CSE110A: Compilers

May 20, 2024

Topics:

• Start of Module 4 - Optimizations

Announcements

- HW 4 is out
 - Due on Friday; If you haven't gotten started, you really should
 - No late assignments accepted.
- Thanks to the TAs for covering the midterm review! Hopefully it was useful
 - Please see us if you want to review your midterm
 - You have 1 week to make a note (either see us in office hours or make a private post on piazza)
- One more homework left on optimizations. Planning for release on Friday

How many virtual registers does the following expression need?

int a, x, y;

a = ((x + 1) * y - 1) / 2.0;

 \bigcirc 1

4

O 5

O 6

How many labels do you need for the following expression?

```
int x, y;
if (x==0){
} else if (y>1) {
}else {
\bigcirc 0
\bigcirc 1
O 2
○ 3
```

The number of virtual registers is equal to the number of nodes in the AST

 \bigcirc True

○ False

Discuss an optimization or two that you could imagine doing as you convert an AST into 3 address code

Start of module 4: optimizations

What are compiler optimizations?

• Why do we want compiler optimizations?

- What are compiler optimizations?
 - automated program transforms designed to make code more optimal
 - optimal can mean different things
 - code optimized for one system might be different for code optimized for a different system
 - we can optimize for speed, for energy efficiency, or for code size. What else?
- Why do we want the compiler to help us optimize?
 - So we can write more maintainable/portable code
 - So we don't have to worry about learning nuanced details about every possible system

• What are some compiler optimizations you know about?

• What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {
   x = x + 1;
}</pre>
```

loop unrolling

```
for (int i = 0; i < 10; i++) {
    x = x + 1;
    i++;
    x = x + 1;
}</pre>
```

• What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {
   x = x + 1;
}</pre>
```

loop unrolling

```
for (int i = 0; i < 10; i++) {
    x = x + 1;
    i++;
    x = x + 1;
}</pre>
```

```
int foo() {
  int i,j,k;
  i = 10;
  j = i;
  k = j;
  return k;
}
```

constant propagation

```
int foo() {
   int i,j,k;
   return 10;
}
```

• What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {
  x = x + 1;
}</pre>
```

loop unrolling

```
for (int i = 0; i < 10; i++) {
    x = x + 1;
    i++;
    x = x + 1;
}</pre>
```

What does this save us?

• What are some compiler optimizations you know about?

```
for (int i = 0; i < 10; i++) {
  x = x + 1;
}</pre>
```

loop unrolling

```
for (int i = 0; i < 10; i++) {
    x = x + 1;
    i++;
    x = x + 1;
}</pre>
```

optimizations at one stage can enable optimizations at another stage:

```
for (int i = 0; i < 10; i+=2) {
   x = x + 2;
}</pre>
```

provides a bigger window for local analysis

• What are some compiler optimizations you know about?

let's do a few more

Function inlining

```
int add(int x, int y) {
  return x + y;
}
int foo(int x, int y, int z) {
  return add(x,y);
}
```

```
int foo(int x, int y, int z) {
  return x + y;
}
```

What does this save us? code size? speed? the ability to debug? local regions to optimize more?

What are some compiler optimizations you know about?

There are many more! This is an active area of research and development

For a rough metric:

git effort shows activities on different files and directories

clang C++/C parser: 3.5K commits

clang AST: 8.7K commits

LLVM transforms/optimizations: 30K commits

The transformation part of the code base has the most activity by far

• How do you enable compiler optimizations?

- How do you enable compiler optimizations?
- most C/++ compilers
 - optimizing for speed
 - -O0, -O1, -O2, -O3
 - what about O4?
 - optimizing for size
 - -Os, -Oz
 - relax some constraints (especially around floating point):
 - Ofast
 - Godbolt example

How do you enable compiler optimizations?

- most C/++ compilers
 - optimizing for speed
 - -O0, -O1, -O2, -O3
 - what about O4?

optimizing for size

- -Os, -Oz
- relax some constraints (especially around floating point):

Does -O3 actually make a difference?

- Ofast
- Godbolt example

STABILIZER: Statistically Sound Performance Evaluation

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2013 research paper

"the performance impact of -03 over -02 optimizations is indistinguishable from random noise."

 What are some of the biggest improvements you've seen from compiler optimizations?

 What are some of the biggest improvements you've seen from compiler optimizations?

 compiler optimizations are great at well-structured, regular loops and arrays

Example: adding together two matrices

• What kind of transforms on your code is the compiler allowed to do?

many_add example

What kind of transforms on your code is the compiler allowed to do?

many_add example

• Why did we get such a dramatic increase?

What kind of transforms on your code is the compiler allowed to do?

many_add example

- Why did we get such a dramatic increase?
 - Programs must maintain their input/output behavior
 - Hard to precisely define (and still being discussed in C++ groups)
 - input/output can be files, volatile memory, console log, etc.

• Extreme example

