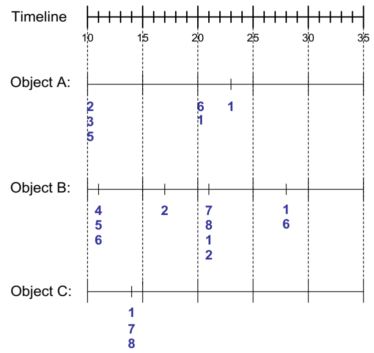
# Data Mining
# Association Rules: Advanced Concepts and Algorithms

Thanks to [Tan,Steinbach, Kumar]

## Sequence Data

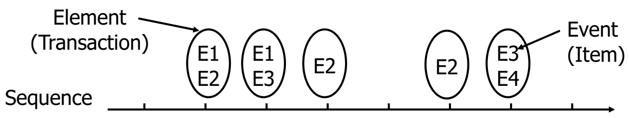
**Sequence Database:**

| Object | Timestamp | Events |
|---|---|---|
| A | 10 | 2, 3, 5 |
| A | 20 | 6, 1 |
| A | 23 | 1 |
| B | 11 | 4, 5, 6 |
| B | 17 | 2 |
| B | 21 | 7, 8, 1, 2 |
| B | 28 | 1, 6 |
| C | 14 | 1, 8, 7 |




## Examples of Sequence Data

| Sequence Database | Sequence | Element (Transaction) | Event (Item) |
|---|---|---|---|
| Customer | Purchase history of a given customer | A set of items bought by a customer at time t | Books, diary products, CDs, etc |
| Web Data | Browsing activity of a particular Web visitor | A collection of files viewed by a Web visitor after a single mouse click | Home page, index page, contact info, etc |
| Event data | History of events generated by a given sensor | Events triggered by a sensor at time t | Types of alarms generated by sensors |
| Genome sequences | DNA sequence of a particular species | An element of the DNA sequence | Bases A,T,G,C |




## Formal Definition of a Sequence

- A sequence is an ordered list of elements (transactions)

  $s = < e_1 \, e_2 \, e_3 \ldots >$

  - Each element contains a collection of events (items)

    $e_i = \{i_1, i_2, \ldots, i_k\}$

  - Each element is attributed to a specific time or location
- Length of a sequence, |s|, is given by the number of elements of the sequence
- A k-sequence is a sequence that contains k events (items)

## Examples of Sequence

- Web sequence:

  < {Homepage} {Electronics} {Digital Cameras} {Canon Digital Camera} {Shopping Cart} {Order Confirmation} {Return to Shopping} >

- Sequence of initiating events causing the nuclear accident at 3-mile Island:
  (http://stellar-one.com/nuclear/staff_reports/summary_SOE_the_initiating_event.htm)

  < {clogged resin} {outlet valve closure} {loss of feedwater} {condenser polisher outlet valve shut} {booster pumps trip} {main waterpump trips} {main turbine trips} {reactor pressure increases}>

- Sequence of books checked out at a library:

  <{Fellowship of the Ring} {The Two Towers} {Return of the King}>

## Formal Definition of a Subsequence

- A sequence $<a_1\, a_2 \ldots a_n>$ is contained in another sequence $<b_1\, b_2 \ldots b_m>$ ($m \geq n$) if there exist integers $i_1 < i_2 < \ldots < i_n$ such that $a_1 \subseteq b_{i1}$ , $a_2 \subseteq b_{i2}$, …, $a_n \subseteq b_{in}$

| Data sequence | Subsequence | Contain? |
|---|---|---|
| < {2,4} {3,5,6} {8} > | < {2} {3,5} > | Yes |
| < {1,2} {3,4} > | < {1} {2} > | No |
| < {2,4} {2,4} {2,5} > | < {2} {4} > | Yes |

- The support of a subsequence w is defined as the fraction of data sequences that contain w
- A *sequential pattern* is a frequent subsequence (i.e., a subsequence whose support is ≥ *minsup*)

## Sequential Pattern Mining: Definition

- Given:
  – a database of sequences
  – a user-specified minimum support threshold, *minsup*

- Task:
  – Find all subsequences with support ≥ *minsup*

## Sequential Pattern Mining: Challenge

- Given a sequence: <{a b} {c d e} {f} {g h i}>
  – Examples of subsequences:

    <{a} {c d} {f} {g} >, < {c d e} >, < {b} {g} >, etc.

