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## 1 Basic

### 1.1 readchar

```
#include <unistd.h>
const int S = 65536;
inline char RC() {
    static char buf[S], *p = buf, *q = buf;
    return p == q and (q = (p = buf) + read(0, buf, S)) == buf ? -1 : *p++;
}
inline int RI() {
    static char c; int a;
    while (((c = RC()) < '0' or c > '9') and c != '-' and c != '.');
    if (c == '-') { a = 0; while ((c = RC()) >= '0' and c <= '9') a *= 10, a -= c ^ '0'; }
    else { a = c ^ '0'; while ((c = RC()) >= '0' and c <= '9') a *= 10, a += c ^ '0'; }
    return a;
}
```

### 1.2 Black Magic

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2; // max heap
    h1.push(1), h1.push(3), h2.push(2), h2.push(4);
    h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
    tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> st;
    tree<int, int, less<int>, rb_tree_tag, tree_order_statistics_node_update> mp;
    for (int x : {0, 2, 3, 4}) st.insert(x);
    cout << *st.find_by_order(2) << st.order_of_key(1) << endl; //31
    rope<char> *root[10]; // nsqrt(n)
    root[0] = new rope<char>();
    root[1] = new rope<char>("root[0]");
    // root[1]->insert(pos, 'a');
    // root[1]->at(pos); 0-base
    // root[1]->erase(pos, size);
}
// __int128_t, __float128_t
// for (int i = bs._Find_first()); i < bs.size(); i = bs._Find_next(i));
```

## 2 Graph

### 2.1 BCC Vertex\*

```
vector<int> G[N]; // 1-base
vector<int> nG[N * 2], bcc[N];
int low[N], dfn[N], Time;
int bcc_id[N], bcc_cnt; // 1-base
bool is_cut[N]; // whether is av
bool cir[N * 2];
int st[N], top;

void dfs(int u, int pa = -1) {
    int child = 0;
    low[u] = dfn[u] = ++Time;
    st[top++] = u;
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_cut[u] = 1;
                bcc[++bcc_cnt].clear();
                int t;
                do {
                    bcc_id[t = st[--top]] = bcc_cnt;
                    bcc[bcc_cnt].eb(t);
                } while (t != v);
            } while (t != v);
            bcc_id[u] = bcc_cnt;
            bcc[bcc_cnt].eb(u);
        }
    } else if (dfn[v] < dfn[u] && v != pa)
        low[u] = min(low[u], dfn[v]);
    if (pa == -1 && child < 2) is_cut[u] = 0;
}

void bcc_init(int n) { // TODO: init {nG, cir}[1..2n]
    Time = bcc_cnt = top = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), dfn[i] = bcc_id[i] = is_cut[i] = 0;
}

void bcc_solve(int n) {
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i);
    // block-cut tree
    for (int i = 1; i <= n; ++i)
        if (is_cut[i])
            bcc_id[i] = ++bcc_cnt, cir[bcc_cnt] = 1;
    for (int i = 1; i <= bcc_cnt && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_cut[j])
                nG[i].eb(bcc_id[j]), nG[bcc_id[j]].eb(i);
}
```

### 2.2 Bridge\*

```

int low[N], dfn[N], Time; // 1-base
vector<pii> G[N], edge;
vector<bool> is_bridge;

void init(int n) {
    Time = 0;
    for (int i = 1; i <= n; ++i)
        G[i].clear(), low[i] = dfn[i] = 0;
}

void add_edge(int a, int b) {
    G[a].eb(pii(b, SZ(edge))), G[b].eb(pii(a, SZ(edge)));
    edge.eb(pii(a, b));
}

void dfs(int u, int f) {
    dfn[u] = low[u] = ++Time;
    for (auto i : G[u])
        if (!dfn[i.X])
            dfs(i.X, i.Y), low[u] = min(low[u], low[i.X]);
        else if (i.Y != f) low[u] = min(low[u], dfn[i.X]);
    if (low[u] == dfn[u] && f != -1) is_bridge[f] = 1;
}

void solve(int n) {
    is_bridge.resize(SZ(edge));
    for (int i = 1; i <= n; ++i)
        if (!dfn[i]) dfs(i, -1);
}

```

## 2.3 2SAT (SCC)\*

```

struct SAT { // 0-base
    int low[N], dfn[N], bln[N], n, Time, nScc;
    bool instack[N], istrue[N];
    stack<int> st;
    vector<int> G[N], SCC[N];
    void init(int _n) {
        n = _n; // assert(n * 2 <= N);
        for (int i = 0; i < n + n; ++i) G[i].clear();
    }
    void add_edge(int a, int b) { G[a].eb(b); }
    int rv(int a) {
        if (a >= n) return a - n;
        return a + n;
    }
    void add_clause(int a, int b) {
        add_edge(rv(a), b), add_edge(rv(b), a);
    }
    void dfs(int u) {
        dfn[u] = low[u] = ++Time;
        instack[u] = 1, st.push(u);
        for (int i : G[u])
            if (!dfn[i])
                dfs(i), low[u] = min(low[u], low[i]);
            else if (instack[i] && dfn[i] < dfn[u])
                low[u] = min(low[u], dfn[i]);
        if (low[u] == dfn[u]) {
            int tmp;
            do {
                tmp = st.top(), st.pop();
                instack[tmp] = 0, bln[tmp] = nScc;
            } while (tmp != u);
            ++nScc;
        }
    }
    bool solve() {
        Time = nScc = 0;
        for (int i = 0; i < n + n; ++i)
            SCC[i].clear(), low[i] = dfn[i] = bln[i] = 0;
        for (int i = 0; i < n + n; ++i)
            if (!dfn[i]) dfs(i);
        for (int i = 0; i < n + n; ++i) SCC[bln[i]].eb(i);
        for (int i = 0; i < n; ++i) {
            if (bln[i] == bln[i + n]) return false;
            istrue[i] = bln[i] < bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};

```

## 2.4 MinimumMeanCycle\*

```

int road[N][N]; // input here
struct MinimumMeanCycle {

```

```

int dp[N + 5][N], n;
pii solve() {
    int a = -1, b = -1, L = n + 1;
    for (int i = 2; i <= L; ++i)
        for (int k = 0; k < n; ++k)
            for (int j = 0; j < n; ++j)
                dp[i][j] =
                    min(dp[i - 1][k] + road[k][j], dp[i][j]);
    for (int i = 0; i < n; ++i) {
        if (dp[L][i] >= INF) continue;
        int ta = 0, tb = 1;
        for (int j = 1; j < n; ++j)
            if (dp[j][i] < INF &&
                ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                ta = dp[L][i] - dp[j][i], tb = L - j;
        if (ta == 0) continue;
        if (a == -1 || a * tb > ta * b) a = ta, b = tb;
    }
    if (a != -1) {
        int g = __gcd(a, b);
        return pii(a / g, b / g);
    }
    return pii(-1LL, -1LL);
}

void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
}
};

```

## 2.5 Virtual Tree\*

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].eb(st[top]), --top;
    if (st[top] != p)
        vG[p].eb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].eb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

## 2.6 Maximum Clique Dyn\*

```

struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(ALL(r),
                [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;

```

```

    if (k > rgt) cs[++rgt + 1].reset();
    cs[k][p] = 1;
    if (k < lft) r[tp++] = p;
}
for (int k = lft; k <= rgt; ++k)
    for (int p = cs[k]._Find_first
          (); p < N; p = cs[k]._Find_next(p))
        r[tp] = p, c[tp] = k, ++tp;
dfs(r, c, l + 1, mask);
}
void dfs(vector<
    int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pb(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr;
        for (int i : r) if (G[p][i]) nr.eb(i);
        if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
        else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pb(), --q;
    }
}
int solve() {
    vector<int> r(n);
    ans = q = 0, iota(ALL(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
}
};

```

## 2.7 Minimum Steiner Tree\*

```

struct SteinerTree { // 0-base
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcst[N]; // the cost of vertexs
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) {
            fill_n(dst[i], n, INF);
            dst[i][i] = vcst[i] = 0;
        }
    }
    void chmin(int &x, int val) {
        x = min(x, val);
    }
    void add_edge(int ui, int vi, int wi) {
        chmin(dst[ui][vi], wi);
    }
    void shortest_path() {
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                for (int j = 0; j < n; ++j)
                    chmin(dst[i][j], dst[i][k] + dst[k][j]);
    }
    int solve(const vector<int> &ter) {
        shortest_path();
        int t = SZ(ter), full = (1 << t) - 1;
        for (int i = 0; i <= full; ++i)
            fill_n(dp[i], n, INF);
        copy_n(vkst, n, dp[0]);
        for (int msk = 1; msk <= full; ++msk) {
            if (!(msk & (msk - 1))) {
                int who = __lg(msk);
                for (int i = 0; i < n; ++i)
                    dp[msk][i] = vcst[ter[who]] + dst[ter[who]][i];
            }
            for (int i = 0; i < n; ++i)
                for (int sub = (
                    msk - 1) & msk; sub; sub = (sub - 1) & msk)
                    chmin(dp[msk][i],
                        dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
            for (int i = 0; i < n; ++i) {
                tdst[i] = INF;
                for (int j = 0; j < n; ++j)
                    chmin(tdst[i], dp[msk][j] + dst[j][i]);
            }
            copy_n(tdst, n, dp[msk]);
        }
        return *min_element(dp[full], dp[full] + n);
    }
}; // O(V 3^AT + V^2 2^AT)

```

## 2.8 Dominator Tree\*

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].eb(v), rG[v].eb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])
                if (v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].eb(i);
            for (auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] =
                    semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for (int i = 2; i <= Time; ++i) {
            if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
            tree[id[idom[i]]].eb(id[i]);
        }
    }
};

```

## 2.9 Minimum Arborescence\*

```

/* TODO
DSU: disjoint set
- DSU(n), .boss(x), .Union(x, y)
min_heap<
    T, Info>: min heap for type {T, Info} with lazy tag
- .push({w, i}),
  .top(), .join(heap), .pop(), .empty(), .add_lazy(v)
*/
struct E { int s, t; int w; }; // 0-base
vector<int> dmst(const vector<E> &e, int n, int root) {
    vector<min_heap<int, int>> h(n * 2);
    for (int i = 0; i < SZ(e); ++i)
        h[e[i].t].push({e[i].w, i});
    DSU dsu(n * 2);
    vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p]
            == -1 || v[p] == i; p = dsu.boss(e[r[p]].s)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].add_lazy(-h[q].top().X);
                    pa[q] = p, dsu.Union(p, q), h[p].join(h[q]);
                } while ((q = dsu.boss(e[r[q]].s)) != p);
            }
        }
    }
}

```

```

    v[p] = i;
    while (!h[p].empty() && dsu.boss(e[h[p].top().Y].s) == p)
        h[p].pop();
    if (h[p].empty()) return {}; // no solution
    r[p] = h[p].top().Y;
}
vector<int> ans;
for (int i = pc - 1; i >= 0; i--) if (i != root && v[i] != n) {
    for (int f = e[r[i]].t; ~f && v[f] != n; f = pa[f])
        v[f] = n;
    ans.pb(r[i]);
}
return ans; // default minimize, returns edgeid array
} // O(Ef(E)), f(E) from min_heap

