Trigonometry

The branch of mathematics that treats the relationships between the angles and sides of triangles is called trigonometry. Basic trigonometric relationships are used to solve some types of physics problems. Figure 1-8 shows a right triangle. Notice that side a is opposite angle θ, side b is adjacent to angle θ, and side c is the hypotenuse opposite the right angle.

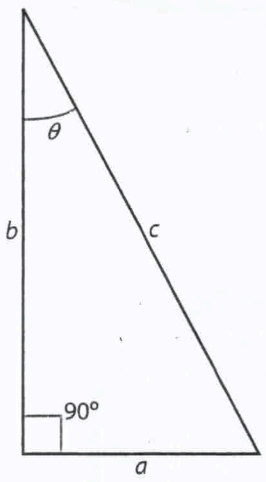


Figure 1-8. Right triangle

Important ratios of the sides of a right triangle in terms of angle θ include the following.

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$

Ⓡ

If the measure of angle θ is 30.°, the ratio of a to c is 0.50 because sin 30.° = 0.50.

If you know the length of any two sides of a right triangle, you can find the length of the third side by using the Pythagorean theorem. The Pythagorean theorem is valid for right triangles only and has the following formula:

$$c^2 = a^2 + b^2$$

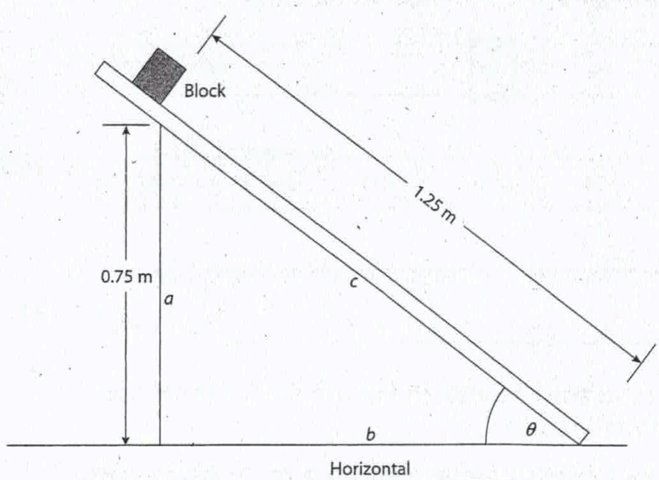
Ⓡ

SAMPLE PROBLEM

A block is displaced a vertical distance of 0.75 meter as it slides down a 1.25-meter long plane inclined to the horizontal, as shown in the following diagram.

(a) Calculate the horizontal displacement of the block.

(b) Calculate the angle the plane makes with the horizontal.



SOLUTION: Relate the Pythagorean theorem to the diagram. Identify the known and unknown values.

Known	Unknown
$a = 0.75$ m	$b = ?$ m
$c = 1.25$ m	$\angle\theta = ?$ °

1. Solve the formula for the Pythagorean theorem for the unknown, b.

$$c^2 = a^2 + b^2$$

$$b = \sqrt{c^2 - a^2}$$

2. Substitute the known values and solve.

$$b = \sqrt{(1.25\text{m})^2 - (0.75\text{ m})^2} = 1.0\text{ m}$$

3. Write the formula for sin θ.

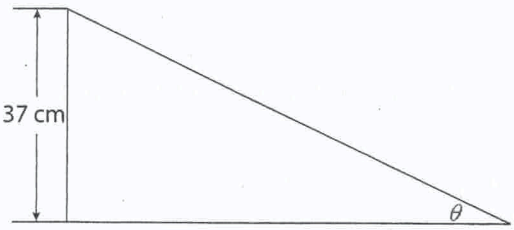
$$\sin\theta = \frac{a}{c}$$

4. Substitute the known values and solve for θ.

$$\sin\theta = \frac{0.75\text{ m}}{1.25\text{ m}}$$

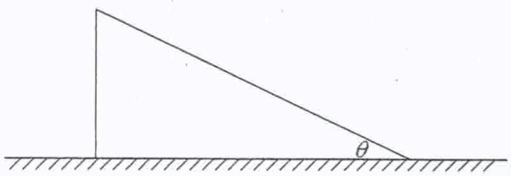
$$\theta = 37°$$

27. The diagram below represents a ramp inclined to the horizontal at angle θ. The upper end of the ramp is 37 centimeters above the horizontal.



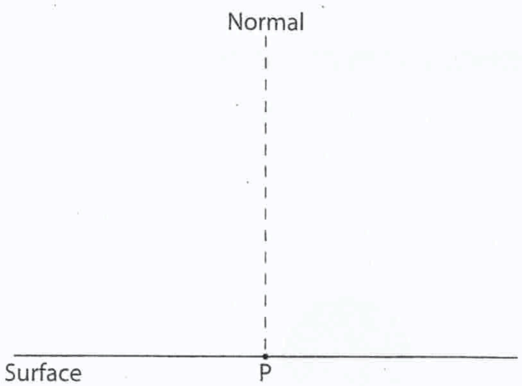
a) Using a protractor measure angle θ.
b) Calculate the length of the ramp.

