Quantum entangling Atomic Clocks to enhance Topological Dark Matter Detection

Bachelor Thesis Research

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Abstract

This research project aims to investigate the potential enhancement of topological dark matter research through quantum entangled atomic clocks. Atomic clocks are locked to specific transition frequencies in atomic systems to provide accurate timekeeping. Topological defects(TD) are a possible explanation for a part of dark matter, constituting to 28% of the universe and are therefore of interest to the scientific community. These defects are hypothesized to alter fundamental constants such as the fine structure constant α, leading to changes in the energy levels of atoms. The passing of topological defects through atomic clocks could therefore be detected due to a phase difference between clocks. To facilitate the detection of topological defects, the atoms within the atomic clocks are entangled to form a Greenberger-Horne-Zeilinger state, which holds the promise of increased clock stability. This hypothesis has been tested by modeling the clock using qubits, applying a dephasing and depolarizing channel and evaluating the resulting Allan deviation in the context of topological dark matter. For a dephasing error probability $\epsilon < 0.0002$, the stability of the entangled clock is up to $2.5 * 10^{-19}$ presenting an improvement of one order of magnitude over the classical comparison. However, it is observed that as the error increases, conventional atomic clocks outperform the proposed entangled clocks. At present, the constraint on a phase difference due to a transient variation of α is $3 * 10^{-18}$, indicating the potential for entangled clocks to detect it. If failing to detect it, it enables them to put new bounds on the fluctuation of α and thus improve the knowledge about topological dark matter. Although quantum entangling atomic clocks does provide the opportunity for novel research of topological dark matter, the approach is presently limited by its reliance on an error-prone system. To advance this research further, the next critical step involves the creation of an entangled network of atomic clocks, which holds the potential to yield even higher levels of clock stability.

Contents

1 Introduction

Time, a concept that has intrigued philosophers since ancient times, has been a subject of continuous investigation and quantification. From its primitive measurement using the length of shadows cast by a sunlit stick to the field of horology, the study of time has evolved significantly. From the invention of sundials, to pendulum clocks, to mechanical clocks and finally atomic clocks: mankind has managed to divide time in smaller and smaller increments. The precision achieved by chronometers has elevated it's utility beyond mere timekeeping, rendering them indispensable for fundamental research and technological advancements. This motivates researchers to push the boundaries of clock stability to the point, where one could possibly even detect dark matter. This is where the question of whether one can entangle clocks to detect the passing of topological defects dark matter comes in. It is a project lying at the interplay of quantum mechanics, cosmology and particle physics. Quantum mechanics provides the basis for the technology of entangled atomic clocks, cosmology encompasses the research field of dark matter, and particle physics governs the interactions between topological defects and clocks.

The universe presents numerous enigmas that continue to elude our grasp. One such mystery is the composition of the universe: what the cosmos consists of. Ordinary matter, forming everything known to mankind, only makes up a very small percentage of space - 5% to be exact. Researching the remaining 95% has therefore been an important endeavour in the field of theoretical physics. It is now known that about 28% of the universe consists of dark matter - and while there are many theories about its exact form and properties, it has thus far not been detected [9]. Among these hypotheses, one suggests that a part of dark matter is made up of topological defects: fields that have undergone symmetry breaking and phase transformations to form certain topologies/shapes [11]. Detecting these topological defects - and simultaneously proving their existence - is of great interest to scientists.

Quantum mechanics, an influential scientific discipline developed over the past century by scientists like Albert Einstein, Niels Bohr, Heisenberg, and Schrödinger, has gained unprecedented relevance

in modern times. Quantum principles underpin everyday technologies, such as cell phones, computers, and medical equipment, and have witnessed a surge in public interest in recent years, particularly in the realm of quantum information. While classical information theory is based on binary bits that either have the value 0 or 1, quantum bits (qubits) can take on a combination of these values - a phenomenon called superposition. Even more fascinating than this is the effect of entanglement: of connecting two or more particles so that a change on one instantaneously affects the other, regardless of the distance between them. These properties of quantum mechanics, driven by the advancement of quantum information science, have started to permeate various technologies, including the field of atomic clocks [16].
Atomic clocks are the most precise time-keeping device available and they are locked to a certain atomic frequency [21]. Beyond establishing the global time standard, they serve as detectors for exploring cosmological concepts, such as topological defects. [30].
Particle physics is the tool used to explain how chronometers are able to detect dark matter. It is the field of fundamental physics that deals with particles, their interactions, their composition and their properties. Its main conclusions are summarised in the standard model of physics, which separates particles into different groups such as leptons, quarks and bosons. Among the last group - bosons - are the force carriers of the 3 fundamental forces: strong nuclear force, weak nuclear force and electromagnetic force. The force propagator of the latter is the photon. Charged particles interact via the electromagnetic force, using photons and their interaction strength is defined by a fundamental constant: the fine structure constant α [14]. It has been hypothesized that topological defects have a very specific interaction with baryonic matter: they alter fundamental constants, including the fine structure constant. Since α is involved in interactions of charged particles, the energy levels in atoms are influenced by it. Changing α means altering the frequencies of atoms, which in turn makes atomic clocks tick differently [4]. To be able to detect these slight changes, a very high accuracy is required for the atomic clocks. One proposed approach to improve their precision is entangling the atoms inside the clock.
Therefore, this thesis will focus on: **Is it possible to improve the stability of atomic clocks by locally entangling them to detect topological defects**? This paper will present an in-depth explanation of the underlying theory, followed by a mathematical model of an entangled clock, evaluated in the context of topological defects. The results will be carefully analysed and interpreted, shedding light on the potential of quantum entangled atomic clocks in furthering our understanding of dark matter.

2 Theory

2.1 Quantum Mechanics

Quantum mechanics is a mere century old field of physics, whose non intuitive rules and magic-like effects have paved the way for the creation of some of today's most revolutionary technologies, including transistors (found in cell phones) and MRI machines. At the core of quantum theory lie properties such as superposition and entanglement [15].
In the 1980s, Richard Feynman proposed the field of quantum computation and information, which takes quantum mechanical phenomena and uses them to create algorithms and machines. David Deutsch then published the idea of a universal quantum computer in 1985. In the following two decades, a lot of theoretical progress was made, leading to famous quantum algorithms such as Shor's factoring algorithm or Grover's search algorithm. In 1998 the first quantum computer, consisting of 2 qubits was built at MIT, the system however was only coherent for a few nanoseconds[16]. Today, quantum computers have improved significantly. IBM's quantum computer 'Eagle' accesses 127 qubits, Quantinuum's H2 model has 32 fully connected qubits and Google's hardware uses 70 qubits - the number of qubits is not the most important benchmark in quantum computing, but certainly a good example of how the field of quantum information has expanded [27] [8][23]. Although commercial applications for current quantum computers remain distant, the field continually progresses, encompassing a diverse range of applications.
Quantum information is based on a 2- state quantum system, making it the ideal starting point to explain basic quantum mechanical concepts [24]. In the last 2 decades, quantum information theory has developed from the pipe dream of a few scientists to a large, swiftly developing field of engineering, computational and theoretical physics. The potential for groundbreaking technologies based on quantum mechanical principles is increasingly pervading scientific research, such as a

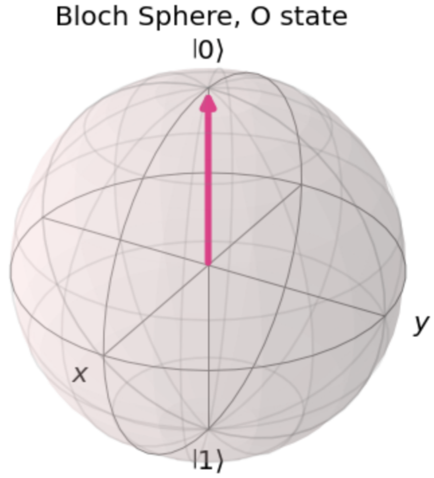


Figure 1: Bloch sphere. $|0\rangle$ and $|1\rangle$are shown

quantum network of clocks or entangling atomic clocks.

2.1.1 Quantum Information Theory

In classical information theory the fundamental unit of information is a bit. In quantum information theory, the analogue is the qubit - a two level quantum system, meaning it has two basis states. Conventionally these states are written in the 0,1 basis using bra-ket notation (Dirac notation), creating the two basis states $|0\rangle$ and $|1\rangle$, where :

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \tag{1}$$

A qubit is in the general state

$$|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{2}$$

where α and β are complex coefficients describing the probability amplitudes of the qubit being in state $|0\rangle$ and $|1\rangle$ respectively. Qubits can easily be depicted using the so-called Bloch-Sphere as shown in Figure 1: one imagine the surface of a unit sphere, where each point is a geometrical representation of a qubit state. The north and south pole are the $|0\rangle$ and $|1\rangle$ state respectively and the points on the xy plane (the equator) are superposition states. The unit vector, symbolizing a state can be rotated using unitary transformations, which in quantum information are often also described as quantum gates. Simple rotations of the Bloch sphere are conducted using the Pauli operators: σ_x (X), σ_y (Y), σ_Z (Z), and the identity I.

$$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}, Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}, Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}, I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \tag{3}$$

The Pauli Matrices are very versatile operators, as any hermitian matrix can be decomposed into a sum of products of Pauli matrices and the Identity, making other transformations (gates in quantum computing) composites of these. The identity matrix I returns the state exactly as is. The X gate induces a π rotation around the X-axis of the Bloch sphere, which effectively switches the state $|0\rangle$ to state $|1\rangle$ and vice versa. The X-gate is therefore also called the NOT-gate, analogous to the classical logical NOT-gate. The Y-operator rotates the qubit by π around the Y-axis, introducing an imaginary phase on the $|1\rangle$ state. The Z-operator, also known as phase-flip gate, performs a π rotation around the Z-axis and flips the phase of the qubit - it adds a value π to the phase. In addition to the Pauli matrices, a crucial operator is the Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} \tag{4}$$

