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## 1 Basic

### 1.1 readchar

```
#include <unistd.h>
const int S = 65536;
inline char RC() {
    static char buf[S], *p = buf, *q = buf;
    return p == q and (q = (p = buf) + read(0, buf, S)) == buf ? -1 : *p++;
}
inline int RI() {
    static char c; int a;
    while (((c = RC()) < '0' or c > '9') and c != '-' and c != '.');
    if (c == '-') { a = 0; while ((c = RC()) >= '0' and c <= '9') a *= 10, a -= c ^ '0'; }
    else { a = c ^ '0'; while ((c = RC()) >= '0' and c <= '9') a *= 10, a += c ^ '0'; }
    return a;
}
```

### 1.2 Black Magic

```
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2; // max heap
    h1.push(1), h1.push(3), h2.push(2), h2.push(4);
    h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
    tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> st;
    tree<int, int, less<int>, rb_tree_tag, tree_order_statistics_node_update> mp;
    for (int x : {0, 3, 20, 50}) st.insert(x);
    assert(st.order_of_key(3) == 1 && st.order_of_key(4) == 2);
    assert(*st.find_by_order(2) == 20 && *st.lower_bound(4) == 20);
    rope<char> *root[10]; // nsqrt(n)
    root[0] = new rope<char>();
    root[1] = new rope<char>(*root[0]);
    // root[1]->insert(pos, 'a');
    // root[1]->at(pos); 0-base
    // root[1]->erase(pos, size);
}
// __int128_t, __float128_t
// for (int i = bs._Find_first(); i < bs.size(); i = bs._Find_next(i));
```

## 2 Graph

### 2.1 BCCVertex\*

```
vector<int> G[N]; // 1-base
vector<int> nG[N * 2], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N * 2];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_cut[u] = 1;
                bcc[++bcc_cnt].clear();
                int t;
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].eb(t);
                } while (t != v);
                bcc_id[u] = bcc_cnt;
                bcc[bcc_cnt].eb(u);
            }
        } else if (dfn[v] < dfn[u] && v != pa)
            low[u] = min(low[u], dfn[v]);
    if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) { // TODO: init {nG, cir}[1..2n]
    Time = bcc_cnt = top = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i);
    // block-cut tree
    for (int i = 1; i <= n; ++i)
        if (is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_cut[j])
                nG[i].eb(bcc_id[j]), nG[bcc_id[j]].eb(i);
}
```

## 2.2 Bridge\*

```
int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].eb(pii(b, SZ(edge))), G[b].eb(pii(a, SZ(edge)));
    edge.eb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for (auto i : G[u])
        if (!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
    if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
```

## 2.3 2SAT (SCC)\*

```
struct SAT { // 0-base
    int low[N], dfn[N], bln[N], n, Time, nScc;
    bool instack[N], istrue[N];
    stack<int> st;
    vector<int> G[N], SCC[N];
    void init(int _n) {
        n = _n; // assert(n * 2 <= N);
        for (int i = 0; i < n + n; ++i) G[i].clear();
    }
    void add_edge(int a, int b) { G[a].eb(b); }
    int rv(int a) {
        if (a >= n) return a - n;
        return a + n;
    }
    void add_clause(int a, int b) {
        add_edge(rv(a), b), add_edge(rv(b), a);
    }
    void dfs(int u) {
        dfn[u] = low[u] = ++Time;
        instack[u] = 1, st.push(u);
        for (int i : G[u])
            if (!dfn[i])
                dfs(i), low[u] = min(low[u], low[i]);
            else if (instack[i] && dfn[i] < dfn[u])
                low[u] = min(low[u], dfn[i]);
        if (low[u] == dfn[u]) {
            int tmp;
            do {
                tmp = st.top(), st.pop();
                instack[tmp] = 0, bln[tmp] = nScc;
            } while (tmp != u);
            ++nScc;
        }
    }
    bool solve() {
        Time = nScc = 0;
        for (int i = 0; i < n + n; ++i)
            SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
        for (int i = 0; i < n + n; ++i)
            if (!dfn[i]) dfs(i);
        for (int i = 0; i < n + n; ++i) SCC[bln[i]].eb(i);
        for (int i = 0; i < n; ++i) {
            if (bln[i] == bln[i + n]) return false;
            istrue[i] = bln[i] < bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};
```

## 2.4 MinimumMeanCycle\*

```
int road[N][N]; // input here
struct MinimumMeanCycle {
    int dp[N + 5][N], n;
    pii solve() {
        int a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            int ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            int g = __gcd(a, b);
            return pii(a / g, b / g);
        }
        return pii(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};
```

## 2.5 Virtual Tree\*

```
vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].eb(st[top]), --top;
    if (st[top] != p)
        vG[p].eb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].eb(st[top]), --top;
    // do something
    reset(v[0]);
}
```

## 2.6 Maximum Clique Dyn\*

```
struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(ALL(r),
                [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
```

```

    int k = 1;
    while ((cs[k] & G[p]).any()) ++k;
    if (k > rgt) cs[++rgt + 1].reset();
    cs[k][p] = 1;
    if (k < lft) r[tp++] = p;
}
for (int k = lft; k <= rgt; ++k)
    for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
dfs(r, c, l + 1, mask);
}
void dfs(vector<
    int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pb(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr;
        for (int i : r) if (G[p][i]) nr.eb(i);
        if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
        else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pb(), --q;
    }
}
int solve() {
    vector<int> r(n);
    ans = q = 0, iota(ALL(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
}
};

```

## 2.7 Minimum Steiner Tree\*

```

struct SteinerTree { // 0-base
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcst[N]; // the cost of vertexs
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            fill_n(dst[i], n, INF);
            dst[i][i] = vcst[i] = 0;
        }
    }
    void chmin(int &x, int val) {
        x = min(x, val);
    }
    void add_edge(int ui, int vi, int wi) {
        chmin(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    chmin(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = SZ(ter), full = (1 << t) - 1;
        for (int i = 0; i <= full; ++i)
            fill_n(dp[i], n, INF);
        copy_n(vkst, n, dp[0]);
        for (int msk = 1; msk <= full; ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk][i] = vcst[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int sub = (msk - 1) & msk; sub; sub = (sub - 1) & msk)
                    chmin(dp[msk][i], dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
            for (int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for (int j = 0; j < n; ++j)
                    chmin(tdst[i], dp[msk][j] + dst[j][i]);
            }
            copy_n(tdst, n, dp[msk]);
        }
        return *min_element(dp[full], dp[full] + n);
    }
}; // O(V 3^T + V^2 2^T)

```

## 2.8 Dominator Tree\*

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].eb(v), rG[v].eb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])
                if (v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].eb(i);
            for (auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] =
                    semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for (int i = 2; i <= Time; ++i) {
            if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
            tree[id[idom[i]]].eb(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence\*

```

/* TODO
DSU: disjoint set
- DSU(n), .boss(x), .Union(x, y)
min_heap<
    T, Info>: min heap for type {T, Info} with lazy tag
- .push({w, i}),
  .top(), .join(heap), .pop(), .empty(), .add_lazy(v)
*/
struct E { int s, t; int w; }; // 0-base
vector<int> dmst(const vector<E> &e, int n, int root) {
    vector<min_heap<int, int>> h(n * 2);
    for (int i = 0; i < SZ(e); ++i)
        h[e[i].t].push({e[i].w, i});
    DSU dsu(n * 2);
    vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p] == -1 || v[p] == i; p = dsu.boss(e[r[p]].s)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].add_lazy(-h[q].top().X);
                    pa[q] = p, dsu.Union(p, q), h[p].join(h[q]);
                } while ((q = dsu.boss(e[r[q]].s)) != p);
            }
        }
    }
}

```

```

    v[p] = i;
    while (!h[p].empty() && dsu.boss(e[h[p].top().Y].s) == p)
        h[p].pop();
    if (h[p].empty()) return {}; // no solution
    r[p] = h[p].top().Y;
}
vector<int> ans;
for (int i = pc - 1; i >= 0; i--) if (i != root && v[i] != n) {
    for (int f = e[r[i]].t; ~f && v[f] != n; f = pa[f])
        v[f] = n;
    ans.pb(r[i]);
}
return ans; // default minimize, returns edgeid array
} // O(Ef(E)), f(E) from min_heap

```

## 2.10 Vizing's theorem\*

```

namespace vizing { // returns
    edge coloring in adjacent matrix G. 1 - based
const int N = 105;
int C[N][N], G[N][N], X[N], vst[N], n;
void init(int _n) { n = _n;
    for (int i = 0; i <= n; ++i)
        for (int j = 0; j <= n; ++j)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pii> &E) {
    auto update = [&](int u)
    { for (X[u] = 1; C[u][X[u]]; ++X[u]); };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    fill_n(X + 1, n, 1);
    for (int t = 0; t < SZ(E); ++t) {
        int u = E[t].X, v0 = E[t].Y, v = v0, c0 = X[u], c = c0, d;
        vector<pii> L;
        fill_n(vst + 1, n, 0);
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for (int a = SZ(L) - 1; a >= 0; --a) c = color(u, L[a].X, c);
            else if (!C[u][d]) for (int a = SZ(L) - 1; a >= 0; --a) color(u, L[a].X, L[a].Y);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (int a; C[u][c0]) {
                for (a = SZ(L) - 2; a >= 0 && L[a].Y != c; --a);
                for (; a >= 0; --a) color(u, L[a].X, L[a].Y);
            }
            else --t;
        }
    }
} // namespace vizing

```

## 2.11 Minimum Clique Cover\*

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
}

```

```

void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
}
int solve() {
    for (int i = 0; i < n; ++i)
        co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
        int t = i & -i;
        dp[i] = -dp[i ^ t];
        co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
        co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
        int sum = 0; // probabilistic
        for (int i = 0; i < (1 << n); ++i)
            sum += (dp[i] * co[i]);
        if (sum) return ans;
    }
    return n;
}

```

## 2.12 NumberofMaximalClique\*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsn = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsn++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsn, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
}

```

## 3 Data Structure

### 3.1 BIT kth\*

```

int bit[N + 1]; // N = 2 ^ k
int query_kth(int k) {
    int res = 0;
    for (int i = N >> 1; i >= 1; i >>= 1)
        if (bit[res + i] < k)
            k -= bit[res + i];
    return res + 1;
}

```

### 3.2 Interval Container\*

```

/* Add and
    remove intervals from a set of disjoint intervals.
* Will merge the added interval with
    any overlapping intervals in the set when adding.
* Intervals are [inclusive, exclusive). */

