

# Errata File

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1. Page 2, (see note0101.pdf at <https://github.com/SorensenD/SLGDS>) for more details on the subject)  
CHANGE: "This requires equating the expected proportion of genotypes shared identical by descent, given a pedigree, with the observed phenotypic correlation between relatives."  
TO: "This requires equating the expected proportion of the genome shared identical by descent, given a pedigree, with the observed phenotypic correlation between relatives."
2. Page 18, CHANGE: "Jeffrey's prior and mathematical form as the likelihood, proportional to  $((1 - \theta)^{27})$  lead to posterior distributions  $Be(0.5, 27.5)$  and  $Be(1, 28)$ , respectively;"  
TO: "Jeffrey's prior and the uniform prior, when combined with the likelihood, proportional to  $(1 - \theta)^{27}$ , lead to posterior distributions  $Be(0.5, 27.5)$  and  $Be(1, 28)$ , respectively;"
3. Page 32, bottom of paragraph, CHANGE:  
"Spurious results may be obtained by complex interactions between a prediction method and a particular structure in the training data at hand that may not be reproduced when the model is deployed using validating data."  
TO: "Spurious results may be obtained by complex interactions between a prediction method and a particular structure in the data at hand that may not be reproduced when the model is deployed using a new sample of data."
4. Page 51, CHANGE  
"... the subject had been in the air long before Fisher, disguised in the terminology of inverse probability."  
TO: "... the subject had been in the air long before Fisher."
5. Page 53, bottom bullet, CHANGE  
"When  $f$  is a function of several variables  $f(x) = f(x_1, x_2, \dots, x_n)$ , the *gradient* of  $f$  is the column vector of  $n$  partial derivatives  $\left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right)'$  and is denoted  $\nabla f$ ."

TO: "When  $f$  is a function of several variables  $f(x) = f(x_1, x_2, \dots, x_n)$ , the *gradient* of  $f$  is the column vector of  $n$  partial derivatives  $\left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right)'$  and is denoted  $\nabla f$ ."

6. Page 63, CHANGE

"There are  $m$  complete bivariate observations  $(x_i, y_i), i = 1, 2, \dots, m$  and  $(n - m)$  univariate records ..."

TO: "There are  $m$  complete independent bivariate observations  $(x_i, y_i), i = 1, 2, \dots, m$  and  $(n - m)$  independent univariate records ..."

7. Page 92, top, CHANGE

"Due to the centring,  $W_{ij}$  has rank  $(n - 1)$ , matrix  $G$  is singular and  $(g|W, \sigma_g^2)$  is singular normally distributed. In practice the expectation and the standard deviation are replaced by their sample estimates."

TO: "In practice,  $E(X_{ij})$  and  $SD(X_{ij})$  are replaced by the  $j$ th column mean and the  $j$ th column standard deviation of  $W$ . Due to the column centring, when  $m > n$ , matrix  $W$  has rank  $(n - 1)$ , matrix  $G$  is singular and  $(g|W, \sigma_g^2)$  is singular normally distributed."

8. Page 121, CHANGE

"... with  $\beta$  and  $a$  viewed as nuisance parameters."

TO: "... with  $\beta$  and  $a$  viewed as nuisance parameters."

9. Page 132, CHANGE in (3.135b)

$i = 1, \dots, n - 1,$

TO:  $i = 1, \dots, n,$

10. Page 139,

Replace the two equations and the last line of the chapter by

$$\frac{1}{n} \sum_{i=1}^n y_i = \hat{\mu}_t,$$

$$\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 = \hat{\sigma}_t^2,$$

where  $\mu_t = \mu + i\sigma$  and  $\sigma_t^2 = \sigma^2(1 - i(i - z))$ . This yields  $\hat{\mu} \approx 9.97$  and  $\hat{\sigma}^2 \approx 3.13$ ."

