set_session

`Model.`**`set_session`**(*session*)

Sets the CAS session for model

Parameters **session** : `swat.cas.connection.CAS` or `saspy.SASsession` objects

CAS or SAS Session object

Notes

- Session of a model can be set at initialization. See `Model`.

sasoptpy.Model.add_constraint

`Model.`**`add_constraint`**(*c*, *name=None*)

Adds a single constraint to the model

Parameters **c** : Constraint

Constraint to be added to the model

name : string, optional

Name of the constraint

Returns `Constraint` object

See also:

Constraint, *Model.include()*

Examples

```
>>> x = m.add_variable(name='x', vartype=so.INT, lb=0, ub=5)
>>> y = m.add_variables(3, name='y', vartype=so.CONT, lb=0, ub=10)
>>> c1 = m.add_constraint(x + y[0] >= 3, name='c1')
>>> print(c1)
 x  +  y[0]  >=  3
```

```
>>> c2 = m.add_constraint(x - y[2] == [4, 10], name='c2')
>>> print(c2)
 -  y[2]  +  x  =  [4, 10]
```

sasoptpy.Model.add_constraints

`Model.add_constraints`(*argv*, *cg=None*, *name=None*)

Adds a set of constraints to the model

Parameters **argv** : Generator type objects

List of constraints as a Generator-type object

cg : *ConstraintGroup* object, optional

An existing list of constraints if an existing group is being added

name : string, optional

Name for the constraint group and individual constraint prefix

Returns *ConstraintGroup* object

A group object for all constraints aded

See also:

ConstraintGroup, *Model.include()*

Examples

```
>>> x = m.add_variable(name='x', vartype=so.INT, lb=0, ub=5)
>>> y = m.add_variables(3, name='y', vartype=so.CONT, lb=0, ub=10)
>>> c = m.add_constraints((x + 2 * y[i] >= 2 for i in [0, 1, 2]),
                          name='c')
>>> print(c)
Constraint Group (c) [
  [0:  2.0 * y[0]  +  x  >=  2]
  [1:  2.0 * y[1]  +  x  >=  2]
  [2:  2.0 * y[2]  +  x  >=  2]
]
```

```
>>> t = m.add_variables(3, 4, name='t')
>>> ct = m.add_constraints((t[i, j] <= x for i in range(3)
                            for j in range(4)), name='ct')
>>> print(ct)
Constraint Group (ct) [
  [(0, 0):  -  x  +  t[0, 0]  <=  0]
  [(0, 1):  t[0, 1]  -  x  <=  0]
  [(0, 2):  -  x  +  t[0, 2]  <=  0]
  [(0, 3):  t[0, 3]  -  x  <=  0]
  [(1, 0):  t[1, 0]  -  x  <=  0]
  [(1, 1):  t[1, 1]  -  x  <=  0]
  [(1, 2):  -  x  +  t[1, 2]  <=  0]
  [(1, 3):  -  x  +  t[1, 3]  <=  0]
  [(2, 0):  -  x  +  t[2, 0]  <=  0]
  [(2, 1):  t[2, 1]  -  x  <=  0]
  [(2, 2):  t[2, 2]  -  x  <=  0]
  [(2, 3):  t[2, 3]  -  x  <=  0]
]
```

sasoptpy.Model.add_variable

`Model.`**`add_variable`**(*var=None*, *vartype='CONT'*, *name=None*, *lb=-inf*, *ub=inf*, *init=None*)

Adds a new variable to the model

New variables can be created via this method or existing variables can be added to the model.

Parameters **var** : `Variable` object, optional

Existing variable to be added to the problem

vartype : string, optional

Type of the variable, either 'BIN', 'INT' or 'CONT'

name : string, optional

Name of the variable to be created

lb : float, optional

Lower bound of the variable

ub : float, optional

Upper bound of the variable

init : float, optional

Initial value of the variable

Returns `Variable` object

Variable that is added to the model

See also:

`Variable`, `Model.include()`

Notes

- If argument *var* is not None, then all other arguments are ignored.

- A generic variable name is generated if name argument is None.

Examples

Adding a variable on the fly

```
>>> m = so.Model(name='demo')
>>> x = m.add_variable(name='x', vartype=so.INT, ub=10, init=2)
>>> print(repr(x))
NOTE: Initialized model demo
sasoptpy.Variable(name='x', lb=0, ub=10, init=2, vartype='INT')
```

Adding an existing variable to a model

```
>>> y = so.Variable(name='y', vartype=so.BIN)
>>> m = so.Model(name='demo')
>>> m.add_variable(var=y)
```

sasoptpy.Model.add_variables

`Model.`**`add_variables`**(**argv*, *vg=None*, *name=None*, *vartype='CONT'*, *lb=None*, *ub=None*, *init=None*, *abstract=None*)

Adds a group of variables to the model

Parameters **argv** : list, dict, `pandas.Index`

Loop index for variable group

vg : *`VariableGroup`* object, optional

An existing object if it is being added to the model

name : string, optional

Name of the variables

vartype : string, optional

Type of variables, *BIN*, *INT*, or *CONT*

lb : list, dict, `pandas.Series`

Lower bounds of variables

ub : list, dict, `pandas.Series`

Upper bounds of variables

init : list, dict, `pandas.Series`

Initial values of variables

See also:

`VariableGroup`, *`Model.include()`*

Notes

If *vg* argument is passed, all other arguments are ignored.

Examples

```
>>> production = m.add_variables(PERIODS, vartype=so.INT,
                                  name='production', lb=min_production)
>>> print(production)
>>> print(repr(production))
Variable Group (production) [
  [Period1: production['Period1',]]
  [Period2: production['Period2',]]
  [Period3: production['Period3',]]
]
sasoptpy.VariableGroup(['Period1', 'Period2', 'Period3'],
name='production')
```

sasoptpy.Model.add_implicit_variable

`Model.add_implicit_variable`(*argv=None*, *name=None*)

Adds an implicit variable to the model

Parameters **argv** : Generator type object

Generator object where each item is an entry

name : string, optional

Name of the implicit variable

Notes

- Based on whether generated by a regular expression or an abstract one, implicit variables may appear in generated OPTMODEL codes.

Examples

```
>>> x = m.add_variables(range(5), name='x')
>>> y = m.add_implicit_variable((
>>>     x[i] + 2 * x[i+1] for i in range(4)), name='y')
>>> print(y[2])
x[2] + 2 * x[3]
```

```
>>> I = m.add_set(name='I')
>>> z = m.add_implicit_variable((x[i] * 2 + 2 for i in I), name='z')
>>> print(z._defn())
impvar z {i_1 in I} = 2 * x[i_1] + 2;
```

sasoptpy.Model.add_set

`Model.add_set`(*name, init=None, settype=['num']*)

Adds a set to the model

Parameters **name** : string, optional

Name of the set

init : *Set*, optional

Initial value of the set

settype : list, optional

Types of the set, a list consists of 'num' and 'str' values

Examples

```
>>> I = m.add_set(name='I')
>>> print(I._defn())
set I;
```

```
>>> J = m.add_set(name='J', settype=['str'])
>>> print(J._defn())
set <str> J;
```

```
>>> N = m.add_parameter(name='N', init=4)
>>> K = m.add_set(name='K', init=so.exp_range(1, N))
>>> print(K._defn())
set K = 1..N;
```

sasoptpy.Model.add_parameter

`Model.add_parameter`(**argv*, *name=None*, *init=None*, *p_type=None*)

Adds a parameter to the model

Parameters **argv** : `sasoptpy.data.Set` object, optional

Key set of the parameter

name : string, optional

Name of the parameter

init : float or expression, optional

Initial value of the parameter

p_type : string, optional

Type of the parameter, 'num' for floats or 'str' for strings

Examples

```
>>> I = m.add_set(name='I')
>>> a = m.add_parameter(I, name='a', init=5)
>>> print(a._defn())
num a {I} init 5 ;
```

sasoptpy.Model.add_statement

`Model.add_statement`(*statement*, *after_solve=False*)

Adds a PROC OPTMODEL statement to the model

Parameters **statement** : `Statement`, *`Expression`* or string

Statement object

after_solve : boolean, optional

Option for putting the statement after 'solve' declaration

Notes

- If the statement string includes 'print', then it is automatically placed after solve.

Examples

```
>>> I = m.add_set(name='I')
>>> x = m.add_variables(I, name='x', vartype=so.INT)
>>> a = m.add_parameter(I, name='a')
>>> c = m.add_constraints((x[i] <= 2 * a[i] for i in I), name='c')
>>> m.add_statement('print x;', after_solve=True)
>>> print(m.to_optmodel())
proc optmodel;
min m_obj = 0;
set I;
var x {I} integer >= 0;
num a {I};
con c {i_1 in I} : x[i_1] - 2.0 * a[i_1] <= 0;
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x;
quit;
```

sasoptpy.Model.set_objective

`Model.`**`set_objective`**`(`*expression*, *sense=None*, *name=None*`)`

Sets the objective function for the model

Parameters **expression** : *`Expression`* object

The objective function as an Expression

sense : string, optional

Objective value direction, 'MIN' or 'MAX'

name : string, optional

Name of the objective value

Returns *`Expression`*

Objective function as an *`Expression`* object

Notes

- Default objective sense is minimization (MIN)

Examples

```
>>> profit = so.Expression(5 * sales - 2 * material, name='profit')
>>> m.set_objective(profit, so.MAX)
>>> print(m.get_objective())
 - 2.0 * material + 5.0 * sales
```

```
>>> m.set_objective(4 * x - 5 * y, name='obj')
>>> print(repr(m.get_objective()))
sasoptpy.Expression(exp = 4.0 * x - 5.0 * y , name='obj')
```

sasoptpy.Model.set_coef

`Model.set_coef`(*var*, *con*, *value*)

Updates the coefficient of a variable inside constraints

Parameters **var** : `Variable` object

Variable whose coefficient will be updated

con : `Constraint` object

Constraint where the coefficient will be updated

value : float

The new value for the coefficient of the variable

See also:

`Constraint.update_var_coef()`

Notes

Variable coefficient inside the constraint is replaced in-place.