```

## 2.10 Vizing's theorem\*

```

namespace vizing { // returns
    edge coloring in adjacent matrix G. 1 - based
const int N = 105;
int C[N][N], G[N][N], X[N], vst[N], n;
void init(int _n) { n = _n;
    for (int i = 0; i <= n; ++i)
        for (int j = 0; j <= n; ++j)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pii> &E) {
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; ++X[u]);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    fill_n(X + 1, n, 1);
    for (int t = 0; t < SZ(E); ++t) {
        int u = E[t].X, v0 = E[t].Y, v = v0, c0 = X[u], c = c0, d;
        vector<pii> L;
        fill_n(vst + 1, n, 0);
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for (int a = SZ(L) - 1; a >= 0; --a) c = color(u, L[a].X, c);
            else if (!C[u][d]) for (int a = SZ(L) - 1; a >= 0; --a) color(u, L[a].X, L[a].Y);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
        if (!G[u][v0]) {
            for (; v; v = flip(v, c, d), swap(c, d));
            if (int a; C[u][c0]) {
                for (a = SZ(L) - 2; a >= 0 && L[a].Y != c; --a);
                for (; a >= 0; --a) color(u, L[a].X, L[a].Y);
            }
            else --t;
        }
    }
} // namespace vizing

```

## 2.11 Minimum Clique Cover\*

```

struct Clique_Cover { // 0-base, O(n^2*n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
}

```

```

void add_edge(int u, int v) {
    E[u] |= 1 << v, E[v] |= 1 << u;
}
int solve() {
    for (int i = 0; i < n; ++i)
        co[1 << i] = E[i] | (1 << i);
    co[0] = (1 << n) - 1;
    dp[0] = (n & 1) * 2 - 1;
    for (int i = 1; i < (1 << n); ++i) {
        int t = i & -i;
        dp[i] = -dp[i ^ t];
        co[i] = co[i ^ t] & co[t];
    }
    for (int i = 0; i < (1 << n); ++i)
        co[i] = (co[i] & i) == i;
    fwt(co, 1 << n, 1);
    for (int ans = 1; ans < n; ++ans) {
        int sum = 0; // probabilistic
        for (int i = 0; i < (1 << n); ++i)
            sum += (dp[i] * co[i]);
        if (sum) return ans;
    }
    return n;
}
}

```

## 2.12 NumberofMaximalClique\*

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsn = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsn++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsn, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
}
}

```

## 3 Data Structure

### 3.1 Interval Container\*

```

/* Add and
    remove intervals from a set of disjoint intervals.
* Will merge the added interval with
    any overlapping intervals in the set when adding.
* Intervals are [inclusive, exclusive). */
set<pii>::
    iterator addInterval(set<pii>& is, int L, int R) {
        if (L == R) return is.end();
        auto it = is.lower_bound({L, R}), before = it;
        while (it != is.end() && it->X <= R) {
            R = max(R, it->Y);
            before = it = is.erase(it);
        }
        if (it != is.begin() && (--it)->Y >= L) {
            L = min(L, it->X);
            R = max(R, it->Y);
        }
    }
}

```

```

    is.erase(it);
}
return is.insert(before, pii(L, R));
}
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->Y;
    if (it->X == L) is.erase(it);
    else (int&)it->Y = L;
    if (R != r2) is.emplace(R, r2);
}

```

## 3.2 Heavy light Decomposition

```

struct Heavy_light_Decomposition { // 1-base
    int n, ulink[N], deep[N], mxson[N], w[N], pa[N];
    int t, pl[N], data[N], dt[N], bln[N], edge[N], et;
    vector<pii> G[N];
    void init(int _n) {
        n = _n, t = 0, et = 1;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b, int w) {
        G[a].eb(pii(b, et));
        G[b].eb(pii(a, et));
        edge[et++] = w;
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (auto &i : G[u])
            if (i.X != f) {
                dfs(i.X, u, d), w[u] += w[i.X];
                if (w[mxson[u]] < w[i.X]) mxson[u] = i.X;
                else bln[i.Y] = u, dt[u] = edge[i.Y];
            }
    }
    void cut(int u, int link) {
        data[pl[u] = t++] = dt[u], ulink[u] = link;
        if (!mxson[u]) return;
        cut(mxson[u], link);
        for (auto i : G[u])
            if (i.X != pa[u] && i.X != mxson[u])
                cut(i.X, i.X);
    }
    void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
    int query(int a, int b) {
        int ta = ulink[a], tb = ulink[b], re = 0;
        while (ta != tb)
            if (deep[ta] < deep[tb])
                /*query*/, tb = ulink[b = pa[tb]];
            else /*query*/, ta = ulink[a = pa[ta]];
        if (a == b) return re;
        if (pl[a] > pl[b]) swap(a, b);
        /*query*/
        return re;
    }
};

```

## 3.3 Centroid Decomposition\*

```

struct Cent_Dec { // 1-base
    vector<pii> G[N];
    pii info[N]; // store info. of itself
    pii upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    int dis[lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].eb(pii(b, w)), G[b].eb(pii(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pii e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
};

```

```

}
void dfs(int u, int f, int d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pii e : G[u])
        if (!done[e.X] && e.X != f)
            dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pii e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pii(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
int query(int u) {
    int rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
};

```

## 3.4 Link cut tree\*

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int _val = 0)
        : val(_val), sum(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void give_tag(int r)
        { if (r) swap(ch[0], ch[1]), rev ^= 1; }
    void push() {
        if (ch[0] != &nil) ch[0]->give_tag(rev);
        if (ch[1] != &nil) ch[1]->give_tag(rev);
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(), x->pull();
};

```

```

}
void splay(Splay *x) {
    vector<Splay *> splayVec;
    for (Splay *q = x;; q = q->f) {
        splayVec.eb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay *access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x, x->setCh(q, 1), q = x);
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x, x->rev ^ 1;
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {
    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay *get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay *lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x, x->val = val, x->pull());
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}
}

```

### 3.5 KDTree

```

namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &)> f =
        [dep](const point &a, const point &b) {
            if (dep & 1) return a.x < b.x;
            else return a.y < b.y;
        };
    int m = (l + r) >> 1;
    nth_element(p + l, p + m, p + r, f);
    xl[m] = xr[m] = p[m].x;
    yl[m] = yr[m] = p[m].y;
    lc[m] = build(l, m, dep + 1);
    if (~lc[m]) {
        xl[m] = min(xl[m], xl[lc[m]]);
        xr[m] = max(xr[m], xr[lc[m]]);
        yl[m] = min(yl[m], yl[lc[m]]);
        yr[m] = max(yr[m], yr[lc[m]]);
    }
}
}

```

```

rc[m] = build(m + 1, r, dep + 1);
if (~rc[m]) {
    xl[m] = min(xl[m], xl[rc[m]]);
    xr[m] = max(xr[m], xr[rc[m]]);
    yl[m] = min(yl[m], yl[rc[m]]);
    yr[m] = max(yr[m], yr[rc[m]]);
}
return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

## 4 Flow/Matching

### 4.1 Kuhn Munkres\*

```

struct KM { // 0-base, maximum matching
    int w[N][N], hl[N], hr[N], slk[N];
    int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
    bool vl[N], vr[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            fill_n(w[i], n, -INF);
    }
    void add_edge(int a, int b, int wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] = 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill_n(slk,
            , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
        ql = qr = 0, qu[qr++] = s, vr[s] = 1;
        for (int d; ; ) {
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] && slk
                        [x] >= (d = hl[x] + hr[y] - w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!Check(x)) return;
                    }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
        }
    }
}

```



```

    }
    for (int x = 0; x < n; ++x)
        if (!vl[x] && !slk[x] && !Check(x)) return;
}
int solve() {
    fill_n(fl,
        , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
    for (int i = 0; i < n; ++i)
        hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    int res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

## 4.2 MincostMaxflow\*

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        int from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    int dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, int d, int cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                int d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax(e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s,
        int _t, int &flow, int &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; BellmanFord(); copy_n(dis, n, pot)) {
            for (int i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
            for (int i = t; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                e.flow += up[t], G[e.to][e.rev].flow -= up[t];
            }
        }
    }
    void init(int _n) {
        n = _n, fill_n(pot, n, 0);
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int cap, int cost) {
        G[a].eb(Edge{a, b, cap, 0, cost, SZ(G[b])});
        G[b].eb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
    }
};

```

## 4.3 Maximum Simple Graph Matching

```

struct GenMatch { // 1-base
    int V, pr[N];
    bool el[N][N], inq[N], inp[N], inb[N];
    int st, ed, nb, bk[N], djs[N], ans;
    void init(int _V) {
        V = _V;
        for (int i = 0; i <= V; ++i) {
            for (int j = 0; j <= V; ++j) el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
    }
    void add_edge(int u, int v) {

```

```

        el[u][v] = el[v][u] = 1;
    }
    int lca(int u, int v) {
        fill_n(inp, V + 1, 0);
        while (1)
            if (u = djs[u], inp[u] = true, u == st) break;
            else u = bk[pr[u]];
        while (1)
            if (v = djs[v], inp[v]) return v;
            else v = bk[pr[v]];
        return v;
    }
    void upd(int u) {
        for (int v; djs[u] != nb;) {
            v = pr[u], inb[djs[u]] = inb[djs[v]] = true;
            u = bk[v];
            if (djs[u] != nb) bk[u] = v;
        }
    }
    void blo(int u, int v, queue<int> &qe) {
        nb = lca(u, v), fill_n(inb, V + 1, 0);
        upd(u), upd(v);
        if (djs[u] != nb) bk[u] = v;
        if (djs[v] != nb) bk[v] = u;
        for (int tu = 1; tu <= V; ++tu)
            if (inb[djs[tu]])
                if (djs[tu] = nb, !inq[tu])
                    qe.push(tu), inq[tu] = 1;
    }
    void flow() {
        fill_n(inq + 1, V, 0), fill_n(bk + 1, V, 0);
        iota(djs + 1, djs + V + 1, 1);
        queue<int> qe;
        qe.push(st), inq[st] = 1, ed = 0;
        while (!qe.empty()) {
            int u = qe.front();
            qe.pop();
            for (int v = 1; v <= V; ++v)
                if (el[u][v] && djs[u] != djs[v] &&
                    pr[u] != v) {
                    if ((v == st) ||
                        (pr[v] > 0 && bk[pr[v]] > 0)) {
                        blo(u, v, qe);
                    } else if (!bk[v]) {
                        if (bk[v] = u, pr[v] > 0) {
                            if (!inq[pr[v]]) qe.push(pr[v]);
                        } else {
                            return ed = v, void();
                        }
                    }
                }
        }
    }
    void aug() {
        for (int u = ed, v, w; u > 0;)
            v = bk[u], w = pr[v], pr[v] = u, pr[u] = v,
            u = w;
    }
    int solve() {
        fill_n(pr, V + 1, 0), ans = 0;
        for (int u = 1; u <= V; ++u)
            if (!pr[u])
                if (st = u, flow(), ed > 0) aug(), ++ans;
        return ans;
    }
};

```

## 4.4 Minimum Weight Matching (Clique version)\*

