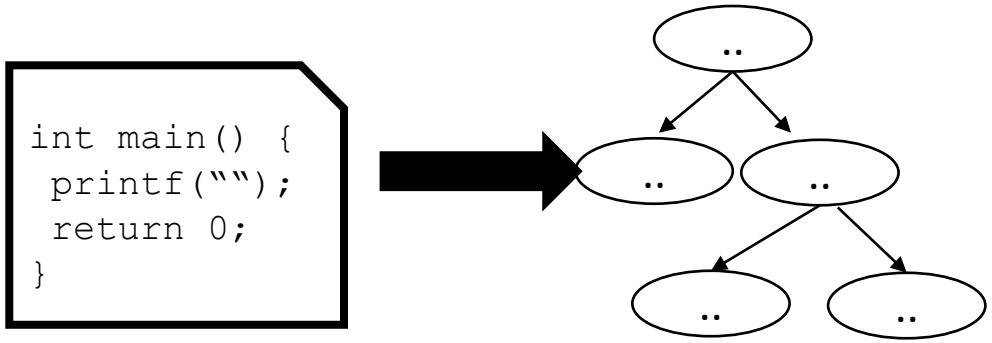
CSE110A: Compilers

April 24, 2024

Topics:

- *Syntactic Analysis continued*
  - *Top down parsing*
    - *Oracle parser*
    - *Rewriting to avoid left recursion*



Announcements

- HW 2 is due on **Monday** by midnight
  - You'll have what you need for part 2 by the end of today (and hopefully part 3)

- For help
  - Ask on Piazza: ***No guaranteed help over the weekend or off business hours***

- ***You do not have to return anything from your parser. Right now it is all about specification. You either match the string or you don't***

Announcements

- Private posts on piazza for any questions, especially about grading.

Midterm study guide (so far)

Any of the following are fair game. Anything not listed below but in the lectures are fair game. Any combination of topics is fair game. This is only meant to be an overview of what we have discussed so far.

Midterm study guide (so far)

- Regular expressions
  - Operators, how to specify, how match vs full match works
- Scanners
  - What the API is, how strings are tokenized, how to specify tokens, token actions
- Grammars
  - How to specify a grammar, how to identify/avoid ambiguous grammars, how to show a derivation for match, parse trees
  - How to re-write grammars not to be left recursive, how to identify first+ sets
  - How the top down parsing algorithm works, how a recursive decent parser works
- Symbol tables
  - How scope can be tracked and manage during parse time, symbol table specification and implementation

- First 2 classes of module 3

Quiz

Which of the following can be sources of ambiguity in grammars?

- ◯ operator associativity not being specified
- ◯ incorrect parenthesis matching
- ◯ operator precedence not being specified
- ◯ operator commutativity not being specified

Quiz

Please make sure to download HW 2 and try writing simple grammars in PLY. Please mark true when you've tried executing some simply programs and have experimented with specifying some tokens and production rules.

