1	تاریخ تحویل: 01/12/15	نام درس: ۳ ش ۱ تکلیف سری اول	سروش مس‌فروش مشهد شماره دانشجویی: 810198472

-۱

$$\vec A(\vec R) = \int_{V'} \frac{\mu_0 \vec J(\vec R')\, e^{-jk|\vec R-\vec R'|}}{4\pi|\vec R-\vec R'|}\, dv' \qquad \text{Lorenz gauge: } \nabla\cdot\vec A + j\omega\mu\epsilon V = 0$$

$$V(\vec R) = \int_{V'} \frac{\rho(\vec R')\, e^{-jk|\vec R-\vec R'|}}{4\pi\epsilon_0|\vec R-\vec R'|}\, dv' \qquad \vec\nabla\cdot\vec A(\vec R) = \vec\nabla\cdot\left(\int_{V'} \underbrace{\frac{\mu_0 \vec J(\vec R')}{4\pi}}_{\vec G(\vec R')}\cdot\underbrace{\frac{e^{-jk|\vec R-\vec R'|}}{|\vec R-\vec R'|}}_{a}\, dv'\right)$$

نوتاسیون دلخواه

$$\to \vec\nabla\cdot\vec A(\vec R) = \nabla\cdot\left(\int_{V'} a\, G(\vec R')\, dv'\right) \qquad \text{می‌دانیم: } \nabla\cdot(a\vec G) = a(\vec\nabla\cdot\vec G) + (\vec\nabla a)\cdot\vec G$$

$$\to \vec\nabla\cdot\vec A(\vec R) = \int_{V'} a\,(\vec\nabla\cdot\vec G(\vec R'))\, dv - \underbrace{\oint_{\partial V} G(\vec R')\, a\cdot d\vec s}_{0}$$

$$\to \vec\nabla\cdot\vec A(\vec R) = \int_{V'} \frac{e^{-jk|\vec R-\vec R'|}}{|\vec R-\vec R'|}\, \mu_0 \frac{\vec\nabla\cdot\vec J(\vec R')}{4\pi}\, dv'$$

Lorenz Gauge: $\vec\nabla\cdot A + j\omega\mu\epsilon V = 0$, $\quad j\omega\mu\epsilon V = ?$ $\quad \mu=\mu_0,\ \epsilon=\epsilon_0$

$$\to j\omega\mu\epsilon V = j\omega\mu_0\epsilon_0 V = \int_{V'} j\omega\mu_0\epsilon_0\rho(\vec R')\, \frac{e^{-jk|\vec R-\vec R'|}}{4\pi\epsilon_0|\vec R-\vec R'|}\, dv'$$

$$= j\omega\int_{V'} \mu_0\rho(\vec R')\, \frac{e^{-jk|\vec R-\vec R'|}}{4\pi|\vec R-\vec R'|}\, dv',$$

عبارت $\vec\nabla\cdot\vec A$ را به فرم فازوری منتقل خواهیم کرد.

$$\vec\nabla\cdot\vec A(\vec R) = \int_{V'} \frac{e^{-jk|\vec R-\vec R'|}}{|\vec R-\vec R'|}\, \frac{\mu_0\vec\nabla\cdot\vec J(\vec R')}{4\pi}\, dv' \qquad \text{در درس داریم: } \vec\nabla\cdot\vec J(\vec R') = -\frac{\partial\rho}{\partial t}$$

$$\xrightarrow{\text{Phasor}} \vec\nabla\cdot\vec J(\vec R') = -j\omega\rho(\vec R') \to \vec\nabla\cdot\vec A(\vec R) = -j\omega\int_{V'} \mu_0\rho(\vec R')\, \frac{e^{-jk|\vec R-\vec R'|}}{4\pi|\vec R-\vec R'|}\, dv'$$

$$\vec\nabla\cdot\vec A(\vec R) + j\omega\mu_0\epsilon_0 V = -j\omega\int_{V'} \mu_0\rho(\vec R')\, \frac{e^{-jk|\vec R-\vec R'|}}{4\pi|\vec R-\vec R'|}\, dv'$$

$$+ j\omega\int_{V'} \mu_0\rho(\vec R')\, \frac{e^{-jk|\vec R-\vec R'|}}{4\pi|\vec R-\vec R'|}\, dv' = 0 \qquad \text{QED}$$

2	تاریخ تحویل: ٩٩/١٢/١٥	نام درس: آنتن ۱ — تکلیف سری اول	سروش مس‌فروش مشهد شماره دانشجویی: 810198472

2 - الف)

$U = \sin\theta \sin\varphi \qquad 0 \leq \theta \leq \pi \qquad 0 \leq \varphi \leq \pi$

راستایی : $D(\theta,\varphi) = 4\pi \dfrac{U(\theta,\varphi)}{P_{rad}} \qquad P_{rad} = \int U(\theta,\varphi)\, d\Omega$

