Problem Set I

Computing Models
April 23, 2023

Alon Filler[1]

1 Automatas

Given the automata $D = (Q^D, \{a, b\}, \delta^D, q_0^D, F^D)$

Problem 1.1.
Imagine a new automata $E = (Q^E, \{a, b\}, \delta^E, q_0^E, F^E)$ *s.t:*

- $Q^E = Q^D \cup \{q_0^E\}$
- $F^E = F^D$
- $\delta^E(q, \sigma) = \begin{cases} \delta^D(q, \sigma) & q \in Q^D \\ q_0^D & q = q_0^E, \sigma = a \\ q_0^E & q = q_0^E, \sigma = b \end{cases}$

Define:

- *L(A)*

Solution.
It would be non but rational to divide this construction into three divisions, each corresponding to a different set of circumstances recognised by the trasitions function.

One of those aforementioned circumstances is $q = q_0^E, \sigma = b$*, the study of such case lead me to determine that for the character input of b, under the assumption that the current state is* q_0^E*, the state would lead back to itself, meaning that that an instance of* $\{b\}^*$ *at the beginning of the input would not affect the output of the automata. And hence* $\{b\}^*$ *should be imbued to the language L(A).*

Another set of circumstances is $q = q_0^E, \sigma = a$*, which implies the current state to be the one added to* Q^D *in order to craft* Q^E*, and that the input chracter is 'a'. Such circumstances appear to be digested by the automata to return* q_0^D*, the first state of the previous automata D. Accordingly, it would only be after the appearence of an 'a' character in the input that the state would be changed. And hence,* $\{a\}$ *must be added to the language L(A).*

[1]With Σorer

The last of such circumstances addressed in δ^E appears to be $q \in Q^D$. For such case, the function would make the transition from the current state to the one returned by δ^D, accordingly, L(D) must be concatenated at the end of L(E)

Hence - I may declare that $L(A) = \{b\}^* \cdot \{a\} \cdot L(D)$. $\blacksquare$

Problem 1.2.
Consider the previous automata[2] $E = (Q^E, \{a, b\}, \delta^E, q_0^E, F^E)$
And a new automata $E' = (Q^E, \{a, b\}, \delta^E, q_0^D, F^E)$

Define:

- $L(E')$

Solution.
In the previous problem[3] I have explained in great detail the effects caused by the declaration of a new state q_0^E to be the initial state of E. Considering q_0^E not to be part of Q^D, it can be seen with vividness that δ^E, known as E''s transitions function would cease to return it as output as soon as it no longer is the current state. Therefore, knowing E' does not use q_0^E as its initial state it is cogent that it would never be returned. Accordingly, the only circumstances relevant to the transitions function would be $q \in Q^D$, and thus the addition of $\{b\}^* \cdot \{a\}$ to the beginning of L(E) must be undone for $L(E')$ to be precise. And hence $L(E') = L(D)$ $\blacksquare$

Problem 1.3.
Consider the previous automata[4] $E = (Q^E, \{a, b\}, \delta^E, q_0^E, F^E)$
And a new automata $E' = (Q^E, \{a, b\}, \delta^E, q_0^D, F^D \cup q_0^E)$

Define:

- $L(E')$

Solution.
Now that q_0^E is an accepting state, I should like for $L(E')$ to support it. Thus, anything that happens to follow $\{b\}^*$ is merely optional. And hence it might be defined as such: $L(E') = \{b\}^* \cdot \{\{\epsilon\} \cup (\{a\} \cdot L(D))\}$ $\blacksquare$

[2] Problem 1.1
[3] Problem 1.1
[4] Problem 1.2