- How many k-subsequences can be extracted from a given n-sequence?

<{a b} {c d e} {f} {g h i}> n = 9

k=4: $\underbrace{\text{Y \_ \_ Y Y \_ \_ \_ Y}}$

<{a} {d e} {i}>

Answer : $\binom{n}{k} = \binom{9}{4} = 126$

## Sequential Pattern Mining: Example

| Object | Timestamp | Events |
|---|---|---|
| A | 1 | 1,2,4 |
| A | 2 | 2,3 |
| A | 3 | 5 |
| B | 1 | 1,2 |
| B | 2 | 2,3,4 |
| C | 1 | 1, 2 |
| C | 2 | 2,3,4 |
| C | 3 | 2,4,5 |
| D | 1 | 2 |
| D | 2 | 3, 4 |
| D | 3 | 4, 5 |
| E | 1 | 1, 3 |
| E | 2 | 2, 4, 5 |

*Minsup* = 50%

**Examples of Frequent Subsequences:**

| | |
|---|---|
| < {1,2} > | s=60% |
| < {2,3} > | s=60% |
| < {2,4}> | s=80% |
| < {3} {5}> | s=80% |
| < {1} {2} > | s=80% |
| < {2} {2} > | s=60% |
| < {1} {2,3} > | s=60% |
| < {2} {2,3} > | s=60% |
| < {1,2} {2,3} > | s=60% |

## Extracting Sequential Patterns

- Given n events: $i_1, i_2, i_3, \ldots, i_n$
- Candidate 1-subsequences:

  $\langle\{i_1\}\rangle, \langle\{i_2\}\rangle, \langle\{i_3\}\rangle, \ldots, \langle\{i_n\}\rangle$
- Candidate 2-subsequences:

  $\langle\{i_1, i_2\}\rangle, \langle\{i_1, i_3\}\rangle, \ldots, \langle\{i_1\}\{i_1\}\rangle, \langle\{i_1\}\{i_2\}\rangle, \ldots, \langle\{i_n\}\{i_n\}\rangle$
- Candidate 3-subsequences:

  $\langle\{i_1, i_2, i_3\}\rangle, \langle\{i_1, i_2, i_4\}\rangle, \ldots, \langle\{i_1, i_2\}\{i_1\}\rangle, \langle\{i_1, i_2\}\{i_2\}\rangle, \ldots,$
  $\langle\{i_1\}\{i_1, i_2\}\rangle, \langle\{i_1\}\{i_1, i_3\}\rangle, \ldots, \langle\{i_1\}\{i_1\}\{i_1\}\rangle, \langle\{i_1\}\{i_1\}\{i_2\}\rangle, \ldots$

## Generalized Sequential Pattern (GSP)

- **Step 1**:
  - Make the first pass over the sequence database D to yield all the 1-element frequent sequences
- **Step 2**:

  Repeat until no new frequent sequences are found
  - **Candidate Generation**:
    - Merge pairs of frequent subsequences found in the (k-1)*th* pass to generate candidate sequences that contain k items
  - **Candidate Pruning**:
    - Prune candidate *k*-sequences that contain infrequent *(k-1)*-subsequences
  - **Support Counting**:
    - Make a new pass over the sequence database D to find the support for these candidate sequences
  - **Candidate Elimination**:
    - Eliminate candidate *k*-sequences whose actual support is less than *minsup*

