Quarto!

A computational game theory project

@GamesCrafters🥰

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Catalog

1. History & Rules

Quarto!

History of Quarto

- Quarto is a board game for two players invented by Swiss mathematician Blaise Müller in 1991.[1]

- It is published and copyrighted by Gigamic.

Rule of the Game

- Board: 4×4 board.
- Pieces: 16 unique pieces, each of which is either: tall or short (height); light or dark (color); square or circular (shape); and hollow-top or solid-top (top).
- Basic Rule: Players take turns choosing a piece which the other player must then place on the board. A player wins by placing a piece on the board which forms a horizontal, vertical, or diagonal row of four pieces, all of which have a common attribute (all short, all circular, etc.).
- Variant rule: includes a second way to win by placing four matching pieces in a 2×2 square.

2. Basic Statistics

Quarto!

Basic Statistics

- For each piece, there are:
  - 4 pieces that share exactly 3 properties
  - 6 pieces that share exactly 2 properties
  - 4 pieces that share exactly 1 property
  - ONLY 1 piece that shares no properties
- For any single piece, there are 14 others that share at least 1 property
- There are 10 winning areas: 4 rows, 4 columns, 2 diagonals
- If it is the advanced version of this game, there are 19 winning areas: 4 rows, 4 columns, 2 diagonals, 9 squares.

3. Advanced Statistics

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Advanced Statistics 1: Game Theory Analysis

- If we regard a player's win as +1, player's lose as -1, and draw as 0, Quarto is a zero-sum game.
- Quarto is a game with perfect information, since both players know the state and the possible actions for the next step.
- Quarto is an impartial game.

Advanced Statistics 2: Game States

- Without reduction and counting impossible cases, the raw answer is:

- 6.1787 x 10^15 <——————

$$\sum_{n=0}^{16} \binom{16}{n} \binom{16}{n} n!$$

- I used enumeration method here. (See details below and next page)

Let's break this formula into four parts:

1. Summation: from 0 to 16, which means n pieces will be placed on the board
2. 1st 16_choose_n: choose n positions from all 16 available positions on the board
3. 2nd 16_choose_n: choose n pieces from all 16 available pieces
4. Factorial n: place the selected n pieces on the n selected positions without replacement

Advanced Statistics 2: Game States

- Here is two screenshots of how I calculate the answer :
- Big thanks to Jupyter Notebook!

```
In [11]: lis = []
         for i in range(1,17):
             temp = 1/(fac(i)*fac(16-i)*fac(16-i))
             lis.append(temp)
             print(i)
             print(temp)
```

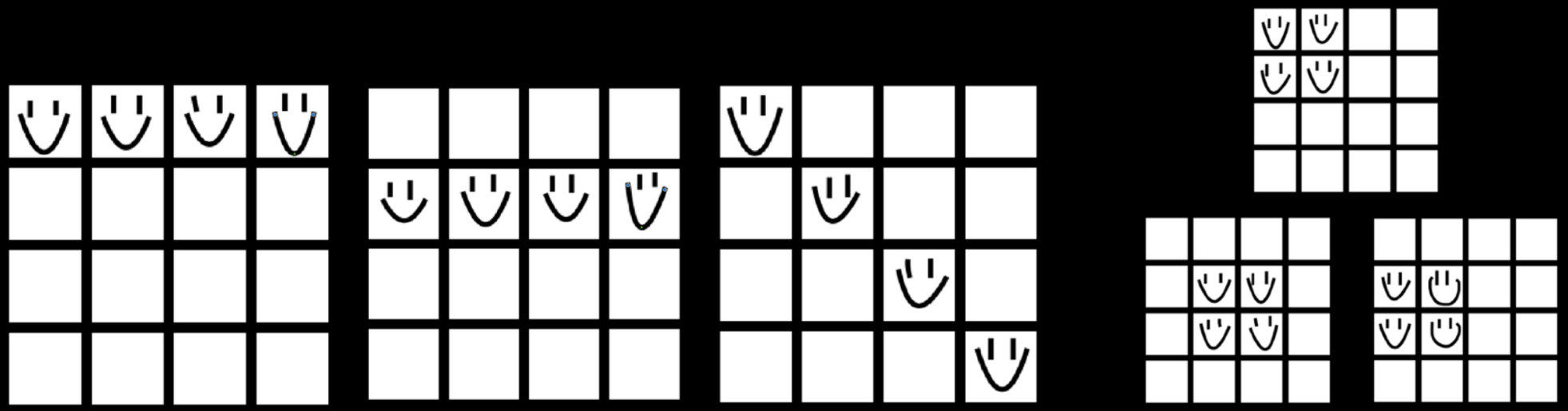
```
1
5.8479113141260385e-25
2
6.578900228391793e-23
3
4.298214815882638e-21
4
1.8159957597104147e-19
5
5.230067787965994e-18
6
1.0547303372398088e-16
7
1.5067576246282983e-15
8
1.525592094936152e-14
9
1.0848654897323748e-13
10
5.315840899688636e-13
11
1.7397297489890083e-12
12
3.624436977060434e-12
13
4.4608455102282266e-12
14
2.867686399432431e-12
15
7.647163731819816e-13
```

```
In [13]: sum(lis)

Out[13]: 1.411435921547385e-11
```

```
In [15]: coef = sum(lis)
         final_result = fac(16) * fac(16) * coef
         final_result

Out[15]: 6178746162639616.0
```