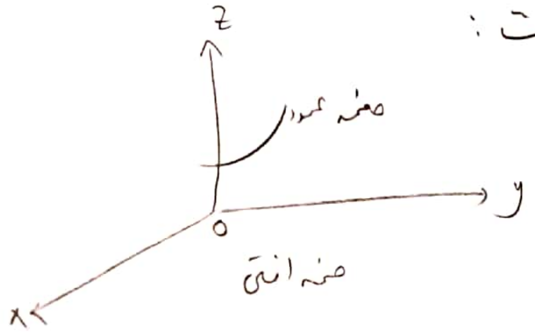
$4\pi U(\theta,\varphi) = 4\pi \sin\theta \sin\varphi \ , \quad P_{rad} = \int_0^\pi \int_0^\pi \sin\theta \sin\varphi \underbrace{\sin\theta\, d\theta\, d\varphi}_{d\Omega}$

$= \int_0^\pi \int_0^\pi \sin^2\theta \sin\varphi \, d\theta\, d\varphi = \int_0^\pi \int_0^\pi \left(\frac{1-\cos 2\theta}{2}\right) \sin\varphi\, d\theta\, d\varphi = \int_0^\pi \sin\varphi \left(\frac{\pi}{2}\right) d\varphi$

$= \pi/2 \, (-\cos\varphi)\Big|_0^\pi = \pi/2 \, (1-(-1)) = \pi \ , \quad D(\theta,\varphi) = \dfrac{4\pi \sin\theta \sin\varphi}{\pi} = 4 \sin\theta \sin\varphi$

$\rightarrow D(\theta,\varphi) = 4 \sin\theta \sin\varphi \ , \quad D_{max} = 4$

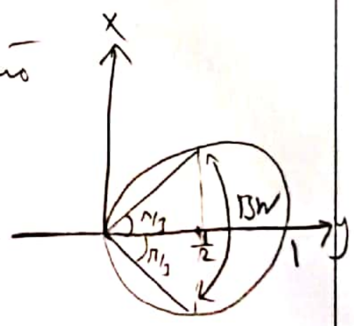
ب) سراغ نمودار برای یافتن Beamwidth خواهیم رفت :



منظور من این است که منظور از صفحه افقی صفحه xoy و صفحه عمودی صفحه zoy باشد. لذا داریم:

صفحه افقی : $xoy \longrightarrow$ یعنی $\longrightarrow \theta = \pi/2$, φ متغیر

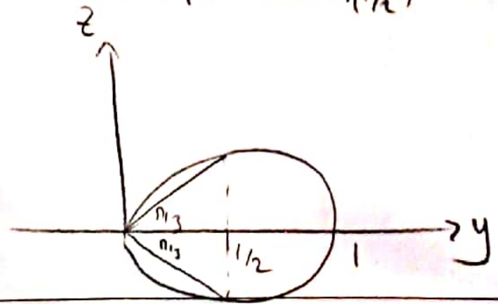
$\rightarrow U(\theta,\varphi) = U(\pi/2,\varphi) = \sin\varphi \quad (0 \leq \varphi \leq \pi)$



$BW = \text{Beamwidth} = 2\pi/3$

صفحه عمودی : $zoy \rightarrow \varphi = \pi/2 \rightarrow \theta$ متغیر $\rightarrow U(\theta,\varphi) = U(\theta,\pi/2)$

$= \sin\theta \longrightarrow BW = \text{Beamwidth} = \dfrac{2\pi}{3}$



3	تاریخ تحویل: ۹ / ۱۲/۱۶	نام درس: آنتن ۱ تکلیف سری اول	سروش مس‌فروش مشهد شماره دانشجویی: 810198472

3 - ۱ الف)

آرایه در مختصی در امتداد z ، $0 \le \theta \le \pi$

$$E = Cs\left[\frac{\pi}{4}(\cos(\theta)-1)\right]\frac{e^{-jkR}}{R}$$

$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{P_{rad}}$$

در درس میدان ها و امواج ، همچنین در کلاس آنتن ۱ آ به مفهوم ~~مفهوم~~ W_{rad} به صورت زیر بدست می‌آید.

