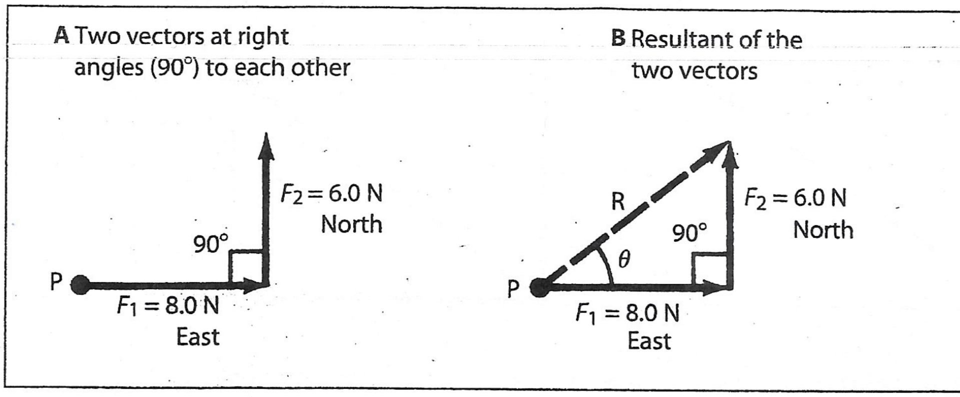


Resultant from the Pythagorean theorem

$R^2 = F_1^{\,2} + F_2^{\,2}$

$R = \sqrt{64\ \text{N}^2 + 36\ \text{N}^2} = 10.\ \text{N}$

The tangent of the angle θ is $\frac{6.0}{8.0}$, or 0.75, making the angle equal to 37°.

Figure 2-6. Finding the resultant of two concurrent forces acting at right angles (90°) to each other

at any angle is the parallelogram method shown in Figure 2-7. The two vectors are drawn to scale with both tails originating at the same point. A parallelogram is then constructed with the force vectors as adjacent sides. Recall that a parallelogram is a quadrilateral having opposite sides parallel and equal in length. The diagonal of the parallelogram drawn from the vertex of the original two vector tails is the resultant.

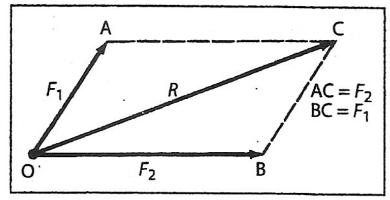


Figure 2-7. Finding the resultant of two concurrent forces at any angle to each other using the parallelogram method

The parallelogram method makes it obvious that as the angle between two vectors increases from 0° to 180.° the magnitude of the resultant decreases from a maximum, $F_1 + F_2$ at 0°, to a minimum, $F_1 - F_2$ at 180°.

Resolution of Forces

Just as force vectors can be added to provide the magnitude and direction of the resultant force, force vectors can be resolved or broken up into component vectors. The process of determining the magnitude and direction of the components of a force is called **resolution of forces.** Although a force vector could be resolved into any number of components, it is usually resolved into two components that are perpendicular to each other. The **vector components** of a force vector F are the concurrent forces whose vector sum is F. If the force is resolved into two components at right angles to each other, vector F is a diagonal of the rectangle formed in the parallelogram method. Perpendicular component forces are usually given directions such as east-west and north-south, perpendicular and parallel to the ground, or perpendicular and parallel to an incline.

Graphical Method of Resolving a Force into Components Figure 2-8 shows a 50.-newton force at 37° north of east being resolved into two perpendicular component forces by the graphical method. Force F_1 is along the north-south axis and force F_2 is along the east-west axis. The magnitude of each force is found by drawing a perpendicular to each axis from the head end of the given vector. The line drawn from the origin, O, to each intersection with the axes determines the magnitude of each component vector. The components are measured to be $F_1 = 30.$ N north and $F_2 = 40.$ N east. Note that the vector sum of the components is equal to the original force F.

