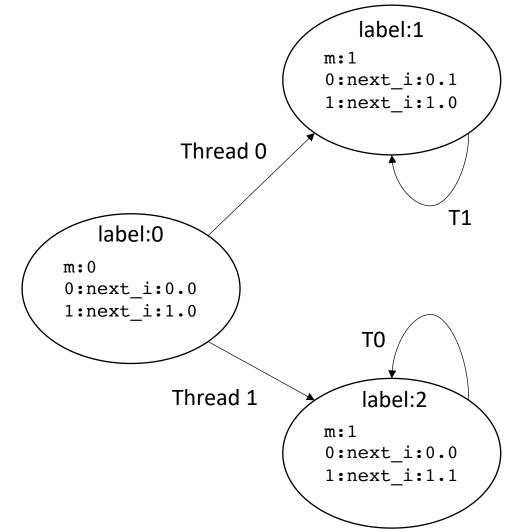
## CSE113: Parallel Programming

Feb. 28, 2022

#### • Topics:

- Forward progress
  - scheduler specifications



#### Announcements

- HW 4 is out
  - Due on Friday
  - You can share timing results
- Grades for HW 2 are out
  - last day to raise issues
- Grades for Midterm are just wrapping up
  - Expect them by the end of the day

## Today's Quiz

• Due Tomorrow by midnight; please do it!

It is safe to run a program written for RMO on a TSO system?

O True

O False

It is safe to run a program written for TSO on an RMO system

○ True

False

The edges	s in a labele	d transition	system	encode wha	t?

A cycle in the labeled transition system (LTS) indicates:

- o a weak memory behavior has occurred
- a bug and the program must abort and retry
- a potential source of non-termination

## Review

## Relaxed memory models and mutexes

```
Thread 0:
m.lock();
display.enq(triangle0);
m.unlock();
```

```
Thread 1:
m.lock();
display.enq(triangle1);
m.unlock();
```

```
Thread 0:
m.lock();
display.enq(triangle0);
m.unlock();
```

```
Thread 1:
m.lock();
display.enq(triangle1);
m.unlock();
```

We know how lock and unlock are implemented

```
Thread 0:
SPIN:CAS(mutex,0,1);
display.enq(triangle0);
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex,0,1);
display.enq(triangle1);
store(mutex,0);
```

We know how lock and unlock are implemented We also know how a queue is implemented

```
Thread 0:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

We know how lock and unlock are implemented We also know how a queue is implemented

What is an execution?

```
Thread 0:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

CAS(mutex, 0, 1);

if blue goes first
it gets to complete
its critical section
while thread 1 is spinning

```
Thread 0:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

```
CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
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store(mutex,0);
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# Thread 0: SPIN:CAS(mutex,0,1); %i = load(head); store(buffer+i, triangle0); store(head, %i+1); store(mutex,0);

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

```
CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

now yellow gets a change to go

# Thread 0: SPIN:CAS(mutex,0,1); %i = load(head); store(buffer+i, triangle0); store(head, %i+1); store(mutex,0);

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

```
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

CAS(mutex, 0, 1);

now yellow gets a change to go

```
Thread 0:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

what can happen in a PSO memory model?

\_

L NO Different address

NO Different address

```
CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 0:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
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```

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

what can happen in a PSO memory model?

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NO Different address

NO Different address

```
CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 0:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex,0,1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
store(mutex,0);
```

what can happen in a PSO memory model?

L S

NO Different address

NO Different address

CAS(mutex, 0, 1);%i = load(head); store(buffer+i, triangle0); store(mutex,0); CAS(mutex, 0, 1);%i = load(head); store(head, %i+1); store(buffer+i, triangle1); store(head, %i+1); store(mutex,0);

What just happened if this store moves?

```
Thread 0:
SPIN:CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle0);
store(head, %i+1);
fence;
                    unlock contains fence
                    before store!
store(mutex,0);
```

```
Thread 1:
SPIN:CAS(mutex, 0, 1);
%i = load(head);
store(buffer+i, triangle1);
store(head, %i+1);
fence;
                    unlock contains fence
                    before store!
store(mutex,0);
```

what can happen in a PSO memory model?

Different NO address Different NO address

S

CAS(mutex, 0, 1);%i = load(head); store(buffer+i, triangle0); store(head, %i+1); fence; No instructions store(mutex,0); can move after the mutex store! CAS(mutex, 0, 1);%i = load(head); store(buffer+i, triangle0); store(head, %i+1);

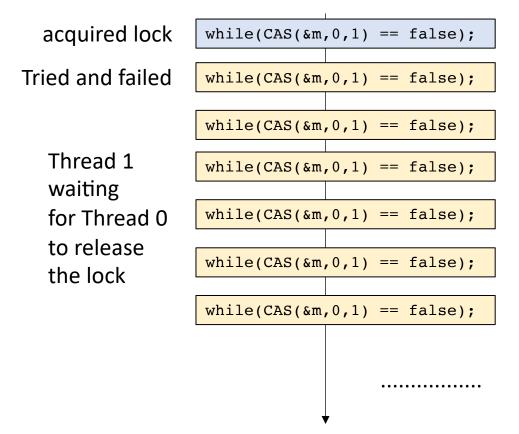
How to fix the issue?

your unlock function should contain a fence!

## Scheduler specifications

```
Thread 0:
//lock
while(CAS(&m,0,1) == false);
int tmp = *bank_account;
tmp++;
*bank_account = tmp;
m.store(0); //unlock
```

```
Thread 1:
//lock
while(CAS(&m,0,1) == false);
int tmp = *bank_account;
tmp--;
*bank_account = tmp;
m.store(0); //unlock
```



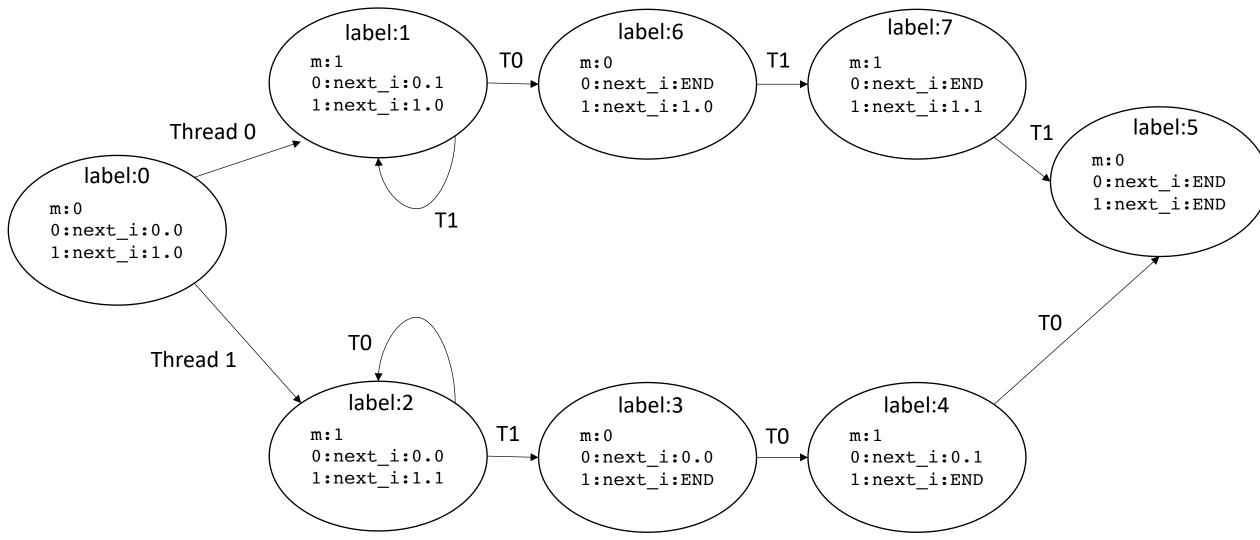
Can this keep going forever?

Is this program guaranteed to terminate?