Examples

```
>>> c1 = m.add_constraint(x + y >= 1, name='c1')
>>> print(c1)
y + x >= 1
>>> m.set_coef(x, c1, 3)
>>> print(c1)
y + 3.0 * x >= 1
```

sasoptpy.Model.drop_constraint

`Model.drop_constraint`(*constraint*)

Drops a constraint from the model

Parameters **constraint** : `Constraint` object

The constraint to be dropped from the model

See also:

Model.drop_constraints(), *Model.drop_variable()*, *Model.drop_variables()*

Examples

```
>>> c1 = m.add_constraint(2 * x + y <= 15, name='c1')
>>> print(m.get_constraint('c1'))
  2.0 * x  +  y  <=  15
>>> m.drop_constraint(c1)
>>> print(m.get_constraint('c1'))
None
```

sasoptpy.Model.drop_constraints

Model.**drop_constraints**(*constraints*)

Drops a constraint group from the model

Parameters **constraints** : *ConstraintGroup* object

The constraint group to be dropped from the model

See also:

Model.drop_constraints(), *Model.drop_variable()*, *Model.drop_variables()*

Examples

```
>>> c1 = m.add_constraints((x[i] + y <= 15 for i in [0, 1]), name='c1')
>>> print(m.get_constraints())
[sasoptpy.Constraint( x[0]  +  y  <=  15, name='c1_0'),
 sasoptpy.Constraint( x[1]  +  y  <=  15, name='c1_1')]
>>> m.drop_constraints(c1)
>>> print(m.get_constraints())
[]
```

sasoptpy.Model.drop_variable

Model.**drop_variable**(*variable*)

Drops a variable from the model

Parameters **variable** : *Variable* object

The variable to be dropped from the model

See also:

Model.drop_variables(), *Model.drop_constraint()*, *Model.drop_constraints()*

Examples

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> print(m.get_variable('x'))
 x
>>> m.drop_variable(x)
>>> print(m.get_variable('x'))
None
```

sasoptpy.Model.drop_variables

`Model.drop_variables(variables)`

Drops a variable group from the model

Parameters **variables** : `VariableGroup` object

The variable group to be dropped from the model

See also:

`Model.drop_variable()`, `Model.drop_constraint()`, `Model.drop_constraints()`

Examples

```
>>> x = m.add_variables(3, name='x')
>>> print(m.get_variables())
[sasoptpy.Variable(name='x_0',  vartype='CONT'),
 sasoptpy.Variable(name='x_1',  vartype='CONT')]
>>> m.drop_variables(x)
>>> print(m.get_variables())
[]
```

sasoptpy.Model.get_constraint

`Model.get_constraint(name)`

Returns the reference to a constraint in the model

Parameters **name** : string

Name of the constraint requested

Returns `Constraint` object

Examples

```
>>> m.add_constraint(2 * x + y <= 15, name='c1')
>>> print(m.get_constraint('c1'))
2.0 * x  +  y  <=  15
```

sasoptpy.Model.get_constraints

`Model.get_constraints()`

Returns a list of constraints in the model

Returns **list** : A list of `Constraint` objects

Examples

```
>>> m.add_constraint(x[0] + y <= 15, name='c1')
>>> m.add_constraints((2 * x[i] - y >= 1 for i in [0, 1]), name='c2')
>>> print(m.get_constraints())
[sasoptpy.Constraint( x[0]  +  y  <=  15, name='c1'),
 sasoptpy.Constraint( 2.0 * x[0]  -  y  >=  1, name='c2_0'),
 sasoptpy.Constraint( 2.0 * x[1]  -  y  >=  1, name='c2_1')]
```

sasoptpy.Model.get_variable

`Model.`**`get_variable`**(*name*)

Returns the reference to a variable in the model

Parameters **name** : string

Name or key of the variable requested

Returns `Variable` object

Examples

```
>>> m.add_variable(name='x', vartype=so.INT, lb=3, ub=5)
>>> var1 = m.get_variable('x')
>>> print(repr(var1))
sasoptpy.Variable(name='x', lb=3, ub=5, vartype='INT')
```

sasoptpy.Model.get_variables

`Model.`**`get_variables`**()

Returns a list of variables

Returns **list** : A list of `Variable` objects

Examples

```
>>> x = m.add_variables(2, name='x')
>>> y = m.add_variable(name='y')
>>> print(m.get_variables())
[sasoptpy.Variable(name='x_0',  vartype='CONT'),
 sasoptpy.Variable(name='x_1',  vartype='CONT'),
 sasoptpy.Variable(name='y',  vartype='CONT')]
```

sasoptpy.Model.get_objective

`Model.`**`get_objective`**()

Returns the objective function as an `Expression` object

Returns *Expression* object

Objective function

Examples

```
>>> m.set_objective(4 * x - 5 * y, name='obj')
>>> print(repr(m.get_objective()))
sasoptpy.Expression(exp =  4.0 * x  -  5.0 * y , name='obj')
```

sasoptpy.Model.get_variable_coef

`Model.get_variable_coef(var)`

Returns the objective value coefficient of a variable

Parameters **var** : *Variable* object or string

Variable whose objective value is requested or its name

Returns float

Objective value coefficient of the given variable

Examples

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> m.set_objective(4 * x - 5 * y, name='obj', sense=so.MAX)
>>> print(m.get_variable_coef(x))
4.0
>>> print(m.get_variable_coef('y'))
-5.0
```

sasoptpy.Model.read_data

`Model.read_data(table, key_set, key_cols=None, option='', params=None)`

Reads a CASTable into PROC OPTMODEL and adds it to the model

Parameters **table** : CASTable

The CAS table to be read to sets and parameters

key_set : *Set*

Set object to be read as the key (index)

key_cols : list or string, optional

Column names of the key columns

option : string, optional

Additional options for read data command

params : list, optional

A list of dictionaries where each dictionary represent parameters

See also:

`sasoptpy.utils.read_data()`

Notes

- This function is intended to be used internally.
- It imitates the `read data` statement of PROC OPTMODEL.
- This function is still under development and subject to change.
- *key_cols* parameters should be a list. When passing a single item, string type can be used instead.
- Values inside each dictionary in `params` list should be as follows:
  - **param** : *Parameter* object

    Paramter object, whose index is the same as table key
  - **column** : string, optional

    Column name to be read
  - **index** : list, optional

    List of sets if the parameter has to be read in a loop

Examples

```
>>> table = session.upload_frame(df, casout='df')
>>> item = m.add_set(name='set_item')
>>> value = m.add_parameter(item, name='value')
>>> m.read_data(table, key_set=item, key_cols=['items'],params=[{'param': value,
↪'column': 'value'}])
>>> print(m.to_optmodel())
proc optmodel;
min m_obj = 0;
set set_item;
num value {set_item};
read data df into set_item=[items] value;
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
quit;
```

sasoptpy.Model.read_table

`Model.`**`read_table`**(*table, key=['_N_'], columns=None, key_type=['num'], col_types=None, upload=False, casout=None*)

Reads a CAS Table or pandas DataFrame into the model

Parameters **table** : `swat.cas.table.CASTable`, `pandas.DataFrame` object or string

Pointer to CAS Table (server data, CASTable), DataFrame (local data) or the name of the table at execution (server data, string)

key : list, optional

List of key columns (for CASTable) or index columns (for DataFrame)

columns : list, optional

List of columns to read into parameters

key_type : list or string, optional

A list of column types consists of 'num' or 'str' values

col_types : dict, optional

Dictionary of column types

upload : boolean, optional

Option for uploading a local data to CAS server first

casout : string or dict, optional

Casout options if data is uploaded

Returns tuple

A tuple where first element is the key (index) and second element is a list of requested columns

See also:

Model.read_data(), *Model.add_parameter()*, *Model.add_set()*

Notes

- This method can take either a `swat.cas.table.CASTable`, a `pandas.DataFrame` or name of the data set as a string as the first argument.
- If the model is running in saspy or MPS mode, then the data is read to client from the CAS server.
- If the model is running in OPTMODEL mode, then this method generates the corresponding optmodel code.
- When table is a CASTable object, since the actual data is stored on the CAS server, some of the functionalities may be limited.
- For the local data, `upload` argument can be passed for performance improvement.
- See `swat.CAS.upload_frame()` and `table.loadtable` CAS action for `casout` options.

Examples

```
>>> info = pd.DataFrame([
        ['clock', 6, 15, 1],
        ['pc', 3, 14, 5],
        ['headphone', 2, 9, 3],
        ['mug', 2, 4, 1],
        ['book', 5, 1, 3],
        ['pen', 1, 1, 4]
        ], columns=['item', 'weight', 'value', 'limit'])
>>> ITEMS, [weight, value, limit] = m.read_table(
        info, key=['item'], columns=['weight', 'value', 'limit'],
        key_stype='str', upload=False)
```

sasoptpy.Model.include

Model.**include**(**argv*)

Adds existing variables and constraints to a model

Parameters argv : *Model*, *Variable*, *Constraint*, *VariableGroup*, *ConstraintGroup*, *Set*, *Parameter*, Statement, *ImplicitVar*

Objects to be included in the model

Notes

- Including a model causes all variables and constraints inside the original model to be included.

Examples

Adding an existing variable

```
>>> x = so.Variable(name='x', vartype=so.CONT)
>>> m.include(x)
```

Adding an existing constraint

```
>>> c1 = so.Constraint(x + y <= 5, name='c1')
>>> m.include(c1)
```

Adding an existing set of variables

```
>>> z = so.VariableGroup(3, 5, name='z', ub=10)
>>> m.include(z)
```

Adding an existing set of constraints

```
>>> c2 = so.ConstraintGroup((x + 2 * z[i, j] >= 2 for i in range(3)
                            for j in range(5)), name='c2')
>>> m.include(c2)
```

Adding an existing model (including all of its elements)

```
>>> new_model = so.Model(name='new_model')
>>> new_model.include(m)
```

8.1.3 Solver calls

Model.solve([options, submit, name, frame, ...])	Solves the model by calling CAS or SAS optimization solvers
Model.solve_on_cas(session, options, submit, ...)	Solves the optimization problem on CAS Servers
Model.solve_on_mva(session, options, submit, ...)	Solves the optimization problem on SAS Clients
Model.get_solution([vtype, solution, pivot])	Returns the solution details associated with the primal or dual solution

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Table 3 – continued from previous page

Model.get_variable_value([var, name])	Returns the value of a variable.
Model.get_objective_value()	Returns the optimal objective value, if it exists
Model.get_solution_summary()	Returns the solution summary table to the user
Model.get_problem_summary()	Returns the problem summary table to the user
Model.print_solution()	Prints the current values of the variables
Model.upload_user_blocks()	Uploads user-defined decomposition blocks to the CAS server

sasoptpy.Model.solve

`Model.`**`solve`**(*options=None*, *submit=True*, *name=None*, *frame=False*, *drop=False*, *replace=True*, *primalin=False*, *milp=None*, *lp=None*, *verbose=False*)

Solves the model by calling CAS or SAS optimization solvers

Parameters **options** : dict, optional

A dictionary solver options

submit : boolean, optional

Switch for calling the solver instantly

name : string, optional

Name of the table name

frame : boolean, optional

Switch for uploading problem as a MPS DataFrame format

drop : boolean, optional

Switch for dropping the MPS table after solve (only CAS)

replace : boolean, optional

Switch for replacing an existing MPS table (only CAS and MPS)

primalin : boolean, optional

Switch for using initial values (only MILP)

verbose : boolean, optional (experimental)

Switch for printing generated OPTMODEL code

Returns `pandas.DataFrame` object

Solution of the optimization model

See also:

`Model.solve_on_cas()`, *`Model.solve_on_mva()`*

Notes

- This method is essentially a wrapper for two other methods.
- Some of the options listed under `options` argument may not be passed based on which CAS Action is being used.
- The `option` argument should be a dictionary, where keys are option names. For example, `m.solve(options={'maxtime': 600})` limits the solution time to 600 seconds.

