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Network community detection from the perspective of time series

Dong Wang, Yi Zhao *

Harbin Institute of Technology (Shenzhen), Shenzhen, 518055, China

HIGHLIGHTS

- Equivalent transformation enables accurate detection of network community from time series.
- Weighting methods are presented to reinforce the quasi-isometry and clustering performance.
- The number of network communities is identified over transformation.

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ABSTRACT

We present a quasi-isometric mapping to transform complex networks into time series, which enables the network distance to be strictly preserved and allows to solve the network clustering problem from the perspective of its time series. In order to reconstruct the network distance characteristics exactly, we weight the network links in several ways and then convert the weighted networks into time series via classical multidimensional scaling (CMDS). Given such a transformation framework, we utilize the criterion of relative eigenvalue gap (REG) to estimate the number of communities of a network. Further, we enunciate that the distributions of two time series from two isomorphic networks are identical. We then apply the distance-based *k*-means algorithm to the generated time series to detect the community structures of complex networks with success. The results of diverse simulated and real networks demonstrate the superiority of quasi-isometry-based time series in network community detection.

1. Introduction

Complex networks and time series both are popular techniques to describe complex systems [1–7]. During the last decade, many methods have been proposed to transform time series into complex networks [8–13], and the dynamics hidden in time series can be characterized by the tools from the network domain. Meanwhile, many efforts [14,15] have been taken to convert networks into time series and the results show that some properties of the networks can be studied by the corresponding time series analysis. Ref. [14] is a pioneer work about how to transform the network to the time series in an explicit way. Their method applies the theory of Toeplitz matrix to make the regular networks (with slight perturbations) to periodic time series (with noise disturbance). So it deals with such regular networks whose matrix is specific Toeplitz matrix (i.e. circular matrix).

Significantly, the equivalence between networks and their resultant time series in some given transformations has been discussed. In particular, the work in Ref. [15] suggests a quasi-isometry between time series and complex networks, thereby

* Corresponding author.
E-mail address: zhao.yi@hit.edu.cn (Y. Zhao).

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giving the theoretical evidence of an equivalent transformation. In this paper, given such a quasi-isometric mapping, we propose to identify the community of a network from its transformed time series. It is demonstrated that the underlying structural character of the network can be exactly captured by its counterpart.

Meanwhile, network clustering is always a popular and challenging problem in network science and practical applications. To address this problem, a number of partitioning algorithms have been proposed. Reference [16] detects communities in large-scale networks by the optimization of modularity. Benson et al. [17] partitions networks based on their higher-order connectivity patterns. Other works [18–20] divide clusters of networks based on spectral graph theory. However, we notice that there are few attempts at detecting the community of networks from the perspectives of time series due to the fact that without an equivalent transformation such characterization is less persuasive or even incorrect.

For this purpose, we apply quasi-isometric transformation method from a network to time series, which makes an equivalent transformation in between feasible. This method is to make an exact configuration of the matrices of weighted networks into a lower dimensional space [21]. In this sense, each network node is coordinated by corresponding vectors, and finally forms a trajectory in this low-dimensional space [15]. As a result of appropriate weighted matrices of the network, the quasi-isometric transformation is established. Based on such transformation, we are able to explore the network community structures from the perspective of time series accurately.

Next, in Section 2, we introduce quasi-isometric mapping from networks to time series and specially, elaborate how to identify the number of communities. In Section 3, we validate the proposed approach with several network examples. Finally, Section 4 presents our conclusions.

2. Method

Let $A = \{a_{ij}\}_{i,j=1}^{n}$ be the adjacency matrix of a given network, where $a_{ij} = 1$, if nodes v_i and v_j are connected, otherwise, $a_{ij} = 0$. In order to get the time series of keeping network distance information more accurately, we assign weights on the network links in several ways. Meanwhile, some methods that describe the graphic distance between nodes have been presented [14,22]. For examples, constant distance (CD): $CD = \{cd\}_{ij}$ is defined as $cd_{ij} = a_{ij}$, if v_i and v_j are adjacent; otherwise $cd_{ij} = w(> 1)$, which is constant (i.e. w is set to 1.01 [14]). The shortest distance ($SD = \{sd\}_{ij}$) directly adopts the shortest path length between network nodes as link weight. Weighted distance (WD), $WD = \{wd_{ij}\}$, for each existing link between v_i and v_j, defines the weight as $wd_{ij} = 1 - \frac{|G_i \cap G_j|}{|G_i \cup G_j|}$, where G_i denotes the set of nodes connected with v_i and $|G_i|$ shows the cardinality of elements in set G_i. Novelty distance (ND): $ND = \{nd_{ij}\}$ is defined as $sd_{ij} - \frac{|G_i^* \cap G_j^*|}{|G_i^* \cup G_j^*|}$, where $G_i^* = \{k : sd_{ik} \leqslant sd_{ij}\}$ represents the number of nodes of reaching v_i through no more than sd_{ij} steps. Gravitational Distance($GD = \{gd_{ij}\}$): $gd_{ij} = 1 - \sum_{r=1}^{R} \frac{(N_{ij}^r/N_i^r)(N_{ji}^r/N_j^r)}{\ln(r^2+1)}$, where R is the boundary of all possible transfer steps (usually R is set to 3), N_{ij}^r denotes the number of paths of node v_i through r steps to v_j, and $N_i^r = \sum_{j=1}^{n} N_{ij}^r$ means the sum of different transfer paths of v_i through r steps to reach other nodes. We assign the weights to the connected links according to the given equation above, and then get the weighted matrix of the network in terms of these weights and the shortest path matrix for any pair of nodes, and further obtain series of the weighted network distance matrices $D = \{d_{ij}\}$, where d_{ij} is the shortest distance between nodes v_i and v_j based on these new link weights.