Why? Why not?

global timeline

```
Thread 0:
0.0: while(CAS(&m,0,1) == false); //lock
    // critical section
0.1: m.store(0); //unlock
label:1
m:1
TO m:0
```



### This is called an LTS

- A graph:
  - Each state encodes all variables/values and what the next instruction to execute is
  - Each edge out of a node is the different threads that can execute
  - A concurrent execution is any path through the LTS

```
Thread 0:
                                                                    Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
                                                                    1.0: while (CAS(&m, 0, 1) == false); //lock
       // critical section
                                                                           // critical section
0.1: m.store(0); //unlock
                                                                    1.1: m.store(0); //unlock
                                                                                             label:7
                                                                label:6
                                   label:1
                                                   T0
                                                                                T1
                               m:1
                                                            m:0
                                                                                         m:1
                               0:next i:0.1
                                                            0:next i:END
                                                                                         0:next i:END
                               1:next_i:1.0
                                                            1:next i:1.0
                                                                                         1:next i:1.1
                                                                                                                       label:5
                Thread 0
                                                                                                            T1
                                                                                                                   m:0
       label:0
                                                                                                                   0:next i:END
                                                                                                                   1:next i:END
   m:0
                                                                    Examples:
                                            T1
   0:next i:0.0
                                                                    0 \rightarrow 1 \rightarrow 6 \rightarrow 7 \rightarrow 5
   1:next i:1.0
                                                                    0 \rightarrow 2 \rightarrow 2 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5
                                                                                                             TO
                                   T0
              Thread 1
                                   label:2
                                                                label:3
                                                                                            label:4
                                                   T1
                                                                                T0
                                                            m:0
                                                                                         m:1
                               m:1
                               0:next i:0.0
                                                            0:next i:0.0
                                                                                         0:next i:0.1
                               1:next i:1.1
                                                            1:next i:END
                                                                                         1:next i:END
```

## Liveness property

Something good will eventually happen

- Examples:
  - The mutex program will eventually terminate
  - The self driving car will eventually reach its destination
- More difficult to reason about that safety properties

```
Thread 0:
                                                            Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
                                                            1.0: while (CAS(&m,0,1) == false); //lock
      // critical section
                                                                  // critical section
0.1: m.store(0); //unlock
                                                            1.1: m.store(0); //unlock
Is this program
guaranteed to
                                                                                 label:7
                                                        label:6
                               label:1
terminate?
                                             T0
                                                                      T1
                           m:1
                                                     m:0
                                                                              m:1
What could go
                           0:next i:0.1
                                                     0:next i:END
                                                                              0:next i:END
wrong?
                           1:next_i:1.0
                                                     1:next i:1.0
                                                                              1:next i:1.1
                                                                                                        label:5
              Thread 0
                                                                                               T1
                                                                                                    m:0
      label:0
                                                                                                    0:next i:END
                                                                                                    1:next_i:END
   m:0
                                       T1
   0:next i:0.0
   1:next i:1.0
                                                                                                TO
                               T0
            Thread 1
                               label:2
                                                        label:3
                                                                                 label:4
                                             T1
                                                                      T0
                                                                              m:1
                           m:1
                                                     m:0
                           0:next i:0.0
                                                     0:next i:0.0
                                                                              0:next i:0.1
                           1:next i:1.1
                                                     1:next i:END
                                                                              1:next i:END
```

```
Thread 0:
                                                                         Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
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       // critical section
                                                                                // critical section
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Is this program
guaranteed to
                                                                                                   label:7
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terminate?
                                                       T0
                                                                                      T1
                                 m:1
                                                                m:0
                                                                                               m:1
What could go
                                 0:next i:0.1
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                                                                                               0:next i:END
wrong?
                                 1:next i:1.0
                                                                1:next i:1.0
                                                                                               1:next i:1.1
                                                                                                                               label:5
                  Thread 0
                                                                                                                    T1
                                                                                                                          m:0
        label:0
                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
    m:0
                                                                         Forever?
                                                T1
    0:next i:0.0
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                     T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                   label:4
                                                       T1
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                                                                m:0
                                                                                               m:1
                                 m:1
                                 0:next i:0.0
                                                                0:next i:0.0
                                                                                               0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

### Liveness

- Starvation cycles
  - There exists a thread that can break the system out of a cycle, but that thread never executes (i.e. it is starved).
- Can starvation cycles happen?

### Liveness

- Starvation cycles
  - There exists a thread that can break the system out of a cycle, but that thread never executes (i.e. it is starved).
- Can starvation cycles happen?
  - Depends on your scheduler!
  - With no scheduler guarantees, they cannot be ruled out!

### Liveness

- Starvation cycles
  - There exists a thread that can break the system out of a cycle, but that thread never executes (i.e. it is starved).
- Can starvation cycles happen?
  - Depends on your scheduler!
  - With no scheduler guarantees, they cannot be ruled out!
- Note that we are talking about scheduler specifications, actual implementations are very complicated (take an OS class to learn more)

## On to new material

## Schedule

- Schedule specifications
  - Fair schedule

### The fair scheduler

- every thread that has not terminated will "eventually" get a chance to execute.
  - "concurrent forward progress": defined by C++
    not guaranteed, but encouraged (and likely what you will observe)
  - "weakly fair scheduler": defined by classic concurrency textbooks
- The fair scheduler disallows starvation cycles
  - waiting will always be finite (but no bounds on time)

```
Thread 0:
                                                                         Thread 1:
0.0: while (CAS(&m,0,1) == false); //lock
                                                                         1.0: while (CAS(&m, 0, 1) == false); //lock
       // critical section
                                                                                // critical section
0.1: m.store(0); //unlock
                                                                         1.1: m.store(0); //unlock
What about a
fair scheduler?
                                                                                                   label:7
                                                                    label:6
                                     label:1
                                                       T0
                                                                                     T1
                                 m:1
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                                 0:next i:0.1
                                                                0:next i:END
                                                                                               0:next i:END
                                 1:next i:1.0
                                                                1:next i:1.0
                                                                                               1:next i:1.1
                                                                                                                               label:5
                 Thread 0
                                                                                                                   T1
                                                                                                                          m:0
        label:0
                                                                                                                          0:next i:END
                                                                                                                          1:next i:END
    m:0
                                                                         Forever?
                                               T1
    0:next i:0.0
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                    T0
                                      T0
               Thread 1
                                      label:2
                                                                    label:3
                                                                                                  label:4
                                                      T1
                                                                                     T0
                                                                                              m:1
                                 m:1
                                                                m:0
                                 0:next i:0.0
                                                                0:next i:0.0
                                                                                              0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                              1:next i:END
```

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Thread 0:
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What about a
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                                                                                                   label:7
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                 Thread 0
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    m:0
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    0:next i:0.0
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    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
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                                      T0
               Thread 1
                                      label:2
                                                                    label:3
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                                                      T1
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                                                                                               0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
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Thread 0:
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What about a
fair scheduler?
                                                                                                   label:7
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                                     label:1
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                                 m:1
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                                 0:next i:0.1
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                                 1:next i:1.0
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                                                                                                                               label:5
                 Thread 0
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        label:0
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    m:0
                                                                         Forever?
                                               T1
    0:next i:0.0
                                                         disallowed!
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                    T0
                                      T0
               Thread 1
                                      label:2
                                                                    label:3
                                                                                                   label:4
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                                                                                               0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

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Thread 0:
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What about a
fair scheduler?
                                                                                                   label:7
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                                     label:1
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                                 m:1
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                                 0:next i:0.1
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                                                                                               0:next i:END
                                 1:next i:1.0
                                                                1:next i:1.0
                                                                                               1:next i:1.1
                                                                                                                               label:5
                 Thread 0
                                                                                                                   T1
                                                                                                                          m:0
        label:0
                                                                                                                          0:next i:END
                                                                                                                          1:next i:END
    m:0
                                                                         Forever?
                                                T1
    0:next i:0.0
                                                         disallowed!
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                    T0
                                      T0
               Thread 1
                                      label:2
                                                                    label:3
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                                 m:1
                                                                m:0
                                 0:next i:0.0
                                                                0:next i:0.0
                                                                                               0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

