Chapter 2

说明:

- 1. \dot{x} 表示x对时间的一阶导数, \ddot{x} 表示x对时间的二阶导数。
- 2. 粗体字母表示矢量。

2-4

解:

取微元dx, 有:

$$dF_T = x \lambda \omega^2 dx$$

其中 $\lambda = m/L$,为绳子的线密度。积分得:

$$F_T(x)=rac{m\omega^2}{2L}(L^2-x^2)$$

2-6

解:

• (1) 有:

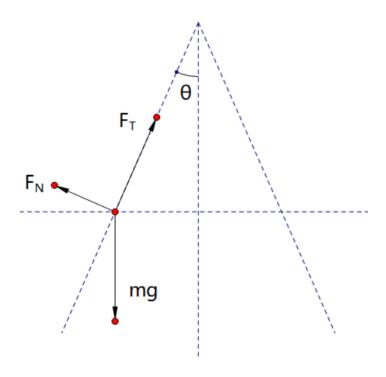
$$egin{cases} mg\cos heta+F_T=mrac{v^2}{R}\ ma_t=mg\sin heta \end{cases} \Rightarrow egin{cases} F_T&=mrac{v^2}{R}-mg\cos heta\ a_t&=g\sin heta \end{cases}$$

• (2) $\label{eq:continuous} \begin{array}{l} \textbf{ 4} : \ |a_t| = g|\sin\theta|, \ \texttt{方向垂直于半径向下}. \end{array}$

2-8

解:

• (1)



沿绳与垂直于面的方向分解,有:

$$egin{cases} F_T = mg\cos heta + m\omega^2l\sin^2 heta \ F_N = mg\sin heta - m\omega^2l\sin heta\cos heta \end{cases}$$

• (2) $\mathbb{p} F_N = 0 \text{ th. } \diamondsuit F_N = 0, \text{ 有: }$

$$mg\sin heta=m\omega_0^2l\sin heta\cos heta \ \Rightarrow \omega_0=\sqrt{rac{g}{l\cos heta}}$$

代入得:

$$F_T = rac{mg}{\cos heta}$$

2-10

解:

$$W = \int oldsymbol{F} \cdot doldsymbol{x} = F(h-h_0)(\csc heta_0 - \csc heta_1) = 18.26(ext{J})$$

解:

由功能原理,得:

$$W_{tot} = \Delta E_k = \frac{1}{2}m(v_2^2 - v_1^2) \tag{1}$$

由运动方程求对时间求一阶导数,得:

$$\begin{cases} \dot{x} = 5\\ \dot{y} = t \end{cases} (SI) \tag{2}$$

联立代入 $t=2s \rightarrow 4s$, 得:

$$W_{tot} = 3 \mathrm{J}$$

2-14

解:

有两颗中子星的速度等大反向,设大小为v。由能量守恒,得:

$$egin{align} 2 imesrac{1}{2}mv^2&=Gm^2(rac{1}{r_1}-rac{1}{r_0})\ &\Rightarrow v&=\sqrt{Gm(rac{1}{r_1}-rac{1}{r_0})}\ &rac{r_1=rac{1}{2}r_0}{r_0}&v&=\sqrt{rac{Gm}{r_0}} \end{aligned}$$

代入得:

$$v = 8.17 \times 10^4 \,\mathrm{m\cdot s^{-2}}$$

2-15

解:

由功能关系,得:

$$rac{1}{2}mv_0^2 = rac{1}{2}m(rac{v_0}{2})^2 + rac{1}{2}rac{F^2}{k}$$

解得:

$$F=\sqrt{rac{3}{4}kmv_0^2}$$

2-16

解:

由能量守恒,有:

$$mgR = rac{1}{2}mv^2 - W_f$$

解得:

$$W_f=-42.4\mathrm{J}$$

2-17

最远位移有 $\dot{x}=0$,即v=0。由能量守恒有:

$$Fx = \mu mgx + rac{1}{2}kx^2$$

排除x = 0的解,解得:

$$x = 2\frac{F - \mu mg}{k}$$

则系统弹性势能:

$$E_{pk} = rac{1}{2}kx^2 = 2rac{(F-\mu mg)^2}{k}$$

2-20

解:

• 动量定理

$$oldsymbol{I} = moldsymbol{v}_2 - moldsymbol{v}_1 = -m\omega Roldsymbol{i} - m\omega Roldsymbol{j}$$

积分法受力分析:

$$m{F} = mm{\omega} imes (m{\omega} imes m{r}) = -m\omega^2m{r} \ \Rightarrow m{F} = m\omega^2R(-m{i}\cos\omega t - m{j}\sin\omega t)$$

代入冲量定义

$$I = m\omega^2 R \int_0^{rac{\pi}{2\omega}} (-m{i}\cos\omega t - m{j}\sin\omega t) dt = -m\omega Rm{i} - m\omega Rm{j}$$

2-22

解:

子弹穿过第一个木块的过程,由冲量定理:

$$(m_1 + m_2)v_A = F\Delta t_1 \tag{1}$$

子弹穿过第二个木块的过程,由冲量定理:

$$m_2(v_B - v_A) = F\Delta t_2 \tag{2}$$

解得:

$$egin{cases} v_A = rac{F\Delta t_1}{m_1+m_2} \ v_B = rac{F\Delta t_1}{m_1+m_2} + rac{F\Delta t_2}{m_2} \end{cases}$$

2-23

解:

$$\left\{egin{aligned} 2ma_0 &= mg \ rac{1}{2}a_0t^2 &= L \ v_0 &= a_0t \ 3mv_1 &= 2mv_0 \end{aligned}
ight.$$

联立解得:

$$v_1 = \frac{2}{3}\sqrt{gL} = \frac{14}{15}\text{m}\cdot\text{s}^{-2}$$

解:

取船的原速方向为正方向。对三艘船分别使用动量守恒(其中b代表前船, m代表中船, a代表后船):

$$egin{cases} (m'+m)v_b = m'v + m(v+u) \ (m'-2m)v_m + m(v+u) + m(v-u) = mv \ (m'+m)v_a = m'v + m(v-u) \end{cases}$$

解得:

$$egin{cases} v_b = v + rac{m}{m'+m} u \ v_m = v \ v_a = v - rac{m}{m'+m} u \end{cases}$$

2-28

解:

设B点后的链条长度为x,链条线密度为 λ ,有:

$$\lambda L \frac{d^2x}{dt^2} = \lambda xg \sin \alpha$$

作变换:

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

得:

$$egin{aligned} Lvdv &= xdxg\sinlpha \ \Rightarrow v &= \sqrt{rac{g\sinlpha}{L}(L^2-a^2)} \end{aligned}$$