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Numerical solution of SIR model for transmission of tuberculosis by Runge-Kutta method

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Numerical solution of SIR model for transmission of tuberculosis by Runge-Kutta method

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Abstract. This paper discusses the numerical solution of the SIR model on the spread of Tuberculosis by the Runge-Kutta method. The data used is secondary data of Tuberculosis cases in Makassar city from Health Department of South Sulawesi Province in 2015. The result obtained is a general solution of SIR model of Tuberculosis transmission by fourth-order Runge-Kutta method. The results of model analysis and simulation using data on the number of tuberculosis cases in Makassar showed that tuberculosis cases increased for some time, then decreased. The results have also shown that the fourth-order Runge-Kutta method can be used to predict the transmission and to be considered for the prevention of Tuberculosis in Makassar.

1. Introduction

The development and progress of the modern world today can’t be separated from mathematics. Almost all human activity is related to mathematics [1]. Mathematical model is one tool that can help facilitate solving problems in real life. The example is the application to determine the spread of infectious disease model in a particular region or region. To know the spread of infectious diseases, there are known models of disease spread, both models are deterministic, as well as models that are stochastic. The models have their own characteristics, based on the type and shape of the spread of infectious diseases observed, for example the SIR Model of Tuberculosis [2,3].

The World Health Organization states that one-third of the world's population has been infected by M. tuberculosis, 9 million new Tuberculosis patients and 3 millions tuberculosis deaths worldwide, 95% of Tuberculosis cases and 98% death of Tuberculosis [4]. Particularly in Kota Makassar, based on data obtained from the Division of Disease Prevention and Environmental Health of Makassar City Health Office, the findings of new TB, TB (+) patients in 2013 were 72.44% (1,811 people were found to be 2,500), and 2014, increased by 78.12%, so the city of Makassar including the city of concern for tuberculosis [2].

The numerical method found only solutions approaching or nearing true solutions so that numerical solutions are also called approximation [5]. In solving numerical differential equations we can use the one-step method, one of which is the Runge-Kutta method [6].

One of the infectious diseases whose models can be resolved numerically is Tuberculosis. Some researchers have examined these issues as follows: [7] discusses the use of Runge-Kutta method in finding solutions of initial value problems; [8] discusses the numerical solution of the Predator-Prey model by comparing the fourth-order Runge-Kutta method and the Runge-Kutta Gill method, the result of which is the fourth-order Runge-Kutta method will yield more precise accuracy in computation and rounding than Runge-Kutta another order; [9,10] discusses the mathematical modeling of SIR and SEIR on the transmission of dengue fever and tuberculosis by using Lyapunov function method to analyze the models. [2] discusses the process of SIR modeling on the spread of Tuberculosis disease in Makassar starting from determining the point of disease-free equilibrium, the point of equilibrium endemic and its numerical simulation but not examining further the numerical solution of the model already obtained. This paper discusses the numerical solution of the SIR model on the spread of tuberculosis from [2] with the fourth-order Runge-Kutta.

2. SIR Model for Transmission of Tuberculosis

The SIR model for tuberculosis spread [2] is presented in the form of a transfer chart as Figure 1.

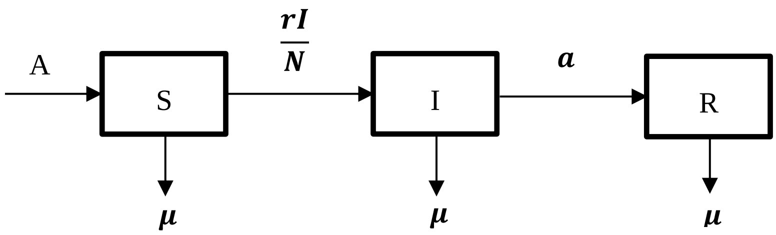

Figure 1. Diagram of SIR Model for Tuberculosis

where : S = Number of vulnerable individuals in the population at time t
I = Number of infected individuals in the population at time t
R = Number of individuals who recover in the population at time t
r = The rate of transmission of the disease from susceptible becomes infectious $(0 \le r \le 1)$
a = The rate of recovery from infectious to recovered
A = Initial value
$\boldsymbol{\mu}$ = Death rate

Based on the assumptions and the relationship between the variables and the parameters in Figure 1, we get the SIR model of tuberculosis dispersion as in (1) - (3) follows:

$$\frac{dS}{dt} = A - \mu S - \frac{rI}{N}S \tag{1}$$

$$\frac{dI}{dt} = \frac{rI}{N}S - (\mu + a)I \tag{2}$$

$$\frac{dR}{dt} = aI - \mu R \tag{3}$$

Where: N = S + I + R

3. Fourth Order Runge-Kutta Method

This method was discovered by German mathematician Carl Runge (1856-1927) and Wilhelm Kutta (1867-1944). The rationale behind this method is to defend almost Taylor, but in the settlement of GDP by Taylor method is not practical because the method requires derivative calculations [7]. The fourth-order Runge-Kutta method is the most rigorous method compared to the previous order Runge-Kutta method. Therefore, the fourth-order Runge-Kutta method is often used to solve a differential equation.

