Decentralized Extrema-finding in Circular Configurations of Processors

Hirschberg, Daniel S., and James Bartlett Sinclair. "Decentralized extrema-finding in circular configurations of processors." *Communications of the ACM* 23.11 (1980): 627-628.

Presented by Fernanda Mora
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“All nodes are equal, but some are more equal than others”

Marco Aiello, Eirini Kaldeli

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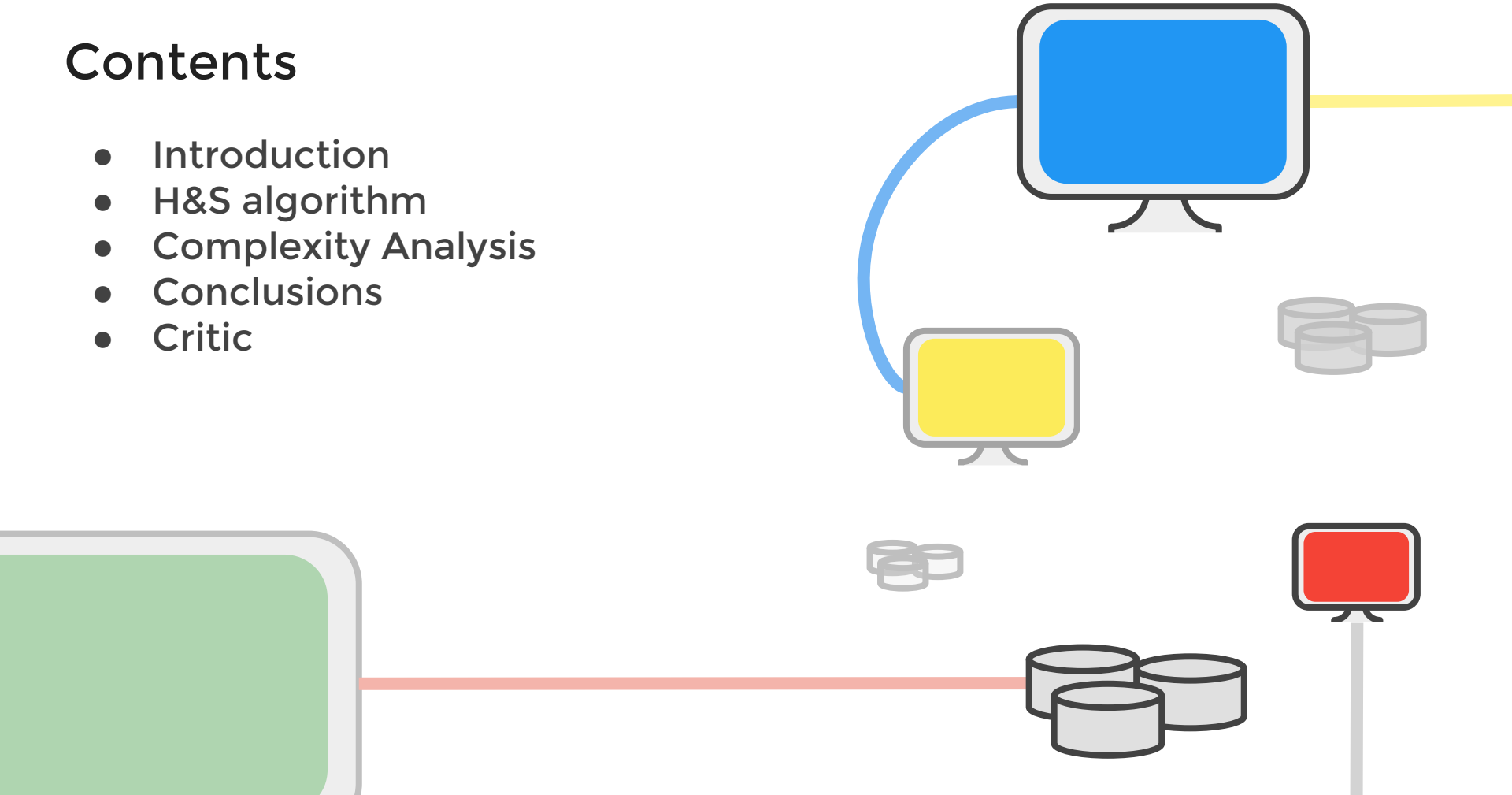