```
void foo(int * arr, int n)
{
    int i, j;
    for (i = 0; i < n - 1; i++)
        for (j = 0; j < n - i - 1; j++)
        if (arr[j] > arr[j + 1]) {
            tmp = arr[j];
            arr[j] = arr[j + 1]);
            arr[j + 1] = tmp;
        }
}
```

```
int p(int arr[], int start, int end)
    int pivot = arr[start];
    int count = 0:
    for (int i = start + 1; i <= end; i++) {
        if (arr[i] <= pivot)</pre>
            count++;
    int pivotIndex = start + count;
    swap(arr[pivotIndex], arr[start]);
    int i = start, j = end;
    while (i < pivotIndex && j > pivotIndex) {
        while (arr[i] <= pivot) {</pre>
            i++;
        while (arr[j] > pivot) {
        if (i < pivotIndex && j > pivotIndex) {
            swap(arr[i++], arr[j--]);
    return pivotIndex;
void foo(int *arr, int n)
    if (start >= end)
        return;
    int p = p(arr, m, n);
    foo(arr, start, p - 1);
    foo(arr, p + 1, end);
```

• Extreme example

```
bubble sort
```

```
void foo(int * arr, int n)
{
    int i, j;
    for (i = 0; i < n - 1; i++)
        for (j = 0; j < n - i - 1; j++)
        if (arr[j] > arr[j + 1]) {
            tmp = arr[j];
            arr[j] = arr[j + 1]);
            arr[j + 1] = tmp;
        }
}
```

```
Yes this transform would be legal!
```

Could any compiler figure it out? currently unlikely..

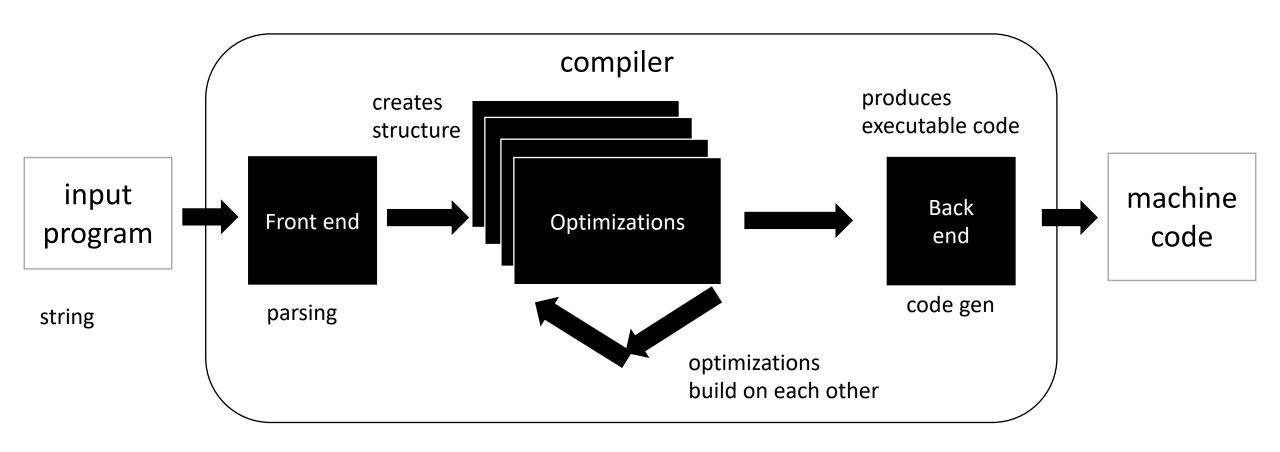
This is a technique called "super optimizing" and it is getting more and more interest