- See *Solver Options* for a list of solver options.

Examples

```
>>> m.solve()
NOTE: Initialized model food_manufacture_1
NOTE: Converting model food_manufacture_1 to DataFrame
NOTE: Added action set 'optimization'.
...
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Dual Simplex solve time is 0.01 seconds.
```

```
>>> m.solve(options={'maxtime': 600})
```

```
>>> m.solve(options={'algorithm': 'ipm'})
```

sasoptpy.Model.solve_on_cas

`Model.`**`solve_on_cas`**(*session*, *options*, *submit*, *name*, *frame*, *drop*, *replace*, *primalin*, *verbose*)

Solves the optimization problem on CAS Servers

See also:

`Model.solve()`

Notes

- This function is not supposed to be used directly. Instead, use the `swat.cas.CAS` type of session for *`Model`* objects and use *`Model.solve()`*.

sasoptpy.Model.solve_on_mva

`Model.`**`solve_on_mva`**(*session*, *options*, *submit*, *name*, *frame*, *drop*, *replace*, *primalin*, *verbose*)

Solves the optimization problem on SAS Clients

See also:

`Model.solve()`

Notes

- This function is not supposed to be used directly. Instead, use the `saspy.SASsession` type of session for *`Model`* objects and use *`Model.solve()`*.

sasoptpy.Model.get_solution

`Model.`**`get_solution`**(*vtype='Primal'*, *solution=None*, *pivot=False*)

Returns the solution details associated with the primal or dual solution

Parameters **vtype** : string, optional

'Primal' or 'Dual'

solution : integer, optional

Solution number to be returned (for MILP)

pivot : boolean, optional

Switch for returning multiple solutions in columns as a pivot table

Returns `pandas.DataFrame` object

Primal or dual solution table returned from the CAS Action

Notes

- If `Model.solve()` method is used with `frame=True` option, MILP solver returns multiple solutions. You can obtain different results using `solution` parameter.

Examples

```
>>> m.solve()
>>> print(m.get_solution('Primal'))
              var   lb             ub  value  solution
0        x[clock]  0.0  1.797693e+308    0.0       1.0
1           x[pc]  0.0  1.797693e+308    5.0       1.0
2    x[headphone]  0.0  1.797693e+308    2.0       1.0
3          x[mug]  0.0  1.797693e+308    0.0       1.0
4         x[book]  0.0  1.797693e+308    0.0       1.0
5          x[pen]  0.0  1.797693e+308    1.0       1.0
6        x[clock]  0.0  1.797693e+308    0.0       2.0
7           x[pc]  0.0  1.797693e+308    5.0       2.0
8    x[headphone]  0.0  1.797693e+308    2.0       2.0
9          x[mug]  0.0  1.797693e+308    0.0       2.0
10        x[book]  0.0  1.797693e+308    0.0       2.0
11         x[pen]  0.0  1.797693e+308    0.0       2.0
12       x[clock]  0.0  1.797693e+308    1.0       3.0
13          x[pc]  0.0  1.797693e+308    4.0       3.0
...
```

```
>>> print(m.get_solution('Primal', solution=2))
              var   lb             ub  value  solution
6        x[clock]  0.0  1.797693e+308    0.0       2.0
7           x[pc]  0.0  1.797693e+308    5.0       2.0
8    x[headphone]  0.0  1.797693e+308    2.0       2.0
9          x[mug]  0.0  1.797693e+308    0.0       2.0
10        x[book]  0.0  1.797693e+308    0.0       2.0
11         x[pen]  0.0  1.797693e+308    0.0       2.0
```

```
>>> print(m.get_solution(pivot=True))
solution      1.0  2.0  3.0  4.0  5.0
var
x[book]       0.0  0.0  0.0  1.0  0.0
x[clock]      0.0  0.0  1.0  1.0  0.0
x[headphone]  2.0  2.0  1.0  1.0  0.0
x[mug]        0.0  0.0  0.0  1.0  0.0
```

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```
x[pc]           5.0   5.0   4.0   1.0   0.0
x[pen]          1.0   0.0   0.0   1.0   0.0
```

```
>>> print(m.get_solution('Dual'))
                     con  value  solution
0             weight_con   20.0       1.0
1       limit_con[clock]    0.0       1.0
2          limit_con[pc]    5.0       1.0
3   limit_con[headphone]    2.0       1.0
4         limit_con[mug]    0.0       1.0
5        limit_con[book]    0.0       1.0
6         limit_con[pen]    1.0       1.0
7             weight_con   19.0       2.0
8       limit_con[clock]    0.0       2.0
9          limit_con[pc]    5.0       2.0
10  limit_con[headphone]    2.0       2.0
11        limit_con[mug]    0.0       2.0
12       limit_con[book]    0.0       2.0
13        limit_con[pen]    0.0       2.0
...
```

```
>>> print(m.get_solution('dual', pivot=True))
solution               1.0   2.0   3.0   4.0  5.0
con
limit_con[book]        0.0   0.0   0.0   1.0  0.0
limit_con[clock]       0.0   0.0   1.0   1.0  0.0
limit_con[headphone]   2.0   2.0   1.0   1.0  0.0
limit_con[mug]         0.0   0.0   0.0   1.0  0.0
limit_con[pc]          5.0   5.0   4.0   1.0  0.0
limit_con[pen]         1.0   0.0   0.0   1.0  0.0
weight_con            20.0  19.0  20.0  19.0  0.0
```

sasoptpy.Model.get_variable_value

Model.**get_variable_value**(*var=None*, *name=None*)

Returns the value of a variable.

Parameters **var** : *Variable* object, optional

Variable object

name : string, optional

Name of the variable

Notes

- It is possible to get a variable's value using *Variable.get_value()* method, if the variable is not abstract.
- This method is a wrapper around *Variable.get_value()* and an overlook function for model components

sasoptpy.Model.get_objective_value

`Model.get_objective_value()`

Returns the optimal objective value, if it exists

Returns float : Objective value at current solution

Notes

- This method should be used for getting the objective value after solve. Using `m.get_objective().get_value()` actually evaluates the expression using optimal variable values. This may not be available for nonlinear expressions.

Examples

```
>>> m.solve()
>>> print(m.get_objective_value())
42.0
```

sasoptpy.Model.get_solution_summary

`Model.get_solution_summary()`

Returns the solution summary table to the user

Returns `swat.dataframe.SASDataFrame` object

Solution summary obtained after solve

Examples

```
>>> m.solve()
>>> soln = m.get_solution_summary()
>>> print(type(soln))
<class 'swat.dataframe.SASDataFrame'>
```

```
>>> print(soln)
Solution Summary
                               Value
Label
Solver                            LP
Algorithm               Dual Simplex
Objective Function               obj
Solution Status              Optimal
Objective Value                   10
Primal Infeasibility               0
Dual Infeasibility                 0
Bound Infeasibility                0
Iterations                         2
Presolve Time                   0.00
Solution Time                   0.01
```

```
>>> print(soln.index)
Index(['Solver', 'Algorithm', 'Objective Function', 'Solution Status',
       'Objective Value', '', 'Primal Infeasibility',
       'Dual Infeasibility', 'Bound Infeasibility', '', 'Iterations',
       'Presolve Time', 'Solution Time'],
      dtype='object', name='Label')
```

```
>>> print(soln.loc['Solution Status', 'Value'])
Optimal
```

sasoptpy.Model.get_problem_summary

`Model.get_problem_summary()`

Returns the problem summary table to the user

Returns `swat.dataframe.SASDataFrame` object

Problem summary obtained after `Model.solve()`

Examples

```
>>> m.solve()
>>> ps = m.get_problem_summary()
>>> print(type(ps))
<class 'swat.dataframe.SASDataFrame'>
```

```
>>> print(ps)
Problem Summary
                                Value
Label
Problem Name                   model1
Objective Sense          Maximization
Objective Function                obj
RHS                               RHS
Number of Variables                 2
Bounded Above                       0
Bounded Below                       2
Bounded Above and Below             0
Free                                0
Fixed                               0
Number of Constraints               2
LE (<=)                             1
EQ (=)                              0
GE (>=)                             1
Range                               0
Constraint Coefficients             4
```

```
>>> print(ps.index)
Index(['Problem Name', 'Objective Sense', 'Objective Function', 'RHS',
'', 'Number of Variables', 'Bounded Above', 'Bounded Below',
'Bounded Above and Below', 'Free', 'Fixed', '',
'Number of Constraints', 'LE (<=)', 'EQ (=)', 'GE (>=)', 'Range', '',
'Constraint Coefficients'],
dtype='object', name='Label')
```

```
>>> print(ps.loc['Number of Variables'])
Value                2
Name: Number of Variables, dtype: object
```

```
>>> print(ps.loc['Constraint Coefficients', 'Value'])
4
```

sasoptpy.Model.print_solution

Model.**print_solution**()

Prints the current values of the variables

See also:

Model.get_solution()

Notes

- This function may not work for abstract variables and nonlinear models.

Examples

```
>>> m.solve()
>>> m.print_solution()
x: 2.0
y: 0.0
```

sasoptpy.Model.upload_user_blocks

Model.**upload_user_blocks**()

Uploads user-defined decomposition blocks to the CAS server

Returns string

CAS table name of the user-defined decomposition blocks

Examples

