Population Genetics

Peerteaching KW24

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Inhalt

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No decendants: $(k = 0)$

$$\mathbb{P}_{\text{Poisson}}(X = 0) = e^{-1} \approx 0.37$$

At least one decendant:

$$1 - 0.37 \approx 0.63$$

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So a population of 10 000 genes would have most likely evolved from just 10 genes over a course of 15 generations.

This means we can now find out how many generations back there was just one individual (the most recent common ancestor of our whole population).

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- Construct previous generations

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Basic idea

- Start with present-day generation
- Construct previous generations
- By randomly choosing parents in the previous generation

The coalescent process

Example

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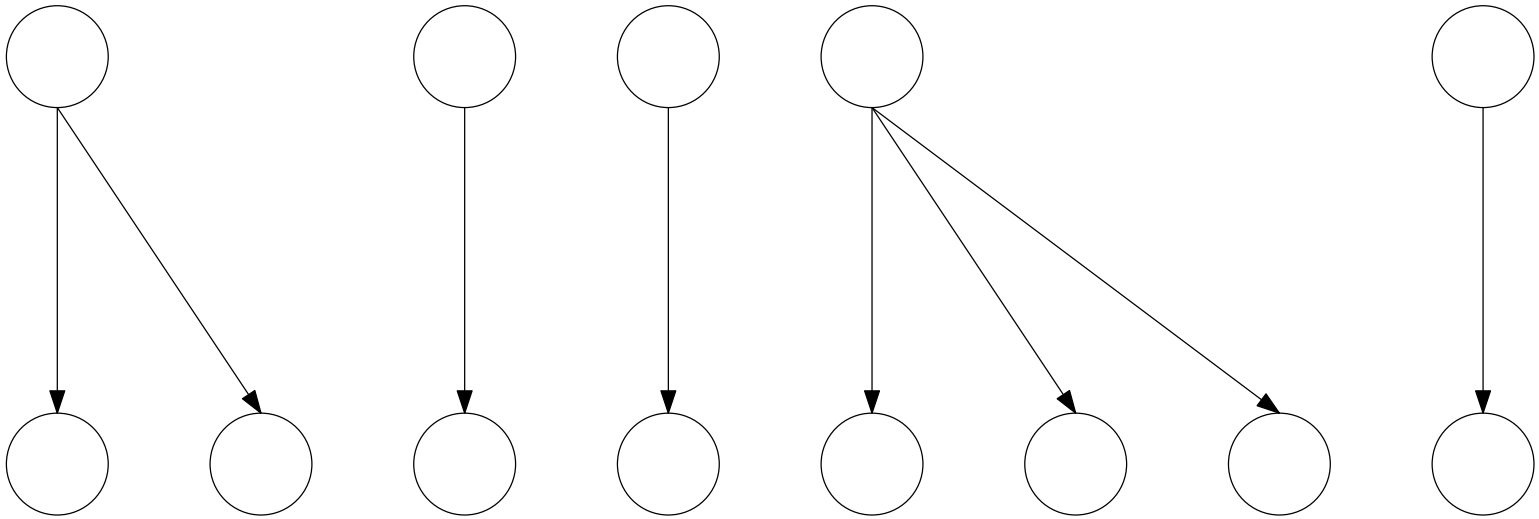