```
int p(int arr[], int start, int end)
    int pivot = arr[start];
    int count = 0:
    for (int i = start + 1; i <= end; i++) {
        if (arr[i] <= pivot)</pre>
            count++;
    int pivotIndex = start + count;
    swap(arr[pivotIndex], arr[start]);
    int i = start, j = end;
    while (i < pivotIndex && j > pivotIndex) {
        while (arr[i] <= pivot) {</pre>
            i++;
        while (arr[j] > pivot) {
        if (i < pivotIndex && j > pivotIndex) {
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    return pivotIndex;
void foo(int *arr, int n)
    if (start >= end)
        return;
    int p = p(arr, m, n);
    foo(arr, start, p - 1);
    foo(arr, p + 1, end);
```

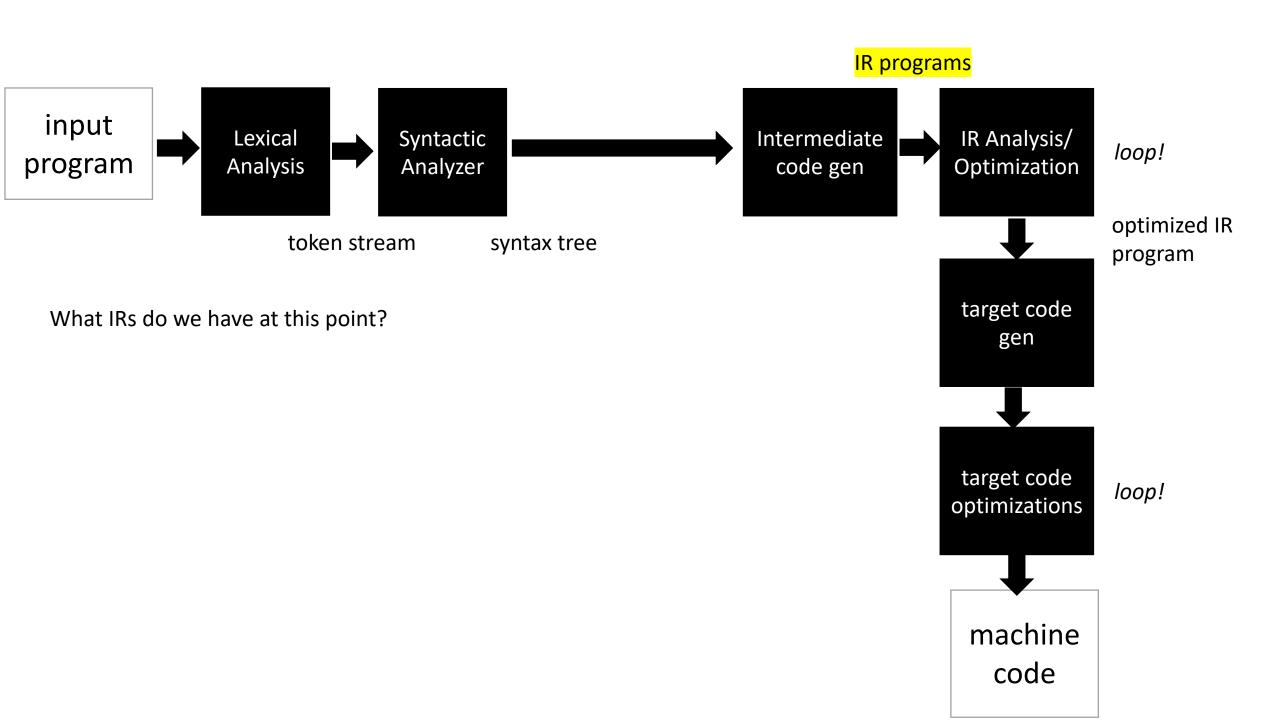
quick sort

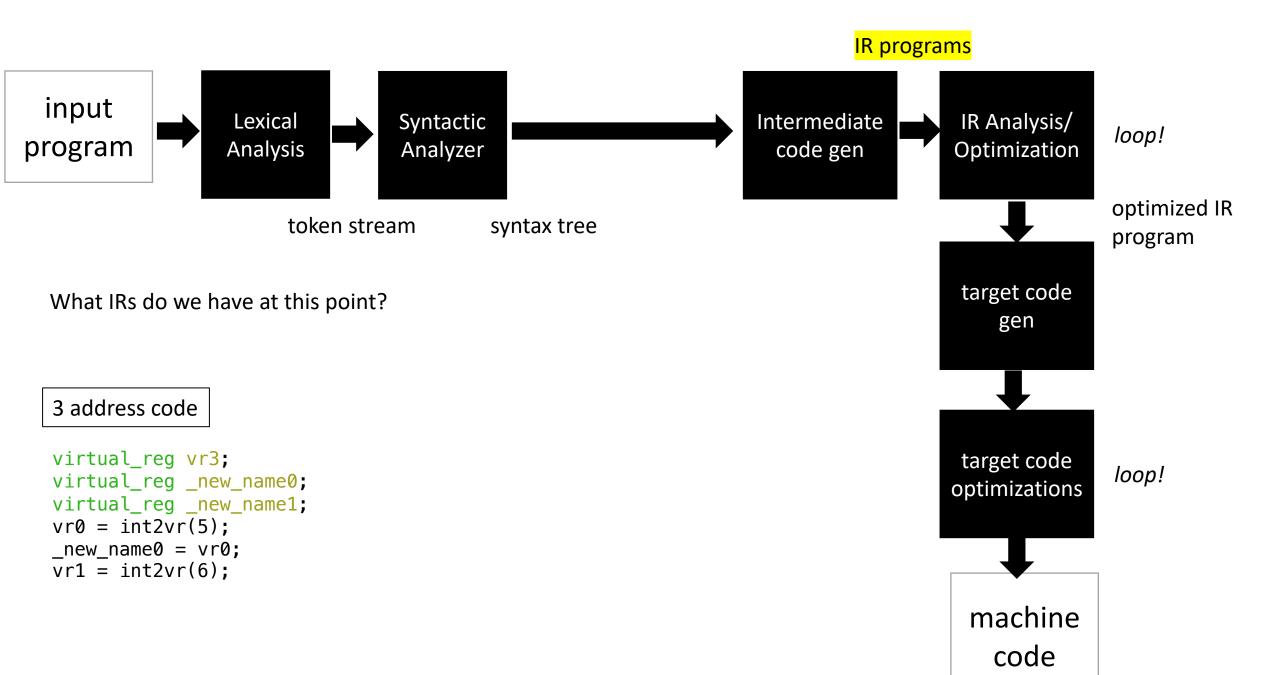
Moving on

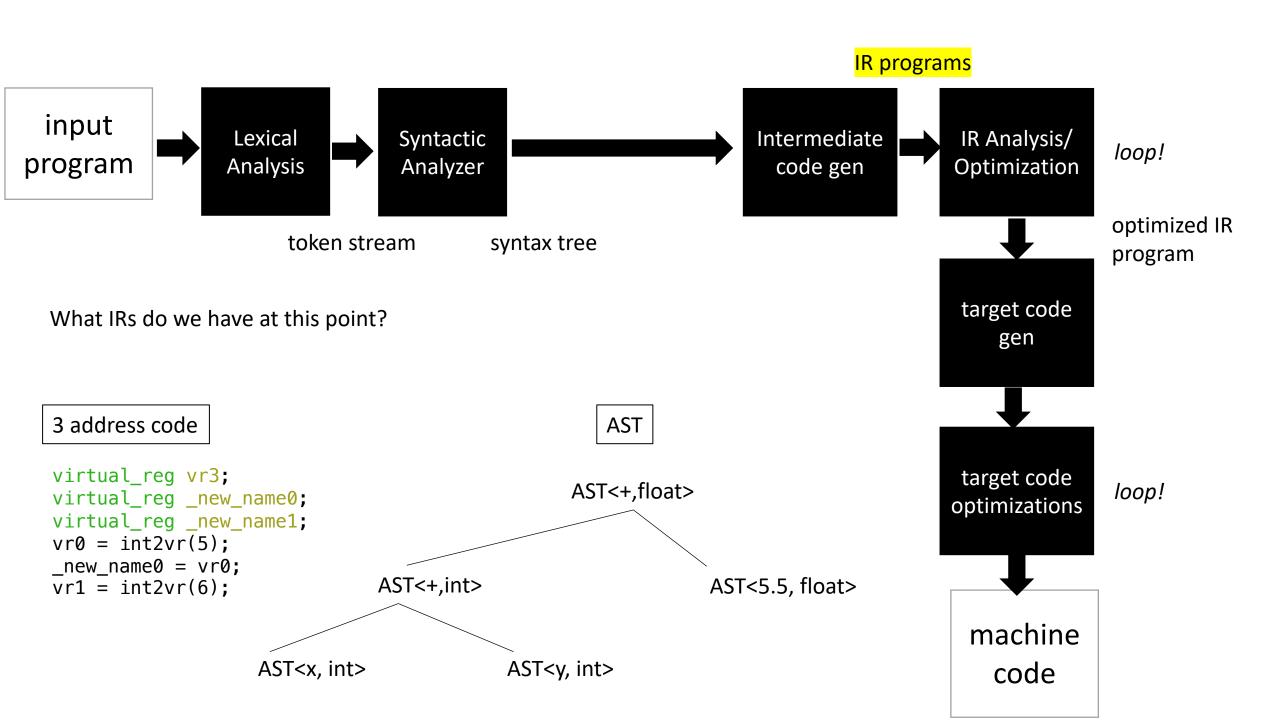
Zooming out again: Compiler Architecture



IRs and type inference type inference are at the boundary of parsing and optimizations





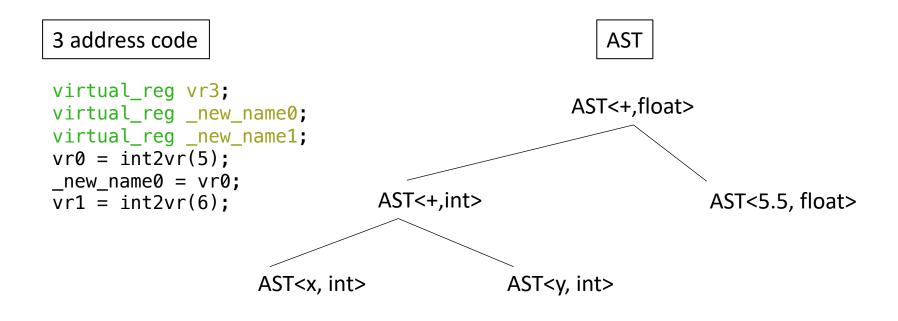


Implicit parse tree