```

```

set<pii>::
    iterator addInterval(set<pii>& is, int L, int R) {
        if (L == R) return is.end();
        auto it = is.lower_bound({L, R}), before = it;
        while (it != is.end() && it->X <= R) {
            R = max(R, it->Y);
            before = it = is.erase(it);
        }
        if (it != is.begin() && (--it)->Y >= L) {
            L = min(L, it->X);
            R = max(R, it->Y);
            is.erase(it);
        }
        return is.insert(before, pii(L, R));
    }
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->Y;
    if (it->X == L) is.erase(it);
    else (int&)it->Y = L;
    if (R != r2) is.emplace(R, r2);
}

```

### 3.3 Centroid Decomposition\*

```

struct Cent_Dec { // 1-base
    vector<pii> G[N];
    pii info[N]; // store info. of itself
    pii upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    int dis[lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].eb(pii(b, w)), G[b].eb(pii(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pii e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
    void dfs(int u, int f, int d, int org) {
        // if required, add self info or climbing info
        dis[layer[org]][u] = d;
        for (pii e : G[u])
            if (!done[e.X] && e.X != f)
                dfs(e.X, u, d + e.Y, org);
    }
    int cut(int u, int f, int num) {
        int mx = 1e9, c = 0, lc;
        get_cent(u, f, mx, c, num);
        done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
        for (pii e : G[c])
            if (!done[e.X]) {
                if (sz[e.X] > sz[c])
                    lc = cut(e.X, c, num - sz[c]);
                else lc = cut(e.X, c, sz[e.X]);
                upinfo[lc] = pii(), dfs(e.X, c, e.Y, c);
            }
        return done[c] = 0, c;
    }
    void build() { cut(1, 0, n); }
    void modify(int u) {
        for (int a = u, ly = layer[a]; a;
            a = pa[a], --ly) {
            info[a].X += dis[ly][u], ++info[a].Y;
            if (pa[a])
                upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
        }
    }
    int query(int u) {
        int rt = 0;
        for (int a = u, ly = layer[a]; a;
            a = pa[a], --ly) {
            rt += info[a].X + info[a].Y * dis[ly][u];
        }
    }
}

```

```

        if (pa[a])
            rt +=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

### 3.4 LiChaoST\*

```

struct L {
    int m, k, id;
    L() : id(-1) {}
    L(int a, int b, int c) : m(a), k(b), id(c) {}
    int at(int x) { return m * x + k; }
};
class LiChao { // maintain max
private:
    int n; vector<L> nodes;
    void insert(int l, int r, int rt, L ln) {
        int m = (l + r) >> 1;
        if (nodes[rt].id == -1)
            return nodes[rt] = ln, void();
        bool atLeft = nodes[rt].at(l) < ln.at(l);
        if (nodes[rt].at(m) < ln.at(m))
            atLeft ^= 1, swap(nodes[rt], ln);
        if (r - l == 1) return;
        if (atLeft) insert(l, m, rt << 1, ln);
        else insert(m, r, rt << 1 | 1, ln);
    }
    int query(int l, int r, int rt, int x) {
        int m = (l + r) >> 1; int ret = -INF;
        if (nodes[rt].id != -1) ret = nodes[rt].at(x);
        if (r - l == 1) return ret;
        if (x
            < m) return max(ret, query(l, m, rt << 1, x));
        return max(ret, query(m, r, rt << 1 | 1, x));
    }
public:
    LiChao(int n_) : n(n_), nodes(n * 4) {}
    void insert(L ln) { insert(0, n, 1, ln); }
    int query(int x) { return query(0, n, 1, x); }
};

```

### 3.5 Link cut tree\*

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int
        _val = 0) : val(_val), sum(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() {
        return f->ch[0] == this ? 0 : 1;
    }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void give_tag(int r) {
        if (r) swap(ch[0], ch[1]), rev ^= 1;
    }
    void push() {
        if (ch[0] != &nil) ch[0]->give_tag(rev);
        if (ch[1] != &nil) ch[1]->give_tag(rev);
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
}

```

```

    p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay*> splayVec;
    for (Splay *q = x; q = q->f) {
        splayVec.eb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay* access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x), x->setCh(q, 1), q = x;
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->give_tag(1);
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay* get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay* lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}
}

```

### 3.6 KDTree

```

namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &)> f =
        [dep](const point &a, const point &b) {
            if (dep & 1) return a.x < b.x;
            else return a.y < b.y;
        };
    int m = (l + r) >> 1;
    nth_element(p + l, p + m, p + r, f);
    xl[m] = xr[m] = p[m].x;
    yl[m] = yr[m] = p[m].y;
    lc[m] = build(l, m, dep + 1);
    if (~lc[m]) {
        xl[m] = min(xl[m], xl[lc[m]]);
        xr[m] = max(xr[m], xr[lc[m]]);
        yl[m] = min(yl[m], yl[lc[m]]);
        yr[m] = max(yr[m], yr[lc[m]]);
    }
}
}

```

```

}
rc[m] = build(m + 1, r, dep + 1);
if (~rc[m]) {
    xl[m] = min(xl[m], xl[rc[m]]);
    xr[m] = max(xr[m], xr[rc[m]]);
    yl[m] = min(yl[m], yl[rc[m]]);
    yr[m] = max(yr[m], yr[rc[m]]);
}
return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

## 4 Flow/Matching

### 4.1 Kuhn Munkres\*

```

struct KM { // 0-base, maximum matching
int w[N][N], hl[N], hr[N], slk[N];
int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
bool vl[N], vr[N];
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
        fill_n(w[i], n, -INF);
}
void add_edge(int a, int b, int wei) {
    w[a][b] = wei;
}
bool Check(int x) {
    if (vl[x] = 1, ~fl[x])
        return vr[qu[qr++]] = fl[x] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
}
void bfs(int s) {
    fill_n(slk, n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    for (int d; ; ) {
        while (ql < qr)
            for (int x = 0, y = qu[ql++]; x < n; ++x)
                if (!vl[x] && slk
                    [x] >= (d = hl[x] + hr[y] - w[x][y])) {
                    if (pre[x] = y, d) slk[x] = d;
                    else if (!Check(x)) return;
                }
        d = INF;
        for (int x = 0; x < n; ++x)
            if (!vl[x] && d > slk[x]) d = slk[x];
        for (int x = 0; x < n; ++x) {
            if (vl[x]) hl[x] += d;
            else slk[x] -= d;
        }
    }
}
}

```



```

        if (vr[x]) hr[x] -= d;
    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !Check(x)) return;
}
int solve() {
    fill_n(fl,
        , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

## 4.2 MincostMaxflow\*

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        int from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    int dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, int d, int cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                int d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax
                    (e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s,
        int _t, int &flow, int &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; BellmanFord(); copy_n(dis, n, pot)) {
            for (int
                i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
            for (int i = t; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                e.flow += up[t], G[e.to][e.rev].flow -= up[t];
            }
        }
    }
    void init(int _n) {
        n = _n, fill_n(pot, n, 0);
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int cap, int cost) {
        G[a].eb(Edge{a, b, cap, 0, cost, SZ(G[b])});
        G[b].eb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
    }
};

```

## 4.3 Maximum Simple Graph Matching

```

struct Matching { // 0-base
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>> G;
    int Find(int u)
    { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
};

```

```

}
}
void Blossom(int x, int y, int l) {
    for (; Find(x) != l; x = pre[y]) {
        pre[x] = y, y = match[x];
        if (s[y] == 1) q.see(y), s[y] = 0;
        for (int z : {x, y}) if (fa[z] == z) fa[z] = l;
    }
}
bool Bfs(int r) {
    iota(ALL(fa), 0); fill(ALL(s), -1);
    q = queue<int>(); q.see(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
        for (int x = q.front(); int u : G[x])
            if (s[u] == -1) {
                if (pre[u] = x, s[u] = 1, match[u] == n) {
                    for (int a = u, b = x, last;
                        b != n; a = last, b = pre[a])
                        last =
                            match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.see(match[u]); s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = LCA(u, x);
                Blossom(x, u, l); Blossom(u, x, l);
            }
        }
    }
    return false;
}
Matching(int _n) : n(_n), fa(n+1), s(n+1),
    vis(n+1), pre(n+1, n), match(n+1, n), G(n) {}
void add_edge(int u, int v)
{ G[u].eb(v), G[v].eb(u); }
int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
        if (match[x] == n) ans += Bfs(x);
    return ans;
} // match[x] == n means not matched
};

```

## 4.4 Maximum Weight Matching\*

```

#define rep(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slack, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int _n) : n(_n), nx(n*2), lab(nx+1),
        g(nx+1, vector<edge>(nx+1)), slack(nx+1),
        flo(nx+1), flo_from(nx+1, vector(n+1, 0)) {
        match = st = pa = S = vis = slack;
        rep(u, 1, n) rep(v, 1, n) g[u][v] = {u, v, 0};
    }
    int ED(edge e)
    { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
    void update_slack(int u, int x, int &s)
    { if (!s || ED(g[u][x]) < ED(g[s][x])) s = u; }
    void set_slack(int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; ++u)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slack(u, x, slack[x]);
    }
    void q_push(int x) {
        if (x <= n) q.see(x);
        else for (int y : flo[x]) q_push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(ALL(f), xr);
        if (auto pr = it - begin(f); pr % 2 == 1)
            reverse(1 + ALL(f), it = f.end() - pr);
        auto res = vector(begin(f), it);
        return f.erase(begin(f), it), res;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
    }
};

```

```

rep(i, 0, SZ(z)-1) set_match(z[i], z[i ^ 1]);
set_match(xr, v); f.insert(f.end(), ALL(z));
}
void augment(int u, int v) {
    for (;;) {
        int xnv = st[match[u]]; set_match(u, v);
        if (!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    }
}
int lca(int u, int v) {
    static int t = 0; ++t;
    for (++t; u || v; swap(u, v)) if (u) {
        if (vis[u] == t) return u;
        vis[u] = t; u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}
void add_blossom(int u, int o, int v) {
    int b = find(n + 1 + ALL(st), 0) - begin(st);
    lab[b] = 0, S[b] = 0; match[b] = match[o];
    vector<int> f = {o};
    for (int x = u, y; x != o; x = st[pa[y]])
        f.pb(x), f.pb(y = st[match[x]]), q_push(y);
    reverse(1 + ALL(f));
    for (int x = v, y; x != o; x = st[pa[y]])
        f.pb(x), f.pb(y = st[match[x]]), q_push(y);
    flo[b] = f; set_st(b, b);
    for (int x = 1; x <= nx; ++x)
        g[b][x].w = g[x][b].w = 0;
    fill(ALL(flo_from[b]), 0);
    for (int xs : flo[b]) {
        for (int x = 1; x <= nx; ++x)
            if (g[
                b][x].w == 0 || ED(g[xs][x]) < ED(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flo_from[xs][x]) flo_from[b][x] = xs;
    }
    set_slack(b);
}
void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
        if (xs == -1) { xs = x; continue; }
        pa[xs] = g[x][xs].u; S[xs] = 1, S[x] = 0;
        slack[xs] = 0; set_slack(x); q_push(x); xs = -1;
    }
    for (int x : flo[b])
        if (x == xr) S[x] = 1, pa[x] = pa[b];
        else S[x] = -1, set_slack(x);
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
        slack[v] = slack[nu] = 0; S[nu] = 0; q_push(nu);
    } else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
}
}
bool matching() {
    fill(ALL(S), -1), fill(ALL(slack), 0);
    q = queue<int>();
    for (int x = 1; x <= nx; ++x)
        if (st[x] == x && !match[x])
            pa[x] = 0, S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
        while (SZ(q)) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (ED(g[u][v]) != 0)
                        update_slack(u, st[v], slack[st[v]]);
                    else if
                        (on_found_edge(g[u][v])) return true;
                }
        }
    }
    int d = INF;

```

