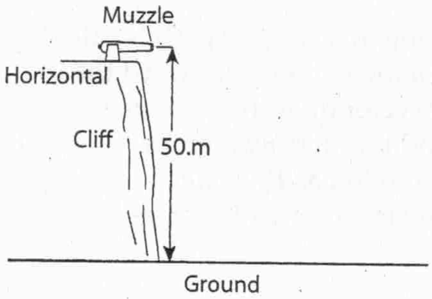
81. The following diagram shows the muzzle of a cannon located 50. meters above the ground. When the cannon is fired, a ball leaves the muzzle with an initial speed of 250 meters per second. [Neglect air resistance].



Which action would most likely increase the time of flight of a ball fired by the cannon?

(1) pointing the muzzle of the cannon toward the ground
(2) moving the cannon closer to the edge of the cliff
(3) positioning the cannon higher above the ground
(4) giving the ball a greater initial horizontal velocity

82. A football player kicks a ball with an initial velocity of 25 meters per second at an angle of 53° above the horizontal. The vertical component of the initial velocity of the ball is

(1) 25 m/s (2) 20. m/s (3) 15 m/s (4) 10. m/s

83. The path of a projectile fired at an angle of 30.° above the horizontal is best described as

(1) parabolic (3) circular
(2) linear (4) hyperbolic

84. A projectile is fired with velocity of 150. meters per second at an angle of 30.° above the horizontal. Calculate the magnitude of the horizontal component of the velocity at the time the projectile is fired.

85. Projectile *A* is fired with velocity *v* at an angle of 30.° above the horizontal. Projectile *B* is fired with velocity *v* at an angle of 40.° above the horizontal. Compared to the magnitude of the horizontal component of *v* at the time projectile *A* is fired, the magnitude of the horizontal component of *v* at the time projectile *B* is fired is

(1) smaller (2) larger (3) the same

86. A projectile is launched at an angle of 60.° above the horizontal. Compared to the vertical component of the initial velocity of the projectile, the vertical component of the projectile's velocity when it has reached its maximum height is

(1) smaller (2) larger (3) the same

87. A projectile is launched at an angle of 30.° above the horizontal. Neglecting air resistance, what are the projectile's horizontal and vertical accelerations when it reaches its maximum height?

Uniform Circular Motion

According to Newton's first law of motion, an unbalanced force acting on an object always produces a change in the object's velocity. If the force has a component in the direction of the object's motion, the magnitude of the velocity changes. However, if the force is applied perpendicular to the direction of motion, only the direction of the velocity changes; its magnitude remains the same. In both instances, the object accelerates because velocity changes with time. If the applied force has a constant magnitude and always acts perpendicular to the direction of the object's velocity, the object moves in a circular path at constant speed, experiencing **uniform circular motion.**

Centripetal Acceleration

An object moving uniformly in a circular path always has **centripetal acceleration,** which is an acceleration directed toward the center of the circle. "Center-seeking" centripetal acceleration is a vector quantity whose magnitude is directly proportional to the square of the speed of the object and inversely proportional to the radius of the circular path in which it travels. Centripetal acceleration is represented by this formula.

$$a_c = \frac{v^2}{r}$$ Ⓡ

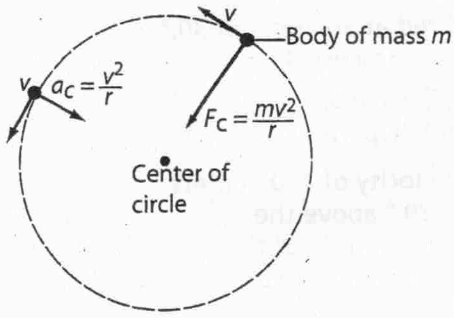


Figure 2-15. The relationship between velocity v, mass m, radius of curvature r, centripetal force F_c, and centripetal acceleration a_c for a body in uniform circular motion: The velocity vector is tangent to the circle. Both the centripetal force and the centripetal acceleration are directed toward the center of the circle. The radius of curvature is the radius of the circle.

The speed v of the object is in meters per second, the radius of curvature r is in meters, and the centripetal acceleration a_c is in meters per second^2. The centripetal acceleration of an object is independent of the mass of the object.

Centripetal Force

The force needed to keep an object moving in a circular path is called **centripetal force,** F_c. Centripetal force is a force directed toward the center of curvature. Centripetal force, a vector quantity, produces centripetal acceleration. Newton's second law, $F = ma$, can be rewritten for the special case of circular motion as $F_c = ma_c$. Substituting in the expression for centripetal acceleration, $a_c = \frac{v^2}{r}$ yields this formula.

$$F_c = \frac{mv^2}{r}$$

Mass m is in kilograms, speed v is in meters per second, radius r is in meters, and centripetal force F_c is in kilogram · meters per second^2, or newtons. Figure 2-15 shows the relationship between these quantities for an object in uniform circular motion.

