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On continuity of moment-generating function of log price at first return in a switching regime

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On continuity of moment-generating function of log price at first return in a switching regime

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Abstract In a recent paper, Kabanov and Pergamenshchikov obtained the rate of convergence to zero of the ruin probabilities as the initial capital tends to infinity for the model based on a hidden Markov process with a finite number of states. In this work we extend the results of the paper [1] assuming that at some states of the economy risky investments are not allowed. The result allows us to investigate the asymptotic behaviour of the ruin probabilities.

Keywords Moment-generating function · Risky investments · Hidden Markov model · Regime switching

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1 Introduction

The moment-generating function is one of the ways to describe the probability distribution. This is a useful tool that can be used to calculate the moments of a random variable, construct an approximation of probability distributions, and prove limit theorems. It sometimes provides better tractability than probability density function and unlike characteristic function allows us not to use complex analysis.

Models with risky investments have been extensively studied in the collective risk theory. In this work we deal with the model introduced in the paper by Kabanov and Pergamenshchikov [1]. The model is based on the regime switching described by an ergodic Markov process θ with a finite number of states forming a single class. In state k the asset price is a geometric Brownian motion with drift a_k and volatility σ_k. We investigate a situation where some $\sigma_k = 0$. It can be interpretated that at some states of the economy risky investments are not allowed.

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Continuity of the moment-generating function for a log price process at first return let us obtain the rate of convergence to zero of the ruin probabilities as the initial reserve increased to infinity.

2 Definitions and required results

Let us consider a stochastic basis $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\in\mathbb{R}_+}, \mathbf{P})$.

Definition 1 The moment-generating function (MGF) of a random variable X is a function $\Upsilon_X(t)$ defined by

$$\Upsilon_X(t) := \mathbf{E}[e^{tX}]$$

provided the expectation is finite.

We can use MGF to find all the moments of a given random variable by differentiating it. Namely,

$$\Upsilon_X'(t) = \frac{d}{dt}\mathbf{E}[e^{tX}] = \mathbf{E}[Xe^{tX}].$$

Put $t = 0$ and get

$$\Upsilon_X'(0) = \mathbf{E}[X].$$

Continuing differentiation we obtain the following result:

Proposition 1 *For all* $n \geq 0$

$$\Upsilon_X^{(n)}(0) = \mathbf{E}[X^n].$$

Definition 2 Process $\theta := \theta_t$ that takes values from the set $\Theta := \{0, 1, 2...\}$ is a continuous-time Markov chain if for every $0 \leq t_1 < t_2 < ... < t_n$ and $j_0, j_1, ...j_n \in \Theta$

$$P(\theta_{t_n} = j_n | \theta_{t_0} = j_0, ..., \theta_{t_{n-1}} = j_{n-1}) = P(\theta_{t_n} = j_n | \theta_{t_{n-1}} = j_{n-1}).$$

Proposition 2 *Let θ be a right-continuous Markov process that takes values from Θ. Then there exists the transition rate matrix Λ described below:*

$$\Lambda := \lim_{h\to 0+} \frac{P(h) - E}{h}.$$

The proof of the proposition can be found in [2] §3.

Let us find the distribution of the length of intervals between jumps. We denote the first jump of θ_t as $\tau_1 := \inf\{t > 0 : \theta_t \neq i\}$, if $\theta_0 = i$. Put $F_i(t) := P(\tau_1 > t \mid \theta_0 = \mathrm{i})$ and $\lambda_i := -\lambda_{ii} = \sum_{i\neq j} \lambda_{ij}$. Then

$$\begin{aligned} F_i(t+h) &= P(\tau_1 > t + h) = P(\theta_s = i, s \in [0, t+h] \mid \theta_0 = i) \\ &= P(\theta_s = i, s \in [0, t) \mid \theta_0 = i)P(\theta_s = i, s \in [t, t+h] \mid \theta_t = i) \\ &= F_i(t)F_i(h) = F_i(t)\left(1 - \lambda_i h + o(h)\right). \end{aligned}$$

From that we obtain that $F_i'(t) = -\lambda_i F_i(t)$. Thus,

Proposition 3 *The continuous-time Markov process θ_t remains in state i for an exponentially distributed time interval with parameter λ_i. These holding times are independent.*

Let us denote τ_n by n^{th} jump of θ_t. Namely,

$$\tau_n := \inf\{t > \tau_{n-1} : \theta_{t-} \neq \theta_t\}, \quad n \geq 1. \quad \tau_0 := 0.$$

Definition 3 A process defined by $\vartheta_n = \theta_{\tau_n}$ is a discrete-time Markov chain. It is an embedded Markov chain for θ_t.