$$\overline{W}_{rad} = \frac{1}{2}\operatorname{Re}[\overline{E}\times \overline{H}^*] = \frac{1}{2}\left|\frac{E^2}{Z_0}\right|\hat{R}$$

$$U(\theta,\varphi) = R^2 W_{rad} = \frac{1}{2}\left|\frac{E^2}{Z_0}\right|R^2 = \frac{1}{2Z_0}|E|^2 R^2$$

$$\rightarrow U(\theta,\varphi) = \frac{1}{2Z_0} Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right] \qquad P_{rad} = \int U(\theta,\varphi)\,d\Omega$$

$$= \iint U(\theta,\varphi)\sin\theta\, d\theta\, d\varphi = \int_0^{2\pi}\int_0^{\pi} \frac{1}{2Z_0} Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right]\sin\theta\, d\theta\, d\varphi$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{1}{2Z_0} Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right]\sin\theta\, d\varphi\, d\theta = \frac{\pi}{Z_0}\int_0^{\pi} Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right]\sin\theta\, d\theta$$

$$= \frac{\pi}{Z_0}\int_0^{\pi}\left[\frac{1+\cos\left[\frac{\pi}{2}(\cos\theta-1)\right]}{2}\right]\sin\theta\, d\theta = \frac{\pi}{2Z_0}\int_0^{\pi}\sin\theta\, d\theta$$

$$+ \frac{\pi}{2Z_0}\underbrace{\int_0^{\pi} \cos\left[\frac{\pi}{2}\cos\theta - \frac{\pi}{2}\right]\sin\theta\, d\theta}_{0} = \frac{\pi}{Z_0} + 0 = \frac{\pi}{Z_0}$$

$$D(\theta,\varphi) = \frac{4\pi U(\theta,\varphi)}{P_{rad}} = \frac{\frac{1}{2Z_0} Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right]\times 4\pi}{\pi/Z_0} = 2Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right]$$

$$\rightarrow D(\theta,\varphi) = 2\cos^2\left[\frac{\pi}{4}(\cos\theta-1)\right] \qquad D_{Max} = 2 \;@\; \cos\theta = 1$$

در کتاب Balanis ترتیب Kraus به صورت زیر است.

$$\text{Kraus}: \quad D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\theta_{1r}\,\theta_{2r}} \qquad \theta_{1r}, \theta_{2r} = ? \qquad \text{Kraus index}: D_0 = \frac{41000}{\theta_1^\circ \theta_2^\circ}$$

حالا باید BW را پیدا کنیم تا θ_{1r}, θ_{2r} را پیدا کنیم.

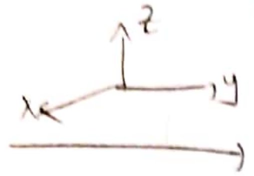
$$U(\theta,\varphi) = \frac{1}{2}\underbrace{U_{max}}_{1/2Z_0} \rightarrow Cs^2\left[\frac{\pi}{4}(\cos\theta-1)\right] = \frac{1}{2} \rightarrow \cos\theta - 1 = -1 \rightarrow \cos\theta = 0 \rightarrow \theta = \frac{\pi}{2}$$

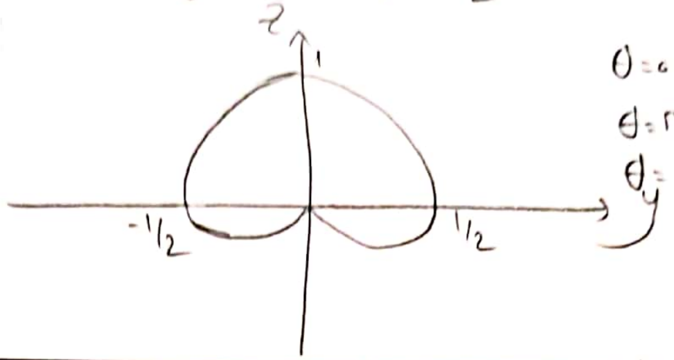
متقارن $\rightarrow \theta_{1r} = \theta_{2r} = \frac{\pi}{2}\times 2 \rightarrow D_0 = \frac{4\pi}{\pi^2} = \frac{4}{\pi}$

ب) برای رسم الگوی تابشی خواهیم داشت:

$$U(\theta,\varphi) = \frac{1}{2Z_0} Cos^2[\pi/_4 (Cos\theta - 1)] \xrightarrow{Normalize} U(\theta,\varphi)_{Norm} = Cos^2[\pi/_4 (Cos\theta-1)]$$

$$U_{Norm}(\theta,\varphi) = \frac{1 + Cos[\pi/_2 (Cos\theta - 1)]}{2} = 1/_2 + Sin(\frac{\pi}{2} Cos\theta)$$





$\theta = 0 \longrightarrow U_{Norm} = 1$

$\theta = \pi/_2 \longrightarrow U_{Norm} = 1/_2$

$\theta = \pi \longrightarrow U_{Norm} = 0$

۴- الف)

$$E = \begin{cases} (Sin\theta\, Cos^2\theta)^{1/2} & 0 \leq \theta \leq \pi,\ 0 \leq \varphi \leq \pi/_2 \\ 0 & \text{o.w.} \end{cases}$$

$$D(\theta,\varphi) = \frac{U(\theta,\varphi) \times 4\pi}{P_{rad}}$$

$$P_{rad} = \int U(\theta,\varphi)\, d\Omega \quad , \quad U(\theta,\varphi) = \frac{1}{2Z_0} |E|^2 R^2 = \frac{1}{2Z_0} Sin\theta\, Cos^2\theta\, R^2$$

