Problem Set II

Linear Algebra
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Problem 1.1. Find the canonical row echelon form of the matrix:

$$\begin{pmatrix} 7 & 7 & 4 & 28 & 21 \\ 3 & 3 & 1 & 12 & 9 \\ 6 & 5 & 2 & 20 & 18 \\ 2 & 3 & 1 & 12 & 6 \\ 4 & 6 & 4 & 24 & 12 \end{pmatrix}$$

Solution.

$$\xRightarrow{\text{swap } R_1 \text{ with } R_5}$$

$$\begin{pmatrix} 4 & 6 & 4 & 24 & 12 \\ 3 & 3 & 1 & 12 & 9 \\ 6 & 5 & 2 & 20 & 18 \\ 2 & 3 & 1 & 12 & 6 \\ 7 & 7 & 4 & 28 & 21 \end{pmatrix}$$

$$\xRightarrow{\text{Divide the new } R_1 \text{ by } R_{1,1}}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 3 & 3 & 1 & 12 & 9 \\ 6 & 5 & 2 & 20 & 18 \\ 2 & 3 & 1 & 12 & 6 \\ 7 & 7 & 4 & 28 & 21 \end{pmatrix}$$

$$\xRightarrow{\text{From each row } R_{i>1} \text{ subtract } R_i * R_1}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & -1.5 & -2 & -6 & 0 \\ 0 & -4 & -4 & -16 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -3.5 & -3 & -14 & 0 \end{pmatrix}$$

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$$\xRightarrow{\text{swap } R_2 \text{ with } R_3}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & -4 & -4 & -16 & 0 \\ 0 & -1.5 & -2 & -6 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -3.5 & -3 & -14 & 0 \end{pmatrix}$$

$$\xRightarrow{\text{Divide the new } R_2 \text{ by } R_{2,2}}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & -1.5 & -2 & -6 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -3.5 & -3 & -14 & 0 \end{pmatrix}$$

$$\xRightarrow{\text{From each row } R_{i>2} \text{ subtract } R_i * R_2}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{pmatrix}$$

$$\xRightarrow{\text{Multiply } R_3 \text{ by -2}}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{pmatrix}$$

$$\xRightarrow{\text{From each row } R_{i>3} \text{ subtract } R_i * R_3}$$

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xRightarrow{\text{From each row } R_{i<2} \text{ subtract } R_{i,2} * R_2}$$

$$\begin{pmatrix} 1 & 0 & -0.5 & 0 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xRightarrow{\text{From each row } R_{i<3} \text{ subtract } R_{i,3} * R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Problem 1.2. Calculate the maximum height the rocket would reach:

Solution.

Firstly, the height reached once the engine failed must be calculated. In order to calculate the latter, one must observe the following equation: $x(t) = x_0 + v_0(t - t_0) + 0.5at^2$*. Applying the equation to the scenario described, we may set* $x_0 = 0, v_0 = 0, t_0 = 0, t = 5, a = 30$ *and therefore:* $x(5) = 0 + 0(5 - 0) + 0.5 * 30 * 5^2 = 15 * 25 = 375_m$

Now, we may calculate the distance that the rocket travelled after the failure of the engine: To do so, we must first calculate the duration of time the rocket's height resumed increasing despite the engine no longer functioning.

We may do so using the following system of equations: $v(t) = 0 \wedge v(t) = v_0 + a(t - t_0)$*, as* $v_0 = 150, a = -9.87, t_0 = 5$*, therefore:* $0 = 150 - 9.87(t - 5) \Longrightarrow t = 20.19$

. We shall use the following equation: $x(t) = x_0 + v_0(t - t_0) + 0.5at^2$*, we may define the parameters to suit our needs:* $x_0 = 375_m, v_0 = 150 m/_{s^2}, t_0 = 5, t = 20.19, a = -9.87 \Longrightarrow$ $x(20.19) = 375 + 150 * (20.19 - 5) + 0.5(-9.87)(15.19)^2 = 1514.8173465$

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