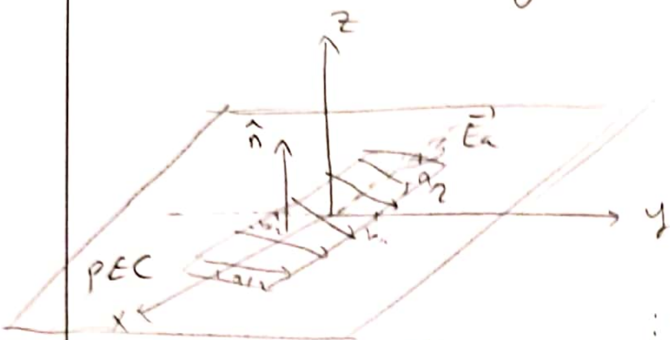
1	تاریخ تحویل: / /	نام درس: آنتن ۱ — تحلیل سری دوم	سروش مس‌فروش مشهد شماره دانشجویی: 810198472

۱- پیش از اقدام به حل، ابتدا هر چه درباره آن می‌دانیم می‌نویسیم

ابعاد روزنه: a, b $\quad |x| < \frac{a}{2}$, $|y| < b/2$ $\quad a > b$ صفحه رسانای بی‌کران PEC داریم

مد غالب: TE_{10}: $\vec{E_a} = \hat{y} E_0 \cos(\frac{\pi}{a} x')$ همینطور که گفتیم،



$\vec{M}$ = Magnetic current Density (MMF) $\left[\frac{Volt}{m^2}\right]$

$\vec{\nabla}\cdot\vec{D'} = 0 \rightarrow \vec{D'} = \nabla \times \vec{F'} : \vec{E}_F = -\frac{1}{\epsilon_0}\nabla\times\vec{F'}$

فضای بدون بار

حال فرض می‌کنیم که $\vec{M'}$ وجود دارد، خواهیم داشت:

Maxwell :
$$\begin{cases} \nabla\times\vec{E'} = -\frac{\partial B}{\partial t} \\ \nabla\times\vec{H'} = \frac{\partial D}{\partial t} + J \\ \vec{\nabla}\cdot\vec{D} = 0 \\ \nabla\cdot H = 0 \\ \nabla\times\vec{J'} = -\frac{\partial \rho}{\partial t} \end{cases}$$

$$\xrightarrow[z=0]{\text{مرزی}} \vec{\nabla}\times H = j\omega D = j\omega\epsilon_0 \vec{E}, \quad \vec{E} = -\frac{1}{\epsilon_0}\vec{\nabla}\times\vec{F}$$

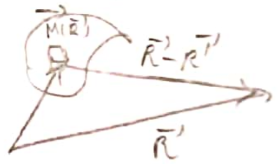
$$\Rightarrow \vec{\nabla}\times(\vec{H'} + j\omega\vec{F'}) = 0 \rightarrow \vec{H'} + j\omega\vec{F'} = -\vec{\nabla}\Phi_m$$

$$\rightarrow \vec{H'}_F = -j\omega\vec{F'} - \vec{\nabla}\Phi_m + \quad \text{معادل لورنتس} : \vec{\nabla}\cdot\vec{F'} + j\omega\mu_0\epsilon_0\Phi_m = 0$$

Φ_m را جایگذاری در #

$$\vec{H}_F = -j\omega\left(\vec{F'} + \frac{1}{k^2}\vec{\nabla}\vec{\nabla}\cdot\vec{F'}\right), \quad k = \omega\sqrt{\mu_0\epsilon_0} \qquad \text{Helmholtz: } \nabla^2\vec{F'} + k^2\vec{F'} = -\epsilon_0\vec{M}$$

$$\rightarrow \vec{F'}(\vec{R}) = \frac{\epsilon_0}{4\pi}\int_{v'} \frac{\vec{M'}(\vec{R'})\, e^{-jk|\vec{R}-\vec{R'}|}}{|\vec{R}-\vec{R'}|} dv'$$



در ادامه از قضیه معادل شکلانوف استفاده می‌کنیم.