```

for (int b = n + 1; b <= nx; ++b)
    if (st[b] == b && S[b] == 1)
        d = min(d, lab[b] / 2);
for (int x = 1; x <= nx; ++x)
    if (int
        s = slack[x]; st[x] == x && s && S[x] <= 0)
        d = min(d, ED(g[s][x]) / (S[x] + 2));
for (int u = 1; u <= n; ++u)
    if (S[st[u]] == 1) lab[u] += d;
    else if (S[st[u]] == 0) {
        if (lab[u] <= d) return false;
        lab[u] -= d;
    }
rep(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
    lab[b] += d * (2 - 4 * S[b]);
for (int x = 1; x <= nx; ++x)
    if (int s = slack[x]; st[x] == x &&
        s && st[s] != x && ED(g[s][x]) == 0)
        if (on_found_edge(g[s][x])) return true;
for (int b = n + 1; b <= nx; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
        expand_blossom(b);
}
return false;
}
pair<int, int> solve() {
    fill(ALL(match), 0);
    rep(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    rep(u, 1, n) rep(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    fill(ALL(lab), w_max);
    int n_matches = 0, tot_weight = 0;
    while (matching()) ++n_matches;
    rep(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return {tot_weight, n_matches};
}
void add_edge(int u, int v, int w)
{ g[u][v].w = g[v][u].w = w; }
};

```

## 4.5 SW-mincut

```

struct SW{ // global min cut, O(V^3)
#define REP for (int i = 0; i < n; ++i)
static const int MXN = 514, INF = 2147483647;
int vst[MXN], edge[MXN][MXN], wei[MXN];
void init(int n) {
    REP fill_n(edge[i], n, 0);
}
void addEdge(int u, int v, int w){
    edge[u][v] += w; edge[v][u] += w;
}
int search(int &s, int &t, int n){
    fill_n(vst, n, 0), fill_n(wei, n, 0);
    s = t = -1;
    int mx, cur;
    for (int j = 0; j < n; ++j) {
        mx = -1, cur = 0;
        REP if (wei[i] > mx) cur = i, mx = wei[i];
        vst[cur] = 1, wei[cur] = -1;
        s = t; t = cur;
        REP if (!vst[i]) wei[i] += edge[cur][i];
    }
    return mx;
}
int solve(int n) {
    int res = INF;
    for (int x, y; n > 1; n--){
        res = min(res, search(x, y, n));
        REP edge[i][x] = (edge[x][i] += edge[y][i]);
        REP {
            edge[y][i] = edge[n - 1][i];
            edge[i][y] = edge[i][n - 1];
        } // edge[y][y] = 0;
    }
    return res;
}
} SW;

```

## 4.6 BoundedFlow\*(Dinic\*)

```

struct BoundedFlow { // 0-base

```



```

struct edge {
    int to, cap, flow, rev;
};
vector<edge> G[N];
int n, s, t, dis[N], cur[N], cnt[N];
void init(int _n) {
    n = _n;
    for (int i = 0; i < n + 2; ++i)
        G[i].clear(), cnt[i] = 0;
}
void add_edge(int u, int v, int lcap, int rcap) {
    cnt[u] -= lcap, cnt[v] += lcap;
    G[u].eb(edge{v, rcap, lcap, SZ(G[v])});
    G[v].eb(edge{u, 0, 0, SZ(G[u]) - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].eb(edge{v, cap, 0, SZ(G[v])});
    G[v].eb(edge{u, 0, 0, SZ(G[u]) - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pb(), G[i].pb();
        else if (cnt[i] < 0)
            G[i].pb(), G[n + 2].pb();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pb(), G[_s].pb(), x;
}
};

```

## 4.7 Gomory Hu tree\*

```

MaxFlow Dinic;
int g[maxn];
void GomoryHu(int n) { // 0-base
    fill_n(g, n, 0);
    for (int i = 1; i < n; ++i) {
        Dinic.reset();

```

```

        add_edge(i, g[i], Dinic.maxflow(i, g[i]));
        for (int j = i + 1; j <= n; ++j)
            if (g[j] == g[i] && ~Dinic.dis[j])
                g[j] = i;
    }
}

```

## 4.8 Minimum Cost Circulation\*

```

struct MinCostCirculation { // 0-base
    struct Edge {
        int from, to, cap, fcap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int dis[N], inq[N], n;
    void BellmanFord(int s) {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, int d, Edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
    }
    void try_edge(Edge &cur) {
        if (cur.cap > cur.flow) return ++cur.cap, void();
        BellmanFord(cur.to);
        if (dis[cur.from] + cur.cost < 0) {
            ++cur.flow, --G[cur.to][cur.rev].flow;
            for (int
                i = cur.from; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                ++e.flow, --G[e.to][e.rev].flow;
            }
        }
        ++cur.cap;
    }
    void solve(int mxlg) {
        for (int b = mxlg; b >= 0; --b) {
            for (int i = 0; i < n; ++i)
                for (auto &e : G[i])
                    e.cap *= 2, e.flow *= 2;
            for (int i = 0; i < n; ++i)
                for (auto &e : G[i])
                    if (e.fcap >> b & 1)
                        try_edge(e);
        }
    }
    void init(int _n) { n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int cap, int cost) {
        G[a].eb(Edge{
            {a, b, 0, cap, 0, cost, SZ(G[b]) + (a == b)};
            G[b].eb(Edge{b, a, 0, 0, 0, -cost, SZ(G[a]) - 1});
        }
    }
} mcmf; // O(VE * ElogC)

```

## 4.9 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.

- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  - For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  - For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  - Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$
  - Construct a max flow model, let  $K$  be the sum of all weights
  - Connect source  $s \rightarrow v, v \in G$  with capacity  $K$
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  - For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  - $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  - Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  - If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
  - Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  - The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  - Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
  - If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\min \sum_{uv} w_{uv} f_{uv} \quad \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$-f_{uv} \geq -c_{uv} \Leftrightarrow \sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0$$

## 4.10 MCMF HLPP

```
// Push-Relabel
// implementation of the cost-scaling algorithm
// Runs in  $O(\max\_flow \cdot \log(V \cdot \max\_edge\_cost)) = O(V^3 \cdot \log(V \cdot C))$ 
// Operates on integers

#include <bits/stdc++.h>
using namespace std;

template<typename flow_t = int, typename cost_t = int>
struct mcSFlow{
    struct Edge{
        cost_t c;
        flow_t f;
        int to, rev;
        Edge(int _to, cost_t _c, flow_t _f,
             int _rev):c(_c), f(_f), to(_to), rev(_rev){}
    };
    const
        cost_t INFCOST = numeric_limits<cost_t>::max()/2;
    const
        cost_t INFFLOW = numeric_limits<flow_t>::max()/2;
    cost_t epsilon;
    int N, S, T;
    vector<vector<Edge>> G;
    vector<unsigned int> isEnqueued, state;
    mcSFlow(int _N, int _S,
             int _T):epsilon(0), N(_N), S(_S), T(_T), G(_N){}
    void add_edge(int a, int b, cost_t cost, flow_t cap){
        if(a==b){assert(cost>=0); return;}
        cost*=N; // to preserve integer-values
        epsilon = max(epsilon, abs(cost));
        assert(a>=0&&a<N&&b>=0&&b<N);
        G[a].emplace_back(b, cost, cap, G[b].size());
        G[b].emplace_back(a, -cost, 0, G[a].size()-1);
    }
};
```

```
flow_t calc_max_flow(){ // Dinic max-flow
    vector<flow_t> dist(N), state(N);
    vector<Edge*> path(N);
    auto cmp = [](Edge*a, Edge*b){return a->f < b->f;};
    flow_t addFlow, retflow=0;
    do{
        fill(dist.begin(), dist.end(), -1);
        dist[S]=0;
        auto head = state.begin(), tail = state.begin();
        for(*tail++ = S; head!=tail; ++head){
            for(Edge const&e:G[*head]){
                if(e.f && dist[e.to]==-1){
                    dist[e.to] = dist[*head]+1;
                    *tail++=e.to;
                }
            }
        }
        addFlow = 0;
        fill(state.begin(), state.end(), 0);
        auto top = path.begin();
        Edge dummy(S, 0, INFFLOW, -1);
        *top++ = &dummy;
        while(top != path.end()){
            int n = (*prev(top))->to;
            if(n==T){
                auto next_top
                    = min_element(path.begin(), top, cmp);
                flow_t flow = (*next_top)->f;
                while(--top!=path.begin()){
                    Edge &e=**top, &f=G[e.to][e.rev];
                    e.f-=flow;
                    f.f+=flow;
                }
                addFlow+=flow;
                retflow+=flow;
                top = next_top;
                continue;
            }
            for(int &i=state[n], i_max
                = G[n].size(), need = dist[n]+1; ++i){
                if(i==i_max){
                    dist[n]=-1;
                    --top;
                    break;
                }
                if(dist[G[n][i].to] == need && G[n][i].f){
                    *top++ = &G[n][i];
                    break;
                }
            }
        }
    }while(addFlow);
    return retflow;
}

vector<flow_t> excess;
vector<cost_t> h;
void push(Edge &e, flow_t amt){
    //cerr << "push: "
    << G[e.to][e.rev].to << " -> " << e.to << " ("
    << e.f << "/" << e.c << ") : " << amt << "\n";
    if(e.f < amt) amt=e.f;
    e.f-=amt;
    excess[e.to]+=amt;
    G[e.to][e.rev].f+=amt;
    excess[G[e.to][e.rev].to]-=amt;
}

void relabel(int vertex){
    cost_t newHeight = -INFCOST;
    for(unsigned int i=0; i<G[vertex].size(); ++i){
        Edge const&e = G[vertex][i];
        if(e.f && newHeight < h[e.to]-e.c){
            newHeight = h[e.to] - e.c;
            state[vertex] = i;
        }
    }
    h[vertex] = newHeight - epsilon;
}

const int scale=2;
pair<flow_t, cost_t> minCostFlow(){
    cost_t retCost = 0;
    for(int i=0; i<N; ++i){
        for(Edge &e:G[i]){
            retCost += e.c*(e.f);
        }
    }
    //find feasible flow
```