```
Thread 0:
                                                                         Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
                                                                         1.0: while (CAS(&m,0,1) == false); //lock
       // critical section
                                                                                // critical section
0.1: m.store(0); //unlock
                                                                         1.1: m.store(0); //unlock
What about a
fair scheduler?
                                                                                                   label:7
                                                                    label:6
                                     label:1
                                                       T0
                                                                                     T1
                                 m:1
                                                                m:0
                                                                                              m:1
                                 0:next i:0.1
                                                                0:next i:END
                                                                                               0:next i:END
                                 1:next i:1.0
                                                                1:next i:1.0
                                                                                               1:next i:1.1
                                                                                                                               label:5
                 Thread 0
                                                                                                                   T1
                                                                                                                          m:0
        label:0
                                                                                                                          0:next i:END
                                                                                                                          1:next i:END
    m:0
                                                                         Forever?
                                               T1
    0:next i:0.0
                                                         disallowed!
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                         disallowed!
                                                                        0 \rightarrow 2 \dots
                                                                             Only other paths
                                                                                                                    T0
                                      TO
                                                                             reach the end
               Thread 1
                                      label:2
                                                                    label:3
                                                                                                  label:4
                                                       T1
                                                                                     T0
                                                                                              m:1
                                 m:1
                                                                m:0
                                 0:next i:0.0
                                                                0:next i:0.0
                                                                                               0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

• A fair scheduler typically requires preemption



Core 0

resources



Operating System

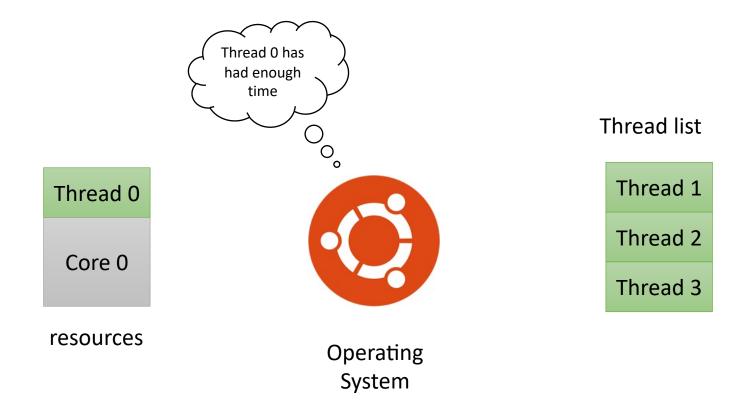
#### Thread list

Thread 1

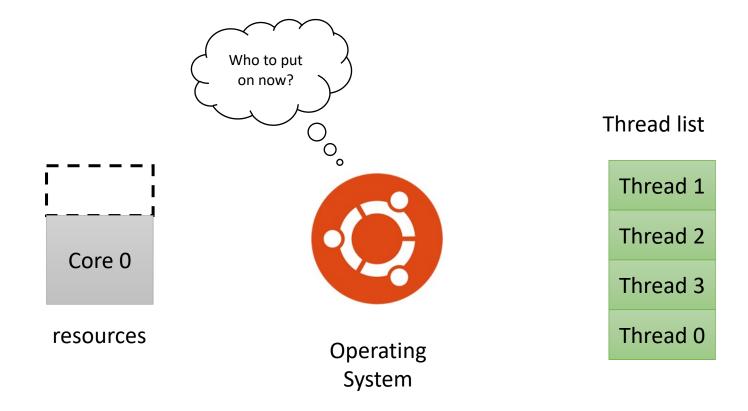
Thread 2

Thread 3

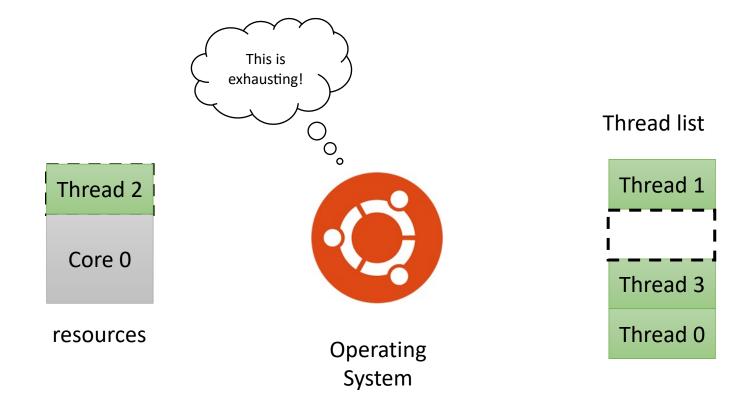
A fair scheduler typically requires preemption



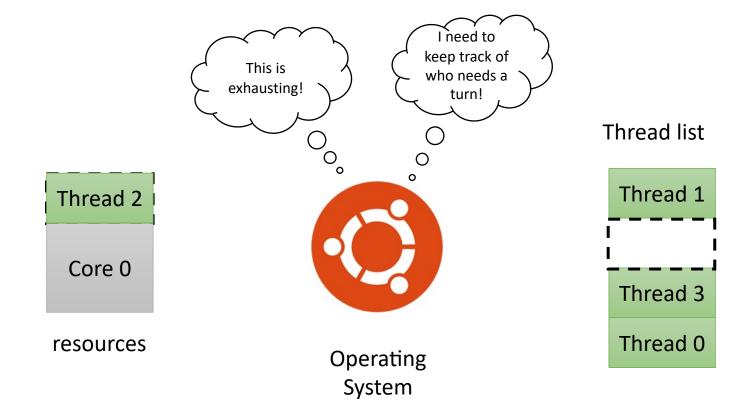
A fair scheduler typically requires preemption



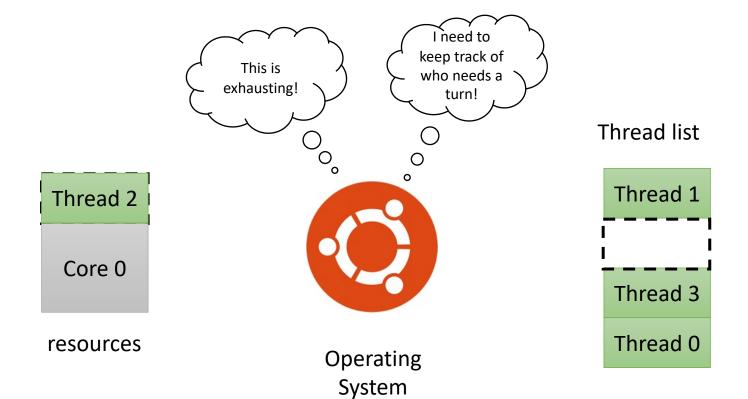
A fair scheduler typically requires preemption



• A fair scheduler typically requires preemption



A fair scheduler typically requires preemption



peak into a thread object:

A fair scheduler typically requires preemption



peak into a thread object:

Thread 1: program data local variables

Estimated to be ~30K cycles to context switch between threads

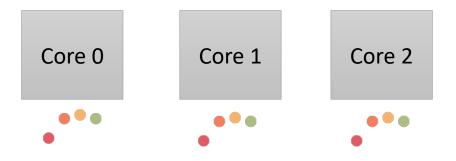
• Systems might not support preemption: e.g. GPUs

# simplified execution model

Program with 5 threads



thread pool



Device with 3 Cores

finished threads

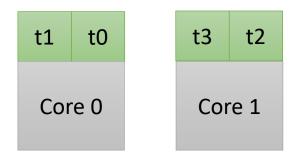
# Solutions?

• I have N cores, only run N threads?

### Solutions?

• I have N cores, only run N threads?

sometimes concurrency can help hide latency! Don't want to completely disallow it!



Device with 2 cores



# Solutions?

I have N cores, only run N threads?

