Measurement and Mathematics

TOPIC 1

How Scientists Study Measurement and Mathematics

Are angles always measured in degrees?

A protractor is a device used to measure the size of an angle in degrees. A protractor that is a semicircle has a range from 0° to 180°, because a circle has 360°. The reason there are 360° in a circle may be related to some ancient calendars using 360 days for a year. Early astronomers noticed that the stars seemed to move $1/360^{th}$ of a circular path each night. Long before there were calculators, it was known that 360 was divisible by every number from 1 to 10 except 7, and that it had a total of 24 factors.

In higher levels of physics and mathematics, angles are often measured in radians. The radian measure of an angle has no dimensions. A full circle is equal to 2π radians, making 1 radian equal to $360°/2\pi$ or approximately 57°.

When using your calculator for angle calculations in this text and on the Regents examination, be sure to confirm that your calculator is in the correct mode. For example, the sine of 20.0 degrees is 0.342 and the sine of 20.0 radians is 0.912.

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Vocabulary

absolute error	fundamental unit	scalar
accepted value	independent variable	scientific notation
accurate	indirect squared proportion	SI prefix
constant proportion	inversely proportional	SI system
dependent variable	line of best fit	significant figures
derived unit	mass	slope
direct squared proportion	mean	standard deviation
directly proportional	percent error	unit
experimental value	precise	variance
extrapolation	range	vector
force		

Topic Overview

Note to student: *This topic explains some of the process skills based on Standards 1, 2, 6, and 7 that you will use in the study of physics. These skills will be applied to specific content in later topics. Material in this topic will not be tested as definitions or on a purely mathematical basis. However, these concepts and process skills are testable when they are incorporated into specific physics content based on Standard 4.*

Physics is based on observations and measurements of the physical world. Consequently, scientists have developed tools for measurement and adopted standard conventions for describing natural phenomena. These conventions are reviewed below.

Units

A **unit** is a standard quantity with which other similar quantities can be compared. All measurements must be made with respect to some standard quantity. For example, it makes no sense to say the distance between two cities is 26. Distance must be stated in terms of a standard unit. The distance between the cities might be 26 miles or 26 kilometers.

The SI System

The **SI system** provides standardized units for scientific measurements. All quantities measured by physicists can be expressed in terms of the seven **fundamental units** listed in Table 1-1. **Derived units** are combinations of two or more of the fundamental units and are used to simplify notation. Other systems of units are sometimes used when they are more appropriate because of the size of the quantity being measured.

Table 1-1. Units of Measure			
Kind of Unit	**Quantity Being Measured**	**Name of Unit**	**Symbol**
Fundamental SI	length	meter	m
	mass	kilogram	kg
	time	second	s
	electric current	ampere	A
	temperature	kelvin	K
	amount of substance*	mole	mol
	luminous intensity*	candela	cd
Derived SI	frequency	hertz	Hz
	force	newton	N
	energy, work	joule	J
	quantity of electric charge	coulomb	C
	electric potential, potential difference	volt	V
	power	watt	W
	electrical resistance	ohm	Ω
	resistivity	ohm · meter	Ω · m
Non-SI	length	centimeter	cm
	mass	gram	g
	mass	universal mass unit	u
	time	hour	h
	energy, work	electronvolt	eV
	angle size	degree	°
*These quantities are not treated in this review.			

SI Prefixes

SI prefixes are prefixes combined with SI base units to form new units that are larger or smaller than the base units by a multiple or submultiple of 10. The symbol for the new unit consists of the symbol for the prefix followed by the symbol for the base unit. Table 1-2 lists some common SI prefixes. For example, 1000 meters can be expressed as 1 kilometer or 1 km, and 0.01 meter can be expressed as 1 centimeter or 1 cm.

Table 1-2. Prefixes for Powers of 10		
Prefix	**Symbol**	**Notation**
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}

Symbols for Units and Quantities Symbols for SI units are printed in normal type. For example, m is the symbol for meter, and A is the symbol for ampere. Letter symbols are also used for the names of quantities in formulas. These symbols are printed in *italic* type. For example, *m* is the symbol for mass, and *A* is the symbol for area. Be careful not to confuse these different meanings of the same letters.

Dimensional Analysis Analyzing units can help in solving problems. The units on the left side of an equation must always be equivalent to the units on the right side of the equation. Quantities can be added or subtracted only if they have the same units. These facts can be used to check whether an answer is reasonable.

Formula for the period of a simple pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Dimensional equation after substituting the units of acceleration due to gravity

$$T = \sqrt{\frac{\ell}{\frac{\ell}{T^2}}}$$

Figure 1-1. Analyze units to solve problems

For example, dimensional analysis for the period of a simple pendulum is shown in Figure 1-1. The time T required to complete one cycle of motion is the period of the pendulum, ℓ is the length, and g is the acceleration due to gravity. Because the period represents time, the expression on the right side of the equation must also have the dimension time. The units of the acceleration due to gravity, m/s^2, can be expressed as length ℓ in meters divided by T^2 in seconds squared, or $\frac{\ell}{T^2}$.