An object in uniform circular motion travels at constant speed in its circular path because there is no net force acting on the object in its direction of motion. That is, the object's tangential velocity is constant. However, the object is not in equilibrium because centripetal force acts perpendicular to the tangential velocity and produces a constant acceleration towards the center of curvature. Thus, although the magnitude of the object's velocity (speed) remains constant, the direction of the object's velocity is always changing, as shown in Figure 2-16.

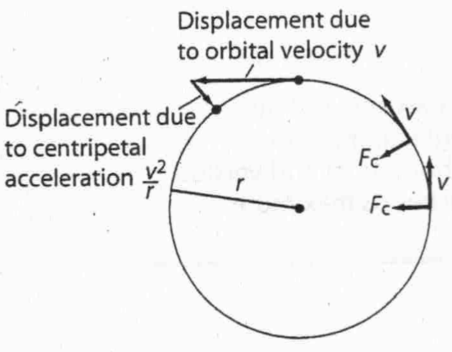


Figure 2-16. The velocity, acceleration, and displacement of a body in uniform circular motion: The velocity vector is always tangent to the circle and perpendicular to the centripetal acceleration. The acceleration causes a continuous change in the direction of the velocity and a continuous displacement to the circular path.

SAMPLE PROBLEM

A 1.5-kilogram cart moves in a horizontal circular path of 1.3-meter radius at a constant speed of 2.0 meters per second.
(a) Calculate the magnitude of the centripetal acceleration of the cart.
(b) Calculate the magnitude of the centripetal force on the cart.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$m = 1.5$ kg	$a_c = ?\ m/s^2$
$r = 1.3$ m	$F_c = ?$ N
$v = 2.0$ m/s	

1. Write the formula for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

2. Substitute the known values and solve.

$$a_c = \frac{(2.0\ m/s)^2}{1.3\ m} = \frac{4.0\ m^2/s^2}{1.3\ m} = 3.1\ m/s^2$$

3. Write the formula for centripetal force.

$$F_c = \frac{mv^2}{r}$$

4. Substitute the known values and solve.

$$F_c = \frac{(1.5\ kg)(2.0\ m/s)^2}{1.3\ m} = \frac{(1.5\ kg)(4.0\ m^2/s^2)}{1.3\ m}$$

$F_c = 4.6$ N

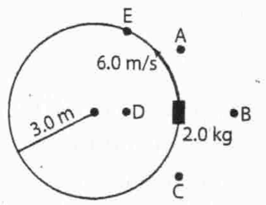
Another way to solve (b) is to substitute the calculated value of a_c from part (a) for v^2/r.

$F_c = ma_c = (1.5\ kg)(3.1\ m/s^2) = 4.7$ N

Review Questions

Base your answers to questions 88 through 96 on the following information and diagram.

A 2.0-kilogram cart travels counter-clockwise at a constant speed of 6.0 meters per second in a horizontal circle of radius 3.0 meters.



88. Calculate the magnitude and direction of the centripetal acceleration of the cart at the position shown.

89. Calculate the magnitude of the centripetal force acting on the cart.

90. If the mass of the cart was doubled, the magnitude of the centripetal force acting on the cart would be

(1) halved (3) quartered
(2) doubled (4) quadrupled

91. If the radius of curvature of the path was doubled, the magnitude of the centripetal acceleration of the cart would be

(1) halved (3) quartered
(2) doubled (4) quadrupled

92. If the speed of the cart was doubled, the magnitude of the centripetal force on the cart would be

(1) halved (3) quartered
(2) doubled (4) quadrupled

93. If the mass of the cart was halved, the magnitude of the centripetal acceleration of the cart would

(1) decrease (2) increase (3) remain the same

94. In the position shown in the diagram, towards which point is the centripetal force acting on the cart directed?

95. In the position shown in the diagram, towards which point is the velocity of the cart directed?

96. Which factor, when doubled, would produce the greatest change in the magnitude of the centripetal force acting on the cart?