```

flow_t retFlow = calc_max_flow();
excess.resize(N); h.resize(N);
queue<int> q;
isEnqueued.assign(N, 0); state.assign(N, 0);
for(; epsilon; epsilon >>= scale){
    //refine
    fill(state.begin(), state.end(), 0);
    for(int i=0; i<N; ++i)
        for(auto &e: G[i])
            if(h[i] + e.c - h[e.to] < 0 && e.f) push(e, e.f);
    for(int i=0; i<N; ++i){
        if(excess[i]>0){
            q.push(i);
            isEnqueued[i]=1;
        }
    }
    while(!q.empty()){
        int cur=q.front(); q.pop();
        isEnqueued[cur]=0;
        // discharge
        while(excess[cur]>0){
            if(state[cur] == G[cur].size()){
                relabel(cur);
            }
            for(unsigned int &i=state[
                cur], max_i = G[cur].size(); i<max_i; ++i){
                Edge &e=G[cur][i];
                if(h[cur] + e.c - h[e.to] < 0){
                    push(e, excess[cur]);
                    if(excess
                        [e.to]>0 && isEnqueued[e.to]==0){
                        q.push(e.to);
                        isEnqueued[e.to]=1;
                    }
                    if(excess[cur]==0) break;
                }
            }
        }
    }
    if(epsilon>1 && epsilon >>= scale==0){
        epsilon = 1<<scale;
    }
}
for(int i=0; i<N; ++i){
    for(Edge &e: G[i]){
        retCost -= e.c*(e.f);
    }
}
//cerr << " -> " << retFlow << " / "
<< retCost << " bzw. " << retCost/2/N << "\n";
return make_pair(retFlow, retCost/2/N);
}
flow_t getFlow(Edge const &e){
    return G[e.to][e.rev].f;
}
};

```

## 5 String

### 5.1 KMP

```

int F[maxn];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.eb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

### 5.2 Z-value\*

```

int z[maxn];
void make_z(const string &s) {
    int l = 0, r = 0;
    for (int i = 1; i < SZ(s); ++i) {
        for (z[i] = max(0, min(r - i + 1, z[i - l]));
            i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
            ++z[i])
    }
}

```

```

;
if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
}
}

```

### 5.3 Manacher\*

```

int z[maxn]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {
    string s = "%";
    int l = 0, r = 0;
    for (char c : tmp) s.eb(c), s.eb('%');
    for (int i = 0; i < SZ(s); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < SZ(s)
            && s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
}

```

### 5.4 SAIS\*

```

namespace sfx {
bool _t[N * 2];
int SA[N * 2], H[N], RA[N];
int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
// zero based, string content MUST > 0
// SA[i]: SA[i]-th
// suffix is the i-th lexicographically smallest suffix.
// H[i]: longest
// common prefix of suffix SA[i] and suffix SA[i - 1].
void pre(int *sa, int *c, int n, int z)
{ fill_n(sa, n, 0), copy_n(c, z, x); }
void induce
    (int *sa, int *c, int *s, bool *t, int n, int z) {
    copy_n(c, z - 1, x + 1);
    for (int i = 0; i < n; ++i)
        if (sa[i] && !t[sa[i] - 1])
            sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
    copy_n(c, z, x);
    for (int i = n - 1; i >= 0; --i)
        if (sa[i] && t[sa[i] - 1])
            sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa
    , int *p, int *q, bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0,
        nmxz = -1, *nsa = sa + n, *ns = s + n, last = -1;
    fill_n(c, z, 0);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
        for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; --i)
        t[i] = (
            s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    pre(sa, c, n, z);
    for (int i = 1; i <= n - 1; ++i)
        if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    for (int i = 0; i < n; ++i)
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            bool neq = last < 0 || !equal
                (s + sa[i], s + p[q[sa[i]] + 1], s + last);
            ns[q[last = sa[i]]] = nmxz += neq;
        }
    sais(ns,
        nsa, p + nn, q + n, t + n, c + z, nn, nmxz + 1);
    pre(sa, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
void mkhei(int n) {
    for (int i = 0, j = 0; i < n; ++i) {
        if (RA[i])
            for (; _s[i + j] == _s[SA[RA[i] - 1] + j]; ++j);
        H[RA[i]] = j, j = max(0, j - 1);
    }
}
}

```

```

}
void build(int *s, int n) {
    copy_n(s, n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    copy_n(SA + 1, n, SA);
    for (int i = 0; i < n; ++i) RA[SA[i]] = i;
    mkhei(n);
}
}

```

## 5.5 Aho-Corasick Automatan

```

const int len = 400000, sigma = 26;
struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], pri[len], top;
    int newnode() {
        fill(nx[top], nx[top] + sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
    int input(
        string &s) { // return the end_node of string
        int X = 1;
        for (char c : s) {
            if (!nx[X][c - 'a']) nx[X][c - 'a'] = newnode();
            X = nx[X][c - 'a'];
        }
        return X;
    }
    void make_fl() {
        queue<int> q;
        q.push(1), fl[1] = 0;
        for (int t = 0; !q.empty(); ) {
            int R = q.front();
            q.pop(), pri[t++] = R;
            for (int i = 0; i < sigma; ++i)
                if (~nx[R][i]) {
                    int X = nx[R][i], Z = fl[R];
                    for (; Z && !nx[Z][i]; Z = fl[Z]);
                    fl[X] = Z ? nx[Z][i] : 1, q.push(X);
                }
        }
    }
    void get_v(string &s) {
        int X = 1;
        fill(cnt, cnt + top, 0);
        for (char c : s) {
            while (X && !nx[X][c - 'a']) X = fl[X];
            X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
        }
        for (int i = top - 2; i > 0; --i)
            cnt[fl[pri[i]]] += cnt[pri[i]];
    }
};

```

## 5.6 De Bruijn sequence\*

```

constexpr int maxc = 10, maxn = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[maxc * maxn]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)
                out[ptr++] = buf[i];
        } else {
            buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
            for (int j = buf[t - p] + 1; j < C; ++j)
                buf[t] = j, dfs(out, t + 1, t, ptr);
        }
    }
    void solve(int _c, int _n, int _k, int *out) {
        int p = 0;
        C = _c, N = _n, K = _k, L = N + K - 1;
        dfs(out, 1, 1, p);
        if (p < L) fill(out + p, out + L, 0);
    }
};

```

## 5.7 Extended SAM\*

```

struct exSAM {
    int len[N * 2], link[N * 2]; // maxlen, suflink
    int next[N * 2][CNUM], tot; // [0, tot), root = 0
    int lenSorted[N * 2]; // topo. order
    int cnt[N * 2]; // occurence

```

```

    int newnode() {
        fill_n(next[tot], CNUM, 0);
        len[tot] = cnt[tot] = link[tot] = 0;
        return tot++;
    }
    void init() { tot = 0, newnode(), link[0] = -1; }
    int insertSAM(int last, int c) {
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len
            [p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < CNUM; ++i)
            next[
                clone][i] = len[next[q][i]] ? next[q][i] : 0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur] = clone] = link[q];
        link[q] = clone;
        return cur;
    }
    void insert(const string &s) {
        int cur = 0;
        for (auto ch : s) {
            int &nxt = next[cur][int(ch - 'a')];
            if (!nxt) nxt = newnode();
            cnt[cur = nxt] += 1;
        }
    }
    void build() {
        queue<int> q;
        q.push(0);
        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            for (int i = 0; i < CNUM; ++i)
                if (next[cur][i])
                    q.push(insertSAM(cur, i));
        }
        vector<int> lc(tot);
        for (int i = 1; i < tot; ++i) ++lc[len[i]];
        partial_sum(ALL(lc), lc.begin());
        for (int i
            = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
    }
    void solve() {
        for (int i = tot - 2; i >= 0; --i)
            cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
    }
};

```

## 5.8 PalTree\*

```

struct palindromic_tree {
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
        // pal. suf.
        node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
            for (int i = 0; i < 26; ++i) next[i] = 0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree() : St(2), last(1), n(0) {
        St[0].fail = 1, St[1].len = -1, s.eb(-1);
    }
    inline void clear() {
        St.clear(), s.clear(), last = 1, n = 0;
        St.eb(0), St.eb(-1);
        St[0].fail = 1, s.eb(-1);
    }
    inline int get_fail(int x) {
        while (s[n - St[x].len - 1] != s[n])
            x = St[x].fail;
        return x;
    }
    inline void add(int c) {
        s.eb(c - 'a'), ++n;

```

```

int cur = get_fail(last);
if (!St[cur].next[c]) {
    int now = SZ(St);
    St.eb(St[cur].len + 2);
    St[now].fail =
        St[get_fail(St[cur].fail)].next[c];
    St[cur].next[c] = now;
    St[now].num = St[St[now].fail].num + 1;
}
last = St[cur].next[c], ++St[last].cnt;
}
inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
        St[i->fail].cnt += i->cnt;
    }
}
inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
}
};

```

## 6 Math

### 6.1 ax+by=gcd(only exgcd \*)

```

pii exgcd(int a, int b) {
    if (b == 0) return pii(1, 0);
    int p = a / b;
    pii q = exgcd(b, a % b);
    return pii(q.Y, q.X - q.Y * p);
}
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */

```

### 6.2 Floor and Ceil

```

int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }

```

### 6.3 Floor Enumeration

```

// enumerating x = floor(n / i), [l, r]
for (int l = 1, r; l <= n; l = r + 1) {
    int x = n / l;
    r = n / x;
}

```

### 6.4 Mod Min

```

// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
int mod_min(int a, int m, int l, int r) {
    if (a == 0) return l ? -1 : 0;
    if (int k = (l + a - 1) / a; k * a <= r)
        return k;
    int b = m / a, c = m % a;
    if (int y = mod_min(c, a, a - r % a, a - l % a))
        return (l + y * c + a - 1) / a + y * b;
    return -1;
}

