README

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1 The Lorenz Attractor

1.0.1 Scientific Computing using Python

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This project aims to implementation a module for the Lorentz Attractor. The module will be used to generate a plot of the attractor and a plot of the time evolution of the attractor. The module will also be used to generate a plot of the attractor for different values of the parameters. Unit testing will be used to ensure proper funcitonallity of the modeul.

1.1 Introduction

The Lorenz Attractor is a system of differential equations that describes a chaotic system. The system was first described by Edward Lorenz in 1963. The system is described by the following equations:

$$\frac{dx}{dt} = \sigma(y - x) \tag{1}$$
$$\frac{dy}{dt} = x(\rho - z) - y \tag{2}$$
$$\frac{dz}{dt} = xy - \beta z \tag{3}$$

where σ, ρ and β are parameters.

Using eulers method, the system can be solved numerically. The system is solved by discretizing the time and using the following equations:

$$x_{n+1} = x_n + \sigma(y_n - x_n)\Delta t \tag{4}$$
$$y_{n+1} = y_n + (x_n(\rho - z_n) - y_n)\Delta t \tag{5}$$
$$z_{n+1} = z_n + (x_n y_n - \beta z_n)\Delta t \tag{6}$$

where Δt is the time step. The system is solved by iterating over the equations for a given number of time steps.

1.2 Core algorithm

```
algorithm lorentz-attractor is
    input: Initial values: x0, y0, z0
           Parameters: sigma, rho, beta
           Time step: dt
           Number of time steps: nstep
    output: x, y, z at time t
    n = 0
    while n < nstep do
        x[n+1] = x[n] + sigma * (y[n] - x[n]) * dt
        y[n+1] = y[n] + (x[n] * (rho - z[n]) - y[n]) * dt
        z[n+1] = z[n] + (x[n] * y[n] - beta * z[n]) * dt
        n += 1
end algorithm
```

1.3 Structure

The project is structured as follows:

```
lorentz-attractor/                  # Modules
   __init__.py
   lorentz.py
   visualisation.py
REAME_files/                        # Figures for the README file
   README_[number].png
test/                               # Test modules
   __init__.py
   test_lorenz.py
simulations/                        # Simulation outputs
   lorentz_attractor[params].npz
project1.pdf                        # The project description
pytest_results.txt                  # Results from running pytest --doctest-modules
README.md                           # This report, markdown format
README.ipynb                        # This report, ipython notebook format
README.pdf                          # This report, pdf format
```

It contains the lorentz-attractor module, the test module and the simulations outputs. Additionally, it contains this report, serving as the README file for the project.

1.4 Design considerations

1.4.1 Content

The module contains the following funcitonailies:

1. Solving the lorentz attractor

The solver of the lorenz attractor is implemented as a class, such every parameter setting can be instasiated and have acces to all parameters doing simulation. The class is also implemented

with the possibility to save the simulation data to a file. The class is implemented such that any extension to the simulation method is easily added.

2. Visualise the lorentz attractor

Two visualisation functions are implemented. The first for simply plotting the simulation of attractor. The second for easily plotting multiple simulation with varying parameters, making it easy to compare the attractor for different parameter settings.

1.5 Test plan

The primary test is in the solver itself. Here i will implement the Lorentz Attractor using SciPy's odeint function. I will then compare the results of the two methods. If the results are similar, the solver is working as intended. additionally, i will use the hypothsis package, to test over a range of parameter settings, to ensure the solver is working as intended. The testing range is similair to the ranges used in this report and the number of steps has been limited, as the difference between the euler solution and odeint increases with increasing number of steps.

No testin has been implemented for figures, but this could be done by comparing the figures to a reference figure, visualy inspected to be valid.

The testing scope is quite limited and could be expanded to test the induvidual step frunction and solver methods, such any breaking changes could more easily be identified.

2 Results

2.1 Floating point precision

First the solver is inspected for any floating point errors. There a many ways to do this, but here i simply compare different precision levels to see if the results are the same. A very low time step has been choosen to ensure a breaking point is reached for atleast f16.

A summary of when the simulation fail is given in the table below.