(1) mass of the cart
(2) radius of curvature of the path
(3) speed of the cart
(4) weight of the cart

97. As the time taken for a car to make one lap around a circular track decreases, the centripetal acceleration of the car

(1) decreases (2) increases (3) remains the same

98. The tangential acceleration of a cart moving at a constant speed in a horizontal circle is

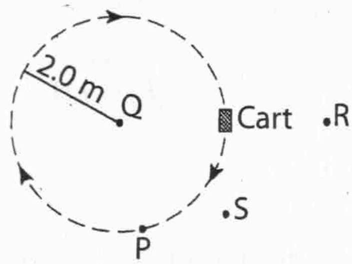
(1) 0.0 m/s^2
(2) 9.8 m/s^2 in the direction of the velocity
(3) constant in magnitude and directed radially toward the center of curvature
(4) constant in magnitude and directed radially away from the center of curvature

99. The centripetal acceleration of a ball of mass *m* moving at constant speed *v* in a horizontal circular path of radius *r* is

(1) zero
(2) constant in direction, but changing in magnitude
(3) constant in magnitude, but changing in direction
(4) changing in both magnitude and direction

Base your answers to questions 100 through 103 on the following information and diagram.

A 5.0-kilogram cart travels clockwise in a horizontal circle of radius 2.0 meters at a constant speed of 4.0 meters per second



100. Towards which point is the velocity of the cart directed at the position shown?

101. Towards which point is the centripetal acceleration of the cart directed at the position shown?

102. If the mass of the cart was doubled, the magnitude of the cart's centripetal acceleration would be

(1) unchanged (3) halved
(2) doubled (4) quadrupled

103. The magnitude of the centripetal force acting on the cart is

(1) 8.0 N (2) 20. N (3) 40. N (4) 50. N

Newton's Universal Law of Gravitation

Every body in the universe exerts a force of attraction on every other body. According to Newton's universal law of gravitation, any two bodies attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The attractive force that one object exerts on another object due to their masses is called **gravitational force,** which is given by this formula.

$$F_g = \frac{Gm_1m_2}{r^2}$$

Ⓡ

In the formula F_g is the gravitational force in newtons, m_1 and m_2 are the masses of the objects in kilograms, r is the distance between the centers of the objects in meters, and G is the universal gravitational constant $6.67 \times 10^{-11}\ \mathrm{N \cdot m^2 / kg^2}$. The universal law of gravitation is valid only for spherical masses of uniform density and masses that are small compared to the distance between their centers.

According to the law, the gravitational force that mass m_1 exerts on mass m_2 is equal in magnitude and opposite in direction to the gravitational force that mass m_2 exerts on mass m_1. If the distance between the centers of the two masses is doubled, the magnitude of the gravitational force is quartered. If one of the two masses is doubled and the distance between their centers remains constant, the magnitude of the gravitational force is doubled.

SAMPLE PROBLEM

Calculate the magnitude of the gravitational force of attraction that Earth exerts on the Moon.

SOLUTION: Identify the known and unknown values. Obtain needed values from the *Reference Tables for Physical Setting/Physics.*

<u>Known</u>

$m_{Earth} = 5.98 \times 10^{24}$ kg

$m_{Moon} = 7.35 \times 10^{22}$ kg

$r_{Earth\ to\ Moon} = 3.84 \times 10^{8}$ m

$G = 6.67 \times 10^{-11}\ \mathrm{N \bullet m^2 / kg^2}$

<u>Unknown</u>

$F_g = ?$ N

1. Write the formula for the gravitational force.

$$F_g = \frac{Gm_1m_2}{r^2}$$

2. Substitute the known values and solve.

$$F_g = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})(7.35 \times 10^{22}\ \mathrm{kg})}{(3.84 \times 10^{8}\ \mathrm{m})^2}$$

$$F_g = 1.99 \times 10^{20}\ \mathrm{N}$$

Gravitational Field Strength A region in space where a test particle would experience a gravitational force is called a **gravitational field.** Every mass is surrounded by a gravitational field. A unit test mass is used to map a gravitational field, such as the one that surrounds Earth.

Figure 2-17A shows gravitational force vectors associated with a test mass at various locations above Earth's surface. The direction of the vectors indicates that the test mass is always attracted to Earth, and the magnitude of the vectors indicates that the force on the test mass increases as it gets closer to Earth. In Figure 2-17B, the force vectors have been joined to form lines of gravitational force. The imaginary line along which a test mass would move in a gravitational field is called a line of gravitational force.

Impulse and Change in Momentum

The product of the net force acting on an object and the time during which the force acts is called **impulse.** Impulse, a vector quantity having the same direction as the net force, is given by this formula.

$$J = F_{net}t$$ Ⓡ

The average force F is in newtons, t is the time during which the force acts in seconds, and J is the impulse in newton · seconds. The SI unit for impulse is the newton · second, N · s.

The impulse imparted to an object can also be determined graphically. A horizontal force of varying magnitude is applied over time to an object on a horizontal surface and a graph of force versus time is plotted. The area under the line equals the impulse imparted to the object.