$$\vec{M}_s = 2\vec{E'}\times\hat{n} \qquad \hat{n} = \hat{z} \qquad \rightarrow \vec{M'_s} = 2E_0\cos(\frac{\pi}{a}x')\,\hat{y}\times\hat{z}, \quad \hat{y}\times\hat{z} = \hat{x}$$

$$\rightarrow \boxed{\vec{M}_s = 2E_0\cos(\frac{\pi}{a}x')\,\hat{x}}$$

در ادامه به دنبال بردار تابش خواهیم بود

$$\vec{L}(\theta,\varphi) = \int_{v'} \vec{M'}(\vec{R'})\, e^{jk\hat{R}\cdot\vec{R'}} dv' \qquad \hat{R}\cdot\vec{R'} = x'\sin\theta\cos\varphi + y'\sin\theta\sin\varphi$$

$$\rightarrow \vec{L}(\theta,\varphi) = 2E_0\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\cos(\frac{\pi}{a}x')\, e^{jk(x'\sin\theta\cos\varphi + y'\sin\theta\sin\varphi)}\, dx'dy'$$

تفکیک انتگرال

$$= 2E_0 \underbrace{\int_{-b/2}^{b/2} e^{jky'\sin\theta\sin\varphi}\, dy'}_{I_1} \times \underbrace{\int_{-a/2}^{a/2}\cos(\frac{\pi}{a}x')\, e^{jkx'\sin\theta\cos\varphi}\, dx'}_{I_2}$$

$$I_1 = \frac{1}{jk\sin\theta\sin\varphi}\, e^{jky'\sin\theta\sin\varphi}\Big|_{-b/2}^{b/2} = 2\,\frac{\sin(\frac{kb}{2}\sin\theta\sin\varphi)}{k\sin\theta\sin\varphi}$$

$$I_2 = \int_{-a/2}^{a/2} \cos\left(\frac{\pi}{a}x'\right) e^{jkx'\sin\theta\cos\varphi}\,dx' = \int_{-a/2}^{a/2} \cos\left(\frac{\pi}{a}x'\right)\left[\cos(kx'\sin\theta\cos\varphi)\right.$$

$$\left.+ j\sin(kx'\sin\theta\cos\varphi)\right] = 2\int_0^{a/2} \cos\left(\frac{\pi}{a}x'\right)\cos(kx'\sin\theta\cos\varphi)\,dx'$$

$$= \int_0^{a/2} \cos\left(\left[\frac{\pi}{a} - k\sin\theta\cos\varphi\right]x'\right) + \cos\left(\left[\frac{\pi}{a} + k\sin\theta\cos\varphi\right]x'\right)dx'$$

$$= \frac{\sin\left(\pi/2 - \frac{ka}{2}\sin\theta\cos\varphi\right)}{\frac{\pi}{a} - k\sin\theta\cos\varphi} + \frac{\sin\left(\frac{\pi}{2} + \frac{ka}{2}\sin\theta\cos\varphi\right)}{\frac{\pi}{a} + k\sin\theta\cos\varphi}$$

$$= \frac{\cos\left(\frac{ka}{2}\sin\theta\cos\varphi\right)}{\frac{\pi}{a} - k\sin\theta\cos\varphi} + \frac{\cos\left(\frac{ka}{2}\sin\theta\cos\varphi\right)}{\frac{\pi}{a} + k\sin\theta\cos\varphi} = \frac{\frac{2\pi}{a}\cos\left(\frac{ka}{2}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}$$

$$\vec{L}'(\theta,\varphi) = \frac{4\pi}{a}\,\frac{\cos\left(\frac{ka}{2}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}\,\frac{\sin\left(\frac{kb}{2}\sin\theta\sin\varphi\right)}{k\sin\theta\sin\varphi} \qquad k = \frac{2\pi}{\lambda}$$

$$\Rightarrow \vec{L}'(\theta,\varphi) = \frac{4\pi}{a}\,\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}\cdot\frac{\sin\left(\frac{\pi b}{\lambda}\sin\theta\sin\varphi\right)}{\frac{2\pi}{\lambda}\sin\theta\sin\varphi} \qquad \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\rightarrow \vec{L}'(\theta,\varphi) = \frac{4\pi E_0 b}{a}\,\mathrm{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\varphi\right)\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}\,\hat{x}$$

$$\hat{x} = \sin\theta\cos\varphi\,\hat{R} + \cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}$$