```

### 6.5 Gaussian integer gcd

```

cpx gaussian_gcd(cpx a, cpx b) {
#define rnd
    (a, b) ((a >= 0 ? a * 2 + b : a * 2 - b) / (b * 2))
    int c = a.real() * b.real() + a.imag() * b.imag();
    int d = a.imag() * b.real() - a.real() * b.imag();
    int r = b.real() * b.real() + b.imag() * b.imag();
    if (c % r == 0 && d % r == 0) return b;
    return gaussian_gcd
        (b, a - cpx(rnd(c, r), rnd(d, r)) * b);
}

```

### 6.6 floor sum\*

```

int floor_sum(int n, int m, int a, int b) {
    int ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    int y_max
        = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum_{i=0}^{n-1} floor((a * i + b) / m) in log(n + m + a + b)

```

### 6.7 Miller Rabin\*

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : primes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(int a, int n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    int tmp = (n - 1) / ((n - 1) & (1 - n));
    int t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp >= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}

```

### 6.8 Simultaneous Equations

```

struct matrix { //m variables, n equations
    int n, m;
    fraction M[maxn][maxn + 1], sol[maxn];
    int solve() { // -1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <=
                    m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv
                < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};

```

### 6.9 Pollard Rho\*

```

map<int, int> cnt;
void PollardRho(int n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2
        == 0) return PollardRho(n / 2), ++cnt[2], void();
    int x = 2, y = 2, d = 1, p = 1;
#define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}

```

## 6.10 chineseRemainder

```
int solve(int x1, int m1, int x2, int m2) {
    int g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pii p = exgcd(m1, m2);
    int lcm = m1 * m2 * g;
    int res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}
```

## 6.11 Factorial without prime factor\*

```
// O(p^k + log^2 n), pk = p^k
int prod[maxp];
int fac_no_p(int n, int p, int pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    int rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k
```

## 6.12 QuadraticResidue\*

```
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL *
                g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL *
            f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

## 6.13 PiCount\*

```
int PrimeCount(int n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<int> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
```

```
    if (smalls[p] > smalls[p - 1]) {
        int q = p * p;
        ++pc;
        if (1LL * q * q > n) break;
        skip[p] = 1;
        for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
        int ns = 0;
        for (int k = 0; k < s; ++k) {
            int i = roughs[k];
            if (skip[i]) continue;
            int d = 1LL * i * p;
            larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
            roughs[ns++] = i;
        }
        s = ns;
        for (int j = v / p; j >= p; --j) {
            int c =
                smalls[j] - pc, e = min(j * p + p, v + 1);
            for (int i = j * p; i < e; ++i) smalls[i] -= c;
        }
    }
    for (int k = 1; k < s; ++k) {
        const int m = n / roughs[k];
        int t = larges[k] - (pc + k - 1);
        for (int l = 1; l < k; ++l) {
            int p = roughs[l];
            if (1LL * p * p > m) break;
            t -= smalls[m / p] - (pc + l - 1);
        }
        larges[0] -= t;
    }
    return larges[0];
}
```

## 6.14 Discrete Log\*

```
int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p;
}
```

## 6.15 Berlekamp Massey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(SZ(output) + 1), me, he;
    for (int f = 0, i = 1; i <= SZ(output); ++i) {
        for (int j = 0; j < SZ(me); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.eb(-k);
        for (T x : he) o.eb(x * k);
        o.resize(max(SZ(o), SZ(me)));
        for (int j = 0; j < SZ(me); ++j) o[j] += me[j];
        if (i - f + SZ(he) >= SZ(me)) he = me, f = i;
        me = o;
    }
```



```

}
return me;
}

```

## 6.16 Characteristic Polynomial

```

template <class Tp>
void hessenberg_reduction(vector<vector<Tp>> &M) {
// assert(M.size() == M[0].size());
const int N = M.size();
for (int r = 0; r < N - 2; r++) {
int piv = -1;
for (int h = r + 1; h < N; ++h) {
if (M[h][r] != 0) {
piv = h;
break;
}
}
if (piv < 0) continue;
for (int i
= 0; i < N; i++) swap(M[r + 1][i], M[piv][i]);
for (int i
= 0; i < N; i++) swap(M[i][r + 1], M[i][piv]);
const auto rin = M[r + 1][r];
for (int i = r + 2; i < N; i++) {
const auto n = M[i][r] * rin;
for (int j
= 0; j < N; j++) M[i][j] -= M[r + 1][j] * n;
for (int j
= 0; j < N; j++) M[j][r + 1] += M[j][i] * n;
}
}
}
template <class Tp> vector
<Tp> characteristic_poly(vector<vector<Tp>> M) {
hessenberg_reduction(M);
const int N = M.size();
vector<vector<Tp>> p(N + 1);
p[0] = {1};
for (int i = 0; i < N; i++) {
p[i + 1].assign(i + 2, 0);
for (int j =
0; j < i + 1; j++) p[i + 1][j + 1] += p[i][j];
for (int j = 0; j
< i + 1; j++) p[i + 1][j] -= p[i][j] * M[i][i];
Tp betas = 1;
for (int j = i - 1; j >= 0; j--) {
betas *= M[j + 1][j];
Tp hb = -M[j][i] * betas;
for (int k = 0;
k < j + 1; k++) p[i + 1][k] += hb * p[j][k];
}
}
return p[N];
}

```

## 6.17 Primes

```

/* 12721 13331 14341 75577 123457 222557
556679 999983 1097774749 1076767633 100102021
999997771 1001010013 1000512343 987654361 999991231
999888733 98789101 987777733 999991921 1010101333
1010102101 1000000000039 100000000000037
2305843009213693951 4611686018427387847
9223372036854775783 18446744073709551557 */

```

## 6.18 Theorem

- Cramer's rule

$$\begin{aligned} ax+by &= e \\ cx+dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed-bf}{ad-bc} \\ y &= \frac{af-ec}{ad-bc} \end{aligned}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

- Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

- Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

- Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each *labeled* vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)! \dots (d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of *labeled* forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

- Erdős–Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if

$$d_1 + \dots + d_n \text{ is even and } \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } 1 \leq k \leq n.$$

- Gale–Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$

is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

- Fulkerson–Chen–Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs

with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and

$$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$$

- Pick's theorem

For simple polygon, when points are all integer, we have  $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$ .

- Möbius inversion formula

$$\begin{aligned} f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

- Spherical cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos\theta)(1 - \cos\theta)^2/3$ .
- Area  $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos\theta)$ .

- Lagrange multiplier

- Optimize  $f(x_1, \dots, x_n)$  when  $k$  constraints  $g_i(x_1, \dots, x_n) = 0$ .
- Lagrangian function  $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$ .
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

- Nearest points of two skew lines

- Line 1:  $v_1 = p + t_1 d_1$
- Line 2:  $v_2 = p + t_2 d_2$
- $n = d_1 \times d_2$
- $n_1 = d_1 \times n$
- $n_2 = d_2 \times n$
- $c_1 = p_1 + \frac{(p_2 - p_1) \cdot n_2}{d_1 \cdot n_2} d_1$
- $c_2 = p_2 + \frac{(p_1 - p_2) \cdot n_1}{d_2 \cdot n_1} d_2$

## 6.19 Estimation

- Estimation

- The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200000 for  $n < 1e19$ .
- The number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands. 1, 1, 2, 3, 5, 7, 11, 15, 22, 30 for  $n = 0 \sim 9$ , 627 for  $n = 20$ ,  $\sim 2e5$  for  $n = 50$ ,  $\sim 2e8$  for  $n = 100$ .
- Total number of partitions of  $n$  distinct elements:  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, \dots$

## 6.20 Euclidean Algorithms

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$$

$$= \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 6.21 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$  j:s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$  j:s s.t.

$$\pi(j) \geq j, k \text{ j:s s.t. } \pi(j) > j.$$

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 6.22 Tips for Generating Functions

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x)' \Rightarrow n a_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x) \Rightarrow n a_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{n-1}{i} x^i$

## 7 Polynomial

### 7.1 Fast Fourier Transform

```
template<int maxn>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[maxn];
    FFT() {
        for (int i = 0; i < maxn; ++i) {
            double arg = 2 * PI * i / maxn;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans
        (val_t *a, int n, bool inv = false); // see NTT;
        // remember to replace LL with val_t
};
```

### 7.2 Number Theory Transform\*

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int maxn, int P, int RT> //maxn must be 2^k
struct NTT {
    int w[maxn];
    int mpow(int a, int n);
    int minv(int a) { return mpow(a, P - 2); }
    NTT() {
        int dw = mpow(RT, (P - 1) / maxn);
        w[0] = 1;
        for (int
            i = 1; i < maxn; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(int *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(int
        *a, int n, bool inv = false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = maxn / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int
                    j = i, x = 0; j < i + dl; ++j, x += dx) {
                    int tmp = a[j + dl] * w[x] % P;
                    if ((a[j
                        + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            int invn = minv(n);
            for (int
                i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};
```

### 7.3 Fast Walsh Transform\*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[
    N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
```

```
// c_k = \sum_{i | j = k, i & j = 0} a_i * b_j
int n = 1 << L;
for (int i = 1; i < n; ++i)
    ct[i] = ct[i & (i - 1)] + 1;
for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
        for (int x = 0; x < n; ++x)
            h[i][x] += f[j][x] * g[i - j][x];
for (int i = 0; i <= L; ++i)
    fwt(h[i], n, -1);
for (int i = 0; i < n; ++i)
    c[i] = h[ct[i]][i];
}
```

## 7.4 Polynomial Operation

```
#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int maxn, int P, int RT> // maxn = 2^k
struct Poly : vector<int> { // coefficients in [0, P)
    using vector<int>::vector;
    static NTT<maxn, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<int>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev()
    { return reverse(data(), data() + n()), *this; }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if
            (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
        return *this;
    }
    Poly& imul(int k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int m = 1;
        while (m < n() + rhs.n() - 1) m <= 1;
        Poly X(*this, m), Y(rhs, m);
        ntt(X.data(), m), ntt(Y.data(), m);
        fi(0, m) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), m, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
        if (n() == 1) return {ntt.minv((*this)[0])};
        int m = 1;
        while (m < n() * 2) m <= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
        Poly Y(*this, m);
        ntt(Xi.data(), m), ntt(Y.data(), m);
        fi(0, m) {
            Xi[i] = (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] % P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), m, true);
        return Xi.isz(n());
    }
    Poly Sqrt()
    const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
        if (n()
            == 1) return {QuadraticResidue((*this)[0], P)};
        Poly
            X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        return
            X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
    }
    pair<Poly, Poly> DivMod
    (const Poly &rhs) const { // (rhs.)back() != 0
        if (n() < rhs.n()) return {0, *this};
        const int m = n() - rhs.n() + 1;
        Poly X(rhs); X.irev().isz(m);
        Poly Y(*this); Y.irev().isz(m);
        Poly Q = Y.Mul(X.Inv()).isz(m).irev();
        X = rhs.Mul(Q), Y = *this;
        fi(0, n()) if ((Y[i] - X[i]) < 0) Y[i] += P;
        return {Q, Y.isz(max(1, rhs.n() - 1))};
    }
    Poly Dx() const {
```

