TOPIC 2

Mechanics

Vocabulary

acceleration
centripetal acceleration
centripetal force
closed system
coefficient of friction
displacement
distance
equilibrium
free fall
free-body diagram
friction
gravitational field
gravitational field strength
gravitational force
gravity
horizontal component
impulse
inertia
instantaneous velocity
kinetic friction
kilogram
law of conservation of momentum
linear motion
mechanics
meter
momentum
net force
newton
normal force
pendulum
period (of a pendulum)
resolution of forces
resultant
second
speed
static friction
tangent
unbalanced force
uniform circular motion
uniform motion
vacuum
vector component
velocity
vertical component
weight

Kinematics

The branch of physics that deals with forces and the way they produce and change motion is called **mechanics.** Kinematics is the mathematical treatment of the motions of bodies without regard to the forces that produce the motion.

Distance and Displacement

When an object moves from one point to another, it experiences a change in position relative to some arbitrary reference point. **Distance** is the total length of a path that an object travels. Distance is a scalar quantity, which means it has magnitude but not direction. **Displacement** is the change in the position of an object described by a vector that begins at the initial position of the object and ends at its final position. Because displacement is a vector quantity, it has both magnitude and direction. Distance and displacement are usually measured in meters, centimeters, or kilometers. The **meter,** m, is the fundamental SI unit of length.

The following example illustrates the difference between distance and displacement. A car is driven on the NYS Thruway from Buffalo to Albany to New York City. The distance traveled by the car is approximately 418 miles or 673 kilometers. The magnitude of the total displacement of the car, however, is only the length of the vector connecting Buffalo and New York City—approximately 313 miles or 504 kilometers. Two or more displacement vectors can be combined to obtain the vector sum, or resultant, as the following sample problem shows.

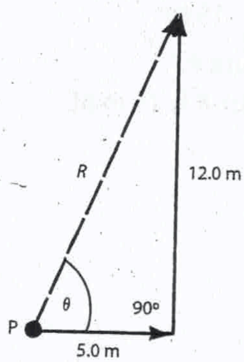
SAMPLE PROBLEM

A student walks 5.0 meters due east and then 12.0 meters due north. Calculate the magnitude and direction of the student's resultant displacement, R.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$d_1 = 5.0$ m east	$R = ?$ m at ?°
$d_2 = 12.0$ m north	

1. Make a sketch of the situation.



Because east and north are perpendicular to each other, the two displacements form the legs of a right triangle. The resultant displacement is the hypotenuse of the triangle.

2. Write the Pythagorean Theorem.

$c^2 = a^2 + b^2$

3. Solve the equation for c.

$c = \sqrt{a^2 + b^2}$

4. Substitute R for c, then substitute the known values and solve.

$R = \sqrt{(5.0\ \text{m})^2 + (12.0\ \text{m})^2} = 13\ \text{m}$

5. Write a trigonometric function to determine θ.

$\tan\theta = \dfrac{\text{side opposite } \angle\theta}{\text{side adjacent to } \angle\theta}$

6. Substitute the known values and solve.

$\tan\theta = \dfrac{12.0\ \text{m}}{5.0\ \text{m}}$

$\theta = 67°$

The resultant is 13 meters at 67° north of east.

Displacements along the same straight line can be combined by simple addition or subtraction to find the resultant. If the successive displacements in the previous problem had been 5.0 meters east and 12.0 meters east, the resultant would have been 17.0 meters east. Also, if the student had walked 5.0 meters east, and then 12.0 meters west, the resultant would have been 7.0 meters west.

When successive displacements are not along the same straight line, the resultant can be found either graphically, by making a scaled vector diagram using a metric ruler and a protractor, or algebraically using the law of cosines and the law of sines. Because the laws of cosines and sines are not provided in the *Reference Tables for Physical Setting/Physics*, this type of algebraic solution is not testable.

Speed and Velocity

The position of an object in motion changes with time. The **speed,** v, of an object is the distance that the object moves in a unit of time. Speed is a scalar quantity. The average speed, $\bar{v}$, of an object is given by this formula

$$\bar{v} = \frac{d}{t}$$ Ⓡ

Distance d is in meters and the time interval t is in seconds. The **second,** s, is the fundamental SI unit of time. Thus, the average speed $\bar{v}$ is in meters per second, or m/s, a derived SI unit. If the object's speed is constant

SAMPLE PROBLEM

A person walks 5.0 meters due east and 12.0 meters at 60.° north of east. Find the magnitude and direction of the person's resultant displacement.