Algebraic Method of Resolving a Force into Components It is also possible to determine algebraically the perpendicular components of a force or any other vector. Figure 2-9 shows how vector A, which is at an angle θ with the horizontal, can be resolved into components at right angles to each other. Recall that in a right triangle, the sine of one of the acute angles is the ratio of the side opposite the angle to the hypotenuse and that the

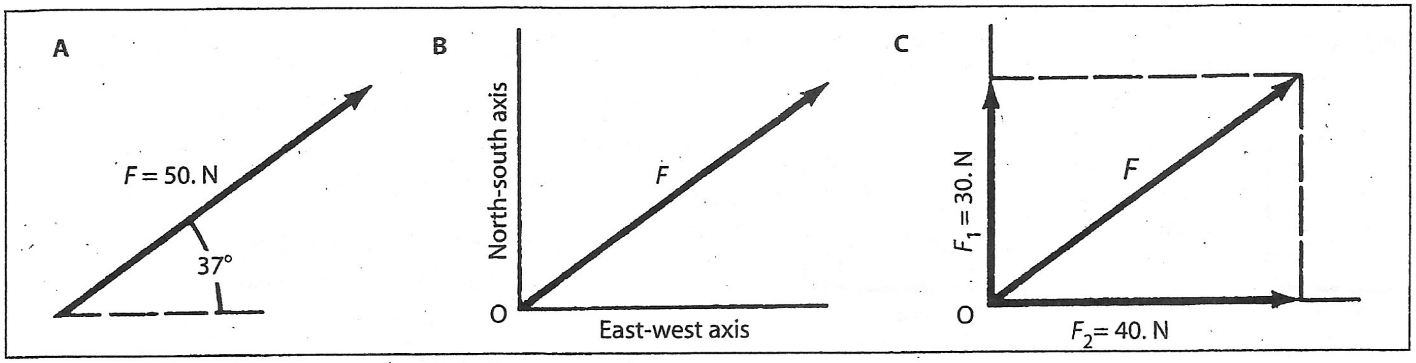


Figure 2-8. Resolution of a force into two components at right angles (90°) to each other: (A) The vector to be resolved is a force vector F of 50. N directed at 37° north of east. (B) Horizontal (east-west) and vertical (north-south) axes are constructed at the tail of the vector. (C) Dashed lines that start at the head of vector F and extend perpendicularly to the axes define two new vectors F_1 and F_2 that are the vertical and horizontal components of the original force vector F. To the scale of the drawing, F_1 measures 30. N north and F_2 measures 40. N east.

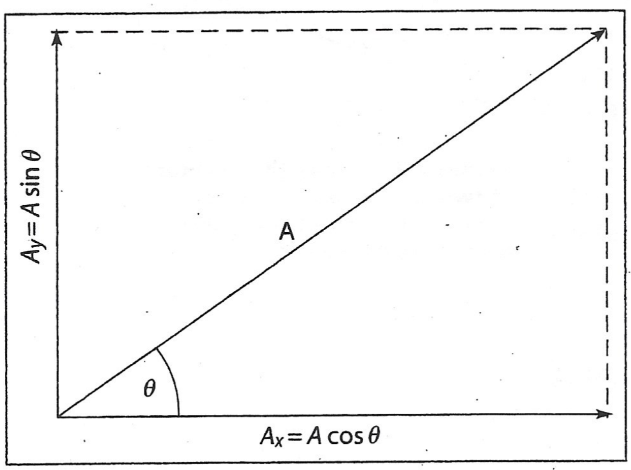


Figure 2-9. A force vector A resolved into horizontal and vertical components

cosine of the angle is the ratio of the adjacent side of the angle to the hypotenuse. Thus, for any vector A, making angle θ with the horizontal, the following apply.

$$A_x = A \cos \theta$$
$$A_y = A \sin \theta$$

Ⓡ

Thus, the components of a vector can be readily determined without making a scale drawing.