Proposition 4 *The elements of the probability transition matrix $\tilde{P}$ of the process ϑ_n are*

$$\tilde{P}_{ij} = \lambda_{ij}/\lambda_i, \; i \neq j, \quad \tilde{P}_{ii} = 0.$$

Proof Consider the probabity $P(\theta_{t+h} = j|\theta_t = i, j \neq i)$. As h tends to 0, this become the probability of transition from state i to state j. Thus, $\lim\limits_{h\to 0+} P(\theta_{t+h} = j|\theta_t = i, j \neq i) = \tilde{P}_{ij}$. We use the definition of conditional probability to compute $\tilde{P}_{ij}$. Namely,

$$P(\theta_{t+h} = j|\theta_t = i, j \neq i) = \frac{P(\theta_{t+h} = j, \theta_t = i, \theta_{t+h} \neq i)}{P(\theta_t = i, \theta_{t+h} \neq i)}$$
$$= \frac{P(\theta_{t+h} = j, \theta_{t+h} \neq i|\theta_t = i)P(\theta_t = i)}{P(\theta_{t+h} \neq i|\theta_t = i)P(\theta_t = i)} = \frac{P_{ij}(h)}{1 - P_{ij}(h)}.$$

The last step is to find the limit:

$$\lim_{h\to 0+} \frac{P_{ij}(h)}{1 - P_{ij}(h)} = \lim_{h\to 0+} \frac{P_{ij}(h)h}{(1 - P_{ij}(h))h} = \frac{\lambda_{ij}}{\lambda_i}.$$

3 The model

Let $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\in\mathbb{R}_+}, \mathbf{P})$ be a stochastic basis with a Wiener process $W = (W_t)$, a Poisson random measure $\pi(dt, dx)$ on $\mathbb{R}_+ \times \mathbb{R}$ with the mean $\tilde{\pi}(dt, dx) = \Pi(dx)dt$, and a right-continuous Markov process $\theta = (\theta_t)$. This process takes values in the finite set $\Theta := \{0, 1, ..., K-1\}$, has the $K \times K$ transition intensity matrix $\Lambda = (\lambda_{jk})$ with communicating states, and the initial value $\theta_0 = i$ (so that $\theta = \theta^i$). Sometimes we will omit the initial state i in the formulae. Although there is a dependence on i, the proof of the theorem and the notation will not change. The σ-algebras generated by W, π, and θ are independent.

Let T_n denote the successive jump times of the Poisson process $N = (N_t)_{t\in\mathbb{R}_+}$ with $N_t := \pi([0, t], \mathbb{R})$ and let τ_n be the successive jumps of θ with the convention $T_0 = 0$ and $\tau_0 = 0$.

The reserve $X = X^u$ of an insurance company evolves not only due to the part of business activity, described as in the classical Cramér–Lundberg model, but also due to the stochastic interest rate. It is fully invested in a risky asset whose price S is a conditional geometric Brownian motion given the Markov process θ. That is, S is given by a so-called hidden Markov model with

$$dS_t = S_t(a_{\theta_t}dt + \sigma_{\theta_t}dW_t), \qquad S_0 = 1,$$

where $a_k \in \mathbb{R}$, $\sigma_k > 0$, $k = 0, ..., K-1$. In this case, X is of the form

$$X_t = u + \int_0^t X_s dR_s + dP_t \tag{1}$$

where $dR_t = a_{\theta_t}dt + \sigma_{\theta_t}dW_t = dS_t/S_t$, that is R is the relative price process, and

$$P_t = ct + \int_0^t \int x\pi(dt, dx) = ct + x * \pi_t. \tag{2}$$

Thus, the reserve evolution is described by the process $(X^u, \theta) = (X^{u,i}, \theta^i)$ where $u > 0$ is the initial capital and i is the initial regime, that is, the initial value of θ.