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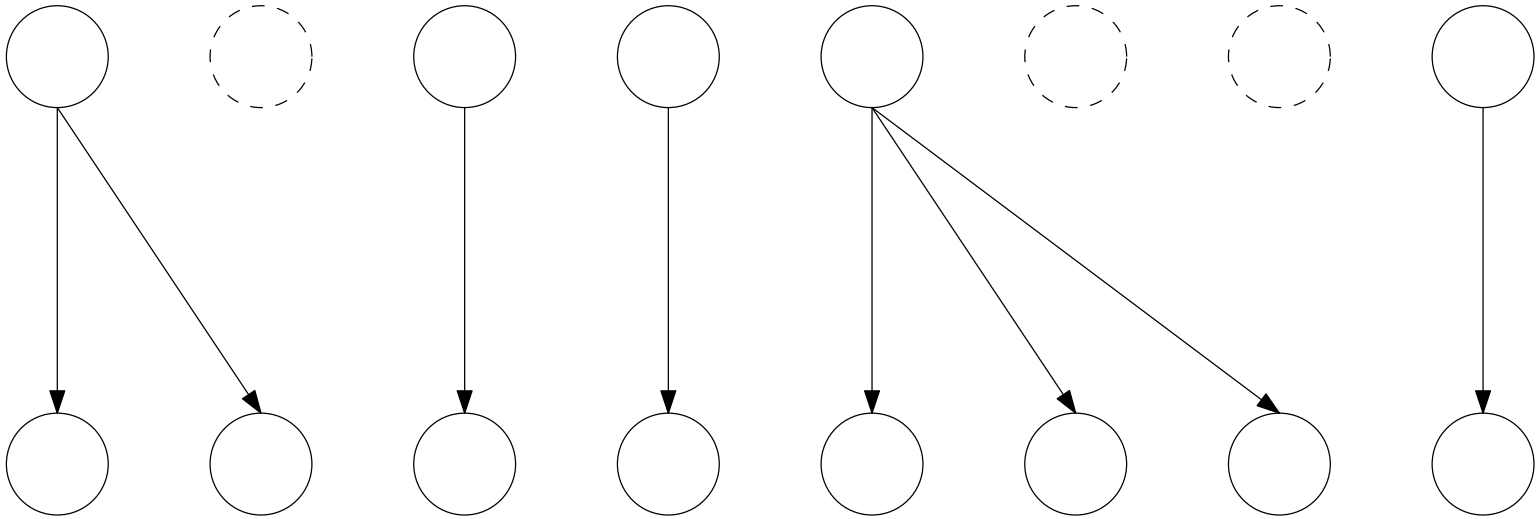

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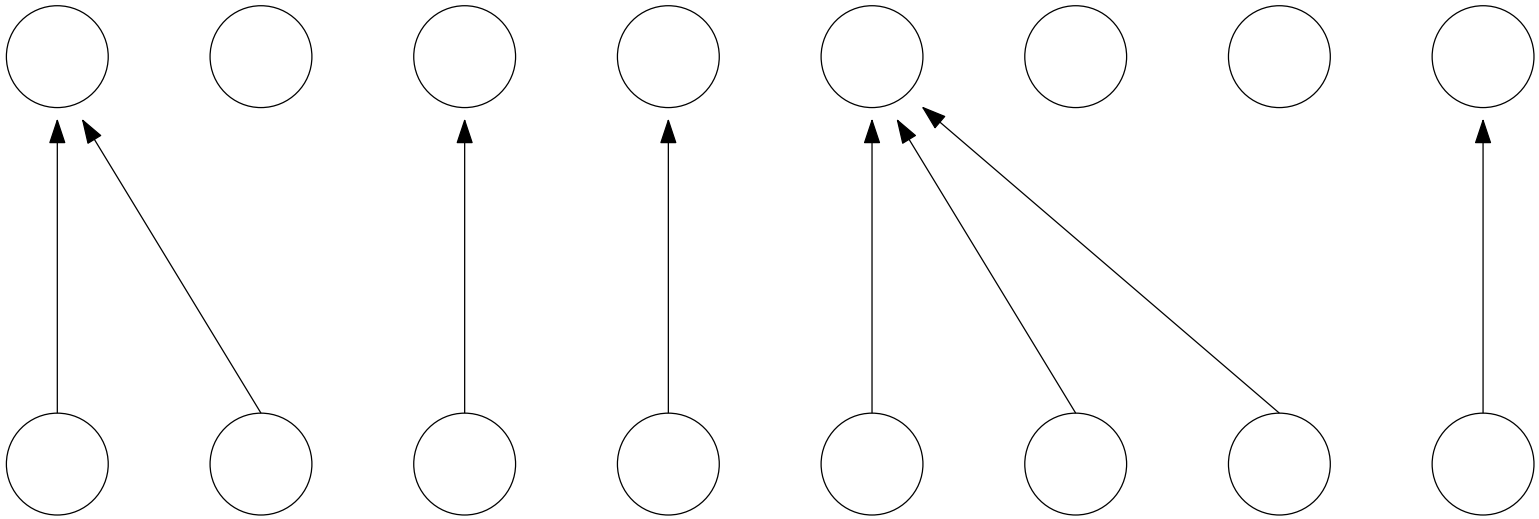