```
Poly ret(n() - 1);
    fi(0,
        ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n())
        ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<int> _eval(const
    vector<int> &x, const vector<Poly> &up) const {
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m *
        2) down[i] = down[i / 2].DivMod(up[i]).second;
    down[1] = Poly(up[1])
        .irev().isz(n()).Inv().irev()._tmul(m, *this);
    fi(2, m * 2) down[i]
        = up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
    vector<int> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<int> &x) {
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = m - 1; i
        > 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
    return up;
}
vector<int>
    > Eval(const vector<int> &x) const { // 1e5, 1s
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector
    <int> &x, const vector<int> &y) { // 1e5, 1.4s
    const int m = (int)x.size();
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<int> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m -
        1; i > 0; --i) down[i] = down[i * 2].Mul(up[i
            * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n())
        if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(int k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (int)n()) >= n()) return Poly(n());
    if (!k) return Poly({1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const int c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul
        (k % P).Exp().imul(c).irev().isz(n()).irev();
}
static int LinearRecursion
    (const vector<int> &a, const vector
    <int> &coef, int n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly{1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
```

```

    if (n % 2) W = W.Mul(M).DivMod(C).second;
    n /= 2, M = M.Mul(M).DivMod(C).second;
}
int ret = 0;
fi(0, k) ret = (ret + W[i] * a[i]) % P;
return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.5 Value Polynomial

```

struct Poly {
    mint base; // f(x) = poly[x - base]
    vector<mint> poly;
    Poly(mint b = 0, mint x = 0): base(b), poly(1, x) {}
    mint get_val(const mint &x) {
        if (x >= base && x < base + SZ(poly))
            return poly[x - base];
        mint rt = 0;
        vector<mint> lmul(SZ(poly), 1), rmul(SZ(poly), 1);
        for (int i = 1; i < SZ(poly); ++i)
            lmul[i] = lmul[i - 1] * (x - (base + i - 1));
        for (int i = SZ(poly) - 2; i >= 0; --i)
            rmul[i] = rmul[i + 1] * (x - (base + i + 1));
        for (int i = 0; i < SZ(poly); ++i)
            rt += poly[i] * ifac[i] * inegfac
                [SZ(poly) - 1 - i] * lmul[i] * rmul[i];
        return rt;
    }
    void raise() { // g(x) = sigma{base:x} f(x)
        if (SZ(poly) == 1 && poly[0] == 0)
            return;
        mint nw = get_val(base + SZ(poly));
        poly.eb(nw);
        for (int i = 1; i < SZ(poly); ++i)
            poly[i] += poly[i - 1];
    }
};

```

## 7.6 Newton's Method

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

# 8 Geometry

## 8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double eps = 1e-8;
pdd operator+(pdd a, pdd b)
{ return pdd(a.X + b.X, a.Y + b.Y); }
pdd operator-(pdd a, pdd b)
{ return pdd(a.X - b.X, a.Y - b.Y); }
pdd operator*(pdd a, double b)
{ return pdd(a.X * b, a.Y * b); }
pdd operator/(pdd a, double b)
{ return pdd(a.X / b, a.Y / b); }
double dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
double cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(pdd a)
{ return dot(a, a); }
double abs(pdd a)
{ return sqrt(dot(a, a)); }
int sign(double a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pdd a, pdd b, pdd c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pdd p1, pdd p2, pdd p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pdd p1, pdd p2, pdd p3) {

```

```

    if (!collinearity(p1, p2, p3)) return 0;
    return sign(dot(p1 - p3, p2 - p3)) <= 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return p4
        * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0
        , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(
        cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r

```

## 8.2 PointSegDist\*

```

double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(dot(q1 - q0,
        p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

## 8.3 Heart

```

pdd circenter
    (pdd p0, pdd p1, pdd p2) { // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
    double m = 2. * (x1 * y2 - y1 * x2);
    center.X = (x1 * x1
        * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
    center.Y = (x1 * x2
        * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
}
pdd incenter
    (pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
    double a =
        abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }

```

## 8.4 point in circle

```

// return q'
    s relation with circumcircle of tri(p[0],p[1],p[2])
bool in_cc(const array<pii, 3> &p, pii q) {
    __int128 det = 0;
    for (int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) *
            cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}

```

## 8.5 Convex hull\*

```
void hull(vector<pii> &dots) { // n=1 => ans = {}
    sort(dots.begin(), dots.end());
    vector<pii> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1,
              t = SZ(ans); i < SZ(dots); ans.pb(dots[i++]))
            while (SZ(ans) > t && ori(
                (ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
                ans.pb();
    ans.pb(), ans.swap(dots);
}
```

## 8.6 PointInConvex\*

```
bool PointInConvex
    (const vector<pii> &C, pii p, bool strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if (SZ(C) == 0) return false;
    if (SZ(C) < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori(
        (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}
```

## 8.7 TangentPointToHull\*

```
/* The point should be strictly out of hull
   return arbitrary point on the tangent line */
pii get_tangent(vector<pii> &C, pii p) {
    auto gao = [&](int s) {
        return cyc_tsearch(SZ(C), [&](int x, int y) {
            { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

## 8.8 Intersection of line and convex

```
int TangentDir(vector<pii> &C, pii dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
}
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pii a, pii b, vector<pii> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if (cmpL(A) < 0 || cmpL(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for (int t = l; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(t) ? l : r) = m;
        }
        return (l + !cmpL(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if (res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if (!
        cmpL(res.X) && !cmpL(res.Y)) // along side i, i+1
        switch ((res.X - res.Y + n + 1) % n) {
            case 0: return pii(res.X, res.X);
            case 2: return pii(res.Y, res.Y);
        }
    /* crossing sides (i, i+1) and (j, j+1)
       crossing corner i is treated as side (i, i+1)
       returned
       in the same order as the line hits the convex */
    return res;
} // convex cut: (r, l]
```

## 8.9 minMaxEnclosingRectangle\*

```
const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pii> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
    hull(dots);
    double Max = 0, Min = INF, deg;
    int n = SZ(dots);
    dots.pb(dots[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        pii nw = vec(i + 1);
        while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
            u = (u + 1) % n;
        while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
            r = (r + 1) % n;
        if (!l) l = (r + 1) % n;
        while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
            l = (l + 1) % n;
        Min = min(Min, (double)(dot(nw, vec(r)) - dot(
            nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
        deg = acos(dot(diff(r, l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
        deg = (qi - deg) / 2;
        Max = max(Max, abs(diff(
            r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
    }
    return pdd(Min, Max);
}
```

## 8.10 VectorInPoly\*

```
// ori(a, b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(pii a, pii b, pii c, pii p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector {cur, p} in counter-clockwise order prv, cur, nxt
bool inside(pii prv, pii cur, pii nxt, pii p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}
```

## 8.11 PolyUnion\*

```
double rat(pii a, pii b) {
    return sign(
        (b.X) ? (double)a.X / b.X : (double)a.Y / b.Y;
} // all poly. should be ccw
double polyUnion(vector<vector<pii>> &poly) {
    double res = 0;
    for (auto &p : poly)
        for (int a = 0; a < SZ(p); ++a) {
            pii A = p[a], B = p[(a + 1) % SZ(p)];
            vector
                <pair<double, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < SZ(q); ++b) {
                    pii C = q[b], D = q[(b + 1) % SZ(q)];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        double sa = cross(D - C, A - C), sb = cross(D - C, B - C);
                        segs.emplace_back(
                            (sa / (sa - sb), sign(sc - sd)));
                    }
                    if (!sc && !sd &&
                        &q < &p && sign(dot(B - A, D - C)) > 0) {
                        segs.emplace_back(rat(C - A, B - A), 1);
                        segs.emplace_back(rat(D - A, B - A), -1);
                    }
                }
            }
        }
    sort(ALL(segs));
    for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    for (int j = 1; j < SZ(segs); ++j) {
        if (!cnt) sum += segs[j].X - segs[j - 1].X;
        cnt += segs[j].Y;
    }
}
```



```

    res += cross(A, B) * sum;
}
return res / 2;
}

```

## 8.12 PolyCut

```

vector<pdd> cut(vector<pdd> poly, pdd s, pdd e) {
    vector<pdd> res;
    for (int i = 0; i < SZ(poly); ++i) {
        pdd cur
            = poly[i], prv = i ? poly[i - 1] : poly.back();
        bool side = ori(s, e, cur) < 0;
        if (side != (ori(s, e, prv) < 0))
            res.eb(intersect(s, e, cur, prv));
        if (side)
            res.eb(cur);
    }
    return res;
}

```

## 8.13 Trapezoidalization

```

template<class T>
struct SweepLine {
    struct cmp {
        cmp(const SweepLine &swp): swp(swp) {}
        bool operator()(int a, int b) const {
            if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)
                return swp.slope_cmp(a, b);
            return swp.get_y(a) + swp.eps < swp.get_y(b);
        }
    };
    const SweepLine &swp;
    T curTime;
    vector<Line> base;
    multiset<int, cmp> sweep;
    multiset<pair<T, int>> event;
    vector<typename multiset<int, cmp>::iterator> its;
    vector<typename multiset<pair<T, int>>::iterator> eits;
    bool slope_cmp(int a, int b) const {
        assert(a != -1);
        if (b == -1) return 0;
        return sign(cross(base
            [a].Y - base[a].X, base[b].Y - base[b].X)) < 0;
    }
    T get_y(int idx) const {
        if (idx == -1) return curQ;
        Line l = base[idx];
        if (l.X.X == l.Y.X) return l.Y.Y;
        return ((curTime - l.X.X) * l.Y.Y
            + (l.Y.X - curTime) * l.X.Y) / (l.Y.X - l.X.X);
    }
    void insert(int idx) {
        its[idx] = sweep.insert(idx);
        if (its[idx] != sweep.begin())
            update_event(*prev(its[idx]));
        update_event(idx);
        event.emplace(base[idx].Y.X, idx + 2 * SZ(base));
    }
    void erase(int idx) {
        assert(eits[idx] == event.end());
        auto p = sweep.erase(its[idx]);
        its[idx] = sweep.end();
        if (p != sweep.begin())
            update_event(*prev(p));
    }
    void update_event(int idx) {
        if (eits[idx] != event.end())
            event.erase(eits[idx]);
        eits[idx] = event.end();
        auto nxt = next(its[idx]);
        if (nxt ==
            sweep.end() || !slope_cmp(idx, *nxt)) return;
        auto t = intersect(base[idx].
            X, base[idx].Y, base[*nxt].X, base[*nxt].Y).X;
        if (t + eps < curTime || t
            >= min(base[idx].Y.X, base[*nxt].Y.X)) return;
        eits[idx] = event.emplace(t, idx + SZ(base));
    }
    void swp(int idx) {
        assert(eits[idx] != event.end());
        eits[idx] = event.end();
        int nxt = *next(its[idx]);
        swap((int&)*its[idx], (int&)*its[nxt]);
    }
}

```