Next, following the procedure of CMDS, we get a squared distance matrix $S = \{d_{ij}^2\}$, and then S is transformed into a Gram matrix $G^c = -\frac{1}{2}J_n S J_n^T$, where $J_n = E - \frac{1}{n}\mathbb{1}_n\mathbb{1}_n^T$, E is an $n \times n$ identity matrix, and $\mathbb{1}_n$ is a column vector with n ones. Then we perform spectral decomposition of the matrix $G^c = XX^T$ to calculate the node coordinate values that preserve the defined distances, where $X = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_h)^T = diag(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots, \sqrt{\lambda_h})(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_h)$, λ_i and $\mathbf{p}_i = (p_{i1}, p_{i2}, \ldots, p_{in})^T (i = 1, \ldots, h)$ are the eigenvalues and eigenvectors of G^c correspondingly, and h is the number of nonzero eigenvalues. So $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{ih})^T$ is the h-dimensional coordinate value of the node v_i, which is regarded as h-dimensional time series. That is, the distance of $\mathbf{x}_i$ and $\mathbf{x}_j$ equals the distance of node v_i and v_j [21]. In practice, we usually choose a low-dimensional optimal matrix of X as time series, i.e., $s_i(t) = \sqrt{\lambda_i} p_{it} (1 \leqslant i \leqslant h, 1 \leqslant t \leqslant n)$, which holds a low-dimensional geometric representation of the given network [15]. As a result, the Euclidean norm of the time series is an approximation of D, thereby making the weighted distances of network (i.e. the network structure in fact) preserved in time series and ready for clustering. Namely, we accomplish the isometric mapping from networks to time series.

We note that CMDS employs the technique of spectral decomposition. But only the spectral decomposition that does not discuss the equivalent transformation between complex network and time series, cannot guarantee exact preservation of nodes' distance. Moreover, technically the spectral decomposition usually deals with the Laplacian matrix of network similarity matrix, rather than the weighted network distance matrix described above. Consequently, we emphasize a quasi-isometric transformation from the network to time series so as to divide the network community from the perspective of time series counterpart.

We first apply amplitude difference method [23] to transform a time series into a network $A = \{a_{ij}\}_{i,j=1}^{n}$, i.e., $a_{ij} = 1$ if $|s_i - s_j| < \varepsilon$, $a_{ij} = 0$ otherwise, where $\{s_i\}_{i=1}^{n}$ is a scalar time series of n observations. Here, we adopt the x-component data of a Lorenz system in chaotic regime as an example,

$$\dot{x} = a(y - x),\ \dot{y} = x(b - z) - y,\ \dot{z} = xy - cz,$$

where a = 10, b = 28, and c = 8/3.