11. Page 152, eqn. above (4.32)

CHANGE:

$$p(\tilde{y}|\sigma^2, y) = \int N(\mu, \sigma^2) N(\mu_1, \sigma_1^2) d\mu$$

TO:

$$p(\tilde{y}|\sigma^2, y) = \int N(\tilde{y}|\mu, \sigma^2) N(\mu|\mu_1, \sigma_1^2) d\mu$$

12. Pages 201-203, REPLACE subsection :  
 "The acceptance probability for a general Metropolis-Hastings algorithm"  
 WITH the content of note0401.pdf
13. Page 228, CHANGE  
 "The last eigenvalue is  $\lambda_n = 0$ ."  
 TO: "The last eigenvalue is  $\lambda_n = 0$  when the rank of  $W$  is equal to  $(n - 1)$ ."
14. Page 231, CHANGE  
 "These expressions make it clear that when  $\lambda_n = 0$ , the mean and variance are zero and the density collapses to a point mass at zero."  
 TO: "These expressions make it clear that when  $\lambda_i = 0$ , the mean and variance are zero and the density collapses to a point mass at zero."
15. Page 233, bottom paragraph, CHANGE  
 "... where  $W$  is the  $n \times p$  centred and scaled matrix of marker genotypes,  $b$  is the  $p \times 1$  vector ..."  
 TO: "... where  $W$  is the  $n \times m$  centred and scaled matrix of marker genotypes,  $b$  is the  $m \times 1$  vector ..."
16. Page 258, eqn. (6.1),  
 CHANGE second line to (brackets missing):
 
$$\begin{aligned} E_{y_0|x} [\text{MSE}(f(x))] &= E_{y_0|x} [(y_0 - f(x))^2] \\ &= E_x [E_{y_0|x} [(y_0 - f(x))^2 | x]] \\ &= E_x [\text{Var}(y_0|x) + (E(y_0|x) - f(x))^2], \end{aligned} \quad (6.1)$$

and a little below,  
 CHANGE: "The error of prediction is"  
 TO: "Once  $y_0$  is observed, the error of prediction is"
17. Page 259, below eqn. (6.7),  
 CHANGE: "that is in the best linear approximation ..."  
 TO: "that is the best linear approximation ..."
18. Page 261, CHANGE from eqn. (6.12) to eqn. (6.15)  
 TO:

$$y_i = \mu + G(z_i) + e_i \quad (6.12)$$

where the continuously distributed environmental effect  $e_i$  has mean zero and variance  $\sigma^2$  and marginally,  $E(y_i) = \mu$ .

The conditional expectation  $\mu + G(z_i)$  may not be linear on  $z_i$  due to the genetic mechanism operating within and across loci. However, one can always define a linear relationship of the form

$$m + \alpha' z_i. \quad (6.13)$$

Then  $m$  and  $\alpha$  are obtained minimising

$$E \left[ (y_i - m - \alpha' z_i)^2 \right]$$

with respect to  $m$  and  $\alpha$ . The expected squared error is a minimum with

$$\begin{aligned} \alpha &= [\text{Var}(z_i)]^{-1} \text{Cov}(y_i, z_i) \\ &= [\text{Var}(z_i)]^{-1} \text{Cov}(z_i, G(z_i)) \end{aligned} \quad (6.14)$$

and

$$m = E(y_i) - \alpha' E(z_i). \quad (6.15)$$

19. Page 263, first paragraph,

CHANGE: ", ... where  $x_i \in \mathbb{R}^p$ , and in this section, they are assumed to have mean zero for all  $i$ ."

TO: ", ... where covariates  $x_i \in \mathbb{R}^p$  are centred."

20. Page 264, bottom

CHANGE: "...as the predictor variables have larger sampling variances and are uncorrelated with each other."

TO: "...as the predictor variables have larger sampling variances and are mutually orthogonal."

CHANGE: "If the covariables are uncorrelated ..."

TO: "If the covariables are orthogonal ..."

CHANGE: "When the covariables are correlated ..."

TO: "When the covariables are not orthogonal ..."

21. Page 265, top

CHANGE: "indicating that when the covariables are correlated ..."

TO: "indicating that when the covariables are not orthogonal ..."

22. Page 270, eqn. (6.31),

CHANGE: the left hand side of the first line:

$$\text{Var}(y_0 - \hat{y}_0 | x_0)$$

TO:

$$\text{Var}[(y_0 - \hat{y}_0) | x_0]$$

(brackets missing)

23. Page 273, just before eqn. (6.40), CHANGE

"As  $n$  increases, ..."

TO: "With centred covariates and data, as  $n$  increases, ..."