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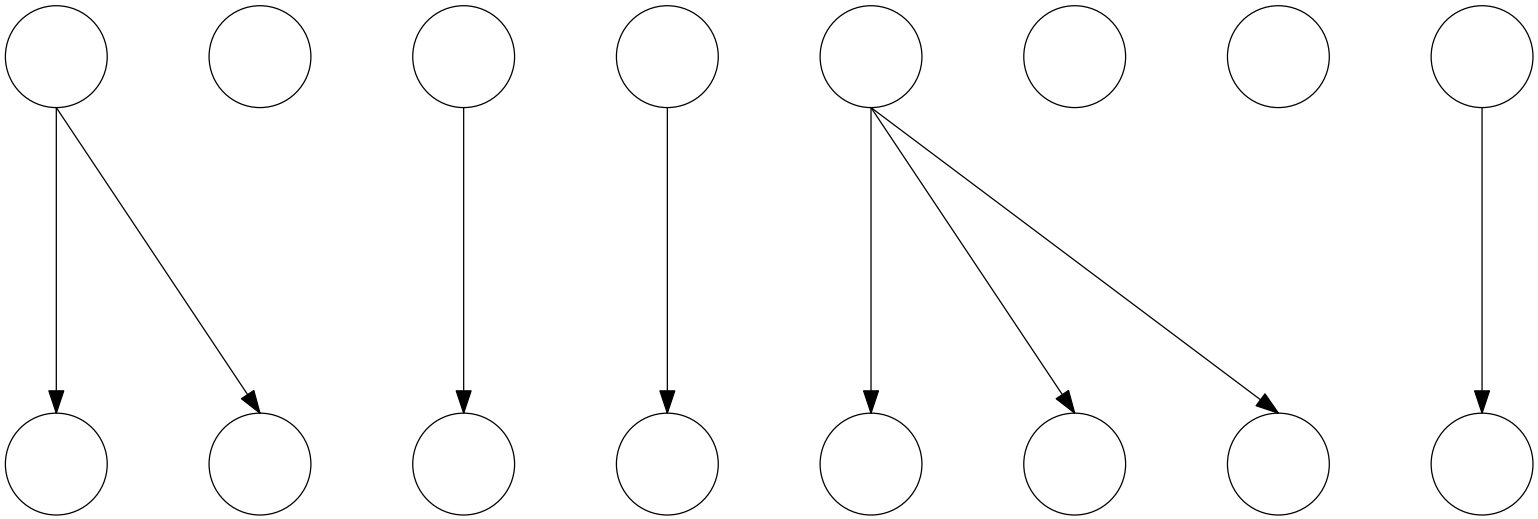

