

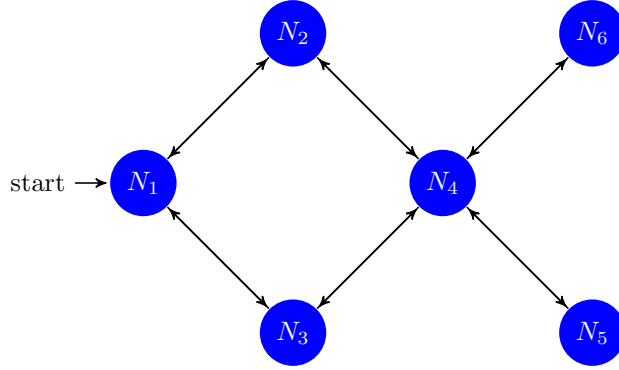
# 1 Write down the settings of the game

source: [http://en.wikipedia.org/wiki/Adjacency\\_matrix](http://en.wikipedia.org/wiki/Adjacency_matrix)

We model the network through an undirected Graph  $G = \langle V, E \rangle$  where  $|V|$  denotes the number of resources in the network and  $|E|$  the number of connections. We can convert this to a adjacent matrix where we can represent which vertices of the graph are neighbours of other vertices.

For our graph we have an  $|V| \times |V|$  matrix with on every entry  $a_{ij}$  a 1 as value if there is a connection between node  $V_i$  and  $V_j$  and with zeros its diagonal. Because our graph is undirected we have a symmetric matrix.

*"If  $A$  is the adjacency matrix of the directed or undirected graph  $G$ , then the matrix  $A^n$  (i.e., the matrix product of  $n$  copies of  $A$ ) has an interesting interpretation: the entry in row  $i$  and column  $j$  gives the number of (directed or undirected) walks of length  $n$  from vertex  $i$  to vertex  $j$ . If  $n$  is the smallest non-negative integer, such that for all  $i, j$ , the  $(i, j)$ -entry of  $A^n > 0$ , then  $n$  is the distance between vertex  $i$  and vertex  $j$ ."* [Wikipedia]



The adjacent matrix becomes this matrix  $[A]$ :

$$\begin{matrix}
 & N_1 & N_2 & N_3 & N_4 & N_5 & N_6 \\
 \begin{matrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}
 \end{matrix}$$

Matrix  $A \times A = A^2$  becomes the matrix with the number of paths with 2 steps from  $N_i$  to  $N_j$ : We denote this matrix as matrix  $[B]$

$$\begin{matrix}
& N_1 & N_2 & N_3 & N_4 & N_5 & N_6 \\
N_1 & \left( \begin{matrix} 2 & 0 & 0 & 2 & 0 & 0 \end{matrix} \right) \\
N_2 & \left( \begin{matrix} 0 & 2 & 2 & 0 & 1 & 1 \end{matrix} \right) \\
N_3 & \left( \begin{matrix} 0 & 2 & 2 & 0 & 1 & 1 \end{matrix} \right) \\
N_4 & \left( \begin{matrix} 2 & 0 & 0 & 4 & 0 & 0 \end{matrix} \right) \\
N_5 & \left( \begin{matrix} 0 & 1 & 1 & 0 & 1 & 1 \end{matrix} \right) \\
N_6 & \left( \begin{matrix} 0 & 1 & 1 & 0 & 1 & 1 \end{matrix} \right)
\end{matrix}$$

Matrix  $A^2 \times A = A^3$  becomes the matrix with the number of paths with 3 steps from  $N_i$  to  $N_j$ : We denote this matrix as matrix  $[C]$

$$\begin{matrix}
& N_1 & N_2 & N_3 & N_4 & N_5 & N_6 \\
N_1 & \left( \begin{matrix} 0 & 4 & 4 & 0 & 2 & 2 \end{matrix} \right) \\
N_2 & \left( \begin{matrix} 4 & 0 & 0 & 6 & 0 & 0 \end{matrix} \right) \\
N_3 & \left( \begin{matrix} 4 & 0 & 0 & 6 & 0 & 0 \end{matrix} \right) \\
N_4 & \left( \begin{matrix} 0 & 6 & 6 & 0 & 4 & 4 \end{matrix} \right) \\
N_5 & \left( \begin{matrix} 2 & 0 & 0 & 4 & 0 & 0 \end{matrix} \right) \\
N_6 & \left( \begin{matrix} 2 & 0 & 0 & 4 & 0 & 0 \end{matrix} \right)
\end{matrix}$$

So for  $A^N$  every  $a_{ij}$  entry gives the number of paths with N steps from  $N_i$  to  $N_j$ .

With this knowledge we can calculate in how many steps a node is infected.  $A$  calculates which nodes are infected after 1 step,  $A^N$  calculates which nodes are infected in N steps.. So if we want to know how many nodes are infected after 3 steps we have to add every matrix ( $A + A^2 + A^3$ ) and see which entry is a non zero entry.

What do we need for an algorithm

Graph network  $G = \langle V, E \rangle$

Graph matrix  $[A]$  which is  $|V| \times |V|$

Attack vector  $[X]$  which is  $1 \times |V|$

cummulative matrix  $[M]$  which is  $|V| \times |V|$

state matrix  $[T]$  which is  $|V| \times |V|$

Reset vector  $[R]$

duration  $d$

time  $n$

rate  $\delta_0$  of defender and  $\delta_1$  of attacker

Initialisation algorithm:

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initialisatie
d=0

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```

A=basismatrix
M=A^{0}
n=0
\delta_{0}
\delta_{1}
X
R
controller = defender

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```

Algorithm
n:= n + 1;
Check who is in control? ( through modulo )
if ( defender & controller=defender)
d:= d + 1;

if ( defender & controller=attacker )
G = X \times R (flippen ten voordele van defender)
d = 0
controller = defender

if ( attacker & controller=defender )
controller=attacker
..

if ( attacker & controller=attacker )
d:= d + 1
M = M x A
T = T + M
G = X x T

```