We assume that P is not an increasing process: otherwise the probability of ruin is zero.

We also assume that $\Pi(\mathbb{R}) < \infty$, that is $\Pi(dx) = \alpha_1 F_1(dx) + \alpha_2 F_2(dx)$ where $F_1(dx)$ is a probability distribution on $(-\infty, 0)$ and $F_2(dx)$ is a probability distribution on $(0, \infty)$. In this case, the integral with respect to the jump measure is simply a difference of two independent compound Poisson processes with intensities α_1, α_2 of jumps downward and upward, and whose absolute values have distributions $F_1(dx)$ and $F_2(dx)$. Although P_t is often considered with only one compound Poisson process, models with two-sided jumps is a classical object of study, see [3].

The solution of the linear equation (1) according to [1] can be represented as

$$X_t^u = \mathcal{E}_t(R)(u - Y_t) = e^{V_t}(u - Y_t) \tag{3}$$

where

$$Y_t := -\int_{[0,t]} \mathcal{E}_s^{-1}(R)dP_s = -\int_{[0,t]} e^{-V_s}dP_s = -e^{-V} \cdot P_t, \tag{4}$$

the stochastic exponential $\mathcal{E}_t(R)$ is equal to S_t. We obtain the SDE for the log price process $V = \ln \mathcal{E}(R)$ using the Ito formula with $f(x) = ln\, x$.

$$d(ln\, S_t) = \frac{1}{S_t}dS_t - \frac{1}{2S_t^2}d < S >_t = \sigma_{\theta_t}dW_t + a_{\theta_t}dt - (1/2)\sigma_{\theta_t}^2 dt.$$

Therefore, the log price process $V = \ln \mathcal{E}(R)$ admits the stochastic differential

$$dV_s = \sigma_{\theta_s}dW_s + (a_{\theta_s} - (1/2)\sigma_{\theta_s}^2)ds, \quad V_0 = 0. \tag{5}$$

Let $\tau^{u,i} := \inf\{t > 0 :\ X_t^{u,i} \le 0\}$ be the instant of ruin corresponding to the initial capital u and the initial regime i. Then $\Psi_i(u) := \mathbf{P}[\tau^{u,i} < \infty]$ is the ruin probability and $\Phi_i(u) := 1 - \Psi_i(u)$ is the survival probability. The moment of ruin can be expressed as $\tau^{u,i} = \inf\{t \ge 0 :\ Y_t^i \ge u\}$.

We can obtain useful assumptions from the model with $K = 1$. Namely, for the model with exponentially distributed jumps and price following a geometric Brownian motion with drift coefficient a and volatility $\sigma > 0$ in the case where $2a/\sigma^2 - 1 > 0$, the ruin probability as a function of the initial capital u, decreases as Cu^{1-2a/σ^2}. If $2a/\sigma^2 - 1 \le 0$ the ruin happens with probability one, see [4], [5].

Therefore, we introduce the condition for our model with a finite-state Markov process

$$\tilde{a}_j := a_j - (1/2)\sigma_j^2 \ge 0, \quad j = 0, ..., K-1, \qquad \max_j \tilde{a}_j \sigma_j > 0. \tag{6}$$

Let $\upsilon_1^i := \inf\{t > 0 \colon \theta_{t-}^i \ne i,\ \theta_t^i = i\}$ be the first return time of the (continuous-time) Markov process $\theta = \theta^i$ to its initial state i. We further consider the consequent return times defined recursively:

$$\upsilon_k^i := \inf\{t > \upsilon_{k-1}^i \colon \theta_{t-}^i \ne i,\ \theta_t^i = i\}, \quad k = 2, ...$$

Recalling that V also depends on i, we introduce the random variable $M_{i1} := e^{-V_{\upsilon_1^i}}$ and define the moment-generating function of $-V_{\upsilon_1^i}$: $\Upsilon_i : \mathbb{R}_+ \to \bar{\mathbb{R}}_+ := \mathbb{R}_+ \cup \{\infty\}$ with

$$\Upsilon_i(q) := \mathbf{E}[M_{i1}^q] = \mathbf{E}\left[\exp\left\{\frac{1}{2}q\int_0^{\upsilon_1^i}(\sigma_{\theta_t}^2 q - 2\tilde{a}_{\theta_t})dt\right\}\right].$$

