1 Regularity of Zigzag

Proof. $Zigzag(L_1, L2) = \{w | w = a_1b_1a_2b_2...a_nb_n, \ such\ that\ a_1...a_n \in L_1\ and\ b_1...b_n \in L_2\}$

Let A, B be finite deterministic automatas defined the following way:
$A = (Q^A, \Sigma, \delta^A, q_0^A, F^A)$
$B = (Q^B, \Sigma, \delta^B, q_0^B, F^B)$

Let L_1, L_2 be two regular languages, accepted by A,B correspondingly, such that:
$L(A) = L_1$
$L(B) = L_2$

Let there be a new automata C, defined as follows:
$C = (Q^C, \Sigma, \delta^C, q_0^C, F^C)$
Let L_3 be a language, we shall prove its regularity by proving C to be a finite deterministic automata.

To prove C to be a finite deterministic automata, we shall prove that:

- $|Q^C| \neq \infty$
- That we have a valid transition function: $\delta^C : Q^C \times \Sigma \to Q^C$
- A valid initial state: q_0^C
- A valid set of accept states: F^C

Knowing the input would be of the following form: $a_1b_1a_2b_2 \dots a_nb_n$,
And that $a_1a_2 \dots a_n$ is a valid word in the language L_1,
And that $b_1b_2 \dots b_n$ is a valid word in the language L_2,

We must somehow be capable of using the current automatas (A,B) in order to distinguish valid words from fallacious ones.

We must be capable of reading $a_1, a_2, \dots, a_n$ using the automata A
And accordingly $b_1, b_2, \dots, b_n$ using the automata B

In accordance to the previous statements, it is as coherent as ever that in order for a set of states to be used in the automata to be groundly esbalished, it must be rendered cogent for the set to include not only the current case; $(q_a, q_b) \mid q_a \in Q^A \wedge q_b \in Q^B$ but also the automata using which it should next read; $(q_a, q_b, automata \mid q_a \in Q^A \wedge q_b \in Q^B \wedge (automata = A \vee automata = B))$

Hence, Q^C must be defined as such: $Q^C = Q^A \times Q^B \times \{A, B\}$
Using our general assumption that both A and B are finite deterministic automatas, we may infer that $|Q^A| \neq \infty \wedge |Q^B| \neq \infty$.
The above is adequate for us to infer that $|Q^C| \neq \infty$ and accordingly is a cogent set to be used as the set of cases for our automata C.

Additionally, we must specify an initial state. For it is a compulsury field of the valid automata. I should like to propose the following:
$q_0^c \in Q^C \mid q_0^C = (q_0^A, q_0^B, A)$.
The aforementioned definition of an initial state not only imbues the two initial states corresponding with automatas A and B, but also the automata from which it should next read. (Automata A).

Having already defined a set of states composed of a defined initial state and others, we may now craft the trasitions function; (δ^C).

$$\delta^C((q_n^A, q_m^B, automata), \sigma) = \begin{cases} (\delta^A(q_n^A, \sigma), q_m^B, B), & automata = A \\ \\ (q_n^A, \delta^B(q_m^B, \sigma), A), & automata = B \end{cases}$$

We have by now defined all that is to be defined for our automata to be valid, but one thing. The set of accepted states. For a state to be adequate to be an accepted one, it must match the following form:
$F^C = \{q_i^C = (q_n^A, q_n^B, A) \in Q^C \mid q_n^A \in F^A \wedge q_n^B \in F^B\}$

Using the above set of proofs I should now like to divulge that the Zigzag automata (automata C) is a finite deterministic automata. $\square$