SOLUTION: Identify the known and unknown values.

Known

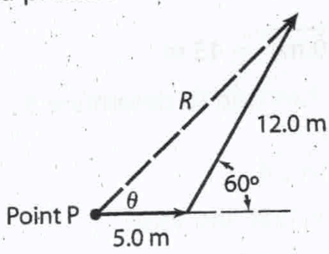
d_1 = 5.0 m east

d_2 = 12.0 m at 60.° north of east

Unknown

R = ? m at ?°

1. Construct a scale drawing. A scale of 1.0 cm = 4.0 m is used, but a scale of 1.0 cm = 2.0 m would provide more accurate results.



Scale: 1.0 cm = 4.0 m

2. Use a ruler to measure the length of the resultant vector.

   Resultant vector R measures 3.75 cm.

3. Use the scale of the drawing to convert R in centimeters to meters.

   R = (3.75 cm)(4.0 m/cm) = 15 m

4. Use a protractor to measure θ.

   θ = 43°. Thus, the resultant R is 15 m at 43° north of east.

during the entire time interval, $\bar{v}$ is its constant speed, and the object is said to be in **uniform motion.** If the speed of the object varies, the motion is nonuniform.

The **velocity** of an object is the time rate of change of its displacement. Velocity is a vector quantity having direction as well as magnitude. The magnitude of an object's velocity is its speed. For example, if one car travels at 88 kilometers per hour due east and a second car travels at 88 kilometers per hour due north, both cars have the same speed. However, the velocities of the cars differ because the direction of travel is not the same. In physics, the terms speed and velocity are not interchangeable.

Linear motion refers to an object's change of position along a straight line. On a straight path, there are only two possible directions for the velocity. One of these is called the positive direction. The opposite direction, then, is the negative direction. Depending upon the direction of the motion, changes in displacement are also positive or negative. When referring to linear motion in this text, the symbol v is used for both velocity and speed, and the symbol d is used for both displacement and distance.

Graphs of Linear Motion Graphs of position versus time are commonly used to represent the linear motion of an object. The independent variable, time, is recorded on the horizontal axis, and the dependent variable, position, is recorded on the vertical axis. Because $\bar{v} = \frac{d}{t}$, the magnitude of the slope of a position versus time graph at any point equals the object's speed at that instant, and the algebraic sign of the slope indicates whether the velocity is in the positive or negative direction. A straight line indicates constant velocity. A straight horizontal line represents zero velocity, that is, an object at rest. If a position-time graph is a curved line, the velocity is not constant. The slope of the tangent to the curve at any point is called the instantaneous velocity of the object. The **tangent** to a curve at any point on the curve is defined as the line passing through the point and having a slope equal to the slope of the curve at that point. **Instantaneous velocity** is the velocity of an object at any particular instant in time. The term is applied to the motion of an object that is not traveling at constant velocity. The steeper the slope of a position versus time graph, the greater the instantaneous speed. Figure 2-1 shows examples of graphs of linear motion.

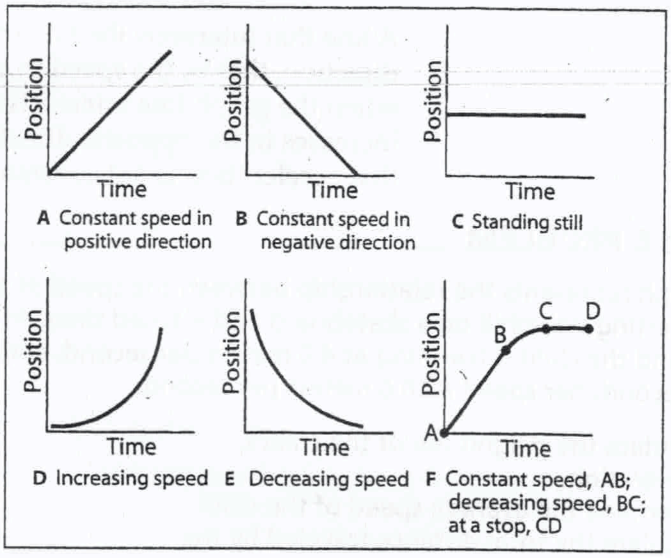


Figure 2-1. Graphs of linear motion, drawn on position-time axes

Acceleration

The time rate of change of velocity is **acceleration,** a, a vector quantity represented by this formula.

$$a = \frac{\Delta v}{t}$$ Ⓡ

The change in velocity Δv is in meters per second and t is the time interval in seconds. Thus, acceleration can be expressed with the unit meters per second per second, or meters per second^2, m/s^2.