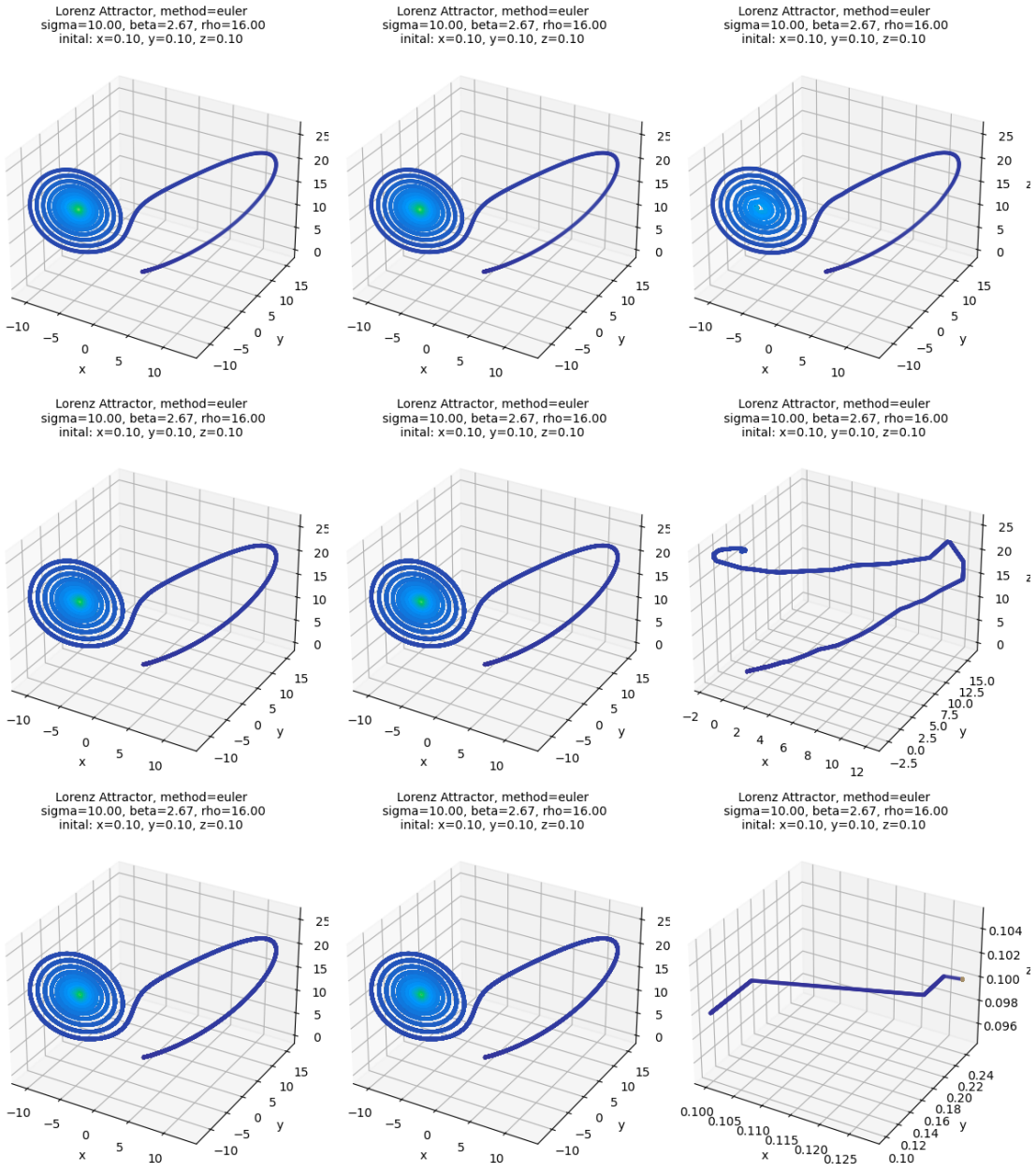
	f64	f32	f16
0.001	pass	pass	pass
0.0001	pass	pass	fail
0.00005	pass	pass	fail

The results are probably different for other parameter settings, but this shows that the solver is working as intended for the parameter settings used in this report using f64.