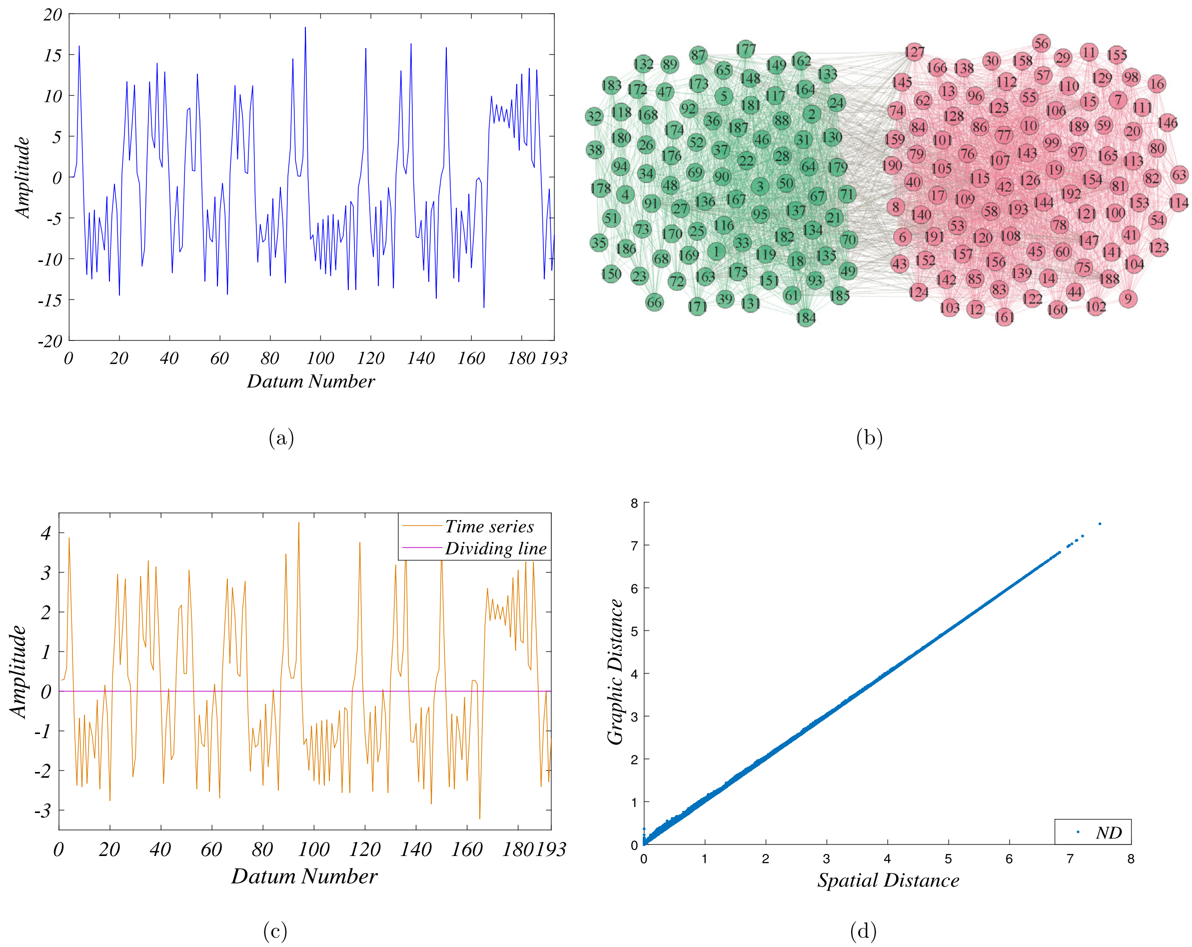

Fig. 1. (a)–(c) Transformation cycle between time series and its network. (a) The original time series of Lorenz system in the chaotic regime, (b) its transformed network, (c) the time series subsequently induced from (b) by ND weighting method and CMDS, and (d) the Euclidean distance of the generated time series in phase space versus graphic distances of the corresponding network nodes. Note that the network nodes' sequences are the time series sequence by default.

A time series of original data points is generated, which is subsequently transformed into a network with $\varepsilon = 3$. Previously, we demonstrate that the tiny transformation threshold (like ε value in this case) enables a strict quasi-isometric transformation from the time series to its network counterpart. That is, the network structure preserves the dynamical features of time series. We then detect the network communities that are determined by the original time series, via using the method described above. We also note that for other transformation methods the theoretical discussion about the equivalence appears to be feasible and promising. The application of quasi-isometric equivalent transformation to community detection just set an example to other methods. Fig. 1 depicts a schematic diagram of quasi-isometry between the time series and transformed network. We just take the eigenvector corresponding to the maximum eigenvalue as the generated time series since the maximal eigenvalue contributes over 90% of the sum of all nonzero eigenvalues in this case.

It is very interesting that through the whole transformation cycle the original time series in Fig. 1(a) and the generated time series in Fig. 1(c) have almost identical waveform but just with different amplitudes, thereby implying that the network structure retains the original time series distance information in terms of the ND aiming to describe the nodes distance. As a result, CMDS can revert the underlying dynamics of original time series through the network precisely. From Fig. 1(d), we observe that there exists a strictly linear relationship between the network and generated time series, which suggests that the time series can preserve the geometrical features of networks. Namely, the network distance among the nodes approximately equals to the corresponding amplitude difference (i.e. the Euclidean distance) of time series points. The other weighting methods have been also validated and their performances are shown in Fig. 2. It is clear that all these methods keep linear relationship or approximately linear tendency, and specifically ND and WD outperform the others. Next, we apply the quasi-isometric time series analysis method to various weighted networks.

Now back to the problem of network community detection, first of all the primary question is how to determine the number of communities. From Fig. 1(a) the chaotic time series fluctuates around two amplitude levels, and the

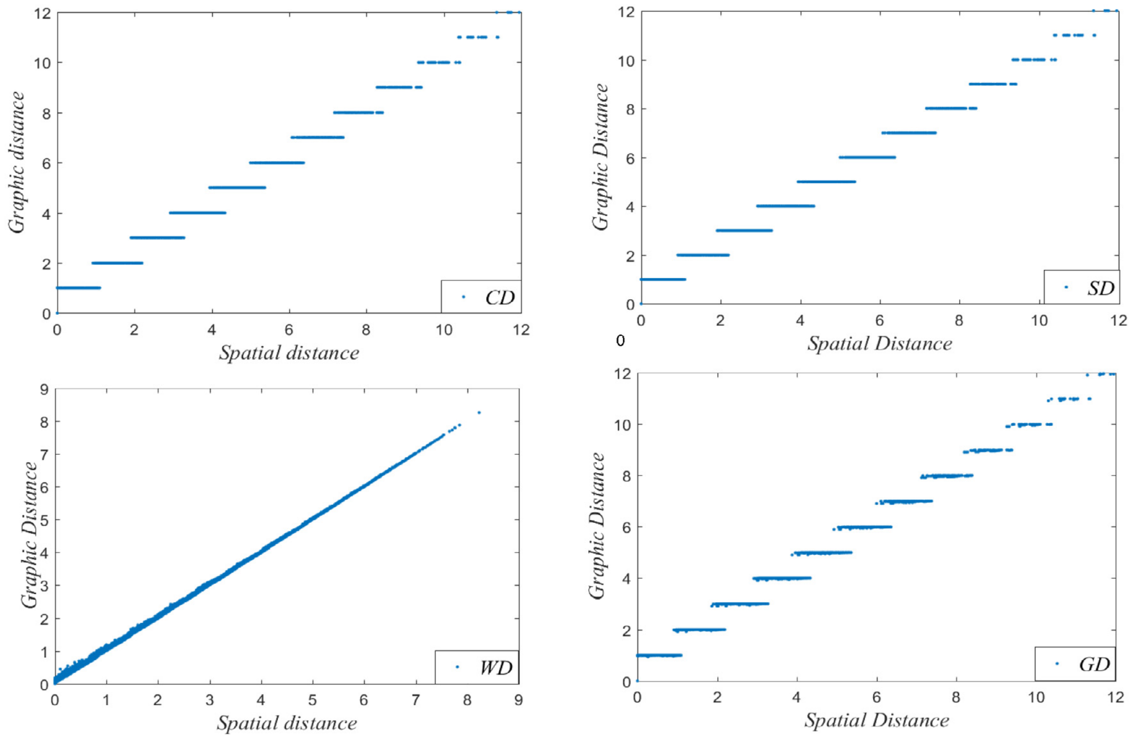

Fig. 2. The Euclidean distance between graphic distances of the corresponding network nodes by using CD, SD, WD and GD versus the generated time series in phase space.