مولفه $\hat{R}$ در راه دور حذف می‌شود.

$$\Rightarrow \vec{L}'(\theta,\varphi) = \frac{4\pi E_0 b}{a}\,\mathrm{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\varphi\right)\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}\left[\cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}\right]$$

$$\vec{H}' = -j\omega\vec{F}_t \qquad \vec{F}_t = -j\omega\epsilon\,\frac{e^{-jkR}}{4\pi R}\,\vec{L}_t(\theta,\varphi) \qquad k = \omega\sqrt{\mu\epsilon} \qquad Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

$$\Rightarrow \vec{H}' = \frac{j4\pi E_0 b k}{a Z_0}\,\mathrm{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\varphi\right)\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}\,\frac{e^{-jkR}}{4\pi R}\left(\cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}\right)$$

$$\vec{E} = Z_0(\vec{H}'\times\hat{R}) \Rightarrow \vec{E}' = \frac{j4\pi E_0 b k}{a}\,\mathrm{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\varphi\right)\frac{\cos\left(\frac{\pi a}{\lambda}\sin\theta\cos\varphi\right)}{\left(\frac{\pi}{a}\right)^2 - (k\sin\theta\cos\varphi)^2}$$

$$\cdot\frac{e^{-jkR}}{4\pi R}\left(+\cos\theta\cos\varphi\,\hat{\varphi} + \sin\varphi\,\hat{\theta}\right)$$

Scanned by CamScanner

$\phi = \frac{\pi}{2} \Rightarrow$ E-plane $\vec{E}' = \frac{j 4\pi E_0 b k}{a} \operatorname{Sinc}\left(\frac{b}{\lambda} \sin\theta\right) \frac{a^2}{\pi^2} \frac{e^{-jkR}}{4\pi R} \hat{\theta}$

$\Rightarrow \vec{E}' = \frac{j E_0 b k a}{\pi^2} \frac{e^{-jkR}}{R} \operatorname{Sinc}\left(\frac{b}{\lambda} S\theta\right) \hat{\theta}$

$\phi = 0 \Rightarrow$ H-plane : $\vec{E}' = \frac{j 4\pi E_0 b k}{a} \frac{\cos\left(\frac{\pi a}{\lambda} S\theta C\varphi\right)}{(\pi/a)^2 - (k S\theta C\varphi)^2} \frac{e^{-jkR}}{4\pi R} C\theta \hat{\varphi}$

$= \frac{j E_0 b k}{R a} \frac{C\left(\frac{\pi a}{\lambda} S\theta C\varphi\right)}{(\pi/a)^2 - (k \sin\theta C\varphi)^2} e^{-jkR} C\theta \hat{\varphi} = \frac{j E_0 b a k}{R} e^{-jkR} \frac{C\left(\frac{\pi a}{\lambda} S\theta C\varphi\right)}{\pi^2 - k^2 a^2 S^2\theta C^2\varphi}$

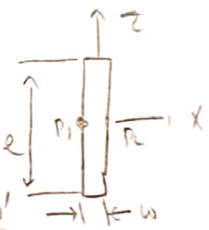
$C\theta \hat{\varphi}$, $D_0 = \frac{4\pi U_{max}}{P_{rad}}$. $P_{rad} = \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} \frac{|E|^2}{2Z_0} dx' dy'$

$= \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} \frac{E_0^2}{2Z_0} C^2\left(\frac{\pi}{a} x'\right) dx' dy' = \frac{1}{4Z_0} E_0^2 ab$ $\qquad D_0 = \frac{4\pi U_{max}}{P_{rad}}$ -

$= \frac{4\pi R^2 \left(\frac{E_0 b k a}{R \pi^2}\right)^2}{2Z_0} \times \frac{4 Z_0}{E_0^2 ab} = \frac{32 ab}{\pi \lambda^2}$