Component forces have practical applications, such as pushing a lawnmower or pulling a suitcase by an extended handle at constant speed along the ground. In pulling a suitcase, the magnitude of the force that needs to be exerted depends upon the angle of the extended handle with the ground. The suitcase is moved only by the component of the applied force parallel to the ground. This component is a smaller fraction of the applied force when the angle the extended handle makes with the ground is larger. Thus, a greater force must be applied as the angle between the extended handle and the ground becomes larger.

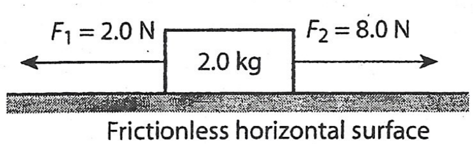
Equilibrium

The vector sum of the concurrent forces acting on an object is called the **net force,** F_{net}. If the net force acting on an object is zero, the object is in **equilibrium.** An object at rest is said to be in static equilibrium.

In the example illustrated in Figure 2-6, the resultant of an 8.0-newton force to the east and a 6.0-newton force to the north is a 10.-newton force at 37° north of east. If a third force of 10. newtons acting at 37° south of west was applied, the net force would be zero. A force that is equal in magnitude and opposite in direction to the resultant of concurrent forces produces equilibrium.

Figure 2-10A shows a sign hanging from the side of a building. Because the sign is at rest, it is in static equilibrium and the net force on the sign is zero. But three forces are acting on the sign. They are its weight F_g acting perpendicular to the ground, the force exerted by the cable F_1 pulling in the direction of the cable toward the building, and the outward push F_2 of the horizontal rod.

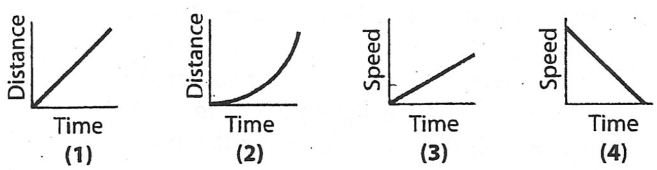
64 Two horizontal forces are applied to a 2.0-kilogram block on a frictionless, horizontal surface, as shown in the following diagram.



The acceleration of the block is

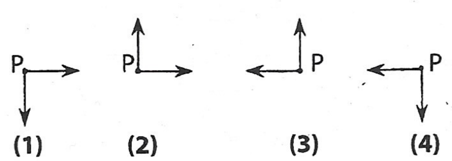
(1) 5.0 m/s^2 to the right
(2) 5.0 m/s^2 to the left
(3) 3.0 m/s^2 to the right
(4) 3.0 m/s^2 to the left

65. Which graph best represents the motion of an object on which the net force is zero?

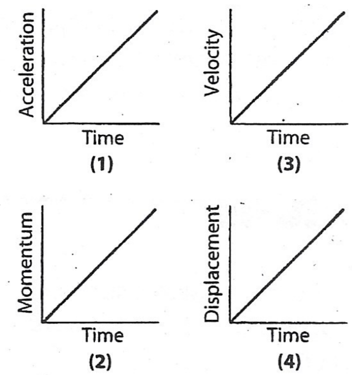


66. The vector diagram below represents force F acting on point P.

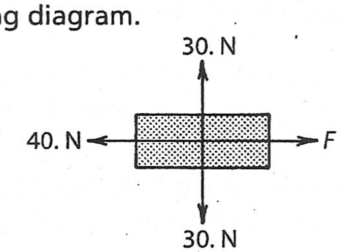
Which vector diagram represents the pair of concurrent forces that would produce equilibrium when added to force F?



67. Which graph best represents the motion of an object that has no unbalanced force acting on it?



68. Four forces act on an object, as shown in the following diagram.

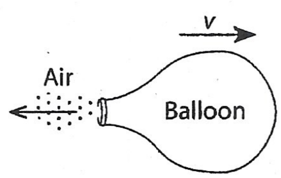


If the object is moving with a constant velocity, what is the magnitude of force F?

69. A 1.0-kilogram book rests on a horizontal tabletop. The magnitude of the force of the tabletop on the book is

(1) 1.0 kg (2) 9.8 kg (3) 1.0 N (4) 9.8 N

70. In the following diagram, an inflated balloon released from rest moves horizontally with velocity v.



What is the most likely cause of this velocity?