```
if else statement := IF LPAR expr RPAR statement ELSE statement
if (program0) {
  program1
else {
  program2
```

We have several structures to utilize to analyze and optimize programs!

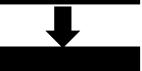
What IRs do we have at this point?



IR programs



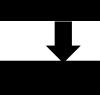
loop!



target code

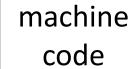
gen

optimized IR program



target code optimizations

loop!



Optimization categories

- Machine-independent these optimizations should work well across many different systems
 - Examples?

- Machine dependent these optimizations start to optimize the code for a given system
 - Examples?

Optimization categories

- Machine-independent these optimizations should work well across many different systems
 - Examples?
 - All the examples we looked at before seem like they will help across many systems

- Machine dependent these optimizations start to optimize the code for a given system
 - Examples?
 - loop chunking for cache line size and vectorization.
 - instruction re-orderings to take advantage of processor pipelines.
 - fused multiply-and-add instructions

- Machine-independent these optimizations should work well across many different systems
 - Examples?
 - All the examples we looked at before seem like they will help across many systems

• In this module we will be looking at machine-independent optimizations.

What are the pros of machine-independent optimizations?

Next category level is how much code we need to reason about for the optimization.

- **local optimizations**: examine a "basic block", i.e. a small region of code with no control flow.
 - Examples?
- Regional optimizations: several basic blocks with simple control flow.
 - Examples?
- Global optimization: optimizes across an entire function

- **local optimizations**: examine a "basic block", i.e. a small region of code with no control flow.
- Regional optimizations: several basic blocks with simple control flow
- Global optimization: optimizes across an entire function

Discussion:

- What are the pros and cons of each?
- Why don't we go further than functions?

- **local optimizations**: examine a "basic block", i.e. a small region of code with no control flow.
- Regional optimizations: several basic blocks with simple control flow
- Global optimization: optimizes across an entire function

For this module:

- We will look at two optimizations in detail:
- A local optimization: Local value numbering
- A regional optimization: Loop unrolling
- We will implement both as homework
- We will discuss several other optimizations and analysis

Basic blocks

A sequence of 3 address instructions

- Programs can be split into **Basic Blocks**:
 - A sequence of 3 address instructions such that:
 - There is a single entry, single exit

• *Important property*: an instruction in a basic block can assume that all preceding instructions will execute

Single Basic Block