-2

$\vec{E}' = \vec{E}'(x, y, z) = \hat{x} \frac{V_m}{w} \sin(k(l - |z|))$ $\quad 2l = \frac{\lambda}{2}$



$\vec{M}_s = 2\vec{E}' \times \hat{n}$ $\qquad \hat{n} = \hat{y} \Rightarrow \vec{M}_s = \frac{2V_m}{w} \sin(k(l - |z|))\hat{z}$

$\vec{L}_{(\theta,\varphi)} = \int_{v'} \vec{M}(\vec{r}') e^{jk\hat{r}\cdot\vec{r}'} dv' = \hat{z}\frac{4V_m}{w} \int_0^l \sin(k(l - z')) \cos(kz' C\theta) dz'$

$= \hat{z}\frac{2V_m}{w} \int_0^l \sin(kl - kz' + kz' C\theta) + \sin(kl - kz' - kz' C\theta)\, dz'$

$= \hat{z}\frac{2V_m}{w}\left[\frac{\cos(kl C\theta) - \cos(kl)}{k C\theta - k} + \frac{\cos(kl \cos\theta) - \cos(kl)}{k C\theta + k}\right]$

$= \hat{z}\frac{4V_m}{w}\left[\frac{\cos(kl C\theta) - \cos(kl)}{k - k C^2\theta}\right]$ $\quad k = \frac{2\pi}{\lambda} \quad 2l = \frac{\lambda}{2} \Rightarrow \vec{L}(\theta, \varphi) = \hat{z}\frac{4V_m}{w} \frac{C\left(\frac{\pi}{2} C\theta\right)}{\frac{2\pi}{\lambda} \sin^2\theta}$

$L_\theta = -\frac{4V_m}{w} \frac{C\left(\frac{\pi}{2} C\theta\right)}{\frac{2\pi}{\lambda} \sin^2\theta} \hat{\theta}$ $\qquad \vec{H}' = -j\omega F_\theta = +j\omega\epsilon_0 \frac{e^{-jkR}}{4\pi R} \frac{4V_m}{w} \frac{C(\pi/2\, C\theta)}{k \sin^2\theta} \hat{\theta}$

$\vec{E}' = Z_0(\vec{H}' \times \hat{R}) = -j\frac{e^{-jkR}}{4\pi R} \cdot \frac{4V_m}{w} \frac{C(\pi/2\, C\theta)}{\sin\theta} \hat{\varphi}$ $\qquad$ کل عبارت در w ضرب شده

$\Rightarrow \vec{E} = -j V_m \frac{e^{-jkR}}{\pi R} \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$ $\qquad W_{avg} = \frac{1}{2Z_0}|E|^2 \Rightarrow P_{rad} = \int_0^{2\pi}\int_0^{\pi} |E|^2 R^2 S\theta d\theta d\varphi$

4	تاریخ تحویل: / /	نام درس: آنتن ۱ — تکالیف سری دهم	سروش مس‌فروش مشهد شماره دانشجویی: 810198472

اگر بخواهیم خیلی دقیق باشیم. لذا باید از مرجع بمانند :

$$E_\varphi = \frac{jV_m e^{-jkr}}{\pi r \omega}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\hat{\varphi}$$

$$\Rightarrow P_{rad} = \frac{\pi}{Z_0}\frac{V_m^2}{\omega^2\pi^2}\int_0^\pi \frac{\cos^2\left(\pi/2\cos\theta\right)}{\sin\theta}\xrightarrow[\text{Calculator}]{\text{integral}} = \frac{\pi}{Z_0}\frac{V_m^2}{\omega^2\pi^2}\frac{1}{4}\Big[\ln(1-\cos\theta)-Ci(\pi\cos\theta-\pi)$$