[47]:
```
import numpy as np
from lorenz_attractor.lorenz import LorenzAttractor
import lorenz_attractor.visualisation as vis
import matplotlib.pyplot as plt
import itertools
```

[48]:
```
combinations = list(itertools.product([0.001, 0.0001, 0.00005], [np.float64, np.
↪float32, np.float16]))
```

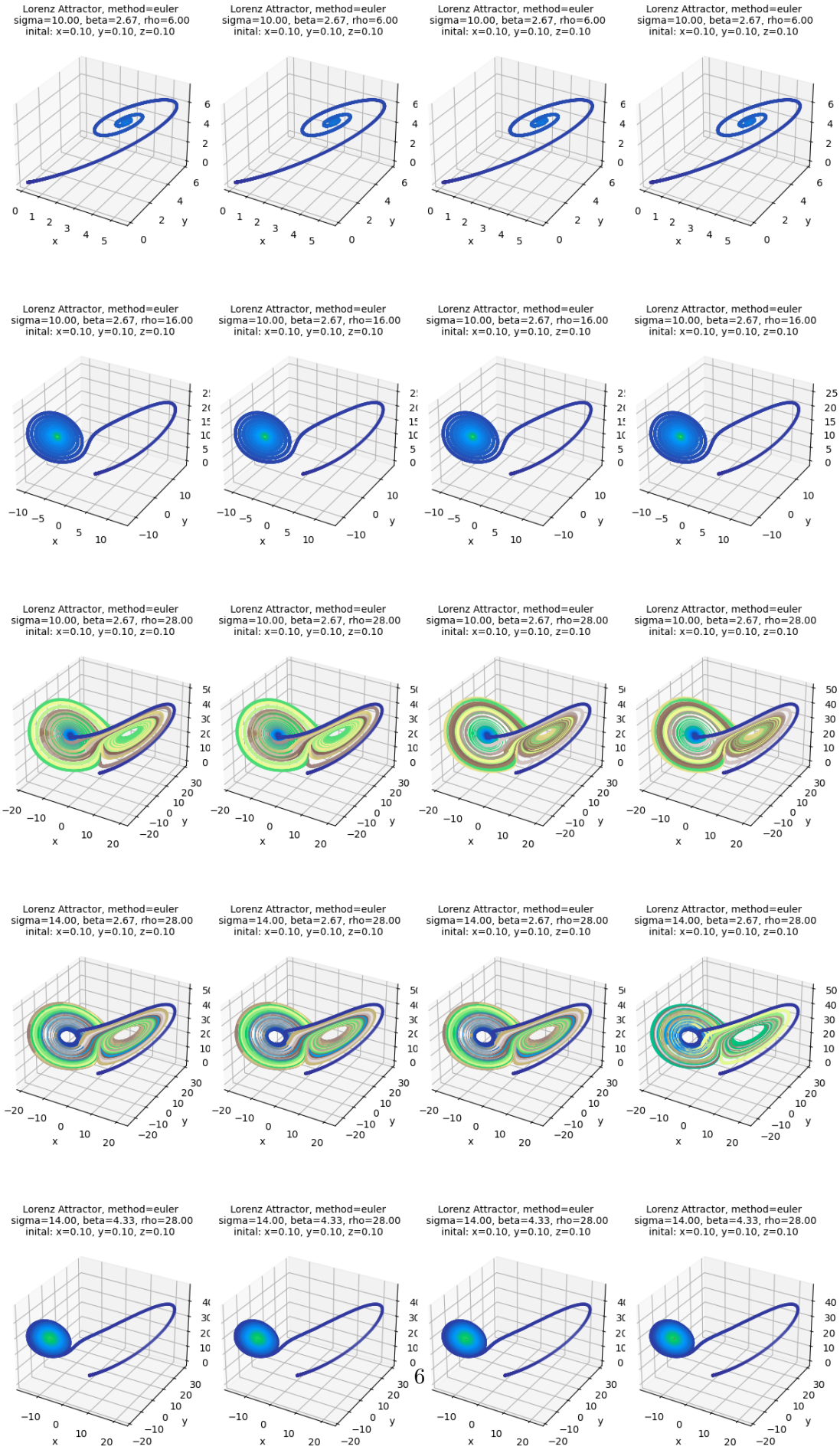
```
vis.plot_multiple_settings(
    dtype=[i[1] for i in combinations],
    dt=[i[0] for i in combinations]
)
```



At a timestep of 0.00005, encoding the values as f16, the results are not the same. This is due to the low precision of f16. At a timestep of 0.0001, the results are the same. This is the lowest precision that can be used for the solver. The results are the same for all higher precisions. This shows that the solver is working as intended.