- GPU examples:
  - Depending on program size Nvidia GPUs support
    - 32 threads per core for small programs
    - 2 threads per core for big programs
- We need a better specification

# Parallel Forward Progress

 "Any thread that has executed at least 1 instruction, is guaranteed to continue to be fairly executed"

### Also called:

- "Parallel Forward Progress": by C++
- "Persistent Thread Model": by GPU programmers
- "Occupancy Bound Execution Model": in some of my papers

```
Thread 0:
                                                                          Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
                                                                          1.0: while (CAS(&m, 0, 1) == false); //lock
        // critical section
                                                                                 // critical section
0.1: m.store(0); //unlock
                                                                          1.1: m.store(0); //unlock
Any thread that has
executed at least 1
                                                                                                    label:7
                                                                     label:6
                                      label:1
instruction, is
                                                        T0
                                                                                       T1
                                  m:1
                                                                 m:0
                                                                                                m:1
guaranteed to
                                  0:next i:0.1
                                                                 0:next i:END
                                                                                                0:next i:END
continue to be fairly
                                  1:next i:1.0
                                                                 1:next i:1.0
                                                                                                1:next i:1.1
executed
                                                                                                                                 label:5
                                                                                                                     T1
                                                                                                                            m:0
        label:0
                           Thread 0
                                                                                                                            0:next i:END
                                                                                                                            1:next i:END
    m:0
                                                                          Forever?
                                                T1
    0:next i:0.0
                                                                          0 \rightarrow 1 \dots
    1:next i:1.0
                                                                          0 \rightarrow 2 \dots
                                                                                                                      T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                    label:4
                                                       T1
                                                                                      T0
                                                                                                m:1
                                  m:1
                                                                 m:0
                                  0:next i:0.0
                                                                 0:next i:0.0
                                                                                                0:next i:0.1
                                  1:next i:1.1
                                                                 1:next i:END
                                                                                                1:next i:END
```

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executed at least 1
                                                                                                    label:7
                                                                     label:6
                                      label:1
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                                                        T0
                                                                                      T1
                                  m:1
                                                                 m:0
                                                                                                m:1
guaranteed to
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                                                                 0:next i:END
                                                                                                0:next i:END
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                                                                 1:next i:1.0
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                                                                                                                                 label:5
                                                                                                                     T1
                                                                                                                            m:0
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                                                                                                                            0:next i:END
                                                                                                                            1:next i:END
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                                                                          Forever?
                                                T1
    0:next i:0.0
                                                                          0 \rightarrow 1 \dots
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                                                                          0 \rightarrow 2 \dots
                                                                                                                      T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                    label:4
                                                       T1
                                                                                      T0
                                                                                                m:1
                                  m:1
                                                                 m:0
                                  0:next i:0.0
                                                                 0:next i:0.0
                                                                                                0:next i:0.1
                                  1:next i:1.1
                                                                 1:next i:END
                                                                                                1:next i:END
```

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                                                                                                    label:7
                                                                     label:6
                                      label:1
instruction, is
                                                        T0
                                                                                      T1
                                  m:1
                                                                 m:0
                                                                                               m:1
guaranteed to
                                  0:next i:0.1
                                                                 0:next i:END
                                                                                                0:next i:END
continue to be fairly
                                  1:next i:1.0
                                                                 1:next i:1.0
                                                                                                1:next i:1.1
executed
                                                                                                                                label:5
                                                                                                                    T1
                                                                                                                           m:0
        label:0
                           Thread 0
                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
    m:0
                                                                          Forever?
                                                T1
    0:next i:0.0
                                                            disallowed 0 \rightarrow 1 \dots
    1:next i:1.0
                                                                          0 \rightarrow 2 \dots
                                                                                                                      T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                    label:4
                                                       T1
                                                                                      T0
                                                                                               m:1
                                  m:1
                                                                 m:0
                                  0:next i:0.0
                                                                 0:next i:0.0
                                                                                                0:next i:0.1
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                                                                                                1:next i:END
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                                                        T0
                                                                                      T1
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                                                                 m:0
                                                                                                m:1
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                                  0:next i:0.1
                                                                 0:next i:END
                                                                                                0:next i:END
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                                  1:next i:1.0
                                                                 1:next i:1.0
                                                                                                1:next i:1.1
executed
                                                                                                                                label:5
                                                                                                                     T1
                                                                                                                            m:0
        label:0
                           Thread 0
                                                                                                                            0:next i:END
                                                                                                                            1:next i:END
    m:0
                                                                          Forever?
                                                T1
    0:next i:0.0
                                                            disallowed 0 \rightarrow 1 \dots
    1:next i:1.0
                                                                          0 \rightarrow 2 \dots
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                                      T0
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                                                                                                    label:4
                                                       T1
                                                                                      T0
                                                                                                m:1
                                  m:1
                                                                 m:0
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                                                                 0:next i:0.0
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                                                                                                1:next i:END
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                                                       T0
                                                                                      T1
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                                                                                               m:1
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                                  0:next i:0.1
                                                                 0:next i:END
                                                                                                0:next i:END
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                                  1:next i:1.0
                                                                 1:next i:1.0
                                                                                                1:next i:1.1
executed
                                                                                                                                label:5
                                                                                                                    T1
                                                                                                                           m:0
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                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
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                                                                          Forever?
                                                T1
    0:next i:0.0
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                                                                                                                     T0
                                      T0
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                                      label:2
                                                                     label:3
                                                                                                    label:4
                                                       T1
                                                                                      T0
                                                                                               m:1
                                  m:1
                                                                 m:0
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                                                                 0:next i:0.0
                                                                                                0:next i:0.1
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                                                                 1:next i:END
                                                                                                1:next i:END
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                                                                                                   label:7
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instruction. is
                                                       T0
                                                                                     T1
                                 m:1
                                                                m:0
                                                                                               m:1
guaranteed to
                                 0:next i:0.1
                                                                0:next i:END
                                                                                               0:next i:END
continue to be fairly
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                                                                1:next i:1.0
                                                                                               1:next i:1.1
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                                                                                                                               label:5
                                                                                                                   T1
                                                                                                                          m:0
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                           Thread 0
                                                                                                                          0:next i:END
                                                                                                                          1:next i:END
    m:0
                                                                         Forever?
                                                T1
    0:next i:0.0
                                                            disallowed 0 \rightarrow 1 \dots
    1:next i:1.0
                                                            disallowed 0 \rightarrow 2 \dots
                                                                        Mutexes will terminate under
                                                                                                                     T0
                                      T0
                                                                        the parallel scheduler
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                   label:4
                                                       T1
                                                                                     T0
                                                                                               m:1
                                 m:1
                                                                m:0
                                 0:next i:0.0
                                                                0:next i:0.0
                                                                                               0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

# Another example

- Producer consumer
  - Thread 0 waits for Thread 1 to write a flag

```
Thread 0:
```

```
0.0: while(flag.load() == 0);
```

```
Thread 1:
1.0: flag.store(1);
```

```
Thread 0:
```

```
0.0: while(flag.load() == 0);
```

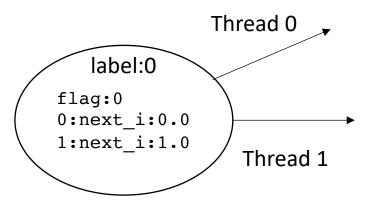
start with initial node

```
flag:0
0:next_i:0.0
1:next_i:1.0
```

```
Thread 1:
1.0: flag.store(1);
```

```
Thread 0:
```

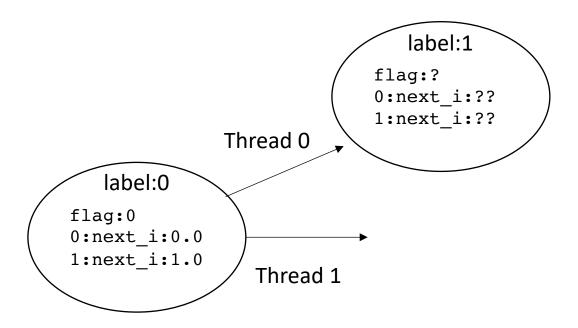
```
0.0: while(flag.load() == 0);
```



# Thread 1: 1.0: flag.store(1);