24. Page 277,  
CHANGE:

$$E_{y_v y}(\text{MSE}_v) = \text{Var}(y_{v,i}|x_i) + \text{bias}^2 + \frac{1}{N} \text{Var}(\hat{f}(x_i)|x_i)$$

and

$$E_{y_v y}(\text{MSE}_t) = E_{y_v y}(\text{MSE}_v) - \frac{2}{N} \sum_i \text{Cov}(y_i, \hat{f}(x_i))$$

TO:

$$E_{y_v y}(\text{MSE}_v) = \frac{1}{N} \sum_i \text{Var}(y_{v,i}|x_i) + \frac{1}{N} \sum_i \text{bias}(i)^2 + \frac{1}{N} \sum_i \text{Var}(\hat{f}(x_i)|x_i)$$

and when covariates in training and validating data take the same values,

$$E_{y_v y}(\text{MSE}_t) = E_{y_v y}(\text{MSE}_v) - \frac{2}{N} \sum_i \text{Cov}(y_i, \hat{f}(x_i))$$

25. Page 286, below eqn. (6.66),  
CHANGE: "and from (6.65)"  
TO: "and from (6.65) when covariates in training and validating data take the same values,"  
(see note0603.pdf for elaborations on the subject).
26. Page 287, below eqn. (6.69),  
CHANGE: "The first term on the right-hand side is a sample estimate of  $E_{y_v y}(\text{MSE}_t)$ , and ..."  
TO: "The first term on the right-hand side is the sample training mean squared error  $\text{MSE}_t$ , and ..."
27. Page 288, incorporate the following paragraph at the end of subsection **Correlated data**  
"In general, the whole setup of cross-validation builds on the assumption that validating data represent an independent draw from the same distribution as the training data, conditional on the model parameters. When available data with a complicated hierarchical structure (such as a pedigree structure) are divided into training and validating sets, the assumption of conditional independence is often violated and the method is not likely to produce reliable results as a model comparison tool. The problem can be partly mitigated by dividing the data judiciously accounting for the structure in the data and the prediction problem at hand. This may not always be possible and results must therefore be interpreted cautiously, and, as a general rule, supplemented with other approaches of model criticism and comparison."

28. Page 290, left hand side of eqn. (6.74)

CHANGE:  $E_{yy^v}(\text{MSE}_t)$

TO:  $E_{yy_v}(\text{MSE}_t)$

and, left hand side of eqn. (6.75)

CHANGE:  $\hat{E}(\text{MSE}_v)$

TO:  $\hat{E}_{yy_v}(\text{MSE}_v)$

29. Page 293, first sentence bottom subsection.

CHANGE: " The simulation example compares the expected optimism (6.68) with its estimate based on (6.75)."

TO: "The simulation example compares the Monte Carlo estimate of the expected optimism (6.68) with its expected value based on the second term of (6.75), for two values of the ratio  $p/N$ ."

30. Page 324, below eqn. (7.37)

CHANGE: " This binary indicator variable with its associated distribution  $\Pr(\delta_i = 1|\pi)$  specifies the a priori probability that a marker effect  $b_i$  is non-zero;"

TO: This binary indicator variable specifies the a priori probability that a marker effect  $b_i$  is non-zero:  $\Pr(\delta_i = 1|\pi)$ ."

31. Page 325, eqn. (7.41),

CHANGE:

The conditional distribution of the data takes the form

$$y|\mu, \alpha, \delta, \sigma^2 \sim N\left(1\mu + \sum_{i=1}^m X_i(\alpha_i \delta_i), I\sigma^2\right), \quad i = 1, \dots, m \quad (7.41)$$

where  $X_i$  is the  $i$ th column of  $X$ .

TO: The conditional distribution of the data takes the form

$$y|\mu, \alpha, \delta, \sigma^2 \sim N\left(1\mu + \sum_{i=1}^m X_i(\alpha_i \delta_i), I\sigma^2\right), \quad i = 1, \dots, m \quad (7.41)$$

where  $X_i$  is the  $i$ th column of  $X$ . (remove extra ", " before  $i = 1, \dots, m$ ).

32. Page 332, CHANGE: "..., while incurring a relatively low proportion of false positives."

TO: "..., while incurring a user-chosen proportion of false positives."

33. Page 339

CHANGE: " Benjamini and Yekutieli (2001) proved that the above theorem holds under certain type of stochastic dependence among the  $p$ -values, ..."

TO: " Benjamini and Yekutieli (2001) proved that the above theorem holds, with a slight redefinition of the rule (8.14), under certain type of stochastic dependence among the  $p$ -values, ..."