Note that the formula, as written without a bar over the a to indicate average, implies constant, or uniform, acceleration. This text does not address nonuniform acceleration.

The average speed $\overline{v}$ of an object accelerating uniformly from an initial speed v_i to a final speed v_f is given by this formula.

$$\overline{v} = \frac{v_i + v_f}{2}$$

This formula is valid only when the acceleration is constant. This formula does not appear on the *Reference Tables for Physical Setting/Physics*, but it is acceptable to use where appropriate.

Velocity versus time graphs can be used to represent accelerated linear motion, as shown in Figure 2-2. The independent variable, time, is measured on the horizontal axis, and the dependent variable, velocity, is recorded on the vertical axis. Because $a = \frac{\Delta v}{t}$, the magnitude of the slope of a velocity versus time graph at any point equals the object's acceleration at that instant, and the algebraic sign of the slope indicates whether the acceleration is in the positive or negative direction. For example, a horizontal line with zero slope indicates constant speed or no acceleration. A straight line with a positive slope shows increasing speed or constant acceleration. A straight line with negative slope shows decreasing speed or constant negative acceleration (deceleration).

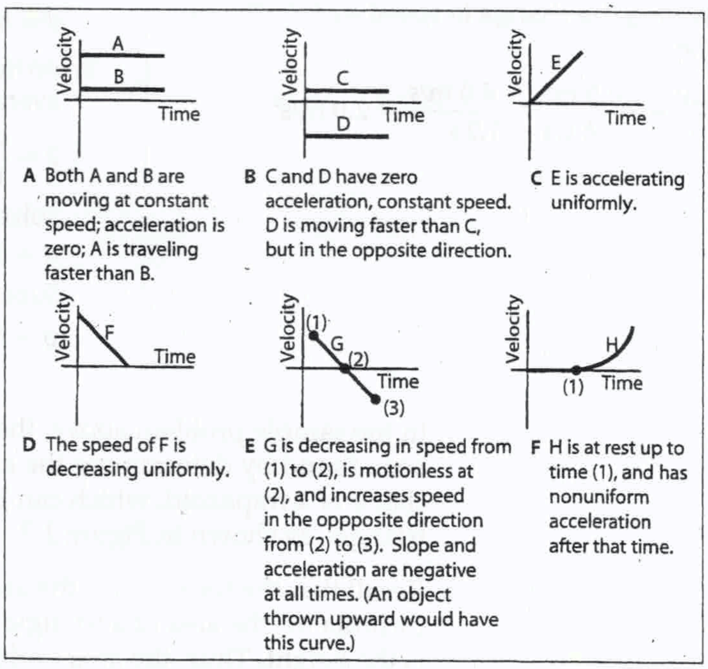


Figure 2-2. Graphs of various types of motion in a straight line path, drawn on velocity-time axes

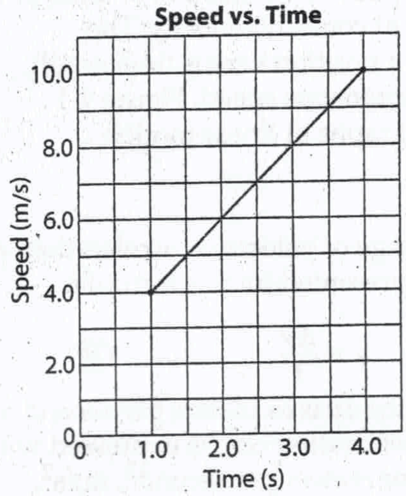
A line that intersects the horizontal time axis indicates a change in direction, that is, the speed in one direction decreases to zero at the time when the graph line intersects the horizontal axis, and then the speed increases in the opposite direction. A curved velocity-time line indicates that acceleration is not constant.

SAMPLE PROBLEM

The graph represents the relationship between the speed of a child coasting downhill on a skateboard and elapsed time. At 1.0 second the child is traveling at 4.0 meters per second, and at 4.0 seconds her speed is 10.0 meters per second.

(a) Calculate the magnitude of the child's acceleration.
(b) Determine the average speed of the child.
(c) Calculate the total distance traveled by the child during this 3.0-second interval.

