63. A person does 100 joules of work in pulling back the string of a bow. What is the initial speed of a 0.5-kilogram arrow when it is fired from the bow?

(1) 20 m/s
(2) 50 m/s
(3) 200 m/s
(4) 400 m/s

64. An 8.0-kilogram object and a 4.0-kilogram object are released simultaneously from a height of 50. meters above the ground. After falling freely for 2.0 seconds, the objects have different

(1) accelerations
(2) speeds
(3) kinetic energies
(4) displacements

65. The work done in raising an object must result in an increase in the object's

(1) internal energy
(2) kinetic energy
(3) gravitational potential energy
(4) elastic potential energy

66. Two cars having different weights are traveling on a level surface at different constant velocities. Within the same time interval, greater force is always required to stop the car that has the greater

(1) weight
(2) kinetic energy
(3) velocity
(4) momentum

Work-Energy Relationship

If there is no friction, all the work done in lifting an object to a new height is equal to the object's increase in gravitational potential energy. The change in potential energy depends only on the change in height, not on the path taken. For example, the work done in lifting a 10.0-kilogram box from the floor to a 0.92-meter high tabletop is equal to the box's change in gravitational potential energy.

$$W = \Delta PE = mg\Delta h = (10.0\text{ kg})(9.81\text{ m/s}^2)(0.92\text{ m}) = 90.\text{ J}$$

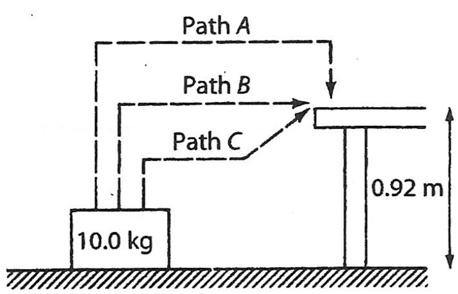


Figure 3-1. A conservative force: Because the force of gravity is a conservative force, the same amount of work is done when raising the box from the floor to the tabletop regardless of which path is followed.

Figure 3-1 shows that the work done in moving the box from the floor to the tabletop is the same regardless of the path taken. When work done against a force is independent of the path taken, the force is said to be a conservative force. The force of gravity is an example of a conservative force. The elastic force of a spring is also a conservative force. Potential energy has meaning only in relation to work done against conservative forces.

Air resistance and friction are examples of nonconservative forces. The work done against a nonconservative force is dependent upon the path taken. In Figure 3-2, the same box is moved from the floor to the tabletop by sliding it along an inclined plane A. Once again, 90. joules of work is done to change the gravitational potential energy of the box, but because additional work must be done against friction, the total work done is greater than 90. joules.

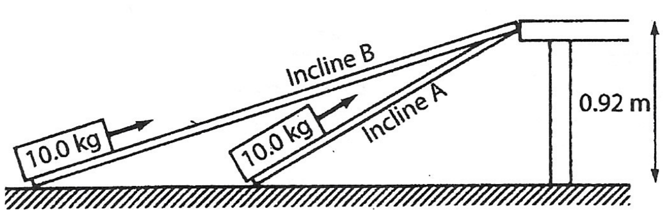


Figure 3-2. A nonconservative force: Because friction is a nonconservative force, moving the box from the floor to the tabletop requires more work on incline B than on incline A. In this case, the path makes a difference in the amount of work required. (Read the explanation in the text.)

If inclined plane B is used instead of inclined plane A, the work done against friction, $W_f = F_f d$, is greater, even though the coefficient of friction is the same for both planes. The force of friction F_f is greater when a plane is inclined at a smaller angle because the normal force F_N for the same object on the incline is larger and $F_f = \mu F_N$. In addition, the frictional force acts over a greater distance on incline B. Because friction is a nonconservative force, the work required to raise the box from the floor to the top of the table on incline B is greater than the work required to raise it on incline A.

Conservation of Energy

A closed system is one in which there are no external forces doing work on the system, no external work being done by the system, and no transfer of energy into or out of the system. In a closed system, the sum of the potential energy (gravitational and/or elastic), kinetic energy, and internal energy remains constant. Although the energy within a closed system may be transformed from one type to another, the total energy of the system always remains the same. These ideas are expressed in the **law of conservation of energy,** which states that energy cannot be created or destroyed. In other words, the sum of the *changes* in energy (potential, kinetic, and internal) within a closed system is zero.

Ideal Mechanical Systems

The sum of the kinetic and potential energies in a system is called the total **mechanical energy.** An **ideal mechanical system** is a closed system in which no friction or other nonconservative force acts. In an ideal mechanical system, the sum of the kinetic and potential energies is constant, or the sum of the *changes* in kinetic and potential energy is zero.

The relationship between the gravitational potential energy and kinetic energy for an ideal simple pendulum is shown in Figure 3-3. A **simple pendulum** consists of a mass (bob) attached to one end of a string or wire that is attached at the other end to a pivot point.

