Uncertainty in Measurement

When a quantity is measured, the measurement always consists of some digits that are certain plus one digit whose value has been estimated. Thus, every measurement has an experimental uncertainty. The uncertainty can result from the quality and limitations of the measuring instrument, the skill of the person using the instrument, and the number of measurements made.

If several measurements taken of the same event are nearly identical, the measurements are said to be **precise.** If a measurement is very close to the accepted value found in a handbook, the measurement is said to be **accurate.** For example, the accepted value for the acceleration due to gravity near Earth's surface is 9.81 m/s^2. If a student measures this quantity as 9.98 m/s^2, 9.98 m/s^2, and 9.99 m/s^2, the measurements are precise, but not accurate.

Significant Figures (Significant Digits)

In a measured value, the digits that are known with certainty plus the one digit whose value has been estimated are called **significant figures** or significant digits. The greater the number of significant digits in a measurement, the greater the accuracy of the measurement.

Nonzero digits in a measurement are always significant. Zeroes appearing in a measurement may or may not be significant. The following rules should be applied *in order* to the zeroes in a measured value:

1. Zeroes that appear *before* a nonzero digit are *not* significant. Examples: 0.002 m (1 significant figure) and 0.13 g (2 significant figures)
2. Zeroes that appear *between* nonzero digits are significant. Examples: 0. 705 kg (3 significant figures) and 2006 km (4 significant figures)
3. Zeroes that appear *after* a nonzero digit are significant *only* if (a) followed by a decimal point. Examples: 40 s (1 significant figure) and 20. m (2 significant figures); *or* if (b) they appear to the right of the decimal point. Examples: 37.0 cm (3 significant figures) and 4.100 m (4 significant figures)

A measurement of 0.040 900 kg utilizes all of the rules for zeroes and contains 5 significant digits.

If a whole-number measurement ends in two or more zeroes, it is not possible to indicate that some, but not all, of the zeroes are significant. For example, a measurement of 5200 m is interpreted to have only two significant figures, although it could actually represent a measurement to the nearest 10 meters. This situation is avoided by the use of scientific notation, which will be discussed later in this topic.

Addition and Subtraction with Measured Values

Measured values must have the same units before they are added or subtracted. For example, if the dimensions of a rectangle are recorded as 4.3 cm and 0.085 m, both measurements must be expressed either in centimeters or in meters before they can be combined by addition to find the perimeter of the rectangle. After adding or subtracting measured values expressed in the same units, the sum or difference is rounded to the same decimal place value as the least sensitive measurement.

Example A below shows that subtracting a measurement known to the nearest thousandth of a meter from a measurement known to the nearest tenth of a meter produces a difference known to the nearest tenth of a meter.

Similarly, in Example B below, adding measurements to the nearest hundredth of a centimeter, tenth of a centimeter, and centimeter produces a sum to the nearest centimeter.

Example A

$$\begin{array}{r} 31.1\ \text{m} \\ -\ 2.461\ \text{m} \\ \hline 28.639\ \text{m} = 28.6\ \text{m} \end{array}$$

Example B

$$\begin{array}{r} 24.82\ \text{cm} \\ 4.7\ \text{cm} \\ +\ 2\ \text{cm} \\ \hline 31.52\ \text{cm} = 32\ \text{cm} \end{array}$$

Multiplication and Division with Measured Values

When multiplying or dividing measured values, the operation is performed and the answer is rounded to the same number of significant figures as appears in the value having the lowest number of significant figures. In the example that follows, 2.6 cm has two significant figures, whereas 200.0 cm has four. Thus, the product of the two values can have only two significant figures.

$$(200.0\ \text{cm})(2.6\ \text{cm}) = 520\ \text{cm}^2$$

Notice that although both measurements are accurate to the nearest tenth of a centimeter, the last significant figure in the product is in the tens place. Thus, the product of a measurement with four significant figures and a measurement with two significant figures has only two significant figures.

Review Questions

33. A student measures the speed of yellow light in water to be 2.00×10^8 meters per second, 1.87×10^8 meters per second, and 2.39×10^8 meters per second. If the accepted value for the speed is 2.25×10^8 meters per second, the student's measurements are

(1) precise, only
(2) accurate, only
(3) both precise and accurate
(4) neither precise nor accurate

34. A student measures the length of a quarter-mile lap around the school's track to be 402.3 meters, 402.3 meters, and 402.5 meters. If the accepted value for the path length is 402.3 meters, the student's measurements are

(1) precise, only
(2) accurate, only
(3) both precise and accurate
(4) neither precise nor accurate