2.2 3D plots

[49]:
```
vis.plot_multiple_settings(
    sigma = [i for i in [10, 10, 10, 14, 14] for _ in range(4)],
    beta =  [i for i in [8/3, 8/3, 8/3, 8/3, 13/3] for _ in range(4)],
    rho =   [i for i in [6, 16, 28, 28,28] for _ in range(4)],
    dtype = [i for i in [float, np.float64, np.float32, np.float16] for _ in␣
 ↪range(5)],
    ncols = 4,
    total_time=[50],
    dt = [0.001]
)
```

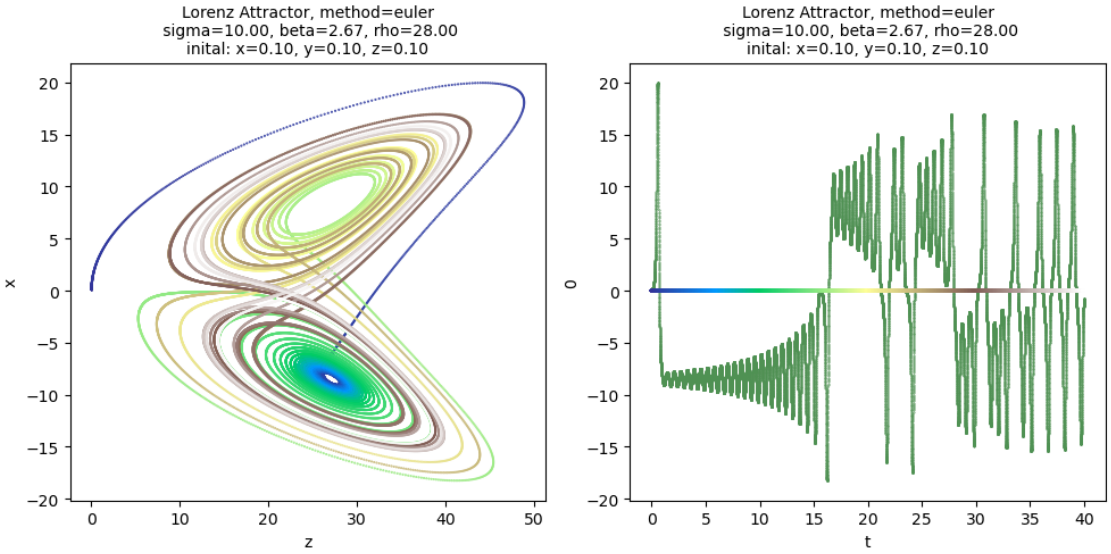

Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=6.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=6.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=6.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=6.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=16.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=16.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=16.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=16.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=4.33, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=4.33, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=4.33, rho=28.00
inital: x=0.10, y=0.10, z=0.10
Lorenz Attractor, method=euler
sigma=14.00, beta=4.33, rho=28.00
inital: x=0.10, y=0.10, z=0.10

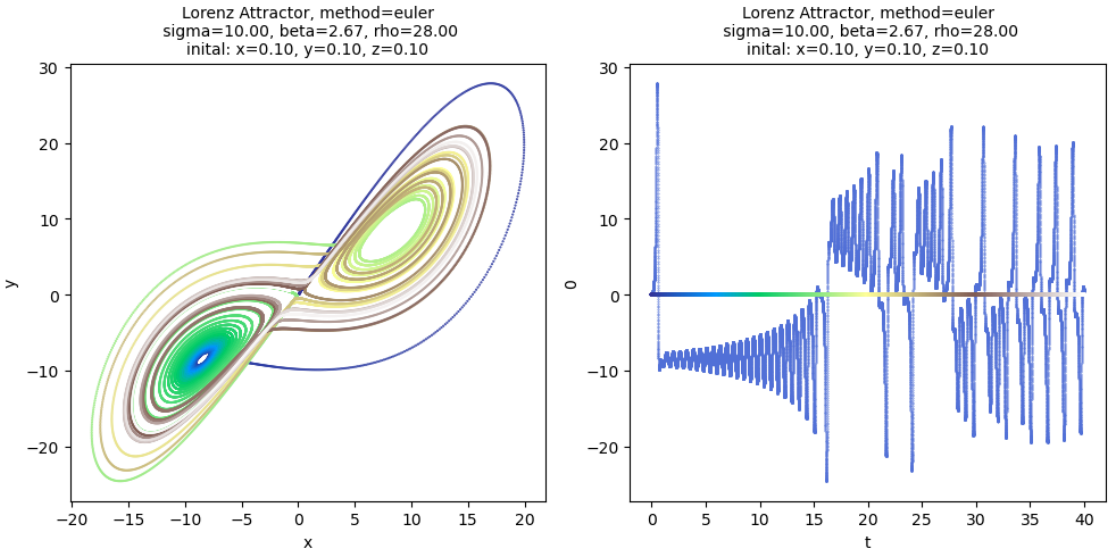
The simulations seems to work nicely.