```
Label_x:
op1;
op2;
op3;
br label_z;
```

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```

Two Basic Blocks

```
Label_x:
op1;
op2;
op3;

Label_y:
op4;
op5;
```

How might they appear in a high-level language? What are some examples?

A sequence of 3 address instructions

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- Programs can be split into Basic Blocks:
 - A sequence of 3 address instructions such that:
 - There is a single entry, single exit

• Important property: an instruction in a basic block can assume that all preceding instructions will execute

How might they appear in a high-level language?

How many basic blocks?

```
...
if (x) {
    ...
}
else {
    ...
}
...
```

Two Basic Blocks

Single Basic Block

```
Label_x:
op1;
op2;
op3;
br label_z;
```

```
Label_x:
op1;
op2;
op3;

Label_y:
op4;
op5;
```

Converting 3 address code into basic blocks

• Let's try an example: test 4 in HW 3:

Converting 3 address code into basic blocks

- Simple algorithm:
 - keep a list of basic blocks
 - a basic block is a list of instructions
 - Iterate over the 3 address instructions
 - if you see a branch or a label, finalize the current basic block and start a new one.

Converting 3 address code into basic blocks

pseudo code

```
basic_blocks = []
bb = []
for instr in program:
    if instr type is in [branch, label]:
        bb.append(instr)
        basic_blocks.append[bb]
        bb = []
else:
        bb.append(instr)
```

Local optimizations:

Optimizes an individual basic block

Regional optimizations:

Combines several basic blocks

Global optimizations:

- operates across an entire procedure
- what about across procedures?

Local optimizations:

Optimizes an individual basic block

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Label_0: x = a + b; y = a + b;

Local optimizations:

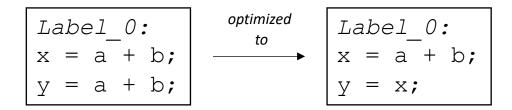
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Local optimizations:

Optimizes an individual basic block

Regional optimizations:

Combines several basic blocks

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- what about across procedures?

Label_0:

$$x = a + b;$$
 $y = a + b;$
optimized
to
$$x = a + b;$$

$$y = x;$$
Label_0:
 $x = a + b;$

$$y = x;$$

code could skip Label_0,
leaving x undefined!

Regional Optimization

```
...
if (x) {
    ...
}
else {
    x = a + b;
}
y = a + b;
...
```

```
we cannot replace:

y = a + b.

with

y = x;
```

Regional Optimization

```
...
if (x) {
    ...
}
else {
    x = a + b;
}
y = a + b;
...
```

```
we cannot replace:
    y = a + b.
    with
    y = x;
```

```
x = a + b;
if (x) {
    ...
}
else {
    ...
}
y = a + b;
...
```

```
But in this case, we can check if a
and b are not redefined, then
y = a + b;
can be replaced with
y = x;
```

This requires regional analysis and optimizations

A local optimization over 3 address code

 Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)

• A local optimization over 3 address code

 Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)

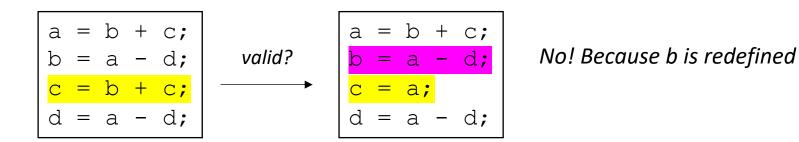
```
a = b + c;
b = a - d;
c = b + c;
d = a - d;
```

• A local optimization over 3 address code

 Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)

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• A local optimization over 3 address code

 Attempts to replace arithmetic operations (expensive) with copy instructions (cheap)

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.
- Keep a global counter; increment with new variables or assignments

Global_counter = 0

Algorithm:

- Provide a number to each variable. Update the number each time the variable is updated.
- Keep a global counter; increment with new variables or assignments

Global_counter = 7

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = ·
```

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
H = \{ b0 + c1" : "a2", \}
```

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
\begin{array}{c} a2 = b0 + c1; \\ b4 = a2 - d3; \\ c5 = b4 + c1; \\ d6 = a2 - d3; \end{array}
```

```
H = {
      "b0 + c1" : "a2",
}
```

Algorithm: Now that variables are numbered

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\begin{array}{c} a2 = b0 + c1; \\ b4 = a2 - d3; \\ c5 = b4 + c1; \\ d6 = a2 - d3; \end{array}
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a2 = b0 + c1;
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c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = \{
    "b0 + c1" : "a2",
    "a2 - d3" : "b4",
}
```

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = \{ \\ \text{"b0} + \text{c1"} : \text{"a2",} \\ \text{"a2} - \text{d3"} : \text{"b4",} \\ \\ \text{numberings!}
```

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = {
        "b0 + c1" : "a2",
        "a2 - d3" : "b4",
        "b4 + c1" : "c5",
}
```

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = a2 - d3;
```

```
H = {
        "b0 + c1" : "a2",
        "a2 - d3" : "b4",
        "b4 + c1" : "c5",
}
```

Local value numbering

Algorithm: Now that variables are numbered

• Iterate sequentially through instructions. Keep a hash table of the rhs (numbered variables and operation) mapped to their lhs.