Speed vs. Time



SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$t_i = 1.0$ s	$a = ?$ m/s^2
$t_f = 4.0$ s	$\overline{v} = ?$ m/s
$v_i = 4.0$ m/s	$d = ?$ m
$v_f = 10.0$ m/s	

1. The slope of the graph is the magnitude of the child's acceleration. Determine the slope by dividing the change in speed by the change in time.

$$a = \frac{\Delta v}{t} = \frac{10.0 \text{ m/s} - 4.0 \text{ m/s}}{4.0 \text{ s} - 1.0 \text{ s}} = 2.0 \text{ m/s}^2$$

2. Write the formula for average speed.

$$\overline{v} = \frac{v_i + v_f}{2}$$

Substitute the known values and solve.

$$\overline{v} = \frac{4.0 \text{ m/s} + 10.0 \text{ m/s}}{2} = 7.00 \text{ m/s}$$

The average speed is the vertical coordinate of the midpoint of the graphed line segment.

3. Write the formula that relates distance, average speed, and time.

$$\overline{v} = \frac{d}{t}$$

Solve the equation for d.

$$d = \overline{v}t$$

Substitute the known values and solve.

$$d = (7.00 \text{ m/s})(3.0 \text{ s}) = 21 \text{ m}$$

In the sample problem above, the distance traveled by the child could have been found by determining the area under the graph line. That area has the shape of a trapezoid, which can be separated into a rectangle and a triangle, as shown in Figure 2-3.

Recall that the formula for the area of a rectangle is $A = bh$ and that the formula for the area of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height. Thus, the area under the line is the sum of the area of the rectangle and the area of the triangle. For the purpose of this calculation, these quantities may be represented as meters. Ⓡ

That is,

$$A_{rectangle} = bh = (3.0 \text{ s})(4.0 \text{ m/s}) = 12 \text{ m}$$

$$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}(3.0\text{s})(6.0 \text{ m/s}) = 9.0 \text{ m}$$

$$A_{total} = 12 \text{ m} + 9.0 \text{ m} = 21 \text{ m}$$

The total distance is 21 meters, as calculated in the sample problem. Thus, the physical significance of the area under the line of a speed versus time graph is the distance traveled.

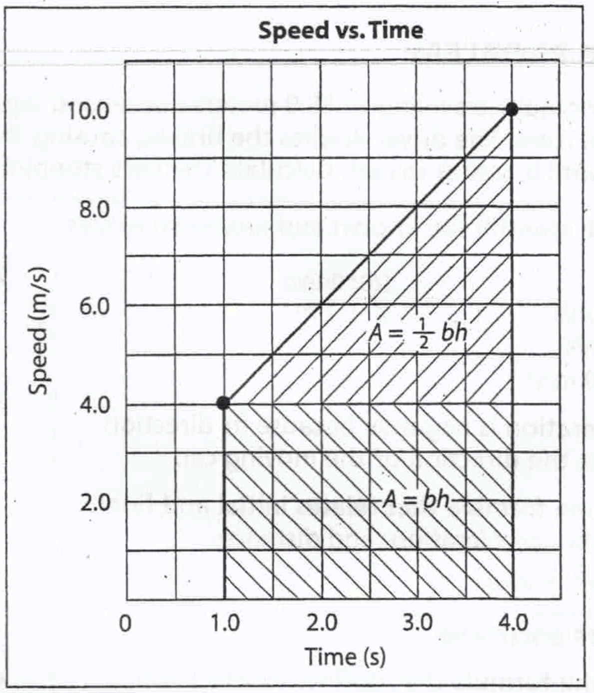


Figure 2-3. The area under a speed vs. time graph: **The magnitude of the area of the rectangle is 12 m and the magnitude of the area of the triangle is 9 m. The magnitude of the total area is 21 m, which is the total distance in meters traveled by the child.**

Final Velocity and Distance Traveled During Constant Acceleration

Acceleration is defined by the formula $a = \frac{\Delta v}{t}$. Because Δ always represents a change in a variable, that is, final conditions minus initial conditions, it follows that $a = \frac{v_f - v_i}{t}$. Solving for the final speed v_f yields this formula.

$$v_f = v_i + at \quad \text{Ⓡ}$$

This expression can be combined with $d = \bar{v}t$ and $\bar{v} = \frac{v_i + v_f}{2}$ to obtain a useful expression for displacement d that involves initial velocity v_i, the acceleration a, and time t.

$$d = \bar{v}t = \left(\frac{v_i + v_f}{2}\right)t = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(v_i + v_i + at)t$$

Thus, the equation becomes the following.

$$d = v_i t + \frac{1}{2}at^2 \quad \text{Ⓡ}$$

This formula is valid only when acceleration is constant.