34. Page 340, second paragraph starting from the bottom  
 CHANGE: " This rule controls FDR at level  $q = 0.15$ ." and  
 " ... that is expected to include  $19 \times 0.15 \approx 3$  false discoveries."  
 TO: " This rule controls FDR at level  $q \leq 0.15$ ." and  
 " ... that is expected to include less than or equal to  $19 \times 0.15 \approx 3$  false discoveries."

35. Page 346, top  
 CHANGE: " This may require first mapping the  $z$ -values to  $p$ -values using (8.7) with  $z$  instead of  $t$ . The numerator of (8.27) is

$$\begin{aligned} p_i &= 1 - F_0(z_i) = \Pr(Z \geq z_i | H = 0) \\ &= \int_{z_i}^{\infty} N(0, 1). \end{aligned}$$

Using the ordered  $p$ -values from smallest to largest, the denominator of (8.27) is"  
 TO: " The numerator of (8.30) is  $\pi_0 p_i$ . Using the ordered  $p$ -values, an estimator of the denominator of (8.27) is"

AND

CHANGE:

" or

$$\widehat{BFDR}(p_i) = \hat{\pi}_0 q \leq q \quad (8.32)$$

as in (8.15). "

TO: " or

$$\widehat{BFDR}(p_i) = \hat{\pi}_0 \frac{mp_i}{i} \leq \hat{\pi}_0 q. \quad (8.32)$$

"

36. Page 349,  
 CHANGE:

"In the example of Table 8.1, where the test statistics are sorted  $p$ -values starting from the largest at the bottom of the list, the  $q_i^*$  is numerically equal to  $q_i\text{-value}(1/\hat{\pi}_0)$ . The regions that reject  $H_{19}$  ( $p$ -values  $\geq 0.02844954$ ) involve all the  $q_i\text{-values}(1/\hat{\pi}_0)$ ,  $i = 19, \dots, 100$  and  $q_{19}\text{-value}(1/\hat{\pi}_0) = 0.14973443$  is the smallest such BFDR. Alternatively,  $\hat{\pi}_0$  can be set equal to 1 and then  $q_i^*$  and  $q_i\text{-value}$  are numerically identical quantities. In this case, if the null hypotheses is rejected for features  $i = 15, 14, \dots, 1$ , then the figures in Table 8.1 indicate that  $q_{15}^* = 0.083$ . This is the estimate of BFDR for  $i = 15$  and for the set comprising all the features that are this significant or more."

TO:

"In the example of Table 8.1, where the test statistics are sorted  $p$ -values starting from the largest at the bottom of the list, the  $q_i^*$  is numerically equal to  $q_i\text{-value}(1/\hat{\pi}_0)$ . The regions that reject  $H_{19}$  ( $p$ -values  $\geq 0.02844954$ ) involve all the  $q_i\text{-values}(1/\hat{\pi}_0)$ ,  $i = 19, \dots, 100$  and  $q_{19}\text{-value}(1/\hat{\pi}_0) = 0.14973443$  is the smallest such BFDR. Alternatively,  $\hat{\pi}_0$  can be set equal to 1 and then  $q_i^*$  and  $q_i\text{-value}$  are numerically identical

quantities. In this case, if the null hypothesis is rejected for features  $i = 15, 14, \dots, 1$ , then the figures in Table 8.1 indicate that  $q_{15}^* = 0.083$ . This is the estimate of BFDR for  $i = 15$  and for the set comprising all the features that are this significant or more; we should expect 8.3% false positives among the  $i = 15$  discoveries (this gives  $15 \times 0.083 \approx 1$  false positive). On the other hand, the corresponding  $p$ -value for  $i = 15$  is 0.01247; this implies that, in conceptual replications of the experiment, the expected number of false positives among all the  $m$  tests with  $p$ -values cutoffs 0.01247 or smaller, is  $0.01247 \times m$  or smaller.”

37. Page 413, last line

CHANGE:

$$f_i^* | \sigma_f^2 \sim N(0, \sigma_f^{2*})$$

TO:

$$f_i^* | \sigma_f^{2*} \sim N(0, \sigma_f^{2*})$$

38. Page 415

CHANGE:

$$\begin{aligned} \hat{E}(\mu|y) &= 0.160, \\ \hat{E}(\sigma_f^2|y) &= 0.0102, \\ \hat{E}(\sigma_e^2|y) &= 0.126, \\ \hat{E}(h^2|y) &= 0.149. \end{aligned}$$

TO:

$$\begin{aligned} \hat{E}(\mu^*|y) &= 0.160, \\ \hat{E}(\sigma_f^{2*}|y) &= 0.0102, \\ \hat{E}(\sigma_e^{2*}|y) &= 0.126, \\ \hat{E}(h_o^2|y) &= 0.149. \end{aligned}$$

39. Page 392

CHANGE: "Consider a person who feels who has been exposed to ..."