Two-Dimensional Motion and Trajectories

The motion of an object traveling in a two-dimensional plane can be described by separating its motion into the horizontal (x) and vertical (y) components of its displacement, velocity, and acceleration. A component parallel to the horizon is a **horizontal component** and a component at right angles to the horizon is a **vertical component.** If air resistance is neglected, an example of two-dimensional motion is the motion of a cannonball projected near the surface of Earth at an angle above the horizontal. If gravity is the only unbalanced force acting on the cannonball, the vertical component of the ball's motion is identical to that of a freely falling body, and the horizontal component is uniform motion. Although the two motions occur simultaneously, the two components of the motion are independent. Thus, if the object's initial velocity is known, the motion of the object in Earth's gravitational field can be described by the superposition of the two motions.

A Projectile Fired Horizontally

An object projected horizontally from some height above Earth's surface obeys Newton's laws of motion. If air resistance is neglected, the horizontal component of the velocity of the object remains constant. The initial vertical velocity of the object is zero but the vertical velocity increases as the object accelerates downward due to gravity. Figure 2-12A shows a ball falling freely straight downward from rest. Figure 2-12B shows a ball that has a horizontal component of velocity as it falls downward.

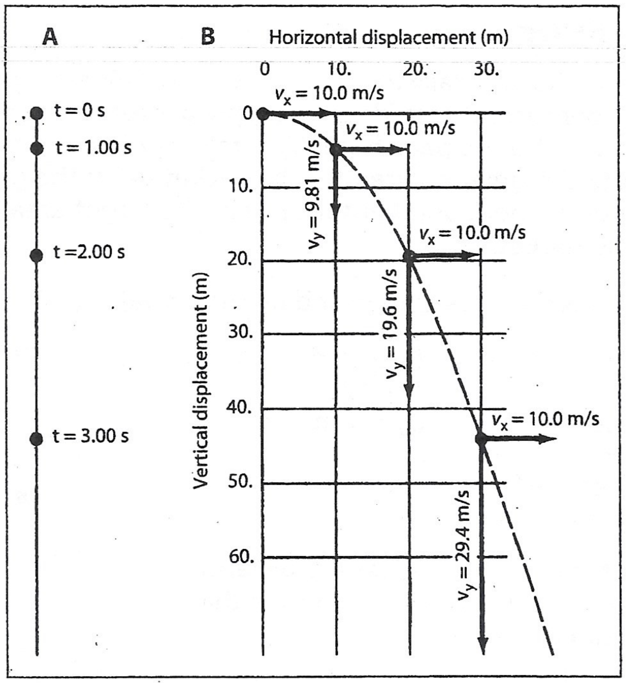


Figure 2-12. (A) The position of a ball at 1.00-second intervals as it falls from rest in a vacuum near Earth's surface. The vertical scale in drawing B also applies to drawing A. (B) The position of the same ball at 1.00-second intervals after it has been rolled off the edge of a building with an initial horizontal velocity of 10.0 m/s. The arrows are velocity vectors giving the horizontal and vertical components of the velocity when the ball is at each position. Note that the horizontal component of the ball's velocity is the same after each second but the vertical component increases with time as the ball is accelerated by gravity. Note also that in both A and B, the vertical distance the ball has fallen at the end of each second is the same.

Whether an object is dropped from rest or projected horizontally, the vertical distance fallen by the object is the same at any particular instant of time, as can be seen by comparing the vertical position of the ball at 1.00-second intervals in Figure 2-12A and B. This example illustrates that a ball thrown horizontally at 10.0 meters per second from a height of 44.1 meters above level ground, will hit the ground at the same time as another ball dropped from the same height at the same time.