• At each step, check to see if the rhs has already been computed.

```
a2 = b0 + c1;
b4 = a2 - d3;
c5 = b4 + c1;
d6 = b4;
```

```
H = \{

"b0 + c1" : "a2",

"a2 - d3" : "b4",

"b4 + c1" : "c5",

match!
```

What else can we do?

What else can we do?

Consider this snippet:

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

Commutative operations

What is the definition of commutative?

Commutative operations

What is the definition of commutative?

$$x OP y == y OP x$$

What operators are commutative? Which ones are not?

Adding commutativity to local value numbering

• For commutative operators (e.g. + *), the analysis should consider a deterministic order of operands.

You can use variable numbers or lexigraphical order

Algorithm optimization:

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = ·
```

Algorithm optimization:

 for commutative operations, re-order operands into a deterministic order

cannot re-order because - is not commutative

```
→ a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

Algorithm optimization:

```
a2 = c1 - b0;

f4 = d3 * a2;

c5 = b0 - c1;

d6 = a2 * d3;
```

Algorithm optimization:

 for commutative operations, re-order operands into a deterministic order

re-ordered because a2 < d3 lexigraphically

```
a2 = c1 - b0;

f4 = d3 * a2;

c5 = b0 - c1;

d6 = a2 * d3;
```

```
H = {
        "c1 - b0" : "a2",
        "a2 * d3" : "f4",
}
```

Algorithm optimization:

```
H = {
        "c1 - b0" : "a2",
        "a2 * d3" : "f4",
}
```

Algorithm optimization:

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = {
        "c1 - b0" : "a2",
        "a2 * d3" : "f4",
        "b0 - c1" : "c5",
}
```

Algorithm optimization:

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = a2 * d3;
```

```
H = {
        "c1 - b0" : "a2",
        "a2 * d3" : "f4",
        "b0 - c1" : "c5",
}
```

Algorithm optimization:

```
a2 = c1 - b0;
f4 = d3 * a2;
c5 = b0 - c1;
d6 = f4;
```

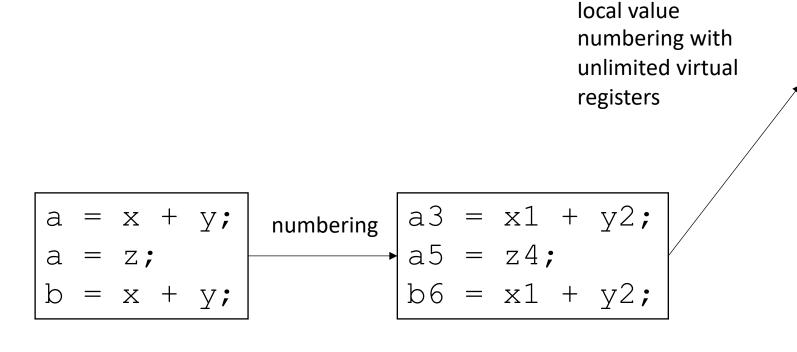
```
H = {
        "c1 - b0" : "a2",
        "a2 * d3" : "f4",
        "b0 - c1" : "c5",
}
```

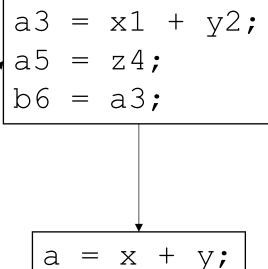
Other considerations?

 We've assumed we have access to an unlimited number of virtual registers.

- In some cases we may not be able to add virtual registers
 - If an expensive register allocation pass has already occurred.
- New constraint:
 - We need to produce a program such that variables without the numbers is still valid.

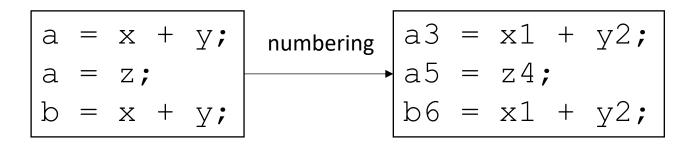
• Example:





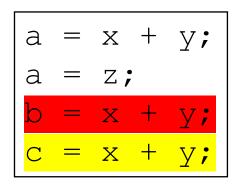
if we drop the numbers, the optimization is invalid.