```

struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    int edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, int w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u) {
        stk[tp++] = u, onstk[u] = 1;
        for (int v = 0; v < n; ++v)
            if (!onstk[v] && match[u] != v) {
                int m = match[v];
                if (dis[m] >

```

```

        dis[u] - edge[v][m] + edge[u][v]) {
        dis[m] = dis[u] - edge[v][m] + edge[u][v];
        onstk[v] = 1, stk[tp++] = v;
        if (onstk[m] || SPFA(m)) return 1;
        --tp, onstk[v] = 0;
    }
}
onstk[u] = 0, --tp;
return 0;
}
int solve() { // find a match
    for (int i = 0; i < n; ++i) match[i] = i ^ 1;
    while (1) {
        int found = 0;
        fill_n(dis, n, 0);
        fill_n(onstk, n, 0);
        for (int i = 0; i < n; ++i)
            if (tp = 0, !onstk[i] && SPFA(i))
                for (found = 1; tp >= 2; ) {
                    int u = stk[--tp];
                    int v = stk[--tp];
                    match[u] = v, match[v] = u;
                }
        if (!found) break;
    }
    int ret = 0;
    for (int i = 0; i < n; ++i)
        ret += edge[i][match[i]];
    return ret >> 1;
}
};

```

## 4.5 SW-mincut

```

struct SW { // global min cut,  $O(V^3)$ 
#define REP for (int i = 0; i < n; ++i)
static const int MXN = 514, INF = 2147483647;
int vst[MXN], edge[MXN][MXN], wei[MXN];
void init(int n) {
    REP fill_n(edge[i], n, 0);
}
void addEdge(int u, int v, int w) {
    edge[u][v] += w; edge[v][u] += w;
}
int search(int &s, int &t, int n) {
    fill_n(vst, n, 0), fill_n(wei, n, 0);
    s = t = -1;
    int mx, cur;
    for (int j = 0; j < n; ++j) {
        mx = -1, cur = 0;
        REP if (wei[i] > mx) cur = i, mx = wei[i];
        vst[cur] = 1, wei[cur] = -1;
        s = t; t = cur;
        REP if (!vst[i]) wei[i] += edge[cur][i];
    }
    return mx;
}
int solve(int n) {
    int res = INF;
    for (int x, y; n > 1; n--) {
        res = min(res, search(x, y, n));
        REP edge[i][x] = (edge[x][i] += edge[y][i]);
        REP {
            edge[y][i] = edge[n - 1][i];
            edge[i][y] = edge[i][n - 1];
        } // edge[y][y] = 0;
    }
    return res;
}
} sw;

```

## 4.6 BoundedFlow\*(Dinic\*)

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
    }
}

```

```

G[u].eb(edge{v, rcap, lcap, SZ(G[v])});
G[v].eb(edge{u, 0, 0, SZ(G[u]) - 1});
}
void add_edge(int u, int v, int cap) {
    G[u].eb(edge{v, cap, 0, SZ(G[v])});
    G[v].eb(edge{u, 0, 0, SZ(G[u]) - 1});
}
int dfs(int u, int cap) {
    if (u == t || !cap) return cap;
    for (int &i = cur[u]; i < SZ(G[u]); ++i) {
        edge &e = G[u][i];
        if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
            int df = dfs(e.to, min(e.cap - e.flow, cap));
            if (df) {
                e.flow += df, G[e.to][e.rev].flow -= df;
                return df;
            }
        }
    }
    dis[u] = -1;
    return 0;
}
bool bfs() {
    fill_n(dis, n + 3, -1);
    queue<int> q;
    q.push(s), dis[s] = 0;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (edge &e : G[u])
            if (!dis[e.to] && e.flow != e.cap)
                q.push(e.to), dis[e.to] = dis[u] + 1;
    }
    return dis[t] != -1;
}
int maxflow(int _s, int _t) {
    s = _s, t = _t;
    int flow = 0, df;
    while (bfs()) {
        fill_n(cur, n + 3, 0);
        while ((df = dfs(s, INF))) flow += df;
    }
    return flow;
}
bool solve() {
    int sum = 0;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            add_edge(n + 1, i, cnt[i]), sum += cnt[i];
        else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
    if (sum != maxflow(n + 1, n + 2)) sum = -1;
    for (int i = 0; i < n; ++i)
        if (cnt[i] > 0)
            G[n + 1].pb(), G[i].pb();
        else if (cnt[i] < 0)
            G[i].pb(), G[n + 2].pb();
    return sum != -1;
}
int solve(int _s, int _t) {
    add_edge(_t, _s, INF);
    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pb(), G[_s].pb(), x;
}
};

```

## 4.7 Gomory Hu tree\*

```

MaxFlow Dinic;
int g[maxn];
void GomoryHu(int n) { // 0-base
    fill_n(g, n, 0);
    for (int i = 1; i < n; ++i) {
        Dinic.reset();
        add_edge(i, g[i], Dinic.maxflow(i, g[i]));
        for (int j = i + 1; j <= n; ++j)
            if (g[j] == g[i] && ~Dinic.dis[j])
                g[j] = i;
    }
}

```

## 4.8 Minimum Cost Circulation\*

```

struct MinCostCirculation { // 0-base
    struct Edge {
        int from, to, cap, fcap, flow, cost, rev;
    };
}

```



```

} *past[N];
vector<Edge> G[N];
int dis[N], inq[N], n;
void BellmanFord(int s) {
    fill_n(dis, n, INF), fill_n(inq, n, 0);
    queue<int> q;
    auto relax = [&](int u, int d, Edge *e) {
        if (dis[u] > d) {
            dis[u] = d, past[u] = e;
            if (!inq[u]) inq[u] = 1, q.push(u);
        }
    };
    relax(s, 0, 0);
    while (!q.empty()) {
        int u = q.front();
        q.pop(), inq[u] = 0;
        for (auto &e : G[u])
            if (e.cap > e.flow)
                relax(e.to, dis[u] + e.cost, &e);
    }
}
void try_edge(Edge &cur) {
    if (cur.cap > cur.flow) return ++cur.cap, void();
    BellmanFord(cur.to);
    if (dis[cur.from] + cur.cost < 0) {
        ++cur.flow, --G[cur.to][cur.rev].flow;
        for (int i = cur.from; past[i]; i = past[i]->from) {
            auto &e = *past[i];
            ++e.flow, --G[e.to][e.rev].flow;
        }
    }
    ++cur.cap;
}
void solve(int mxlg) {
    for (int b = mxlg; b >= 0; --b) {
        for (int i = 0; i < n; ++i)
            for (auto &e : G[i])
                e.cap *= 2, e.flow *= 2;
        for (int i = 0; i < n; ++i)
            for (auto &e : G[i])
                if (e.fcap >> b & 1)
                    try_edge(e);
    }
}
void init(int _n) { n = _n;
    for (int i = 0; i < n; ++i) G[i].clear();
}
void add_edge(int a, int b, int cap, int cost) {
    G[a].eb(Edge{a, b, 0, cap, 0, cost, SZ(G[b]) + (a == b)});
    G[b].eb(Edge{b, a, 0, 0, 0, -cost, SZ(G[a]) - 1});
}
} mcmf; // O(VE * ElogC)

```

## 4.9 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1

- For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
- For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
- Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$
  - Construct a max flow model, let  $K$  be the sum of all weights
  - Connect source  $s \rightarrow v, v \in G$  with capacity  $K$
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$
  - For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  - $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
  - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
  - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
  - Find the minimum weight perfect matching on  $G'$ .
- Project selection problem
  - If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
  - Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
  - The mincut is equivalent to the maximum profit of a subset of projects.
- Dual of minimum cost maximum flow
  - Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
  - If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\min \sum_{uv} w_{uv} f_{uv} - f_{uv} \geq -c_{uv} \Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv})$$

$$\sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0$$

## 4.10 MCMF HLPP

```

// Push-Relabel
// implementation of the cost-scaling algorithm
// Runs in O( <max_flow
// * log(V * max_edge_cost)) = O( V^3 * log(V * C))
// Operates on integers

#include <bits/stdc++.h>
using namespace std;

template<typename flow_t = int, typename cost_t = int>
struct mcsFlow {
    struct Edge {
        cost_t c;
        flow_t f;
        int to, rev;
        Edge(int _to, cost_t _c, flow_t _f,
            , int _rev): c(_c), f(_f), to(_to), rev(_rev) {}
    };
    const
        cost_t INFCOST = numeric_limits<cost_t>::max()/2;
    const
        cost_t INFFLOW = numeric_limits<flow_t>::max()/2;
    cost_t epsilon;
    int N, S, T;
    vector<vector<Edge>> G;
    vector<unsigned int> isEnqueued, state;
    mcsFlow(int _N, int _S,
        int _T): epsilon(0), N(_N), S(_S), T(_T), G(_N) {}
    void add_edge(int a, int b, cost_t cost, flow_t cap) {
        if (a == b) { assert(cost == 0); return; }
        cost *= N; // to preserve integer-values
        epsilon = max(epsilon, abs(cost));
        assert(a >= 0 && a < N && b >= 0 && b < N);
        G[a].emplace_back(b, cost, cap, G[b].size());
        G[b].emplace_back(a, -cost, 0, G[a].size() - 1);
    }
    flow_t calc_max_flow() { // Dinic max-flow
        vector<flow_t> dist(N), state(N);
        vector<Edge*> path(N);
        auto cmp = [](Edge*a, Edge*b) { return a->f < b->f; };
        flow_t addFlow, retflow = 0;
        do {
            fill(dist.begin(), dist.end(), -1);
            dist[S] = 0;
            auto head = state.begin(), tail = state.begin();
            for (*tail++ = S; head != tail; ++head) {
                for (Edge const& e : G[*head]) {
                    if (e.f && dist[e.to] == -1) {

```

```

        dist[e.to] = dist[*head]+1;
        *tail++=e.to;
    }
}
addFlow = 0;
fill(state.begin(), state.end(), 0);
auto top = path.begin();
Edge dummy(S, 0, INFLOW, -1);
*top++ = &dummy;
while(top != path.begin()){
    int n = (*prev(top))->to;
    if(n==T){
        auto next_top
            = min_element(path.begin(), top, cmp);
        flow_t flow = (*next_top)->f;
        while(--top!=path.begin()){
            Edge &e=*top, &f=G[e.to][e.rev];
            e.f-=flow;
            f.f+=flow;
        }
        addFlow+=1;
        retflow+=flow;
        top = next_top;
        continue;
    }
    for(int &i=state[n], i_max
        = G[n].size(), need = dist[n]+1; ++i){
        if(i==i_max){
            dist[n]=-1;
            --top;
            break;
        }
        if(dist[G[n][i].to] == need && G[n][i].f){
            *top++ = &G[n][i];
            break;
        }
    }
}
}
}
while(addFlow);
return retflow;
}
vector<flow_t> excess;
vector<cost_t> h;
void push(Edge &e, flow_t amt){
    //cerr << "push: "
    << G[e.to][e.rev].to << " -> " << e.to << " ("
    << e.f << "/" << e.c << ") : " << amt << "\n";
    if(e.f < amt) amt=e.f;
    e.f-=amt;
    excess[e.to]+=amt;
    G[e.to][e.rev].f+=amt;
    excess[G[e.to][e.rev].to]-=amt;
}
void relabel(int vertex){
    cost_t newHeight = -INFCOST;
    for(unsigned int i=0; i<G[vertex].size(); ++i){
        Edge const&e = G[vertex][i];
        if(e.f && newHeight < h[e.to]-e.c){
            newHeight = h[e.to] - e.c;
            state[vertex] = i;
        }
    }
    h[vertex] = newHeight - epsilon;
}
const int scale=2;
pair<flow_t, cost_t> minCostFlow(){
    cost_t retCost = 0;
    for(int i=0; i<N; ++i){
        for(Edge &e:G[i]){
            retCost += e.c*(e.f);
        }
    }
    //find feasible flow
    flow_t retFlow = calc_max_flow();
    excess.resize(N); h.resize(N);
    queue<int> q;
    isEnqueued.assign(N, 0); state.assign(N, 0);
    for(epsilons; epsilon>=scale){
        //refine
        fill(state.begin(), state.end(), 0);
        for(int i=0; i<N; ++i)
            for(auto &e:G[i])
                if(h[i]
                    + e.c - h[e.to] < 0 && e.f) push(e, e.f);
        for(int i=0; i<N; ++i){