The units of length divide out and $T = \sqrt{T^2}$. The factor 2π has no units so it is not considered in the analysis.

Review Questions

1. Which term is not a fundamental unit?
   (1) kilogram (2) meter (3) second (4) watt

2. Which quantity and unit are correctly paired?
   (1) electric current — coulomb
   (2) frequency — hertz
   (3) power — joule
   (4) resistivity — ohm

3. Which amount of power is the *smallest*?
   (1) 1 gigawatt
   (2) 2 kilowatts
   (3) 3 megawatts
   (4) 4 watts

4. Which length is the *shortest*?
   (1) 1 μm (2) 2 mm (3) 3 nm (4) 4 cm

5. Continental drift speed is 1×10^{-9} meter per second. This is equivalent to a speed of
   (1) 1 Tm/s (2) 1 Gm/s (3) 1 nm/s (4) 1 pm/s

6. The diameter of 12-gauge wire is 2.053×10^{-3} meter. This is equivalent to 2.053
   (1) km (2) mm (3) μm (4) nm

7. The energy in half a tank of gasoline is 1,000,000,000 joules. Express this value in gigajoules.

8. The mean radius of Earth is 6,000,000 meters. Express this value in kilometers.

9. Which length is 10^6 times greater than a nanometer?
   (1) μm (2) mm (3) cm (4) km

10. The period of rotation of the Sun is 2.125×10^6 seconds. This is equivalent to 2.125
    (1) μs (2) ms (3) Ms (4) Ts

11. Human hair grows at the rate of 3 nanometers per second. This rate is equivalent to
    (1) 3×10^{-3} m/s (2) 3×10^{-6} m/s (3) 3×10^{-9} m/s (4) 3×10^{-12} m/s

12. The wavelength of red light is 7×10^{-7} meter. Express this value in nanometers.

13. If m represents mass in kg, v represents speed in m/s, and r represents radius in m, show that the force F in the formula $F = \frac{mv^2}{r}$ can be expressed in the unit kg · m/s^2.

14. If PE_s represents the potential energy stored in a spring in kg · m^2/s^2, and x represents the change in spring length from its equilibrium position in m, what is the unit for the spring constant k in the formula $PE_s = \frac{1}{2}kx^2$?

15. If F_e represents the electrostatic force in N that point charge q_1 in C exerts on point charge q_2 in C, and r represents the distance between the point charges in m, what is the unit for the electrostatic constant k in the formula $F = \frac{kq_1q_2}{r^2}$?
    (1) N · m^2/C^2
    (2) N · m^2
    (3) N · C^2/m^2
    (4) N · m^2/C

16. Using dimensional analysis, show that the expression $\frac{v^2}{d}$ has the same units as acceleration.

Tools for Measurement

In most laboratory investigations, you will make observations and measurements of physical quantities. You will be expected to select the appropriate piece of equipment, determine its scale, and make measurements to the proper number of significant figures.

Measuring Length

The length of an object or the total length of a path an object moves is measured with a metric ruler or meter stick. Path length is usually measured in meters, but occasionally centimeters are more appropriate. You can convert a measurement in centimeters to meters by dividing by 100. The piece of wire in Figure 1-2 has a length of 5.20 cm or 0.0520 m.

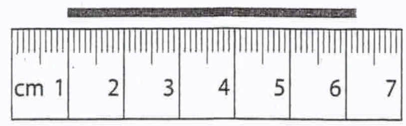


Figure 1-2. Metric ruler: The length of the wire is 5.20 cm.

Measuring Mass

The **mass,** or amount of matter contained in an object, can be measured with an electronic balance or a triple-beam balance. It is important that the balance be zeroed before determining the mass of an object.

The steel ball on the electronic balance in Figure 1-3 has a mass of 115.2 g or 0.1152 kg.

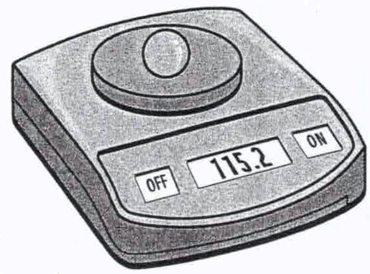


Figure 1-3. Electronic balance: The steel ball has a mass of 115.2 g.

The block of wood on the triple-beam balance in Figure 1-4 has a mass of 208.50 g or 0.20850 kg. A mass that is determined in grams can be converted to kilograms by dividing by 1000.