```

swap(its[idx], its[nxt]);
if (its[nxt] != sweep.begin())
    update_event(*prev(its[nxt]));
update_event(idx);
}
// only expected to call the functions below
SweepLine(T t, T e, vector
    <Line> vec): _cmp(*this), curTime(t), eps(e)
    , curQ(), base(vec), sweep(_cmp), event(), its(SZ
    (vec), sweep.end()), eits(SZ(vec), event.end()) {
    for (int i = 0; i < SZ(base); ++i) {
        auto &[p, q] = base[i];
        if (p > q) swap(p, q);
        if (p.X <= curTime && curTime <= q.X)
            insert(i);
        else if (curTime < p.X)
            event.emplace(p.X, i);
    }
}
void setTime(T t, bool ers = false) {
    assert(t >= curTime);
    while (!event.empty() && event.begin()->X <= t) {
        auto [et, idx] = *event.begin();
        int s = idx / SZ(base);
        idx %= SZ(base);
        if (abs(et - t) <= eps && s == 2 && !ers) break;
        curTime = et;
        event.erase(event.begin());
        if (s == 2) erase(idx);
        else if (s == 1) swp(idx);
        else insert(idx);
    }
    curTime = t;
}
T nextEvent() {
    if (event.empty()) return INF;
    return event.begin()->X;
}
int lower_bound(T y) {
    curQ = y;
    auto p = sweep.lower_bound(-1);
    if (p == sweep.end()) return -1;
    return *p;
}
};

```

## 8.14 Polar Angle Sort\*

```

int cmp(pii a, pii b, bool same = true) {
#define is_neg(k) (
    sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
    int A = is_neg(a), B = is_neg(b);
    if (A != B)
        return A < B;
    if (sign(cross(a, b)) == 0)
        return same ? abs2(a) < abs2(b) : -1;
    return sign(cross(a, b)) > 0;
}

```

## 8.15 Half plane intersection\*

```

pii area_pair(Line a, Line b)
{ return pii(cross(a.Y
    - a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return (__int128)
        a02Y * a12X - (__int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(ALL(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    for (auto p : arr) {
        if (cmp(
            dq.back().Y - dq.back().X, p.Y - p.X, 0) == -1)
            continue;
        while (SZ(dq)
            ) >= 2 && !isin(p, dq[SZ(dq) - 2], dq.back())
    }
}

```



```

    dq.pb();
    while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
        dq.pop_front();
    dq.eb(p);
}
while (SZ(dq)
    >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back()))
    dq.pb();
while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
    dq.pop_front();
return vector<Line>(ALL(dq));
}

```

## 8.16 RotatingSweepLine

```

void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps), m = 0;
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1));
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            if (i != j) line[m++] = pii(i, j);
    sort(ALL(line), [&](pii a, pii b) {
        return cmp(ps[a.Y] - ps[a.X], ps[b.Y] - ps[b.X]);
    }); // cmp(): polar angle compare
    iota(ALL(id), 0);
    sort(ALL(id), [&](int a, int b) {
        if (ps[a.Y] != ps[b.Y]) return ps[a.Y] < ps[b.Y];
        return ps[a] < ps[b];
    }); // initial order, since (1, 0) is the smallest
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // do something
        tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]
            ]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
    }
}

```

## 8.17 Minimum Enclosing Circle\*

```

pdd Minimum_Enclosing_Circle
(vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter
                                (dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

## 8.18 Intersection of two circles\*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
    if (d < max(r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
    pdd u = (o1 + o2) * 0.5
        + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

## 8.19 Intersection of polygon and circle\*

```

// Divides into multiple triangle, and sum up
const double PI=acos(-1);
double _area(pdd pa, pdd eb, double r){
    if(abs(pa)<abs(eb)) swap(pa, eb);

```

```

    if(abs(eb)<eps) return 0;
    double S, h, theta;
    double a=abs(eb),b=abs(pa),c=abs(eb-pa);
    double cosB = dot(eb,eb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,eb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const
    vector<pdd> poly,const pdd &O,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)]-O,r);
    return fabs(S);
}

```

## 8.20 Intersection of line and circle\*

```

vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
    pdd p = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
    double s = cross(b - a, c - a), h2 = r * r - s * s / abs2(b - a);
    if (h2 < 0) return {};
    if (h2 == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}

```

## 8.21 Tangent line of two circles

```

vector<Line>
    > go(const Cir& c1, const Cir& c2, int sign1){
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = abs2(c1.O - c2.O);
    if (sign1*d_sq == 0) return ret;
    double d = sqrt(d_sq);
    pdd v = (c2.O - c1.O) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        pdd n = pdd(v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X);
        pdd p1 = c1.O + n * c1.R;
        pdd p2 = c2.O + n * (c2.R * sign1);
        if (sign(p1.X - p2.X) == 0 and
            sign(p1.Y - p2.Y) == 0)
            p2 = p1 + perp(c2.O - c1.O);
        ret.eb(Line(p1, p2));
    }
    return ret;
}

```

## 8.22 CircleCover\*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[N];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang;
        }
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x)

```

```

{return sign(abs(a.O - b.O) - a.R - b.R) > x;}
bool contain(Cir &a, Cir &b, int x)
{return sign(a.R - b.R - abs(a.O - b.O)) > x;}
bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign
        (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].
            R) == 0 && i < j)) && contain(c[i], c[j], -1);
}
void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                disjunct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
        int E = 0, cnt = 1;
        for(int j = 0; j < C; ++j)
            if(j != i && overlap[j][i])
                ++cnt;
        for(int j = 0; j < C; ++j)
            if(i != j && g[i][j]) {
                pdd aa, bb;
                CCinter(c[i], c[j], aa, bb);
                double A =
                    atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                double B =
                    atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                eve[E++] = Teve
                    (bb, B, 1), eve[E++] = Teve(aa, A, -1);
                if(B > A) ++cnt;
            }
        if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
        else{
            sort(eve, eve + E);
            eve[E] = eve[0];
            for(int j = 0; j < E; ++j){
                cnt += eve[j].add;
                Area[cnt]
                    += cross(eve[j].p, eve[j + 1].p) * .5;
                double theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] += (theta
                    - sin(theta)) * c[i].R * c[i].R * .5;
            }
        }
    }
}
};

```

## 8.23 3Dpoint\*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double
        _y = 0, double _z = 0): x(_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(Point p1, Point p2)
{ return
    Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point operator+(Point p1, Point p2)
{ return
    Point(p1.x + p2.x, p1.y + p2.y, p1.z + p2.z); }
Point operator*(Point p1, double v)
{ return Point(p1.x * v, p1.y * v, p1.z * v); }
Point operator/(Point p1, double v)
{ return Point(p1.x / v, p1.y / v, p1.z / v); }
Point cross(Point p1, Point p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z
    * p2.x - p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(Point p1, Point p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(Point a)
{ return sqrt(dot(a, a)); }
Point cross3(Point a, Point b, Point c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
//Azimuthal
    angle (longitude) to x-axis in interval [-pi, pi]

```

```

double phi(Point p) { return atan2(p.y, p.x); }
//Zenith
    angle (latitude) to the z-axis in interval [0, pi]
double theta(Point p)
{ return atan2(sqrt(p.x * p.x + p.y * p.y), p.z); }
Point masscenter(Point a, Point b, Point c, Point d)
{ return (a + b + c + d) / 4; }
pdd proj(Point a, Point b, Point c, Point u) {
    // proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}
Point
    rotate_around(Point p, double angle, Point axis) {
    double s = sin(angle), c = cos(angle);
    Point u = axis / abs(axis);
    return u
        * dot(u, p) * (1 - c) + p * c + cross(u, p) * s;
}

```

## 8.24 Convexhull3D\*

```

struct convex_hull_3D {
    struct Face {
        int a, b, c;
        Face(int ta, int tb, int tc): a(ta), b(tb), c(tc) {}
    }; // return the faces with pt indexes
    vector<Face> res;
    vector<Point> P;
    convex_hull_3D(const vector<Point> &P): res(), P(P) {
        // all points coplanar case will WA, O(n^2)
        int n = SZ(P);
        if (n <= 2) return; // be careful about edge case
        // ensure first 4 points are not coplanar
        swap(P[1], *find_if(ALL(P), [&](
            auto p) { return sign(abs2(P[0] - p)) != 0; }));
        swap(P[2], *find_if(ALL(P), [&](auto p) { return
            sign(abs2(cross3(p, P[0], P[1]))) != 0; }));
        swap(P[3], *find_if(ALL(P), [&](auto p) { return
            sign(volume(P[0], P[1], P[2], p)) != 0; }));
        vector<vector<int>> flag(n, vector<int>(n));
        res.emplace_back(0, 1, 2); res.emplace_back(2, 1, 0);
        for (int i = 3; i < n; ++i) {
            vector<Face> next;
            for (auto f : res) {
                int d
                    = sign(volume(P[f.a], P[f.b], P[f.c], P[i]));
                if (d <= 0) next.emplace_back(f);
                int ff = (d > 0) - (d < 0);
                flag[f.a][
                    f.b] = flag[f.b][f.c] = flag[f.c][f.a] = ff;
            }
            for (auto f : res) {
                auto F = [&](int x, int y) {
                    if (flag[x][y] > 0 && flag[y][x] <= 0)
                        next.emplace_back(x, y, i);
                };
                F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
            }
            res = next;
        }
    }
    bool same(Face s, Face t) {
        if (sign(volume
            (P[s.a], P[s.b], P[s.c], P[t.a])) != 0) return 0;
        if (sign(volume
            (P[s.a], P[s.b], P[s.c], P[t.b])) != 0) return 0;
        if (sign(volume
            (P[s.a], P[s.b], P[s.c], P[t.c])) != 0) return 0;
        return 1;
    }
    int polygon_face_num() {
        int ans = 0;
        for (int i = 0; i < SZ(res); ++i)
            ans += none_of(res.begin(), res.begin()
                + i, [&](Face g) { return same(res[i], g); });
        return ans;
    }
    double get_volume() {
        double ans = 0;
        for (auto f : res)

