Technical University of Denmark

46750 - Optimization in Modern Energy Systems
Assignment 1
Deadline: October 24, 2023 (12:01am)

Instructions: This assignment evaluates the topics covered in **Lectures 1 - 7** as well as programming and writing skills. It should be carried out in groups. Each group should provide a single submission, including the following:

- A concise project report (10 - 14 pages), detailing the mathematical models developed and presenting and analysing the main results. An appendix can be included (outside of page limit).
- A participation table accompanying the report, detailing the participation of each group member (as provided in Lecture 1)
- A working and well-documented code in the programming language of your choice.
- Additional relevant files and data.

All relevant files should be uploaded on DTU Learn. This assignment will count towards 50% of the final grade. The assessment will be based on the grading guide provided.

1. **Single-time Step Optimal Power Flow (OPF)**

   We consider the following power system, based on a modified IEEE 24-node reliability test system, including 12 conventional generating units, 6 wind farms of 300 MW each (at nodes n_3, n_5, n_7, n_16, n_21, n_23), 17 demands, and 34 transmission lines, as illustrated in Figure 1. Data on the techno-economic characteristics of the generating units, transmission lines, and load profiles is provided in [1]. Please consider that the capacity of the transmission lines connecting the node pairs $(15, 21)$, $(14, 16)$ and $(13, 23)$ is reduced to 400 MW, 250 MW and 250 MW, respectively. You can assume the production cost of wind farms is zero. Wind data can be found at [2], which can be normalized for your case study. All loads are considered inflexible, however, a load-shedding variable (one for each load) can be introduced, considering a load-shedding cost sufficiently high compared to the production costs of the generators.

   In lecture 4 you learned about the DC-OPF problem.

   (a) Please write the optimal flow for a single hour, in a compact way, and then derive its dual formulation and the corresponding KKT conditions.

   (b) Please solve this OPF and compute the values for the locational marginal prices (LMPs), the production cost, and the profit of every supplier.

2. **Decomposition of OPF**

   In Lectures 6 and 7 you learned about decomposition methods for solving large-scale optimization problems.

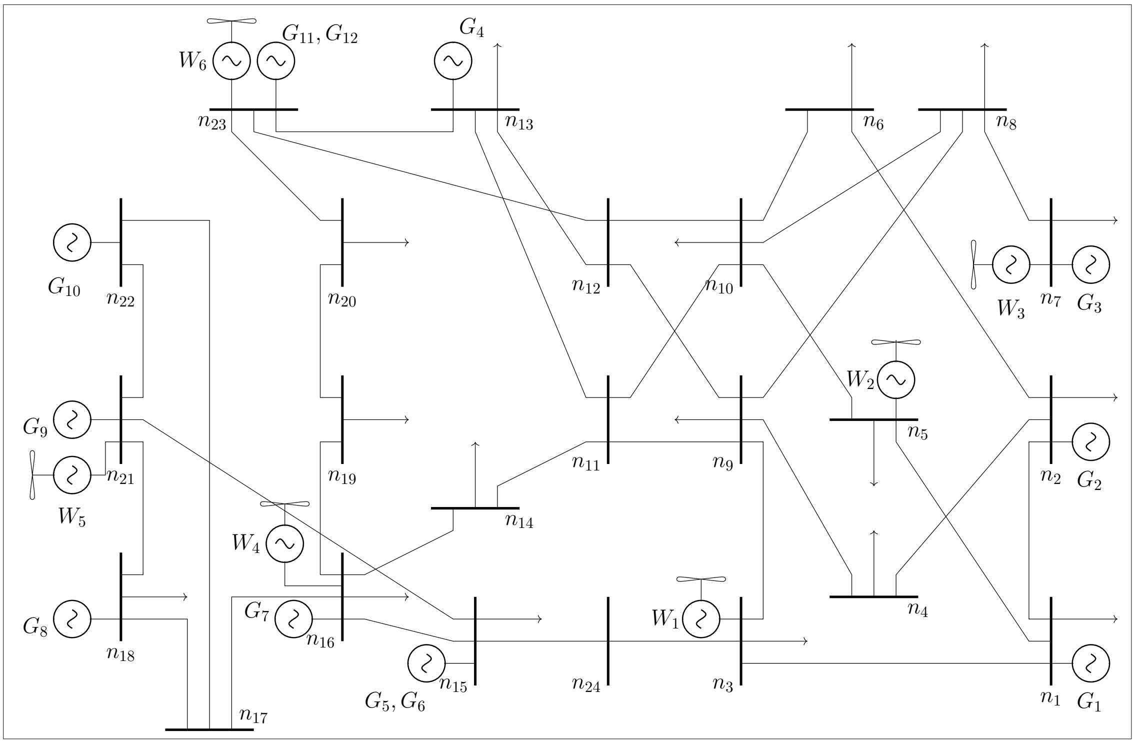

Figure 1: Modified IEEE 24-node electricity system with 6 wind farms

(a) Please, discuss whether the structure of the single-time step OPF in Question 1 can be decomposed, and if so, in how many subproblems.

(b) If this problem can be decomposed, please select and implement a suitable decomposition algorithm. Provide an outline of the proposed algorithm, and a detailed formulation of the different steps.

(c) Present and analyse results about the convergence, and computational tractability of the algorithm.

3. **Multi-time Step OPF with Battery Energy Storage Systems (BESSs)**

Please, consider now that 3 of the 6 wind farms can be equipped with battery energy storage systems (BESSs), with a maximum storage capacity of 450 MWh each, a charging (discharging) power of 150 MW, and a charging and discharging efficiency of 0.9 and 1.1, respectively. You can consider the operating costs of thes BESSs to be 0. You can select arbitrarily the placement, and initial state of charge of these BESSs.

In Lecture 5 you learned how to generalize the formulation of the single-hour OPF with inter-temporal constraints, such as BESSs operation.

(a) Please extend the formulation of the single-hour OPF in Question 1(a) for 24 hours with these 3 energy storage units. Provide a compact formulation of this multi-time step OPF. The dual formulation and KKT conditions do not need to be repeated in this question.

(b) Please solve this multi-time step OPF with and without energy storage units, and compare the solutions. How does the introduction of these BESSs impact the LMPs, production cost, congestion, and wind utilization in the system?

4. **Optimal Placement of BESSs**

As a central planner/system operator, we now consider how to optimally place these 3 BESSs in the power system in order to minimize the system's production cost. Each BESS cannot be split between multiple locations in the power system, however, multiple BESSs can be placed in the same location. As a result the storage capacity installed at each node of the system is either 0 MWh (0 BESS), 450 MWh (1 BESS), 900 MWh (2 BESSs) or 1350 MWh (3 BESSs). The system operation can be modelled over a single (representative) day.

(a) Please formulate this optimal placement problem for BESSs. specify the decision variables and their type. Is this optimization problem convex?

(b) Pleae solve this optimal placement problem and analyze the solutions.

References

[1] C. Ordoudis, P. Pinson, J. M. Morales, and M. Zugno, "An updated version of the ieee rts 24-bus system for electricity market and power system operation studies," *Technical University of Denmark*, vol. 13, 2016.

[2] W. Bukhsh, "Data for stochastic multiperiod optimal power flow problem," 2025. [Online]. Available: https://sites.google.com/site/datasmopf/