Advanced Statistics 3: Symmetry

- Due to symmetry, we can reduce the winning areas into 3 categories: outboard line x 4, inboard line x 4, diagonal x 2
- Advanced version (Won't be discussed in this report): 3 more categories (6 in total) with corner square x 4, middle square x 4, center square x 1



Advanced Statistics 4: Encode the Game

- At this stage, I suddenly realize that it is very difficult to think in a way like dark-square-tall-hollow. Human brains naturally prefer a more uniform way to process multiple properties all at a same time.
- For such reason, I converted all 16 pieces into 4-char-long strings like gene type. (More details in the next slide)

Advanced Statistics 4: Encode the Game

- A: Tall / a: Short
- B: Dark / b: Light
- C: Circular / c: Square
- D: Solid / d: Hallow

- For example, the piece on the left is AbCD, and the piece on the right is aBcd. (Yes, they are perfect complement of each other, more info about this later)

Advanced Statistics 5: Strategy

- If both players play optimally, the result will be a tie.
- Personally, I think it is better to play to not lose than it is to play to win.
- In real game: Hand your opponent lots of pieces of the same attribute until he places two in the same line; Count the number of pieces remaining that are not in your chosen attribute; Hand your opponent another piece of the chosen attribute if the number of remaining opposite pieces is even; Continue handing your opponent off-attribute pieces once he has made three in a row of your chosen attribute; Pay particular attention to any row or column with three pieces in it.s

4. Quarto x CogSci

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Quarto x CogSci

- In general, "Quarto x CogSci" is not a precisely designed experiment with all variables well-controlled. For example, environment and time.
- However, it is a very interesting idea that comes in my mind when I play the game. Also, I believe the results will still be statistically significant if the process is conducted in a more scientific way.

Quarto x CogSci: Experiment Design

- I invited 10 of my friends (and myself, 11 people in total) to play Quarto level 6 (the hardest mode) with computer on a website, and recorded the failure cases. We played more than 200 times of Quarto in total, and 177 valid failure cases were recorded.
- Before the experiment, I divided the failure cases into two categories: forced fail (after the last move of ourselves, all remaining pieces make the computer win) & accidental fail (after the last move of ourselves, at least one remaining pieces prevents the computer from winning in the next round)

Quarto x CogSci: Data Collection

- My goal is to collect the information that human players ignore which cause their failure.
- For such reason, if the failure is forced, the players record the piece they handed to the opponent which cause the forced-win scenario. In other words, the penultimate piece of the opponent.
- If the failure is accidental, the players record the piece they handed to the computer which immediately cause the failure. In other words, the last piece of the opponent. Also, the winning areas of accidental failure are recorded.

Quarto x CogSci: Results

- Statistically, if players are paying same attention to each feature, the four features should cause roughly equal amount of failure cases. In other words, the ratio is 1:1:1:1.
- Statistically, if the players are paying same attention to each winning area, the three kinds winning area should cause failure cases with ratio of 4:4:2 (Outboard : Inboard: Diagonal)
- However, the results show some interesting pattern. The below ratio take the mean as denominator. For example, in 177 failure cases, the mean should be 177 / 4 = 44.25. Then the 0.66 of color comes from 29 (the actual number of failure) / 44.25 (the mean)
- Result 1: Color : Shape : Height : Top = 0.66: 1.18 : 0.75 : 1.43
- Result 2: Outboard : Inboard: Diagonal = 0.73 : 1.04: 1 (adjusted by diagonal's ratio) or Outboard : Inboard: Diagonal = 0.66 : 0.95: 1.79 (real value / expected value if no bias)

Quarto x CogSci: Results

	Accidental	Forced	Subtotal	Percentage (Subtotal/177)
Color	28	1	29	16.38%
Shape	47	5	52	29.38%
Height	30	3	33	18.64%
Top	59	4	63	35.60%
Total	164	13	177	100%

- Upper Pic:
- Failure Cases by Feature

	Accidental Failure	Ratio(Category/Total)
Outboard	43	26.22%
Inboard	62	37.80%
Diagonal	59	35.98%
Total	164	100%

- Bottom Pic:
- Failure Cases by Position

Quarto x CogSci: Conclusion

- From the data collected, we can see that:
- The feature which easiest to be ignored is the top feature (solid / hallow). The feature which easiest to be noticed is the color feature (light / dark).
- In regards of position, people more often to forget to check the inboard lines, and more often to remember to check the outboard lines (which is actually the boundary).