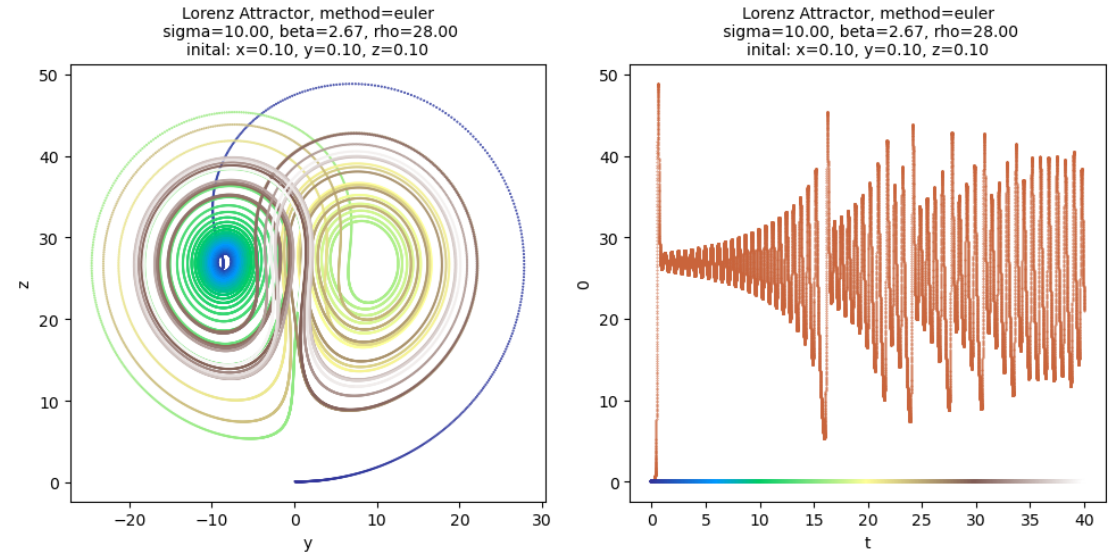
2.3 2D plots

Lets pick one parameter setting and plot the 2D versions and the temporal evolution of each axis.

[50]:
```
fig, axs = plt.subplots(3,2, figsize=(10, 15))
dt = 0.001
total_time = 40
nstep = int(total_time/dt)
sim = LorenzAttractor(sigma = 10, rho = 28, beta = 8/3, dt=dt, nstep=nstep)
sim.solve()
vis.plot_simulation_color_2d(sim, "z", "x", "t", ax=axs[0,0], size=1)
vis.plot_simulation_color_2d(sim, "t", "x", "#4f9153", ax=axs[0,1], size=0.5,␣
 ↪label = "z")
vis.plot_simulation_color_2d(sim, "x", "y", "t", ax=axs[1,0], size=1)
vis.plot_simulation_color_2d(sim, "t", "y", "#5170d7", ax=axs[1,1], size=0.5,␣
 ↪label = "x")
vis.plot_simulation_color_2d(sim, "y", "z", "t", ax=axs[2,0], size=1)
vis.plot_simulation_color_2d(sim, "t", "z", "#c9643b", ax=axs[2,1], size=0.5,␣
 ↪label = "y")
vis.plot_simulation_color_2d(sim, "t", 0, "t", ax=axs[0,1], size=9)
vis.plot_simulation_color_2d(sim, "t", 0, "t", ax=axs[1,1], size=9)
vis.plot_simulation_color_2d(sim, "t", 0, "t", ax=axs[2,1], size=9)
plt.tight_layout()
```


Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
x
z
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
0
t


Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
y
x
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
0
t


Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
z
y
Lorenz Attractor, method=euler
sigma=10.00, beta=2.67, rho=28.00
inital: x=0.10, y=0.10, z=0.10
0
t

By breaking the simulation down into ”d and temporal plots, we can really see the chaotic nature of the system. And how it oscillates between different states. Also how Z always osciallates around a center point, while X and Y shift between two different center points.