The Hadamard gate puts the qubit in equal superposition of $|0\rangle$ and $|1\rangle$, or rotates it onto the equator of the Bloch sphere - if it was originally in state $|0\rangle$ or $|1\rangle$. Superposition is one of the main properties of quantum mechanics. It allows a quantum system to exist in multiple states simultaneously, meaning that until a measurement or observation is made, the quantum system exists in a state that is a combination of all possible outcomes. While the majority of operations is done in superposition, one needs to measure the qubit to get out any state information and thus a result of the calculation. Once a measurement is done, the superposition gets destroyed and the wave function describing the state collapses. The outcome of the measurement is either 0 or 1 (or equivalent states in other bases). Prior to measurement, there exist distinct probabilities associated with the quantum states of the qubits. The pre-collapse probabilities of the qubits can be ascertained by performing measurements on several qubits, enabling the retrieval of the needed information. Typical quantum computing measurement is done by applying a measurement operator. The probability of measuring a system in state m is given by:

$$P(m) = \langle\psi| M_m^\dagger M_m |\psi\rangle \tag{5}$$

The measurement operator M_m obeys the completeness theorem, meaning $\sum_m M_m^\dagger M_m = I$. For measuring a state in the $|0\rangle$, $|1\rangle$ basis, the following is true: $M_0^\dagger M_0 + M_1^\dagger M_1 = M_0 + M_1 = I$. As the operators are hermitian, the modulo squared returns the original operator. It follows from this that if $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$:

$$P(0) = \langle\psi| M_0 |\psi\rangle = |\alpha|^2 \tag{6}$$

$$P(1) = \langle\psi| M_1 |\psi\rangle = |\beta|^2 \tag{7}$$

Essentially, to find the probability of measuring 0, one modulo squares the respective coefficient: $P(0) = |\alpha|^2$. The normalization condition requires that $|\alpha|^2 + |\beta|^2 = 1$.
These basic properties of qubits and qubit operations enable the explanation of another postulate of quantum mechanics: Entanglement. Entanglement refers to a phenomenon where two or more particles become correlated in such a way that the state of one cannot be described independently of the state of the other. The quantum states of the particles are intrinsically linked, regardless of the distance between them. When one operation is performed on one particle, it instantaneously affects the state of the other particle(s). This effect is known as quantum non-locality and gives rise to many promising technologies and theories [12]. More intuitively, if we cut an entangled state in two, something will be missing. Entangling two qubits would for example create the following joint qubit state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle) \tag{8}$$

When particles become entangled, their individual quantum states form a joint or composite state that cannot be expressed as a simple combination of the individual states. This means one cannot find two separate state vectors for the two qubits, such that tensor multiplied they give the state at hand: $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$. If it is possible to find such $|\Psi_A\rangle$ and $|\Psi_B\rangle$, it is a product state but not an entangled state [24].

While entangling two qubits is already of great interest to scientists and has many useful applications, multi- party entanglement proves to be just as, maybe even more fascinating. One particularly notable multi-party entangled state is the Greenberger-Horne-Zeilinger (GHZ) state. The GHZ state is a maximally entangled state involving three or more particles. In the case of three particles, the GHZ state can be represented as:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle \otimes |1\rangle) = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{9}$$

The GHZ state exhibits a unique property where all the particles are in a symmetric superposition of being in the state $|0\rangle$ or $|1\rangle$ simultaneously. Measuring one qubit gives the same result as measuring any other qubit, if measured in the Z basis [13].
Entanglement - especially multiparty entanglement is a crucial component of many quantum technologies and algorithms. It's non-locality gives rise to many protocols that improve their classical counterpart.

So far, all equations have been written out in state space formalism, it is however often advantageous to utilise the density operator notation, especially when systems interact with the environment. Once the state is no longer pure, density matrices become the preferred formalism. The density matrix ρ is defined as :

$$\rho = \sum_i p_i |\Psi_i\rangle \langle\Psi_i| \tag{10}$$

where $|\Psi_i\rangle$ are pure states and p_i are their respective probabilities. A simple example is the density matrix of the pure state $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$:

$$\rho = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} * \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix} = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{bmatrix} \tag{11}$$

The diagonals of the density matrix represent the probabilities of the states, which makes the condition $Tr(\rho) = 1$ obvious: the sum of the probabilities must be 1. When evaluating whether the state of a quantum system is pure, the trace of the square of ρ is important. If a state is pure, $Tr(\rho^2) = 1$, if it is a mixed state $Tr(\rho^2) < 1$. One can also apply unitary operators on density matrices. While for a pure state, the transformation is $|\Psi_t\rangle = U|\Psi\rangle$, for a density matrix it is $\rho_t = U\rho U^\dagger$ [24]. The density operator enables the calculation of the interaction between a state and the environment. Such interplay between two systems oftentimes leads to errors in quantum technologies. It is at this time unavoidable to deal with noisy systems, meaning it is necessary to account for errors when evaluating the quantum advantage of protocols and crucial to figure out error correction/mitigation approaches.

2.1.2 Error Channel

Quantum noise and errors are a constant issue within the field of quantum technologies. It is common practice to apply so-called error channels to a given model to test its validity in the presence of noise. Among the possible errors, the dephasing and depolarizing channel are some of the most common ones. In the dephasing channel (also called phase-flip channel), the quantum state dephases with a certain probability. More precisely, its gets a phase shift of π with probability ϵ and it is left unperturbed with probability $1-\epsilon$. Mathematically this takes the following form:

$$\Lambda(\rho) = (1-\epsilon)\rho + \epsilon\sigma_z\rho\sigma_z \tag{12}$$

The second notable error channel is called depolarizing channel. Again, with probability ϵ the state depolarizes, meaning it turns into the completely mixed state $\frac{I}{2}$ and with probability $(1-\epsilon)$ nothing happens to the state. The state after this occurs is of the following form:

$$\Lambda(\rho) = (1-\epsilon)\rho + \epsilon\frac{I}{2} \tag{13}$$

Applying it to a part of a multi-qubit system introduces a partial trace, yielding this:

$$\Lambda(\rho) = (1-\epsilon)\rho + \epsilon\frac{I}{d} \otimes Tr(\rho) \tag{14}$$

.

(d=2 for single qubit). In the Bloch sphere imagery, the Bloch sphere contracts uniformly [24].

2.2 Atomic Clocks

Atomic clocks are the most accurate time-keeping device currently available. These systems operate on the same basic principle as a regular pendulum clock: time is quantified by counting the oscillations of a periodic signal with a known frequency. Unlike pendulum clocks, atomic clocks use light as a clock signal. They utilize a laser or microwave source as a local oscillator (LO), whose oscillations are counted to determine the length of 1 second. Since lasers are prone to errors and frequency fluctuations, one needs a feedback loop to stabilize them. To this end, atomic systems are placed in the path of the laser and the clock signal is locked to an ultra stable transition frequency of that atom or ion [21].

The process of interrogating atoms to adjust the frequency of the laser is called Ramsey spectroscopy [28].

2.2.1 Ramsey Spectroscopy

As aforementioned, a local oscillator is coupled to N atoms, driving the transition from the ground state $|0\rangle$ to the first excited state $|1\rangle$. The evolution of this system is described by the following

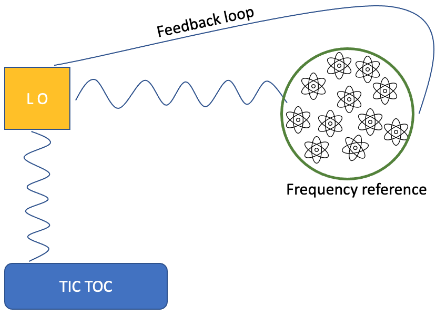


Figure 2: Theoretical atomical clock, LO=local oscillator

Hamiltonian, which is written in the rotating frame of the LO:

$$H(t) = \hbar(\omega_0 - \omega(t)) \sum_{j=1}^{N} |1\rangle_j \langle 1| + \hbar\Omega(t) \sum_{j=1}^{N} |1\rangle_j \langle 0| + |0\rangle_j \langle 1| \tag{15}$$

ω_0 is the transition frequency of between $|0\rangle$ and $|1\rangle$, $\omega(t)$ is the frequency of the local oscillator, and $\Omega(t)$ is the coupling strength between the LO and the transition. Using this system, Ramsey spectroscopy(RS) can be conducted. It consists of three general steps: The LO applies a $\frac{\pi}{2}$pulse to the atom, which, assuming all atoms started in the ground state, leads to the following:

$$|\Psi(0)\rangle = \frac{1}{2^{\frac{N}{2}}} \prod_{j}^{\otimes N} (|0\rangle_j + i\,|1\rangle_j) \tag{16}$$

The atoms now experience a free evolution time and are thus picking up a phase $\phi(t) = \int_0^T (\omega_0 - \omega(t))dt$ compared to the clock signal . It is important to remember that this calculation is done from the rotating frame of the local oscillator, meaning the LO's frequency will be the one undergoing variations, but it is simpler to calculate it acting as if the atomic systems do:

$$|\Psi(T)\rangle = \frac{1}{2^{\frac{N}{2}}} \prod_{j}^{\otimes N} (|0\rangle_j + ie^{-i\phi(T)}\,|1\rangle_j) \tag{17}$$

Another $\frac{\pi}{2}$ pulse around the y-axis of the Bloch sphere induced by the clock signal leads to this final state:

$$\frac{1}{2^{\frac{N}{2}}} \prod_{j}^{\otimes N} (1 + ie^{-i\phi(T)})\,|0\rangle_j + (1 - ie^{-i\phi(T)})\,|1\rangle_j) \tag{18}$$

To estimate the accumulated phase, which can be approximated using the small angle approximation $\sin(\phi) \approx \phi(t)$, the measurement operator $O = \prod_j^N (|0\rangle_j \langle 0|_j - |1\rangle_j \langle 1|_j)$ is applied. This operator measures the atomic population of ground and excited state, enabling an estimate of $\phi(t)$, which in turn can be used to adjust the frequency of the laser. Figure 2 also describes Ramsey spectroscopy in a more intuitive way. The RS is repeated many times to accurately compensate for any fluctuations [28]. Different types of atomic clocks have been designed, all based on this same underlying principle. Among these are optical lattice clocks, which trap ions in a optical traps made of lasers at a so called 'magic wavelength', that does not interfere with the LO but keeps the atoms stuck in place, which reduces the effect of atomic motion on the stability of the clock [25]. Trapped Ion clocks utilise a similar approach as they trap Ions in electromagnetic fields, again increasing the fractional uncertainty of the clock [7].