transformed network naturally takes the presence of two communities by using strict quasi-isometric transformation, indicating that the network should be divided into two communities. We improve the eigenvalue gap statistic [24] by $k := \arg\max\{i|\max\{\mathrm{REG}_i = \frac{\lambda_{i-1}-\lambda_i}{\lambda_i}, i = 2, \ldots, h, \lambda_1 > \lambda_2 > \cdots > \lambda_h\}\}$ which is regarded as the number of communities. We notice that the more the number of communities are divided, the more detailed the network structure can be showed. Here we are inclined to explore the precise community structure of networks. So the larger k is then selected if there are two close large REG. Note that the more time series are utilized, the more accurate the network distance is preserved. We are inclined to employ more time series to cluster the network communities. In the following examples, for simplicity and fair comparison, the top $(k-1)$ time series corresponding to eigenvalues $\lambda_1, \ldots, \lambda_{k-1}$ are used to detect the k communities of the network once the value of k is determined. Note that we just show these selected time series in the following figures if the generated time series is plotted. Fig. 3 presents the results about the estimation of the number of communities.

It is consistent that the REG_2 is the maximal relative eigenvalue-gap from all weighting methods so the network is divided into two communities. In the following we just show the estimation results of ND weighting methods since it realizes the most accurate preservation of geometrical distances. As a result of the quasi-isometric transformation from the network to its time series, we can just employ the common k-means algorithm to the generated time series, thereby exactly revealing the network community structures.

For a complex network of n nodes, the adjacent matrices may vary due to different displacement of node orders, which have $n!$ forms in total. The time series from the various forms of distance matrices caused by such displacement, take distinct waveform. If network B is shuffled from network A, namely, A and B are isomorphic, then there exists a permutation matrix P such that $B = PAP^T$. Apparently, their distance matrices are still isomorphic, that is $D(B) = PD(A)P^T$. Further, by CMDS we can obtain $G_A^c = PG_B^cP^T$, and $G_A^c = XX^T$, $G_B^c = YY^T$. Let $X = PY$, and then network A and its shuffled network B have the same time series. Since P is a permutation matrix and the Euclidean distances of two nodes in two isomorphic networks are the same in amplitude, it will not change their own low-dimensional coordinate values. So, we merely use the amplitudes of the time series which exactly describe the network distance to detect its community structures. Accordingly, by employing the k-means clustering algorithm that is amplitude-based (i.e. distance-based), we are able to detect the communities based on the geometrically equivalent transformation even though the time series appears to be peculiar.

The purple line (i.e. the average location of two community centers) in Fig. 1(c) is regard as the divisional line based on the $k(=2)$-means algorithm, thereby classifying the data points into two categories, which exactly match those nodes from the two communities in Fig. 1(b). This horizontal line is determined by the k-means algorithm, which is only applicable to two-classification problems. Fig. 1(b) displays the network transformed from the original time series in Fig. 1(a) by using a tiny amplitude difference (i.e. ε). Owing to this amplitude difference transformation, the resulting network will naturally

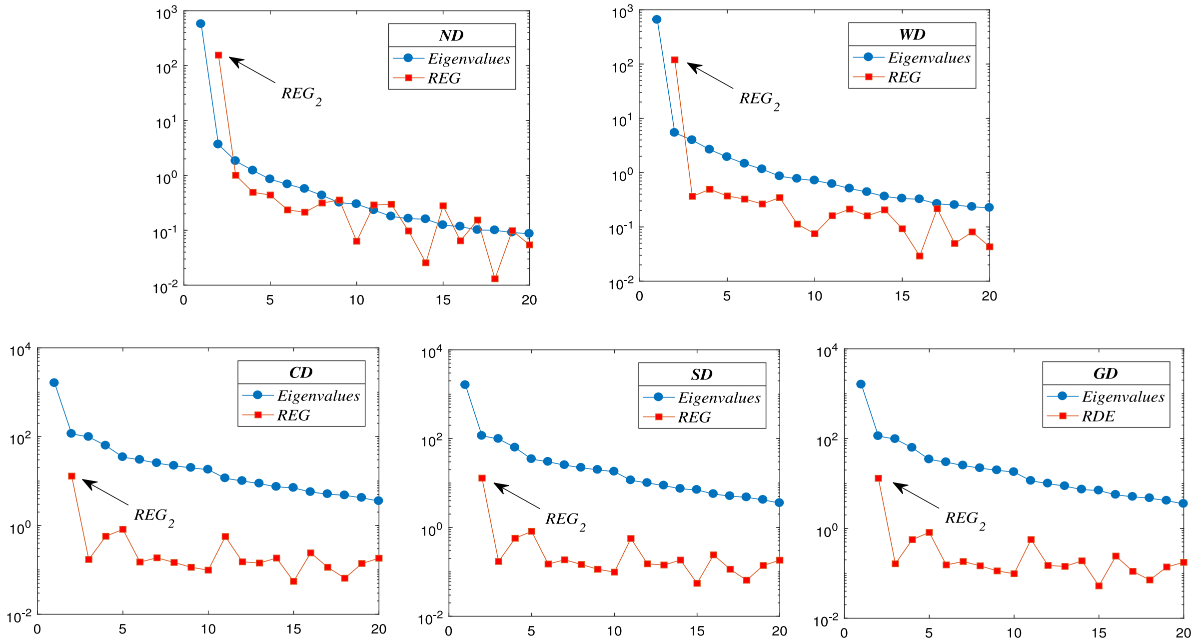

Fig. 3. The eigenvalue characters and relative eigenvalue gap (REG) for different weighting methods.