5.Quarto x UI Design

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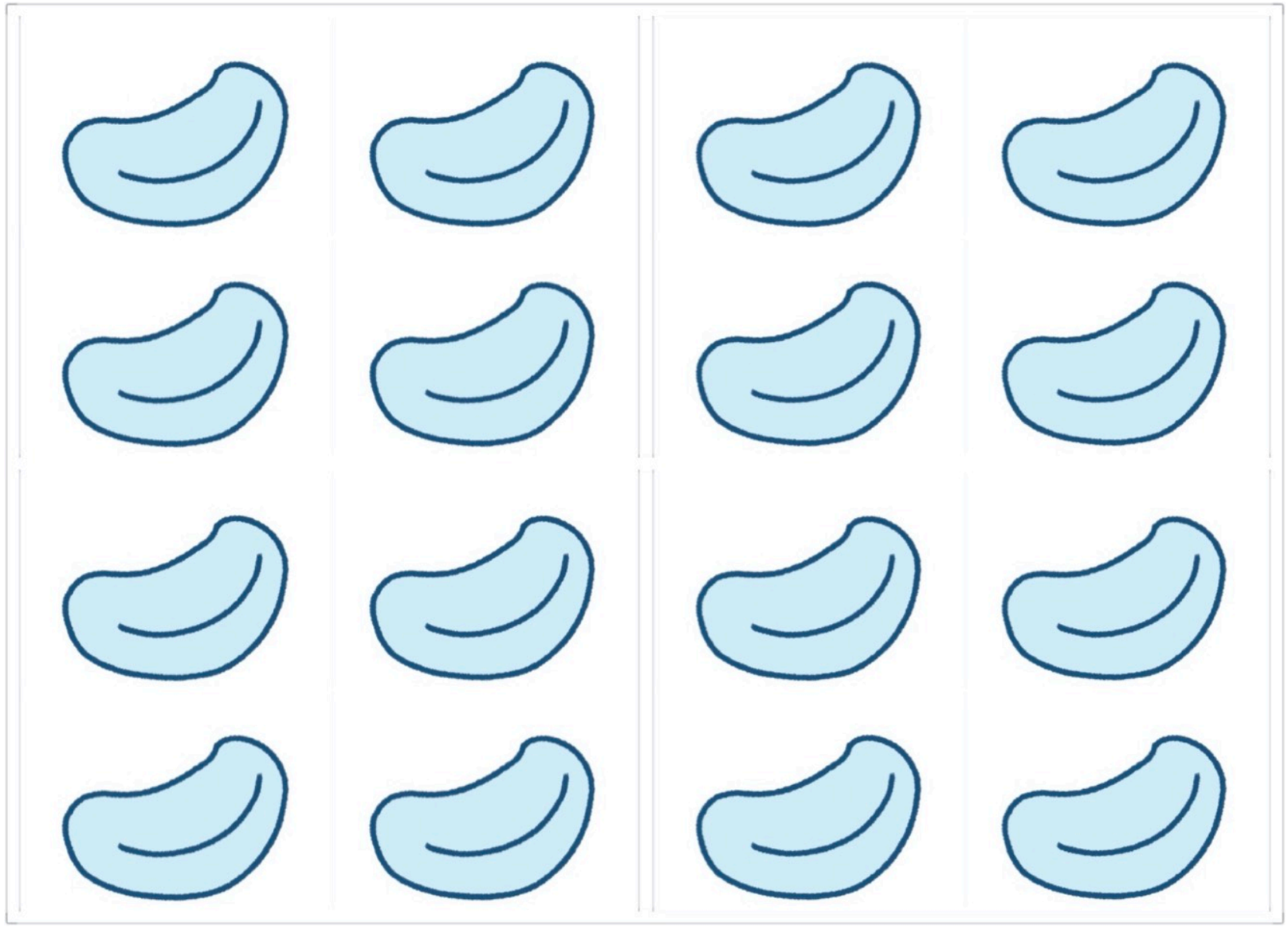
UI Design: Cato!

- Height/Color/Fish/Spot

UI Design: Cato!

- Chessboard —> 16 Cat Cushions



6.Quarto x Product

Quarto!

A Product for Focusing

- I am developing a product which originated from Quarto but have multiple variation.
- The aim of this product is to improve concentration and help people focus.

A Product for Focusing

- 1. Variation 1: More variables, and randomly choose 3/4 variables among the n variables to form a game.
- 2. Variation 2: Make it a larger game with more chess, such as 5x5, without changing the rule. It will not filled up all the spots in the end.

A Product for Focusing

- 3. Variation 3: Reverse the rule. If you get 4 of a kind in a row, you lose.
- 4. Variation 4: Randomly shuffle all the pieces and make them in a line. Players can only pick the left-most or the right-most by themselves.
- Similar game: Set (a card game)

7.Reference

Quarto!

Reference

- 1. https://en.wikipedia.org/wiki/Quarto_(board_game), Wikipedia, 21 Sept. 2020.
- 2. http://quarto.freehostia.com/en/, QUARTO! - Play the game against the computer, 8 Apr. 2020.
- 3. Michael Neumann, “An artificial intelligence for the board game 'Quarto!' in Java”, International Conference on Principles and Practices of Programming on the Java, 2013.