The fourth-order Runge-Kutta method has the form as follows equation (4):

$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{4}$$

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with

$$k_1 = hf(x_r, y_r)$$

$$k_2 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_1)$$

$$k_3 = hf(x_r + \frac{1}{2}h, y_r + \frac{1}{2}k_2)$$

$$k_4 = hf(x_r + h, y_r + k_3)$$

Source [8].

In addition to having a better accuracy of the solution than the previous order Runge-Kutta method, this method is also easy to program, stable, small cut errors and also small rounding errors.

4. Method

This research is an experimental research by conducting a study of theories about tuberculosis transmission model. The numerical solution of the SIR model is obtained using the fourth-order Runge-Kutta method. The model was then analyzed and simulated using secondary data on the number of tuberculosis cases from the Ministry of Health of South Sulawesi Province in 2015 [2]. The simulation was done using MATLAB software.

5. Results and Discussion

5.1. Numerical Solution of SIR Model forTuberculosis by fourth order Runge-Kutta Method

The SIR model on the spread of tuberculosis diseases present in equations (1) - (3) will be further identified into the form of ordinary differential equations as in equations (5) - (7) as follows:

$$\frac{dS}{dt} = f(t, S, I, R) = A - \mu S - \frac{rI}{N}S \tag{5}$$

$$\frac{dI}{dt} = g(t, S, I, R) = \frac{rI}{N}S - (\mu + a)I \tag{6}$$

$$\frac{dR}{dt} = i(t, S, I, R) = aI - \mu R \tag{7}$$

The equation (5) - (7) above is solved using the fourth-order Runge-Kutta method based on equation (4) by substitution, so that the following equation (8) - (10) is obtained:

$$S_{r+1} = S_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{8}$$

$$I_{r+1} = I_r + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)h \tag{9}$$

$$R_{r+1} = R_r + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h \tag{10}$$

with is the step of time

$$k_1 = A - \mu S_r - \frac{r}{N} I_r S_r$$

$$l_1 = \frac{rI_r}{N} S_r - (\mu + a) I_r$$

$$m_1 = aI_r - \mu R_r$$

$$k_2 = A - \mu\left(S_r + k_1 \frac{h}{2}\right) - \frac{r}{N}\left(I_r + l_1 \frac{h}{2}\right)\left(S_r + k_1 \frac{h}{2}\right)$$

$$l_2 = \frac{r}{N}\left(I_r + l_1 \frac{h}{2}\right)\left(S_r + k_1 \frac{h}{2}\right) - (\mu + a)\left(I_r + l_1 \frac{h}{2}\right)$$

$$m_2 = a\left(I_r + l_1 \frac{h}{2}\right) - \mu\left(R_r + m_1 \frac{h}{2}\right)$$

$$k_3 = A - \mu\left(S_r + k_2 \frac{h}{2}\right) - \frac{r}{N}\left(I_r + l_2 \frac{h}{2}\right)\left(S_r + k_2 \frac{h}{2}\right)$$

$$l_3 = \frac{r}{N}\left(I_r + l_2 \frac{h}{2}\right)\left(S_r + k_2 \frac{h}{2}\right) - (\mu + a)\left(I_r + l_2 \frac{h}{2}\right)$$

$$m_3 = a\left(I_r + l_2 \frac{h}{2}\right) - \mu\left(R_r + m_2 \frac{h}{2}\right)$$

$$k_4 = A - \mu(S_r + k_3 h) - \frac{r}{N}(I_r + l_3 h)(S_r + k_3 h)$$

$$l_4 = \frac{r}{N}(I_r + l_3 h)(S_r + k_3 h) - (\mu + a)(I_r + l_3 h)$$

$$m_4 = a(I_r + l_3 h) - \mu(R_r + m_3 h)$$

Equation (8) - (10) is a numerical solution of the SIR model on the spread of tuberculosis.