4 Continuity of $\boldsymbol{\Upsilon(q)}$

Theorem 1 *The function Υ_i is continuous.*

Let us introduce the embedded Markov chain $\vartheta_n := \theta_{\tau_n}$, $n = 0, 1, ...$, where τ_n denotes the time of the n^{th} jump of θ defined above. Without loss of generality we assume that the initial state of θ is 0. The elements of the transition matrix P of the embedded Markov chain can be expressed in terms of the transition intensity matrix Λ. That is, according to Proposition 4, $P_{kl} = \lambda_{kl}/\lambda_k, k \neq l$ and $P_{kk} = 0$. Let $\varpi := \inf\{j \geq 2 \colon \vartheta_j = 0\}$ be the first return time of the (discrete-time) Markov chain ϑ to the initial point 0 and $\upsilon_1 = \tau_\varpi$. We define $A = \{k : \sigma_k = 0\}$ as the set of states in which risky investments are not allowed. We can easily compute the solution to the SDE (5). Put

$$\begin{aligned} Z_t^j &:= \sigma_j W_t + (a_j - \sigma_j^2/2)t = \sigma_j W_t + \tilde{a}_j t, && j \in \Theta\backslash A,\\ Z_t^j &:= \tilde{a}_j t, && j \in A. \end{aligned}$$

The random variable M_1 can be represented using law of total probability and partition to increments as follows:

$$M_1 = \sum_{k\geq 2}\sum_{i_1\neq 0, i_2\neq 0,\dots,i_k=0} I_{\{\vartheta_1=i_1,\vartheta_2=i_2,\dots,\vartheta_k=i_k\}} e^{-\zeta_1^0} e^{-\zeta_2^{i_1}} \dots e^{-\zeta_k^{i_{k-1}}}$$

where

$$\zeta_1^0 := Z_{\tau_1}^0 - Z_{\tau_0}^0, \quad \zeta_2^{i_1} := Z_{\tau_2}^{i_1} - Z_{\tau_1}^{i_1}, \quad \dots, \quad \zeta_k^{i_{k-1}} := Z_{\tau_k}^{i_{k-1}} - Z_{\tau_{k-1}}^{i_{k-1}}.$$

We denote the holding times $\widehat{\tau_n} := \tau_{n+1} - \tau_n$. It is known that $\{\widehat{\tau_n}\}_{n=1}^\infty$ lengths of the intervals between consecutive jumps are independent exponential random variables with parameters λ_j (Proposition 3) and independent of the Wiener process W. Therefore, the conditional law of random variables ζ_1^0,...,$\zeta_k^{i_{k-1}}$ given $\vartheta_1 = i_1, \vartheta_2 = i_2, ..., \vartheta_k = i_k$ is the same as the unconditional law of independent random variables $\tilde{\zeta}_1^0$,...,$\tilde{\zeta}_k^{i_{k-1}}$, which is $\mathcal{L}(\tilde{\zeta}_m^j) = \mathcal{L}(\sigma_j W_{\widehat{\tau_n}} + \tilde{a}_j\widehat{\tau_n})$, for $j \in \Theta\backslash A$.

From the independence of random variables and properties of expectation it follows that

$$\Upsilon(q) = \mathbf{E}[M_1^q] = \sum_{k\geq 2}\sum_{i_1\neq 0, i_2\neq 0,\dots,i_k=0} P_{0i_1}P_{i_1i_2}\dots P_{i_{k-1}0} f_0(q) f_{i_1}(q) \dots f_{i_{k-1}}(q), \tag{7}$$

where $f_j(q) = \mathbf{E}[e^{-q\tilde{\zeta}_m^j}]$.

For $j \in A$, we obtain that

$$f_j(q) = \lambda_j \int_0^\infty e^{-q(a_j t)} e^{-\lambda_j t} dt = \frac{\lambda_j}{\lambda_j + a_j q},$$

if the denominator is positive and $f_j(q) = \infty$ otherwise.