Table 1
The MEC for different weighting methods.

Methods	CD	SD	WD	ND	GD
MEC	0.4641	0.4641	0.4184	0.4087	0.4637

form two community structures, each of which just corresponds to the high and low fluctuation ranges of the time series. That is, the network in Fig. 1(b) includes two communities really. Next, we apply the proposed equivalent transformation to obtain the generated time series in Fig. 1(c). These two network communities are successfully identified by classifying this time series. Accordingly, the different communities are marked by different color.

The mean error criterion (MEC) is a popular criterion to evaluate the performance of network community clustering: $E = \sum_{i=1}^{k} \sum_{\mathbf{x}\in C_i} |\mathbf{x} - \overline{\mathbf{x}}_i|^2$, where $\mathbf{x}$ and $\overline{\mathbf{x}}_i$ are the coordinates of network's nodes and the average coordinates in the communities of C_i, respectively. Note that $\mathbf{x}$ and $\overline{\mathbf{x}}_i$ could be vectors. This criterion aims to describe the lower distances within groups but larger distances between groups so MEC is defined as the ratio of distance between intra-community and inter-community based on weighting methods. The smaller the MEC is, the better the clustering performance is. The results of the MEC for different weighting methods are presented in Table 1. With reference to Table 1 in conjunction with Figs. 1(d) and 2, we notice that the values of MEC are highly consistent with the quasi-isometric mapping performance, i.e., the clustering performance of ND is prominent. Therefore, for the sake of brevity and readability, in the following experiments, we focus on the ND method to partition networks and other methods may be used for comparison.

3. Results

In order to demonstrate the utility of the proposed approach, let us study four typical network examples, one artificial model and three real networks.

3.1. Artificial networks

We first validate the proposed methods with a simulated complex network constructed with 4 communities, in which a regular network (i.e., ring lattice network with $N = 20$, $K = 7$), an ER network (with $N = 20$, $p = 0.33$) [25], a scale-free network ($N = 20$, $m = 7$, initially with a 7-node fully connected network) [26], a small world network (a regular network with a disturbance probability 0.1) [27] and few (i.e. 2 in this paper) links among them form the whole network. Namely, the procedure of generating a network with community structures is that we firstly generate four networks by the previous models respectively as the subnetworks and then assign some links between subnetworks so as to construct a network with different community structures. A constructed network and its matrix as examples are presented in Figs. 4(a) and 4(b).