Solutions?



```
a = x + y;
a = z;
b = x + y;
c = x + y;
```

Keep another hash table to keep the current variable number



We cannot optimize the first line, but we can optimize the second

```
a = x + y;
a = z;
b = x + y;
c = x + y;
```

Keep another hash table to keep the current variable number

$$a = x + y;$$
 $a = z;$
 $b = x + y;$
 $c = x + y;$

First we number

```
a3 = x1 + y2;

a5 = z4;

b6 = x1 + y2;

c7 = x1 + y2;
```

```
Current_val = {
}

H = {
}
```

```
a3 = x1 + y2;

a5 = z4;

b6 = x1 + y2;

c7 = x1 + y2;
```

```
a3 = x1 + y2;

a5 = z4;

b6 = x1 + y2;

c7 = b6;
```

Anything else we can add to local value numbering?

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• Final heuristic: keep sets of possible values

Local value numbering: value sets

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```
Current_val = {
}

H = {
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```

```
a = x + y;
b = x + y;
a = z;
c = x + y;
```

```
Current_val = {
}

H = {
}
```

```
a3 = x1 + y2;

b4 = x1 + y2;

a6 = z5;

c7 = x1 + y2;
```

```
a3 = x1 + y2;
b4 = a3;
a6 = z5;
c7 = x1 + y2;
H = \{
x1 + y2'' : "a3''
```

```
a3 = x1 + y2;
b4 = a3;
a6 = z5;
c7 = x1 + y2;
H = \{
x1 + y2'' : "a3''
```

```
a3 = x1 + y2;

b4 = a3;

a6 = z5;

c7 = x1 + y2;
```

```
Current_val = {
                       "a" : 3,
                         "b" : 4
hash a list of possible values
```

```
fast forward again

\begin{array}{rcl}
a3 &=& x1 + y2; \\
b4 &=& a3; \\
a6 &=& z5; \\
c7 &=& x1 + y2;
\end{array}
```

```
fast forward again

\begin{array}{rcl}
a3 &=& x1 + y2; \\
b4 &=& a3; \\
a6 &=& z5; \\
c7 &=& b4;
\end{array}
```

Consider a 3 address code that allows memory accesses

```
a[i] = x[j] + y[k];

b[i] = x[j] + y[k];
```

is this transformation allowed?
No!

only if the compiler can prove that a does not alias \boldsymbol{x} and \boldsymbol{y}

$$a[i] = x[j] + y[k];$$

 $b[i] = a[i];$

In the worst case, every time a memory location is updated, the compiler must update the value for all pointers.

- How to number:
 - Number each pointer/index pair

```
(a[i],3) = (x[j],1) + (y[k],2);

b[i] = x[j] + y[k];
```

- How to number:
 - Number each pointer/index pair
 - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);

(b[i],6) = (x[j],4) + (y[k],5);
```

- How to number:
 - Number each pointer/index pair
 - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);

(b[i],6) = (x[j],4) + (y[k],5);
```

compiler analysis:

```
can we trace a, x, y to
a = malloc(...);
x = malloc(...);
y = malloc(...);
// a, x, y are never overwritten
```

- How to number:
 - Number each pointer/index pair
 - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);

(b[i],6) = (x[j],1) + (y[k],2);
```

in this case we do not have to update the number

compiler analysis:

```
can we trace a, x, y to
a = malloc(...);
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```
(a[i],3) = (x[j],1) + (y[k],2);

(b[i],6) = (x[j],4) + (y[k],5);
```

programmer annotations can also tell the compiler that no other pointer can access the memory pointed to by a

- How to number:
 - Number each pointer/index pair
 - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);

(b[i],6) = (x[j],4) + (y[k],5);
```

in this case we do not have to update the number

restrict a

the compiler that no other pointer can access the memory pointed to by a

- How to number:
 - Number each pointer/index pair
 - Any pointer/index pair that might alias must be incremented at each instruction

```
(a[i],3) = (x[j],1) + (y[k],2);

(b[i],6) = (a[i],3);
```

Optimizing over wider regions

Local value numbering operated over just one basic block.

 We want optimizations that operate over several basic blocks (a region), or across an entire procedure (global)

- For this, we need Control Flow Graphs and Flow Analysis
 - We may have time to discuss this later in the module

See everyone on Wednesday

More about optimizations!