```

```

            if(excess[i]>0){
                q.push(i);
                isEnqueued[i]=1;
            }
        }
        while(!q.empty()){
            int cur=q.front(); q.pop();
            isEnqueued[cur]=0;
            // discharge
            while(excess[cur]>0){
                if(state[cur] == G[cur].size()){
                    relabel(cur);
                }
                for(unsigned int &i=state[
                    cur], max_i = G[cur].size(); i<max_i; ++i){
                    Edge &e=G[cur][i];
                    if(h[cur] + e.c - h[e.to] < 0){
                        push(e, excess[cur]);
                        if(excess
                            [e.to]>0 && isEnqueued[e.to]==0){
                            q.push(e.to);
                            isEnqueued[e.to]=1;
                        }
                        if(excess[cur]==0) break;
                    }
                }
            }
        }
        if(epsilon>1 && epsilon>>scale==0){
            epsilon = 1<<scale;
        }
    }
    for(int i=0; i<N; ++i){
        for(Edge &e:G[i]){
            retCost -= e.c*(e.f);
        }
    }
    //cerr << " -> " << retFlow << " / "
    << retCost << " bzw. " << retCost/2/N << "\n";
    return make_pair(retFlow, retCost/2/N);
}
flow_t getFlow(Edge const &e){
    return G[e.to][e.rev].f;
}
};

```

## 5 String

### 5.1 KMP

```

int F[maxn];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

### 5.2 Z-value\*

```

int z[maxn];
void make_z(const string &s) {
    int l = 0, r = 0;
    for (int i = 1; i < SZ(s); ++i) {
        for (z[i] = max(0, min(r - i + 1, z[i - l]));
            i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
            ++z[i]);
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

### 5.3 Manacher\*

```

int z[maxn]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {

```

```

string s = "%";
int l = 0, r = 0;
for (char c : tmp) s.eb(c), s.eb('%');
for (int i = 0; i < SZ(s); ++i) {
    z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
    while (i - z[i] >= 0 && i + z[i] < SZ(s)
            && s[i + z[i]] == s[i - z[i]]) ++z[i];
    if (z[i] + i > r) r = z[i] + i, l = i;
}
}

```

## 5.4 SAIS\*

```

namespace sfx {
bool _t[N * 2];
int SA[N * 2], H[N], RA[N];
int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
// zero based, string content MUST > 0
// SA[i]: SA[i]-th
// suffix is the i-th lexicographically smallest suffix.
// H[i]: longest
// common prefix of suffix SA[i] and suffix SA[i - 1].
void pre(int *sa, int *c, int n, int z)
{ fill_n(sa, n, 0), copy_n(c, z, x); }
void induce
    (int *sa, int *c, int *s, bool *t, int n, int z) {
    copy_n(c, z - 1, x + 1);
    for (int i = 0; i < n; ++i)
        if (sa[i] && !t[sa[i] - 1])
            sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
    copy_n(c, z, x);
    for (int i = n - 1; i >= 0; --i)
        if (sa[i] && t[sa[i] - 1])
            sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
}
void sais(int *s, int *sa
    , int *p, int *q, bool *t, int *c, int n, int z) {
    bool uniq = t[n - 1] = true;
    int nn = 0,
        nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
    fill_n(c, z, 0);
    for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
    partial_sum(c, c + z, c);
    if (uniq) {
        for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
        return;
    }
    for (int i = n - 2; i >= 0; --i)
        t[i] = (
            s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    pre(sa, c, n, z);
    for (int i = 1; i <= n - 1; ++i)
        if (t[i] && !t[i - 1])
            sa[--x[s[i]]] = p[q[i] = nn++] = i;
    induce(sa, c, s, t, n, z);
    for (int i = 0; i < n; ++i)
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            bool neq = last < 0 || !equal
                (s + sa[i], s + p[q[sa[i]] + 1], s + last);
            ns[q[last = sa[i]]] = nmzx += neq;
        }
    sais(ns,
        nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    pre(sa, c, n, z);
    for (int i = nn - 1; i >= 0; --i)
        sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
    induce(sa, c, s, t, n, z);
}
void mkhei(int n) {
    for (int i = 0, j = 0; i < n; ++i) {
        if (RA[i])
            for (; _s[i + j] == _s[SA[RA[i] - 1] + j]; ++j);
        H[RA[i]] = j, j = max(0, j - 1);
    }
}
void build(int *s, int n) {
    copy_n(s, n, _s), _s[n] = 0;
    sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
    copy_n(SA + 1, n, SA);
    for (int i = 0; i < n; ++i) RA[SA[i]] = i;
    mkhei(n);
}
}

```

## 5.5 Aho-Corasick Automatan

```
const int len = 400000, sigma = 26;
```

```

struct AC_Automatan {
int nx[len][sigma], fl[len], cnt[len], pri[len], top;
int newnode() {
    fill(nx[top], nx[top] + sigma, -1);
    return top++;
}
void init() { top = 1, newnode(); }
int input(
    string &s) { // return the end_node of string
    int X = 1;
    for (char c : s) {
        if (!nx[X][c - 'a']) nx[X][c - 'a'] = newnode();
        X = nx[X][c - 'a'];
    }
    return X;
}
void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty(); ) {
        int R = q.front();
        q.pop(), pri[t++] = R;
        for (int i = 0; i < sigma; ++i)
            if (~nx[R][i]) {
                int X = nx[R][i], Z = fl[R];
                for (; Z && !nx[Z][i];) Z = fl[Z];
                fl[X] = Z ? nx[Z][i] : 1, q.push(X);
            }
    }
}
void get_v(string &s) {
    int X = 1;
    fill(cnt, cnt + top, 0);
    for (char c : s) {
        while (X && !nx[X][c - 'a']) X = fl[X];
        X = X ? nx[X][c - 'a'] : 1, ++cnt[X];
    }
    for (int i = top - 2; i > 0; --i)
        cnt[fl[pri[i]]] += cnt[pri[i]];
}
};

```

## 5.6 De Bruijn sequence\*

```

constexpr int maxc = 10, maxn = 1e5 + 10;
struct DBSeq {
int C, N, K, L, buf[maxc * maxn]; // K <= C^N
void dfs(int *out, int t, int p, int &ptr) {
    if (ptr >= L) return;
    if (t > N) {
        if (N % p) return;
        for (int i = 1; i <= p && ptr < L; ++i)
            out[ptr++] = buf[i];
    } else {
        buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
        for (int j = buf[t - p] + 1; j < C; ++j)
            buf[t] = j, dfs(out, t + 1, t, ptr);
    }
}
void solve(int _c, int _n, int _k, int *out) {
    int p = 0;
    C = _c, N = _n, K = _k, L = N + K - 1;
    dfs(out, 1, 1, p);
    if (p < L) fill(out + p, out + L, 0);
}
} db;

```

## 5.7 Extended SAM\*

```

struct exSAM {
int len[N * 2], link[N * 2]; // maxlength, suflink
int next[N * 2][CNUM], tot; // [0, tot), root = 0
int lenSorted[N * 2]; // topo. order
int cnt[N * 2]; // occurence
int newnode() {
    fill_n(next[tot], CNUM, 0);
    len[tot] = cnt[tot] = link[tot] = 0;
    return tot++;
}
void init() { tot = 0, newnode(), link[0] = -1; }
int insertSAM(int last, int c) {
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
        next[p][c] = cur, p = link[p];
}

```

```

    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len
        [p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
        next[
            clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
        next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
}
void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
        int &nxt = next[cur][int(ch - 'a')];
        if (!nxt) nxt = newnode();
        cnt[cur = nxt] += 1;
    }
}
void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (int i = 0; i < CNUM; ++i)
            if (next[cur][i])
                q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(ALL(lc), lc.begin());
    for (int i
        = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
}
void solve() {
    for (int i = tot - 2; i >= 0; --i)
        cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
}
};

```

## 5.8 PalTree\*

```

struct palindromic_tree {
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
                      // pal. suf.
        node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
            for (int i = 0; i < 26; ++i) next[i] = 0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree() : St(2), last(1), n(0) {
        St[0].fail = 1, St[1].len = -1, s.eb(-1);
    }
    inline void clear() {
        St.clear(), s.clear(), last = 1, n = 0;
        St.eb(0), St.eb(-1);
        St[0].fail = 1, s.eb(-1);
    }
    inline int get_fail(int x) {
        while (s[n - St[x].len - 1] != s[n])
            x = St[x].fail;
        return x;
    }
    inline void add(int c) {
        s.eb(c - 'a'), ++n;
        int cur = get_fail(last);
        if (!St[cur].next[c]) {
            int now = SZ(St);
            St.eb(St[cur].len + 2);
            St[now].fail =
                St[get_fail(St[cur].fail)].next[c];
            St[cur].next[c] = now;
            St[now].num = St[St[now].fail].num + 1;
        }
        last = St[cur].next[c], ++St[last].cnt;
    }
    inline void count() { // counting cnt

```

```

        auto i = St.rbegin();
        for (; i != St.rend(); ++i) {
            St[i->fail].cnt += i->cnt;
        }
    }
    inline int size() { // The number of diff. pal.
        return SZ(St) - 2;
    }
};

```

## 6 Math

### 6.1 ax+by=gcd(only exgcd\*)

```

pii exgcd(int a, int b) {
    if (b == 0) return pii(1, 0);
    int p = a / b;
    pii q = exgcd(b, a % b);
    return pii(q.Y, q.X - q.Y * p);
}
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t /

```

### 6.2 Floor and Ceil

```

int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }

```

### 6.3 Floor Enumeration

```

// enumerating x = floor(n / i), [l, r]
for (int l = 1, r; l <= n; l = r + 1) {
    int x = n / l;
    r = n / x;
}

```

### 6.4 Mod Min

```

// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
int mod_min(int a, int m, int l, int r) {
    if (a == 0) return l ? -1 : 0;
    if (int k = (l + a - 1) / a; k * a <= r)
        return k;
    int b = m / a, c = m % a;
    if (int y = mod_min(c, a, a - r % a, a - l % a))
        return (l + y * c + a - 1) / a + y * b;
    return -1;
}