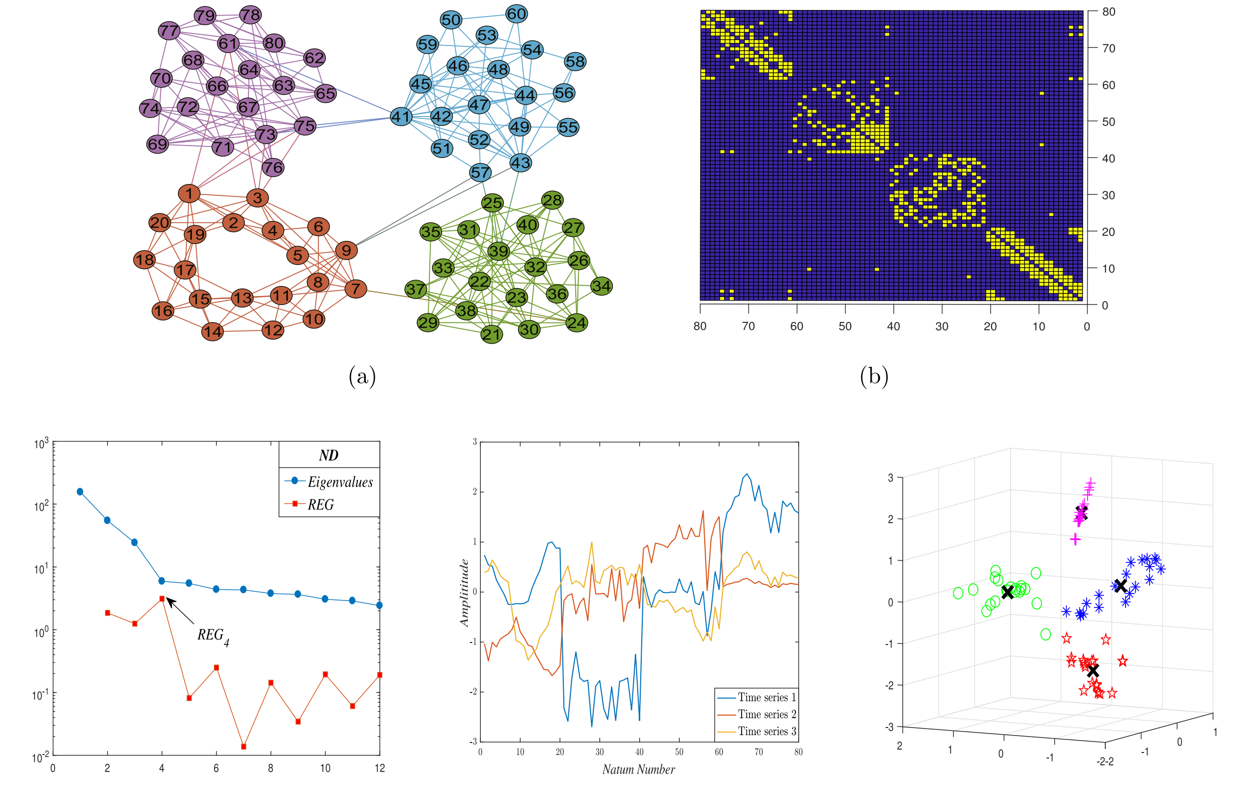

Fig. 4. (a) An artificial network with 4 communities, (b) the pattern of this network matrix, (c) the plot of eigenvalues and REG of the network, (d) the generated first three-dimensional time series, and (e) clustering results of the generated time series by $k(=4)$-means algorithm.

Table 2
The MECs for different weighting methods, which correspond to the three cases.

Methods	CD	SD	WD	ND	GD
MEC1	0.1617	0.1617	0.1502	0.1453	0.1600
MEC2	0.1393	0.1392	0.1302	0.1212	0.1387
MEC3	0.1395	0.1392	0.1335	0.1135	0.1410

As presented in Fig. 4(c), the maximal REG appears at REG_4 so the network is suggested to be separated into 4 clusters. We regard the 3 eigenvectors corresponding to first three largest eigenvalues as the time series of this network. By applying the $k(=4)$-means algorithm to the time series, we can observe that the data points in each cluster correspond to the nodes of each community in Fig. 4(e).

Given four weighting strategies (i.e. ND, WD, SD and GD), we compare the relationship between the graphic distances on the original network and the Euclidean distance of the transformed time series in phase space, as presented in Fig. 5. We then observe that ND and WD weighting methods retain better quasi-isometric effects.

Similarly, we further explore the network in which the subnetworks have the same structure. That is, four ER and SF subnetworks are utilized to construct the networks with 4 communities, respectively. Note that the presented method and the conclusive results are applicable to the generated networks with different combinations of subnetwork models so the time series-based clustering method in terms of the equivalent transformation appears to be independent of individual network models.

Next, we compute the MEC for these weighting methods in the proceeding three cases [i.e. the network with four different communities (MEC1), network with four ER communities (MEC2), and network with four SF communities (MEC3)]. To achieve a reliable MEC, we randomly generate 1000 networks for each case. The results on average are shown in Table 2. We find that the clustering performance of ND weighting method outperforms the others due to the fact that it achieves the exact quasi-isometry from the network to the time series in comparison with other weighting methods.