An object falling freely from rest in a vacuum is another example of an ideal mechanical system. If a stationary object having mass m is located a vertical distance h above Earth's surface, the object has initial gravitational potential energy, $PE_i = mgh$ with respect to Earth and kinetic energy, $KE_i = 0$. As the object falls, its gravitational potential energy decreases, but because its speed increases, the object's kinetic

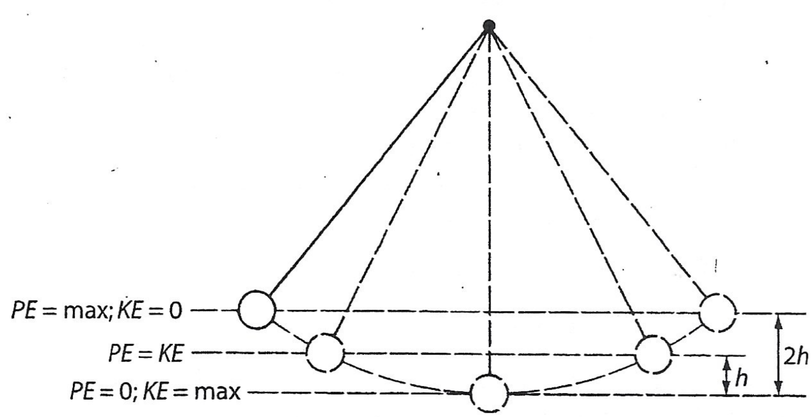


Figure 3-3. The relationship between gravitational potential energy with respect to the lowest point and kinetic energy for an ideal simple pendulum

energy increases. These energy changes can be expressed by the law of conservation of energy:

$$\Delta PE + \Delta KE = 0$$

or

$$\Delta KE = -\Delta PE$$

As the object falls from rest, its change in gravitational potential energy is given by $\Delta PE = -mgh$, and its change in kinetic energy is $\Delta KE = \frac{1}{2}mv^2$. These expressions can be substituted into the previous equations:

$$\frac{1}{2}mv^2 - mgh = 0$$

or

$$\frac{1}{2}mv^2 = mgh$$

The common factor, m, can be eliminated:

$$\frac{1}{2}v^2 = gh, \text{ so } v^2 = 2gh \text{ and } v = \sqrt{2gh}$$

The acceleration due to gravity, g, can be considered constant near Earth's surface, so the last equation can be used to determine the speed of an object falling from rest from a known height. Note that the speed of the object is independent of its mass.

Nonideal Mechanical Systems

When a system is acted upon by a nonconservative force, such as friction, it is called a **nonideal mechanical system.** In reality, friction opposes the motion of two objects in contact with each other and moving relative to each other. Frictional force converts some or all of the kinetic energy of a moving object into internal energy, that is, potential or kinetic energy of the individual particles that comprise the object. The "lost" kinetic energy usually appears as an increase in temperature of the objects in contact. For example, a simple pendulum set in motion in air does not swing back to its original release point. The pendulum experiences both friction at the pivot point and air resistance. A piece of paper dropped to the ground from some height has more initial gravitational potential energy with respect to the ground than it has kinetic energy at the instant it reaches the ground. A lead sphere dropped from some height onto a steel surface does not bounce; all of its initial gravitational potential energy with respect to the steel surface is converted into internal energy when it hits the steel.

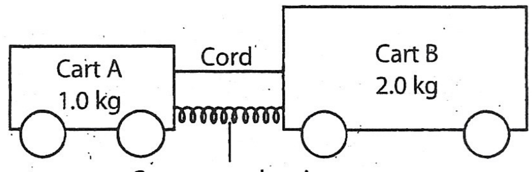
The **total energy** of a nonideal system is given by this formula:

$$E_T = PE + KE + Q$$ Ⓡ

E_T represents the total energy, PE is potential energy, KE is kinetic energy, and Q is internal energy. All quantities are expressed in joules.

SAMPLE PROBLEM

A 1.0-kilogram cart A and a 2.0-kilogram cart B are at rest on a frictionless table, as shown in the diagram. A cord and a spring of negligible mass join the two carts. The spring is compressed 0.060 meter between the two carts until the elastic's potential energy stored in the spring is 12 joules. When the cord is cut, the spring will force the carts apart.