```

### 6.5 Gaussian integer gcd

```

cpx gaussian_gcd(cpx a, cpx b) {
#define rnd
    (a, b) ((a >= 0 ? a * 2 + b : a * 2 - b) / (b * 2))
    int c = a.real() * b.real() + a.imag() * b.imag();
    int d = a.imag() * b.real() - a.real() * b.imag();
    int r = b.real() * b.real() + b.imag() * b.imag();
    if (c % r == 0 && d % r == 0) return b;
    return gaussian_gcd
        (b, a - cpx(rnd(c, r), rnd(d, r)) * b);
}

```

### 6.6 floor sum\*

```

int floor_sum(int n, int m, int a, int b) {
    int ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    int y_max
        = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
}
// sum^{
n-1}_{0} floor((a * i + b) / m) in log(n + m + a + b)

```

## 6.7 Miller Rabin\*

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : primes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(int a, int n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    int tmp = (n - 1) / ((n - 1) & (1 - n));
    int t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
```

## 6.8 Simultaneous Equations

```
struct matrix { //m variables, n equations
    int n, m;
    fraction M[maxn][maxn + 1], sol[maxn];
    int solve() { //-1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m) continue;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                fraction tmp = -M[j][piv] / M[i][piv];
                for (int k = 0; k <=
                    m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
            }
        }
        int rank = 0;
        for (int i = 0; i < n; ++i) {
            int piv = 0;
            while (piv < m && !M[i][piv].n) ++piv;
            if (piv == m && M[i][m].n) return -1;
            else if (piv
                < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
        }
        return rank;
    }
};
```

## 6.9 Pollard Rho\*

```
map<int, int> cnt;
void PollardRho(int n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2
        == 0) return PollardRho(n / 2), ++cnt[2], void();
    int x = 2, y = 2, d = 1, p = 1;
    #define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}
```

## 6.10 Simplex Algorithm

```
const int maxn = 11000, maxm = 405;
const double eps = 1E-10;
double a[maxn][maxm], b[maxn], c[maxm];
double d[maxn][maxm], x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(int n, int m) {
    ++m;
    fill_n(d[n], m + 1, 0);
```

```
fill_n(d[n + 1], m + 1, 0);
iota(ix, ix + n + m, 0);
int r = n, s = m - 1;
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
    d[i][m - 1] = 1;
    d[i][m] = b[i];
    if (d[r][m] > d[i][m]) r = i;
}
copy_n(c, m - 1, d[n]);
d[n + 1][m - 1] = -1;
for (double dd; ) {
    if (r < n) {
        swap(ix[s], ix[r + m]);
        d[r][s] = 1.0 / d[r][s];
        for (int j = 0; j <= m; ++j)
            if (j != s) d[r][j] *= -d[r][s];
        for (int i = 0; i <= n + 1; ++i) if (i != r) {
            for (int j = 0; j <= m; ++j) if (j != s)
                d[i][j] += d[r][j] * d[i][s];
            d[i][s] *= d[r][s];
        }
    }
    r = s = -1;
    for (int j = 0; j < m; ++j)
        if (s < 0 || ix[s] > ix[j]) {
            if (d[n + 1][j] > eps ||
                (d[n + 1][j] > -eps && d[n][j] > eps))
                s = j;
        }
    if (s < 0) break;
    for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
        if (r < 0 ||
            (dd = d[r][m]
                / d[r][s] - d[i][m] / d[i][s]) < -eps ||
            (dd < eps && ix[r + m] > ix[i + m]))
            r = i;
    }
    if (r < 0) return -1; // not bounded
}
if (d[n + 1][m] < -eps) return -1; // not executable
double ans = 0;
fill_n(x, m, 0);
for (int i = m; i <
    n + m; ++i) { // the missing enumerated x[i] = 0
    if (ix[i] < m - 1) {
        ans += d[i - m][m] * c[ix[i]];
        x[ix[i]] = d[i - m][m];
    }
}
return ans;
}
```

### 6.10.1 Construction

Standard form: maximize  $\mathbf{c}^T \mathbf{x}$  subject to  $\mathbf{A}\mathbf{x} \leq \mathbf{b}$  and  $\mathbf{x} \geq 0$ .

Dual LP: minimize  $\mathbf{b}^T \mathbf{y}$  subject to  $\mathbf{A}^T \mathbf{y} \geq \mathbf{c}$  and  $\mathbf{y} \geq 0$ .

$\bar{\mathbf{x}}$  and  $\bar{\mathbf{y}}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

1. In case of minimization, let  $c'_i = -c_i$
2.  $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
3.  $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
4. If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 6.11 chineseRemainder

```
int solve(int x1, int m1, int x2, int m2) {
    int g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pii p = exgcd(m1, m2);
    int lcm = m1 * m2 * g;
    int res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}
```

## 6.12 Factorial without prime factor\*

```
// O(p^k + log^2 n), pk = p^k
int prod[maxp];
int fac_no_p(int n, int p, int pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    int rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k
```

## 6.13 QuadraticResidue\*

```
int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL *
                g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL *
            f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}
```

## 6.14 PiCount\*

```
int PrimeCount(int n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<int> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                int d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges[smalls[d] - pc] : smalls[n / d]) + pc;
            }
        }
    }
}
```

```
        roughs[ns++] = i;
    }
    s = ns;
    for (int j = v / p; j >= p; --j) {
        int c =
            smalls[j] - pc, e = min(j * p + p, v + 1);
        for (int i = j * p; i < e; ++i) smalls[i] -= c;
    }
}
for (int k = 1; k < s; ++k) {
    const int m = n / roughs[k];
    int t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
}
return larges[0];
}
```

## 6.15 Discrete Log\*

```
int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p;
}
```

## 6.16 Berlekamp Massey

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(SZ(output) + 1), me, he;
    for (int f = 0, i = 1; i <= SZ(output); ++i) {
        for (int j = 0; j < SZ(me); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] -= output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.emb(-k);
        for (T x : he) o.emb(x * k);
        o.resize(max(SZ(o), SZ(me)));
        for (int j = 0; j < SZ(me); ++j) o[j] += me[j];
        if (i - f + SZ(he) >= SZ(me)) he = me, f = i;
        me = o;
    }
    return me;
}
```

## 6.17 Primes

```
/* 12721 13331 14341 75577 123457 222557
556679 999983 1097774749 1076767633 100102021
999997771 1001010013 1000512343 987654361 999991231
999888733 98789101 987777733 999991921 1010101333
1010102101 1000000000039 100000000000037
2305843009213693951 4611686018427387847
9223372036854775783 18446744073709551557 */
```



## 6.18 Theorem

- Cramer's rule

$$\begin{aligned} ax+by &= e \\ cx+dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed-bf}{ad-bc} \\ y &= \frac{af-ec}{ad-bc} \end{aligned}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

- Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

- Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

- Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each labeled vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of labeled forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

- Erdős–Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if

$d_1 + \dots + d_n$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$  holds for every

$1 \leq k \leq n$ .

- Gale–Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$  is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

- Fulkerson–Chen–Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and

$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

- Möbius inversion formula

- $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
- $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$

- Spherical cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$ .
- Area  $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$ .

- Lagrange multiplier

- Optimize  $f(x_1, \dots, x_n)$  when  $k$  constraints  $g_i(x_1, \dots, x_n) = 0$ .
- Lagrangian function  $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$ .
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

## 6.19 Estimation

- Estimation

- The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200000 for  $n < 1e19$ .
- The number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands. 1, 1, 2, 3, 5, 7, 11, 15, 22, 30 for  $n = 0 \sim 9$ , 627 for  $n = 20$ ,  $\sim 2e5$  for  $n = 50$ ,  $\sim 2e8$  for  $n = 100$ .
- Total number of partitions of  $n$  distinct elements:  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, \dots$

## 6.20 Euclidean Algorithms

- $m = \lfloor \frac{a+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor = \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)}{2} + \lfloor \frac{b}{c} \rfloor \cdot (n+1) \\ + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor = \begin{cases} \lfloor \frac{a}{c} \rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1)) \\ - h(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2 = \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 6.21 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1 = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1-x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

## 6.22 Tips for Generating Functions

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x)' \Rightarrow n a_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $x A(x) \Rightarrow n a_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{n-1}{i} x^i$

## 7 Polynomial

### 7.1 Fast Fourier Transform

```
template<int maxn>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[maxn];
    FFT() {
        for (int i = 0; i < maxn; ++i) {
            double arg = 2 * PI * i / maxn;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans
        (val_t *a, int n, bool inv = false); // see NTT;
    // remember to replace LL with val_t
};
```

### 7.2 Number Theory Transform\*

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int maxn, int P, int RT> //maxn must be 2^k
struct NTT {
    int w[maxn];
    int mpow(int a, int n);
    int minv(int a) { return mpow(a, P - 2); }
    NTT() {
        int dw = mpow(RT, (P - 1) / maxn);
        w[0] = 1;
        for (int i = 1; i < maxn; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(int *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(int
        *a, int n, bool inv = false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = maxn / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    int tmp = a[j + dl] * w[x] % P;
                    if ((a[j]
                        + dl) = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);
            int invn = minv(n);
            for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
        }
    }
};
```

### 7.3 Fast Walsh Transform\*

```
/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[
    N][1 <= N], g[N][1 <= N], h[N][1 <= N], ct[1 <= N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
```

```
// c_k = \sum_{i+j=k, i&j=0} a_i * b_j
int n = 1 <= L;
for (int i = 1; i < n; ++i)
    ct[i] = ct[i & (i - 1)] + 1;
for (int i = 0; i < n; ++i)
    f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
for (int i = 0; i <= L; ++i)
    fwt(f[i], n, 1), fwt(g[i], n, 1);
for (int i = 0; i <= L; ++i)
    for (int j = 0; j <= i; ++j)
        for (int x = 0; x < n; ++x)
            h[i][x] += f[j][x] * g[i - j][x];
for (int i = 0; i <= L; ++i)
    fwt(h[i], n, -1);
for (int i = 0; i < n; ++i)
    c[i] = h[ct[i]][i];
}
```

### 7.4 Polynomial Operation

```
#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int maxn, int P, int RT> // maxn = 2^k
struct Poly : vector<int> { // coefficients in [0, P)
    using vector<int>::vector;
    static NTT<maxn, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<int>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev()
        { return reverse(data(), data() + n(), *this); }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if
            (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
        return *this;
    }
    Poly& imul(int k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int m = 1;
        while (m < n() + rhs.n() - 1) m <= 1;
        Poly X(*this, m), Y(rhs, m);
        ntt(X.data(), m), ntt(Y.data(), m);
        fi(0, m) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), m, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
        if (n() == 1) return {ntt.minv((*this)[0])};
        int m = 1;
        while (m < n() * 2) m <= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
        Poly Y(*this, m);
        ntt(Xi.data(), m), ntt(Y.data(), m);
        fi(0, m) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] % = P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), m, true);
        return Xi.isz(n());
    }
    Poly Sqrt()
        const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
        if (n()
            == 1) return {QuadraticResidue((*this)[0], P)};
        Poly
            X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        return
            X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
    }
    pair<Poly, Poly> DivMod
        (const Poly &rhs) const { // (rhs.)back() != 0
        if (n() < rhs.n()) return {{0}, *this};
        const int m = n() - rhs.n() + 1;
        Poly X(rhs); X.irev().isz(m);
        Poly Y(*this); Y.irev().isz(m);
        Poly Q = Y.Mul(X.Inv()).isz(m).irev();
        X = rhs.Mul(Q), Y = *this;
        fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
        return {Q, Y.isz(max(1, rhs.n() - 1))};
    }
    Poly Dx() const {
```