Example

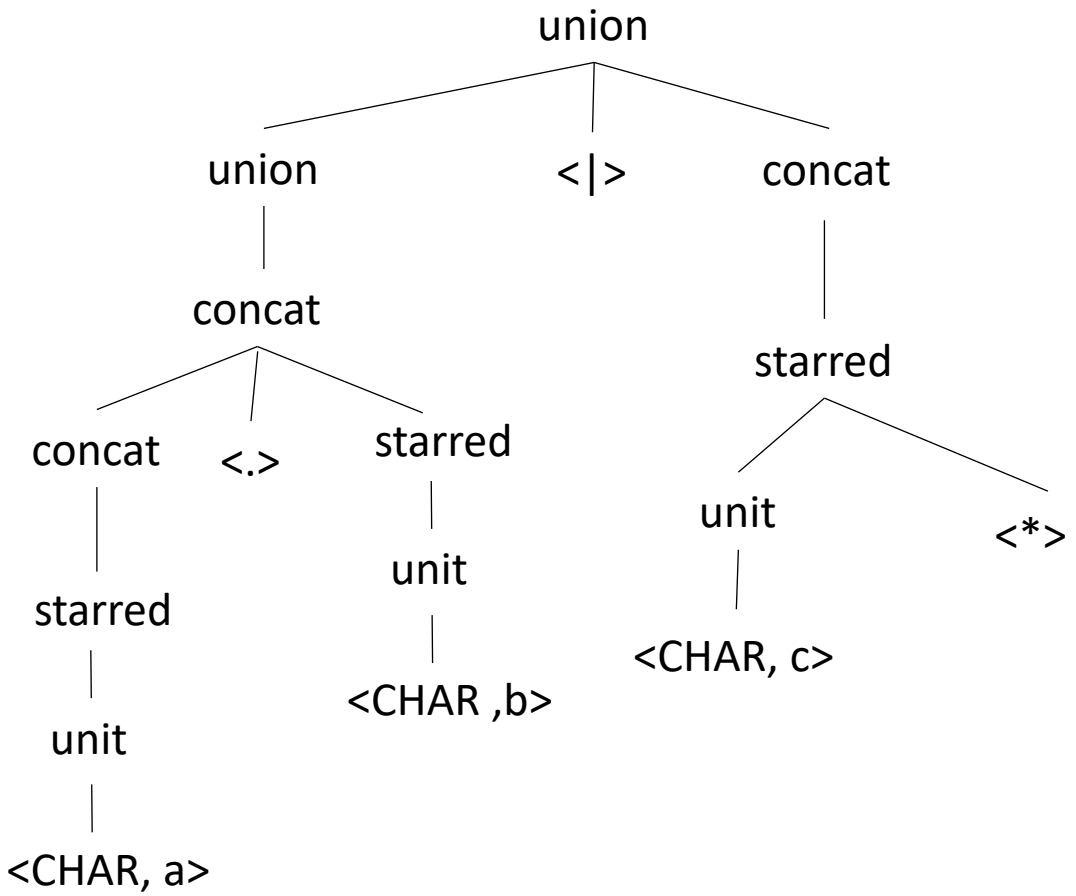
Parsing REs

Let's try it for regular expressions, {| . * ()}

- *Assume . is concat*

Operator	Name	Productions
|	`union`	`: union PIPE concat` `| concat`
.	`concat`	`: concat DOT starred` `| starred`
*	`starred`	`: starred STAR` `| unit`
()	`unit`	`: LPAREN union RPAREN` `| CHAR`

`input: a.b | c*`



Review

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Variable	Value
focus	
to_match	
s.istring	
stack	

```
1: Expr   ::= Expr Op Unit
2:        |   Unit
3: Unit   ::= '(' Expr ')'
4:        |    ID
5: Op     ::= '+'
6:        |   '*'
```

Can we derive the string `(a+b)*c`

Expanded Rule	Sentential Form
start	Expr

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Currently we assume this is magic and picks the right rule every time

Variable	Value
focus	
to_match	
s.istring	
stack	

```
1: Expr   ::= Expr Op Unit
2:        |   Unit
3: Unit   ::= '(' Expr ')'
4:        |    ID
5: Op     ::= '+'
6:        |   '*'
```

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root = start symbol;
focus = root;
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while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

What can go wrong if we don't have a magic choice

Variable	Value
focus	
to_match	
s.istring	
stack	

```
1: Expr   ::= Expr Op Unit
2:        |   Unit
3: Unit   ::= '(' Expr ')'
4:        |    ID
5: Op     ::= '+'
6:        |   '*'
```

Can we derive the string `(a+b)*c`

Expanded Rule	Sentential Form
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root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

What can go wrong

Variable	Value
focus	
to_match	
s.istring	
stack	

```
1: Expr   ::= Expr Op Unit
2:        |   Unit
3: Unit   ::= '(' Expr ')'
4:        |    ID
5: Op     ::= '+'
6:        |   '*'
```

Can we derive the string `(a+b)*c`

Expanded Rule	Sentential Form
start	Expr
2	Expr Op Unit
2	Expr Op Unit Op Unit
2	Expr Op Unit Op Unit Op Unit
2	Expr Op Unit ....

Infinite recursion!

Top down parsing does not handle left recursion

```
1: Expr   ::= Expr Op Unit
2:        |   Unit
3: Unit   ::= '(' Expr ')'
4:        |    ID
5: Op     ::= '+'
6:        |   '*'
```

direct left recursion

```
1: Expr_base ::= Unit
2:           |   Expr_op
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:           |    ID
6: Op        ::= '+'
7:           |   '*'
```

indirect left recursion

Top down parsing cannot handle either

Top down parsing does not handle left recursion

- In general, any CFG can be re-written without left recursion

Eliminating direct left recursion

```
Fee ::= Fee “a”
    |   “b”
```

What does this grammar describe?