5.2. SIR Model Simulation Using MATLAB Software

Initial values and parameters used in the SIR model on the spread of Tuberculosis disease are presented in Table 1 below:

Table 1. Initial condition of SIR model for Tuberculosis

Initial Condition of SIR Model		
Variable/Parameter	**Value**	**Source**
$S(0) = S(2015)$	$\frac{1446093}{1449401}$	KKRI, 2015
$I(0) = I(2015)$	$\frac{1885}{1449401}$	KKRI, 2015
$R(0) = R(2015)$	$\frac{1423}{1449401}$	KKRI, 2015
r	$\frac{1}{2} = 0.5 / month$	KKRI, 2015

Initial Condition of SIR Model		
Variable/Parameter	**Value**	**Source**
a	$\frac{1}{9} = 0.111111 / month$	
μ	$\frac{1}{71.41 \times 12} = 0.001167 / bulan$	BPS, South Sulawesi, 2015 [11]
$N = A$	1449401	

The SIR model formulation on the spread of Tuberculosis in Makassar is as in equations (11) - (13).

$$\frac{dS}{dt} = 1449401 - 0.001167\,S - \frac{(0.5)}{1449401}IS \quad (11)$$

$$\frac{dI}{dt} = \frac{(0.5)}{1449401}IS - (0.001167 + 0.111111)I \quad (12)$$

$$\frac{dR}{dt} = 0.111111I - 0.001167\,R \quad (13)$$

The result of simulation equation (8) - (10) by using time interval per month, time step, and initial condition in the form of parameter value and initial value as in Table 1 are shown in Figure 2, Figure 3, Figure 4 and Figure 5 below:

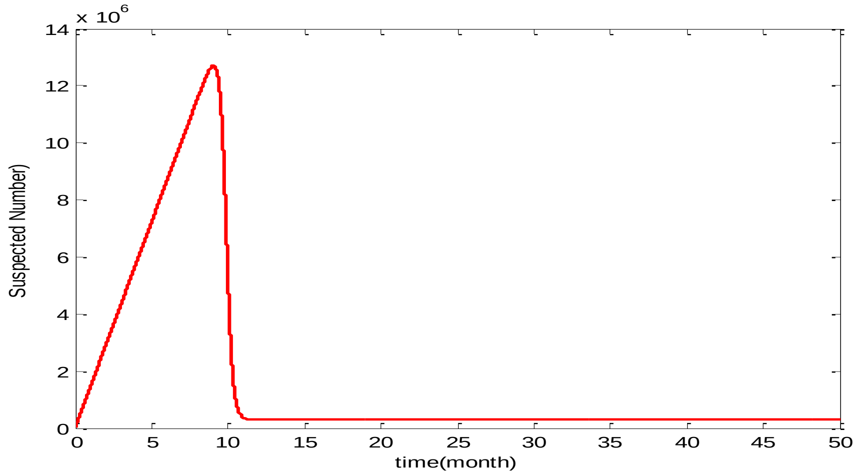

Figure 2. Suspected Population Model SIR for Tuberculosis

According figure 2, the number of susceptible population of 1446094 from 1449401 people, then increased until the ninth month reached the peak of the suspected population about 1,272 x 10^7 population. The increase is due to the amount of population or the initial value used is greater than the amount of population decline in the suspected class in the form of disease transmission rate from suspected to infectious and the rate of death of suspected class. After passing the peak point, there is a continuous decline and the line will converge to a point, indicating that in some time later, the suspected population will decrease.