```

```

    ans +=
        volume(Point(0, 0, 0), P[f.a], P[f.b], P[f.c]);
    return fabs(ans / 6);
}

double get_dis(Point p, Face f) {
    Point p1 = P[f.a], p2 = P[f.b], p3 = P[f.c];
    double a = (p2.y - p1.y)
        * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
    double b = (p2.z - p1.z)
        * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
    double c = (p2.x - p1.x)
        * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
    double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
    return fabs(a * p.x + b *
        p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
}

// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case

```

## 8.25 Minkowski Sum\*

```

vector<pii> Minkowski(vector<pii> A, vector<pii> B) {
    hull(A), hull(B);
    vector<pii> C(1, A[0] + B[0]), s1, s2;
    for (int i = 0; i < SZ(A); ++i)
        s1.eb(A[(i + 1) % SZ(A)] - A[i]);
    for (int i = 0; i < SZ(B); ++i)
        s2.eb(B[(i + 1) % SZ(B)] - B[i]);
    for (int i = 0, j = 0; i < SZ(A) || j < SZ(B);)
        if (j >= SZ
            (B) || (i < SZ(A) && cross(s1[i], s2[j]) >= 0))
            C.eb(B[j % SZ(B)] + A[i++]);
        else
            C.eb(A[i % SZ(A)] + B[j++]);
    return hull(C), C;
}

```

## 9 Else

### 9.1 Cyclic Ternary Search\*

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}

```

### 9.2 Mo's Alogrithm(With modification)

```

/*
Mo's Algorithm With modification
Block: N^{2/3}, Complexity: N^{5/3}
*/
struct Query {
    int L, R, LBid, RBid, T;
    Query(int l, int r, int t):
        L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        if (RBid != q.RBid) return RBid < q.RBid;
        return T < b.T;
    }
};

void solve(vector<Query> query) {
    sort(ALL(query));
    int L=0, R=0, T=-1;
    for (auto q : query) {
        while (T < q.T) addTime(L, R, ++T); // TODO
        while (T > q.T) subTime(L, R, T--); // TODO
        while (R < q.R) add(arr[++R]); // TODO
        while (L > q.L) add(arr[--L]); // TODO
        while (R > q.R) sub(arr[R--]); // TODO
        while (L < q.L) sub(arr[L++]); // TODO
        // answer query
    }
}

```

## 9.3 Mo's Alogrithm On Tree

```

/*
Mo's Algorithm On Tree
Preprocess:
1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<maxn> inset
*/
struct Query {
    int L, R, LBid, lca;
    Query(int u, int v) {
        int c = LCA(u, v);
        if (c == u || c == v)
            q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
        else if (out[u] < in[v])
            q.lca = c, q.L = out[u], q.R = in[v];
        else
            q.lca = c, q.L = out[v], q.R = in[u];
        q.Lid = q.L / blk;
    }
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        return R < q.R;
    }
};

void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}

void solve(vector<Query> query) {
    sort(ALL(query));
    int L = 0, R = 0;
    for (auto q : query) {
        while (R < q.R) flip(ord[++R]);
        while (L > q.L) flip(ord[--L]);
        while (R > q.R) flip(ord[R--]);
        while (L < q.L) flip(ord[L++]);
        if (~q.lca) add(arr[q.lca]);
        // answer query
        if (~q.lca) sub(arr[q.lca]);
    }
}

```

## 9.4 Additional Mo's Algorithm Trick

### • Mo's Algorithm With Addition Only

- Sort queries same as the normal Mo's algorithm.
- For each query  $[l, r]$ :
- If  $l/blk = r/blk$ , brute-force.
- If  $l/blk \neq curL/blk$ , initialize  $curL := (l/blk + 1) \cdot blk$ ,  $curR := curL - 1$
- If  $r > curR$ , increase  $curR$
- decrease  $curL$  to fit  $l$ , and then undo after answering

### • Mo's Algorithm With Offline Second Time

- Require: Changing answer  $\equiv$  adding  $f([l, r], r+1)$ .
- Require:  $f([l, r], r+1) = f([1, r], r+1) - f([1, l], r+1)$ .
- Part1: Answer all  $f([1, r], r+1)$  first.
- Part2: Store  $curR \rightarrow R$  for  $curL$  (reduce the space to  $O(N)$ ), and then answer them by the second offline algorithm.
- Note: You must do the above symmetrically for the left boundaries.

## 9.5 Hilbert Curve

```

int hilbert(int n, int x, int y) {
    int res = 0;
    for (int s = n / 2; s >= 1; s /= 2) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 1ll * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
}
// n = 2^k

```

## 9.6 DynamicConvexTrick\*

```
// only works for integer coordinates!! maintain max
struct Line {
    mutable int a, b, p;
    bool operator
        <(const Line &rhs) const { return a < rhs.a; }
    bool operator<(int x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const int kInf = 1e18;
    int Div(int a,
            int b) { return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x
            ->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(int a, int b) {
        auto z = insert({a, b, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin
            ()) && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin
            ()) && (--x)->p >= y->p) isect(x, erase(y));
    }
    int query(int x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

## 9.7 All LCS\*

```
void all_lcs(string s, string t) { // 0-base
    vector<int> h(SZ(t));
    iota(ALL(h), 0);
    for (int a = 0; a < SZ(s); ++a) {
        int v = -1;
        for (int c = 0; c < SZ(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}
```

## 9.8 DLX\*

```
#define TRAV(i, link, start)
    for (int i = link[start]; i != start; i = link[i])
template<
    bool E> // E: Exact, NN: num of 1s, RR: num of rows
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN],
        rw[NN], cl[NN], bt[NN], s[NN], head, sz, ans;
    int rows, columns;
    bool vis[NN];
    bitset<RR> sol, cur; // not sure
    void remove(int c) {
        if (E) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        TRAV(i, dn, c) {
            if (E) {
                TRAV(j, rg, i)
                    up[dn[j]]
                        = up[j], dn[up[j]] = dn[j], --s[cl[j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        TRAV(i, up, c) {
            if (E) {
                TRAV(j, lt, i)
                    ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (E) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
```

```
rows = 0, columns = c;
for (int i = 0; i < c; ++i) {
    up[i] = dn[i] = bt[i] = i;
    lt[i] = i == 0 ? c : i - 1;
    rg[i] = i == c - 1 ? c : i + 1;
    s[i] = 0;
}
rg[c] = 0, lt[c] = c - 1;
up[c] = dn[c] = -1;
head = c, sz = c + 1;
}
void insert(const vector<int> &col) {
    if (col.empty()) return;
    int f = sz;
    for (int i = 0; i < (int)col.size(); ++i) {
        int c = col[i], v = sz++;
        dn[bt[c]] = v;
        up[v] = bt[c], bt[c] = v;
        rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
        rw[v] = rows, cl[v] = c;
        ++s[c];
        if (i > 0) lt[v] = v - 1;
    }
    ++rows, lt[f] = sz - 1;
}
int h() {
    int ret = 0;
    fill_n(vis, sz, false);
    TRAV(x, rg, head) {
        if (vis[x]) continue;
        vis[x] = true, ++ret;
        TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
    }
    return ret;
}
void dfs(int dep) {
    if (dep + (E ? 0 : h()) >= ans) return;
    if (rg[head]
        ] == head) return sol = cur, ans = dep, void();
    if (dn[rg[head]] == rg[head]) return;
    int w = rg[head];
    TRAV(x, rg, head) if (s[x] < s[w]) w = x;
    if (E) remove(w);
    TRAV(i, dn, w) {
        if (!E) remove(i);
        TRAV(j, rg, i) remove(E ? cl[j] : j);
        cur.set(rw[i]), dfs(dep + 1), cur.reset(rw[i]);
        TRAV(j, lt, i) restore(E ? cl[j] : j);
        if (!E) restore(i);
    }
    if (E) restore(w);
}
int solve() {
    for (int i = 0; i < columns; ++i)
        dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, sol.reset(), dfs(0);
    return ans;
}
```

## 9.9 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.

## 9.10 AdaptiveSimpson\*

```
template<typename Func, typename d = double>
struct Simpson {
    using pdd = pair<d, d>;
    Func f;
    pdd mix(pdd l, pdd r, optional<d> fm = {}) {
        d h = (r.X - l.X) / 2, v = fm.value_or(f(l.X + h));
        return {v, h / 3 * (l.Y + 4 * v + r.Y)};
    }
    d eval(pdd l, pdd r, d fm, d eps) {
```

```

pdd m((l.X + r.X) / 2, fm);
d s = mix(l, r, fm).second;
auto [flm, sl] = mix(l, m);
auto [fmr, sr] = mix(m, r);
d delta = sl + sr - s;
if (abs(delta)
    ) <= 15 * eps) return sl + sr + delta / 15;
return eval(l, m, flm, eps / 2) +
    eval(m, r, fmr, eps / 2);
}
d eval(d l, d r, d eps) {
    return eval
        ({l, f(l)}, {r, f(r)}, f((l + r) / 2), eps);
}
d eval2(d l, d r, d eps, int k = 997) {
    d h = (r - l) / k, s = 0;
    for (int i = 0; i < k; ++i, l += h)
        s += eval(l, l + h, eps / k);
    return s;
}
};
template<typename Func>
Simpson<Func> make_simpson(Func f) { return {f}; }

```

## 9.11 Simulated Annealing

```

double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans:
    answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans)
        ) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}

```

## 9.12 Tree Hash\*

```

ull seed;
ull shift(ull x) {
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    return x;
}
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u])
        if (i != f)
            sum += shift(dfs(i, u));
    return sum;
}

```

## 9.13 Binary Search On Fraction

```

struct Q {
    int p, q;
    Q go(Q b, int d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(int N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        int len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !!len;
    }
    return dir ? hi : lo;
}

```

## 9.14 Min Plus Convolution\*

```

// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution
    (vector<int> &a, vector<int> &b) {

```

```

int n = SZ(a), m = SZ(b);
vector<int> c(n + m - 1, INF);
auto dc = [&](auto Y, int l, int r, int jl, int jr) {
    if (l > r) return;
    int mid = (l + r) / 2, from = -1, &best = c[mid];
    for (int j = jl; j <= jr; ++j)
        if (int i = mid - j; i >= 0 && i < n)
            if (best > a[i] + b[j])
                best = a[i] + b[j], from = j;
    Y(Y, l,
        mid - 1, jl, from), Y(Y, mid + 1, r, from, jr);
};
return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}

```

## 9.15 Bitset LCS

```

cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';

```

# 10 Python

## 10.1 Misc

```

from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction
    ('3.14159').limit_denominator(10).numerator # 22

```

# 11 Yamada