Eliminating direct left recursion

```
Fee ::= Fee “a”
    |   “b”
```

The grammar can be rewritten as

```
Fee  ::= “b” Fee2

Fee2 ::= “a” Fee2
     |   “”
```

Eliminating direct left recursion

In general, A and B can be any sequence of non-terminals and terminals

```
Fee ::= Fee A
    |   B
```

```
Fee  ::= B Fee2

Fee2 ::= A Fee2
     |   “”
```

Eliminating direct left recursion

```
1: Expr  ::= Expr Op Unit
2:       |   Unit
3: Unit  ::= '(' Expr ')'
4:       |    ID
5: Op    ::= '+'
6:       |   '*'
```

Lets do this one as an example:

```
Fee ::= Fee A
    |   B
```

```
Fee  ::= B Fee2

Fee2 ::= A Fee2
     |    ""
```

Eliminating direct left recursion

```
A = ?
B = ?
```

```
1: Expr  ::= Expr Op Unit
2:       |   Unit
3: Unit  ::= '(' Expr ')'
4:       |    ID
5: Op    ::= '+'
6:       |   '*'
```

```
1: Expr  ::= ?
2: Expr2 ::= ?
3:       | ?
4: Unit  ::= '(' Expr ')'
5:       |    ID
6: Op    ::= '+'
7:       |   '*'
```

Lets do this one as an example:

```
Fee ::= Fee A
    |   B
```

```
Fee  ::= B Fee2

Fee2 ::= A Fee2
     |    “”
```

Eliminating direct left recursion

```
A = Op Unit
B = Unit
```

```
1: Expr  ::= Expr Op Unit
2:       |   Unit
3: Unit  ::= '(' Expr ')'
4:       |    ID
5: Op    ::= '+'
6:       |   '*'
```

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | ""
4: Unit  ::= '(' Expr ')'
5:       |    ID
6: Op    ::= '+'
7:       |   '*'
```

Lets do this one as an example:

```
Fee ::= Fee A
    |   B
```

```
Fee  ::= B Fee2

Fee2 ::= A Fee2
     |    ""
```

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Variable	Value
focus	
to_match	
s.istring	
stack	

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:         |    ID
6: Op    ::= ‘+’
7:         |   ‘*’
```

Expanded Rule	Sentential Form
start	Expr

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Variable	Value
focus	
to_match	
s.istring	
stack	

How to handle this case?

```
1: Expr   ::= Unit Expr2
2: Expr2  ::= Op Unit Expr2
3:          | ""
4: Unit   ::= '(' Expr ')'
5:          |   ID
6: Op     ::= '+'
7:          |   '*'
```

Expanded Rule	Sentential Form
start	Expr

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    if A == "": focus=pop(); continue;
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Variable	Value
focus	
to_match	
s.istring	
stack	

How to handle this case?

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | ""
4: Unit  ::= '(' Expr ')'
5:         |   ID
6: Op    ::= '+'
7:         |   '*'
```

Expanded Rule	Sentential Form
start	Expr

How about indirect left recursion?

```
1: Expr   ::= Expr Op Unit
2:        |   Unit
3: Unit   ::= '(' Expr ')'
4:        |    ID
5: Op     ::= '+'
6:        |   '*'
```

direct left recursion

```
1: Expr_base ::= Unit
2:           |   Expr_op
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:           |    ID
6: Op        ::= '+'
7:           |   '*'
```

indirect left recursion

Top down parsing cannot handle either

How about indirect left recursion?

```
1: Expr_base ::= Unit
2:           |   Expr_op
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:           |    ID
6: Op        ::= '+'
7:           |   '*'
```

Identify indirect left left recursion

$$Expr\_base \rightarrow_{lhs} Expr\_op \rightarrow_{lhs} Expr\_base$$

How about indirect left recursion?

