Solving Proximity Constraints

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Motivation

For proving theorems, a frequently occurring problem is to find common instances of formulae.

Example 1

Let f be a function, a, b constants and x a variable. The two expressions

$$f(a,x) \quad \text{and} \quad f(a,b)$$

can be unified with $\{x \mapsto b\}$.

Motivation

For proving theorems, a frequently occurring problem is to find common instances of formulae.

Example 2

Let f, g be functions, a, b constants and x a variable. The two expressions

$$f(a, x) \quad \text{and} \quad g(a, b)$$

cannot be unified as $f \neq g$.

Motivation

In 1965 Robinson presented his unification algorithm and solved this problem, his algorithm was improved for better(=faster) performance since.

If we consider now the unification problem

$$f(a, x) \simeq^? g(a, b)$$

again, we might wonder, if we could not ignore $f \neq g$, if they are "close" to each other, i.e. if they are equal in a fuzzy logic sense. Being close is represented as a proximity relation, which are symmetric and reflexive, but not necessarily transitive. C. Pau and T. Kutsia solved this problem, presenting an algorithm, which we implemented.

Introduction

The Algorithm consists of two sub-algorithms and works on (modifies) 4 sets:

- P: unification problem to be solved ,
- C: neighbourhood constraint,
- σ: set of pre-unifier,
- Φ: name-class mapping,

where Algorithm 1 modifies P, C, and σ and Algorithm 2 modifies C and Φ. If Algorithm 1 was successful, $P = \emptyset$, if Algorithm 2 was successful $C = \emptyset$.

Pre-Unification rules

(Tri) $\{x \simeq^? x\} \uplus P; C; \sigma \Rightarrow P; C; \sigma$

(Dec)
$\{F(\overline{s_n}) \simeq^? G(\overline{t_n})\} \uplus P; C; \sigma \Rightarrow \{\overline{s_n \simeq^? t_n}\} \cup P; \{F \approx^? G\} \cup C; \sigma$

(VE) $\{x \simeq^? t\} \uplus P; C; \sigma \Rightarrow \{t' \simeq^? t\} \cup Px \mapsto t'; C; \sigma\{x \mapsto t'\}$

(Ori) $\{t \simeq^? x\} \uplus P; C; \sigma \Rightarrow \{x \simeq^? t\} \cup P; C; \sigma$

(Cla) $\{F(\overline{s_n}) \simeq^? G(\overline{t_n})\} \uplus P; C; \sigma \Rightarrow \bot$ if $m \neq n$

(Occ)
$\{x \simeq^? t\} \uplus P; C; \sigma \Rightarrow \bot$ if there is an occurrence cycle of x in t

(VO)
$\{x \simeq^? y, \overline{x_n \simeq^? y_n}\}; C; \sigma \Rightarrow \{\overline{x_n \simeq^? y_n}\}\{x \mapsto y\}; C; \sigma\{x \mapsto y\}$

Rules for Neighbourhood Constraints

(FFS) $\{f \approx^? g\} \uplus C; \Phi \Rightarrow C; \Phi;$ if $\mathcal{R}(f, g) \geq \lambda$

(NFS) $\{N \approx^? g\} \uplus C; \Phi \Rightarrow C; update(\Phi, N \to \mathbf{pc}(g, \mathcal{R}, \lambda))$

(FSN) $\{g \approx^? N\} \uplus C; \Phi \Rightarrow \{N \approx^? g\} \cup C; \Phi$

(NN1)
$\{N \approx^? M\} \uplus C; \Phi \Rightarrow C; update(\Phi, N \to f, M \to \mathbf{pc}(f, \mathcal{R}, \lambda)),$
where $N \in dom(\Phi)$, $f \in \Phi(N)$

(NN2) $\{M \approx^? N\} \uplus C; \Phi \Rightarrow \{N \approx^? M\} \cup C; \Phi$, where
$M \notin dom(\Phi)$, $N \in dom(\Phi)$

(Fail1) $\{f \approx^? g\} \uplus C; \Phi \Rightarrow \perp$, if $\mathcal{R}(f, g) < \lambda$

(Fail2) $C; \Phi \Rightarrow \perp$, if there exists $N \in dom(\Phi)$ such that $\Phi(N) = \emptyset$

Simple example

Example - both success

$p(x, z) =^? q(f(b), f(x))$ and $R = \{(b, c'), (c', c), (p, q)\}$

Pre Unification

$P = \{p \simeq^? q\}$
$C = \{\}$
$\sigma = \{\}$
$\Rightarrow^{\text{Dec}}$
$P = \{x \simeq^? f, z \simeq^? f\}$
$C = \{p \approx^? q\}$
$\sigma = \{\}$
$\Rightarrow^{\text{VE}}$

Simple example cont.