```
>>> userblocks = m.upload_user_blocks()
>>> m.solve(milp={'decomp': {'blocks': userblocks}})
```

8.1.4 Export

Model.to_frame([constant])	Converts the Python model into a DataFrame object in MPS format
Model.to_optmodel([header, expand, ordered, ...])	Generates the equivalent PROC OPTMODEL code for the model.

sasoptpy.Model.to_frame

`Model.to_frame(constant=False)`

Converts the Python model into a DataFrame object in MPS format

Parameters constant : boolean, optional

Switching for using objConstant argument for solveMilp, solveLp. Adds the constant as an auxiliary variable if value is True.

Returns `pandas.DataFrame` object

Problem in strict MPS format

Notes

- This method is called inside `Model.solve()`.

Examples

```
>>> df = m.to_frame()
>>> print(df)
     Field1 Field2  Field3 Field4 Field5 Field6 _id_
0      NAME         model1      0             0    1
1      ROWS                                        2
2       MAX    obj                                 3
3         L     c1                                 4
4   COLUMNS                                        5
5                x     obj      4                  6
6                x      c1      3                  7
7                y     obj     -5                  8
8                y      c1      1                  9
9       RHS                                       10
10             RHS      c1      6                 11
11   RANGES                                       12
12   BOUNDS                                       13
13   ENDATA                     0             0   14
```

sasoptpy.Model.to_optmodel

`Model.to_optmodel(header=True, expand=False, ordered=False, ods=False, options={})`

Generates the equivalent PROC OPTMODEL code for the model.

Parameters header : boolean, optional

Option to include PROC headers

expand : boolean, optional

Option to include 'expand' command to OPTMODEL code

ordered : boolean, optional

Option to generate OPTMODEL code in a specific order (True) or in creation order (False)

options : dict, optional

Solver options for the OPTMODEL solve command

Returns string

PROC OPTMODEL representation of the model

Notes

- This method is called inside `Model.solve()`.

Examples

```
>>> print(m.to_optmodel())
proc optmodel;
var get {{'clock','mug','headphone','book','pen'}} integer >= 0;
get['clock'] = 3.0;
get['mug'] = 4.0;
get['headphone'] = 2.0;
get['book'] = -0.0;
get['pen'] = 5.0;
con limit_con_clock : get['clock'] <= 3;
con limit_con_mug : get['mug'] <= 5;
con limit_con_headphone : get['headphone'] <= 2;
con limit_con_book : get['book'] <= 10;
con limit_con_pen : get['pen'] <= 15;
con weight_con : 4 * get['clock'] + 6 * get['mug'] + 7 * get['headphone'] + 12 *
↪get['book'] + get['pen'] <= 55;
max total_value = 8 * get['clock'] + 10 * get['mug'] + 15 * get['headphone'] + 20
↪* get['book'] + get['pen'];
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
quit;
```

8.1.5 Internal functions

`Model.upload_model`([name, replace, constant])	Converts internal model to MPS table and upload to CAS session
`Model.test_session`()	Tests if the model session is defined and still active
`Model._is_linear`()	Checks if the model can be written as a linear model (in MPS format)

sasoptpy.Model.upload_model

`Model.upload_model`(*name=None*, *replace=True*, *constant=False*)

Converts internal model to MPS table and upload to CAS session

Parameters **name** : string, optional

Desired name of the MPS table on the server

replace : boolean, optional

Option to replace the existing MPS table

Returns `swat.cas.table.CASTable` object

Reference to the uploaded CAS Table

Notes

- This method returns None if the model session is not valid.
- Name of the table is randomly assigned if name argument is None or not given.
- This method should not be used if `Model.solve()` is going to be used. `Model.solve()` calls this method internally.

sasoptpy.Model.test_session

`Model.test_session()`

Tests if the model session is defined and still active

Returns string

'CAS' for CAS sessions, 'SAS' for SAS sessions, None otherwise

sasoptpy.Model._is_linear

`Model._is_linear()`

Checks if the model can be written as a linear model (in MPS format)

Returns boolean

True if model does not have any nonlinear components or abstract operations, False otherwise

8.2 Expression

8.2.1 Constructor

`Expression`([exp, name, temp])	Creates a mathematical expression to represent model components

sasoptpy.Expression

class `sasoptpy.Expression`(*exp=None*, *name=None*, *temp=False*)

Bases: `object`

Creates a mathematical expression to represent model components

Parameters **exp** : `Expression` object, optional

An existing expression where arguments are being passed

name : string, optional

A local name for the expression

temp : boolean, optional

A boolean shows whether expression is temporary or permanent

Notes

- Two other classes (`Variable` and `Constraint`) are subclasses of this class.
- Expressions are created automatically after linear math operations with variables.
- An expression object can be called when defining constraints and other expressions.

Examples

```
>>> x = so.Variable(name='x')
>>> y = so.VariableGroup(3, name='y')
>>> e = so.Expression(exp=x + 3 * y[0] - 5 * y[1], name='exp1')
>>> print(e)
 -  5.0 * y[1]  +  3.0 * y[0]  +  x
>>> print(repr(e))
sasoptpy.Expression(exp =  -  5.0 * y[1]  +  3.0 * y[0]  +  x ,
                    name='exp1')
```

```
>>> sales = so.Variable(name='sales')
>>> material = so.Variable(name='material')
>>> profit = 5 * sales - 3 * material
>>> print(profit)
 5.0 * sales  -  3.0 * material
>>> print(repr(profit))
sasoptpy.Expression(exp =  5.0 * sales  -  3.0 * material , name=None)
```

```
>>> import sasoptpy.math as sm
>>> f = sm.sin(x) + sm.min(y[1],1) ** 2
>>> print(type(f))
<class 'sasoptpy.components.Expression'>
>>> print(f)
sin(x) + (min(y[1] , 1)) ** (2)
```

8.2.2 Methods

`Expression.add`(other[, sign])	Combines two expressions and produces a new one
`Expression.copy`([name])	Returns a copy of the `Expression` object
`Expression.get_name`()	Returns the name of the expression
`Expression.get_value`()	Calculates and returns the value of the linear expression
`Expression.mult`(other)	Multiplies the `Expression` with a scalar value
`Expression.set_name`([name])	Sets the name of the expression
`Expression.set_permanent`([name])	Converts a temporary expression into a permanent one

sasoptpy.Expression.add

`Expression.add`(*other*, *sign=1*)

Combines two expressions and produces a new one

Parameters **other** : float or *Expression* object

Second expression or constant value to be added

sign : int, optional

Sign of the addition, 1 or -1

in_place : boolean, optional

Whether the addition will be performed in place or not

Returns *Expression* object

Notes

- This method is mainly for internal use.
- Adding an expression is equivalent to calling this method: (x-y)+(3*x-2*y) and (x-y).add(3*x-2*y) are interchangeable.

sasoptpy.Expression.copy

`Expression.copy(name=None)`

Returns a copy of the *Expression* object

Parameters **name** : string, optional

Name for the copy

Returns *Expression* object

Copy of the object

Examples

```
>>> e = so.Expression(7 * x - y[0], name='e')
>>> print(repr(e))
sasoptpy.Expression(exp =  -  y[0]  +  7.0 * x , name='e')
>>> f = e.copy(name='f')
>>> print(repr(f))
sasoptpy.Expression(exp =  -  y[0]  +  7.0 * x , name='f')
```

sasoptpy.Expression.get_name

`Expression.get_name()`

Returns the name of the expression

Returns string

Name of the expression

Examples

```
>>> var1 = m.add_variables(name='x')
>>> print(var1.get_name())
x
```

sasoptpy.Expression.get_value

`Expression.get_value()`

Calculates and returns the value of the linear expression

Returns float

Value of the expression

Notes

- Nonlinear expressions may not be evaluated.

Examples

```
>>> profit = so.Expression(5 * sales - 3 * material)
>>> m.solve()
>>> print(profit.get_value())
41.0
```

sasoptpy.Expression.mult

`Expression.mult(other)`

Multiplies the `Expression` with a scalar value

Parameters **other** : `Expression` or int

Second expression to be multiplied

Returns `Expression` object

A new `Expression` that represents the multiplication

Notes

- This method is mainly for internal use.
- Multiplying an expression is equivalent to calling this method: 3*(x-y) and (x-y).mult(3) are interchangeable.

sasoptpy.Expression.set_name

`Expression.set_name(name=None)`

Sets the name of the expression

Parameters **name** : string

Name of the expression

Returns string

Name of the expression after resolving conflicts

Examples

```
>>> e = x + 2*y
>>> e.set_name('objective')
```

sasoptpy.Expression.set_permanent

`Expression.set_permanent(name=None)`

Converts a temporary expression into a permanent one

Parameters **name** : string, optional

Name of the expression

Returns string

Name of the expression in the namespace

8.2.3 Private Methods

Expression._expr()	Generates the OPTMODEL compatible string representation of the object.
Expression._is_linear()	Checks if the expression is composed of linear components
Expression._relational(other, direction_)	Creates a logical relation between *Expression* objects
Expression.__repr__()	Returns a string representation of the object.
Expression.__str__()	Generates a representation string that is Python compatible

sasoptpy.Expression._expr

`Expression._expr()`

Generates the OPTMODEL compatible string representation of the object.

Examples

```
>>> f = x + y ** 2
>>> print(f)
x + (y) ** (2)
>>> print(f._expr())
x + (y) ^ (2)
```

sasoptpy.Expression._is_linear

`Expression._is_linear()`

Checks if the expression is composed of linear components

Returns boolean

True if the expression is linear, False otherwise

Examples

```
>>> x = so.Variable()
>>> e = x*x
>>> print(e.is_linear())
False
```

```
>>> f = x*x + x*x - 2*x*x + 5
>>> print(f.is_linear())
True
```

sasoptpy.Expression._relational

`Expression._relational(other, direction_)`

Creates a logical relation between `Expression` objects

Parameters **other** : `Expression` object

Expression on the other side of the relation wrt self

direction_ : string

Direction of the logical relation, either 'E', 'L', or 'G'

Returns `Constraint`

Constraint generated as a result of linear relation

sasoptpy.Expression.__repr__

`Expression.__repr__()`

Returns a string representation of the object.

Examples

```
>>> f = x + y ** 2
>>> print(repr(f))
sasoptpy.Expression(exp = x + (y) ** (2), name=None)
```

sasoptpy.Expression.__str__

`Expression.__str__()`

Generates a representation string that is Python compatible

Examples