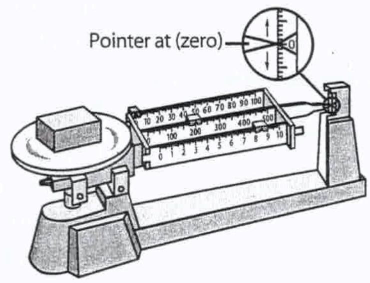


Figure 1-4. Triple-beam balance: The beam must be at zero when a reading of the mass is made.

Measuring Time

Elapsed time can be measured with a clock or stopwatch. As you know, one hour equals sixty minutes and one minute equals sixty seconds. Because many of the events you will be measuring in physics occur quickly, you may be asked to record elapsed time to the nearest hundredth of a second. The stopwatch in Figure 1-5 shows an elapsed time of 37.08 s.

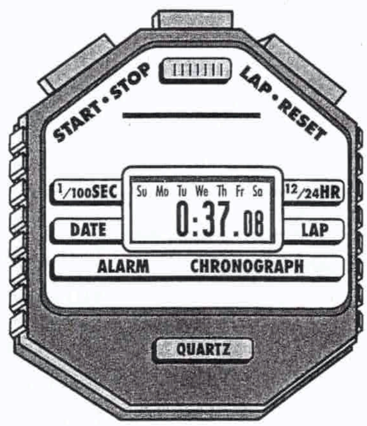


Figure 1-5. Stopwatch: Minutes are recorded to the left of the colon. Seconds (to the one-hundreth place) are recorded to the right of the colon. The elapsed time is 37.08 s.

Measuring Force

A push or pull on a mass is called a **force.** Forces are measured with a spring scale. Ranges on spring scales typically vary from 2.5 newtons to 20.0 newtons. Figure 1-6 shows a spring scale recording a force of 4.5 N as a block is lifted at constant speed.

Measuring an Angle

A common unit for measuring angles is the degree (°), which is one ninetieth of a right angle. The protractor is an instrument used for measuring angles in degrees. Figure 1-7 shows a protractor being used to measure angle *AOB*. The wedge point of the protractor is on *O*, and the diameter of the semicircle lies on *OA*, one side of the angle. The other side of the angle intersects the semicircle at 47°. This reading gives the number of degrees in the angle. If the sides of the angle to be measured are too short to intersect the semicircle, they can be extended.

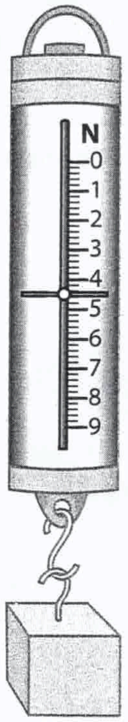


Figure 1-6. Spring scale: Force or weight is measured with a spring scale. This scale reads 4.5 N.

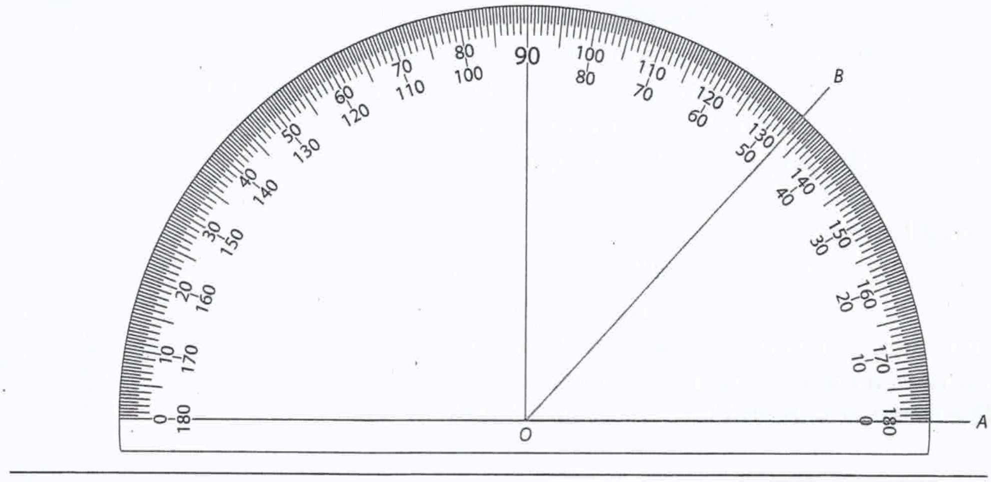
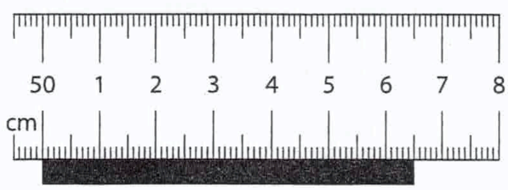


Figure 1-7. Protractor: Angle *AOB* has a measure of 47°.