Introduction

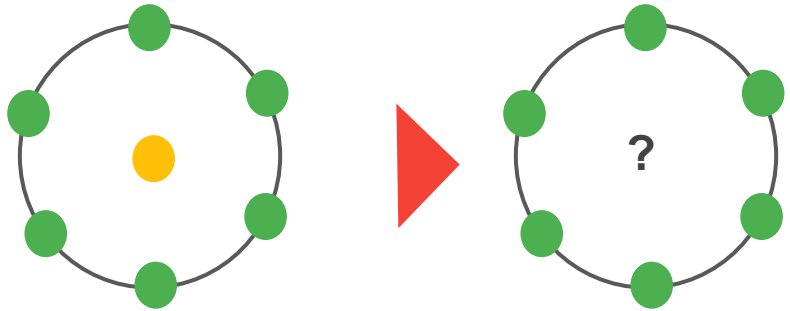
Problem definition & relevance

Leader election: designating a process as the organizer of a given task in a distributed system

LeLann, 1977

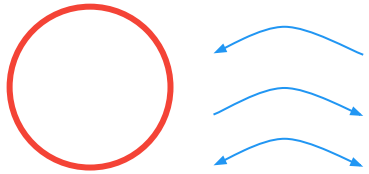
Many DS are client-server based: leader can fail

Challenge in DS: every processor should agree on the new leader

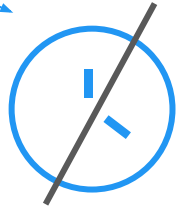


Assumptions

Ring topology



Asynchronous communication
(but reliable + FIFO. More realistic)



Decentralized system
(agreement and election)

Unique identification number
(asymmetry)

1 6 2 8
3

Max=8

Unknown number of processors

Redefining our problem...

Given *n* (unknown) processors, each with a unique value arranged in a ring and work asynchronously, we want to designate by consensus a unique processor from the ring.

Basic idea: Choose the processor with the highest value.

Which algorithm? -> Complexity: messages passed, other measures (number of phases, time)

First attempts

- LeLann (1977): $O(n^2)$ worst case

  Idea: send a list with everyone's id. Leader will have the highest.

- Chang and Roberts (1979): O(nlogn) average

  Worst case $O(n^2)$!

  Idea: similar but...If I receive a token with a smaller id then I'll drop it!

Can we do better?

H&S algorithm

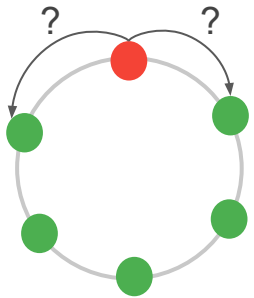
The H&S algorithm

Add. assumption: Bidirectional message passing

Idea: Smaller ids travel less-> elections on increasingly larger sets

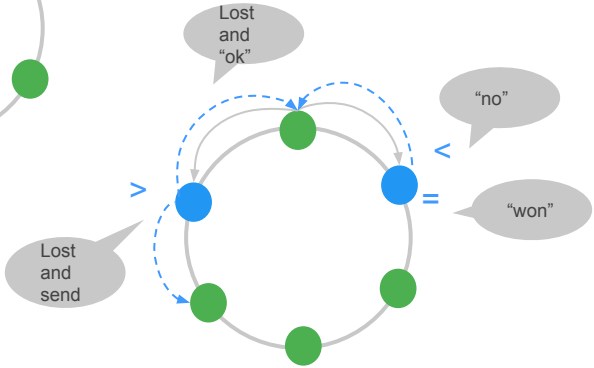
The Algorithm
To run for election:
status ← "candidate"
maxnum ← 1
WHILE status = "candidate" DO
sendboth ("from", myvalue, 0, maxnum)
await both replies (but react to other messages)
IF either reply is "no" THEN status ← "lost"
Else maxnum ← 2•maxnum
OD