The velocity of an object as a function of its displacement can be determined without knowing the elapsed time. From the previous derivation, the following equation can be written.

$$d = \frac{1}{2}(v_i + v_f)t$$

Solving the equation $v_f = v_i + at$ for t yields

$$t = \frac{v_f - v_i}{a}$$

Combining these expressions yields

$$d = \frac{1}{2}(v_f + v_i)\left(\frac{v_f - v_i}{a}\right) = \frac{v_f^2 - v_i^2}{2a}$$

Solving for the final velocity yields

$$v_f^2 = v_i^2 + 2ad \quad \text{Ⓡ}$$

This formula is valid only for constant acceleration.

In many problems involving motion, the object is initially at rest and v_i is zero. In such cases, terms containing v_i drop out of the motion formulas

SAMPLE PROBLEM

A car is originally traveling at 15.0 meters per second (approximately 34 miles per hour) on a straight, horizontal road. The driver applies the brakes, causing the car to decelerate uniformly at 4.00 meters per second² until it comes to rest. Calculate the car's stopping distance.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$v_i = 15.0 \text{ m/s}$	$d = ?\text{ m}$
$v_f = 0.0 \text{ m/s}$	
$a = -4.00 \text{ m/s}^2$	

The acceleration is negative because its direction is opposite the direction of the moving car.

1. Write the formula that relates initial and final velocities, acceleration, and distance.

$$v_f^2 = v_i^2 + 2ad$$

2. Solve the equation for distance, d.

$$d = \frac{v_f^2 - v_i^2}{2a}$$

3. Substitute the known values and solve.

$$d = \frac{(0.0 \text{ m/s})^2 - (15.0 \text{ m/s})^2}{2(-4.00 \text{ m/s}^2)}$$

$$d = \frac{-225 \text{ m}^2/\text{s}^2}{-8.00 \text{ m/s}^2} = 28.1 \text{ m}$$

ALTERNATE SOLUTION:

1. Write the formula that defines acceleration

$$a = \frac{\Delta v}{t}.$$

2. Solve the equation for t.

$$t = \frac{\Delta v}{a}$$

3. Substitute the known values and solve.

$$t = \frac{0.0 \text{ m/s} - 15.0 \text{ m/s}}{-4.00 \text{ m/s}^2} = 3.75 \text{ s}$$

4. Write the formula that relates distance, initial velocity, acceleration, and time.

$$d = v_i t + \frac{1}{2}at^2$$

5. Substitute the known values and solve for d.

$$d = (15.0 \text{ m/s})(3.75 \text{ s}) + \tfrac{1}{2}(-4.00 \text{ m/s}^2)(3.75 \text{ s})^2$$

$$d = 56.3 \text{ m} + (-28.1 \text{ m}) = 28.2 \text{ m}$$

If the initial speed was doubled to 30.0 meters per second (or 67 miles per hour), the stopping distance would quadruple!

and the equations are simplified. In addition, the symbol for v_f can be written as v. Thus, for objects starting from rest and accelerating uniformly,

$$a = \frac{v}{t} \qquad d = \frac{1}{2}at^2 \qquad \overline{v} = \frac{v}{2} \qquad v^2 = 2ad$$

Freely Falling Objects

In a **vacuum,** which is a space in which there is no matter, a coin and a feather fall with the same acceleration due to gravity, g. **Gravity** is the force between the mass of Earth and the mass of any object in the vicinity of Earth. According to the *Reference Tables for Physical Setting/Physics,* near the surface of Earth g is a constant 9.81 meters per second². The ideal falling motion of an object acted upon only by the force of gravity is called **free fall.** (R)

Although the acceleration due to gravity is the same for all objects in a vacuum, in air the acceleration of the feather is less than that of the coin because the shape and exposed area of the feather result in greater air resistance.

If an object falls freely from rest (air resistance is neglected), its speed and position at any instant in time are given by $v = gt$ and $d = \frac{1}{2}gt^2$. Table 2-1 shows the speed and distance traveled by an object falling freely from rest near Earth's surface in the absence of air resistance.