For $j \in \Theta\backslash A$ we obtain $f_j(q) = \displaystyle\int_0^\infty \mathbf{E}\left[e^{-q\tilde{\zeta}_m^j}|\widehat{\tau} = t\right] p(\widehat{\tau})dt$, using the total expectation law. $\mathbf{E}\left[e^{-q(\sigma_j W_t + \tilde{a}_j t)}\right]$ can be calculated if we look at that random process as a geometric Brownian motion with drift $\mu = -q\tilde{a}_j$ and volatility $\sigma = -q\sigma_j$. Then,

$$\mathbf{E}\left[e^{-q(\sigma_j W_t + \tilde{a}_j t)}\right] = e^{(\mu + \frac{\sigma^2}{2})t}.$$

Thus, it follows that:

$$f_j(q) = \lambda_j \int_0^\infty e^{(-q\tilde{a}_j + q^2\sigma_j^2/2)t} e^{-\lambda_j t} dt = \frac{\lambda_j}{\lambda_j + (1/2)q(2\tilde{a}_j - q\sigma_j^2)},$$

if the denominator is positive, and $f_j(q) = \infty$ otherwise.

Let us look at the domain of $f_j(q)$. First, consider $j \in A$. $f_j(q) < \infty$, if $q > -\lambda_j/\tilde{a}_j$ for $\tilde{a}_j > 0$ and $f_j(q) = 1$ for $\tilde{a}_j = 0$.
For $j \in \Theta\backslash A$ the situation is a bit more complicated. $f_j(q) < \infty$, if $q \in [0, r_j)$, $f_j(q) = \infty$, if $q \in [r_j, \infty)$, and $f_j(r_j-) = \infty$, where r_j is the positive root of the equation

$$q^2 - 2\tilde{a}_j\sigma_j^{-2}q - 2\lambda_j\sigma_j^{-2} = 0,$$

that is, $r_j = r(\lambda_j, \tilde{\alpha}_j, \sigma_j)$,

$$r(\lambda, \alpha, \sigma) := \alpha\sigma^{-2} + \sqrt{\alpha^2\sigma^{-4} + 2\lambda\sigma^{-2}}. \tag{8}$$

The formula (7) can be written in a shorter form where the states are not specified :

$$\Upsilon(q) = \mathbf{E}\left[\sum_{k=2}^{\infty} f_0(q) f_{\vartheta_1}(q), ..., f_{\vartheta_{k-1}}(q) I_{\{\varpi \geq k,\, \vartheta_k = 0\}}\right]. \tag{9}$$

Since we assume that any state of θ can be reached from any other state, $\operatorname{dom}\Upsilon \subseteq [0, \underline{r})$ where $\underline{r} := inf\{q \geq 0 : \Upsilon(q) = \infty\} \leq r_* = \min_j r_j$.

To prove the continuity of the function Υ let us consider rather simple case where $K = 3$ and then generalise the given idea. There are four ways to return to an initial state without repeat. All other cases are these four with a certain number of $1 \to 2 \to 1$ paths. Thus, we can take outside the sum sign these four terms. Using description of regrouping terms above we get the representation of the formula (7). That is

$$\begin{aligned}\Upsilon(q) = &\big[P_{0,1}f_0(q)P_{1,0}f_1(q) + P_{0,2}f_0(q)P_{2,0}f_2(q) + P_{0,1}P_{1,2}P_{2,0}f_0(q)f_1(q)f_2(q) \\ &+ P_{0,2}P_{2,1}P_{1,0}f_0(q)f_2(q)f_1(q)\big] \sum_{k=0}^{\infty}(P_{1,2}f_1(q)P_{2,1}f_2(q))^k.\end{aligned}$$

Note that if $P_{1,2}f_1(q)P_{2,1}f_2(q) < 1$, then

$$\sum_{k=0}^{\infty}(P_{1,2}f_1(q)P_{2,1}f_2(q))^k = \frac{1}{1 - P_{1,2}f_1(q)P_{2,1}f_2(q)},$$

otherwise the above sum is equal to infinity. Thus, Υ is a product of two continuous functions with values in $\bar{\mathbb{R}}_+$, hence it is continuous as well.