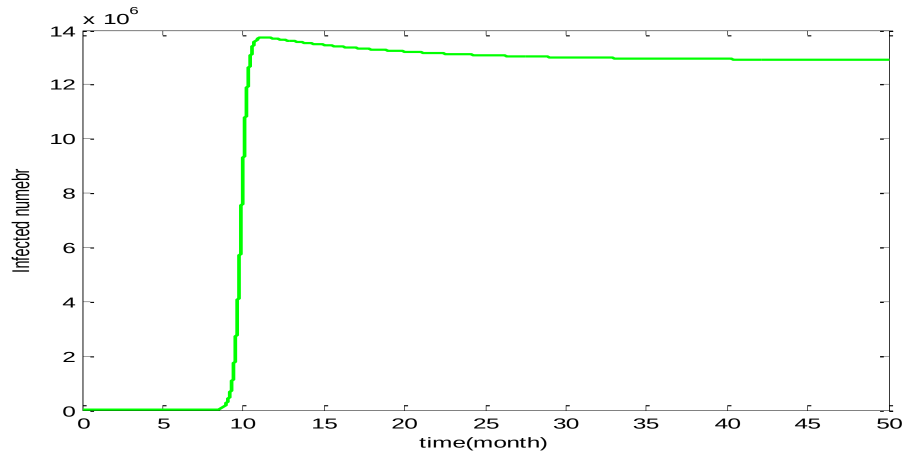

Figure 3. Total Population Infected Model SIR spread of Tuberculosis

According to the Figure 3, the infected population amounted to 1885 from 1449401 inhabitants, which then increased steadily until the eleventh month peaked around 1,375 x 107 populations. The increase is due to the rate of transmission of the disease from suspected to infectious greater than the rate of recovery from infected to recover, and infected class mortality rate. After the hemisphere, the population in the infected class begins to decline and then the line converges to a certain point until sometime later.

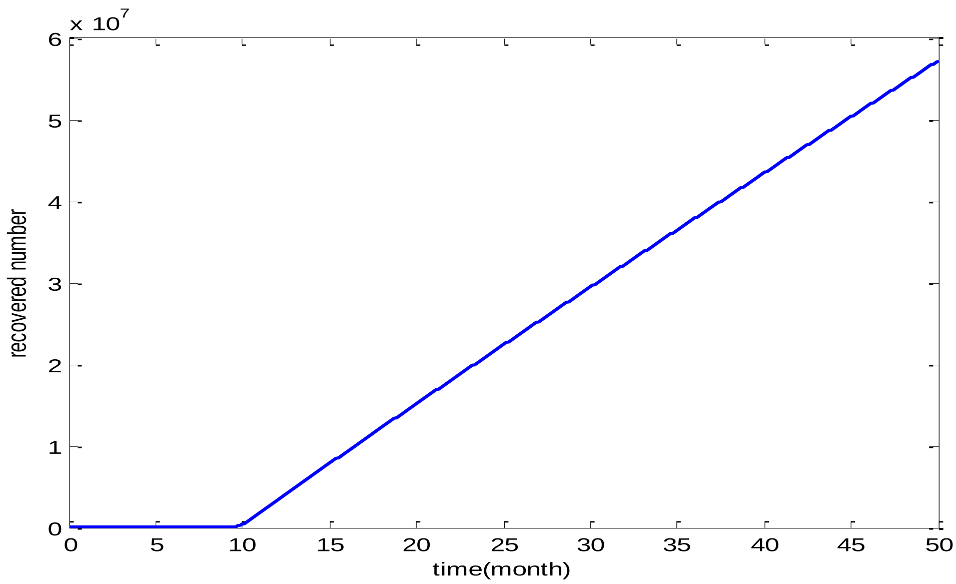

Figure 4. Total Population Recovered SIR Model of Tuberculosis Deployment

Based on Figure 4, Recovered population amounted to 1423 from 1449401 inhabitants, it is seen that the number of healthy population has increased over time. The increase is caused by the rate of recovery rate from infected to recover greater than the rate of death of the recovered class and the inability of the disease transmission rate from suspected to infected.

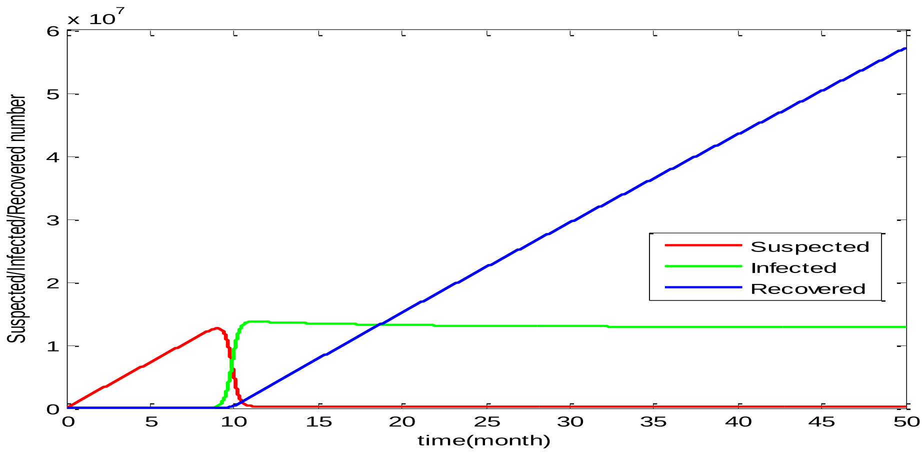

Figure 5. Total Population Suspected, Infected dan Recovered (SIR) Model of Tuberculosis