```
Thread 0:
```

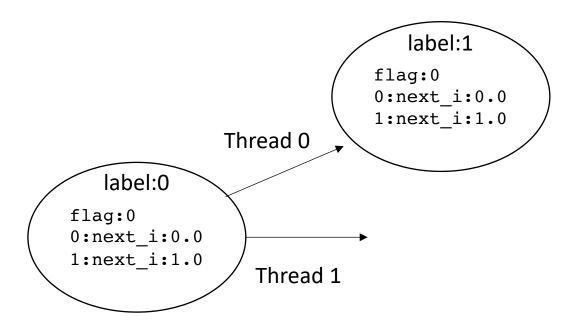
```
0.0: while(flag.load() == 0);
```



# Thread 1: 1.0: flag.store(1);

```
Thread 0:
```

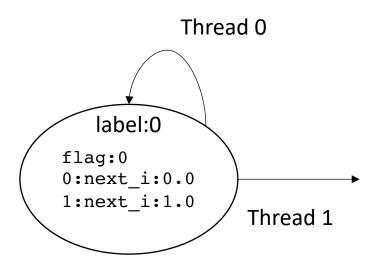
```
0.0: while(flag.load() == 0);
```



# Thread 1: 1.0: flag.store(1);

#### Thread 0:

0.0: while(flag.load() == 0);

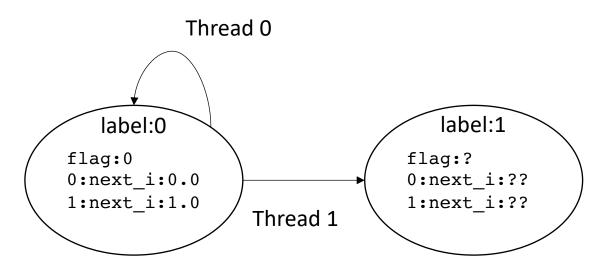


#### Thread 1:

1.0: flag.store(1);

```
Thread 0:
```

```
0.0: while(flag.load() == 0);
```

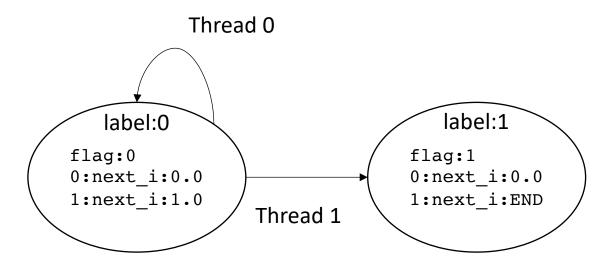


#### Thread 1:

1.0: flag.store(1);

```
<u>Thread 0:</u>
```

```
0.0: while(flag.load() == 0);
```



#### Thread 1:

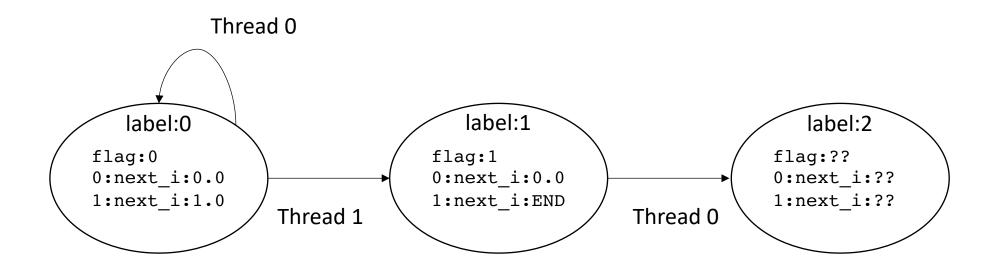
```
1.0: flag.store(1);
```

```
Thread 0:
```

0.0: while(flag.load() == 0);

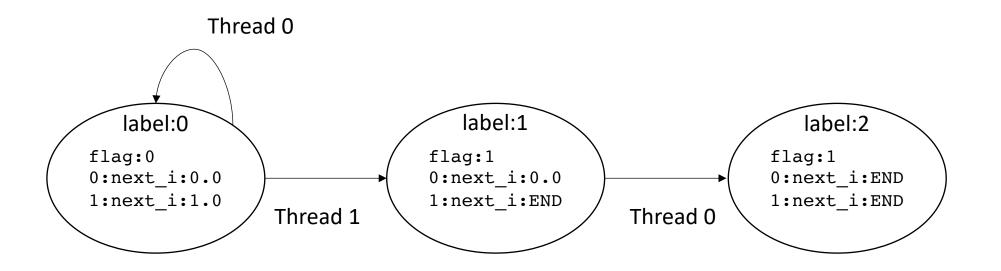
#### Thread 1:

1.0: flag.store(1);



# Thread 0: 0.0: while(flag.load() == 0);

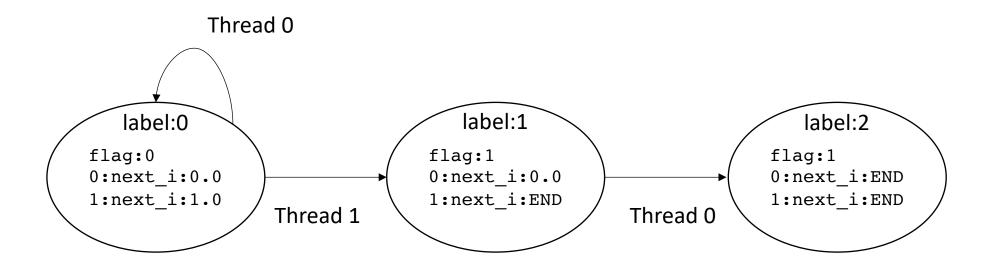
```
Thread 1:
1.0: flag.store(1);
```



```
Thread 0:
0.0: while(flag.load() == 0);
```

```
Thread 1:
1.0: flag.store(1);
```

*Is this program guaranteed to terminate under the fair scheduler?* 

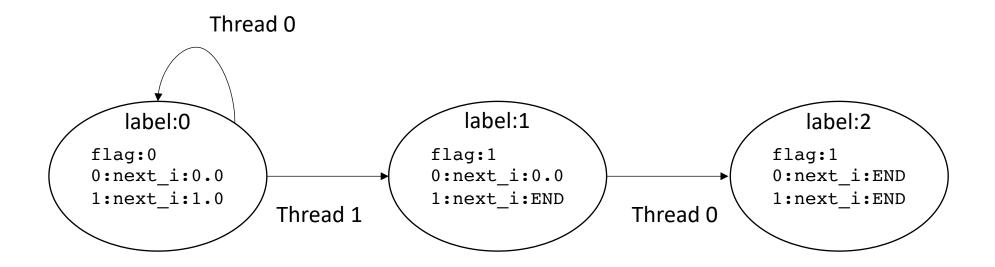