28. The diagram below represents a ramp inclined at angle θ to the horizontal.



a) Using a protractor measure angle θ to the nearest degree.
b) What is $\sin\theta$?
c) What is $\cos\theta$?

29. On the diagram below, use a protractor and a straightedge to construct an angle of 40.° with the normal to the surface at point *P*.



2

Scalar and Vector Quantities

Physical quantities can be categorized as either scalar or vector quantities. As physical quantities are introduced in this text, their scalar or vector nature will be indicated.

A **scalar** quantity has magnitude only, with no direction specified. Time and mass are scalar quantities. For example, 30 seconds and 45 kilograms are scalar quantities. The measurement of a scalar quantity is indicated by a number with an appropriate unit. Scalar quantities are added and subtracted according to the rules of arithmetic.

A **vector** quantity has both magnitude and direction. Velocity is a vector quantity because it must be described not only by a number with an appropriate unit, but also by a specified direction. For example, the velocity of a car might be described as 25 meters per second, due north. Vector quantities are added and subtracted using geometric or algebraic methods. These methods will be illustrated later in the text.

Solving Equations Using Algebra

Several axioms or statements are used in solving an equation for an unknown quantity. These axioms, which can be assumed to be true, include the following.

- If equals are added to equals, the sums are equal.
- If equals are subtracted from equals, the remainders are equal.
- If equals are multiplied by equals, the products are equal.
- If equals are divided by equals, the quotients are equal.
- A quantity may be substituted for its equal.
- Like powers or like roots of equals are equal.

You should make use of these axioms to isolate the unknown on the left side of an equation before substituting known values. Always include the units with the values in an equation. Although it is not necessary to align equal signs in the solution of an equation, it may help you keep your work orderly.

Mathematicians have agreed on the following order to be used in performing a series of operations:

1. Simplify the expression within each set of parentheses.
2. Perform exponents.
3. Perform the multiplications and divisions in order from left to right.
4. Do the additions and subtractions from left to right.

"*P*lease *e*xcuse *m*y *d*ear *A*unt *S*ue" is a useful memory device for this order: *p*arentheses, *e*xponents, *m*ultiplication and *d*ivision in order, and finally *a*ddition and *s*ubtraction in order.

Review Questions

94. Solve the following formulas for r.

(a) $F = \dfrac{mv^2}{r}$

(b) $A = \pi r^2$

(c) $C = 2\pi r$

(d) $F = G\dfrac{m_1 m_2}{r^2}$

95. Solve the following formulas for d.

(a) $\bar{v} = \dfrac{d}{t}$

(b) $P = \dfrac{Fd}{t}$

(c) $v_f^2 = v_i^2 + 2ad$

96. Solve the following formulas for v.

(a) $KE = \dfrac{1}{2}mv^2$

(b) $p = mv$

(c) $n = \dfrac{c}{v}$

97. Solve the following formulas for I.

(a) $R = \dfrac{V}{I}$

(b) $W = VIt$

(c) $P = I^2R$

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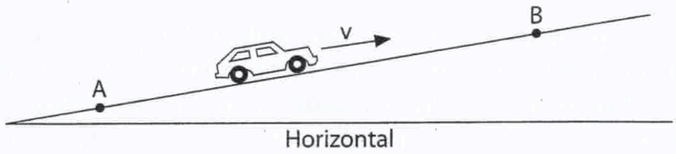
98. Express 1/299 792 458 second, the time it takes light to travel one meter in a vacuum, in scientific notation.

99. What is the approximate length of a baseball bat?

(1) 10^{-1} m
(2) 10^{0} m
(3) 10^{1} m
(4) 10^{2} m

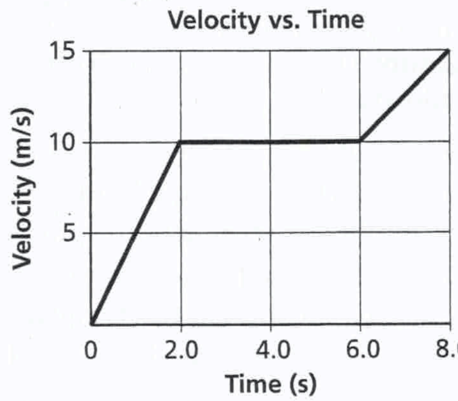
Base your answers to questions 100 and 101 on the diagram below, which represents a toy car traveling at constant speed v up an incline from point A to point B, a distance of 1.4 meters.



100. Determine the measure of the angle that the incline makes with the horizontal.

101. Calculate the vertical change in the car's position.

Base your answers to questions 102 and 103 on the graph below, which represents the velocity of an object traveling in a straight line as a function of time.



102. Calculate the total area under the curve.

103. Calculate the slope of the line in the time interval 6.0 seconds to 8.0 seconds.