```

Poly ret(n() - 1);
fi(0,
    ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n())
        ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<int> _eval(const
    vector<int> &x, const vector<Poly> &up) const {
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m *
        2) down[i] = down[i / 2].DivMod(up[i]).second;
    down[1] = Poly(up[1])
        .irev().isz(n()).Inv().irev()._tmul(m, *this);
    fi(2, m * 2) down[i]
        = up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
    vector<int> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<int> &x) {
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = m - 1; i
        > 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
    return up;
}
vector<int>
    > Eval(const vector<int> &x) const { // 1e5, 1s
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector
    <int> &x, const vector<int> &y) { // 1e5, 1.4s
    const int m = (int)x.size();
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<int> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m -
        1; i > 0; --i) down[i] = down[i * 2].Mul(up[i
        * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n())
        if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(int k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (int)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly{1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const int c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul
        (k % P).Exp().imul(c).irev().isz(n()).irev();
}
static int LinearRecursion
    (const vector<int> &a, const vector
    <int> &coef, int n) { // a_n = \sum c_j a_{n-j}
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly{1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {

```

```

        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    int ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.5 Value Polynomial

```

struct Poly {
    mint base; // f(x) = poly[x - base]
    vector<mint> poly;
    Poly(mint b = 0, mint x = 0): base(b), poly(1, x) {}
    mint get_val(const mint &x) {
        if (x >= base && x < base + SZ(poly))
            return poly[x - base];
        mint rt = 0;
        vector<mint> lmul(SZ(poly), 1), rmul(SZ(poly), 1);
        for (int i = 1; i < SZ(poly); ++i)
            lmul[i] = lmul[i - 1] * (x - (base + i - 1));
        for (int i = SZ(poly) - 2; i >= 0; --i)
            rmul[i] = rmul[i + 1] * (x - (base + i + 1));
        for (int i = 0; i < SZ(poly); ++i)
            rt += poly[i] * ifac[i] * inegfac
                [SZ(poly) - 1 - i] * lmul[i] * rmul[i];
        return rt;
    }
    void raise() { // g(x) = sigma{base:x} f(x)
        if (SZ(poly) == 1 && poly[0] == 0)
            return;
        mint nw = get_val(base + SZ(poly));
        poly.eb(nw);
        for (int i = 1; i < SZ(poly); ++i)
            poly[i] += poly[i - 1];
    }
};

```

## 7.6 Newton's Method

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

## 8 Geometry

### 8.1 Default Code

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double eps = 1e-8;
pdd operator+(pdd a, pdd b)
{ return pdd(a.X + b.X, a.Y + b.Y); }
pdd operator-(pdd a, pdd b)
{ return pdd(a.X - b.X, a.Y - b.Y); }
pdd operator*(pdd a, double b)
{ return pdd(a.X * b, a.Y * b); }
pdd operator/(pdd a, double b)
{ return pdd(a.X / b, a.Y / b); }
double dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
double cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(pdd a)
{ return dot(a, a); }
double abs(pdd a)
{ return sqrt(dot(a, a)); }
int sign(double a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pdd a, pdd b, pdd c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pdd p1, pdd p2, pdd p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pdd p1, pdd p2, pdd p3) {

```

```

if (!collinearity(p1, p2, p3)) return 0;
return sign(dot(p1 - p3, p2 - p3)) <= 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return (p4
        * a123 - p3 * a124) / (a123 - a124); // C^3 / C^2
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0
        , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(
        cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r

```

## 8.2 PointSegDist\*

```

double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(dot(q1 - q0,
        p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

## 8.3 Heart

```

pdd circenter
    (pdd p0, pdd p1, pdd p2) { // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
    double m = 2. * (x1 * y2 - y1 * x2);
    center.X = (x1 * x1
        * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
    center.Y = (x1 * x2
        * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
}
pdd incenter
    (pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
    double a =
        abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }

```

## 8.4 point in circle

```

// return
// p4 is strictly in circumcircle of tri(p1,p2,p3)
int sqr(int x) { return x * x; }
bool in_cc(const pii&
    p1, const pii& p2, const pii& p3, const pii& p4) {
    int u11 = p1.X - p4.X; int u12 = p1.Y - p4.Y;
    int u21 = p2.X - p4.X; int u22 = p2.Y - p4.Y;
    int u31 = p3.X - p4.X; int u32 = p3.Y - p4.Y;
    int u13
        = sqr(p1.X) - sqr(p4.X) + sqr(p1.Y) - sqr(p4.Y);
    int u23
        = sqr(p2.X) - sqr(p4.X) + sqr(p2.Y) - sqr(p4.Y);
    int u33
        = sqr(p3.X) - sqr(p4.X) + sqr(p3.Y) - sqr(p4.Y);
}

```

```

__int128 det = (__int128)-u13 * u22 * u31
    + (__int128)u12 * u23 * u31 + (__int128)u13 *
    u21 * u32 - (__int128)u11 * u23 * u32 - (__int128
    )u12 * u21 * u33 + (__int128)u11 * u22 * u33;
return det > eps;
}

```

## 8.5 Convex hull\*

```

void hull(vector<pii> &dots) { // n=1 => ans = {}
    sort(dots.begin(), dots.end());
    vector<pii> ans(1, dots[0]);
    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1,
            t = SZ(ans); i < SZ(dots); ans.pb(dots[i++]))
            while (SZ(ans) > t && ori
                (ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
                ans.pb();
    ans.pb(), ans.swap(dots);
}

```

## 8.6 PointInConvex\*

```

bool PointInConvex
    (const vector<pii> &C, pii p, bool strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if (SZ(C) == 0) return false;
    if (SZ(C) < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori
        (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}

```

## 8.7 TangentPointToHull\*

```

/* The point should be strictly out of hull
return arbitrary point on the tangent line */
pii get_tangent(vector<pii> &C, pii p) {
    auto gao = [&](int s) {
        return cyc_tsearch(SZ(C), [&](int x, int y)
            { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0

```

## 8.8 Intersection of line and convex

```

int TangentDir(vector<pii> &C, pii dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
}
#define cml(i) sign(cross(C[i] - a, b - a))
pii lineHull(pii a, pii b, vector<pii> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if (cml(A) < 0 || cml(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for (int t = l; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cml(m) == cml(t) ? l : r) = m;
        }
        return (l + !cml(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if (res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if (!
        cml(res.X) && !cml(res.Y)) // along side i, i+1
        switch ((res.X - res.Y + n + 1) % n) {
            case 0: return pii(res.X, res.X);
            case 2: return pii(res.Y, res.Y);
        }
    /* crossing sides (i, i+1) and (j, j+1)
crossing corner i is treated as side (i, i+1)
returned
in the same order as the line hits the convex */
    return res;
} // convex cut: (r, l]

```

## 8.9 minMaxEnclosingRectangle\*

```
const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pii> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
    hull(dots);
    double Max = 0, Min = INF, deg;
    int n = SZ(dots);
    dots.eb(dots[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        pii nw = vec(i + 1);
        while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
            u = (u + 1) % n;
        while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
            r = (r + 1) % n;
        if (!i) l = (r + 1) % n;
        while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
            l = (l + 1) % n;
        Min = min(Min, (double)(dot(nw, vec(r)) - dot(
            nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
        deg = acos(dot(diff(r, l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
        deg = (qi - deg) / 2;
        Max = max(Max, abs(diff(
            r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
    }
    return pdd(Min, Max);
}
```

## 8.10 VectorInPoly\*

```
// ori(a, b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(pii a, pii b, pii c, pii p, int strict) {
    return ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector {cur, p} in counter-clockwise order prv, cur, nxt
bool inside(pii prv, pii cur, pii nxt, pii p, int strict) {
    if (ori(cur, prv, p) >= 0)
        return btwangle(cur, prv, p, !strict);
    return !btwangle(cur, prv, p, !strict);
}
```

## 8.11 PolyUnion\*

```
double rat(pii a, pii b) {
    return sign(
        (b.X ? (double)a.X / b.X : (double)a.Y / b.Y);
} // all poly. should be ccw
double polyUnion(vector<vector<pii>> &poly) {
    double res = 0;
    for (auto &p : poly)
        for (int a = 0; a < SZ(p); ++a) {
            pii A = p[a], B = p[(a + 1) % SZ(p)];
            vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < SZ(q); ++b) {
                    pii C = q[b], D = q[(b + 1) % SZ(q)];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        double sa = cross(D - C, A - C), sb = cross(D - C, B - C);
                        segs.emplace_back(
                            (sa / (sa - sb), sign(sc - sd)));
                    }
                    if (!sc && !sd &&
                        &q < &p && sign(dot(B - A, D - C)) > 0) {
                        segs.emplace_back(rat(C - A, B - A), 1);
                        segs.emplace_back(rat(D - A, B - A), -1);
                    }
                }
            }
        }
    sort(ALL(segs));
    for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
    double sum = 0;
    int cnt = segs[0].second;
    for (int j = 1; j < SZ(segs); ++j) {
        if (!cnt) sum += segs[j].X - segs[j - 1].X;
        cnt += segs[j].Y;
    }
}
```

```
res += cross(A, B) * sum;
}
return res / 2;
}
```

## 8.12 PolyCut

```
vector<pdd> cut(vector<pdd> poly, pdd s, pdd e) {
    vector<pdd> res;
    for (int i = 0; i < SZ(poly); ++i) {
        pdd cur = poly[i], prv = i ? poly[i - 1] : poly.back();
        bool side = ori(s, e, cur) < 0;
        if (side != (ori(s, e, prv) < 0))
            res.eb(intersect(s, e, cur, prv));
        if (side)
            res.eb(cur);
    }
    return res;
}
```

## 8.13 Polar Angle Sort\*

```
int cmp(pii a, pii b, bool same = true) {
#define is_neg(k) (
    sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
    int A = is_neg(a), B = is_neg(b);
    if (A != B)
        return A < B;
    if (sign(cross(a, b)) == 0)
        return same ? abs2(a) < abs2(b) : -1;
    return sign(cross(a, b)) > 0;
}
```

## 8.14 Half plane intersection\*

```
pii area_pair(Line a, Line b) {
    return pii(cross(a.Y - a.X, b.Y - a.X));
}
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return (int128)
        a02Y * a12X - (int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --- Line.X --- Line.Y --- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(ALL(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    for (auto p : arr) {
        if (cmp(
            dq.back().Y - dq.back().X, p.Y - p.X, 0) == -1)
            continue;
        while (SZ(dq)
            ) >= 2 && !isin(p, dq[SZ(dq) - 2], dq.back())
            dq.pb();
        while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.eb(p);
    }
    while (SZ(dq)
        ) >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back())
        dq.pb();
    while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(ALL(dq));
}
```