In addition to showing the positions of the ball after an elapsed time of 1.00, 2.00, and 3.00 seconds, Figure 2-12B shows (by means of velocity vectors) the vertical and horizontal components of the velocity. At any particular time after the ball is released, the vertical component of velocity of an object projected horizontally is the same as the vertical component of velocity of an object dropped from rest. However, the vertical component of the ball's velocity increases, as the ball is accelerated by gravity. Thus, the vertical distance the ball falls in the third second is greater than the vertical distance the ball falls in the second second.

When air resistance is neglected, there is no acceleration or change in velocity in the horizontal direction. In Figure 2-12B, the horizontal displacement of the ball is a constant 10.0 meters in each 1.00-second interval. If the horizontal velocity of the ball had been greater, the object would have traveled a greater horizontal distance in the first 3.00 seconds of travel.

A Projectile Fired at an Angle

A golf ball is an example of an object that is projected with an initial velocity at an angle above the horizontal. Such a projectile rises to some height above Earth and then falls back to the ground. The projectile's motion can be studied by resolving the initial velocity into its horizontal and vertical components and then calculating the motions resulting from the two components. If air resistance is ignored, the horizontal component

SAMPLE PROBLEM

A plane flying horizontally at an altitude of 490 meters and having a velocity of 250 meters per second east, drops a supply packet to a work crew on the ground. It falls freely without a parachute. [Assume no wind and negligible air resistance.]
(a) Calculate the time required for the packet to hit the ground.
(b) Calculate the horizontal distance from the target area that the plane must drop the packet.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$d_y = 490$ m	$t = ?$ s
$v_x = 250$ m/s	$d_x = ?$ m
$v_{i_y} = 0.0$ m/s	
$a_y = g = 9.81$ m/s^2	
$a_x = 0.00$ m/s^2	

1. Write a formula that relates the distance, acceleration, and time for motion in the vertical direction.

$$d_y = v_{i_y}t + \tfrac{1}{2}a_y t^2$$

Because v_{i_y} is zero, the equation becomes

$$d_y = \frac{1}{2}a_y t^2$$

2. Solve the equation for t and substitute g for acceleration.

$$t = \sqrt{\frac{2d_y}{g}}$$

3. Substitute the known values and solve.

$$t = \sqrt{\frac{2(490\text{ m})}{9.81\text{ m/s}^2}} = 10.\text{s}$$

4. Write a formula that relates the distance, acceleration, and time for motion in the horizontal direction.

$$d_x = v_{i_x}t + \tfrac{1}{2}a_x t^2$$

Because $a_x = 0.0$ m/s^2 the equation becomes $d_x = v_{i_x}t$.

5. Substitute the known values and solve.

$$d_x = (250\text{ m/s})(10.\text{ s}) = 2.5 \times 10^3\text{ m}$$

The plane must drop the packet 2.5×10^3 meters west of the target.

of the velocity remains constant. The object's vertical motion is accelerated by the force of gravity.

If a golf ball is projected with initial velocity v_i at an angle θ above the horizontal, v_i can be separated into perpendicular components, as shown in Figure 2-13.

Recall from page 36 that any vector, A, making an angle θ with the horizontal can be resolved into horizontal and vertical components. The two components of velocity can be determined using these formulas.

Horizontal component: $v_{i_x} = v_i \cos\theta$
Vertical component: $v_{i_y} = v_i \sin\theta$

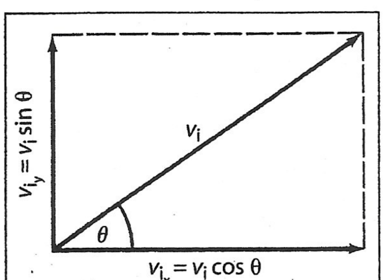


Figure 2-13. An initial velocity vector resolved into horizontal and vertical components

