Chapter 7

7-1

解:

规定垂直纸面向外为正。

经分析,CA段与BD段对O点磁场贡献为0,设导线线电阻率为 λ ,得:

$$B_o = rac{\mu_0}{4\pi} rac{i_1 l_1}{R^2} - rac{\mu_0}{4\pi} rac{i_2 l_2}{R^2} \ \lambda l_1 i_1 = \lambda l_2 i_2$$

解得:

$$B_o = 0$$

7-5

解:

球面电势为U,得:

$$U = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$
$$\sigma = \frac{Q}{4\pi R^2}$$

取 θ 处 $d\theta$ 的圆环,由毕奥萨伐尔定律:

$$dB_z = rac{\mu_0}{4\pi} rac{\sigma R d heta \cdot \omega R \sin heta \cdot 2\pi R \sin heta}{R^2} \sin heta$$

解得:

$$dB_z = rac{1}{2}\mu_0\epsilon_0\omega U\sin^3\theta d\theta$$

 $\Rightarrow B_z = rac{1}{2c_0^2}\omega U\int_0^\pi\sin^3\theta d\theta$

解得:

$$B_z=rac{2}{3c_0^2}\omega U$$

其中 c_0 表示真空中的光速。

7-6

解:

在半球面的底部加上一圆平面组成闭合曲面S,设半球面为 S_1 ,圆平面为 S_2 。由 $abla \cdot oldsymbol{B} = 0$,有:

$$\oint \int_S m{B} \cdot dm{S} = 0$$

即:

$$\iint_{S_1} oldsymbol{B} \cdot doldsymbol{S} + \iint_{S_2} oldsymbol{B} \cdot doldsymbol{S} = 0$$

故:

$$egin{aligned} \Phi_1 &= - \iint_{S_2} m{B} \cdot dm{S} \ &= -\pi R^2 c \end{aligned}$$

7-8

解:

由安培环路定理:

$$\oint_L m{B} \cdot dm{l} = \mu_0 N I$$

得:

$$B = rac{\mu_0 NI}{2\pi r}$$

$$egin{align} \Phi = \iint_S oldsymbol{B} \cdot doldsymbol{S} &= \int_{R_1}^{R_2} rac{\mu_0 NI}{2\pi r} h dr \ &= rac{\mu_0 NIh}{2\pi r} \ln rac{R_2}{R_1} \end{aligned}$$

7-9

解:

先求解距离O点 $r(r \in [a,a+b])$ 处电流强度:

$$i=rac{\omega\lambda dr}{2\pi}$$

• (1) *r*处电流对O点磁感应强度贡献为:

$$dm{B_0} = rac{\mu_0}{2}rac{i}{r}(-m{k})$$

得:

$$egin{aligned} m{B_0} &= -rac{\omega\lambda\mu_0}{4\pi}m{k}\int_a^{a+b}rac{dr}{r}\ &= -rac{\omega\lambda\mu_0}{4\pi}\lnrac{a+b}{a}m{k} \end{aligned}$$

• (2) r处电流的磁矩贡献为:

$$dm{m}=\pi r^2i(-m{k})$$

得:

$$egin{aligned} m{m} &= -rac{\omega\lambda}{2}m{k}\int_a^{a+b}r^2dr \ &= -rac{1}{6}\omega\lambda[(a+b)^3-a^3]m{k} \end{aligned}$$

• (3) 若a >> b, 有:

$$oldsymbol{B_0} = -rac{\omega\lambda\mu_0 b}{4\pi a} oldsymbol{k} \ oldsymbol{m} = -rac{1}{2}\omega\lambda a^2 b oldsymbol{k}$$

7-10

解:

由安培环路:

$$B \cdot 2\pi \cdot 3r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{6\pi r}$$
,方向垂直纸面向外

电子所受磁场力:

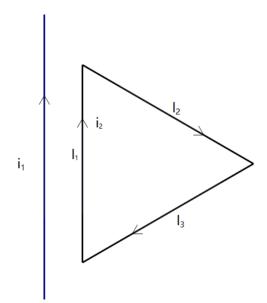
$$oldsymbol{F} = -eoldsymbol{v} imes oldsymbol{B}$$

即:

$$F=\mu_0I\Rightarrow B=rac{\mu_0Iev}{6\pi r},$$
方向垂直 OO_1 向左

7-11

解:



由对称性,合力垂直于 I_1 。

对 l_1 :

_ -

$$F_{1x}=-rac{\mu_0I_1I_2a}{2\pi d}$$

对 l_2 :

$$F_{2x} = \int_{l_2} I_2 dy rac{\mu_0 I_1}{2\pi (d+\sqrt{3}y)} = rac{\mu_0 I_1 I_2}{2\sqrt{3}\pi} \ln rac{2d+\sqrt{3}a}{2d}$$

同理,对 l_3 :

$$F_{3x}=rac{\mu_0I_1I_2}{2\sqrt{3}\pi}\lnrac{2d+\sqrt{3}a}{2d}$$

得:

$$F = F_{1x} + F_{2x} + F_{3x} = rac{\mu_0 I_1 I_2}{2\pi} (rac{2}{\sqrt{3}} \ln rac{2d + \sqrt{3}a}{2d} - rac{a}{d})$$

7-14

解:

由受力平衡得:

$$f = mg\sin\theta$$

由力矩平衡得:

$$fr = NBI \cdot 2rl \cdot \sin \theta$$

联立得:

$$I = \frac{mg}{2NBl}$$

7-15

解:

以柱的中轴线为z轴,向上为正方向建立柱坐标系。 由安培环路定理,得:

•
$$r \in [0, R]$$

$$B\cdot 2\pi r = \mu_0rac{Ir^2}{R^2} \Rightarrow oldsymbol{B} = rac{\mu_0}{2\pi}rac{I}{R^2}roldsymbol{e}_{oldsymbol{ heta}}$$

•
$$r \in (R, \infty)$$

$$B\cdot 2\pi r=\mu_0 I\Rightarrow oldsymbol{B}=rac{\mu_0}{2\pi}rac{I}{r}oldsymbol{e}_{oldsymbol{ heta}}$$

综上:

$$m{B} = egin{cases} rac{\mu_0}{2\pi}rac{I}{R^2}rm{e_{m{ heta}}}, & r \in [0,R] \ rac{\mu_0}{2\pi}rac{I}{r}m{e_{m{ heta}}}, & r \in (R,\infty) \end{cases}$$

7-17

由力矩平衡:

$$2\rho SLgL\sin\alpha = BIL^2\cos\alpha$$

联立得:

$$B = \frac{2\rho Sg \tan \alpha}{I} = 9.3 \times 10^{-3} \text{ T}$$