```
>>> f = x + y ** 2
>>> print(str(f))
x + (y) ** (2)
```

8.3 Variable

8.3.1 Constructor

Variable(name[, vartype, lb, ub, init, . . .])	Creates an optimization variable to be used inside models

sasoptpy.Variable

class sasoptpy.**Variable**(*name*, *vartype='CONT'*, *lb=-inf*, *ub=inf*, *init=None*, *abstract=False*, *shadow=False*, *key=None*)

Bases: `sasoptpy.components.Expression`

Creates an optimization variable to be used inside models

Parameters name : string

Name of the variable

vartype : string, optional

Type of the variable

lb : float, optional

Lower bound of the variable

ub : float, optional

Upper bound of the variable

init : float, optional

Initial value of the variable

abstract : boolean, optional

Indicator of whether the variable is abstract or not

shadow : boolean, optional

Indicator of whether the variable is shadow or not Used for internal purposes

See also:

sasoptpy.Model.add_variable()

Examples

```
>>> x = so.Variable(name='x', lb=0, ub=20, vartype=so.CONT)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=0, ub=20, vartype='CONT')
```

```
>>> y = so.Variable(name='y', init=1, vartype=so.INT)
>>> print(repr(y))
sasoptpy.Variable(name='y', lb=0, ub=inf, init=1, vartype='INT')
```

8.3.2 Methods

`Variable.set_bounds`([lb, ub])	Changes bounds on a variable
`Variable.set_init`([init])	Changes initial value of a variable

sasoptpy.Variable.set_bounds

`Variable.`**`set_bounds`**(*lb=None*, *ub=None*)

Changes bounds on a variable

Parameters **lb** : float

Lower bound of the variable

ub : float

Upper bound of the variable

Examples

```
>>> x = so.Variable(name='x', lb=0, ub=20)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=0, ub=20, vartype='CONT')
>>> x.set_bounds(lb=5, ub=15)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=5, ub=15, vartype='CONT')
```

sasoptpy.Variable.set_init

`Variable.`**`set_init`**(*init=None*)

Changes initial value of a variable

Parameters **init** : float or None

Initial value of the variable

Examples

```
>>> x = so.Variable(name='x')
>>> x.set_init(5)
```

```
>>> y = so.Variable(name='y', init=3)
>>> y.set_init()
```

8.3.3 Inherited Methods

Variable.add(other[, sign])	Combines two expressions and produces a new one
Variable.copy([name])	Returns a copy of the Expression object
Variable.get_dual()	Returns the dual value
Variable.get_name()	Returns the name of the expression
Variable.get_value()	Calculates and returns the value of the linear expression

sasoptpy.Variable.add

Variable.**add**(*other*, *sign=1*)

Combines two expressions and produces a new one

Parameters **other** : float or Expression object

Second expression or constant value to be added

sign : int, optional

Sign of the addition, 1 or -1

in_place : boolean, optional

Whether the addition will be performed in place or not

Returns Expression object

Notes

- This method is mainly for internal use.
- Adding an expression is equivalent to calling this method: (x-y)+(3*x-2*y) and (x-y).add(3*x-2*y) are interchangeable.

sasoptpy.Variable.copy

Variable.**copy**(*name=None*)

Returns a copy of the Expression object

Parameters **name** : string, optional

Name for the copy

Returns Expression object

Copy of the object

Examples

```
>>> e = so.Expression(7 * x - y[0], name='e')
>>> print(repr(e))
sasoptpy.Expression(exp =  -  y[0]  +  7.0 * x , name='e')
>>> f = e.copy(name='f')
>>> print(repr(f))
sasoptpy.Expression(exp =  -  y[0]  +  7.0 * x , name='f')
```

sasoptpy.Variable.get_dual

`Variable.get_dual()`

Returns the dual value

Returns float

Dual value of the variable

sasoptpy.Variable.get_name

`Variable.get_name()`

Returns the name of the expression

Returns string

Name of the expression

Examples

```
>>> var1 = m.add_variables(name='x')
>>> print(var1.get_name())
x
```

sasoptpy.Variable.get_value

`Variable.get_value()`

Calculates and returns the value of the linear expression

Returns float

Value of the expression

Notes

- Nonlinear expressions may not be evaluated.

Examples

```
>>> profit = so.Expression(5 * sales - 3 * material)
>>> m.solve()
>>> print(profit.get_value())
41.0
```

8.4 Variable Group

8.4.1 Constructor

VariableGroup(*argv, name[, vartype, lb, ...])	Creates a group of Variable objects

sasoptpy.VariableGroup

class sasoptpy.**VariableGroup**(**argv, name, vartype='CONT', lb=-inf, ub=inf, init=None, abstract=False*)

Bases: object

Creates a group of Variable objects

Parameters **argv** : list, dict, int, pandas.Index

Loop index for variable group

name : string, optional

Name (prefix) of the variables

vartype : string, optional

Type of variables, *BIN*, *INT*, or *CONT*

lb : list, dict, pandas.Series, optional

Lower bounds of variables

ub : list, dict, pandas.Series, optional

Upper bounds of variables

init : float, optional

Initial values of variables

See also:

sasoptpy.Model.add_variables(), *sasoptpy.Model.include()*

Notes

- When working with a single model, use the *sasoptpy.Model.add_variables()* method.
- If a variable group object is created, it can be added to a model using the *sasoptpy.Model.include()* method.
- An individual variable inside the group can be accessed using indices.

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> print(repr(z[0, 'a']))
sasoptpy.Variable(name='z_0_a', lb=0, ub=10, vartype='CONT')
```

Examples

```
>>> PERIODS = ['Period1', 'Period2', 'Period3']
>>> production = so.VariableGroup(PERIODS, vartype=so.INT,
                                  name='production', lb=10)
>>> print(production)
Variable Group (production) [
  [Period1: production['Period1']]
```

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```
  [Period2: production['Period2']]
  [Period3: production['Period3']]
]
```

```
>>> x = so.VariableGroup(4, vartype=so.BIN, name='x')
>>> print(x)
Variable Group (x) [
  [0: x[0]]
  [1: x[1]]
  [2: x[2]]
  [3: x[3]]
]
```

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z')
>>> print(z)
Variable Group (z) [
  [(0, 'a'): z[0, 'a']]
  [(0, 'b'): z[0, 'b']]
  [(0, 'c'): z[0, 'c']]
  [(1, 'a'): z[1, 'a']]
  [(1, 'b'): z[1, 'b']]
  [(1, 'c'): z[1, 'c']]
]
>>> print(repr(z))
sasoptpy.VariableGroup([0, 1], ['a', 'b', 'c'], name='z')
```

8.4.2 Methods

VariableGroup.get_name()	Returns the name of the variable group
VariableGroup.set_bounds([lb, ub])	Sets / updates bounds for the given variable
VariableGroup.set_init(init)	Sets / updates initial value for the given variable
VariableGroup.mult(vector)	Quick multiplication method for the variable groups
VariableGroup.sum(*argv)	Quick sum method for the variable groups

sasoptpy.VariableGroup.get_name

`VariableGroup.`**`get_name`**`()`

Returns the name of the variable group

Returns string

Name of the variable group

Examples

```
>>> var1 = m.add_variables(4, name='x')
>>> print(var1.get_name())
x
```

sasoptpy.VariableGroup.set_bounds

`VariableGroup.`**`set_bounds`**`(`*lb=None*, *ub=None*`)`

Sets / updates bounds for the given variable

Parameters **lb** : float, `pandas.Series`, optional

Lower bound

ub : float, `pandas.Series`, optional

Upper bound

Examples

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> print(repr(z[0, 'a']))
sasoptpy.Variable(name='z_0_a', lb=0, ub=10, vartype='CONT')
>>> z.set_bounds(lb=3, ub=5)
>>> print(repr(z[0, 'a']))
sasoptpy.Variable(name='z_0_a', lb=3, ub=5, vartype='CONT')
```

```
>>> u = so.VariableGroup(['a', 'b', 'c', 'd'], name='u')
>>> lb_vals = pd.Series([1, 4, 0, -1], index=['a', 'b', 'c', 'd'])
>>> u.set_bounds(lb=lb_vals)
>>> print(repr(u['b']))
sasoptpy.Variable(name='u_b', lb=4, ub=inf, vartype='CONT')
```

sasoptpy.VariableGroup.set_init

`VariableGroup.`**`set_init`**`(`*init*`)`

Sets / updates initial value for the given variable

Parameters **init** : float, list, dict, `pandas.Series`

Initial value of the variables

Examples

```
>>> y = m.add_variables(3, name='y')
>>> print(y._defn())
var y {{0,1,2}};
>>> y.set_init(5)
>>> print(y._defn())
var y {{0,1,2}} init 5;
```

sasoptpy.VariableGroup.mult

`VariableGroup.`**`mult`**`(`*vector*`)`

Quick multiplication method for the variable groups

Parameters **vector** : list, dictionary, `pandas.Series` object, or `pandas.DataFrame` object

Vector to be multiplied with the variable group

Returns *Expression* object

An expression that is the product of the variable group with the given vector

Examples

Multiplying with a list

```
>>> x = so.VariableGroup(4, vartype=so.BIN, name='x')
>>> e1 = x.mult([1, 5, 6, 10])
>>> print(e1)
 10.0 * x[3]  +  6.0 * x[2]  +  x[0]  +  5.0 * x[1]
```

Multiplying with a dictionary

```
>>> y = so.VariableGroup([0, 1], ['a', 'b'], name='y', lb=0, ub=10)
>>> dvals = {(0, 'a'): 1, (0, 'b'): 2, (1, 'a'): -1, (1, 'b'): 5}
>>> e2 = y.mult(dvals)
>>> print(e2)
 2.0 * y[0, 'b']  -  y[1, 'a']  +  y[0, 'a']  +  5.0 * y[1, 'b']
```

Multiplying with a pandas.Series object

```
>>> u = so.VariableGroup(['a', 'b', 'c', 'd'], name='u')
>>> ps = pd.Series([0.1, 1.5, -0.2, 0.3], index=['a', 'b', 'c', 'd'])
>>> e3 = u.mult(ps)
>>> print(e3)
 1.5 * u['b']  +  0.1 * u['a']  -  0.2 * u['c']  +  0.3 * u['d']
```

Multiplying with a pandas.DataFrame object