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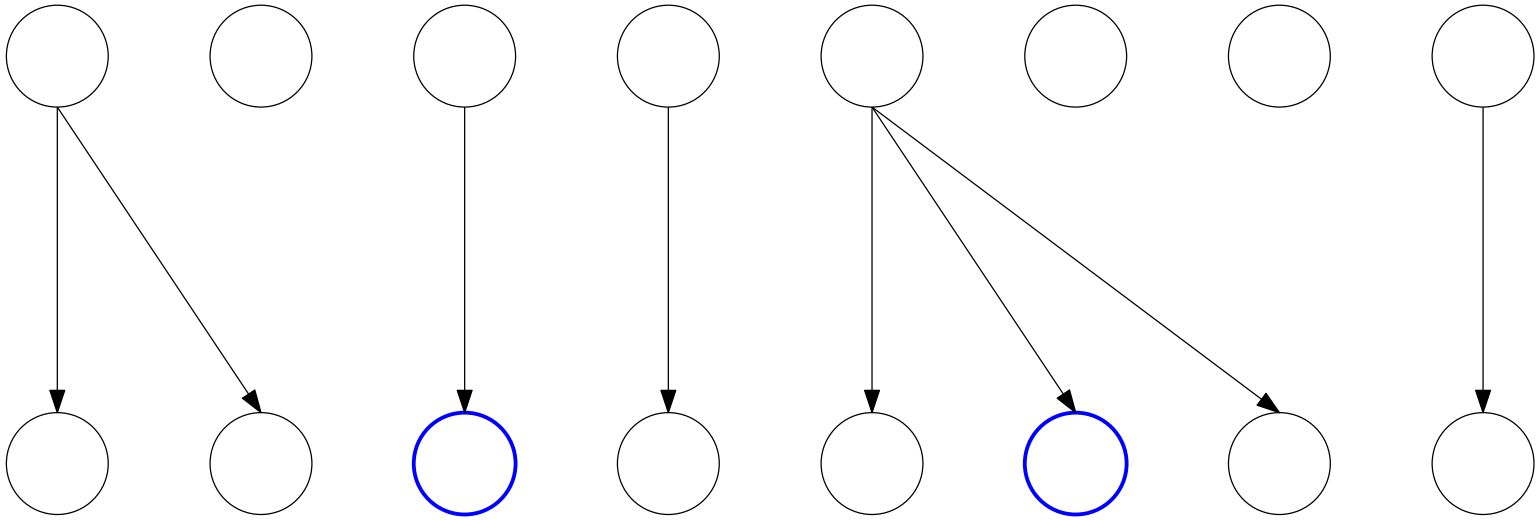