2.2.2 Allan deviation

To evaluate the stability of clocks and compare them effectively, the Allan deviation comes into play. By letting a clock run for time $\tau = mT$, where m refers to the number of Ramsey sequence with free evolution time T, one can estimate the accumulated phase $\phi_{est}(T_j)$ between time $t = (j-1)T$

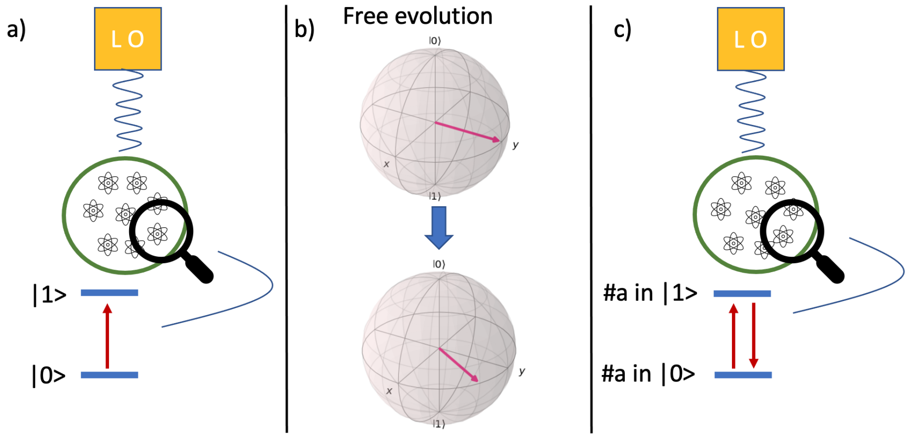


Figure 3: Ramsey sequence: a) LO applies $\frac{\pi}{2}$ pulse to atomic systems, putting them in superposition b) free evolution time: atoms accumulate phase by rotating around bloch sphere c) LO applies another $\frac{\pi}{2}$ pulse, population in $|0\rangle$ and $|1\rangle$ gets measured to find phase

and $t = jT$ [2]. The Allan deviation $\sigma_y(\tau)$ then refers to the long term stability of clocks. The general equation of $\sigma_y(\tau)$ of a clock operated for time τ is :

$$\sigma_y(\tau) = \left\langle (\bar{\omega}(\tau)/\omega_0)^2 \right\rangle^{\frac{1}{2}} \tag{19}$$

where $\bar{\omega}(\tau) = \frac{1}{m}\sum_j^m \frac{\phi(\tau_j)-\phi_e(T_j)}{T}$, which corresponds to the mean frequency offset. Plugging that into the first equation gives:

$$\sigma_y(\tau) = \frac{1}{\omega_0}\Big\langle \frac{1}{m}\sum_j^m \frac{\phi(T_j)-\phi_e(T_j)}{T}\Big\rangle^{\frac{1}{2}} \tag{20}$$

$$= \frac{1}{\omega_0\sqrt{\tau T}}\Big\langle \frac{1}{m}\sum_j^m \frac{\phi(T_j)-\phi_e(T_j)}{T}\Big\rangle^{\frac{1}{2}} = \frac{1}{\omega_0\sqrt{\tau T}}\sigma_\phi(T) \tag{21}$$

This is a widely accepted form of the equation for the allan deviation [5]. One can also rewrite the average phase estimation uncertainty σ_ϕ as:

$$\sigma_\phi = \left\langle (\phi_0 - \phi_{est})^2 \right\rangle = \left\langle \phi_o^2 \right\rangle + \left\langle \phi_{est}^2 \right\rangle - 2\left\langle \phi_0\phi_{est} \right\rangle \tag{22}$$

where ϕ_0 is the actual phase and ϕ_{est} is the measured phase. This second equation is a more intuitive version of the formula.
The Allan deviation is a useful tool to evaluate the performance of a time keeping device and compare different chronometers. The allan deviation of a regular atomic clock with N atoms scales as $\propto \frac{1}{\sqrt{N}}$ [20].

2.2.3 Standard quantum limit and Heisenberg limit

The standard quantum limit (SQL) is a fundamental limit in the precision of measurements imposed by quantum mechanics. In the context of atomic clocks it refers to a limit on the clock's ability to accurately measure time intervals. It arises because of the Heisenberg uncertainty principle, which states that there is a fundamental trade-off between the precision of measurements of two complementary observables, such as position and momentum or time and energy. In the case of atomic clocks, this uncertainty manifests as constraints on the precision of frequency measurements associated with atomic oscillations. This means that even though it is possible to improve a clock's Allan deviation by increasing the number of atoms, it is constrained by the SQL. The standard quantum limit for atomic clocks is $\frac{1}{\sqrt{N}}$ [26]. Heisenberg limited clocks have been hypothesized, that can surpass the SQL and thus have a higher stability. The stability is supposed to scale as $\frac{1}{N}$. One such ansatz is entangling the clock atoms and thus gaining a lower uncertainty [5].

2.3 Topological Dark Matter

The universe consists of 5 % baryonic matter, 28 % dark matter, and 68 % dark energy[9]. While the standard model of physics aids in our understanding of baryonic matter, there is little known about dark matter and even less about dark energy. Dark matter does not interact via the electromagnetic force, meaning photons have no effect on it, which is the origin of the name. The dark sector (dark matter and dark energy) is therefore invisible to human eyes. The existence of dark matter has hence only been deduced by its gravitational effect, such as on the motions of galaxies or gravitational lensing. There are numerous theories of the composition of dark matter, some focusing on particle dark matter, others on very light boson fields (mass<<1GeV). Among the latter there is the possibility that, depending on the circumstances during early cosmological times, these fields could create dark matter by forming 3-dimensional stable configurations which are referred to as topological defects (TD). Topological defects can arise in certain physical systems when symmetry-breaking processes or phase transitions occur. Depending on their dimensionality, they can be put into three groups: Monopoles (0-dimensional), Strings (1-dimensional) and Domain Walls (2-dimensional). It is assumed that a higher dimensional TD decays into a lower one - so domain wall to string to monopole [11]. To understand the way dark matter interacts with the standard model matter, so-called 'portals' have been introduced. These consist of gauge invariant standard model operators that are coupled to dark sector operators. This effectively means they enable the calculation of a possible effect of TD's on the standard model. The portals create multiple individual portal interactions whose sum is the interaction Lagrangian between the dark sector field and the SM matter. An interaction Lagrangian is a mathematical expression that describes how different fields or particles in a physical system interact with each other. The most widely accepted and applied portal is the quadratic scalar portal. From its interaction Lagrangian one can calculate the effect of topological defects on the standard model, or more accurately on the fundamental constants α and μ. The fine structure constant α is the coupling constant of the electromagnetic force and thus quantifying the strength of EM interactions. It has the approximate value of $\alpha = \frac{1}{137}$. μ is the proton-electron mass ratio is $\mu = \frac{m_p}{m_e} \approx 1836$ [10]. When a topological defect passes through baryonic matter, these fundamental constants change in the following way:

$$\alpha_{TD} = \frac{\alpha}{1 - \phi^2/\Lambda_\gamma^2} \tag{23}$$

$$\mu_{TD} = \mu(1 + \frac{\phi^2}{\Lambda_{e,p}^2}) \tag{24}$$

As visible from these two equations, the effect of the TD is dependent on the energy scale and the field. The energy scale refers to the necessary energy associated with the formation or presence of these defects [11]. It represents the scale at which the topological defect's properties become significant and observable. Previous research has managed to put some bounds on the energy scales, for example $\Lambda_\alpha > 10^{10} TeV$ for a field diameter of 10^7km [29]. TD's have however not been detected yet. Since they interact neither gravitationally nor electromagnetically, one must utilise their effect on the fundamental constants. The energy levels of atoms are set by the Rydberg constant which is dependent on α and μ: $R_\infty = \frac{c}{4\pi\hbar}\alpha^2 m_e$, where c is the speed of light, $\hbar$ is the reudced planck constant and m_e is the electron mass. To detect topological defects, one must sense the change in atomic energy levels. The dependence of atomic clocks on energy levels as well as their sensitivity and precision make them promising tools for detecting TDs, as even subtle changes in the fundamental constants can be measured. By continuously monitoring the time difference of a network of atomic clocks, researchers can search for transient variations that indicate the presence of TDs and contribute to our understanding of the elusive dark matter components of the universe [4]. To detect topological defects using atomic clocks, scientists would need to compare and correlate the measurements from multiple clocks distributed in different locations. This approach allows them to differentiate between local effects such as gravitational time dilation and true variations caused by the passage of a TD. A network of atomic clocks therefore becomes the natural choice for a detector.

2.4 Detecting Topological Defects using Networks of Atomic Clocks

Several protocols have been proposed and tested to detect topological defects using a network of atomic clocks. Their exceptional accuracy and reliance on the stability of fundamental constants

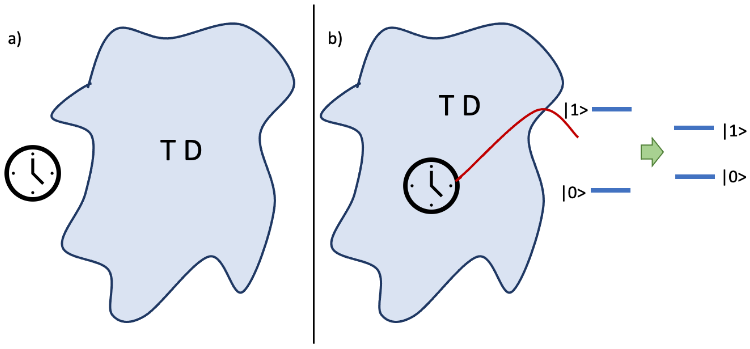


Figure 4: Topological defects: a)Topological defect approaches clock b) topological defect passes through clock, changes energy levels of atomic systems

make them the ideal tool for observing any variations in α and μ . Detecting dark matter is made possible by registering a time difference between clocks. To effectively eliminate frequency variations originating from other sources and ensure that not all clocks are simultaneously affected by the topological defects, a network of atomic clocks in space spanning significant distances is necessary. The ability to detect dark matter through time differences between clocks is contingent on the specific clock used, as each may exhibit varying degrees of sensitivity to these fundamental constant changes. Therefore the concept of sensitivity coefficients of atomic or molecular transitions ν_i to the transient variation of fundamental constants has been introduced (K_X where $X = \alpha, \mu$). K_X is defined as:

$$K_X = \frac{\delta ln(\frac{\nu_i}{cR_\infty})}{\delta lnX} \tag{25}$$

where ν_i is the frequency of the molecular transition, c is the speed of light, R_∞ is the Rydberg constant, X is the fundamental constant in question. Different clock types have different sensitivity coefficients, making the choice of which clock to use as a detector a crucial one [4]. To determine how a given clock(or network of clocks) performs in presence of a topological defect, a signal to noise ratio (SNR) comes into play. The SNR is dependent on both the strength of the signal, so the size, energy density and energy scale of the topological defect, as well as the noise, introduced by the Allan deviation, the sensitivity coefficients and the time of operation. :

$$\frac{S}{N} = \frac{c\hbar\rho_{TDM}Td^2}{T\sigma_y(T)\sqrt{2Tv_g/l}} \sum_X K_X \Lambda_X^{-2} \tag{26}$$

here c is the speed of light, $\hbar$ the planck constant, ρ_{TDM} the energy density of the TD, T the time of operation, d the size of the TD, σ_y the allan deviation of the clocks, v_g the galactic velocity, at which the topological defects are considered to propagating, l the distance between the clocks, K_X the sensitivity coefficients and Λ_X the energy scale. As it is difficult to know the topological defect parameters, a figure of merit solely dependent on clock attributes has been developed from this SNR:

$$F = \frac{K_X}{T^{\frac{3}{2}}\sigma_y(T)} \tag{27}$$

This enables a clear comparison between different clocks, making informed decisions possible as to which clock to use in which case. This shows that a low Allan deviation does not suffice, as the sensitivity coefficient also plays an important role. Additionally, the figure of merit increases in size as the clock becomes better at detecting transient variations of fundamental constants, making it a more intuitive measure than the allan deviation, which gets smaller the better the clock is [11]. Attempts have been made at utilising existing networks of clocks to find topological defects. One such project utilised the global positioning system (GPS) as a dark matter detector with a 50.000 km wide aperture. The GPS consists of 32 satellites, which carry atomic clocks that use either Rb or Cs atoms as frequency references. In this set up, time differences between a satellite and a stationary clock can be found up to less than $1s * 10^{-9}$. The Allan deviation of the Rb clocks was of the order of 10^{-11} for a data sampling interval of 30 seconds. They used data provided by the Jet Propulsion Laboratory and focused their search on domain wall dark matter. Their investigation turned out unsuccessful, which does not mean that topological defects do not exist

but rather that the very limited stability of the GPS does not suffice to detect them [30].
In 2020, a research group used data from a European network of fiber-linked optical atomic clocks of 40 days in 2017 to search for topological defects. The network consists of clocks based on Yb^+, Sr and Hg clocks that are positioned above France, Germany and the United Kingdom. The sampling interval was 1s, the average sampling period 60s. Their network's stability was limited by the clock uncertainties, as the optical fibre uncertainties were smaller than the ones of the clocks. The Allan deviation for the clocks was around 10^{-16} for averaging times of more than 100s. They did not detect any topological defects using this network, but it enabled the placing on constraints for the transient variation of α and in turn on the energy scale Λ_α: $\frac{\delta\alpha}{\alpha} < 5 * 10^{-17}$ for transients of $10^3 s$ and $\Lambda_\alpha \geq 10^{10}$TeV for a topological defect with diameter $d \sim 10^7$ [29].
The research company QSNET is currently working on building a network of atomic clocks, specifically designed to detect variations in fundamental constants due to a variety of reasons, among which topological dark matter is listed. They have however not yet published any results[4].
As the previous section showcases, the current classical technologies do not suffice for detecting topological defects. It is therefore necessary to investigate emerging technologies such as quantum sensing for their application in fundamental physics research. In the following section, the possibility of entangling clocks locally - meaning entangling the atoms that serve as frequency references within the clock - and using these to build a network of atomic clocks will be investigated, as well as applying them to the detection of topological defects.

3 Model Clock

The objective of this thesis is to model the local entanglement of an atomic clock to investigate the possible improved stability and its application to the transient variation of fundamental constants due to the passing of topological defects dark matter. To this end, the atoms within the atomic clock will be seen as qubits, with their ground state and first excited state being the two possible states of a qubit. In the following section, a toy model of the clock will be created, using quantum information formalism and evaluating its performance by applying generic noise channels and subsequently calculating its Allan deviation.
A regular atomic clock consists of N atoms, which are going to be entangled in this project. A GHZ state is therefore created by entangling the clock qubits using a standard entangling protocol. The state of the qubits after entangling is $|\Psi_{ent}\rangle$, which shows that there is equal probability of all qubits being in the ground state $|0\rangle$ or another state $|1\rangle$.

$$|\Psi_{ent}\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + i\,|1\rangle^{\otimes N}) \tag{28}$$

Now it is assumed that these qubits evolve through time, which can be imagined as a rotation with a phase, just as in a regular Ramsey Sequence. This effectively means that one of the states picks up a relative phase $\phi(t)$:

$$|\Psi_t\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + e^{i*N*\phi(t)}i\,|1\rangle^{\otimes N}) \tag{29}$$

To simplify the following calculations, Ψ gets rewritten as the density matrix ρ, where $\rho = |\Psi\rangle\langle\Psi|$:

$$\rho = \frac{1}{2}\begin{bmatrix} |0\rangle^{\otimes N}\langle 0|^{\otimes N} & -ie^{-iN\phi(t)}\,|0\rangle^{\otimes N}\langle 1|^{\otimes N} \\ ie^{iN\phi(t)}\,|1\rangle^{\otimes N}\langle 0|^{\otimes N} & |1\rangle^{\otimes N}\langle 1|^{\otimes N} \end{bmatrix} \tag{30}$$

This density matrix enables the application of error channels.

3.1 Dephasing noise

Since the system is not noiseless, it is necessary to model the possibility of a(or n) qubits dephasing, meaning their phase will change, leading to an error. As we are dealing with an entangled clock, the noise scales exponentially with N - this would not be the case for a non entangled clock. The noise channel is typically written as $\Lambda(\rho) = (1-\epsilon)\rho + \epsilon * \sigma_z\rho\sigma_z$, where with a probability $1-\epsilon$ nothing happens to the qubits (the state of the density matrix) , but with probability ϵ the phase picks up a minus sign. A minus sign is equivalent to a phase change of π. A single qubit dephasing channel is applied to the state of the clock/ the density matrix ρ. Due to the entanglement of the atoms, the noise scales exponentially with N. This gives the following result:

$$\Lambda(\rho) = \frac{1}{2}\begin{bmatrix} |0\rangle^{\otimes N}\langle 0|^{\otimes N} & -(2\epsilon-1)^N ie^{-iN\phi(t)}\,|0\rangle^{\otimes N}\langle 1|^{\otimes N} \\ (1-2\epsilon)^N ie^{iN\phi(t)}\,|1\rangle^{\otimes N}\langle 0|^{\otimes N} & |1\rangle^{\otimes N}\langle 1|^{\otimes N} \end{bmatrix} \tag{31}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -(2\epsilon-1)^N ie^{-iN\phi(t)} \\ (1-2\epsilon)^N ie^{iN\phi(t)} & 1 \end{bmatrix} \tag{32}$$

Non-entangled clocks also experience dephasing noise, it however does not accumulate exponentially. The Density matrix for the non-entangled clock is therefore:

$$\frac{1}{2}\begin{bmatrix} |0\rangle\langle 0| & -(2\epsilon-1)ie^{-iN\phi(t)}\,|0\rangle\langle 1| \\ (1-2\epsilon)ie^{iN\phi(t)}\,|1\rangle\langle 0| & |1\rangle\langle 1| \end{bmatrix}^{\otimes N} \tag{33}$$

In addition to the dephasing channel, the depolarization channel also needs to be considered. For entangled clocks however, one qubit depolarizing leads to the creation of a fully mixed state, meaning all phase information gets lost. This derivation is shown in the appendix, it is not considered in the following stability analysis as even the smallest probability of error destroys the state. It will be taken into account in the evaluation of the results however.
Once the noise channel has been applied to the entangled clock in this form, the state gets rewritten in state space formalism by changing basis: before the basis consisted of $|0\rangle$ and $|1\rangle$, it is however

advantageous to change to the $|GHZ_\pm\rangle$ basis, where $|GHZ_\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} \pm ie^{iN\phi(t)}|1\rangle^{\otimes N})$ This operation turns the density matrix into $\rho = p_+|GHZ_+\rangle\langle GHZ_+| + p_-|GHZ_-\rangle\langle GHZ_-|$ or:

$$\begin{aligned}\rho = 1/2((p_+ + p_-)|0\rangle^{\otimes N}\langle 0|^{\otimes N} - (p_- - p_+)ie^{-iN\phi(t)}|0\rangle^{\otimes N}\langle 1|^{\otimes N} + \\ (p_+ - p_-)ie^{iN\phi(t)}|0\rangle^{\otimes N}\langle 1|^{\otimes N} + (p_+ + p_-)|1\rangle^{\otimes N}\langle 1|^{\otimes N})\end{aligned} \tag{34}$$

From the density matrix , it can be read off that that $(p_+ + p_-) = 1$ and $(p_+ - p_-) = (1-2\epsilon)^N$ from which it was calculated that $p_\pm = \frac{1}{2} \pm \frac{(1-2\epsilon)^N}{2}$.

To approximate $\phi(t)$, a $\frac{\pi}{2}$ pulse around the y axis is applied on each atom subsequently such that $|0\rangle \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Then it is measured whether the atom is in state $|0\rangle$ or $|1\rangle$.

In the following, $|GHZ_+\rangle$and $|GHZ_-\rangle$ need to be treated separately. Applying $R_y(\frac{\pi}{2})$ on the first atom leads to the following state:

$$|GHZ_\pm\rangle = \frac{1}{2}(|0\rangle + |1\rangle)|0\rangle^{\otimes N-1} \pm (|0\rangle - |1\rangle)ie^{iN\phi(t)}|1\rangle^{\otimes N-1}) \tag{35}$$

This operation is repeated with every atom, meaning the state of the clock qubits might acquire a phase shift of π. These repeated rotations and measurements lead to the following state of the final qubit, with the parity P denoting whether there has been an even or odd number of atoms in state 1 (P=0,1 respectively):

$$|GHZ_\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm (-1)^P ie^{iN\phi(t)}|1\rangle) \tag{36}$$

Now another $\frac{\pi}{2}$pulse around the y axis is applied leading to:

$$|GHZ_{\pm,\frac{\pi}{2}}\rangle = \frac{1}{2}(|0\rangle + |1\rangle) \pm (-1)^P ie^{iN\phi(t)}(|0\rangle - |1\rangle) \tag{37}$$

$$= \frac{1}{2}(1 \pm (-1)^P ie^{iN\phi(t)})|0\rangle + (1 \mp (-1)^P ie^{iN\phi(t)})|1\rangle \tag{38}$$

The next step is calculating the probability of measuring 1 and the probability of measuring 0. Since the system consist of two states, the total probability of measuring a qubit in state $|0\rangle$ or $|1\rangle$ is the sum of the respective probabilities for $|GHZ_\pm\rangle$ multiplied by the dephasing probability:

$$P(0,1) = p_+ * P(0,1|GHZ_+) + p_- * P(0,1|GHZ_-) \tag{39}$$

First the respective probabilities of measuring 0 and 1 are calculated for the separate states:

$$P(0|GHZ_+, 1|GHZ_-) = \frac{1}{2}(1 - (-1)^P \sin(N\phi(t))) \tag{40}$$

$$P(0|GHZ_-, 1|GHZ_+) = \frac{1}{2}(1 + (-1)^P \sin(N\phi(t))) \tag{41}$$

Adding these together and multiplying by $p_\pm$ leads to the following probabilities of measuring 0 and 1.