```
1: Expr_base ::= Unit
2:           |   Expr_op
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:           |    ID
6: Op        ::= '+'
7:           |   '*'
```

Identify indirect left left recursion

$$Expr\_base \rightarrow_{lhs} Expr\_op \rightarrow_{lhs} Expr\_base$$

Substitute indirect non-terminal closer to initial non-terminal

How about indirect left recursion?

```
1: Expr_base ::= Unit
2:           |   Expr_op
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:           |   ID
6: Op        ::= '+'
7:           |   '*'
```

```
1: Expr_base ::= Unit
2:           |   Expr_base Op Unit
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:           |   ID
6: Op        ::= '+'
7:           |   '*'
```

Identify indirect left left recursion

What to do with production rule 3?

$$Expr\_base \rightarrow_{lhs} Expr\_op \rightarrow_{lhs} Expr\_base$$

Substitute indirect non-terminal closer to initial non-terminal

How about indirect left recursion?

```
1: Expr_base ::= Unit
2:           |   Expr_op
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:            |  ID
6: Op        ::= '+'
7:           |   '*'
```

```
1: Expr_base ::= Unit
2:           |   Expr_base Op Unit
3: Expr_op   ::= Expr_base Op Unit
4: Unit      ::= '(' Expr_base ')'
5:            |  ID
6: Op        ::= '+'
7:           |   '*'
```

Identify indirect left left recursion

What to do with production rule 3?
It may need to stay if another production rule references it!

$$Expr\_base \rightarrow_{lhs} Expr\_op \rightarrow_{lhs} Expr\_base$$

Substitute indirect non-terminal closer to initial non-terminal

It is always possible to eliminate left recursion

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    cache_state();
    pick next rule (A ::= B1,B2,B3...BN);
    if B1 == "": focus=pop(); continue;
    push(BN... B3, B2);
    focus = B1

 else if (to_match == None and focus == None)
    Accept

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (we have a cached state)
     backtrack();

  else
    parser_error()
```

Keep track of what choices we've done

```
1: Expr  ::= ID Expr2
2: Expr2 ::= '+' Expr2
         |   ""
```

Can we match: "a"?

Expanded Rule	Sentential Form
start	Expr

Backtracking gets complicated...

- Do we need to backtrack?
  - In the general case, **yes**
  - In many useful cases, **no**

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    if B1 == "": focus=pop(); continue;
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Variable	Value
focus	Expr2
to_match	None
s.istring	""
stack	None

```
1: Expr  ::= ID Expr2
2: Expr2 ::= '+' Expr2
3:       |   ""
```

Could we make a smarter choice here?

Can we match: "a"?

Expanded Rule	Sentential Form
start	Expr
1	ID Expr2

The First Set

For each production choice, find the set of tokens that each production can start with

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

```
First sets:
1: {}
2: {}
3: {}
4: {}
5: {}
6: {}
7: {}
```

The First Set

For each production choice, find the set of tokens that each production can start with

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:         |    ID
6: Op    ::= ‘+’
7:         |   ‘*’
```

```
First sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {“”}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

We can use first sets to decide which rule to pick!

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

Variable	Value
focus	
to_match	
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```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

```
First sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {“”}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

We simply use to_match and compare it to the first sets for each choice

For example, Op and Unit

The Follow Set

Rules with “” in their First set need special attention

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

```
First sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {“”}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

```
Follow sets:
1: NA
2: NA
3: {}
4: NA
5: NA
6: NA
7: NA
```

We need to find the tokens that any string that follows the production can start with.

The Follow Set

Rules with “” in their First set need special attention

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

```
First sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {“”}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

```
Follow sets:
1: NA
2: NA
3: {None, ’)’}
4: NA
5: NA
6: NA
7: NA
```

We need to find the tokens that any string that follows the production can start with.

The First+ Set

The First+ set is the combination of First and Follow sets

```
                                First sets:    Follow sets:     First+ sets:
1: Expr  ::= Unit Expr2         1: {'(', ID}   1: NA            1: {'(', ID}
2: Expr2 ::= Op Unit Expr2      2: {'+', '*'}  2: NA            2: {'+', '*'}
3:       | ""                   3: {""}        3: {None, ')'}   3: {None, ')'}
4: Unit  ::= '(' Expr ')'       4: {'('}       4: NA            4: {'('}
5:       |    ID                5: {ID}        5: NA            5: {ID}
6: Op    ::= '+'                6: {'+'}       6: NA            6: {'+'}
7:       |   '*'                7: {'*'}       7: NA            7: {'*'}
```

Do we need backtracking?

The First+ set is the combination of First and Follow sets