35. Which length measurement contains three significant figures?

(1) 0.203 m (3) 34.70 km
(2) 0.54 m (4) 570 cm

36. To what number of significant figures is a measurement of 14,020 grams expressed?

(1) 5 (2) 2 (3) 3 (4) 4

37. What is the area of a rectangle having dimensions of 9.8 meters and 12.7 meters?

(1) 100 m^2 (3) 124 m^2
(2) 120 m^2 (4) 124.46 m^2

38. Which mass measurement is expressed to two significant figures?

(1) 0.040 kg (3) 40 kg
(2) 0.4 kg (4) 405 kg

39. To what number of significant figures is a measurement of 7002 meters expressed?

(1) 1 (3) 3
(2) 2 (4) 4

40. A car travels 685 meters in 27 seconds. What is the average speed of the car?

(1) 25.370 m/s (3) 25.4 m/s
(2) 25.37 m/s (4) 25 m/s

41. A radar signal traveling at 3.00×10^8 meters per second is sent from Earth to the Moon and is received back at Earth in 2.56 seconds. What is the distance from Earth to the Moon?

(1) 7.68×10^8 m (3) 8×10^8 m
(2) 3.84×10^8 m (4) 4×10^8 m

42. What is the sum of 3.04 meters, 4.134 meters, and 6.1 meters?

43. What is the sum of 0.027 kilogram and 0.0023 kilogram?

44. To what number of significant figures is the measurement 0.705 meter expressed?

45. To what number of significant figures is the measurement 470 meters expressed?

46. Express forty meters with four significant figures.

47. Calculate the area of a rectangle having a length of 41.6 centimeters and a width of 2.3 centimeters.

48. Safety guidelines recommend an area of 5.6 meters^2 per student in a laboratory setting. Would a room having dimensions of 13.2 meters and 10.6 meters accommodate 24 students and comply with these guidelines? Justify your answer.

Scientific Notation

Measurements that have very large or very small values are usually expressed in **scientific notation.** Scientific notation consists of a number equal to or greater than one and less than ten followed by a multiplication sign and the base ten raised to some integral power. The general form of a number expressed in scientific notion is $A \times 10^n$. All of the digits in A are significant. For numbers having an absolute value greater than one, n is positive. For numbers having an absolute value less than one, n is negative. For a number having an absolute value of one, n is zero. For example, the mean radius of Earth is 6,370,000 m or 6.37×10^6 m (3 significant figures). The universal gravitational constant is

0.000 000 000 066 7 $N \cdot m^2/kg^2$ or 6.67×10^{-11} $N \cdot m^2/kg^2$ (3 significant figures). The height of a physics student might be 1.75 meters or 1.75×10^0 meters (3 significant figures).

Addition and Subtraction Measurements written in scientific notation can be added or subtracted only if they are expressed in the same units and to the same power of ten. Sometimes, as in the example below, the power of ten must be changed first before adding or subtracting.

$$3.2 \times 10^2 \text{ m} + 4.73 \times 10^3 \text{ m} = 0.32 \times 10^3 \text{ m} + 4.73 \times 10^3 \text{ m} = 5.05 \times 10^3 \text{ m}$$

Multiplication and Division The commutative and associative laws for multiplication are used to find products and quotients of physical quantities written in scientific notation. Recall that the exponents are added when like bases are multiplied and the exponents are subtracted when like bases are divided. The general rule is as follows.

$$(A \times 10^n)(B \times 10^m) = (A \times B)(10^{n+m})$$

and

$$\frac{(A \times 10^n)}{(B \times 10^m)} = \frac{A}{B} \times 10^{n-m}$$

When multiplying and dividing measured values, the rules for significant figures apply to values expressed in scientific notation. Some examples follow.

$$(1.3 \times 10^5 \text{ m})(3.47 \times 10^2 \text{ m}) = 4.5 \times 10^7 \text{ m}^2$$

$$(1.3 \times 10^{-5} \text{ m})(3.47 \times 10^2 \text{ m}) = 4.5 \times 10^{-3} \text{ m}^2$$

$$(4.73 \times 10^5 \text{ m})(5.2 \times 10^2 \text{ m}) = 25 \times 10^7 \text{ m}^2 = 2.5 \times 10^8 \text{ m}^2$$

$$(8.4 \times 10^5 \text{ m}) \div (2.10 \times 10^2 \text{ m}) = 4.0 \times 10^3$$

$$(8.4 \times 10^5 \text{ m}) \div (2.10 \times 10^{-2} \text{ m}) = 4.0 \times 10^7$$

$$(2.10 \times 10^2 \text{ m}) \div (8.4 \times 10^5 \text{ m}) = 0.25 \times 10^{-3} = 2.5 \times 10^{-4}$$

SAMPLE PROBLEM A

As the Voyager spacecraft passed the planet Uranus, it sent signals back to Earth. Determine the order of magnitude of the time in seconds for a signal to reach Earth. The distance from Earth to Uranus is 2.71×10^{12} meters. The speed of light in a vacuum is 3.00×10^8 meters per second.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$d = 2.71 \times 10^{12}$ m	$t = ?$ s
$v = 3.00 \times 10^8$ m/s	

1. Round the known values to the nearest whole numbers and find the formula relating distance, time, and average speed.

$$\bar{v} = \frac{d}{t}$$

2. Solve for t. Substitute the rounded values and solve.

$$t = \frac{d}{\bar{v}} = \frac{3 \times 10^{12} \text{ m}}{3 \times 10^8 \text{ m/s}} = 10^4 \text{ s}$$

The order of magnitude is 10^4.