TO: "Consider a person who has been exposed to ..."

40. Top of page 393, eqn. (9.30),

CHANGE:

An unbiased estimator of incidence (given  $n$ ,  $sen$ ,  $spe$  and bounded between 0 and 1) is

$$\frac{\Pr(\hat{Y} = 1) - (1 - spe)}{sen + spe - 1} = \frac{T - n(1 - spe)}{n(sen + spe - 1)}. \quad (9.30)$$



TO: A corrected estimator of prevalence (given  $n$ ,  $sen$ ,  $spe$ ) is

$$\frac{\widehat{\Pr}(\widehat{Y} = 1) - (1 - spe)}{sen + spe - 1}, \widehat{\Pr}(\widehat{Y} = 1) > (1 - spe). \quad (9.30)$$

(change "incidence" to "prevalence", "hat" missing on the corrected estimator of prevalence and remove "and bounded between 0 and 1"; see note0901.pdf for further details).

41. Page 415,

CHANGE: "These can be transformed to the estimates on the underlying scale using (9.69):"

TO: "These can be transformed to estimates on the underlying scale using (9.69)"

AND BELOW,

CHANGE:

$$\begin{aligned} \widehat{E}(\mu|y) &= -0.995, \\ \widehat{E}(\sigma_f^2|y) &= 0.174, \\ \widehat{E}(h^2|y) &= 0.291. \end{aligned}$$

TO:

$$\begin{aligned} \widehat{E}(\mu|y) &= -0.995, \\ \widehat{E}(\sigma_f^2|y) &= 0.174, \\ \widehat{E}(h^2|y) &= 0.291, \end{aligned}$$

42. Page 429, in the second lines of the right hand side of equations (10.19) and (10.20),

CHANGE:  $\sigma^2$

TO:  $E_{\sigma^2|y_t}(\sigma^2)$

43. Page 429,

CHANGE: "The step from the first to the second line uses the decomposition of the posterior predictive variance (10.4b)."

TO: "The step from the first to the second line uses the decomposition of the posterior predictive variance (10.4b), here accounting for the unknown variance  $\sigma^2$ ."

44. Page 486,

CHANGE: "The three operational models used to analyse the data assumed that the liability can be written in terms of the linear structure:

$$u_i = \mu + x_i' \beta + \varepsilon_i, \quad i = 1, 2, \dots, 1279,"$$

TO: "The three operational models used to analyse the data assumed that the liability can be written in terms of the linear structure:

$$u_i = \mu + x_i' \beta + \varepsilon_i, \quad i = 1, 2, \dots, 599,"$$

45. Page 501,

CHANGE in first line of eqn. (11.79)

$$a^2 = \dots$$

TO:  $a^{(2)} = \dots$

46. Page 525,

CHANGE:

" A matrix of genotypic markers (of order *number of individuals* (sample size)  $\times$  *number of markers* = 1000) is generated ..."

TO: "A matrix of genotypic markers (of order *number of individuals* (sample size)  $\times$  *number of markers* = 1000) is generated ..."

47. Page 670,

CHANGE:

$$\text{MSE}_v = \text{MSE}_t + \frac{2}{n} \text{tr} (\text{Cov} (y, \hat{y}'))$$

TO:

$$\widehat{\text{MSE}}_v = \text{MSE}_t + \frac{2}{n} \text{tr} (\text{Cov} (y, \hat{y}'))$$

48. Page 675,

CHANGE: "Boyle EA, Li YI, Pritchard JK (2017) An expanded view of complex **trits**: from polygenic to omnigenic. Cell 169:1177–1186"

TO: "Boyle EA, Li YI, Pritchard JK (2017) An expanded view of complex traits: from polygenic to omnigenic. Cell 169:1177–1186"

49. Page 680,

CHANGE: "Storey JD, Tibshirani R (2007) Statistical significance for genomewide studies. Proceedings of the National Academy of Sciences 100:9440–9445"

TO: "Storey JD, Tibshirani R (2003) Statistical significance for genomewide studies. Proceedings of the National Academy of Sciences 100:9440–9445"