```
1: Expr  ::= Unit Expr2          First+ sets:
2: Expr2 ::= Op Unit Expr2       1: {‘(‘, ID}
3:       | “”                    2: {‘+’, ‘*’}
4: Unit  ::= ‘(‘ Expr ‘)’        3: {None, ’)’}
5:       |    ID                 4: {‘(‘}
6: Op    ::= ‘+’                 5: {ID}
7:       |   ‘*’                 6: {‘+’}
                                 7: {‘*’}
```

For each non-terminal: if every production has a disjoint First+ set then we do not need any backtracking!

Do we need backtracking?

The First+ set is the combination of First and Follow sets

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

First+ sets:

1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}

For each non-terminal: if every production has a disjoint First+ set then we do not need any backtracking!

Do we need backtracking?

The First+ set is the combination of First and Follow sets

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

First+ sets:

```
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

These grammars are called LL(1)

- L - scanning the input left to right
- L - left derivation
- 1 - how many look ahead symbols

They are also called predictive grammars

Many programming languages are LL(1)

For each non-terminal: if every production has a disjoint First+ set then we do not need any backtracking!

Sometimes the grammar needs to be refactored

```
1: Factor ::= ID
2:          |   ID ‘[‘ Args ‘]’
3:          |   ID ‘(‘ Args ‘)’
...
```

Sometimes the grammar needs to be refactored

```
                                        First
1: Factor ::= ID                        1: {}
2:          |   ID '[' Args ']'         2: {}
3:          |   ID '(' Args ')'         3: {}
...                                     ...
```

Sometimes the grammar needs to be refactored

```
1: Factor ::= ID                        First
2:          |   ID '[' Args ']'          1: {ID}
3:          |   ID '(' Args ')'          2: {ID}
...                                      3: {ID}
                                         ...
```

We cannot select the next rule based on a single look ahead token!

Sometimes the grammar needs to be refactored

```
                                    First
1: Factor ::= ID                    1: {ID}
2:          |   ID ‘[‘ Args ‘]’      2: {ID}
3:          |   ID ‘(‘ Args ‘)’      3: {ID}
...                                 ...
```

We can refactor

```
                                          First
1: Factor      ::= ID Option_args         1: {}
2: Option_args ::= ‘[‘ Args ‘]’           2: {}
3:             |   ‘(‘ Args ‘)’           3: {}
4:             |   “”                     4: {}
```

Sometimes the grammar needs to be refactored

```
1: Factor ::= ID                    First
2:          |   ID '[' Args ']'     1: {ID}
3:          |   ID '(' Args ')'     2: {ID}
...                                 3: {ID}
                                    ...
```

We can refactor

```
1: Factor      ::= ID Option_args       First
2: Option_args ::= '[' Args ']'         1: {ID}
3:             |   '(' Args ')'         2: {'['}
4:             |   ""                   3: {'('}
                                        4: {""}   // We will need to compute the follow set
```

Sometimes the grammar needs to be refactored

```
1: Factor ::= ID
2:          |   ID '[' Args ']'
3:          |   ID '(' Args ')'
...
```

```
First
1: {ID}
2: {ID}
3: {ID}
...
```

It is not always possible to rewrite grammars into a predictive form, but many programming languages can be.

We can refactor

```
1: Factor      ::= ID Option_args
2: Option_args ::= '[' Args ']'
3:             |   '(' Args ')'
4:             |   ""
```

```
First
1: {ID}
2: {'['}
3: {'('}
4: {""}
```

// We will need to compute the follow set

We now have a full top-down parsing algorithm!

```
root = start symbol;
focus = root;
push(None);
to_match = s.token();

while (true):
  if (focus is a nonterminal)
    pick next rule (A ::= B1,B2,B3...BN);
    push(BN... B3, B2);
    focus = B1

  else if (focus == to_match)
    to_match = s.token()
    focus = pop()

  else if (to_match == None and focus == None)
    Accept
```

```
First+ sets:
1: {'(', ID}
2: {'+', '*'}
3: {None, ')'}
4: {'('}
5: {ID}
6: {'+'}
7: {'*'}
```

