Алгоритм K-SVD и его усовершенствования в задачах разреженного представления.

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AIMasters

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Содержание

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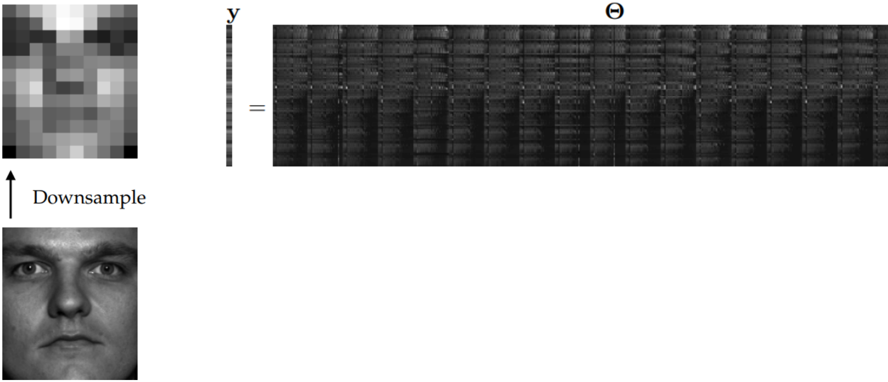
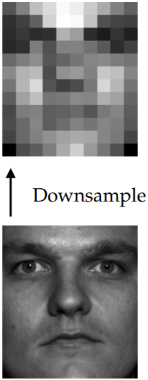
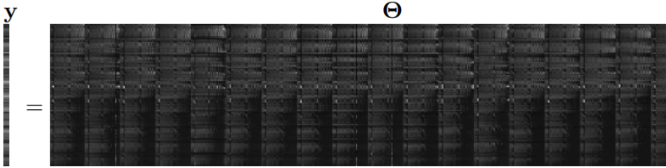
SRC







Algorithm SRC

1: Input: a matrix of training samples $D = [D_1, D_2, \ldots, D_k] \in \mathbb{R}^{m \times n}$ for k classes, a test sample $y \in \mathbb{R}^m$, (and an optional error tolerance $\varepsilon > 0$.)

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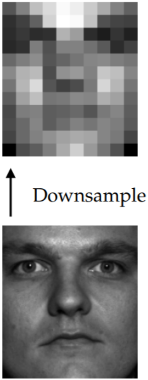

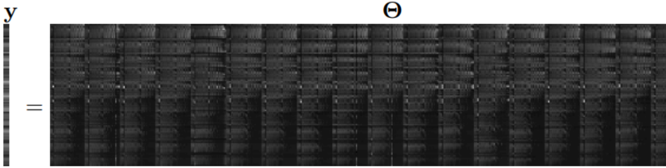

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2: Normalize the columns of D to have unit ℓ^2-norm.

3: Solve the ℓ^1-minimization problem:

$\hat{x}_1 = \arg\min_x \|x\|_1$ subject to $\|Dx - y\|_2 \leq \varepsilon$

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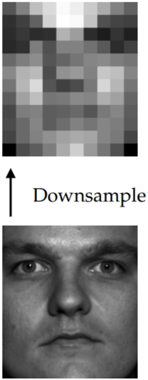

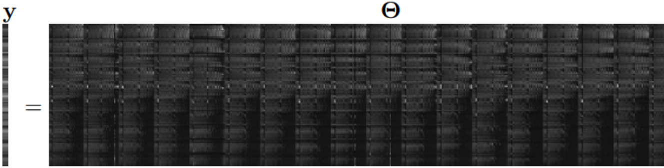

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4: Compute the residuals $r_i(y) = \|\boldsymbol{y} - D\delta_i(\hat{x}_1)\|_2$ for $i = 1, \dots, k$.

5: Output: predicted $y = \arg\min_i r_i(\boldsymbol{y})$.

Улучшения

$\hat{x}_0 = \arg\min_x \|x\|_0$ при $\|Dx - y\|_2 \leq \varepsilon$, где $\|x\|_0$ - количество ненулевых элементов.

Orthogonal Matching Pursuit

Result: x_k
Initialization $\mathrm{r}_0 = \mathrm{y}, \Lambda_0 = \varnothing$;
Normalize all columns of D to unit L_2 norm;
while $r_k > \varepsilon$ and $k < num$ do
Step-1. $\lambda_k = \underset{j \notin \Lambda_{k-1}}{\mathrm{argmax}} |\langle \mathrm{d}_j, \mathrm{r}_{k-1} \rangle|$;
Step-2. $\Lambda_k = \Lambda_{k-1} \cup \{\lambda_k\}$;
Step-3. $\mathrm{x}_k\,(i \in \Lambda_k) = \underset{\mathrm{x}}{\mathrm{argmin}} \left\|\mathrm{D}_{\Lambda_k}\mathrm{x} - \mathrm{y}\right\|_2, \quad \mathrm{x}_k\,(i \notin \Lambda_k) = 0$;
Step-4. $\hat{\mathrm{y}}_k = \mathrm{D}\mathrm{x}_k$;
Step-5. $\mathrm{r}_k \leftarrow \mathrm{y} - \hat{\mathrm{y}}_k$; $k+ = 1$;

Содержание

KSVD

$$D, X = \mathrm{argmin}_{\mathrm{D},\mathrm{X}} \|\mathrm{Y} - \mathrm{D}\mathrm{X}\|_F^2 \quad \text{subject to } \forall i, \|\mathrm{x}_i\|_0 \leq T$$

KSVD

1) Initialize $\mathrm{D}^{(0)} \in \mathbb{R}^{m \times K}$ with ℓ^2 normalized columns. Set $J = 1$.