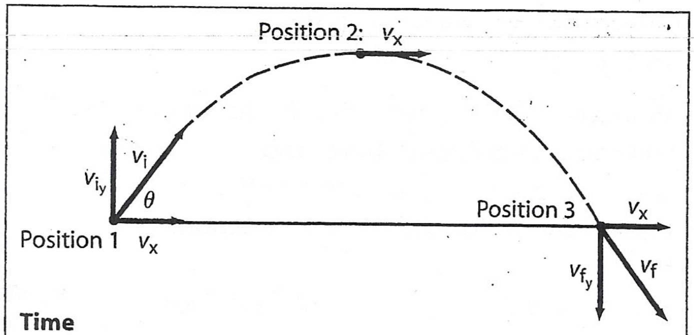
The vertical component of the velocity gradually decreases to zero as the golf ball reaches the highest point in its trajectory. When the vertical component of the velocity is zero, all of the velocity is in the horizontal. Then the vertical component gradually increases along the ball's downward path due to the constant acceleration of gravity. See Figure 2-14.

To find the time t for the projectile to reach its maximum height, solve the equation $v_f = v_i + at$ for t.

$$t = \frac{v_f - v_i}{a}$$

Then, substitute the appropriate values for the vertical velocity and acceleration. If upward in the vertical direction is considered positive, then g, a downward acceleration, is negative. At the highest point, the vertical velocity of the projectile is zero. Thus $t = \frac{v_i \sin\theta}{g}$. It can be shown that the time for the projectile to reach its maximum height is the same as the time to fall back to the ground from that height. Therefore the total time of travel to return to ground level is $\frac{2v_i \sin\theta}{g}$.

The horizontal distance traveled by a projectile is called its range. For any given initial velocity, the range is a maximum when $\theta = 45°$. For any given initial speed of a projectile, the range of the projectile is the same for complementary angles above the horizontal, neglecting friction. For example, a projectile launched with an initial velocity of 15 meters per second at 20.° above the horizontal has the same range as a projectile launched with an initial velocity of 15 meters per second at 70.° above the horizontal. However, the time of flight and maximum height above the horizontal are greater for the launch at 70.°. The actual range of a projectile (when air resistance is present) is shorter than the calculated ideal range.



Time

Position 1:	Position 2:	Position 3:
$t = 0$	$t = \frac{v_i \sin\theta}{g}$	$t = \frac{2\,v_i \sin\theta}{g}$

Vertical motion

Position 1:	Position 2:	Position 3:
$v_{i_y} = v_i \sin\theta$	$v_y = 0$	$v_{f_y} = -\,v_i \sin\theta$
$a_y = -g$	$a_y = -g$	$a_y = -g$

Horizontal motion

Position 1: Position 2: Position 3:

$v_x = v_i \cos\theta$ in all positions

$a_x = 0$ in all positions

Figure 2-14. The motion of a projectile that is fired at angle θ above the horizontal and returns to the horizontal. [Neglect friction.]

SAMPLE PROBLEM

A small missile is fired with a velocity of 300. meters per second at an angle of 30.0° above the ground. After a total flight time of 30.6 seconds, the missile returns to the level ground. [Neglect air resistance.]

(a) Calculate the initial horizontal and vertical components of the velocity.
(b) Calculate the maximum height of the missile above the ground.
(c) Calculate the total horizontal range of the missile.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$v_i = 300.$ m/s	$v_{i_x} = ?$ m/s
$\theta = 30.0°$	$v_{i_y} = ?$ m/s
$t = 30.6$ s	$d_y = ?$ m
$a_y = g = -9.81$ m/s^2	$d_x = ?$ m
$a_x = 0.00$ m/s^2	

1. Write the formulas that resolve the initial velocity vector into horizontal and vertical components.

$v_{i_x} = v_i \cos\theta$
$v_{i_y} = v_i \sin\theta$

2. Substitute the known values and solve.

$v_{i_x} = (300.\text{ m/s})(\cos 30.0°) = 260.\text{ m/s}$
$v_{i_y} = (300.\text{ m/s})(\sin 30.0°) = 150.\text{ m/s}$

3. Find the time for the missile to reach its highest point.

Because the total time of flight is 30.6 s, the time to reach the maximum height is $\frac{1}{2}(30.6\text{ s}) = 15.3$ s.