(1) Determine the total amount of work done in compressing the spring.
(2) Calculate the spring constant for the spring.
(3) Calculate the magnitude of the average force required to compress the spring 0.060 meter.
(4) Compare the following quantities while the spring is pushing the carts apart:
  (a) the forces acting on the two carts
  (b) the change in momentum of the two carts
  (c) the total initial and final momentum of the two carts
  (d) the acceleration of the two carts
(5) Calculate the final velocity of cart A.
(6) Determine the ratio of the maximum kinetic energy of cart A to the maximum kinetic energy of cart B.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$m_A = 1.0$ kg	$W = ?$ J
$m_B = 2.0$ kg	$k = ?$ N/m
$x = 0.060$ m	$F_s = ?$ N
$PE_s = 12$ J	$v_{f_A} = ?$ m/s

1. The work done in compressing the spring is equal to the potential energy stored in the spring.

$W = PE_s = 12$ J

2. Write the formula for the potential energy of a spring.

$$PE_s = \frac{1}{2}kx^2$$

Solve the equation for k.

$$k = \frac{2PE_s}{x^2}$$

Substitute the known values and solve.

$$k = \frac{2(12\text{ J})}{(0.060\text{ m})^2}$$

$$k = 6.7 \times 10^3\text{ N/m}$$

3. Write the formula for the average force needed to compress the spring.

$$F_s = kx$$

Substitute the known values and solve.

$$F_s = (6.7 \times 10^3\text{ N/m})(0.060\text{ m})$$

$$F_s = 4.0 \times 10^2\text{ N}$$

4. The forces are equal in magnitude and opposite in direction.

Momentum must be conserved. Thus, the change in momentum is equal in magnitude and opposite in direction for the two carts at all times.

The total momentum is zero at all times, because the carts were initially at rest.

The forces on the two carts are equal in magnitude and the mass of A is one half the mass of B. Thus, the acceleration of cart A is twice that of cart B and opposite in direction.

5. Write an equation for the relationship between the initial and final momentum of the system. Because momentum must be conserved, the initial momentum of the system, which is zero, must equal the final momentum.

$p_{before} = p_{after} = 0$

Write this equality in terms of mass and velocity.

$m_A v_A + m_B v_B = 0$

Solve the equation for v_B.

$m_B v_B = -m_A v_A$

$$v_B = -\frac{m_A v_A}{m_B}$$

Substitute known values and solve.

$$v_B = -\frac{(1.0\ \text{kg})v_A}{2.0\ \text{kg}}$$

$$v_B = -\frac{1}{2}v_A$$

Recognizing that energy is conserved, write an equation that equates the total initial energy of the system and the total final energy of the system.

$PE_i + KE_i + PE_{s_i} = PE_f + KE_f + PE_{s_f}$

$PE_i = PE_f$, $PE_{s_f} = 0$, and $KE_i = 0$

Thus, because energy is conserved, the final kinetic energy of the two carts equals the initial potential energy of the spring.

$PE_{s_i} = KE_f$

Write an equation in terms of mass and velocity that states this relationship.

$$PE_{s_i} = \frac{1}{2}m_A(v_A)^2 + \frac{1}{2}m_B(v_B)^2$$

Substitute known values and solve for v_A.

$$12\ \text{J} = \frac{1}{2}(1.0\ \text{kg})(v_A)^2 + \frac{1}{2}(2.0\ \text{kg})\left(-\frac{v_A}{2}\right)^2$$

$$12\ \text{J} = (0.50\ \text{kg})(v_A)^2 + (1.0\ \text{kg})\left(\frac{v_A^2}{4}\right)$$

$$12\ \text{J} = (0.75\ \text{kg})v_A^2$$

$$v_A^2 = 16\ \text{J/kg} = 16\ \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{kg}}$$

$$v_A = 4.0\ \text{m/s}$$

6. It has already been determined that the speed of cart B is one-half that of cart A, thus

$$\frac{KE_A}{KE_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{\frac{1}{2}(1.0\ \text{kg})(4.0\ \text{m/s})^2}{\frac{1}{2}(2.0\ \text{kg})(2.0\ \text{m/s})^2} = \frac{2}{1}$$

Review Questions

67. As the speed of an object falling toward Earth increases, the gravitational potential energy of the object with respect to Earth

(1) decreases
(2) increases
(3) remains the same

68. At what point in its fall does the kinetic energy of a freely falling object equal its gravitational potential energy with respect to the ground?

(1) at the start of the fall
(2) halfway between the start and the end
(3) at the end of the fall
(4) at all points during the fall

69. A 2.0-kilogram mass falls freely for 10. meters near the surface of Earth. The total kinetic energy gained by the object during its free fall is approximately

(1) 400 J (2) 200 J (3) 100 J (4) 50 J

70. A 20.0-kilogram object falls freely from rest and strikes the ground with 1,962 joules of kinetic energy. Calculate how far above the ground the object was when it was released.

71. A 1.0-kilogram mass gains kinetic energy as it falls freely from rest a vertical distance d. How far would a 2.0-kilogram mass have to fall freely from rest to gain the same amount of kinetic energy?

(1) d (2) $2d$ (3) $\frac{d}{2}$ (4) $\frac{d}{4}$

72. A basketball player, who weighs 600 newtons, jumps 0.5 meter vertically off the floor. Calculate her kinetic energy just before hitting the floor.

73. As an object falls freely in a vacuum, the total mechanical energy of the object

(1) decreases
(2) increases
(3) remains the same