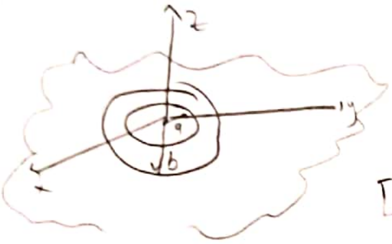
$$-\ln(\cos\theta+1)+Ci(\pi\cos\theta+\pi)\Big] = \frac{\pi}{Z_0}\frac{V_m^2}{\cancel{\omega^2\pi^2}}\times 1{,}2188766965786013$$

$$\frac{1}{2}\frac{|V_m|^2}{R_{rad}} = \frac{\pi}{Z_0}\frac{V_m^2}{\pi^2}\times 1{,}2188766965786013 \Rightarrow R_{rad} = \frac{\pi Z_0}{2\times 1{,}2188766965786013}$$

$Z_0 = 120\pi$

$$\Rightarrow R_{rad} = 485.8576411412\ \Omega$$

3- ابتدا شکل را مجددا رسم می‌کنیم



$$\vec{E}_a = -\hat{r}\frac{V}{\epsilon\ln\frac{b}{a}}\frac{1}{r'},\quad a<r'<b$$

$$a,b \ll \lambda \qquad \vec{M}_s = 2E_a\times\hat{n} \qquad \hat{n}=\hat{z}$$

$$\vec{L}(\theta,\varphi) = \int \vec{M}(r')\,e^{jk\hat{R}\cdot\vec{r}'} \quad , \vec{M}_s = 2E_a\hat{\varphi}'$$

$$\Rightarrow \vec{M}_s = \frac{-2V}{\epsilon\ln\left(\frac{b}{a}\right)}\cdot\frac{1}{\rho'}$$

در $z=0$ هستیم از استوانه‌ای/کارتزین استفاده $r'=\rho'$ می‌گیریم.

$$\vec{R}' = \rho'\cos\varphi'\,\hat{x} + \rho'\sin\varphi'\,\hat{y} \Rightarrow \vec{R}'\cdot\hat{R} = \rho'\sin\theta\cos(\varphi-\varphi')$$

$$\vec{L}(\theta,\varphi) = \int_0^{2\pi}\int_a^b \frac{-2V}{\epsilon\ln(b/a)\rho'}\,e^{jk\rho'\sin\theta\cos(\varphi-\varphi')}\,\rho'\,d\rho'\,d\varphi'\ \hat{\varphi}'$$

$$= \frac{-2V}{\epsilon\ln\left(\frac{b}{a}\right)}\int_0^{2\pi}\int_a^b \left(1+jk\rho'\sin\theta\cos(\varphi-\varphi')\right)\left(-\sin\varphi'\hat{x}+\cos\varphi'\hat{y}\right)d\rho'\,d\varphi'$$

$$= \frac{-2Vjk}{\epsilon\ln(b/a)}\int_a^b \rho'\sin\theta\int_0^\pi \frac{1}{2}\Big[-\big(\sin\varphi+\cancel{\sin(2\varphi-\varphi')}^{\,0}\big)\hat{x}+\big(\cos\varphi+\cancel{\cos(2\varphi-\varphi')}^{\,0}\big)\hat{y}\Big]$$

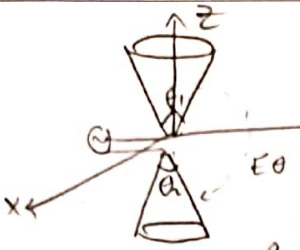
$$d\varphi'\,d\rho' \Rightarrow \vec{L} = \frac{-2Vjk}{2\epsilon\ln(b/a)}\,\pi\sin\theta\,(b^2-a^2)\,\hat{\varphi}$$

R
$\varphi \quad \theta$

$$\Rightarrow \vec{L}_t = \frac{-Vjk\pi\sin\theta}{\epsilon\ln(b/a)}(b^2-a^2)\,\hat{\varphi} \qquad \vec{H}' = -j\omega\vec{F}_t = \frac{-Vk^2(b^2-a^2)\sin\theta\, e^{-jkR}}{4\epsilon R\ln(b/a)Z_0}\hat{\theta}$$

$$\vec{E}' = Z_0(\vec{H}'\times\hat{R}) = \frac{Vk^2(b^2-a^2)\sin\theta\, e^{-jkR}}{4\epsilon R\ln(b/a)}\,\hat{\varphi}$$

5	تاریخ تحویل: / /	نام درس: آنتن ۱ — تکلیف سری هشتم	سروش مس‌فروش مشهد شماره دانشجویی: 810198472



H_φ

۴- آنتن دو مخروطی را بررسی خواهیم کرد.