$$P(0) = \frac{1}{2}(1 - (1-2\epsilon)^N(-1)^P \sin(N\phi)) \tag{42}$$

$$P(1) = \frac{1}{2}(1 + (1-2\epsilon)^N(-1)^P \sin(N\phi)) \tag{43}$$

The measurement operator is $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$, which effectively means the probability of measuring a qubit in state 0 minus the probability of measuring it in state 1: $P(0) - P(1)$. Finding the average value of an operator is done by multiplying it by the bra and ket of the state it acts on. For $|GHZ_\pm\rangle$ this looks as follows:

$$\langle Z_\pm\rangle = \langle GHZ_\pm| Z |GHZ_\pm\rangle = \pm(-1)^P \sin(N\phi(t)) \tag{44}$$

As for the probabilities, the calculation needs to be done both for $|GHZ_-\rangle$and $|GHZ_+\rangle$, subsequently they are multiplied by $p_\pm$ respectively and summed to find $\langle Z\rangle$:

$$\langle Z\rangle = p_+\langle Z_+\rangle + p_-\langle Z_-\rangle \tag{45}$$

$$\langle Z \rangle = (-1)^P \sin(N\phi(t)) * [\frac{1}{2} + \frac{(1-2\epsilon)^N}{2}] - [\frac{1}{2} - \frac{(1-2\epsilon)^N}{2}] \tag{46}$$

$$\langle Z \rangle = (-1)^P \sin(N\phi(t)) * (1-2\epsilon)^N \tag{47}$$

Since it is assumed that $N * \phi << 1$, the small angle approximation can be utilised, meaning $\sin(N\phi) \approx N\phi$:

$$\langle Z \rangle = (1-2\epsilon)^N (-1)^P N\phi(t) \tag{48}$$

While it is important to know the expected value of a certain measurement, the next step is finding the stability of the entangled clock. This is done by calculating the Allan deviation:

$$\sigma_y(T) = \frac{1}{\omega_o * \sqrt{\tau T}} * \sigma_\phi(t) \tag{49}$$

where ω_0 is the resonance frequency of the clock atoms, τ is the duration of the ramsey cycle, T is the free evolution time and σ_ϕ is the average phase estimation uncertainty after a ramsey cycle. ω_0, τ, T can be found or researched easily, σ_ϕ however needs to be calculated as it is model clock dependent. σ_ϕ can also be expressed as the mean frequency offset (or average phase change after a Ramsey cycle), for which the following holds true:

$$\sigma_\phi = \sqrt{\langle (\phi_0 - \phi_{est})^2 \rangle} = \sqrt{\langle \phi_o^2 \rangle + \langle \phi_{est}^2 \rangle - 2\langle \phi_0 \phi_{est} \rangle}$$

(50)

It is known that $\langle \phi_0^2 \rangle = \gamma T$, where γ is the linewidth of the resonance frequency and $\langle \phi_{est}^2 \rangle = \frac{\langle Z_{est}^2 \rangle}{N^2}$. Analogue to that, $\langle \phi_{est} \rangle = \frac{\langle Z_{est} \rangle}{N}$. To find $\langle Z_{est} \rangle$, the measurement is repeated on m subsets of atoms to find the phase with more accuracy. Using the fact that the expectation value for measuring 0 and 1 can be calculated using the binomial formula where the expected value of a variable is $E = np$, $\langle Z_{est} \rangle$ is :

$$\langle Z_{est} \rangle = \frac{2\langle n_0 \rangle - m}{m} = \frac{2m * P(0) - m}{m} = 2P(0) - 1 \tag{51}$$

$$= (1-2\epsilon)^N (-1)^P \sin(N\phi) \tag{52}$$

applying the small angle approximation:

$$\langle Z_{est} \rangle = (1-2\epsilon)^N (-1)^P N\phi \tag{53}$$

$$\langle \phi_{est} \rangle = (1-2\epsilon)^N (-1)^P \phi \tag{54}$$

This is the same result as the average value of the measurement Z, which makes sense. To find $\langle \phi_{est}^2 \rangle$ one first needs to calculate $\langle Z_{est}^2 \rangle$, which is $\frac{4\langle n_0^2 \rangle - 4*\langle n_0 \rangle m + m^2}{m^2}$ with $\langle n_0^2 \rangle = mpq + m^2p^2$ and $\langle n_0 \rangle = mp$, as in the equation for $\langle Z_{est} \rangle$:

$$4 * \langle n_0^2 \rangle = m(1 - (1-2\epsilon)^{2N} N^2\phi^2) + m^2((1 - 2(1-2\epsilon)^N (-1)^P N\phi + (1-2\epsilon)^{2N} N^2\phi^2) \tag{55}$$

$$4 * \langle n_0 \rangle = 2m^2(1 - (-1)^P (1-2\epsilon)^N N\phi) \tag{56}$$

Putting these two equations together for $\langle Z_{est}^2 \rangle$ yields:

$\langle Z_{est}^2 \rangle = \frac{1}{m} + (1-2\epsilon)^{2N} N^2\phi^2(1 - \frac{1}{m})$(57) To get an estimate of the phase - which is necessary - one divides by N:

$$\langle \phi_{est}^2 \rangle = \frac{1}{mN^2} + (1-2\epsilon)^{2N} \phi^2 (1 - \frac{1}{m}) \tag{58}$$

This result enables the calculation of the mean frequency offset of the entangled clock σ_ϕ:

$$\sigma_\phi = \sqrt{\langle (\phi_0 - \phi_{est})^2 \rangle} = \sqrt{\langle \phi_o^2 \rangle + \langle \phi_{est}^2 \rangle - 2\langle \phi_0 \phi_{est} \rangle} \tag{59}$$

Plugging in the results from equations 54 and 58 gives:

$$\sigma_\phi = \sqrt{\gamma T + \frac{1}{mN^2} + (1-2\epsilon)^{2N}\phi^2(1-\frac{1}{m}) - 2(1-2\epsilon)^N(-1)^P\phi^2} \tag{60}$$

$$= \sqrt{\gamma T(1 + (1-2\epsilon)^{2N}(1-\frac{1}{m}) - 2(1-2\epsilon)^N(-1)^P) + \frac{1}{mN^2}} \tag{61}$$

If there was no dephasing at all ($\epsilon = 0$), the mean frequency would scale as $\sqrt{\frac{1}{mN^2} - \frac{\gamma T}{m}}$. Normal clocks scale as $\frac{1}{\sqrt{mN}}$. The entangled one effectively scales as $\frac{1}{\sqrt{mN}}$, meaning there is an increase in accuracy of factor $\frac{1}{\sqrt{N}}$. If there was maximal dephasing ($\epsilon = 0.5$), it would scale as $\sqrt{\frac{1}{mN^2} + \gamma T}$, which is a significantly larger value than no dephasing. To compare the entangled clock with the one posed by classical comparisons, an example Allan deviation for Sr clocks is calculated. Strontium optical clocks are very insensitive to background changes in electric or magnetic field, making them a fitting candidate. For ^{87}Sr the resonance frequency is $\omega_0 = 4,2693409742120343 * 10^{14}$, the time the clock is running τ=1s and the length of a Ramsey sequence is $T_{opt} = \gamma_{LO}^{-1}\frac{\pi^2}{2}(log(\gamma_L O * \tau N))^{-1}$ [17]. γ_{LO} is the linewidth of the local oscillator, in this case it is ^{87}Sr: $2\pi * 1mHz$ [4]. It was decided to use $m = 1000$ and $N = 10$ to keep it comparable to classical stabilities, as $m * N = 10^4$, which is the amount of atoms the QSNET group uses for their Strontium clocks [4]. Calculating the optimal T and σ_ϕ, while still keeping ϵ as a variable.

$$T_{opt} = (2\pi * 1mHz)^{-1}\frac{\pi^2}{2}((log(2\pi * 1mHz * 1s * 10^4))^{-1} = 379,37 \tag{62}$$

$$\sigma_\phi(T) = \sqrt{2\pi * 10^{-3}(1 + (1-2\epsilon)^{2*10^4}(1-\frac{1}{100}) - 2(1-2\epsilon)^{10^4}) + \frac{1}{10^7}} = 0.08 - 0.0019 \tag{63}$$

To find the minimal and maximal value of this mean frequency offset, $\epsilon = 0$ and $\epsilon = 0.5$ where plugged into the equation. The calculation of these two variables enables the calculation of the Allan deviation for the entangled ^{87}Sr clock.

$$\sigma_y(T) = \frac{1}{4,269 * 10^{14}Hz * \sqrt{1s * 379,37s}} * \sigma_\phi = 1 * 10^{-17} - 2.5 * 10^{-19} \tag{64}$$

The Allan deviation of the classical ensemble is $2.0 * 10^{-18}$, meaning locally entangling clocks has the possibility to give increased stability, as long as dephasing is not too high [4]. Figure 5 shows how the stability of the entangled clocks scales dependent on phase error probability. Overall, the more atoms, the smaller the uncertainty. The point at which classical clocks become better than entangled ones due to noise, is shown in red. It is at $\epsilon = 0.0002$, this means for a larger dephasing probability non-entangled clocks outperform this approach. Figure 6 shows the comparison of m=1000, N=10 and m=100, N=100, clearly displaying how having many subensembles is advantageous. It is also visible that by increasing m, the stability decreases slower, making it preferable over classical clocks for a higher error.