4. Write the formula that relates distance, average velocity, and time in the vertical direction.

$d_y = \bar{v}_y t_{rise}$

At the highest point, velocity in the vertical direction is zero. Average velocity $= \frac{1}{2}(v_i + v_f) = \frac{1}{2}(150.\text{ m/s} + 0\text{ m/s}) = 75.0\text{ m/s}.$

5. Substitute the known values and solve.
$d_y = (75.0\ \text{m/s})(15.3\ \text{s}) = 1150\ \text{m}$

- An alternate solution is to write the formula that relates displacement, time, and acceleration, $d = v_i t + \frac{1}{2}at^2$ and substitute the known values for the vertical direction. That is

$d_y = (150.\ \text{m/s})(15.3\ \text{s}) + \frac{1}{2}(-9.81\ \text{m/s}^2)(15.3\ \text{s})^2$
$d_y = 1150\ \text{m}$

- Another alternate solution is to write the formula that relates final velocity, initial velocity, acceleration, and displacement, $v_f^2 = v_i^2 + 2ad$. Solve the equation for d to yield $d = \frac{v_f^2 - v_i^2}{2a}$. Then substitute the known values for the vertical direction. That is $d = \frac{(0\ \text{m/s})^2 - (150.\ \text{m/s})^2}{2(-9.81\ \text{m/s}^2)} = 1150\ \text{m}$

6. Write the equation that relates distance, velocity, and time in the horizontal direction.
$d_x = \bar{v}_x t_{total}$

7. Substitute the known values and solve.
$d_x = (260.\ \text{m/s})(30.6\ \text{s}) = 7960\ \text{m}$

Review Questions

Base your answers to questions 71 through 75 on the following information.

A ball of mass m is thrown horizontally with speed v from a height h above level ground. [Neglect friction.]

71. If the height above the ground from which the ball is thrown was increased, the time of flight of the ball would
(1) decrease (2) increase (3) remain the same

72. If the initial speed of the ball was increased, the time of flight of the ball would
(1) decrease (2) increase (3) remain the same

73. If the initial speed of the ball was increased, the horizontal distance traveled by the ball would
(1) decrease (2) increase (3) remain the same

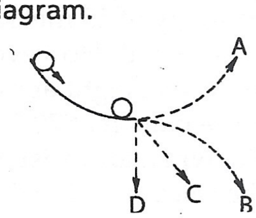
74. As time elapses before the ball strikes the ground, the horizontal velocity of the ball
(1) decreases (3) remains the same
(2) increases

75. Compared to the total horizontal distance traveled by the ball in the absence of air resistance, the total horizontal distance traveled by the ball with air resistance is
(1) shorter (2) longer (3) the same

---

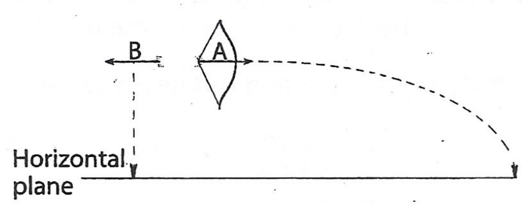
76. A student throws a baseball horizontally at 25 meters per second from a cliff 45 meters above the level ground. Approximately how far from the base of the cliff does the ball hit the ground? [Neglect air resistance.]
(1) 45 m (2) 75 m (3) 140 m (4) 230 m

77. A ball rolls down a curved ramp, as shown in the following diagram.



Which dotted line best represents the path of the ball after leaving the ramp?

78. Above a flat horizontal plane, arrow A is shot horizontally from a bow at a speed of 20 meters per second, as shown in the following diagram. A second arrow B is dropped from the same height and at the same instant as A is fired.



Compare the amount of time A takes to strike the plane to the amount of time B takes to strike the plane. [Neglect friction.]

79. A rock is thrown horizontally from the top of a cliff at 12 meters per second. Calculate the time required for the rock to fall 45 meters vertically.

80. A ball is thrown horizontally at a speed of 24 meters per second from the top of a cliff. If the ball hits the ground 4.0 seconds later, approximately how high is the cliff?
(1) 6.0 m (2) 39 m (3) 78 m (4) 96 m