## 8.15 RotatingSweepLine

```
void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps), m = 0;
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1));
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            if (i != j) line[m++] = pii(i, j);
    sort(ALL(line), [&](pii a, pii b) {
        return cmp(ps[a.Y] - ps[a.X], ps[b.Y] - ps[b.X]);
    });
}
```

```

}); // cmp(): polar angle compare
iota(ALL(id), 0);
sort(ALL(id), [&](int a, int b) {
    if (ps[a].Y != ps[b].Y) return ps[a].Y < ps[b].Y;
    return ps[a] < ps[b];
}); // initial order, since (1, 0) is the smallest
for (int i = 0; i < n; ++i) pos[id[i]] = i;
for (int i = 0; i < m; ++i) {
    auto l = line[i];
    // do something
    tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]
    ]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
}
}
}

```

## 8.16 Minimum Enclosing Circle\*

```

pdd Minimum_Enclosing_Circle
(vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter
                                (dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

## 8.17 Intersection of two circles\*

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 = a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
    if (d < max(r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
    pdd u = (o1 + o2) * 0.5
        + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) *
        (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v
        = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

## 8.18 Intersection of polygon and circle\*

```

// Divides into multiple triangle, and sum up
const double PI = acos(-1);
double _area(pdd pa, pdd eb, double r) {
    if (abs(pa) < abs(eb)) swap(pa, eb);
    if (abs(eb) < eps) return 0;
    double S, h, theta;
    double a = abs(eb), b = abs(pa), c = abs(eb - pa);
    double cosB = dot(eb, eb - pa) / a / c, B = acos(cosB);
    double cosC = dot(pa, eb) / a / b, C = acos(cosC);
    if (a > r) {
        S = (C/2) * r * r;
        h = a * b * sin(C) / c;
        if (h < r && B < PI/2) S -= (acos(h/r) * r * r - h * sqrt(r * r - h * h));
    }
    else if (b > r) {
        theta = PI - B - asin(sin(B) / r * a);
        S = .5 * a * r * sin(theta) + (C - theta) / 2 * r * r;
    }
    else S = .5 * sin(C) * a * b;
    return S;
}
double area_poly_circle(const
    vector<pdd> poly, const pdd &o, const double r) {
    double S = 0;
    for (int i = 0; i < SZ(poly); ++i)
        S += _area(poly[i] - o, poly[(i + 1) % SZ(poly)
        ] - o, r) * ori(0, poly[i], poly[(i + 1) % SZ(poly)]);
    return fabs(S);
}

```

## 8.19 Intersection of line and circle\*

```

vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
    pdd p = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
    double s = cross(b - a, c - a), h2 = r * r - s * s / abs2(b - a);
    if (h2 < 0) return {};
    if (h2 == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}

```

## 8.20 Tangent line of two circles

```

vector<Line>
    > go(const Cir& c1, const Cir& c2, int sign1) {
    // sign1 = 1 for outer tang, -1 for inner tang
    vector<Line> ret;
    double d_sq = abs2(c1.O - c2.O);
    if (sign(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    pdd v = (c2.O - c1.O) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        pdd n = pdd(v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X);
        pdd p1 = c1.O + n * c1.R;
        pdd p2 = c2.O + n * (c2.R * sign1);
        if (sign(p1.X - p2.X) == 0 and
            sign(p1.Y - p2.Y) == 0)
            p2 = p1 + perp(c2.O - c1.O);
        ret.eb(Line(p1, p2));
    }
    return ret;
}

```

## 8.21 CircleCover\*

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[N];
    void init(int _C) { C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a, double _b, int _c) : p(_a), ang(_b), add(_c) {}
        bool operator<(const Teve &a) const {
            return ang < a.ang;
        }
    } eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x) {
        return sign(abs(a.O - b.O) - b.R) > x;
    }
    bool contain(Cir &a, Cir &b, int x) {
        return sign(a.R - b.R - abs(a.O - b.O)) > x;
    }
    bool contain(int i, int j) {
        /* c[j] is non-strictly in c[i]. */
        return (sign
            (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].R) == 0 && i < j)) && contain(c[i], c[j], -1);
    }
    void solve() {
        fill_n(Area, C + 2, 0);
        for (int i = 0; i < C; ++i)
            for (int j = 0; j < C; ++j)
                overlap[i][j] = contain(i, j);
        for (int i = 0; i < C; ++i)
            for (int j = 0; j < C; ++j)
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjunct(c[i], c[j], -1));
        for (int i = 0; i < C; ++i) {
            int E = 0, cnt = 1;
            for (int j = 0; j < C; ++j)
                if (j != i && overlap[j][i])
                    ++cnt;
            for (int j = 0; j < C; ++j)
                if (i != j && g[i][j]) {
                    pdd aa, bb;
                    CCinter(c[i], c[j], aa, bb);

```



```

    double A =
        atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
    double B =
        atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
    eve[E++] = Teve
        (bb, B, 1), eve[E++] = Teve(aa, A, -1);
    if(B > A) ++cnt;
}
if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
else{
    sort(eve, eve + E);
    eve[E] = eve[0];
    for(int j = 0; j < E; ++j){
        cnt += eve[j].add;
        Area[cnt]
            += cross(eve[j].p, eve[j + 1].p) * .5;
        double theta = eve[j + 1].ang - eve[j].ang;
        if (theta < 0) theta += 2. * pi;
        Area[cnt] += (theta
            - sin(theta)) * c[i].R * c[i].R * .5;
    }
}
}
};

```

## 8.22 3Dpoint\*

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double
        _y = 0, double _z = 0): x(_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(Point p1, Point p2)
{ return
    Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point operator+(Point p1, Point p2)
{ return
    Point(p1.x + p2.x, p1.y + p2.y, p1.z + p2.z); }
Point operator*(Point p1, double v)
{ return Point(p1.x * v, p1.y * v, p1.z * v); }
Point operator/(Point p1, double v)
{ return Point(p1.x / v, p1.y / v, p1.z / v); }
Point cross(Point p1, Point p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z
    * p2.x - p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(Point p1, Point p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(Point a)
{ return sqrt(dot(a, a)); }
Point cross3(Point a, Point b, Point c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
//Azimuthal
    angle (longitude) to x-axis in interval [-pi, pi]
double phi(Point p) { return atan2(p.y, p.x); }
//Zenith
    angle (latitude) to the z-axis in interval [0, pi]
double theta(Point p)
{ return atan2(sqrt(p.x * p.x + p.y * p.y), p.z); }
Point masscenter(Point a, Point b, Point c, Point d)
{ return (a + b + c + d) / 4; }
pdd proj(Point a, Point b, Point c, Point u) {
// proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}
Point
    rotate_around(Point p, double angle, Point axis) {
    double s = sin(angle), c = cos(angle);
    Point u = axis / abs(axis);
    return u
        * dot(u, p) * (1 - c) + p * c + cross(u, p) * s;
}

```

## 8.23 Convexhull3D\*

```

struct convex_hull_3D {
struct Face {
    int a, b, c;
    Face(int ta, int tb, int tc): a(ta), b(tb), c(tc) {}
}; // return the faces with pt indexes
vector<Face> res;
vector<Point> P;
convex_hull_3D(const vector<Point> &P): res(), P(_P) {
// all points coplanar case will WA, O(n^2)
    int n = SZ(P);
    if (n <= 2) return; // be careful about edge case
    // ensure first 4 points are not coplanar
    swap(P[1], *find_if(ALL(P), [&](
        auto p) { return sign(abs2(P[0] - p)) != 0; }));
    swap(P[2], *find_if(ALL(P), [&](auto p) { return
        sign(abs2(cross3(p, P[0], P[1]))) != 0; }));
    swap(P[3], *find_if(ALL(P), [&](auto p) { return
        sign(volume(P[0], P[1], P[2], p)) != 0; }));
    vector<vector<int>> flag(n, vector<int>(n));
    res.emplace_back(0, 1, 2); res.emplace_back(2, 1, 0);
    for (int i = 3; i < n; ++i) {
        vector<Face> next;
        for (auto f : res) {
            int d
                = sign(volume(P[f.a], P[f.b], P[f.c], P[i]));
            if (d <= 0) next.emplace_back(f);
            int ff = (d > 0) - (d < 0);
            flag[f.a][
                f.b] = flag[f.b][f.c] = flag[f.c][f.a] = ff;
        }
        for (auto f : res) {
            auto F = [&](int x, int y) {
                if (flag[x][y] > 0 && flag[y][x] <= 0)
                    next.emplace_back(x, y, i);
            };
            F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
        }
        res = next;
    }
}
bool same(Face s, Face t) {
    if (sign(volume
        (P[s.a], P[s.b], P[s.c], P[t.a])) != 0) return 0;
    if (sign(volume
        (P[s.a], P[s.b], P[s.c], P[t.b])) != 0) return 0;
    if (sign(volume
        (P[s.a], P[s.b], P[s.c], P[t.c])) != 0) return 0;
    return 1;
}
int polygon_face_num() {
    int ans = 0;
    for (int i = 0; i < SZ(res); ++i)
        ans += none_of(res.begin(), res.begin()
            + i, [&](Face g) { return same(res[i], g); });
    return ans;
}
double get_volume() {
    double ans = 0;
    for (auto f : res)
        ans +=
            volume(Point(0, 0, 0), P[f.a], P[f.b], P[f.c]);
    return fabs(ans / 6);
}
double get_dis(Point p, Face f) {
    Point p1 = P[f.a], p2 = P[f.b], p3 = P[f.c];
    double a = (p2.y - p1.y)
        * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
    double b = (p2.z - p1.z)
        * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
    double c = (p2.x - p1.x)
        * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
    double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
    return fabs(a * p.x + b *
        p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
}
// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case

```

## 8.24 DelaunayTriangulation\*

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.