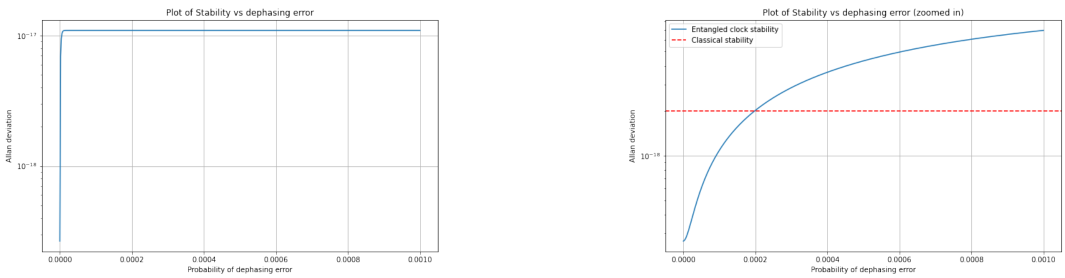
3.2 Application to topological defects

To find out how the entangled clocks would perform for detecting topological dark matter, one can use the following figure of merit [11].

$$F = \frac{K_x}{T^{\frac{3}{2}} * \sigma_y(T)} \tag{65}$$

where K_x is the sensitivity to the fundamental constant. There are two fundamental constant influencing the clock resonance frequency: the fine structure constant α and the electron/proton mass ratio μ. The ^{87}Sr clock is however only sensitive to a variation in the fine structure constant, which is why the figure of merit F_α is the value describing the sensitivity of this specific entangled clock to topological defects.

$$F = \frac{0.06}{1s^{\frac{3}{2}} * \sigma_y(T)} = 5.8 * 10^{15} - 2.4 * 10^{17} \tag{66}$$



(a) (b)

Figure 5: a) This figure shows the general stability of entangled clock dependent on the dephasing error b) This graph is a zoomed in version of a, it also shows the dephasing error ϵ at which a non-entangled clock is better than the proposed entangled one

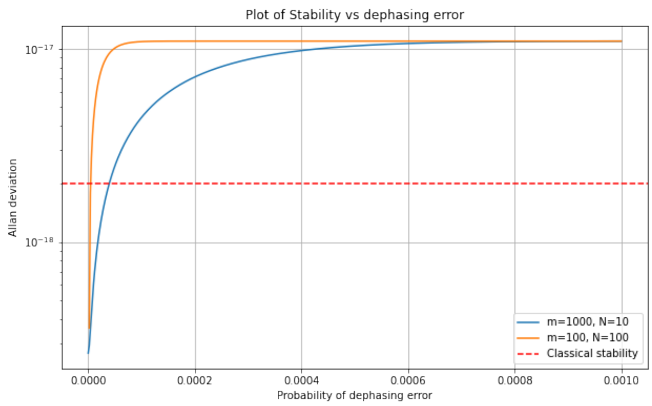


Figure 6: Comparison of different number of subensembles, while keeping the total number of atoms (m*N) constant

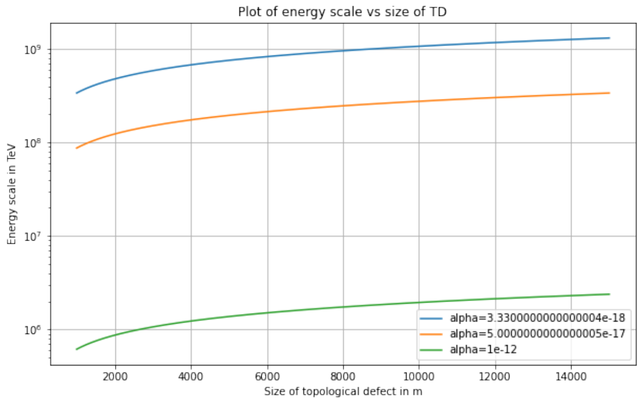


Figure 7: Plot of the energy scale for topological defects that could be detected using the entangled clocks, the orange and green line are constraints from previous research(orange: using GPS network [30], green: using an european network [29]

The signal to noise ratio scales as F. An easier interpretable result comes from checking whether one can detect the phase shift that the topological defect would cause using the entangled clock. Following a paper that tried to detect topological dark matter, a bound was found on the variation of the fine structure constant: $\frac{\delta\alpha}{\alpha} = 5 * 10^{-17}$. For ^{87}Sr clocks the sensitivity constant is $K_\alpha = +0.06$ They also propose the following equation to calculate the variation in the frequency ratio of a clock in normal phase and one out of phase $\frac{\nu_a}{\nu_b} = \gamma_{ab}$:

$$\frac{\delta\gamma}{\gamma_{ab}} = \frac{1}{T}\int_0^T K_{ab} * \frac{\delta\alpha(r,t)}{\alpha} t \tag{67}$$

for large T it turns into:

$$\frac{\delta\gamma}{\gamma_{ab}} = K_{ab} * \frac{\delta\alpha}{\alpha} = 0.06 * 5 * 10^{-17} = 3 * 10^{-18} = \omega_{var} \tag{68}$$

$$\Phi = \int_0^\tau (\omega_0 - \omega(t))dt = \int_0^\tau (\omega_0 - \omega_0 + \omega_{var})dt = 3 * 10^{-18} * \tau \tag{69}$$

This phase $\phi = 3*10^{-18}$ is within the possible range of stability for the entangled clock, meaning that depending on the dephasing error, one could detect such and slightly smaller variations of the fine structure constant [29]. To evaluate the range of variations on α that entangled clocks could detect, the maximal Allan deviation is taken as the variation in the frequency ratio: $\frac{\delta\gamma}{\gamma_{ab}} = 2.5 * 10^{-19}$. Using this, one can calculate $\frac{\delta\alpha}{\alpha}$ with equation 68:

$$\frac{\delta\alpha}{\alpha} = \frac{\delta\gamma}{\gamma_{ab}}/K_{ab} = \frac{2.5 * 10^{-19}}{0.06} = 3.333 * 10^{-18} \tag{70}$$

This variation of α is now utilised to find constraints on the energy scale Λ_α, as $\frac{\delta\alpha}{\alpha} = \frac{\hbar c \rho_{DM} v_g T d}{\Lambda_\alpha^2}$, where $\hbar$ is the reduced planck constant, c is the speed of light, ρ_{DM} is the energy density of dark matter: 0.3 GeV/cm^3, v_g is the galactic velocity: 300 km/s and T (time)will be taken as 12 hours. This yields:

$$\Lambda_\alpha^2 = \frac{\hbar c \rho_{DM} v_g T d}{\frac{\delta\alpha}{\alpha}} = 0.36 * 10^{11} * d \tag{71}$$

Using this equation, figure 7 was created. It includes results from previous studies that have found constraints on $\frac{\delta\alpha}{\alpha}$: These constraints on Λ_α as displayed in Figure 7 do not necessarily precisely correspond to the ones found by the respective groups, as they used different assumptions/different clocks. Here merely their constraints on the variation of α is used, to properly compare the entangled clock to previous research. As this graph shows, using entangled clocks with a very small dephasing error yields higher constraints on the energy scale, such as $\Lambda_\alpha > 10^9$TeV for a size of the topological defect of d=10km.

4 Discussion

4.1 Impact of research

Locally entangled clocks exhibit a possible increase of one order of magnitude for the clock stability compared to classical clocks. It is important to note that this is the case for clocks with a high number of atoms and a low probability of dephasing. The smaller the clock, the less the entanglement matters, making the absolute quantum advantage less apparent. As shown in the part describing the Model clock, entangling the clocks leads to a decreased uncertainty of a factor of $\frac{1}{\sqrt{N}}$. This equation also showcases the dependence on the number of clocks and the number of sub ensembles m of clocks on which the measurement is done, as N*m is the total number of atoms. Entangling clocks makes more sense, the higher number of clock qubits one is presented with. For molecular ion clocks that typically only use one ion, this entanglement scheme makes no sense as one needs at least two systems to create an entangled state. Furthermore, the higher the dephasing error probability ϵ is, the less entanglement should be used, meaning one should have many subensembles and little qubits per ensemble. If the error is however very small, larger GHZ states are preferable, meaning a slightly lower m and larger N.
As previously mentioned, the dephasing error significantly impacts the stability of entangled clocks. Since the error accumulates exponentially, even a tiny probability of an error, leads to a large difference in uncertainty. Due to the impossibility of completely shielding the clocks from the environment, it is unfeasible to avoid all noise. Additionally, if one qubit depolarizes, all phase information vanishes. Apart from dephasing and depolarization, decoherence is another error that cannot be ignored. Decoherence is a process by which a system loses its quantum behavior and becomes classical due to interactions with the environment. The environment 'measures' the state of the qubit, leading to a loss of information. Once a clock qubit decoheres, all phase information is lost, making it useless [24]. This compilation of possible errors negates the quantum advantage to an extent - if error mitigation and correction protocols are applied to avoid some of these errors, entangling clocks is definitely an important step toward more accurate timekeeping and its possible other applications. Otherwise, it is uncertain at this point if this entangling scheme will lead to any improvement.
Assuming the error issue is solved, it becomes necessary to evaluate whether entangled clocks can be used to detect topological defects. According to our momentary estimation (upper bound) for transient variation of α and μ, entangled clocks are one order of magnitude more sensitive than is required. This number however stems from the fact that the most sensitive clocks used had a stability of around 10^{-16}, which is how they put an upper limit on the change of fundamental constants. It is in no way given, that within the extra order of magnitude of stability it will be possible to detect TD's, since it is not known how small the effect of topological defects is or if they even exist. What can be achieved by using a network of locally entangled clocks is placing new constraints on the size and strength of TD's, which still gives novel information about the universe and especially its condition at an early cosmological time. So even not detecting any TD's using a network of locally entangled clocks, is still an important step in Dark sector research.

While this quantum advantage might not yield enough sensitivity for detecting topological defects, it could prove very useful in other domains. Among such use-cases are gravimetry missions, which investigate the gravitational field. Gravimetry relies on the phenomenon of gravitational time dilation, as predicted by general relativity, wherein time elapses differently at various locations due to variations in gravitational potential. Atomic clocks are instrumental in measuring this time dilation, enabling the detection of differences in the gravitational field. The precise timekeeping provided by atomic clocks plays a vital role in determining satellite-to-satellite distances and extracting gravitational data from satellite measurements. Gravimetry missions serve as crucial tools in monitoring Earth's climate-related changes, such as the melting of glaciers and ice sheets, rising sea levels, droughts, floods, as well as the impacts of tsunamis and earthquakes. Greater understanding of these processes is certain to have positive effects on society, including operational flood and drought forecasting, monitoring and prediction of sea level rise, and water management applications. Satellite missions equipped with atomic clocks, such as the Gravity Recovery and Climate Experiment (GRACE), its successor GRACE Follow-On (GRACE-FO) and the Gravity field and steady-state Ocean Circulation Explorer (GOCE), are capable of capturing minuscule variations in Earth's gravitational field by precisely tracking the distance between orbiting satellites. Increasing the precision of the atomic clocks used in these past missions, could give rise to a new level of detail on essential climate variables which is crucial in battling climate change and its effects on the earth. This is where entangling clocks could contribute as a very useful novel technology [3].