$P = \{N_1(N_2) \simeq^? f(b), z \simeq^? f(N_1(N_2))\}$
$C = \{p \approx^? q\}$
$\sigma = \{x \mapsto N_1(N_2)\}$
$\Rightarrow^{\text{Dec}^2}$
$p(x,z) =^? q(f(b), f(x)); R = \{(a,a'), (a',b), (b,c'), (c',c), (p,q)\}$
$P = \{z \simeq^? f\}$
$C = \{p \approx^? q, N_1 \approx^? f, N_2 \approx^? b\}$
$\sigma = \{x \mapsto N_1(N_2)\}$
$\Rightarrow^{\text{VE}}$
$P = \{N_3(N_4(N_5)) \simeq^? f(N_1(N_2))\}$
$C = \{p \approx^? q, N_1 \approx^? f, N_2 \approx^? b\}$
$\sigma = \{x \mapsto N_1(N_2), z \mapsto N_3(N_4(N_5))\}$
$\Rightarrow^{\text{Dec}^3}$

Simple example cont.

$$P = \{\}$$
$$C = \{p \approx^? q, N_1 \approx^? f, N_2 \approx^? b, N_3 \approx^? f, N_4 \approx^? N_1, N_5 \approx^? N_2\}$$
$$\sigma = \{x \mapsto N_1(N_2), z \mapsto N_3(N_4(N_5))\}$$

Constraint Simplification

$$C = \{p \approx^? q, N_1 \approx^? f, N_2 \approx^? b, N_3 \approx^? f, N_4 \approx^? N_1, N_5 \approx^? N_2\}$$
$$\Phi = \{\}$$
$$\Rightarrow^{\mathsf{FFS}}$$
$$C = \{N_1 \approx^? f, N_2 \approx^? b, N_3 \approx^? f, N_4 \approx^? N_1, N_5 \approx^? N_2\}$$
$$\Phi = \{\}$$
$$\Rightarrow^{\mathsf{NFS}^3}$$

Simple example cont.

$$R = \{(b, c'), (c', c), (p, q)\}$$

$$C = \{N_4 \approx^? N_1, N_5 \approx^? N_2\}$$
$$\Phi = \{N1 \mapsto \{f\}, N2 \mapsto \{b, c'\}, N3 \mapsto \{f\}\}$$
$$\Rightarrow^{\text{NN2}}$$
$$C = \{N_1 \approx^? N_4, N_5 \approx^? N_2\}$$
$$\Phi = \{N1 \mapsto \{f\}, N2 \mapsto \{b, c'\}, N3 \mapsto \{f\}\}$$
$$\Rightarrow^{\text{NN1}}$$
$$C = \{N_5 \approx^? N_2\}$$
$$\Phi = \{N1 \mapsto \{f\}, N2 \mapsto \{b, c'\}, N3 \mapsto \{f\}, N4 \mapsto \{f\}\}$$
$$\Rightarrow^{\text{NN2,NN1}}$$

Simple example cont.

$C = \{\}$
$\Phi_1 = \{N1 \mapsto \{f\}, N2 \mapsto \{b\}, N3 \mapsto \{f\}, N4 \mapsto \{f\}, N5 \mapsto \{b, c'\}\}$
$\Phi_2 = \{N1 \mapsto \{f\}, N2 \mapsto \{c'\}, N3 \mapsto \{f\}, N4 \mapsto \{f\}, N5 \mapsto \{b, c', c\}\}$

Solution

$\Phi_1 = \{N1 \mapsto \{f\}, N2 \mapsto \{b\}, N3 \mapsto \{f\}, N4 \mapsto \{f\}, N5 \mapsto \{b, c'\}\}$
$\Phi_2 = \{N1 \mapsto \{f\}, N2 \mapsto \{c'\}, N3 \mapsto \{f\}, N4 \mapsto \{f\}, N5 \mapsto \{b, c', c\}\}$
$\sigma = \{x \mapsto N_1(N_2), z \mapsto N_3(N_4(N_5))\}$