```
>>> data = np.random.rand(3, 3)
>>> df = pd.DataFrame(data, columns=['a', 'b', 'c'])
>>> print(df)
>>> NOTE: Initialized model model1
          a         b         c
0  0.966524  0.237081  0.944630
1  0.821356  0.074753  0.345596
2  0.065229  0.037212  0.136644
>>> y = m.add_variables(3, ['a', 'b', 'c'], name='y')
>>> e = y.mult(df)
>>> print(e)
 0.9665237354418064 * y[0, 'a']  +  0.23708064143289442 * y[0, 'b']  +
0.944629500537536 * y[0, 'c']  +  0.8213562592159828 * y[1, 'a']  +
0.07475256894157478 * y[1, 'b']  +  0.3455957019116668 * y[1, 'c']  +
0.06522945752546017 * y[2, 'a']  +  0.03721153533250843 * y[2, 'b']  +
0.13664422498043194 * y[2, 'c']
```

sasoptpy.VariableGroup.sum

`VariableGroup.`**sum**(**argv*)

Quick sum method for the variable groups

Parameters **argv** : Arguments

List of indices for the sum

Returns *Expression* object

Expression that represents the sum of all variables in the group

Examples

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> e1 = z.sum('*', '*')
>>> print(e1)
z[1, 'c']  +  z[1, 'a']  +  z[1, 'b']  +  z[0, 'a']  +  z[0, 'b']  +
z[0, 'c']
>>> e2 = z.sum('*', 'a')
>>> print(e2)
 z[1, 'a']  +  z[0, 'a']
>>> e3 = z.sum('*', ['a', 'b'])
>>> print(e3)
 z[1, 'a']  +  z[0, 'b']  +  z[1, 'b']  +  z[0, 'a']
```

8.5 Constraint

8.5.1 Constructor

Constraint(exp[, direction, name, crange])	Creates a linear or quadratic constraint for optimization models

sasoptpy.Constraint

class sasoptpy.**Constraint**(*exp*, *direction=None*, *name=None*, *crange=0*)

Bases: `sasoptpy.components.Expression`

Creates a linear or quadratic constraint for optimization models

Constraints should be created by adding logical relations to *Expression* objects.

Parameters **exp** : *Expression*

A logical expression that forms the constraint

direction : string

Direction of the logical expression, 'E' (=), 'L' (<=) or 'G' (>=)

name : string, optional

Name of the constraint object

crange : float, optional

Range for ranged constraints

See also:

sasoptpy.Model.add_constraint()

Notes

- A constraint can be generated in multiple ways:
  1. Using the *sasoptpy.Model.add_constraint()* method

  ```
  >>> c1 = m.add_constraint(3 * x - 5 * y <= 10, name='c1')
  >>> print(repr(c1))
  sasoptpy.Constraint( -  5.0 * y  +  3.0 * x  <=  10, name='c1')
  ```

  2. Using the constructor

  ```
  >>> c1 = sasoptpy.Constraint(3 * x - 5 * y <= 10, name='c1')
  >>> print(repr(c1))
  sasoptpy.Constraint( -  5.0 * y  +  3.0 * x  <=  10, name='c1')
  ```

- The same constraint can be included into other models using the *Model.include()* method.

Examples

```
>>> c1 = so.Constraint( 3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( -  5.0 * y  +  3.0 * x  <=  10, name='c1')
```

```
>>> c2 = so.Constraint( - x + 2 * y - 5, direction='L', name='c2')
sasoptpy.Constraint( -  x  +  2.0 * y  <=  5, name='c2')
```

8.5.2 Methods

Constraint.get_value([rhs])	Returns the current value of the constraint
Constraint.set_block(block_number)	Sets the decomposition block number for a constraint
Constraint.set_direction(direction)	Changes the direction of a constraint
Constraint.set_rhs(value)	Changes the RHS of a constraint

sasoptpy.Constraint.get_value

Constraint.**get_value**(*rhs=False*)

Returns the current value of the constraint

Parameters **rhs** : boolean, optional

Whether constant values (RHS) will be included in the value or not. Default is false

Examples

```
>>> m.solve()
>>> print(c1.get_value())
6.0
>>> print(c1.get_value(rhs=True))
0.0
```

sasoptpy.Constraint.set_block

`Constraint.`**`set_block`**(*block_number*)

Sets the decomposition block number for a constraint

Parameters **block_number** : int

Block number of the constraint

Examples

```
>>> c1 = m.add_constraints((x + 2 * y[i] <= 5 for i in NODES),
                           name='c1')
>>> for i in NODES:
      c1[i].set_block(i)
```

sasoptpy.Constraint.set_direction

`Constraint.`**`set_direction`**(*direction*)

Changes the direction of a constraint

Parameters **direction** : string

Direction of the constraint, 'E', 'L', or 'G' for equal to, less than or equal to, and greater than or equal to, respectively

Examples

```
>>> c1 = so.Constraint(exp=3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( 3.0 * x  -  5.0 * y  <=  10, name='c1')
>>> c1.set_direction('G')
>>> print(repr(c1))
sasoptpy.Constraint( 3.0 * x  -  5.0 * y  >=  10, name='c1')
```

sasoptpy.Constraint.set_rhs

`Constraint.`**`set_rhs`**(*value*)

Changes the RHS of a constraint

Parameters **value** : float

New RHS value for the constraint

Examples

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> c = m.add_constraint(x + 3*y <= 10, name='con_1')
>>> print(c)
x  +  3.0 * y  <=  10
```

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```
>>> c.set_rhs(5)
>>> print(c)
x  +  3.0 * y  <=  5
```

8.6 Constraint Group

8.6.1 Constructor

`ConstraintGroup`(argv, name)	Creates a group of `Constraint` objects

sasoptpy.ConstraintGroup

class sasoptpy.**ConstraintGroup**(*argv*, *name*)

Bases: `object`

Creates a group of `Constraint` objects

Parameters **argv** : GeneratorType object

A Python generator that includes `sasoptpy.Expression` objects

name : string, optional

Name (prefix) of the constraints

See also:

sasoptpy.Model.add_constraints(), *sasoptpy.Model.include()*

Notes

Use *sasoptpy.Model.add_constraints()* when working with a single model.

Examples

```
>>> var_ind = ['a', 'b', 'c', 'd']
>>> u = so.VariableGroup(var_ind, name='u')
>>> t = so.Variable(name='t')
>>> cg = so.ConstraintGroup((u[i] + 2 * t <= 5 for i in var_ind),
                            name='cg')
>>> print(cg)
Constraint Group (cg) [
  [a:  2.0 * t  +  u['a']  <=  5]
  [b:  u['b']  +  2.0 * t  <=  5]
  [c:  2.0 * t  +  u['c']  <=  5]
  [d:  2.0 * t  +  u['d']  <=  5]
]
```

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> cg2 = so.ConstraintGroup((2 * z[i, j] + 3 * z[i-1, j] >= 2 for i in
                              [1] for j in ['a', 'b', 'c']), name='cg2')
```

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```
>>> print(cg2)
Constraint Group (cg2) [
  [(1, 'a'):  3.0 * z[0, 'a']  +  2.0 * z[1, 'a']  >=  2]
  [(1, 'b'):  2.0 * z[1, 'b']  +  3.0 * z[0, 'b']  >=  2]
  [(1, 'c'):  2.0 * z[1, 'c']  +  3.0 * z[0, 'c']  >=  2]
]
```

8.6.2 Methods

ConstraintGroup.get_name()	Returns the name of the constraint group
ConstraintGroup.get_expressions([rhs])	Returns constraints as a list of expressions

sasoptpy.ConstraintGroup.get_name

ConstraintGroup.**get_name**()

Returns the name of the constraint group

Returns string

Name of the constraint group

Examples

```
>>> c1 = m.add_constraints((x + y[i] <= 4 for i in indices),
                           name='con1')
>>> print(c1.get_name())
con1
```

sasoptpy.ConstraintGroup.get_expressions

ConstraintGroup.**get_expressions**(*rhs=False*)

Returns constraints as a list of expressions

Parameters **rhs** : boolean, optional

Whether to pass the constant part (rhs) of the constraint or not

Returns `pandas.DataFrame`

Returns a DataFrame consisting of constraints as expressions

Examples

```
>>> cg = so.ConstraintGroup((u[i] + 2 * t <= 5 for i in var_ind),
                            name='cg')
>>> ce = cg.get_expressions()
>>> print(ce)
                     cg
c    u['c']  +  2.0 * t
b    u['b']  +  2.0 * t
```

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```
d   u['d']  +  2.0 * t
a   u['a']  +  2.0 * t
>>> ce_rhs = cg.get_expressions(rhs=True)
>>> print(ce_rhs)
                                  cg
b      u['b']  -  5  +  2.0 * t
c   -  5  +  u['c']  +  2.0 * t
d   -  5  +  u['d']  +  2.0 * t
a   -  5  +  2.0 * t  +  u['a']
```

8.7 Others

8.7.1 Constructors

ExpressionDict([name])	Creates a dictionary of *Expression* objects
ImplicitVar([argv, name])	Creates an implicit variable
Set(name[, init, settype])	Creates an index set to be represented inside PROC OPTMODEL
SetIterator(initset[, conditions, datatype, . . .])	Creates an iterator object for a given Set
Parameter(name[, keys, order, init, p_type])	Creates a parameter to be represented inside PROC OPTMODEL
ParameterValue(param[, key, prefix, suffix])	Represents a single value of a parameter

sasoptpy.ExpressionDict

class sasoptpy.**ExpressionDict**(*name=None*)

Bases: object

Creates a dictionary of *Expression* objects

Parameters name : string

Name of the object

Notes

- ExpressionDict is the underlying class for *ImplicitVar*.
- It behaves as a regular dictionary for client-side models.

Examples

```
>>> e[0] = x + 2*y
>>> e[1] = 2*x + y**2
>>> print(e.get_keys())
>>> for i in e:
>>>     print(i, e[i])
(0,) x + 2 * y
(1,) 2 * x + (y) ** (2)
```

sasoptpy.ImplicitVar

class sasoptpy.**ImplicitVar**(*argv=None*, *name=None*)

Bases: sasoptpy.data.ExpressionDict

Creates an implicit variable

Parameters argv : Generator, optional

Generator object for the implicit variable

name : string, optional

Name of the implicit variable

Notes

- If the loop inside generator is over an abstract object, a definition for the object will be created inside *Model.to_optmodel()* method.

Examples

Regular Implicit Variable

```
>>> I = range(5)
>>> x = so.Variable(name='x')
>>> y = so.VariableGroup(I, name='y')
>>> z = so.ImplicitVar((x + i * y[i] for i in I), name='z')
>>> for i in z:
>>>     print(i, z[i])
(0,) x
(1,) x + y[1]
(2,) x + 2 * y[2]
(3,) x + 3 * y[3]
(4,) x + 4 * y[4]
```

Abstract Implicit Variable