```
\frac{Thread 0:}{0.0: while(flag.load() == 0);}
```

```
Thread 1:
1.0: flag.store(1);
```

*Is this program guaranteed to terminate under the fair scheduler?* 

Is this program guaranteed to terminate under the parallel scheduler?



### Thread 0:

0.0: while(flag.load() == 0);

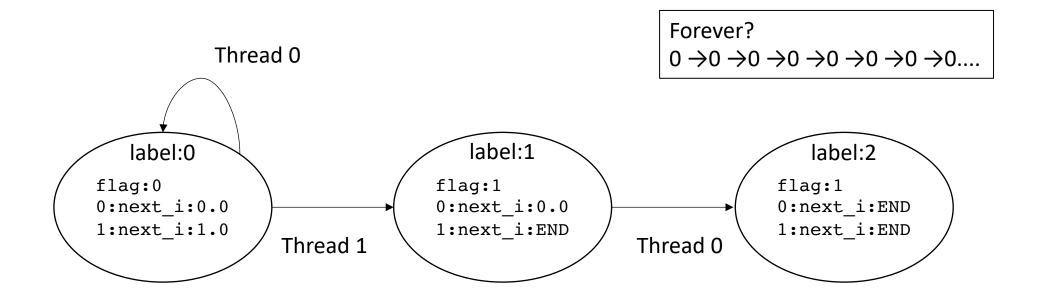
```
Thread 1:
```

1.0: flag.store(1);

*Is this program guaranteed to terminate under the fair scheduler?* 

Is this program guaranteed to terminate under the parallel scheduler?

Any thread that has executed at least 1 instruction, is guaranteed to continue to be fairly executed



### Thread 0:

0.0: while(flag.load() == 0);

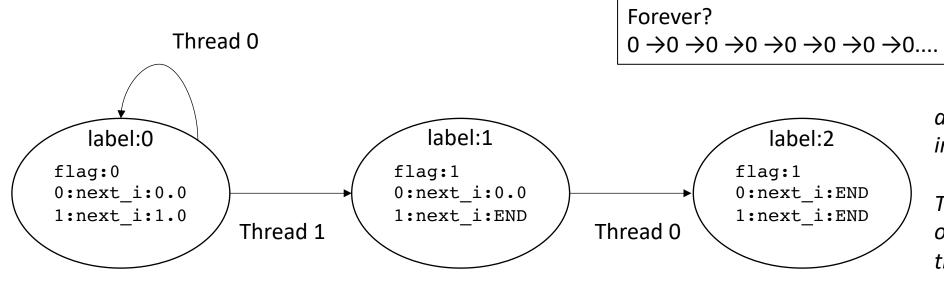
### Thread 1:

1.0: flag.store(1);

*Is this program guaranteed to terminate under the fair scheduler?* 

Is this program guaranteed to terminate under the parallel scheduler?

Any thread that has executed at least 1 instruction, is guaranteed to continue to be fairly executed



allowed to spin forever in the parallel scheduler!

Thread 0 could be scheduled on the only core while thread 1 spins

• In some cases the Parallel scheduler might be too strong

For example dynamic power management on mobile devices

Program with 5 threads



thread pool

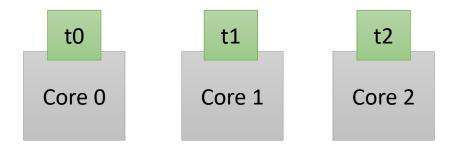
Core 0

Core 1

Core 2

Device with 3 Cores

Program with 5 threads thread pool

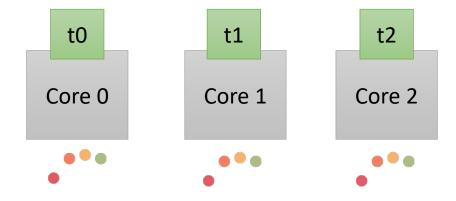


Device with 3 Cores

Program with 5 threads



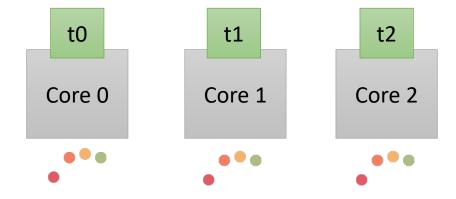
thread pool



Device with 3 Cores

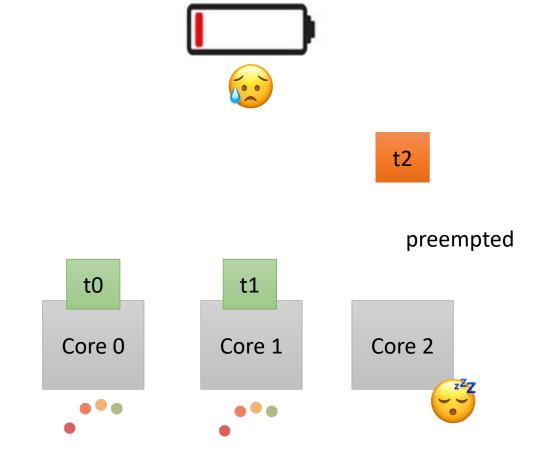
Program with 5 threads thread pool





Device with 3 Cores

Program with 5 threads thread pool



Device with 3 Cores

Program with 5 threads thread pool

t2 preempted t0 Core 1 Core 2 Core 0

Device with 3 Cores

t1

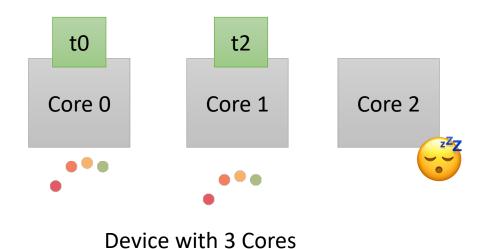
Program with 5 threads

t4 t3

thread pool



t1



 This power-saving optimization messes up the Parallel Scheduler guarantees

Can we do anything interesting with a scheduler like this?

 This power-saving optimization messes up the Parallel Scheduler guarantees

Can we do anything interesting with a scheduler like this?

 The OS can give guarantees about the threads that it preempts for energy savings.

 This power-saving optimization messes up the Parallel Scheduler guarantees

Can we do anything interesting with a scheduler like this?

 The OS can give guarantees about the threads that it preempts for energy savings.

• The OS could target threads with higher ids and give priority with threads with the lower id.

## The HSA scheduler

• The thread with the lowest ID that hasn't terminated is guaranteed to eventually be executed.

### • Called:

- "HSA" Heterogeneous System Architecture, programming language proposed by AMD for new systems.
- The HSA language appears to be defunct now, but the scheduler is a good fit for mobile devices (esp. mobile GPUs).

```
Thread 0:
                                                                         Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
                                                                         1.0: while (CAS(&m,0,1) == false); //lock
        // critical section
                                                                                 // critical section
0.1: m.store(0); //unlock
                                                                         1.1: m.store(0); //unlock
The thread with the
lowest ID that hasn't
                                                                                                    label:7
                                                                     label:6
                                      label:1
terminated is
                                                       T0
                                                                                      T1
                                  m:1
                                                                m:0
                                                                                               m:1
guaranteed to
                                  0:next i:0.1
                                                                 0:next i:END
                                                                                                0:next i:END
eventually be
                                  1:next i:1.0
                                                                1:next i:1.0
                                                                                                1:next i:1.1
executed.
                                                                                                                                label:5
                                                                                                                    T1
                                                                                                                           m:0
        label:0
                           Thread 0
                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
    m:0
                                                                          Forever?
                                                T1
    0:next i:0.0
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                     T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                   label:4
                                                       T1
                                                                                      T0
                                                                                               m:1
                                  m:1
                                                                m:0
                                  0:next i:0.0
                                                                 0:next i:0.0
                                                                                               0:next i:0.1
                                  1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

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                                                                                                1:next i:1.1
executed.
                                                                                                                                label:5
                                                                                                                    T1
                                                                                                                           m:0
        label:0
                           Thread 0
                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
    m:0
                                                                          Forever?
                                                T1
    0:next i:0.0
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                     T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                   label:4
                                                       T1
                                                                                      T0
                                                                                               m:1
                                  m:1
                                                                 m:0
                                  0:next i:0.0
                                                                 0:next i:0.0
                                                                                               0:next i:0.1
                                  1:next i:1.1
                                                                 1:next i:END
                                                                                               1:next i:END
```

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                                                       T0
                                                                                      T1
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                                                                                               m:1
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                                                                0:next i:END
                                                                                               0:next i:END
eventually be
                                  1:next i:1.0
                                                                1:next i:1.0
                                                                                               1:next i:1.1
executed.
                                                                                                                                label:5
                                                                                                                    T1
                                                                                                                           m:0
        label:0
                           Thread 0
                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
    m:0
                                                                         Forever?
                                                T1
    0:next i:0.0
                                                            disallowed
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                     T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                   label:4
                                                       T1
                                                                                     T0
                                                                                               m:1
                                 m:1
                                                                m:0
                                  0:next i:0.0
                                                                0:next i:0.0
                                                                                               0:next i:0.1
                                  1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