```

```

find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const
    int inf = maxc * maxc * 100; // lower_bound unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
    pii p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
    Tri(const pii& p0, const pii& p1, const pii& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return !!chd[0] + !!chd[1] + !!chd[2];
    }
    bool contains(pii const& q) const {
        for (int i = 0; i < 3; ++i)
            if (ori(p[i], p[(i + 1) % 3], q) < 0)
                return 0;
        return 1;
    }
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root
            = // Tri should at least contain all points
              new(tris++) Tri(pii(-inf, -inf),
                              pii(inf + inf, -inf), pii(-inf, inf + inf));
    }
    Tri* find(pii p) { return find(the_root, p); }
    void add_point(const pii &p) { add_point(find(the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pii &p) {
        while (1) {
            if (!root->has_chd())
                return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, pii const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris
                ++) Tri(root->p[i], root->p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
        for (int i = 0; i < 3; ++i)
            root->chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p
            [0], tri->p[1], tri->p[2], trj->p[pj])) return;
        /* flip edge between tri, trj */
    }
};

```

```

Tri* trk = new(tris++) Tri
    (tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
Tri* trl = new(tris++) Tri
    (trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
edge(Edge(trk, 0), Edge(trl, 0));
edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
tri->chd
    [0] = trk; tri->chd[1] = trl; tri->chd[2] = 0;
trj->chd
    [0] = trk; trj->chd[1] = trl; trj->chd[2] = 0;
flip(trk, 1); flip(trk, 2);
flip(trl, 1); flip(trl, 2);
}
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;
    vst.insert(now);
    if (!now->has_chd())
        return triang.eb(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}
void build(int n, pii* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}

```

## 8.25 Triangulation Voronoi\*

```

vector<Line> ls[N];
pii arr[N];
Line make_line(pdd p, Line l) {
    pdd d = l.Y - l.X; d = perp(d);
    pdd m = (l.X + l.Y) / 2;
    l = Line(m, m + d);
    if (ori(l.X, l.Y, p) < 0)
        l = Line(m + d, m);
    return l;
}
double calc_area(int id) {
    // use to calculate
    // the area of point "strictly in the convex hull"
    vector<Line> hpi = halfPlaneInter(ls[id]);
    vector<pdd> ps;
    for (int i = 0; i < SZ(hpi); ++i)
        ps.eb(intersect(hpi[i].X, hpi[i].Y, hpi[(i
            + 1) % SZ(hpi)].X, hpi[(i + 1) % SZ(hpi)].Y));
    double rt = 0;
    for (int i = 0; i < SZ(ps); ++i)
        rt += cross(ps[i], ps[(i + 1) % SZ(ps)]);
    return fabs(rt) / 2;
}
void solve(int n, pii *oarr) {
    map<pii, int> mp;
    for (int i = 0; i < n; ++i)
        arr[i] = pii(oarr[i].X, oarr[i].Y), mp[arr[i]] = i;
    build(n, arr); // Triangulation
    for (auto *t : triang) {
        vector<int> p;
        for (int i = 0; i < 3; ++i)
            if (mp.find(t->p[i]) != mp.end())
                p.eb(mp[t->p[i]]);
        for (int i = 0; i < SZ(p); ++i)
            for (int j = i + 1; j < SZ(p); ++j) {
                Line l(oarr[p[i]], oarr[p[j]]);
                ls[p[i]].eb(make_line(oarr[p[i]], l));
                ls[p[j]].eb(make_line(oarr[p[j]], l));
            }
    }
}

```

## 8.26 Minkowski Sum\*

```

vector<pii> Minkowski(vector<pii> A, vector<pii> B) {
    hull(A), hull(B);
    vector<pii> C(1, A[0] + B[0]), s1, s2;
}

```

```

for (int i = 0; i < SZ(A); ++i)
    s1.eb(A[(i + 1) % SZ(A)] - A[i]);
for (int i = 0; i < SZ(B); i++)
    s2.eb(B[(i + 1) % SZ(B)] - B[i]);
for (int i = 0, j = 0; i < SZ(A) || j < SZ(B);)
    if (j >= SZ(B) || (i < SZ(A) && cross(s1[i], s2[j]) >= 0))
        C.eb(B[j % SZ(B)] + A[i++]);
    else
        C.eb(A[i % SZ(A)] + B[j++]);
return hull(C), C;
}

```

## 9 Else

### 9.1 Cyclic Ternary Search\*

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}

```

### 9.2 Mo's Algorithm(With modification)

```

/*
Mo's Algorithm With modification
Block:  $N^{\frac{2}{3}}$ , Complexity:  $N^{\frac{5}{3}}$ 
*/
struct Query {
    int L, R, LBid, RBid, T;
    Query(int l, int r, int t):
        L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        if (RBid != q.RBid) return RBid < q.RBid;
        return T < b.T;
    }
};
void solve(vector<Query> query) {
    sort(ALL(query));
    int L=0, R=0, T=-1;
    for (auto q : query) {
        while (T < q.T) addTime(L, R, ++T); // TODO
        while (T > q.T) subTime(L, R, T--); // TODO
        while (R < q.R) add(arr[++R]); // TODO
        while (L > q.L) add(arr[--L]); // TODO
        while (R > q.R) sub(arr[R--]); // TODO
        while (L < q.L) sub(arr[L--]); // TODO
        // answer query
    }
}

```

### 9.3 Mo's Algorithm On Tree

```

/*
Mo's Algorithm On Tree
Preprocess:
1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<maxn> inset
*/
struct Query {
    int L, R, LBid, lca;
    Query(int u, int v) {
        int c = LCA(u, v);
        if (c == u || c == v)
            q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
        else if (out[u] < in[v])
            q.lca = c, q.L = out[u], q.R = in[v];
        else
            q.lca = c, q.L = out[v], q.R = in[u];
        q.Lid = q.L / blk;
    }
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        return R < q.R;
    }
}

```

```

}
};
void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}
void solve(vector<Query> query) {
    sort(ALL(query));
    int L = 0, R = 0;
    for (auto q : query) {
        while (R < q.R) flip(ord[++R]);
        while (L > q.L) flip(ord[--L]);
        while (R > q.R) flip(ord[R--]);
        while (L < q.L) flip(ord[L--]);
        if (~q.lca) add(arr[q.lca]);
        // answer query
        if (~q.lca) sub(arr[q.lca]);
    }
}

```

### 9.4 Additional Mo's Algorithm Trick

- Mo's Algorithm With Addition Only
  - Sort queries same as the normal Mo's algorithm.
  - For each query  $[l, r]$ :
    - If  $l/blk = r/blk$ , brute-force.
    - If  $l/blk \neq curL/blk$ , initialize  $curL := (l/blk + 1) \cdot blk$ ,  $curR := curL - 1$
    - If  $r > curR$ , increase  $curR$
    - decrease  $curL$  to fit  $l$ , and then undo after answering
- Mo's Algorithm With Offline Second Time
  - Require: Changing answer  $\equiv$  adding  $f([l, r], r+1)$ .
  - Require:  $f([l, r], r+1) = f([1, r], r+1) - f([1, l], r+1)$ .
  - Part1: Answer all  $f([1, r], r+1)$  first.
  - Part2: Store  $curR \rightarrow R$  for  $curL$  (reduce the space to  $O(N)$ ), and then answer them by the second offline algorithm.
  - Note: You must do the above symmetrically for the left boundaries.

### 9.5 Hilbert Curve

```

int hilbert(int n, int x, int y) {
    int res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 1ll * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
    return res;
} // n = 2^k

```

### 9.6 DynamicConvexTrick\*

```

// only works for integer coordinates!! maintain max
struct Line {
    mutable int a, b, p;
    bool operator<(const Line &rhs) const { return a < rhs.a; }
    bool operator<(int x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const int kInf = 1e18;
    int Div(int a,
            int b) { return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x
            ->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(int a, int b) {
        auto z = insert({a, b, 0}); y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin
            () && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin
            () && (--x)->p >= y->p) isect(x, erase(y));
    }
    int query(int x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};

```

## 9.7 All LCS\*

```
void all_lcs(string s, string t) { // 0-base
    vector<int> h(SZ(t));
    iota(ALL(h), 0);
    for (int a = 0; a < SZ(s); ++a) {
        int v = -1;
        for (int c = 0; c < SZ(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}
```

## 9.8 DLX\*

```
#define TRAV(i, link, start)
    for (int i = link[start]; i != start; i = link[i])
template<bool A, bool B = !A> // A: Exact
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN]
    ], cl[NN], rw[NN], bt[NN], s[NN], head, sz, ans;
    int columns;
    bool vis[NN];
    void remove(int c) {
        if (A) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        TRAV(i, dn, c) {
            if (A) {
                TRAV(j, rg, i)
                up[dn[j]]
                = up[j], dn[up[j]] = dn[j], --s[cl[j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        TRAV(i, up, c) {
            if (A) {
                TRAV(j, lt, i)
                ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (A) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
        columns = c;
        for (int i = 0; i < c; ++i) {
            up[i] = dn[i] = bt[i] = i;
            lt[i] = i == 0 ? c : i - 1;
            rg[i] = i == c - 1 ? c : i + 1;
            s[i] = 0;
        }
        rg[c] = 0, lt[c] = c - 1;
        up[c] = dn[c] = -1;
        head = c, sz = c + 1;
    }
    void insert(int r, const vector<int> &col) {
        if (col.empty()) return;
        int f = sz;
        for (int i = 0; i < (int)col.size(); ++i) {
            int c = col[i], v = sz++;
            dn[bt[c]] = v;
            up[v] = bt[c], bt[c] = v;
            rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
            rw[v] = r, cl[v] = c;
            ++s[c];
            if (i > 0) lt[v] = v - 1;
        }
        lt[f] = sz - 1;
    }
    int h() {
        int ret = 0;
        memset(vis, 0, sizeof(bool) * sz);
        TRAV(x, rg, head) {
            if (vis[x]) continue;
            vis[x] = true, ++ret;
            TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
        }
        return ret;
    }
    void dfs(int dep) {
```

```
        if (dep + (A ? 0 : h()) >= ans) return;
        if (rg[head] == head) return ans = dep, void();
        if (dn[rg[head]] == rg[head]) return;
        int w = rg[head];
        TRAV(x, rg, head) if (s[x] < s[w]) w = x;
        if (A) remove(w);
        TRAV(i, dn, w) {
            if (B) remove(i);
            TRAV(j, rg, i) remove(A ? cl[j] : j);
            dfs(dep + 1);
            TRAV(j, lt, i) restore(A ? cl[j] : j);
            if (B) restore(i);
        }
        if (A) restore(w);
    }
    int solve() {
        for (int i = 0; i < columns; ++i)
            dn[bt[i]] = i, up[i] = bt[i];
        ans = 1e9, dfs(0);
        return ans;
    }
};
```

## 9.9 Matroid Intersection

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.

## 9.10 AdaptiveSimpson

```
using F_t = function<double(double)>;
pdd simpson(const F_t &f, double l, double r,
    double fl, double fr, double fm = nan("")) {
    if (isnan(fm)) fm = f((l + r) / 2);
    return {fm, (r - l) / 6 * (fl + 4 * fm + fr)};
}
double simpson_ada(const F_t &f, double l, double r,
    double fl, double fm, double fr, double eps) {
    double m = (l + r) / 2,
        s = simpson(f, l, r, fl, fr, fm).second;
    auto [flm, sl] = simpson(f, l, m, fl, fm);
    auto [fmr, sr] = simpson(f, m, r, fm, fr);
    double delta = sl + sr - s;
    if (abs(delta) <= 15 * eps)
        return sl + sr + delta / 15;
    return simpson_ada(f, l, m, fl, flm, fm, eps / 2) +
        simpson_ada(f, m, r, fm, fmr, fr, eps / 2);
}
double simpson_ada(const F_t &f, double l, double r) {
    return simpson_ada(
        f, l, r, f(l), f((l + r) / 2), f(r), 1e-9 / 7122);
}
double simpson_ada2(const F_t &f, double l, double r) {
    double h = (r - l) / 7122, s = 0;
    for (int i = 0; i < 7122; ++i, l += h)
        s += simpson_ada(f, l, l + h);
    return s;
}
```

## 9.11 Simulated Annealing

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans:
    answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans
        ) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

## 9.12 Tree Hash\*

```
ull seed;
ull shift(ull x) {
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    return x;
}
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u])
        if (i != f)
            sum += shift(dfs(i, u));
    return sum;
}
```

## 9.13 Binary Search On Fraction

```
struct Q {
    int p, q;
    Q go(Q b, int d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(int N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        int len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !!len;
    }
    return dir ? hi : lo;
}
```

# 10 Python

## 10.1 Misc

```
from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction
    ('3.14159').limit_denominator(10).numerator # 22
```

# 11 Yamada