```
>>> I = so.Set(name='I')
>>> x = so.Variable(name='x')
>>> y = so.VariableGroup(I, name='y')
>>> z = so.ImplicitVar((x + i * y[i] for i in I), name='z')
>>> print(z._defn())
impvar z {i_1 in I} = x + i_1 * y[i_1];
>>> for i in z:
>>>     print(i, z[i])
(sasoptpy.data.SetIterator(name=i_1, ...),) x + i_1 * y[i_1]
```

sasoptpy.Set

class sasoptpy.**Set**(*name*, *init=None*, *settype=['num']*)

Bases: sasoptpy.components.Expression

Creates an index set to be represented inside PROC OPTMODEL

Parameters name : string

Name of the parameter

init : *Expression*, optional

Initial value expression of the parameter

settype : list, optional

List of types for the set, consisting of 'num' and 'str' values

Examples

```
>>> I = so.Set('I')
>>> print(I._defn())
set I;
```

```
>>> J = so.Set('J', settype=['num', 'str'])
>>> print(J._defn())
set <num, str> J;
```

```
>>> N = so.Parameter(name='N', init=5)
>>> K = so.Set('K', init=so.exp_range(1,N))
>>> print(K._defn())
set K = 1..N;
```

sasoptpy.SetIterator

class sasoptpy.**SetIterator**(*initset, conditions=None, datatype='num', group={'id': 0, 'order': 1, 'outof': 1}, multi_index=False*)

Bases: `sasoptpy.components.Expression`

Creates an iterator object for a given Set

Parameters initset : *Set*

Set to be iterated on

conditions : list, optional

List of conditions on the iterator

datatype : string, optional

Type of the iterator

group : dict, optional

Dictionary representing the order of iterator inside multi-index sets

multi_index : boolean, optional

Switch for representing multi-index iterators

Notes

- SetIterator objects are automatically created when looping over a *Set*.
- This class is mainly intended for internal use.

- The `group` parameter consists of following keys
  - **order** : int Order of the parameter inside the group
  - **outof** : int Total number of indices inside the group
  - **id** : int ID number assigned to group by Python

sasoptpy.Parameter

class `sasoptpy.`**`Parameter`**(*name*, *keys=None*, *order=1*, *init=None*, *p_type=None*)

Bases: `object`

Creates a parameter to be represented inside PROC OPTMODEL

Parameters **name** : string

Name of the parameter

keys : list, optional

List of *`Set`* to be used as keys for multi-index parameters

init : *`Expression`*, optional

Initial value expression of the parameter

p_type : string, optional

Type of the parameter, 'num' or 'str'

See also:

`read_table()`, *`Model.read_table()`*

Examples

```
>>> p = so.Parameter('p', init=x + 2*y)
>>> print(p._defn())
num p = x + 2 * y;
```

```
>>> I = so.Set('I')
>>> r = so.Parameter('r', keys=I, p_type='str')
>>> print(r._defn())
str r {I};
```

sasoptpy.ParameterValue

class `sasoptpy.`**`ParameterValue`**(*param*, *key=None*, *prefix="*, *suffix="*)

Bases: `sasoptpy.components.Expression`

Represents a single value of a parameter

Parameters **param** : *`Parameter`*

Parameter that the value belongs to

key : tuple, optional

Key of the parameter value in the multi-index parameter

prefix : string

Prefix of the parameter

suffix : string

Suffix of the parameter, such as `.lb` and `.ub`

Notes

- Parameter values are mainly used in abstract expressions

8.7.2 Methods

ParameterValue.set_init(val)	Sets the initial value of the parameter

sasoptpy.ParameterValue.set_init

ParameterValue.**set_init**(*val*)

Sets the initial value of the parameter

Parameters **val** : *Expression*

Initial value

Notes

- This method is only available for parameters without index/key.

Examples

```
>>> p = so.Parameter(name='p')
>>> print(p._defn())
num p;
>>> p.set_init(10)
>>> print(p._defn())
num p = 10;
```

8.8 Functions

8.8.1 Utility Functions

check_name(name[, ctype])	Checks if a name is valid and returns a random string if not
dict_to_frame(dictobj[, cols])	Converts dictionaries to DataFrame objects for pretty printing
exp_range(start, stop[, step])	Creates a set within given range

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Table 20 – continued from previous page

`extract_list_value`(tuplist, listname)	Extracts values inside various object types
`flatten_frame`(df[, swap])	Converts a `pandas.DataFrame` object into `pandas.Series`
`flatten_tuple`(tp)	Flattens nested tuples
`get_counter`(ctrtype)	Returns and increments the list counter for naming
`get_len`(i)	Safe wrapper of len() function
`get_mutable`(exp)	Returns a mutable copy of the given expression if it is immutable
`get_namespace`()	Prints details of components registered to the global name dictionary
`get_solution_table`(*argv[, key, sort, rhs])	Returns the requested variable names as a DataFrame table
`list_length`(listobj)	Returns the length of an object if it is a list, tuple or dict
`list_pack`(obj)	Converts a given object to a list
`print_model_mps`(model)	Prints the MPS representation of the model
`quick_sum`(argv)	Quick summation function for `Expression` objects
`read_data`(table, key_set[, key_cols, …])	(Experimental) Reads a CASTable into PROC OPTMODEL sets
`read_frame`(df[, cols])	Reads each column in `pandas.DataFrame` into a list of `pandas.Series` objects
`read_table`(table[, session, key, columns, …])	Reads a CAS Table or pandas DataFrame
`recursive_walk`(obj, func[, attr, alt])	Calls a given method recursively for given objects
`register_name`(name, obj)	Adds the name and order of a component into the global reference list
`reset_globals`()	Deletes the references inside the global dictionary and restarts counters
`tuple_pack`(obj)	Converts a given object to a tuple object
`tuple_unpack`(tp)	Grabs the first element in a tuple, if a tuple is given as argument
`union`(*args)	Returns a union of `Set`, list or set objects
`wrap`(e[, abstract])	Wraps expression inside another expression

sasoptpy.check_name

`sasoptpy.`**`check_name`**(*name*, *ctype=None*)

Checks if a name is valid and returns a random string if not

Parameters name : str

Name to be checked if unique

Returns str : The given name if valid, a random string otherwise

sasoptpy.dict_to_frame

`sasoptpy.`**`dict_to_frame`**(*dictobj*, *cols=None*)

Converts dictionaries to DataFrame objects for pretty printing

Parameters dictobj : dict

Dictionary to be converted

cols : list, optional

Column names

Returns `DataFrame` object

DataFrame representation of the dictionary

Examples

```
>>> d = {'coal': {'period1': 1, 'period2': 5, 'period3': 7},
>>>      'steel': {'period1': 8, 'period2': 4, 'period3': 3},
>>>      'copper': {'period1': 5, 'period2': 7, 'period3': 9}}
>>> df = so.dict_to_frame(d)
>>> print(df)
        period1  period2  period3
coal          1        5        7
copper        5        7        9
steel         8        4        3
```

sasoptpy.exp_range

`sasoptpy.`**`exp_range`**(*start*, *stop*, *step=1*)

Creates a set within given range

Parameters **start** : *`Expression`*

First value of the range

stop : *`Expression`*

Last value of the range

step : *`Expression`*, optional

Step size of the range

Returns *`Set`*

Set that represents the range

Examples

```
>>> N = so.Parameter(name='N')
>>> p = so.exp_range(1, N)
>>> print(p._defn())
set 1..N;
```

sasoptpy.extract_list_value

`sasoptpy.`**`extract_list_value`**(*tuplist*, *listname*)

Extracts values inside various object types

Parameters **tuplist** : tuple

Key combination to be extracted

listname : dict or list or int or float or DataFrame or Series object

List where the value will be extracted

Returns object

Corresponding value inside listname

sasoptpy.flatten_frame

sasoptpy.**flatten_frame**(*df*, *swap=False*)

Converts a `pandas.DataFrame` object into `pandas.Series`

Parameters **df** : `pandas.DataFrame` object

DataFrame object to be flattened

swap : boolean, optional

Option to use columns as first index

Returns `pandas.DataFrame` object

A new DataFrame where indices consist of index and columns names as tuples

Examples

```
>>> price = pd.DataFrame([
>>>     [1, 5, 7],
>>>     [8, 4, 3],
>>>     [5, 7, 9]], columns=['period1', 'period2', 'period3']).\
>>>     set_index([['coal', 'steel', 'copper']])
>>> print('Price data: \n{}'.format(price))
>>> price_f = so.flatten_frame(price)
>>> print('Price data: \n{}'.format(price_f))
Price data:
        period1  period2  period3
coal          1        5        7
steel         8        4        3
copper        5        7        9
Price data:
(coal, period1)      1
(coal, period2)      5
(coal, period3)      7
(steel, period1)     8
(steel, period2)     4
(steel, period3)     3
(copper, period1)    5
(copper, period2)    7
(copper, period3)    9
dtype: int64
```

sasoptpy.flatten_tuple

sasoptpy.**flatten_tuple**(*tp*)

Flattens nested tuples

Parameters **tp** : tuple

Nested tuple to be flattened

Returns Generator

A generator object representing the flat tuple

Examples