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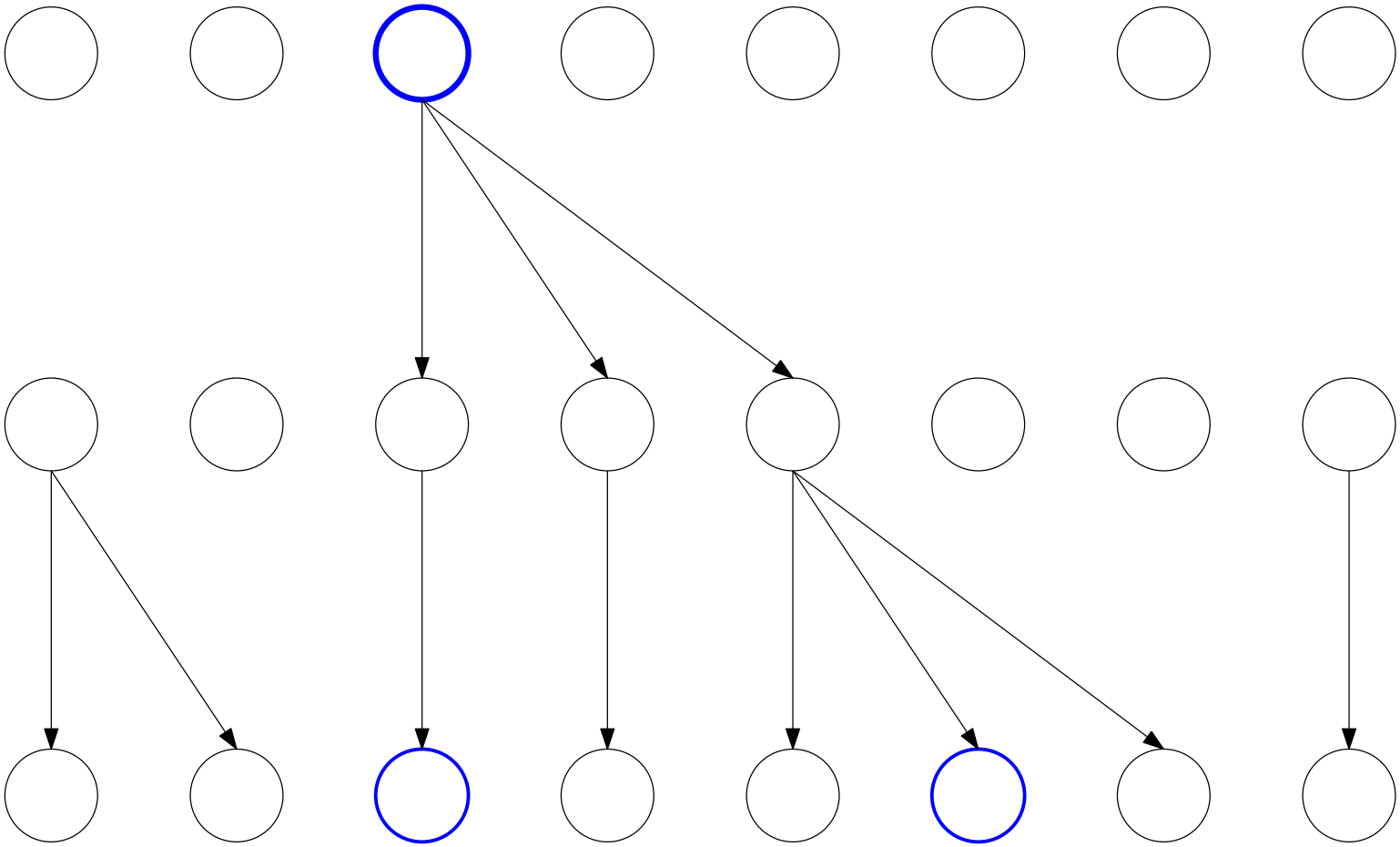

The standard coalescent model

Construct a tree based on estimated coalescent events.
Needs more content

Coalescence of 2 genes

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2. How long is the waiting time until the two genes coalesced?
   $\rightarrow$ How many generations back is their common ancestor?

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Probability to select the right ancestor of j is

$$P(T_2 = 1) = \frac{1}{n}$$

since we need to "hit" the ancestor we've chosen for i.

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($n-1$ failures following one success)

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$$P(T_2 = t) = \left(1 - \frac{1}{n}\right)^{n-1}$$

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$$P(T_2 = t) = \left(1 - \frac{1}{n}\right)^{n-1} \cdot \left(\frac{1}{n}\right)$$

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Exercise

Let's assume $n = 50$
What is the probability that two genes coalesce exactly 5 generations in the past?

$$P(T_2 = 5) =$$

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Let's assume $n = 50$
What is the probability that two genes coalesce exactly 5 generations in the past?

$$P(T_2 = 5) = \left(1 - \frac{1}{50}\right)^{5-1} \cdot \frac{1}{50}$$

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Exercise

Let's assume $n = 50$
What is the probability that two genes coalesce exactly 5 generations in the past?

$$P(T_2 = 5) = \left(1 - \frac{1}{50}\right)^{5-1} \cdot \frac{1}{50} \approx 0.01845 = 1.845\%$$

Coalescence of 2 genes

Exercise

Let's assume $n = 50$
What is the probability that it takes them at least 5 generations to coalesce?

$$P(T_2 > 5) =$$

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What is the probability that it takes them at least 5 generations to coalesce?

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Let's assume $n = 50$
What is the probability that it takes them at least 5 generations to coalesce?

$$P(T_2 > 5) = \left(1 - \frac{1}{50}\right)^{5-1} \approx 0.92237 = 92.237\%$$