## Candidate Generation

- Base case (k=2):
  - Merging two frequent 1-sequences $\langle\{i_1\}\rangle$ and $\langle\{i_2\}\rangle$ will produce two candidate 2-sequences: $\langle\{i_1\}\{i_2\}\rangle$ and $\langle\{i_1\, i_2\}\rangle$
- General case (k>2):
  - A frequent *(k-1)*-sequence $w_1$ is merged with another frequent *(k-1)*-sequence $w_2$ to produce a candidate *k*-sequence if the subsequence obtained by removing the first event in $w_1$ is the same as the subsequence obtained by removing the last event in $w_2$
    - The resulting candidate after merging is given by the sequence $w_1$ extended with the last event of $w_2$.
      - If the last two events in $w_2$ belong to the same element, then the last event in $w_2$ becomes part of the last element in $w_1$
      - Otherwise, the last event in $w_2$ becomes a separate element appended to the end of $w_1$

## Candidate Generation Examples

- Merging the sequences
  $w_1$=<{1} {2 3} {4}> and $w_2$ =<{2 3} {4 5}>
  will produce the candidate sequence < {1} {2 3} {4 5}> because the last two events in $w_2$ (4 and 5) belong to the same element

- Merging the sequences
  $w_1$=<{1} {2 3} {4}> and $w_2$ =<{2 3} {4} {5}>
  will produce the candidate sequence < {1} {2 3} {4} {5}> because the last two events in $w_2$ (4 and 5) do not belong to the same element

- We do not have to merge the sequences
  $w_1$ =<{1} {2 6} {4}> and $w_2$ =<{1} {2} {4 5}>
  to produce the candidate < {1} {2 6} {4 5}> because if the latter is a viable candidate, then it can be obtained by merging $w_1$ with <{2 6} {4 5}>

## GSP Example

Frequent 3-sequences

< {1} {2} {3} >
< {1} {2 5} >
< {1} {5} {3} >
< {2} {3} {4} >
< {2 5} {3} >
< {3} {4} {5} >
< {5} {3 4} >

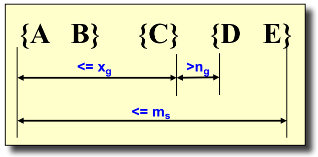
Candidate Generation

< {1} {2} {3} {4} >
< {1} {2 5} {3} >
< {1} {5} {3 4} >
< {2} {3} {4} {5} >
< {2 5} {3 4} >

Candidate Pruning

< {1} {2 5} {3} >

## Timing Constraints (I)




**$x_g$: max-gap**

**$n_g$: min-gap**

**$m_s$: maximum span**

**$x_g$ = 2, $n_g$ = 0, $m_s$= 4**

| Data sequence | Subsequence | Contain? |
|---|---|---|
| < {2,4} {3,5,6} {4,7} {4,5} {8} > | < {6} {5} > | Yes |
| < {1} {2} {3} {4} {5}> | < {1} {4} > | No |
| < {1} {2,3} {3,4} {4,5}> | < {2} {3} {5} > | Yes |
| < {1,2} {3} {2,3} {3,4} {2,4} {4,5}> | < {1,2} {5} > | No |

## Mining Sequential Patterns with Timing Constraints

- Approach 1:
  - Mine sequential patterns without timing constraints
  - Postprocess the discovered patterns

- Approach 2:
  - Modify GSP to directly prune candidates that violate timing constraints
  - Question:
    - Does Apriori principle still hold?