```
Thread 0:
                                                                         Thread 1:
0.0: while (CAS(\&m, 0, 1) == false); //lock
                                                                         1.0: while (CAS(&m,0,1) == false); //lock
        // critical section
                                                                                // critical section
0.1: m.store(0); //unlock
                                                                         1.1: m.store(0); //unlock
The thread with the
lowest ID that hasn't
                                                                                                    label:7
                                                                     label:6
                                      label:1
terminated is
                                                       T0
                                                                                      T1
                                 m:1
                                                                m:0
                                                                                               m:1
guaranteed to
                                  0:next i:0.1
                                                                0:next i:END
                                                                                               0:next i:END
eventually be
                                  1:next i:1.0
                                                                1:next i:1.0
                                                                                               1:next i:1.1
executed.
                                                                                                                                label:5
                                                                                                                    T1
                                                                                                                           m:0
        label:0
                           Thread 0
                                                                                                                           0:next i:END
                                                                                                                           1:next i:END
    m:0
                                                                         Forever?
                                                T1
    0:next i:0.0
                                                            disallowed
                                                                         0 \rightarrow 1 \dots
    1:next i:1.0
                                                                         0 \rightarrow 2 \dots
                                                                                                                     T0
                                      T0
               Thread 1
                                      label:2
                                                                     label:3
                                                                                                   label:4
                                                       T1
                                                                                      T0
                                                                                               m:1
                                 m:1
                                                                m:0
                                  0:next i:0.0
                                                                0:next i:0.0
                                                                                               0:next i:0.1
                                  1:next i:1.1
                                                                1:next i:END
                                                                                               1:next i:END
```

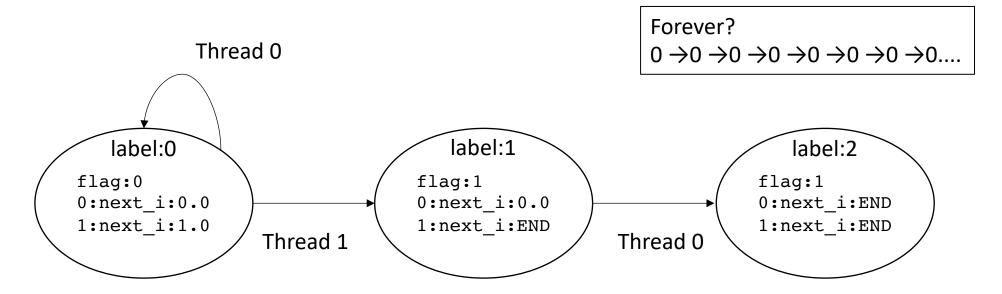
```
Thread 0:
                                                                        Thread 1:
0.0: while (CAS(&m, 0, 1) == false); //lock
                                                                        1.0: while (CAS(&m,0,1) == false); //lock
        // critical section
                                                                               // critical section
0.1: m.store(0); //unlock
                                                                        1.1: m.store(0); //unlock
The thread with the
lowest ID that hasn't
                                                                                                  label:7
                                                                    label:6
                                     label:1
terminated is
                                                       T0
                                                                                     T1
                                 m:1
                                                               m:0
                                                                                              m:1
guaranteed to
                                 0:next i:0.1
                                                                0:next i:END
                                                                                              0:next i:END
eventually be
                                 1:next i:1.0
                                                                1:next i:1.0
                                                                                              1:next i:1.1
executed.
                                                                                                                              label:5
                                                                                                                  T1
                                                                                                                         m:0
        label:0
                          Thread 0
                                                                                                                         0:next i:END
                                                                                                                         1:next i:END
    m:0
                                                                        Forever?
                                               T1
    0:next i:0.0
                                                           disallowed
                                                                        0 \rightarrow 1 \dots
    1:next i:1.0
                                                                        0 \rightarrow 2 \dots
                                                             allowed
                                                                                                                   T0
                                                                        Mutexes might not terminate under
                                      T0
                                                                        HSA scheduler!
               Thread 1
                                     label:2
                                                                    label:3
                                                                                                  label:4
                                                      T1
                                                                                    T0
                                                                                              m:1
                                 m:1
                                                               m:0
                                 0:next i:0.0
                                                                0:next i:0.0
                                                                                              0:next i:0.1
                                 1:next i:1.1
                                                                1:next i:END
                                                                                              1:next i:END
```

# Thread 0: 0.0: while(flag.load() == 0);

```
Thread 1:
1.0: flag.store(1);
```

*Is this program guaranteed to terminate under the HSA scheduler* 

The thread with the lowest ID that hasn't terminated is guaranteed to eventually be executed.

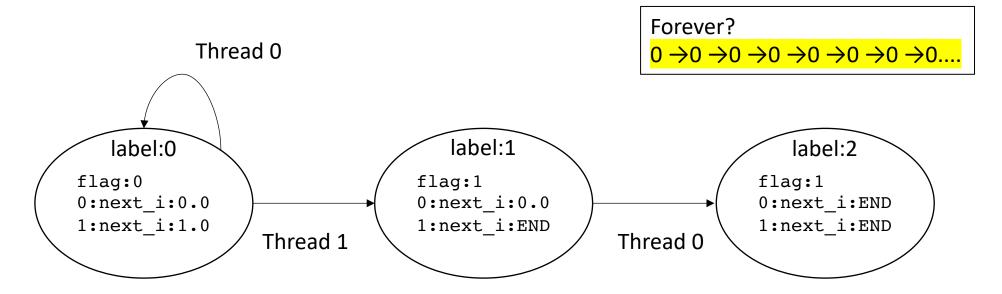


# Thread 0: 0.0: while(flag.load() == 0);

```
Thread 1:
1.0: flag.store(1);
```

*Is this program guaranteed to terminate under the HSA scheduler* 

The thread with the lowest ID that hasn't terminated is guaranteed to eventually be executed.

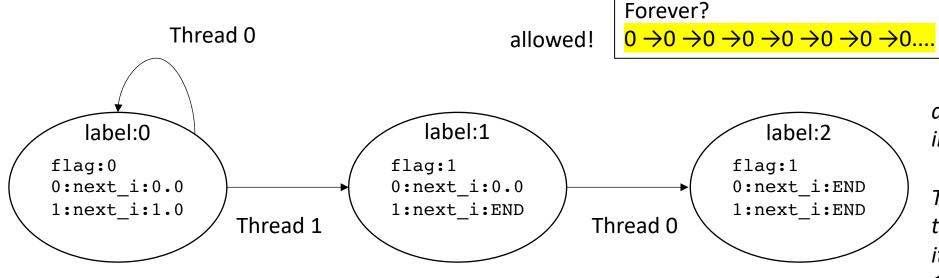


# Thread 0: 0.0: while(flag.load() == 0);

```
Thread 1:
1.0: flag.store(1);
```

*Is this program guaranteed to terminate under the HSA scheduler* 

The thread with the lowest ID that hasn't terminated is guaranteed to eventually be executed.



allowed to spin forever in the HSA scheduler!

Thread 0 is guaranteed to be executed because it has the lowest id. Thread 1 is not!

```
Thread 0:
0.0: while(flag.load() == 0);
```

What if we switch the threads? Thread 1 waits for Thread 0?