در درس دیدیم که $\vec{E}$ مولفه θ دارد و همچنین $\vec{H}$ مولفه φ دارد.

جزوه
$$\begin{cases} \vec{\nabla}\times\vec{E}' = -j\omega\mu_0\vec{H}' \\ \vec{\nabla}\times\vec{H}' = -j\omega\epsilon\vec{E}' \end{cases} \longrightarrow \begin{cases} \frac{1}{R}\frac{\partial}{\partial R}(RE_\theta) = -j\omega\mu_0 H_\varphi & \mathrm{I} \\ \frac{\partial}{\partial\theta}(R\sin\theta H_\varphi) = 0 & \mathrm{II} \\ \frac{1}{R\sin\theta}\frac{\partial}{\partial R}(R\sin\theta H_\varphi) = -j\omega\epsilon_0 E_\theta & \mathrm{III} \end{cases}$$

$$\mathrm{II} \Rightarrow H_\varphi = \frac{f(R)}{\sin\theta} \qquad \frac{\partial H_\varphi}{\partial R} + \frac{1}{R}H_\varphi = -j\omega\epsilon_0 E_\theta$$

$$-\frac{1}{j\omega\epsilon_0}\left\{\frac{\partial^2}{\partial R^2}H_\varphi + \frac{\partial}{\partial R}\left(\frac{1}{R}H_\varphi\right) + \frac{\partial H_\varphi}{\partial R}\cdot\frac{1}{R} + \frac{1}{R^2}H_\varphi\right)$$

$$\frac{\partial^2}{\partial R^2}H_\varphi + \frac{2}{R}\frac{\partial H_\varphi}{\partial R} + k^2 H_\varphi = 0 \Rightarrow R\frac{\partial^2}{\partial R^2}f(R) + 2\frac{\partial f(R)}{\partial R} + k^2 R f(R) = 0$$

$$\longrightarrow H_\varphi = A\frac{e^{-jkR}}{R\sin\theta} \qquad E_\theta = Z_0\frac{A e^{-jkR}}{R\sin\theta} = Z_0 H_\varphi$$

$$P_{Rad} = \int U(\theta,\varphi)\,d\Omega \ , \ d\Omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\varphi \qquad U(\theta,\varphi) = R^2 W_{avg}$$

$$W_{avg} = \frac{|E|^2}{2Z_0} \Rightarrow U(\theta,\varphi) = R^2 W_{avg} = R^2\frac{|E|^2}{2Z_0} \ , \quad |E|^2 = \frac{Z_0^2 A^2}{R^2\sin^2\theta}$$

$$\Rightarrow U(\theta,\varphi) = \frac{Z_0 A^2}{2\sin^2\theta} \qquad P_{Rad} = \int_0^{2\pi}\int_{\frac{\theta_1}{2}}^{\pi-\frac{\theta_2}{2}} \frac{Z_0 A^2}{2\sin^2(\theta)}\sin\theta\, d\theta\, d\varphi$$

$$= \int_0^{2\pi}\int_{\frac{\theta_1}{2}}^{\pi-\frac{\theta_2}{2}} \frac{Z_0 A^2}{2\sin\theta}\, d\theta\, d\varphi = \pi Z_0 A^2 \int_{\frac{\theta_1}{2}}^{\pi-\frac{\theta_2}{2}} \frac{1}{\sin\theta}\, d\theta = \pi Z_0 A^2 \ln\left(\tan\left(\frac{\theta}{2}\right)\right)\Big|_{\frac{\theta_1}{2}}^{\pi-\frac{\theta_2}{2}}$$

$$= \pi Z_0 A^2\left[\ln\left(\tan\left(\pi/2 - \theta_2/4\right)\right) - \ln\left(\tan(\theta_1/4)\right)\right] \ , \quad D_0 = \frac{4\pi U_{max}}{P_{rad}}$$

$$= \frac{4\pi r^2 |E|^2}{2Z_0 P_{rad}} = \frac{4\pi A^2 Z_0^2}{\sin^2\theta\, 2Z_0\pi A^2\left[\ln\left(\cot\left(\frac{\theta_2}{4}\right)\right) + \ln\left(\cot(\theta_1/4)\right)\right]}$$

$$= \frac{2}{\sin^2\theta'\left[\ln(\cot(\theta_2/4)) + \ln(\cot(\theta_1/4))\right]} \quad \left\{\theta' = \min(\theta_1, \theta_2)\right.$$