Entangling clocks that are situated on satellites also have applications in fundamental physics research as well as provide the opportunity for exploring new physics. Among these research projects is for example testing the Einstein equivalence principal, which states that it is not possible to distinguish between the fictitious force one experiences when being accelerated and the the gravitational 'force' of standing on a massive body. Following from that is the local lorentz invariance, meaning the outcomes of an experiment being conducted in a free falling apparatus should give the same outcomes as a stationary one. This can be tested using atomic clocks - and the outcomes are more precise when using entangled clocks. Another possibility would be measuring the gravitational redshift with higher accuracy. Gravitational redshift refers to the aforementioned phenomenon that time passes differently at different location in space. Finding a more accurate gravitational redshift is a fundamental tool to understanding gravity better. This extra order of magnitude gained through entangling clocks, enables improved calculations of the gravitational red shift, as well as better mapping of the gravitational field of the earth or of other celestial bodies in the vicinity of the clocks[6]. There have also been propositions to detect gravitational waves using optical lattice clocks. Gravitational waves are ripples in spacetime that occur when very heavy objects collide, such as black holes or neutron stars. They were proposed at the beginning of the 20th century and detected for the first time in 2015[1]. They have become a major topic in theoretical physics, as gravitational waves have the possibility to teach scientists many things about early cosmological times, deep space and other topics. Gravitational waves stretch space and time on their path through the universe, meaning they can be detected using very precise, linked timekeeping devices [18]. Entangling atomic clocks locally could definitely pose advantageous for such detectors.
Another aspect could be quantum gravity. Quantum gravity is a great field of interest to many scientists, as it would combine two major fields of theoretical physics that have so far eluded all attempts at combining them to one theory. Investigating the influence of gravity on prolonged entangled states using such sensitive machines is thus very interesting [4].

4.2 Future outlooks

Local entanglement of clocks represents a significant advancement in augmenting clock networks. Nevertheless, the communication between these clocks remains constrained by classical technologies. The subsequent proposed undertaking entails establishing a Greenberger-Horne-Zeilinger (GHZ) state across all clock qubits in each clock to enhance the accuracy of the center of mass (COM) time [22]. One method of doing so uses k number of atomic clocks made of N atoms. Each clock has its own local oscillator. Within each clock, some qubits are allocated to form entangled states with the other clocks. More precisely, one atomic clock is chosen as the COM clock and gets additional ancilla (auxiliary) qubits. First, one half of the ancilla qubits gets put into a complete entangled state with the first clock qubit, resulting in a GHZ state. Parallel to this, the other half of the qubits is used to create single Einstein podolsky rosen (EPR) pairs with all remaining atomic clocks [12]. EPR pairs are a specific type of entangled particle pairs introduced in the Einstein-Podolsky-Rosen paper. In the next step, the state of the qubits gets teleported to the COM qubit and as such a collective GHZ state across all atomic clocks gets made. Finally, each entangled qubit per atomic clock extends the entanglement to all other clock qubits within its clock - creating locally entangled clocks within this network of entangled clocks.
In the interrogation step of the Ramsey sequence, entanglement becomes useful as it enables a faster accumulation of the phase difference. Say one qubit picks up a relative phase of $\phi = \delta T$ during the interrogation time T, which is displayed in the total entangled state as $\frac{1}{\sqrt{2}}(|0\rangle^{\otimes N} + ie^{i\phi}|1\rangle^{\otimes N})$. After the free evolution time, each clock measures its qubits in the X-basis to find the parity (how many qubits in the states $|0\rangle$ and $|1\rangle$). Once the parity from every single clock is evaluated, the phase information can be extracted. As the phase accumulates faster than in regular clocks, the problem of phase slips arises: if the phase exceeds the interval $[-\pi, \pi]$, the phase estimation is false. Due to this, the previous protocol needs to be adjusted: instead of having one general GHZ state, one requires a product of j increasingly large GHZ ensembles. With this approach it is possible to find the bits of the binary fraction that make up the laser phase directly, as each level of j gives a measurement. A locally entangled clock as proposed in the model clock section is also in danger

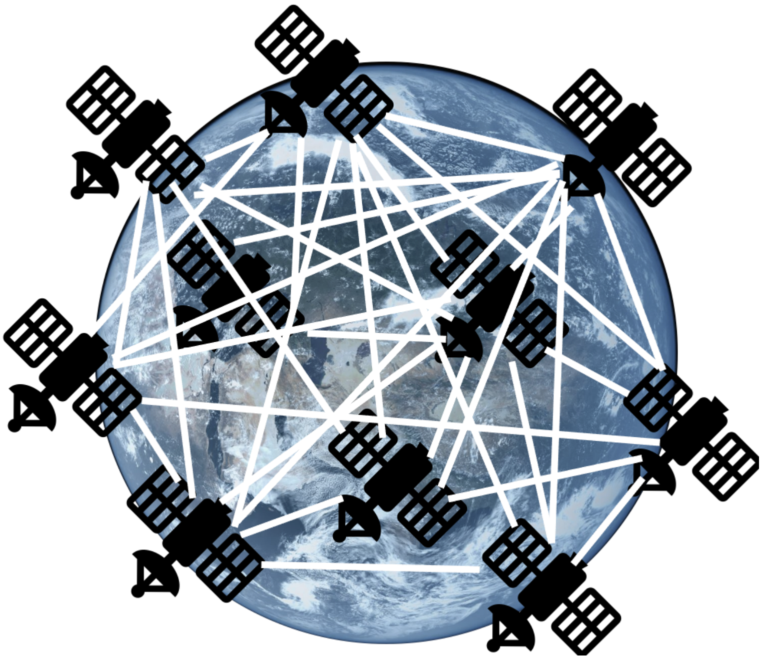


Figure 8: Network of entangled clocks around earth

of phase slips, due to the smallness of the phase differences expected by topological defects, the cascaded GHZ scheme was however not implemented in the presented approach. The stability of the network of entangled clocks scales as $\sigma_y(\tau) \sim \frac{\sqrt{log(N)}}{\omega_0 N \tau}$, which is an improved stability compared to the locally entangled clocks [19]. Additionally it has a more exact measurement of the COM time, since the phase differences of the various clocks do not need to be transmitted classically, making it a very useful instrument to probe space for topological defects.

4.3 Ethical consideration

There are no ethical considerations to be taken in terms of privacy or test subjects as this is purely theoretical research. While the prioritization of spending money on theoretical physics over other fields can be debated, it is worth noting that the technologies developed for particle physics or astrophysics have often found practical applications. Investing in the research of atomic clocks can for example, as mentioned earlier, further the technology of gravimetry, enabling progress in areas such as the observation of essential climate variables, making this thesis a valuable and worthwhile investment.

5 Conclusion

This thesis set out to investigate the applicability of entangling atomic clocks to detect the transient variation of fundamental constants due to topological defects dark matter. By utilising the formalism of quantum information theory (qubits), an analytical model of a clock was created, put through a dephasing noise channel and subsequently evaluated in the context of topological defects. It was hypothesized, that by locally entangling atomic clocks, the precision would increase and approach the Heisenberg limit for clocks. For a noiseless clock, this was shown. Even for a clock that falls victim to some dephasing, it is still an improvement to non entangled clocks. At a certain probability of dephasing however, not entangling clocks yields better results. Its possible implementation for dark matter research is equally ambivalent. While the higher precision enables scientists to put constraints on the energy scale for topological defects, it does not guarantee finding any topological defects. Not detecting any is however also valid research, making this project a good theoretical basis for implementing such a detection protocol.
Regardless of its direct application to topological defects, entangling clocks has numerous other possible use-cases such as gravimetry or fundamental physics research. This theoretical project has reached a successful conclusion, offering a mathematical foundation and a comprehensive compilation of pertinent literature on the subject. These outcomes serve as valuable resources for other researchers undertaking further scientific investigations in this domain.

6 Critical Reflection

This thesis has been a challenging endeavour, both factually as well as on a more personal level. I went into this project, not really knowing what to expect. I had already done my Honours research project on a quantum computing project with Gideon Koekoek, so I thought my background in QC would be sufficient for the thesis. Oh how wrong I was. While I was indeed familiar with the general postulates and techniques, I had never actually done any mathematics in this field apart from algebraically changing given equations to fit another use case. Additionally, I spent one week during the first month of the thesis on a quantum hackathon in Italy, making me essentially miss a week of work. While I learned a lot about quantum computing, which would prove helpful in gaining some intuition later, it made my work on the proposal all the more stressful. When I was then tasked to construct my own toy model of a clock using qubits, I did not know where to start, making my lacking quantum computing background obvious. Even though I took an advanced course in quantum mechanics, I had skipped its prerequisite so even though my understanding was good, I never had enough practice with the actual formalism. So I turned to reading numerous different papers with similar approaches and resources provided by Johannes Borregaard and trying to mimic their steps. Many times I had to however take a step back and read chapters in the book "Quantum computation and information"[24] to grasp the concepts described in the papers as well as gain a deeper understanding of quantum computing.
After the topic of trying to detect dark matter using entangled atomic clocks had crystallized I went straight into writing my proposal, thinking the focus of my thesis would be situated at the interplay of quantum mechanics and general relativity. I realised after my submission, that topological dark matter does not interact gravitationally but rather has an effect on fundamental constants, meaning, my background research had to start anew after the proposal. Once I had understood in what way topological defects affect atoms, I had a clearer image of where the thesis would lead. This lacking background knowledge or structure in my research in the first few weeks led to my going down numerous very wrong roads. I for example spent a week trying to determine the degree of entanglement of my model clock after applying the noise channels using the Von Neumann entropy or the simple purity, as well as a partial transpose. Things became a lot clearer, once Johannes sent me his draft of a book he is working on, focusing on the necessary quantum mechanics for quantum computing. This, together with the book by Nielsen & Chuang and other lecture notes and scripts I found online, finally helped me understand different concepts such as the density matrix, noise channels, measurement, changing basis as well as how to deal with equations describing entangled states. A main problem towards the end was a misunderstanding on how to calculate the mean frequency offset of the entangled clock. I tried numerous slightly different approaches to calculating it, some leading to horrendous 8 line long equations but in the end I figured it out. This was however only 2.5 weeks before the deadline of the thesis, making it rather stressful. Luckily I had started transferring my mathematical notes to latex a few weeks prior, meaning the main objective became simply writing down my knowledge and filling in the gaps between the mathematical equations. So now I am writing this reflection two weeks before the final deadline - while I am still stressed and have a long way to go, I am certain I can do it.
Now why did it become so stressful? Throughout the thesis I was struggling with dealing with a lot of pressure, much of it stemming from other aspects of life, or pent up stress from the previous years of studying. This thesis project was also the first time I had to work this independently. My research experience with Gideon varied significantly. The majority of the work and progress there was done during group meetings, with the downtime being utilised for double-checking the calculations and conducting literary research. It was challenging switching from this closely supervised style to a thesis where I worked independently and only briefly presented my work once a week online to receive feedback. An additional problem was that I was intimidated to ask my supervisor anything outside of these meetings because I knew I would be judged on my participation and the calibre of my work. Simple inquiries seemed improper at the time. Additionally the simple fact that my thesis was fully online, meaning I did not meet a single other person in the research group, made me feel very on my own. ChatGPT became a very helpful resource to ask questions to. After the midterm participation feedback however, when my supervisor encouraged me to initiate asynchronous contact via email, I asked more questions and thus progressed significantly faster.
Aspects to improve on in a next thesis are therefore definitely asking more questions and communicating to my supervisor if I need more assistance. Another way of solving this issue could have been asking in the beginning for a connection with a masters or PhD student who would

have been able to answer my simple general questions on quantum computing. My initial literary research should have also been more diligent, as well as realising my gaps in quantum computing and mathematics at an earlier time.
This thesis taught me many things. It gave me the confidence of knowing that I can work independently and alone - and the realisation that I do not want to do that, as iI thrive when I have somebody to bounce ideas off of or simply talk things through. I also learned a lot about quantum computing, making it a topic I am now seriously considering for a master's degree. Another aspect I noticed that reading many challenging papers improved my skills of understanding the gist of them significantly. Additionally it increased my attention span. This enabled me to read non-fiction books without getting bored or distracted - something I have never been able to before. Overall this thesis has been a very good experience, I started thoroughly enjoying physics again - something I had not done for quite a while now - and it sparked my interest in pursuing it in more depth.