Table 2-1. Free Fall of an Object Starting from Rest

Time of fall (s)	Speed (m/s)	Distance traveled (m)
0.00	0.00	0.00
1.00	9.81	4.91
2.00	19.6	19.6
3.00	29.4	44.1
4.00	39.2	78.5
5.00	49.1	123

Review Questions

1. If a boy runs 125 meters north, and then 75 meters south, his total displacement is
(1) 50. m north
(2) 50. m south
(3) 200. m north
(4) 200. m south

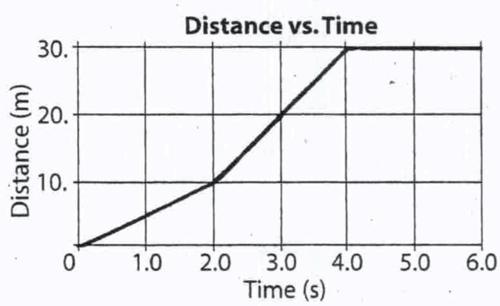
2. A student walks 3 blocks south, 4 blocks west, and 3 blocks north. What is the resultant displacement of the student?
(1) 10. blocks east
(2) 10. blocks west
(3) 4 blocks east
(4) 4 blocks west

3. A girl attempts to swim directly across a stream 15 meters wide. When she reaches the other side, she is 15 meters downstream. Calculate the magnitude of her displacement.

4. What is the average speed of an object that travels 6.00 meters north in 2.00 seconds and then travels 3.00 meters east in 1.00 second?
(1) 9.00 m/s
(2) 0.333 m/s
(3) 3.00 m/s
(4) 4.24 m/s

5. In a 4.0-kilometer race, a runner completes the first kilometer in 5.9 minutes, the second kilometer in 6.2 minutes, the third kilometer in 6.3 minutes, and the final kilometer in 6.0 minutes. The average speed of the runner for the race is approximately
(1) 0.16 km/min
(2) 0.33 km/min
(3) 12 km/min
(4) 24 km/min

6. The graph below shows the relationship between the position of an object moving in a straight line and elapsed time.



What is the speed of the object during the time interval $t = 2.0$ seconds to $t = 4.0$ seconds?
(1) 0.0 m/s (2) 5.0 m/s (3) 7.5 m/s (4) 10. m/s

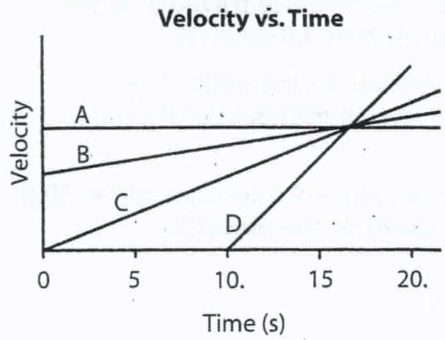
7. A particle is accelerated uniformly from rest to a speed of 50. meters per second in 5.0 seconds. The average speed of the particle during this 5.0-second time interval is
(1) 5.0 m/s (2) 10. m/s (3) 25 m/s (4) 50. m/s

8. Which statement best describes the movement of an object with zero acceleration?
(1) The object must be at rest.
(2) The object must be slowing down.
(3) The object may be speeding up.
(4) The object may be in motion.

9. A particle has a constant acceleration of 2.0 meters per second^2. Calculate the time required for the particle to accelerate from 8.0 meters per second to 28 meters per second.

10. If an object is traveling east with a decreasing speed, the direction of the object's acceleration is
(1) north (2) south (3) east (4) west

Base your answers to questions 11 and 12 on the following graph, which represents the relationship between velocity and time of travel for four cars, A, B, C, and D, in straight-line motion.



11. Which car has the greatest acceleration during the time interval 10. seconds to 15 seconds?

12. Which car travels the greatest distance during the time interval 0 second to 10. seconds?
(1) A only
(2) B only
(3) C only
(4) The distance traveled is the same for cars A, B, and C.

13. Starting from rest, an object rolls freely down a 10.-meter long incline in 2.0 seconds. The acceleration of the object is
(1) 5.0 m/s
(2) 5.0 m/s^2
(3) 10. m/s
(4) 10. m/s^2

14. A car accelerates uniformly from rest at 3.2 meters per second^2. Calculate the speed of the car when it has traveled a distance of 40. meters.