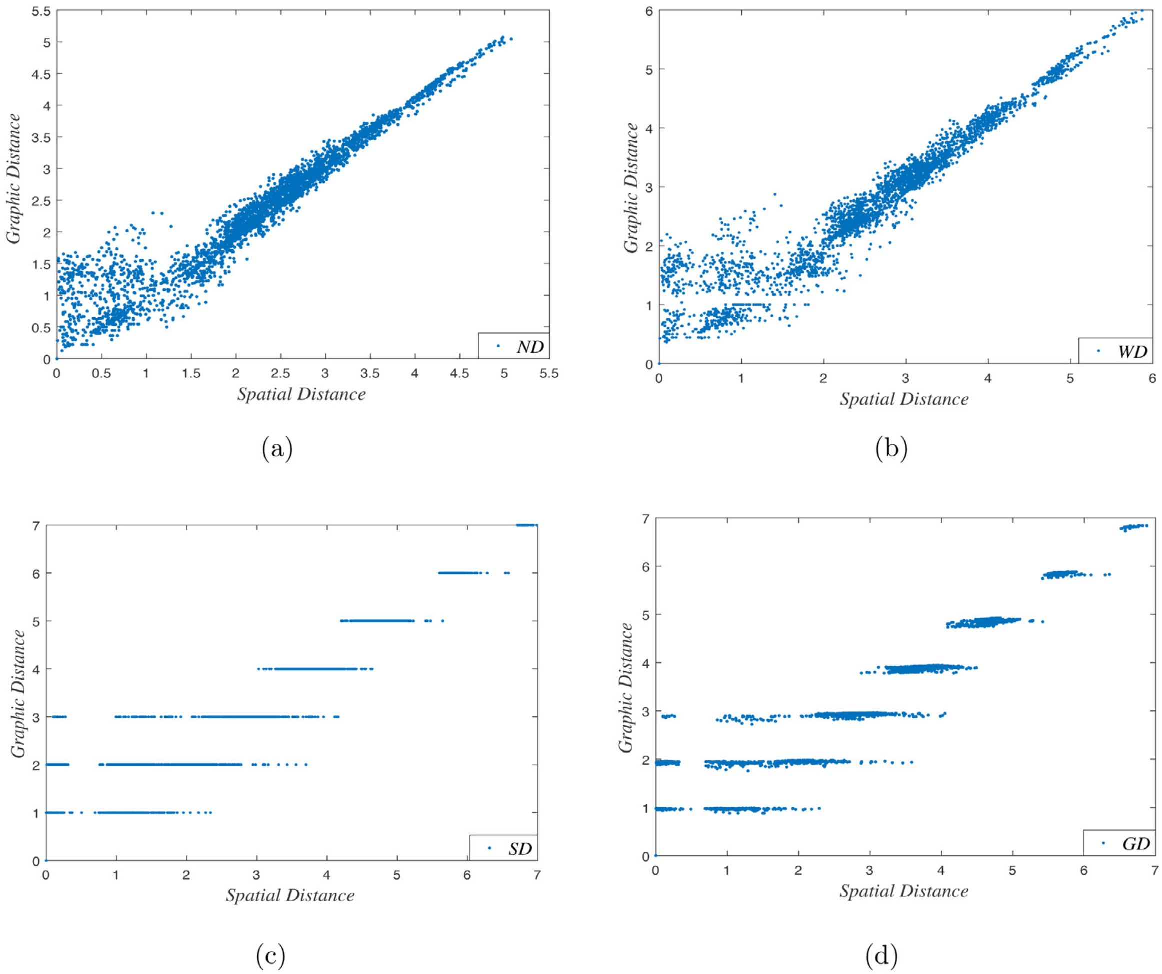

Fig. 5. The Euclidean distance between graphic distances of the corresponding network nodes versus the generated time series in phase space in terms of ND, WD, SD, and GD.

Table 3
The MEC for different weighting methods.

Methods	CD	SD	WD	ND	GD
MEC	0.1481	0.1450	0.1301	0.1199	0.1402

3.2. Les Miserables network

Here, we take a network with multiple communities, Les Miserables network [28] (i.e. the co-appearance network of characters in the novel *Les Miserables*) as a case study. Note that its real communities are unknown. The proposed time series-based network community detection method is employed to explore the community structure of this network.

In Fig. 6(a), two large REG appeared, REG_3 and REG_6, which means that three or six communities are two reasonable options for this network community detection. For REG_3 the network is identified as three communities in the relatively coarse level while for REG_6 the six communities are achieved in the refined level. It is observed that the two large communities in Fig. 6(e) are divided into five new communities in Fig. 6(f), thereby making more structural characters revealed. The two clustering results as well as their meaning are confirmed in Ref. [29], where this network was categorized into three clusters in the initial large scale, and then into six in small scale. The quasi-isometric effects of the two kinds of clustering are presented in Figs. 6(b) and 6(c). According to the two quasi-isometric effects, it appears to be more reasonable to partition the network into six communities. As comparisons, we summarize the MEC of the other different weighting methods in Table 3. It indicates that the better the quasi-isometric effect of weighting methods obtains, the more accurate the network division is.

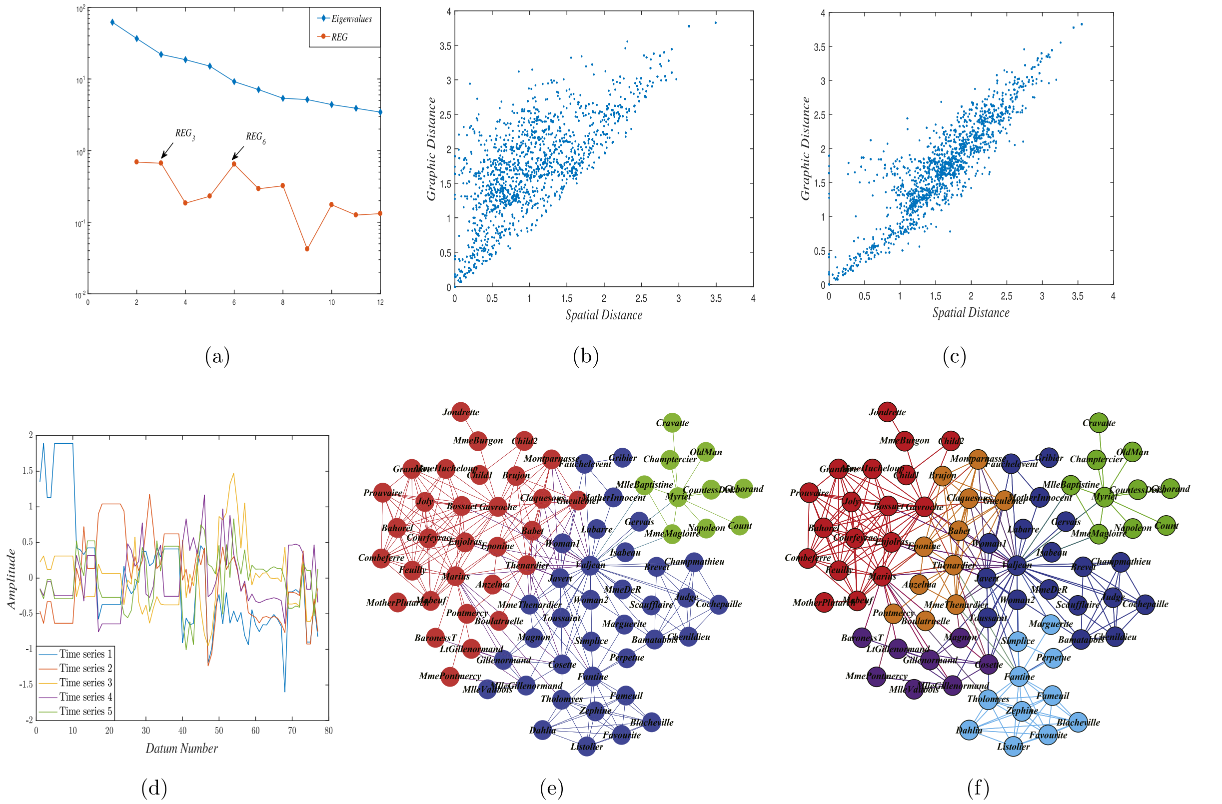

Fig. 6. (a) Eigenvalues and REG, (b) and (c) quasi-isometry corresponding to REG_3 and REG_6, (d) the generated top five-dimensional time series, (e) the community structure of Les Miserables network according to REG_3, and (f) the community structure of *Les Miserables* network according to REG_6.