```
Thread 1:
1.0: flag.store(1);
```

```
Thread 0:
```

```
0.0: flag.store(1);
```

What if we switch the threads? Thread 1 waits for Thread 0?

### Thread 1:

```
1.0: while(flag.load() == 0);
```

#### Thread 0:

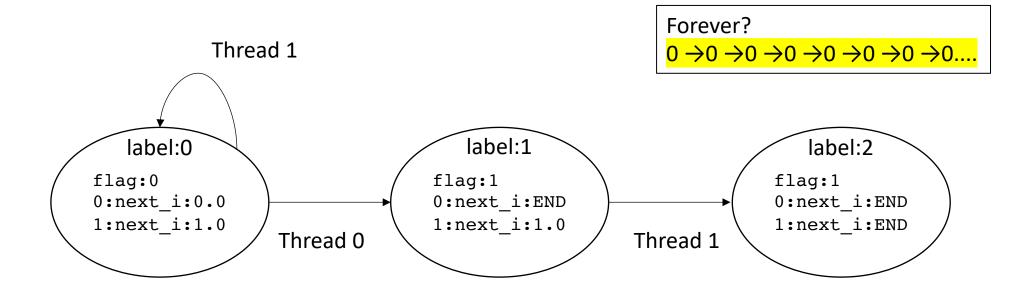
0.0: flag.store(1);

Thread 1:

1.0: while(flag.load() == 0);

What if we switch the threads? Thread 1 waits for Thread 0?

The thread with the lowest ID that hasn't terminated is guaranteed to eventually be executed.



thread 0
has the lowest
id so it is guaranteed
to eventually be
executed

#### Thread 0:

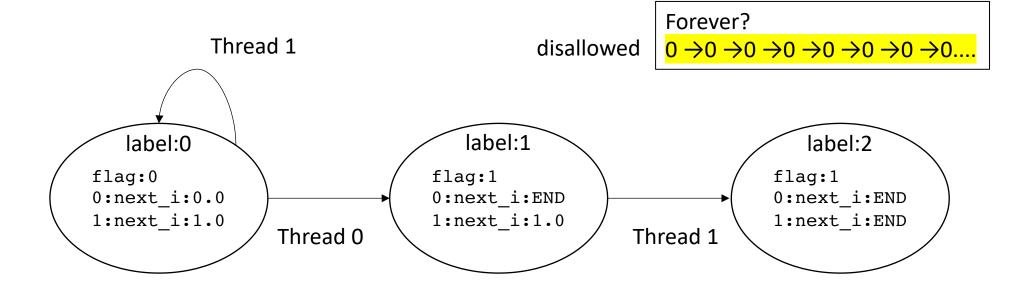
0.0: flag.store(1);

### Thread 1:

1.0: while(flag.load() == 0);

What if we switch the threads? Thread 1 waits for Thread 0?

The thread with the lowest ID that hasn't terminated is guaranteed to eventually be executed.



thread 0
has the lowest
id so it is guaranteed
to eventually be
executed

### Liveness

So where are we now?

- C++ gives 3 degrees of progress guarantees:
  - Concurrent scheduler
    - what you will likely see on your machine; fair scheduler!
  - Parallel scheduler
    - Threads that start executing will continue to be fairly executed. Allows mutexes!
  - Weakly parallel scheduler
    - No guarantees. Any cycle in the LTS can potentially execute forever!

### Liveness

So where are we now?

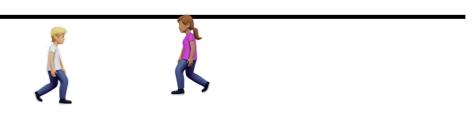
- GPU schedulers:
  - Nvidia provides Parallel Forward Progress
    - Allows mutexes, concurrent data structures, etc.
  - OpenCL, Vulkan, and Metal provide no documentation on scheduler behaviors.
    - In practice, many assume parallel forward progress
    - This is not portable (esp. to ARM and Apple)
    - Working with specification groups to try and provide these

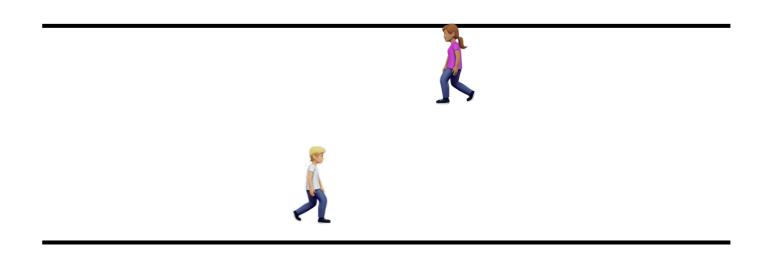
## Demo

• Demo about how things can go wrong on Ipad.

















Hallway problem



Can they dance around each other forever?

```
Thread 0:
... do {
0.0    x.store(0);
0.1 } while (x.load() != 0)
```

```
Thread 1:
... do {
1.0    x.store(1);
1.1 } while (x.load() != 1)
```

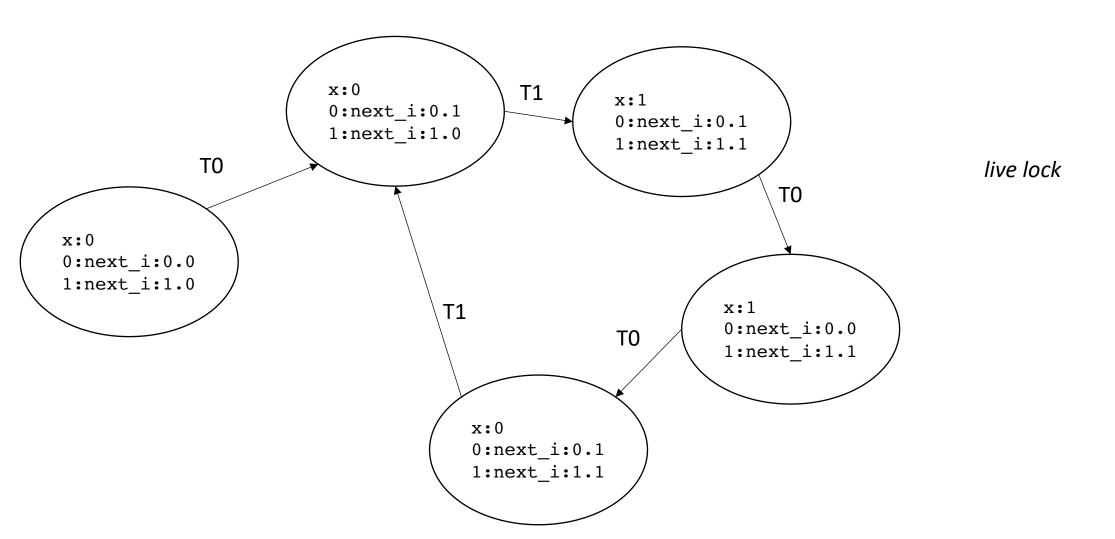
Each thread stores their thread id, and then loads the thread id. It loops while it doesn't see its id

Each thread gets a chance to execute, but they get in each others way.

This is called a livelock

```
Thread 0:
    ... do {
    0.0     x.store(0);
    0.1 } while (x.load() != 0)
```

```
Thread 1:
... do {
1.0    x.store(1);
1.1 } while (x.load() != 1)
```



## Conclusion

- Schedulers are becoming more aggressive
  - Preemption is expensive
  - Power saving shut downs are possible

- Concurrent objects require different amounts of fairness
  - Mutexes require parallel forward progress
  - Producer Consumer requires HSA forward progress
- Be careful that the programs you are writing make the correct assumptions about the underlying scheduler!

# See you on Wednesday!

Homework 4 due on Friday

• Expect midterm grades by end of day.

• Last day of module 4, moving on to last module next week!

```
Thread 0:
                                                          Thread 1:
... do {
                                                          ... do {
                                                          1.0 x.store(1);
0.0 x.store(0);
0.1 \} \text{ while } (x.load() != 0)
                                                          1.1 } while (x.load() != 1)
                                                                                           T1
                                            T0
                                                                    T1
                                                                                                  x:1
                           x:0
                                                   x:0
                                                                          x:1
                                                                                                  0:next_i:END
                           0:next i:0.1
                                                                           0:next i:END
                                                    0:next i:END
                                                                                                  1:next_i:END
                           1:next_i:1.0
                                                    1:next_i:1.0
                                                                           1:next_i:1.1
                                         T1
               T0
                                                 x:1
                                                 0:next i:0.1
                                                 1:next_i:1.1
   x:0
   0:next i:0.0
   1:next_i:1.0
               T1
                                                                                          T0
                                                                    T0
                                            T1
                                                                                                 x:0
                          x:1
                                                                          x:0
                                                   x:1
                          0:next i:0.0
                                                                                                 0:next i:END
                                                                          0:next i:0.1
                                                   0:next_i:0.0
                          1:next i:1.1
                                                                                                 1:next i:END
                                                                          1:next i:END
                                                   1:next_i:END
```