Run Elections



On receiving message ("from", value, num, maxnum):
IF value < myvalue THEN *sendecho* ("no", value)
IF value > myvalue THEN DO
status ← "lost"
num ← num + 1
IF num < maxnum THEN *sendpass* ("from", value, num, maxnum)
ELSE *sendecho* ("ok", value)
OD
IF value = myvalue THEN status ← "won"

Upon receiving a message for election (left/right)



On receiving message ("no", value) or ("ok", value)
IF value ≠ myvalue THEN *sendpass* the message
ELSE this is a reply the processor was awaiting

Upon receiving a reply message

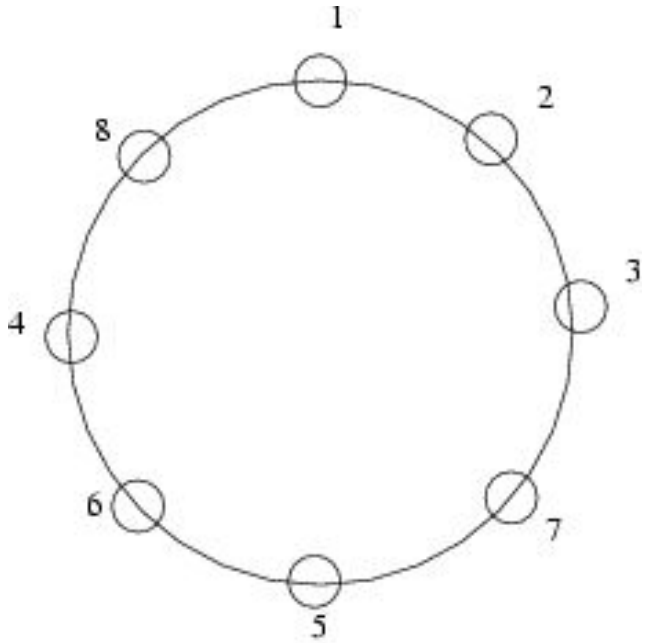
Example

Initially: all processors are leaders

Round 0: 6, 7 and 8 leaders

Round 1: 7 and 8 leaders

Round 2: 8 only leader



Complexity analysis (messages)

Messages passed per phase

- At phase k, at most $4*2^k$ messages are passed (elections and replies) by candidate.
- How many candidates do we have in phase k? (worst case)
- If k=0, n candidates
- If k>0, *floor*$(n/2^{k-1}+1)$ candidates
- Total messages at phase k: $(4*2^k)*$ *floor*$(n/2^{k-1}+1) < 8n$

Complexity analysis (messages)

Number of phases

- roof(logn)+1 phases until leader election
- In last phase 2n messages sent (no replies)
- Total number of messages (worst case) is:

$$4n + \sum_{k=1}^{\lceil \log n \rceil - 1} (4 \cdot 2^k \frac{n}{2^{k-1}+1}) + 2n \leq 6n + 8n(\lceil \log n \rceil - 1).$$

- Message complexity: O(nlogn)

Can we do better?

- Unidirectional passing worst case is quadratic -> true?
- Theorem (Burns): Any leader election algorithm for asynchronous rings whose size is not known a priori has $\Omega(n \log n)$ message complexity (holds also for unidirectional rings).
- So we cannot do better!

Conclusions

H&S Conclusions

- Lower complexity in the worst case than LeLann and C&R
- In fact, achieves the minimum message passes possible
- H&S is O(nlogn) & Ω(nlogn) -> Θ(nlogn)
- Assumes bidirectional passing: how strong is this?

Critic

Is election useful in practice?

- If a processor fails, the ring fails, election fails!
- Is a external fixer a strong assumption?
- Is it reasonable to assume ids have ordered and finite ids?
- If there are no IDs (it can happen!) election (deterministic) is impossible!

Thanks