

Problem Set II

Linear Algebra

July 9, 2023

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1 :

Problem 1.1. Find the canonical row echelon form of the matrix:

$$\begin{pmatrix} 7 & 7 & 4 & 28 & 21 \\ 3 & 3 & 1 & 12 & 9 \\ 6 & 5 & 2 & 20 & 18 \\ 2 & 3 & 1 & 12 & 6 \\ 4 & 6 & 4 & 24 & 12 \end{pmatrix}$$

Solution.

swap R_1 with R_5 \rightarrow

$$\begin{pmatrix} 4 & 6 & 4 & 24 & 12 \\ 3 & 3 & 1 & 12 & 9 \\ 6 & 5 & 2 & 20 & 18 \\ 2 & 3 & 1 & 12 & 6 \\ 7 & 7 & 4 & 28 & 21 \end{pmatrix}$$

Divide the new R_1 by $R_{1,1}$ \rightarrow

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 3 & 3 & 1 & 12 & 9 \\ 6 & 5 & 2 & 20 & 18 \\ 2 & 3 & 1 & 12 & 6 \\ 7 & 7 & 4 & 28 & 21 \end{pmatrix}$$

From each row $R_{i>1}$ subtract $R_i * R_1$ \rightarrow

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & -1.5 & -2 & -6 & 0 \\ 0 & -4 & -4 & -16 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -3.5 & -3 & -14 & 0 \end{pmatrix}$$

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swap R_2 with R_3
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$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & -4 & -4 & -16 & 0 \\ 0 & -1.5 & -2 & -6 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -3.5 & -3 & -14 & 0 \end{pmatrix}$$

Divide the new R_2 by $R_{2,2}$
→

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & -1.5 & -2 & -6 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -3.5 & -3 & -14 & 0 \end{pmatrix}$$

From each row $R_{i>2}$ subtract $R_i * R_2$
→

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{pmatrix}$$

Multiply R_3 by -2
→

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \end{pmatrix}$$

From each row $R_{i>3}$ subtract $R_i * R_3$
→

$$\begin{pmatrix} 1 & 1.5 & 1 & 6 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

From each row $R_{i<2}$ subtract $R_{i,2} * R_2$
→

$$\begin{pmatrix} 1 & 0 & -0.5 & 0 & 3 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

From each row $R_{i < 3}$ subtract $R_{i,3} * R_3$ \rightarrow

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Problem 1.2. Solve the System of Equations above \mathbb{R} :

$$\begin{aligned} 2x + 7y + 13z &= 33 \\ 2x + 4y + 7z &= 15 \\ 1x + 2y + 4z &= 7 \end{aligned}$$

Solution.

Display as Matrix \rightarrow

$$\left(\begin{array}{ccc|c} 2 & 7 & 13 & 33 \\ 2 & 4 & 7 & 15 \\ 1 & 2 & 4 & 7 \end{array} \right)$$

$R_2 = R_2 - R_1$ \rightarrow

$$\left(\begin{array}{ccc|c} 2 & 7 & 13 & 33 \\ 0 & -3 & -6 & -18 \\ 1 & 2 & 4 & 7 \end{array} \right)$$

$R_3 = 2 * R_3 - R_1$ \rightarrow

$$\left(\begin{array}{ccc|c} 2 & 7 & 13 & 33 \\ 0 & -3 & -6 & -18 \\ 0 & -3 & -5 & -19 \end{array} \right)$$

$R_3 = R_3 - R_2$ \rightarrow

$$\left(\begin{array}{ccc|c} 2 & 7 & 13 & 33 \\ 0 & -3 & -6 & -18 \\ 0 & 0 & 1 & -1 \end{array} \right)$$

$$\underline{\underline{R_1 = 3 * R_1 + 7 * R_2 \rightarrow}}$$

$$\left(\begin{array}{ccc|c} 6 & 0 & -3 & -27 \\ 0 & -3 & -6 & -18 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\underline{\underline{R_1 = R_1 + 3 * R_3 \rightarrow}}$$

$$\left(\begin{array}{ccc|c} 6 & 0 & 0 & -30 \\ 0 & -3 & -6 & -18 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\underline{\underline{R_2 = R_2 + 6 * R_3 \rightarrow}}$$

$$\left(\begin{array}{ccc|c} 6 & 0 & 0 & -30 \\ 0 & -3 & 0 & -24 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\underline{\underline{\text{Display as System of Equations} \rightarrow}}$$

$$\begin{array}{rcl} 2x + 7y + 13z & = & 33 \\ -3y + -6z & = & -18 \\ 1z & = & -1 \end{array}$$

$$\underline{\underline{z = -1 \rightarrow}}$$

$$\begin{array}{rcl} 2x + 7y + 13z & = & 33 \\ y & = & 8 \\ z & = & -1 \end{array}$$

$$\underline{\underline{y = 8, z = -1 \rightarrow}}$$

$$\begin{array}{rcl} x & = & -5 \\ y & = & 8 \\ z & = & -1 \end{array}$$

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