Statistical Physics

Homework #1

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Problem 5

We start the exercise by writing a simple python script that calculates the likelihood for a specific position within the the maximum range of 'n' steps using the function:

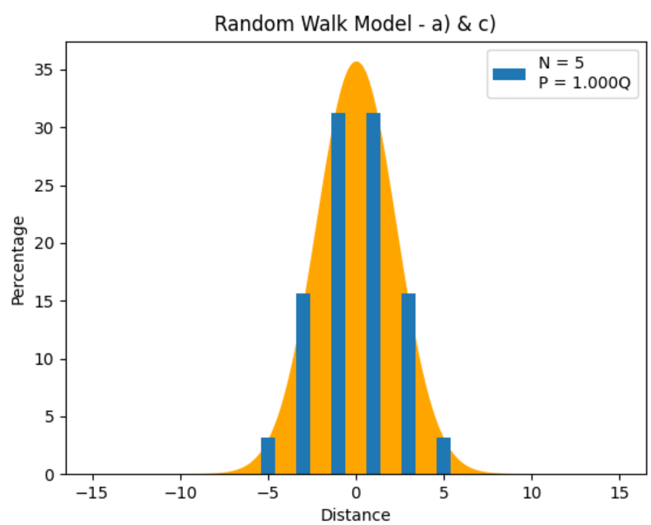
$$W_N(n_+) = \frac{N!}{\frac{(N+m)!}{2}\frac{(N-m)!}{2}} p^{\frac{(N+m)}{2}} q^{\frac{(N-m)}{2}}$$

Where P is the likelihood for a step in the positive direction, and Q is the likelihood for a step in the negative direction.
However since the given p and q values aren't normalized we must first calculate a p' and q' such that:

$$p' = \frac{p}{p+q}$$
$$q' = \frac{q}{p+q}$$

And use these instead of the given ones.

Assuming $p = q$:
For $N = 5$:



$\overline{n_+} = 2.50$ $\Delta n^2 = 1.25$ $\Delta n = 1.12$

$\overline{m} = 0.00$ $\Delta m^2 = 5.00$ $\Delta m = 2.24$

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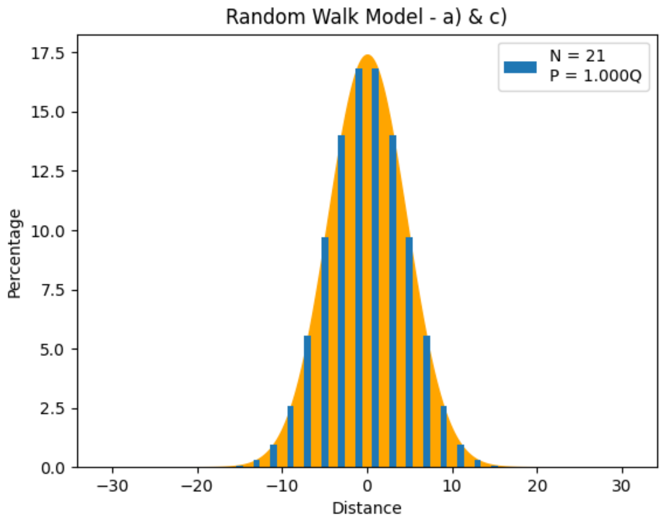
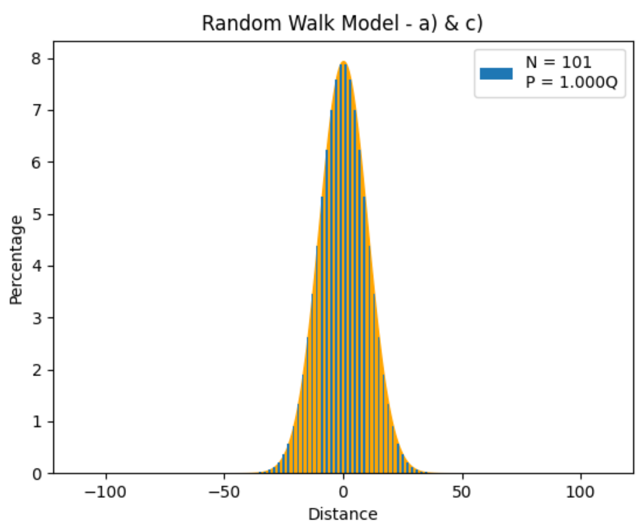
For $N = 21$:



$\overline{n_+} = 10.50$ $\Delta n^2 = 5.25$ $\Delta n = 2.29$

$\overline{m} = 0.00$ $\Delta m^2 = 21.00$ $\Delta m = 4.58$

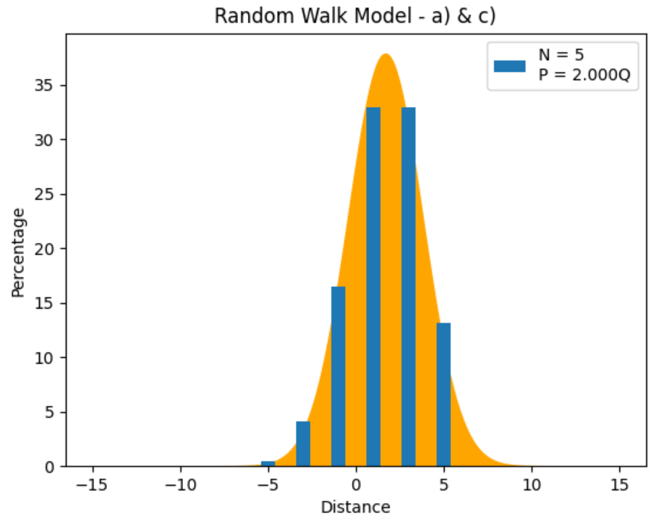
For $N = 101$:



$\overline{n_+} = 50.50$ $\Delta n^2 = 25.25$ $\Delta n = 5.02$

$\overline{m} = 0.00$ $\Delta m^2 = 101.00$ $\Delta m = 10.05$

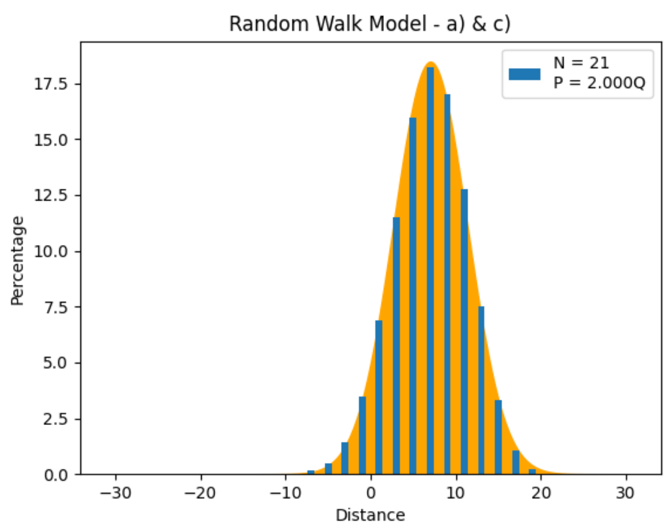
Assuming $p = 2q$:
For $N = 5$:



$\overline{n_+} = 3.33$ $\Delta n^2 = 1.11$ $\Delta n = 1.05$

$\overline{m} = 1.66$ $\Delta m^2 = 4.44$ $\Delta m = 2.11$

For N = 21:



$\overline{n_+} = 14.00$ $\Delta n^2 = 4.67$ $\Delta n = 2.16$

$\overline{m} = 7.00$ $\Delta m^2 = 18.67$ $\Delta m = 4.32$

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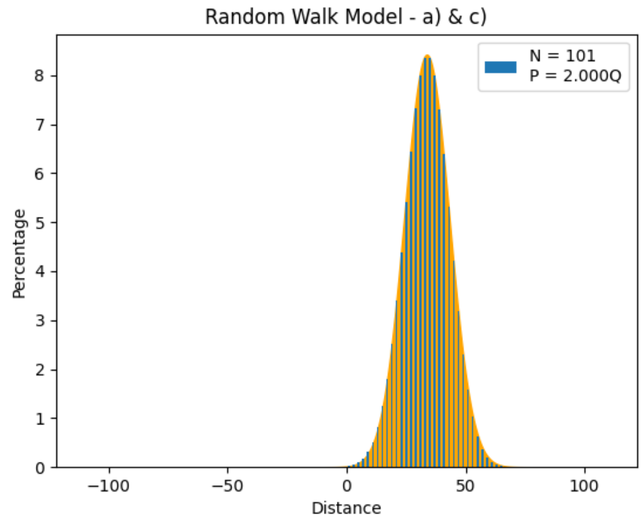
For $N = 101$:



$\overline{n_+} = 67.33$ $\Delta n^2 = 22.44$ $\Delta n = 4.74$

$\overline{m} = 33.67$ $\Delta m^2 = 89.77$ $\Delta m = 9.48$

Discussion

As we could see from the plots, as we increase the N, the binomial distribution starts being well described by a Gaussian distribution, this is quiet the phenomena, since this implies that we can calculate where a single particle will most likely be over time, the only thing we need to do, is say that n amount of steps of length l, happen in t seconds, then we just use $N = nt$, and create this simple distribution, from which we can find out where the particle should be at after a given time.

However things could get interesting really quick, if we decide to expand this to a system that under the same rules has something similar to a singularity, a point alike range that we can get to be smaller and smaller until we can approximate the extremes of the range to be around the same value, while holding within a large enough number of particles, we can use this model to predict how the system will evolve, therefore understanding the distribution of particles through the space over time.

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We can then expand the later idea into a 3 dimensional space by implementing p_x, p_y, p_z and q_x, q_y, q_z, where 'p' represents the likelihood of taking a positive step in the given direction, and 'q' the likelihood of taking a negative step in the given direction, making a much more complete distribution.
Nevertheless we can point out there's a slight difference between the binomial distribution model and the Gaussian distribution, this being due to the fact that there are 'forbidden' values of m since if we do 'n' even steps we can only finish at even numbers, and if we do 'n' odd steps we can only finish at odd numbers, however the difference is also due to the fact that N may be far too small, this is the case for $N = 5$, although we can already see some sort of Gaussian distribution it is still not clear if it isn't following something else, that passes through those values.

But we can fix the first of the problems, the 'forbidden' values, can be removed from the distribution by yet again implementing another probability s_x, s_y, and s_z, where 's' represents the likelihood of not changing the particle's position in the given direction, however this will force us to change the original probability function, so instead we can reduce the step's length to a much smaller value, which is equivalent to increasing the number of 'N' Steps.

Finally if we decide to neglect the missing values of m, we can try to create a relationship between the average error on the values of the binomial distribution and the Gaussian distribution, finding out an optimal starting N for a given P and Q that allows us to get an error inferior to some ϵ, from which we can use this model to predict how a system will evolve with great certainty.

The python scrip use can be found in the following github repository:
`https://github.com/Sorah-Darkhat/FE-Problems`