Other examples

Simple examples where the pre-unification fails:

(Occ) $p(x) =^? q(f(x))$

(Cla) $p(a, b) =^? q(f(x))$

Example - PU success - CS fail

$p(x, y, x) =^? q(f(a), g(b), y); R = \{(b, c), (c, d), (f, g), (p, q)\}$

$P = \{p \simeq^? q\}$

Test 4 in Test.java

Pre - ok, CS - fail: - des ging bei mir, i hätt 5 Schritte und 2 Branches, die aber failen-

$p(x, y, x) =^? q(f(a), g(b), y); R = \{(b, c), (c, d), (f, g), (p, q)\}$

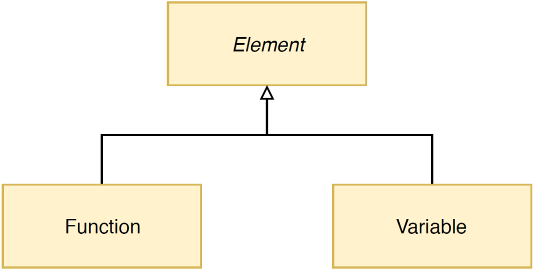
Solution:

System Model

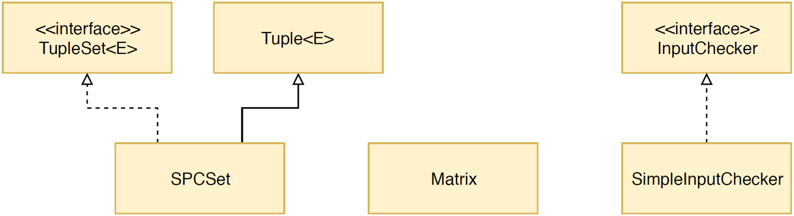
Project consists of 4 packages:

- elements : deals with elements
- tool : offers important tools (e.g. read input)
- unificationProblem : treats the unification problem
- userInterfaces : allow user interfaces

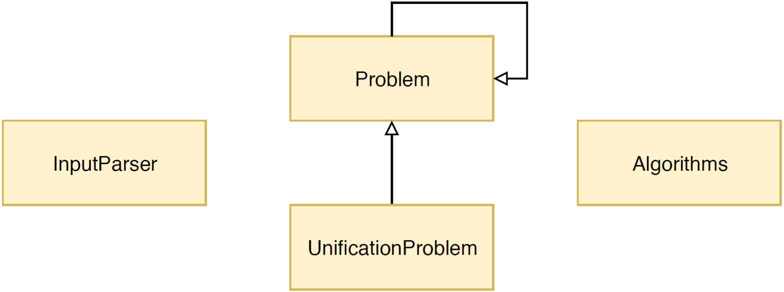
Package elements



Package tool

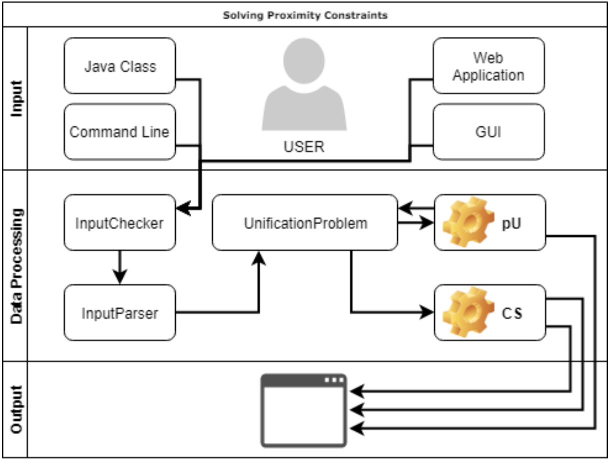


Package `unificationProblem`



Package `userInterfaces`

Workflow


Solving Proximity Constraints
Input
Java Class
Command Line
USER
Web Application
GUI
Data Processing
InputChecker
UnificationProblem
pU
InputParser
CS
Output