1) $i = 1, 2, \ldots, N, \quad \min_{\mathrm{x}_i} \left\{ \|\mathrm{y}_i - \mathrm{D}\mathrm{x}_i\|_2^2 \right\} \quad$ subject to $\quad \|\mathrm{x}_i\|_0 \leq T$.

2) For each column $j = 1, 2, \ldots, K$ in $\mathrm{D}^{(J-1)}$, update it by

$\omega_j = \left\{ i \mid 1 \leq i \leq N, \mathrm{x}_T^j(i) \neq 0 \right\}$

$\|\mathrm{Y} - \mathrm{D}\mathrm{X}\|_F^2 = \left\| \left(\mathrm{Y} - \sum_{j \neq k} \mathrm{d}_j \mathrm{x}_T^j \right) - \mathrm{d}_k \mathrm{x}_T^k \right\|_F^2 = \left\| \mathrm{E}_k - \mathrm{d}_k \mathrm{x}_T^k \right\|_F^2.$

$\mathrm{E}_j = \mathrm{Y} - \sum_{j \neq j} \mathrm{d}_j \mathrm{x}_T^j.$

Restrict E_j : $\quad \mathrm{E}_j^R = \mathrm{E}_j \mid_{\omega_j}$

Apply SVD: $\quad \mathrm{E}_j^R = \mathrm{U}\boldsymbol{\Sigma}\mathrm{V}^T. \quad d_j := U_1 \quad \mathrm{x}^j := V_1 \sigma_1$

Set $J = J + 1$

Содержание

DKSVD

$$D, W, X = \operatorname{argmin} \| \begin{pmatrix} Y \\ \sqrt{\gamma}H \end{pmatrix} - \begin{pmatrix} D \\ \sqrt{\gamma}W \end{pmatrix} \mathrm{X} \|_F$$

subject to $\|x_i\|_0 \leq T$

DKSVD

KSVD with $D' = \begin{pmatrix} D \\ \sqrt{\gamma}W \end{pmatrix}$, $Y' = \begin{pmatrix} Y \\ \sqrt{\gamma}H \end{pmatrix}$

Classification:

1) For y obtain x from OMP
2) $l = Wx$
3) predicted class: $\operatorname{argmax}_i(l_i)$

Содержание

LCKSVD

$$< D, W, A, X >= \text{argmin}_{D,W,A,X} \| \begin{pmatrix} Y \\ \sqrt{\alpha}Q \\ \sqrt{\beta}H \end{pmatrix} - \begin{pmatrix} D \\ \sqrt{\alpha}A \\ \sqrt{\beta}W \end{pmatrix} X\|_2^2$$

s.t. $\forall i \|x_i\|_0 \leq T$

LCKSVD

KSVD with $D' = \begin{pmatrix} D \\ \sqrt{\alpha}A \\ \sqrt{\beta}W \end{pmatrix}$ $Y' = \begin{pmatrix} Y \\ \sqrt{\alpha}Q \\ \sqrt{\beta}H \end{pmatrix}$

Содержание

Extended Yale B Dataset

Таблица: Train=38, Test=20

Метод, словарь	504*760	504*570	126*570	126*380
SRC	0.963	0.932	0.826	0.775
KSVD	0.03	-	-	-
DKSVD	0.962	0.951	0.841	0.794
LCKSVD	0.976	0.968	0.876	0.817

Таблица: Train=38, Test=20, salt paper=0.05

Метод, словарь	504*760	504*570	126*570	126*380
SRC	0.732	0.718	0.453	0.451
DKSVD	0.791	0.773	0.512	0.502
LCKSVD	0.866	0.859	0.579	0.567

Сравнение с другими алгоритмами

Таблица: Extended Yale B Database

Метод	результат	размерность
ADL+SVM	0.829	504
DADL	0.973	504
LCKSVD	0.976	504
LBP+SVM	0.971	16*16 blocks
CNN	up to 0.99	all
KNN	up to 0.85	all

Содержание

RGB-SRC

1: Input: a matrix of training samples
$D^R = \left[D_1^R, D_2^R, \ldots, D_k^R\right], D^G, D^B \in \mathbb{R}^{m \times n}$ for k classes, a test sample
$y \in \mathbb{R}^m$, (and an optional error tolerance $\varepsilon > 0$.)
2: Normalize the columns of D's to have unit ℓ^2-norm.
3: Solve the ℓ^1-minimization problems for D^G, D^B, D^R:
$\hat{x}_1 = \arg\min_x \|x\|_1$ subject to $\|D^* x - y\|_2 \leq \varepsilon$
4: Compute the residuals $r_i(y) = \sum_{R,G,B} \|\boldsymbol{y} - D^* \delta_i\left(\hat{x}_1\right)\|_2$ for
$i = 1, \ldots, k$.
5: Output: predicted $y = \arg\min_i r_i(\boldsymbol{y})$.

Сравнение подходов

The MUCT Face Database

1) Grayscale, 1 dictionary, 504*540
2) RGB, 1 dictionary, 504*540
3) RGB, 3 dictionaries, 3*168*540

Таблица: Train=10, Test=5, salt & pepper= None, 0.05

Метод	1	2	3
SRC	0.833	0.871	0.890
DKSVD	0.870	0.895	0.902
LCKSVD	0.922	0.936	0.929
SRC	0.746	0.748	0.684
DKSVD	0.781	0.746	0.729
LCKSVD	0.889	0.904	0.862

Содержание

Выводы

Полученные результаты показали, что алгоритмы, основанные на KSVD, позволяют значительно улучшить качество классификации.

Как улучшить результат?

1) $D' = (D, I)$

2) Изменение инициализации словаря

3) TenSR, K-TSVD

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