Estimation and Orders of Magnitude

The technique of estimating the answer to a problem before performing the calculations makes it possible to quickly verify the procedures to be used and determine the reasonableness of the answer as in Sample Problem A. Estimating answers using orders of magnitude also helps in evaluating the reasonableness of an answer, as illustrated in Sample Problem B.

Review Questions

49. Express the diameter of a nickel, 0.021 meter, in scientific notation.

50. Express the mass of a car, 1500 kilograms, in scientific notation.

51. The jet engines of a 747 exert a force of 770,000 newtons. Express this value in scientific notation.

52. Divide $1.494\,57 \times 10^{11}$ meters, the average distance from the Sun to Earth, by 3.00×10^{8} meters per second, the speed of light in a vacuum. Write your answer in scientific notation with the correct units and the appropriate number of significant figures.

53. What is the approximate width of a person's little finger?

(1) 1 m (2) 0.1 m (3) 0.01 m (4) 0.001 m

54. The length of a high school classroom is probably closest to

(1) 10^{-2} m (2) 10^{-1} m (3) 10^{1} m (4) 10^{4} m

55. The thickness of a dollar bill is closest to

(1) 1×10^{-4} m
(2) 1×10^{-2} m
(3) 1×10^{-1} m
(4) 1×10^{1} m

56. Which measurement of an average classroom door is closest to 10^{0} meter?

(1) thickness
(2) width
(3) height
(4) surface area

57. A flowerpot falls from a third-story window ledge to the ground. The total distance the flowerpot falls is closest to

(1) 10^{0} m (2) 10^{1} m (3) 10^{2} m (4) 10^{3} m

58. The approximate diameter of a 12-ounce can of root beer is

(1) 6.7×10^{-3} m
(2) 6.7×10^{-2} m
(3) 6.7×10^{-1} m
(4) 6.7×10^{0} m

59. What is the approximate mass of a chicken egg?

(1) 1×10^{1} kg
(2) 1×10^{2} kg
(3) 1×10^{-1} kg
(4) 1×10^{-4} kg

60. A mass of one kilogram of nickels has a monetary value in United States dollars of approximately

(1) \$1.00
(2) \$0.10
(3) \$10.00
(4) \$1000.00

61. The mass of a physics textbook is closest to

(1) 10^{3} kg (2) 10^{1} kg (3) 10^{0} kg (4) 10^{-2} kg

62. The mass of a high-school football player is approximately

(1) 10^{0} kg (2) 10^{1} kg (3) 10^{2} kg (4) 10^{3} kg

63. What is the approximate mass of an automobile?

(1) 10^{1} kg (2) 10^{2} kg (3) 10^{3} kg (4) 10^{6} kg

64. Approximately how many seconds are in three hours?

(1) 10^{2} s (2) 10^{3} s (3) 10^{4} s (4) 10^{5} s

65. The weight of an apple is closest to

(1) 10^{-2} N (2) 10^{0} N (3) 10^{2} N (4) 10^{4} N

66. Which object weighs approximately 1 newton?

(1) dime
(2) paper clip
(3) physics student
(4) golf ball

67. The weight of a chicken egg is approximately

(1) 10^{-3} N (2) 10^{-2} N (3) 10^{0} N (4) 10^{2} N

68. The speed of a rifle bullet is 7×10^{2} meters per second and the speed of a snail is 1×10^{-3} meter per second. How many times faster than the snail does the bullet travel?

69. The power of sunlight striking Earth is 1.7×10^{17} watts. How many 100-watt incandescent light bulbs would produce this amount of power?

Note: Use information found on the first page of the *Reference Tables for Physical Setting/Physics* in answering questions 70 through 73.

70. The acceleration due to gravity is approximately

(1) 10^{-1} m/s^2
(2) 10^{0} m/s^2
(3) 10^{1} m/s^2
(4) 10^{3} m/s^2

71. What is the order of magnitude of the ratio of the charge on an electron to the mass of an electron?

72. What is the order of magnitude of the ratio of the speed of light in a vacuum to the speed of sound in air at STP?

73. What is the order of magnitude of the ratio of the mass of the Moon to the mass of Earth?