For the case with an arbitrary K we get the continuity of Υ from the continuity result for more general functions.

Let us consider a subset $A \subset \{0, 1, ..., K-1\}$. For $i, k \notin A$ we denote by Γ_{ik}^A the set of vectors $(i, i_1, i_2, \ldots, i_m, k)$, $i_j \in A$, $j = 1, \ldots, m$, $m \in \mathbb{N}$. The elements of Γ_{ik}^A are interpreted as parts of sample paths of the Markov chain, that enter A from the state i and leave A to the state k.

We introduce the notation $h_{i,j}(q) = P_{i,j} f_i(q)$ with the natural convention $0 \times \infty = 0$ to shorten the following formulae. We associate with elements of Γ_{ik}^A the continuous functions $q \mapsto h_{i,k}(q)$,

$$q \mapsto h_{i,i_1}(q) h_{i_1,i_2}(q) \ldots h_{i_{m-1},i_m}(q) h_{i_m,k}(q), \quad m \geq 1,$$

with values in $\bar{\mathbb{R}}_+$ and consider the sum (over states i_m and m) of all these functions

$$U_{ik}^A \colon q \mapsto \sum h_{i,i_1}(q) h_{i_1,i_2}(q) \ldots h_{i_{m-1},i_m}(q) h_{i_m,k}(q).$$

We show by induction that $U_{ik}^A \colon \mathbb{R}_+ \to \bar{\mathbb{R}}_+$ is a continuous function and thus prove the theorem since $\Upsilon = U_{00}^{A\setminus\{0\}}$. The result is obvious when $|A| = 1$. Supposing that the assertion is already proven for the case where $|A| = K_1 - 1$ we consider the case where $|A| = K_1$.

In order to express U_{ik}^A through U of sets with smaller size we denote a partition of Γ_{ik}^A. Namely, for $i_1 \in A$ and $n \geq 0$ we define the sets $\Delta_{ik}^{i_1,0} := \{i, k\}$,

$$\Delta_{ik}^{i_1,n} := \{i\} \times \Gamma_{i_1,i_1}^{A\setminus\{i_1\}} \times \cdots \times \Gamma_{i_1,i_1}^{A\setminus\{i_1\}} \times \Gamma_{i_1,k}^{A\setminus\{i_1\}},$$

composed by the vectors with the first component i, followed by $n \geq 0$ blocks formed by vectors from $\Gamma_{i_1,i_1}^{A\setminus\{i_1\}}$, and completed by vectors from $\Gamma_{i_1,k}^{A\setminus\{i_1\}}$. This countable family $\Delta_{ik}^{i_1,n}$, $i_1 \in A$, $n \geq 0$, is a partition of Γ_{ik}^A and thus

$$U_{ik}^A(q) = \sum_{i_1 \in A} h_{i,i_1}(q) U_{i_1 k}^{A\setminus\{i_1\}}(q) \sum_{n=0}^{\infty} \left[U_{i_1 i_1}^{A\setminus\{i_1\}}(q) \right]^n. \tag{10}$$

By the induction hypothesis $U_{i_1 m}^{A\setminus i_1} \colon \mathbb{R}_+ \to \bar{\mathbb{R}}_+$ is a continuous function for every $i_1 \in A$, $m \notin A \setminus i_1$. Using the formula for geometric series, once again, we obtain the desired result. □

5 Use of the result

With the proven continuity of the function $\Upsilon(q)$, one could investigate other properties of this function, proving that there is a unique value γ such that $\Upsilon(\gamma) = 1$. That value γ is represented in the rate of convergence to zero of the ruin probabilities. Namely,

Theorem 2 *Suppose that (6) holds and* $\Pi(|x|^{\gamma_i}) := \int |x|^{\gamma_i} \Pi(dx) < \infty$. *Then*

$$0 < \liminf_{u \to \infty} u^{\gamma_i} \Psi_i(u) \leq \limsup_{u \to \infty} u^{\gamma_i} \Psi_i(u) < \infty.$$

The condition (6) defined above is essential. If all $\sigma_k = 0$ then the ruin probability will be exponentially decreasing similarly to Lundeberg model with relative safety loading in [6] (Section 1.1). So that the Theorem 2 will not hold anymore.

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