## Apriori Principle for Sequence Data

| Object | Timestamp | Events |
|---|---|---|
| A | 1 | 1,2,4 |
| A | 2 | 2,3 |
| A | 3 | 5 |
| B | 1 | 1,2 |
| B | 2 | 2,3,4 |
| C | 1 | 1, 2 |
| C | 2 | 2,3,4 |
| C | 3 | 2,4,5 |
| D | 1 | 2 |
| D | 2 | 3, 4 |
| D | 3 | 4, 5 |
| E | 1 | 1, 3 |
| E | 2 | 2, 4, 5 |

Suppose:

$x_g$ = 1 (max-gap)

$n_g$ = 0 (min-gap)

$m_s$ = 5 (maximum span)

*minsup* = 60%

<{2} {5}> support = 40%

but

<{2} {3} {5}> support = 60%

**Problem exists because of max-gap constraint**

**No such problem if max-gap is infinite**

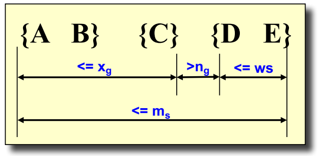
## Contiguous Subsequences

- s is a contiguous subsequence of
  w = $<e_1>< e_2>\ldots< e_k>$
  if any of the following conditions hold:
  1. s is obtained from w by deleting an item from either $e_1$ or $e_k$
  2. s is obtained from w by deleting an item from any element $e_i$ that contains more than 2 items
  3. s is a contiguous subsequence of s' and s' is a contiguous subsequence of w (recursive definition)
- Examples: s = < {1} {2} >
  – is a contiguous subsequence of
    < {1} {2 3}>, < {1 2} {2} {3}>, and < {3 4} {1 2} {2 3} {4} >
  – is not a contiguous subsequence of
    < {1} {3} {2}> and < {2} {1} {3} {2}>

## Modified Candidate Pruning Step

- Without maxgap constraint:
  – A candidate k-sequence is pruned if at least one of its (k-1)-subsequences is infrequent
- With maxgap constraint:
  – A candidate *k*-sequence is pruned if at least one of its **contiguous** (*k-1*)-subsequences is infrequent

## Timing Constraints (II)




**$x_g$: max-gap**

**$n_g$: min-gap**

**ws: window size**

**$m_s$: maximum span**

**$x_g$ = 2, $n_g$ = 0, ws = 1, $m_s$= 5**

| Data sequence | Subsequence | Contain? |
|---|---|---|
| < {2,4} {3,5,6} {4,7} {4,6} {8} > | < {3} {5} > | No |
| < {1} {2} {3} {4} {5}> | < {1,2} {3} > | Yes |
| < {1,2} {2,3} {3,4} {4,5}> | < {1,2} {3,4} > | Yes |

## Modified Support Counting Step

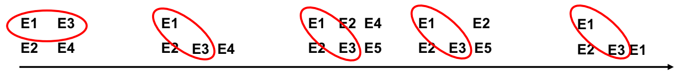
- Given a candidate pattern: <{a, c}>
  - Any data sequences that contain

    <… {a c} … >,
    <… {a} … {c}…> ( where time({c}) – time({a}) ≤ ws)
    <…{c} … {a} …> (where time({a}) – time({c}) ≤ ws)

    will contribute to the support count of candidate pattern

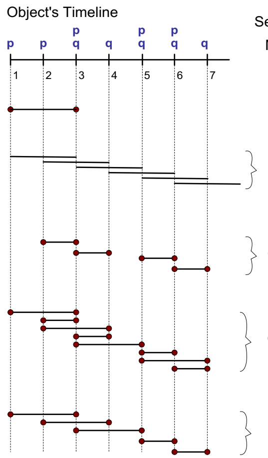
## Other Formulation

- In some domains, we may have only one very long time series
  - Example:
    - monitoring network traffic events for attacks
    - monitoring telecommunication alarm signals
- Goal is to find frequent sequences of events in the time series
  - This problem is also known as frequent episode mining




Pattern: <E1> <E3>

## General Support Counting Schemes




Sequence: (p) (q)

| Method | Support Count |
|---|---|
| COBJ | 1 |
| CWIN | 6 |
| CMINWIN | 4 |
| CDIST_O | 8 |
| CDIST | 5 |

Assume:
$x_g$ = 2 (max-gap)
$n_g$ = 0 (min-gap)
ws = 0 (window size)
$m_s$ = 2 (maximum span)