Based on Figure 5, The number of suspected increased to 9,022 <t <9.069 months later after reaching the peak then the population will decrease and close to zero, this is because some populations move from the suspected class to the infected class, the infected class increases until finally convergent the next time, this is because some populations move to recovered classes through several factors, one of which is healing, and in the recovered class, from the first month to the fiftieth month continues to increase, this explains that the rate of recovery over time has increased

5.3. Discussion
In [7] study only deals with the use of the Runge-Kutta method in finding solutions of initial value problems in ordinary differential equations. [8] discusses the comparative analysis of numerical solutions of the Predator-Prey model by comparing the fourth-order Runge-Kutta method and the Runge-Kutta Gill method. [9,10] discusses the mathematical modeling of SIR and SEIR on the transmission of Tuberculosis and dengue fever disease using Lyapunov function method. The method analyze both models can be used to determine Tuberculosis disease status in a region by using data simulation from certain region.

In [2] describes the SIR model on the spread of Tuberculosis with its assumptions, determining the equilibrium point of the model so as to obtain two equilibrium points, stability and basic reproduction number, and simulation using secondary data of Tuberculosis (+) disease case in Makassar. The simulation results explain that the transmission of Tuberculosis disease has a great influence on the model and explains that Tuberculosis disease in Makassar City decreases or even disappears if its basic reproduction number is less than one.

This paper describes numerical solutions and SIR model analysis using the fourth-order Runge-Kutta method. Then, establish the parameters and initial values used by using real data from the South Sulawesi Provincial Health Office. The model is then simulated to predict and see the trend of the number of tuberculosis cases in Makassar.

6. Conclusion
The numerical simulation results of the SIR model on the dispersion of Tuberculosis by the fourth-order Runge-Kutta method indicates that: a) the larger the time interval used, the clearer the movement of each class. There is an increase in susceptible, infected, and very long increase classes in the recovered class, but for the susceptible class after reaching its peak it will move down while the infected class and the recovered class will move steadily at a later time; b) Infected classes will experience an increase influenced by the rate of transmission of the disease from susceptible to infected. Infected classes will decrease influenced by the rate of recovery of the disease from infected to recovered; c) Tuberculosis cases in Makassar have an incubation period and a longer recovery

process than the average data obtained; and d) The fourth-order Runge-Kutta method can be used to find numerical solutions of the SIR model and predict the number of Tuberculosis cases in Makassar.

Acknowledgments
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References

[1] Belo Y J T 2016 *SIRv Modeling of Swine Flu (H1N1) with Vaccination in Constant Populations* (Thesis) Universitas Negeri Makassar
[2] Hasrina 2015 *SIR (Susceptible, Infectious And Recovered) Model for Spreading Tuberculosis Disease in Makassar* (Thesis) Universitas Negeri Makassar
[3] Tracy A 2008 *Modeling transmission dynamics of tuberculosis inclunding various latent periods* Spring Term
[4] Fardinah 2009 *Solution of Ordinary Differential Equations using Runge-Kutta Method of Old Order and Euler Method* (Thesis) Universitas Negeri Makassar
[5] Side S 2015 A Susceptible-Infected_Recovered Model and Simulation for Transmission of Tuberculosis *Advance Science Letters* **21**(2) 137-139.
[6] Mursalin 2006 *Numerical Solutions for Differential Equations using Ordinary Multi-Step Methods* (Thesis) Universitas Negeri Makassar
[7] Asmah S 2004 *Numerical Solution of Initial Value Problems by Runge-Kutta* (Thesis) Universitas Negeri Makassar
[8] Putri P P 2013 *Analysis of Numerical Solution of Model Predator-Prey with Runge-Kutta Method of Order Four and Gill* (Thesis) Universitas Jember.
[9] Side S, Sanusi W, Aidid M K and Sidjara S 2016 Global Stability of SIR and SEIR Model for Tuberculosis Disease Transmission with Lyapunov Function Method *Asian Journal of Applied Sciences* **9**(3) 87-96.
[10] Syafruddin S and Noorani M S M 2013 Lyapunov function of SIR and SEIR model for transmission of dengue fever disease *International Journal of Simulation and Process Modelling* **8** 177-184.
[11] BPS South Sulawesi 2016 *South Sulawesi Province in Number* (Makassar: BPS)