According to Newton's second law an unbalanced force acting on an object causes it to accelerate. This acceleration produces a change in the object's velocity and consequently its momentum, as shown by the following equations.

$$F_{net} = ma = m\frac{\Delta v}{t} \text{ or } F_{net}t = m\Delta v$$

Because $F_{net}t$ equals the impulse and $m\Delta v$ equals the change in momentum, it follows that

$$J = F_{net}t = \Delta p$$ Ⓡ

The direction of the impulse imparted to an object is the same as the direction of the object's change in momentum. If an object is in equilibrium, there is no change in its momentum and, thus, no impulse imparted to it.

SAMPLE PROBLEM

A 5.0-kilogram object has an initial velocity of 8.0 meters per second due east. An unbalanced force acts on the object for 3.0 seconds, causing its velocity to decrease to 2.0 meters per second east. Calculate the magnitude and direction of the unbalanced force.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$m = 5.0$ kg	F_{net} = ? N to the ?
$v_i = 8.0$ m/s east	
$v_f = 2.0$ m/s east	
$t = 3.0$ s	

1. Write the formula relating impulse and change in momentum and the formula for momentum.

   $J = F_{net}t = \Delta p$ and $p = mv$

2. Combine the formulas.

   $F_{net}t = m\Delta v$

3. Solve the equation for F.

   $$F_{net} = \frac{m\Delta v}{t} = \frac{m(v_f - v_i)}{t}$$

4. Substitute the known values and solve.

   $$F_{net} = \frac{(5.0\text{ kg})(2.0\text{ m/s} - 8.0\text{ m/s})}{3.0\text{ s}} = \frac{-30.\text{ kg}\cdot\text{m/s}}{3.0\text{ s}}$$

   $F_{net} = -10.\text{ kg} \bullet \text{m/s}^2 = -10.\text{ N}$

   The force is 10. N directed to the west.

In baseball, both the batter hitting the ball and the outfielder catching the ball are aware of the relationship between impulse and momentum. The batter "follows through" to keep the bat in contact with the ball as long as possible. The greater the time during which the force of impact acts on the ball, the larger the impulse imparted to it, the greater its final momentum, and the longer the distance of travel. The outfielder catching the ball tries to prolong the time of slowing the ball by moving the gloved hand back in the direction of the ball's motion. By increasing the time during which the gloved hand acts on the ball to reduce its momentum to zero, the force needed to produce the necessary impulse and the "sting" are reduced.

Conservation of Momentum

A group of objects, not acted upon by any external force, is called a **closed system.** According to Newton's third law, within such a system the force F exerted by one mass m_1 in the system on a second mass m_2 must be equal in magnitude and opposite in direction to the force that m_2 exerts on m_1. Because the force F acts on both masses for exactly the same amount of time, the magnitude of the impulse on each mass is the same. Consequently, the change in momentum for each mass has the same magnitude, but they are in opposite directions. The relationship is expressed in this way.

$$m_1 \Delta v_1 = -m_2 \Delta v_2, \text{ or } m_1 \Delta v_1 + m_2 \Delta v_2 = 0$$

The total change in momentum due to the interaction of masses m_1 and m_2 is zero. This relationship is summed up in the **law of conservation of momentum** which states that the total momentum of the objects in a closed system is constant. The law is given by this formula.

$$p_{before} = p_{after}$$

Momentum p is in kilogram • meters per second, kg • m/s.

SAMPLE PROBLEM

A 1.0-kilogram cart A is initially at rest on a horizontal frictionless air track. A 0.20-kilogram cart B is moving to the right at 10.0 meters per second on the same track. Cart B collides with cart A causing cart A to move to the right at 3.0 meters per second. Calculate the velocity of cart B after the collision.

SOLUTION: Identify the known and unknown values. Let velocity to the right be positive.

Known	Unknown
$m_A = 1.0$ kg	$v_{B_f} = ?$ m/s to the ?
$m_B = 0.20$ kg	
$v_{A_i} = 0.0$ m/s	
$v_{B_i} = 10.0$ m/s	
$v_{A_f} = 3.0$ m/s	

1. Write the formula that equates the momentum of the system before and after the collision.

$p_{after} = p_{before}$

$m_A v_{A_f} + m_B v_{B_f} = m_A v_{A_i} + m_B v_{B_i}$

2. Solve the equation for v_{B_f}.

$$v_{B_f} = \frac{m_A v_{A_i} + m_B v_{B_i} - m_A v_{A_f}}{m_B}$$

3. Substitute the known values and solve.

$$v_{B_f} = \frac{(1.0 \text{ kg})(0.0 \text{ m/s}) + (0.20 \text{ kg})(10.0 \text{ m/s}) - (1.0 \text{ kg})(3.0 \text{ m/s})}{0.20 \text{ kg}}$$

$v_{B_f} = -5.0$ m/s

The velocity of cart B after the collision is 5.0 m/s to the left.