Drawing an Angle To draw an angle of 25° with its vertex at point *P*, draw a line segment originating at *P*. Place the wedge point of the protractor on *P* and the diameter of the protractor semicircle along the line segment. Make a dot on the paper at the 25° mark on the inner set of degree readings. Draw a line from this point to *P*.

Review Questions

17. A student measures a strip of metal using a metric ruler, as shown in the diagram below.



What is the length of the strip?

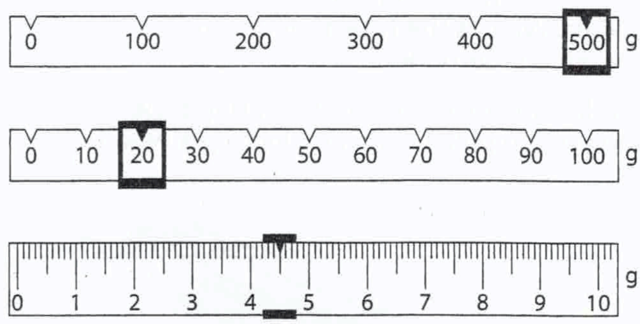
(1) 6.50 cm (3) 56.5 cm
(2) 56.5 mm (4) 5065 mm

18. The diagram below shows the cross-sectional area of a dowel.

Use a metric ruler to measure the diameter of the dowel to the nearest tenth of a centimeter.

19. Express a length of 52.5 centimeters in meters.

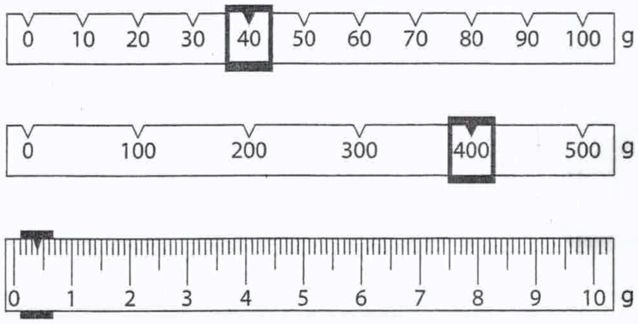
20. The diagram below shows an enlarged view of the beams on a triple-beam balance.



What is the reading for the mass that is being measured?

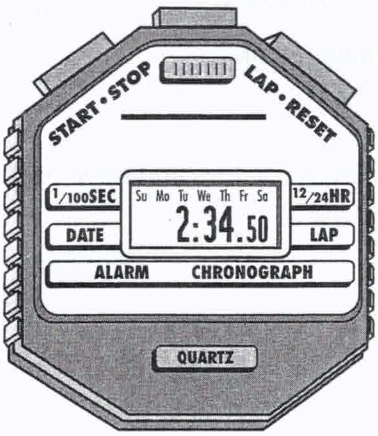
(1) 251.0 g (3) 5245 g
(2) 524.5 g (4) 5,002,045 g

21. The diagram below shows an enlarged view of the beams of a triple-beam balance.



What is the reading, in kilograms, for the mass being measured?

22. The stopwatch below was used to time an event.



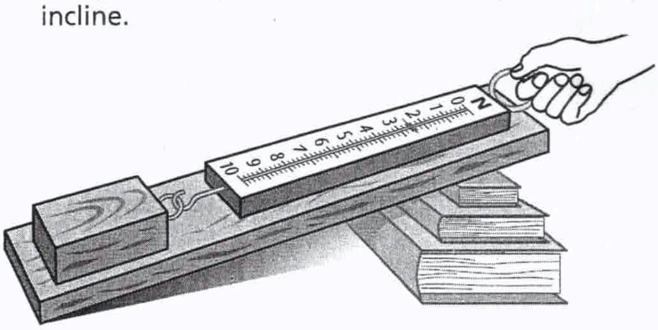
What is the elapsed time in seconds?

(1) 24.450 s
(2) 154.50 s
(3) 234.50 s
(4) 23,450 s

23. An electric lightbulb operates for 1 hour 15 minutes. What is the total time the light bulb operates in seconds?

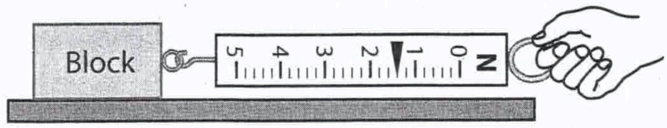
24. An electric iron is operated for 18 minutes at 120 volts. What is the total time the iron is operated in seconds?

25. The following diagram shows a spring scale being used to pull a wooden block up a wooden incline.



What is the magnitude of the force recorded on the spring scale?

26. The diagram below shows a spring scale attached to a wooden block as it is being pulled across a horizontal surface.



What is the magnitude of the force exerted on the spring scale?