Zero inflated Poisson distribution

For Master 2 Math SV

January 2024

1. The data

We study the abundance of fish species at $n = 89$ sites in the Barents Sea (Fossheim, Nilssen, and Aschan (2006)). The data are available in the file BarentsFish.csv where the first 4 columns correspond to four environmental covariates covariates (latitude, longitude, depth, temperature) and the next 30 columns are the abundances of 30 species.

```r
abundance <- read.csv("BarentsFish.csv", sep=";")
View(abundance)
```

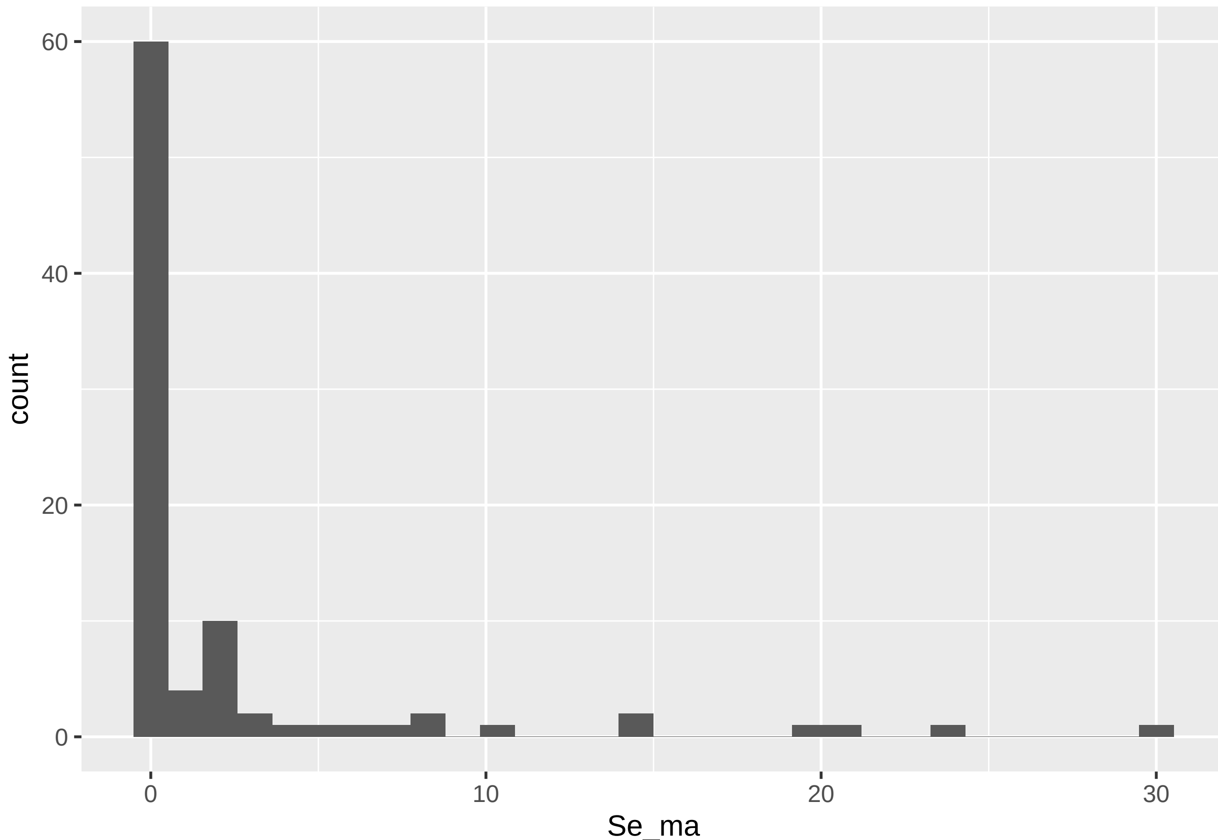
In the following, we will consider only one fish species, for example the 20th ('Se_ma = Sebastes marinus = Golden redfish) and we will note $1 \leq i \leq n$.

$$Y_i = \text{ abundance of golden redfish in station i.}$$

1. Explore the data with standard tools (means, histograms…)

```r
library(ggplot2)
ggplot(abundance,aes(Se_ma))+geom_histogram()
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



Observe an over-representation of null values. We propose to modelize this over inflation of 0.

2. Zero-inflated Poisson model

We propose to consider the following Zero Inflation Poisson distribution (ZIP) Let Z_i be a latent variable such that

$$Z_i \sim_{i.i.d} \mathcal{B}ern(1-\pi)$$

Then

$$Y_i | Z_i \sim (1 - Z_i)\delta_{\{0\}} + Z_i \mathcal{P}(\mu_i) \tag{1}$$

where $\mathcal{P}$ is the Poisson distribution.

2. Write the marginal distribution of Y_i
3. Derive $\mathbb{E}[Y_i]$ and $P(Y_i = 0)$
4. Write the complete log likelihood $\log p_\theta(\mathbf{Y}, \mathbf{Z})$ of the model where $\theta = (\pi, \mu)$.

We propose to maximize likelihood with respect to the parameters using the EM algorithm

5. Write the corresponding E-step.
6. Write the corresponding M-step.
7. Suggest an initial value for the parameter θ.
8. Code the EM algorithm.

3. ZIP with covariates

We now consider a model similar to ZIP but taking into account the environmental covariates. We note x_i the vector comprising these covariates for the site i:

$$x_i = [1, \text{latitude}_i, \text{longitude}_i, \text{depth}_i, \text{temperature}_i].$$

We therefore pose : $(Z_i)_{1,\leq i \leq n}$ independent,$(Y_i|Z_i)_{1,\leq i \leq n}$ independent and

$$\begin{array}{rclll} Z_i & \sim & \mathcal{B}\text{ern}(\pi_i) & \text{with} & \log\left(\frac{\pi_i}{1-\pi_i}\right) & = & x_i^T \alpha \\ Y_i|Z_i & \sim & (1-Z_i)\delta_{\{0\}} + Z_i \mathcal{P}(\mu_i) & \text{with} & \log \mu_i & = & x_i^T \beta \end{array} \tag{2}$$

The vectors α and β contain the regression coefficients to predict absence and abon- dance conditional on the presence of the species at each site.

9. Write the full log likelihood $p_\theta(\mathbf{Y}, \mathbf{Z})$ of this new model as a function of the parameter $\theta = (\alpha, \beta)$.
10. Write the E-step.
11. Write the M-step.
12. Propose an initial value for the parameter θ.
13. Code the EM algorithm

References

Fossheim, Maria, Einar M. Nilssen, and Michaela Aschan. 2006. "Fish Assemblages in the Barents Sea." *Marine Biology Research* 2 (4): 260–69. https://doi.org/10.1080/17451000600815698.