First+ sets for each production rule

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | ""
4: Unit  ::= '(' Expr ')'
5:         |    ID
6: Op    ::= '+'
7:         |   '*'
```

input grammar, refactored to remove left recursion

To pick the next rule, compare `to_match` with the possible `first+` sets.
Pick the rule whose `first+` set contains `to_match`.

If there is no such rule then it is a parsing error.

Moving on to a simpler implementation:

Recursive Descent Parser

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

How do we parse an Expr?

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

How do we parse an Expr?
We parse a Unit followed by an Expr2

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

How do we parse an Expr?
We parse a Unit followed by an Expr2

We can just write exactly that!

```
def parse_Expr(self):
        self.parse_Unit();
        self.parse_Expr2();
        return
```

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:         |    ID
6: Op    ::= ‘+’
7:         |   ‘*’
```

How do we parse an Expr2?

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:         |    ID
6: Op    ::= ‘+’
7:         |   ‘*’
```

How do we parse an Expr2?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

Let's look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:         |    ID
6: Op    ::= ‘+’
7:         |   ‘*’
```

How do we parse an Expr2?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

```
def parse_Expr2(self):

    token_id = get_token_id(self.to_match)

    # Expr2 ::= Op Unit Expr2
    if token_id in ["PLUS", "MULT"]:
        self.parse_Op()
        self.parse_Unit()
        self.parse_Expr2()
        return

    # Expr2 ::= ""
    if token_id in [None, ”RPAR"]:
        return

    raise ParserException(-1,                          # line number (for you to do)
                          self.to_match,               # observed token
                          ["PLUS", "MULT", ”RPAR"])    # expected token
```

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

How do we parse a Unit?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

Let's look at the grammar

```
1: Expr   ::= Unit Expr2
2: Expr2  ::= Op Unit Expr2
3:          | “”
4: Unit   ::= ‘(‘ Expr ‘)’
5:          |    ID
6: Op     ::= ‘+’
7:          |   ‘*’
```

How do we parse a Unit?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

```
def parse_Unit(self):

    token_id = get_token_id(self.to_match)

    # Unit  ::= ‘(‘ Expr ‘)’
    if token_id == "LPAR":
        self.eat("LPAR")
        self.parse_Expr()
        self.eat("RPAR")
        return

    # Unit :: = ID
    if token_id == "ID":
        self.eat("ID")
        return

    raise ParserException(-1,                  # line number (for you to do)
                          self.to_match,       # observed token
                          ["LPAR", "ID"])      # expected token
```

Let's look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:         | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:         |    ID
6: Op    ::= ‘+’
7:         |   ‘*’
```

How do we parse a Unit?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

```
def parse_Unit(self):

    token_id = get_token_id(self.to_match)

    # Unit  ::= ‘(‘ Expr ‘)’
    if token_id == "LPAR":
        self.eat("LPAR")
        self.parse_Expr()
        self.eat("RPAR")
        return

    # Unit :: = ID
    if token_id == "ID":
        self.eat("ID")
        return

    raise ParserException(-1,                  # line number (for you to do)
                          self.to_match,       # observed token
                          ["LPAR", "ID"])      # expected token
```

ensure that to_match has token ID of “LPAREN” and get the next token

Let’s look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

How do we parse an Op?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

Let's look at the grammar

```
1: Expr  ::= Unit Expr2
2: Expr2 ::= Op Unit Expr2
3:       | “”
4: Unit  ::= ‘(‘ Expr ‘)’
5:       |    ID
6: Op    ::= ‘+’
7:       |   ‘*’
```

How do we parse an Op?

```
First+ sets:
1: {‘(‘, ID}
2: {‘+’, ‘*’}
3: {None, ’)’}
4: {‘(‘}
5: {ID}
6: {‘+’}
7: {‘*’}
```

```
def parse_Op(self):

    token_id = get_token_id(self.to_match)

    # Op  ::= '+'
    if token_id == "PLUS":
        self.eat("PLUS")
        return

    # Op  ::= '*'
    if token_id == "MULT":
        self.eat("MULT")
        return


    raise ParserException(-1,                  # line number (for you to do)
                          self.to_match,       # observed token
                          ["MULT", "PLUS"]) # expected token
```