سروش مس‌فروش مشهد شماره دانشجویی: 810198472	نام درس: آنتن ۱ تکلیف سری هشتم	تاریخ تحویل: / /	

5-

$$P_R = P_t D_r D_t \left(\frac{\lambda}{4\pi R}\right)^2 P \cdot q \qquad \lambda = c/f = \frac{3\times10^8}{10^{10}} = 0.03\,m$$

$$D_r = D_t = 15\,dB = 10^{1.5} \qquad R = 3m, \quad P = q = 1 - 0.1^2$$

$$\Rightarrow P_R = 10 \times 10^{1.5} \times 10^{1.5} \times \left(\frac{0.03}{4\pi \times 3}\right)^2 (1-0.1^2)^2$$

$$= 0.0062\,W = -22.07\,dB = 7.93\,dBm$$

6- این مثالی است که جناب دکتر در جلسه درس نیز به آن اشاره کردند که از آن روش MoM بررسی شده است (مناسب دانشجویان کارشناسی)

ابتدا به بررسی انگیزه این مقاله پرداخته شده و بیان می‌شود که کاربردهای این روش در مهندسی چه می‌تواند باشد. پس در ادامه فرمولاسیون روش MoM به شکل زیر مطرح می‌شود.

عملگر

$$L(f) = g \qquad f = \sum_{n=1}^{N} a_n f_n, \qquad \sum_{n=1}^{N} a_n (L f_n) \simeq g$$

$$\sum_{n=1}^{N} a_n \langle L f_n, w_m \rangle = \langle g, w_m \rangle$$

در ادامه فرمولاسیون ماتریسی ارائه می‌شود

$$\underset{\sim}{L} = \begin{bmatrix} \langle L f_1, w_1 \rangle & \cdots & \langle L f_n, w_1 \rangle \\ \langle L f_1, w_2 \rangle & & \\ \langle L f_1, w_n \rangle & & \langle L f_n, w_n \rangle \end{bmatrix} \qquad [a_n] = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad [g_m] = \begin{bmatrix} \langle g, w_1 \rangle \\ \vdots \\ \langle g, w_N \rangle \end{bmatrix}$$

$$\Rightarrow [a_n] \simeq \underset{\sim}{L}^{-1} [g_m]$$

در ادامه مسئله‌های متنوعی حل شده است که یکی از آن به شرح زیر است.

$$-\frac{\partial^2 f(x)}{\partial x^2} = g(x) \qquad f(0) = f(1) = 0 \qquad L = -\frac{\partial^2}{\partial x^2} \qquad f_n = x - x^{n+1}$$

$$g = 6x - 2 \qquad w_n = w_n(x) = f_n(x) \;(\text{Galerkin}) \qquad l_{mn} = \langle L f_n, w_m \rangle = \int_0^1 -\frac{\partial^2}{\partial x^2}(x - x^{n+1})$$

$$\cdot (x - x^{m+1})\, dx = \frac{mn}{m+n+1}, \quad g_m = \langle g, w_m \rangle = \int_0^1 (6x-2)(x - x^{m+1})\, dx = \frac{m(m-1)}{(m+2)(m+3)}$$

$$N = 2 \Rightarrow \underset{\sim}{L} = \begin{bmatrix} 0.333 & 0.5 \\ 0.5 & 0.8 \end{bmatrix} \quad [g_m] = \begin{bmatrix} 0.167 \\ 0.2 \end{bmatrix} \Rightarrow [a_n] = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow f = \sum_{n=1}^{2} a_n f_n = x^2(1-x) = \text{حل تحلیلی}$$