Problem Set I

Computing Models
April 23, 2023

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1 Automatas

Given the automata $D = (Q^D, \{a, b\}, \delta^D, q_0^D, F^D)$

Problem 1.1.
Imagine a new automata $E = (Q^E, \{a, b\}, \delta^E, q_0^E, F^E)$ *s.t:*

- $Q^E = Q^D \cup \{q_0^E\}$
- $F^E = F^D$
- $\delta^E(q, \sigma) = \begin{cases} \delta^D(q, \sigma) & q \in Q^D \\ q_0^D & q = q_0^E, \sigma = a \\ q_0^E & q = q_0^E, \sigma = b \end{cases}$

Define:

- *L(A)*

Solution.
It would be non but rational to divide this construction into three divisions, each corresponding to a different set of circumstances recognised by the trasitions function.

One of those aforementioned circumstances is $q = q_0^E, \sigma = b$*, the study of such case lead me to determine that for the character input of b, under the assumption that the current state is* q_0^E*, the state would lead back to itself, meaning that that an instance of* $\{b\}^*$ *at the beginning of the input would not affect the output of the automata. And hence* $\{b\}^*$ *should be imbued to the language L(A).*

Another set of circumstances is $q = q_0^E, \sigma = a$*, which implies the current state to be the one added to* Q^D *in order to craft* Q^E*, and that the input chracter is 'a'. Such circumstances appear to be digested by the automata to return* q_0^D*, the first state of the previous automata D. Accordingly, it would only be after the appearence of an 'a' character in the input that the*

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state would be changed. And hence, $\{a\}$ must be added to the language L(A).

The last of such circumstances addressed in δ^E appears to be $q \in Q^D$. For such case, the function would make the transition from the current state to the one returned by δ^D, accordingly, L(D) must be concatenated at the end of L(E)

Hence - I may declare that $L(A) = \{b\}^* \cdot \{a\} \cdot L(D)$. ■

Problem 1.2.
Consider the previous automata $E = (Q^E, \{a, b\}, \delta^E, q_0^E, F^E)$
And a new automata $E' = (Q^E, \{a, b\}, \delta^E, q_0^D, F^E)$

Define:

- $L(E')$

Solution.
In the previous problem I have explained in great detail the effects caused by the declaration of a new state q_0^E to be the initial state of E. Considering q_0^E not to be part of Q^D, it can be seen with vividness that δ^E, known as E''s transitions function would cease to return it as output as soon as it no longer is the current state. Therefore, knowing E' does not use q_0^E as its initial state it is cogent that it would never be returned. Accordingly, the only circumstances relevant to the transitions function would be $q \in Q^D$, and thus the addition of $\{b\}^* \cdot \{a\}$ to the beginning of L(E) must be undone for $L(E')$ to be precise. And hence $L(E') = L(D)$ ■