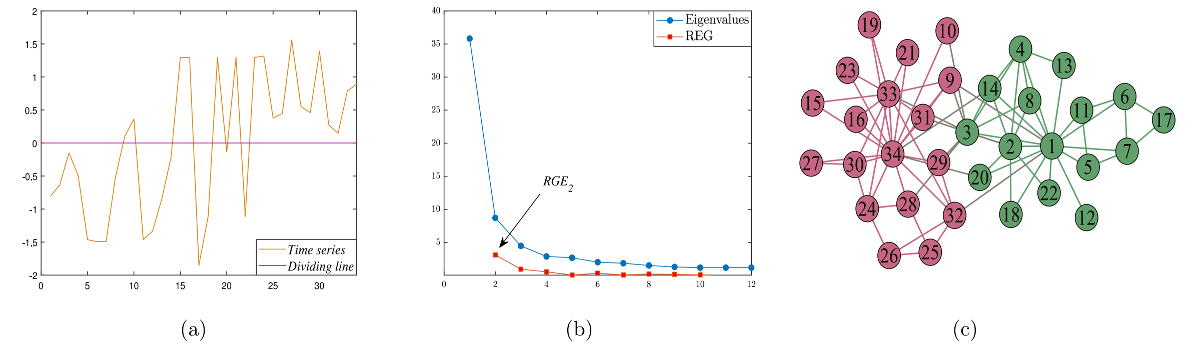

Fig. 7. (a) Time series from karate network, (b) eigenvalues and REG, and (c) the karate network with two communities detected.

3.3. Karate club network

Zachary's karate club network [30] is usually a typical benchmark for community detection. As shown in Fig. 7(b), the largest REG_2 indicates that this network should be divided into two groups. The k-means algorithm gives horizontal line to partition the time series into two categories, thereby identifying the two communities in the network, as shown in Fig. 7(c).

For the sake of brevity, we do not show the quasi-isometric effects. Based on the generated time series of karate network, the accurate community structure is detected. We also compute the MEC of the karate club network versus different weighting methods. Again, the ND method ensures the strictest quasi-isometric transformation, thereby making the best clustering performance by using k-means algorithm. It is conclusive that the identification of network communities through the generated time series is feasible and effective as the quasi-isometric mapping enables time series to preserve the network structure distances in terms of these weighting methods (see Table 4).

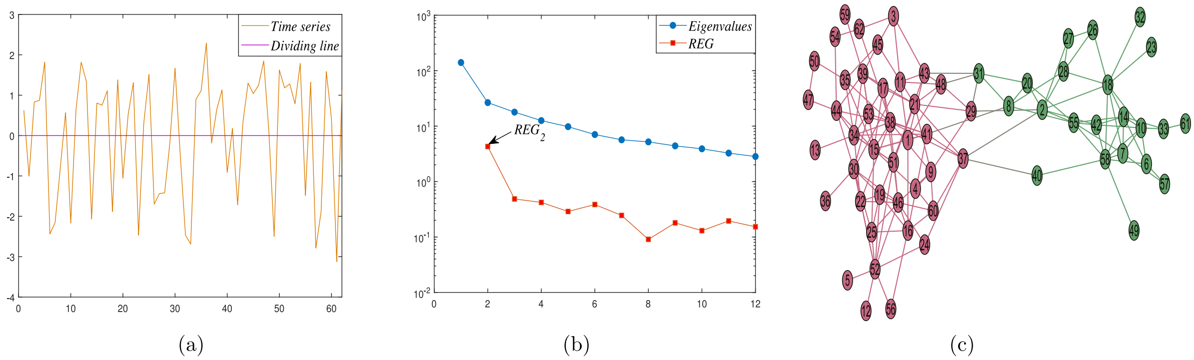

Fig. 8. (a) Time series from dolphin network, (b) eigenvalues and its REG, and (c) the prediction structure of dolphin network.

Table 4
The MEC of different weighting methods.

Methods	CD	SD	WD	ND	GD
MEC	0.6007	0.6007	0.5868	0.5765	0.5950

Table 5
The MEC for different weighting methods.

Methods	CD	SD	WD	ND	GD
MEC	0.5994	0.5994	0.5780	0.5746	0.5967

3.4. Dolphin network

We employ the preceding method to another real complex network, dolphin network [31], which is naturally divided into two groups: female and male. In Fig. 8, we show the community detection results.

From the detected community structure, we find that the *k*-means algorithm accurately categorizes the data points of generated time series that belong to male and female groups. MEC of different weighting methods is shown in Table 5. As all the weighting methods keep approximately or even strictly linear relationship, the quasi-isometric transformation that is valid with their helps then ensures reliable clustering results.

4. Conclusion

In summary, according to the quasi-isometric mapping between time series and complex networks, we present a novel perspective to detect network communities via the generated time series. By various weighting methods and CMDS approach, we describe the network distance and demonstrate a quasi-isometric transformation, which serves as a basis for identifying communities. Significantly, we propose the relative eigenvalue gap to determine the number of communities. Employing the distance-based *k*-means algorithm, various simulated and realistic networks are used to identify their communities, thereby verifying the feasibility and superiority of the proposed method. Moreover, the MEC criterion as a measure of partition accuracy, confirms the importance of a quasi-isometric transformation from a network to its time series for network community detection.

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