7 Appendix

7.1 Depolarizing channel

When creating a clock model, one should in addition to dephasing noise also consider the depolarizing channel, that puts the affected qubits in a completely mixed state. Applying depolarizing channel to one qubit yields:

$$\Lambda(\rho) = (1-\epsilon)\rho + \epsilon\frac{I}{2} \otimes Tr(\rho) \tag{72}$$

and exerting it onto the density matrix EQUATION X ρ:

$$\begin{aligned}\rho = \frac{1}{2}(1-\epsilon)&\begin{bmatrix} |0\rangle^{\otimes N}\langle 0|^{\otimes N} & -ie^{-iN\phi(t)}|0\rangle^{\otimes N}\langle 1|^{\otimes N} \\ ie^{iN\phi(t)}|1\rangle^{\otimes N}\langle 0|^{\otimes N} & |1\rangle^{\otimes N}\langle 1|^{\otimes N}\end{bmatrix} \\ +\frac{\epsilon}{2}&\begin{bmatrix} |0\rangle\langle 0| & 0 \\ 0 & |1\rangle\langle 1|\end{bmatrix} \otimes \begin{bmatrix} |0\rangle^{\otimes N-1}\langle 0|^{\otimes N-1} & 0 \\ 0 & |1\rangle^{\otimes N-1}\langle 1|^{\otimes N-1}\end{bmatrix}\end{aligned} \tag{73}$$

If this depolarizing channel is applied on a random number, it results in the following:

$$\begin{aligned}\Lambda(\rho) = (1-\epsilon)^N\rho + \sum_{i=0}^{N}\epsilon(1-\epsilon)^{N-1}\frac{1}{2}\otimes Tr_i(\rho)+ \\ \sum_i^N\sum_j^N \epsilon^2(1-\epsilon)^{N-2}\frac{1}{2}_i \otimes \frac{1}{2}_j \otimes Tr_{i,j}(\rho)+ \\ \sum_i^N\sum_j^N\sum_k^n \epsilon^3(1-\epsilon)^{N-3}\frac{1}{2}_i \otimes \frac{1}{2}_j \otimes \frac{1}{2}_k \otimes Tr_{i,j,k}(\rho)+ \\ ... + \epsilon^N\frac{1}{2}^{\otimes N}\end{aligned} \tag{74}$$

While this seems like a difficult sum of sums of tensor products, it instigated an important deduction regarding the impact of depolarization. In the case of any depolarization on any qubit, every qubit will lose their phase information. This is true even for the qubits that have not actually depolarized - that are not part of the qubits whose state is the identity matrix. The number of qubits experiencing dephasing, whether one, ten, or even a hundred is of no importance as they lead to an identical outcome: the eradication of phase information. Consequently, when measuring the final state, an equiprobable distribution between 0 and 1 is anticipated.

This observation implies that only when none of the qubits undergo depolarization can one effectively proceed with the subsequent steps outlined earlier. The depolarizing channel can therefore be disregarded for the stability calculation.

7.2 Figures in more quality

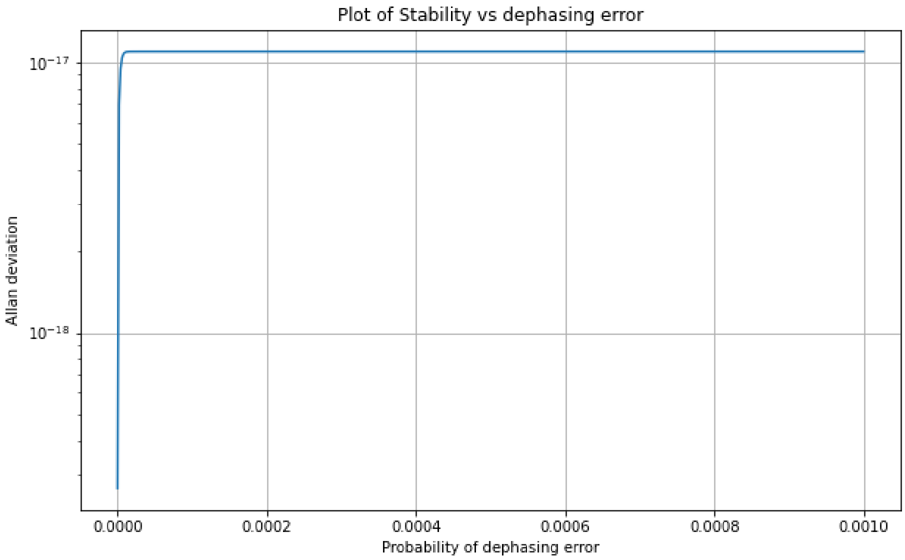


Figure 9: Stability of entangled clocks versus ϵ

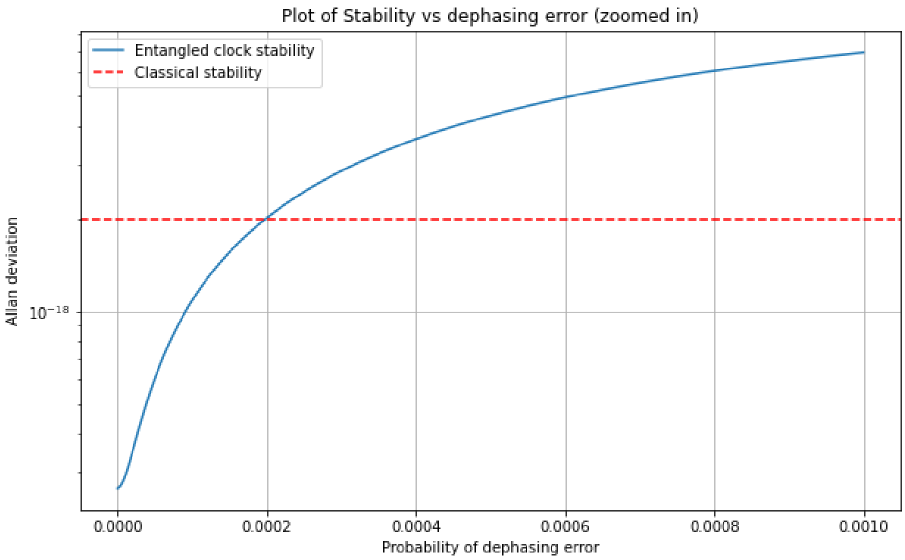


Figure 10: Zoomed in version of stability of entangled and unentangled clock

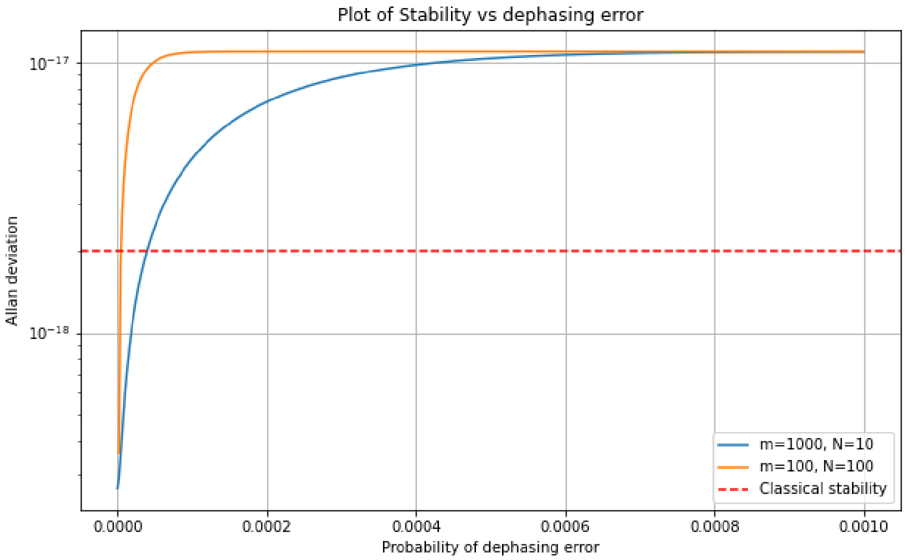


Figure 11: Plot of stability vs dephasing error for two different sizes of subensembles

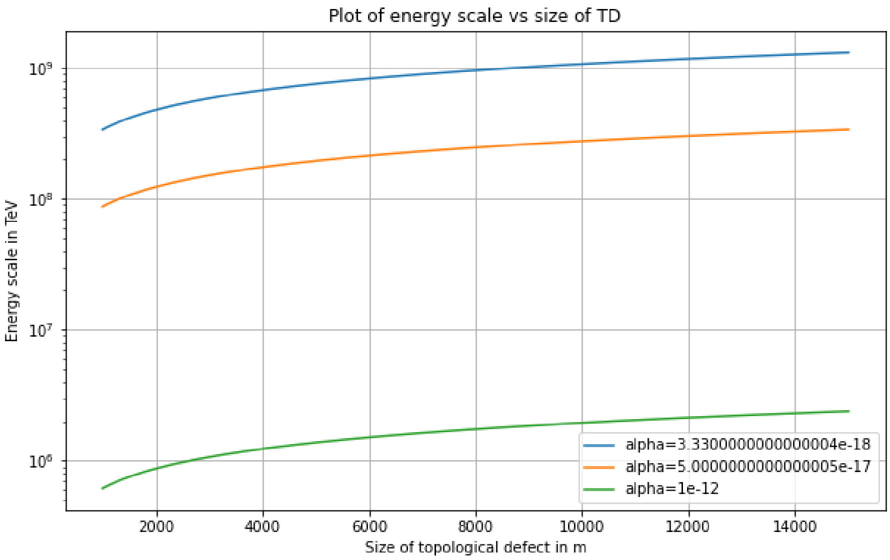


Figure 12: Plot of energy scale vs size of TD

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