```
>>> tp = (3, 4, (5, (1, 0), 2))
>>> print(list(so.flatten_tuple(tp)))
[3, 4, 5, 1, 0, 2]
```

sasoptpy.get_counter

`sasoptpy.`**`get_counter`**(*ctrtype*)

Returns and increments the list counter for naming

Parameters **ctrtype** : string

Type of the counter, 'obj', 'var', 'con' or 'expr'

Returns int

Current value of the counter

sasoptpy.get_len

`sasoptpy.`**`get_len`**(*i*)

Safe wrapper of len() function

Returns int

len(i) if parameter i has len() function defined, othwerwise 1

sasoptpy.get_mutable

`sasoptpy.`**`get_mutable`**(*exp*)

Returns a mutable copy of the given expression if it is immutable

Parameters **exp** : `Variable` or `Expression`

Object to be wrapped

Returns `Expression`

Mutable copy of the expression, if the original is immutable

sasoptpy.get_namespace

`sasoptpy.`**`get_namespace`**()

Prints details of components registered to the global name dictionary

The list includes models, variables, constraints and expressions

Returns string

A string representation of the namespace

sasoptpy.get_solution_table

`sasoptpy.get_solution_table(*argv, key=None, sort=True, rhs=False)`

Returns the requested variable names as a DataFrame table

Parameters **key** : list, optional

Keys for objects

sort : bool, optional

Option for sorting the keys

rhs : bool, optional

Option for including constant values

Returns `pandas.DataFrame`

DataFrame object that holds keys and values

sasoptpy.list_length

`sasoptpy.list_length(listobj)`

Returns the length of an object if it is a list, tuple or dict

Parameters **listobj** : list, tuple or dict

Object whose length will be returned

Returns int

Length of the list, tuple or dict

sasoptpy.list_pack

`sasoptpy.list_pack(obj)`

Converts a given object to a list

If the object is already a list, the function returns the input, otherwise creates a list

Parameters **obj** : Object

Object that is converted to a list

Returns list

List that includes the original object

sasoptpy.print_model_mps

`sasoptpy.print_model_mps(model)`

Prints the MPS representation of the model

Parameters **model** : `Model`

Model whose MPS format will be printed

See also:

`sasoptpy.Model.to_frame()`

Examples

```
>>> m = so.Model(name='print_example', session=s)
>>> x = m.add_variable(lb=1, name='x')
>>> y = m.add_variables(2, name='y', ub=3, vartype=so.INT)
>>> m.add_constraint(x + y.sum('*') <= 9, name='c1')
>>> m.add_constraints((x + y[i] >= 2 for i in [0, 1]), name='c2')
>>> m.set_objective(x+3*y[0], sense=so.MAX, name='obj')
>>> so.print_model_mps(m)
NOTE: Initialized model print_example
     Field1    Field2         Field3 Field4    Field5 Field6 _id_
0      NAME            print_example      0                0    1
1      ROWS                                                     2
2       MAX       obj                                           3
3         L        c1                                           4
4         G      c2_0                                           5
5         G      c2_1                                           6
6   COLUMNS                                                     7
7                   x            obj      1                     8
8                   x             c1      1                     9
9                   x           c2_0      1                    10
10                  x           c2_1      1                    11
11           MARK0000       'MARKER'          'INTORG'         12
12                y_0            obj      3                    13
13                y_0             c1      1                    14
14                y_0           c2_0      1                    15
15                y_1             c1      1                    16
16                y_1           c2_1      1                    17
17           MARK0001       'MARKER'          'INTEND'         18
18       RHS                                                   19
19                RHS             c1      9                    20
20                RHS           c2_0      2                    21
21                RHS           c2_1      2                    22
22   RANGES                                                    23
23   BOUNDS                                                    24
24        LO      BND              x      1                    25
25        UP      BND            y_0      3                    26
26        LO      BND            y_0      0                    27
27        UP      BND            y_1      3                    28
28        LO      BND            y_1      0                    29
29   ENDATA                               0                0   30
```

sasoptpy.quick_sum

`sasoptpy.`**`quick_sum`**(*argv*)

Quick summation function for `Expression` objects

Returns `Expression` object

Sum of given arguments

Notes

This function is faster for expressions compared to Python’s native sum() function.

Examples

```
>>> x = so.VariableGroup(10000, name='x')
>>> y = so.quick_sum(2*x[i] for i in range(10000))
```

sasoptpy.read_data

sasoptpy.**read_data**(*table*, *key_set*, *key_cols=None*, *option=''*, *params=None*)

(Experimental) Reads a CASTable into PROC OPTMODEL sets

Parameters **table** : CASTable

The CAS table to be read to sets and parameters

key_set : `sasoptpy.data.Set`

Set object to be read as the key (index)

key_cols : list or string, optional

Column names of the key columns

option : string, optional

Additional options for read data command

params : list, optional

A list of dictionaries where each dictionary represent parameters

Notes

- *key_set* and *key_cols* parameters should be a list. When passing a single item, string type can be used instead.

sasoptpy.read_frame

sasoptpy.**read_frame**(*df*, *cols=None*)

Reads each column in `pandas.DataFrame` into a list of `pandas.Series` objects

Parameters **df** : `pandas.DataFrame` object

DataFrame to be read

cols : list of strings, optional

Column names to be read. By default, it reads all columns

Returns list

List of `pandas.Series` objects

Examples

```
>>> price = pd.DataFrame([
>>>     [1, 5, 7],
>>>     [8, 4, 3],
>>>     [5, 7, 9]], columns=['period1', 'period2', 'period3']).\
>>>     set_index([['coal', 'steel', 'copper']])
>>> [period2, period3] = so.read_frame(price, ['period2', 'period3'])
>>> print(period2)
coal      5
steel     4
copper    7
Name: period2, dtype: int64
```

sasoptpy.read_table

`sasoptpy.`**`read_table`**(*table, session=None, key=['_N_'], columns=None, key_type=['num'], col_types=None, upload=False, casout=None, ref=True*)

Reads a CAS Table or pandas DataFrame

Parameters **table** : `swat.cas.table.CASTable`, `pandas.DataFrame` object or string

Pointer to CAS Table (server data, CASTable), DataFrame (local data) or the name of the table at execution (server data, string)

session : `swat.CAS` or `saspy.SASsession` object

Session object if the table will be uploaded

key : list, optional

List of key columns (for CASTable) or index columns (for DataFrame)

columns : list, optional

List of columns to read into parameters

key_type : list or string, optional

A list of column types consists of 'num' or 'str' values

col_types : dict, optional

Dictionary of column types

upload : boolean, optional

Option for uploading a local data to CAS server first

casout : string or dict, optional

Casout options if data is uploaded

ref : boolean, optional

Switch for returning the read data statement generated by the function

Returns tuple

A tuple where first element is the key (index), second element is a list of requested columns and the last element is reference to the original

See also:

`Model.read_table()`, *`Model.read_data()`*

sasoptpy.recursive_walk

sasoptpy.**recursive_walk**(*obj*, *func*, *attr=None*, *alt=None*)

Calls a given method recursively for given objects

Parameters **func** : string

Name of the method / function be called

attr : string, optional

An attribute which triggers an alternative method to be called if exists

alt : string, optional

Name of the alternative method / function to be called if passed attr exists for given objects

Notes

- This function is for internal consumption.

sasoptpy.register_name

sasoptpy.**register_name**(*name*, *obj*)

Adds the name and order of a component into the global reference list

Parameters **name** : string

Name of the object

obj : object

Object to be registered to the global name dictionary

Returns int

Unique object number to represent creation order

sasoptpy.reset_globals

sasoptpy.**reset_globals**()

Deletes the references inside the global dictionary and restarts counters

See also:

get_namespace()

Examples

```
>>> import sasoptpy as so
>>> m = so.Model(name='my_model')
>>> print(so.get_namespace())
Global namespace:
    Model
           0 my_model <class 'sasoptpy.model.Model'>,                sasoptpy.
↪Model(name='my_model', session=None)
```

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```
    VariableGroup
    ConstraintGroup
    Expression
    Variable
    Constraint
>>> so.reset_globals()
>>> print(so.get_namespace())
Global namespace:
    Model
    VariableGroup
    ConstraintGroup
    Expression
    Variable
    Constraint
```

sasoptpy.tuple_pack

sasoptpy.**tuple_pack**(*obj*)

Converts a given object to a tuple object

If the object is a tuple, the function returns the input, otherwise creates a single dimensional tuple

Parameters obj : Object

Object that is converted to a tuple

Returns tuple

Tuple that includes the original object

sasoptpy.tuple_unpack

sasoptpy.**tuple_unpack**(*tp*)

Grabs the first element in a tuple, if a tuple is given as argument

Parameters tp : tuple

Returns object

The first object inside the tuple.

sasoptpy.union

sasoptpy.**union**(**args*)

Returns a union of `Set`, list or set objects

sasoptpy.wrap

sasoptpy.**wrap**(*e*, *abstract=False*)

Wraps expression inside another expression

8.8.2 Math Functions

`math.math_func`(exp, op, *args)	Function wrapper for math functions
`math.abs`(exp)	Absolute value function
`math.log`(exp)	Natural logarithm function
`math.log2`(exp)	Logarithm function to the base 2
`math.log10`(exp)	Logarithm function to the base 10
`math.exp`(exp)	Exponential function
`math.sqrt`(exp)	Square root function
`math.mod`(exp, divisor)	Modulo function
`math.int`(exp)	Integer value function
`math.sign`(exp)	Sign value function
`math.max`(exp, *args)	Largest value function
`math.min`(exp, *args)	Smallest value function
`math.sin`(exp)	Sine function
`math.cos`(exp)	Cosine function
`math.tan`(exp)	Tangent function

sasoptpy.math.math_func

`sasoptpy.math.`**`math_func`**(*exp*, *op*, **args*)

Function wrapper for math functions

Parameters **exp** : `Expression`

Expression where the math func will be applied

op : string

String representation of the math function

args : float, optional

Additional arguments

sasoptpy.math.abs

`sasoptpy.math.`**`abs`**(*exp*)

Absolute value function

sasoptpy.math.log

`sasoptpy.math.`**`log`**(*exp*)

Natural logarithm function

sasoptpy.math.log2

`sasoptpy.math.`**`log2`**(*exp*)

Logarithm function to the base 2

sasoptpy.math.log10

`sasoptpy.math.`**`log10`**(*exp*)

Logarithm function to the base 10

sasoptpy.math.exp

`sasoptpy.math.exp(exp)`
Exponential function

sasoptpy.math.sqrt

`sasoptpy.math.sqrt(exp)`
Square root function

sasoptpy.math.mod

`sasoptpy.math.mod(exp, divisor)`
Modulo function

Parameters **exp** : `Expression`

Dividend

divisor : `Expression`

Divisor

sasoptpy.math.int

`sasoptpy.math.int(exp)`
Integer value function

sasoptpy.math.sign

`sasoptpy.math.sign(exp)`
Sign value function

sasoptpy.math.max

`sasoptpy.math.max(exp, *args)`
Largest value function

sasoptpy.math.min

`sasoptpy.math.min(exp, *args)`
Smallest value function

sasoptpy.math.sin

`sasoptpy.math.sin(exp)`
Sine function

sasoptpy.math.cos

`sasoptpy.math.cos(exp)`

Cosine function

sasoptpy.math.tan

`sasoptpy.math.tan(exp)`

Tangent function

PYTHON MODULE INDEX

S

INDEX

Symbols

A

C

D

E

F

G

I

L

M

P

Q

R

S

T

U

V

W