میدان دور: $R^2 = 1 \longrightarrow U(\theta,\varphi) = \frac{1}{2Z_0} Sin\theta\, Cos^2\theta \quad , \quad P_{rad} = \int_0^{\pi/2}\int_0^{\pi} \frac{1}{2Z_0} Sin^2\theta\, Cos^2\theta\, d\theta\, d\varphi$

$$= \int_0^{\pi/2}\int_0^{\pi} \frac{1}{8Z_0} Sin^2(2\theta)\, d\theta\, d\varphi = \frac{\pi}{16Z_0}\int_0^{\pi} Sin^2(2\theta)\, d\theta = \frac{\pi}{16Z_0}\int_0^{\pi} \left(\frac{1 - Cos(4\theta)}{2}\right) d\theta$$

$$= \frac{\pi^2}{32Z_0} \quad , \quad D(\theta,\varphi) = \frac{4\pi \times \frac{1}{2Z_0} \times Sin\theta\, Cos^2\theta}{\frac{\pi^2}{32Z_0}} = \frac{64}{\pi} Sin\theta\, Cos^2\theta$$

$$\longrightarrow D(\theta,\varphi) = \frac{64}{\pi} Sin\theta\, Cos^2\theta \qquad D_{Max} = ? \qquad \frac{\partial D(\theta,\varphi)}{\partial\theta} = 0 \longrightarrow Cos^3\theta - 2Sin^2\theta\, Cos\theta = 0$$

$$\longrightarrow Cos\theta(Cos^2\theta - 2Sin^2\theta) = 0 \begin{cases} Cos\theta = 0 \longrightarrow \theta = \pi/_2 \longrightarrow D(\theta,\varphi) = 0 \ \times \text{ نیست} \\ Cos^2\theta = 2Sin^2\theta \longrightarrow 3Sin^2\theta = 1 \longrightarrow Sin^2\theta = 1/_3 \end{cases}$$

$$\longrightarrow Sin\theta = \pm\sqrt{1/_3} \longrightarrow \theta = arcSin(\pm 1/\sqrt{3}) = \begin{cases} \theta = 35.26^\circ \\ \theta = 144.74^\circ \ (-35.26^\circ) \end{cases}$$

$$\longrightarrow D_{Max} = \frac{64}{\pi} \times Sin(35.26^\circ) \times Cos^2(35.26) = 7.841121930$$

ب) مجدداً از رابطه Kraus کمک خواهیم گرفت.

$$U(\theta,\varphi) = 1/_2 U_{max}$$

$$\frac{1}{\cancel{2Z_0}} Sin\theta\, Cos^2\theta = 1/_2 \times \frac{1}{\cancel{2Z_0}} \times \frac{1}{\sqrt{3}} \times \frac{2}{3} \longrightarrow \frac{3\sqrt{3}}{2} Sin\theta\, Cos^2\theta = 1/_2 \longrightarrow 3\sqrt{3}\, Sin\theta\, Cos^2\theta = 1$$

5	تاریخ تحویل: 01/12/15	نام درس: آنتن ۱	سروش مس‌فروش مشهد شماره دانشجویی: 810198472

$\rightarrow \sin\theta\cos^2\theta = \frac{1}{3\sqrt3} \rightarrow \sin\theta - \sin^3\theta = \frac{1}{3\sqrt3} \rightarrow \begin{cases}\theta = 0.2\ rad = 17.2^\circ \\ \theta = 1.1\ rad = 68.8^\circ\end{cases}$

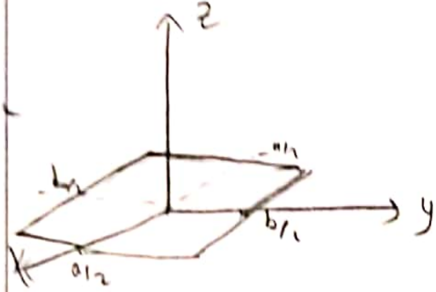
$\theta_{1r} = 68.8 - 17.2 = 51.6, \quad \theta_{2r} = 96.26 \quad$ Kraus: $D_{max} = \frac{41000}{51.6\times 96.26} = 8.25$

ب) رابطه Tai , Pereira به صورت زیر است: $D_{max} = \frac{32\ln 2}{\theta_{1r}^2+\theta_{2r}^2} = \frac{22.181}{\theta_{1r}^2+\theta_{2r}^2}$

$\rightarrow D_{max} = \frac{32\ln 2}{0.81+(1.68)^2} = 6.1$

5-

$\vec J(\vec R') = K_0\,\delta(z'-0)\,\hat x \quad -\frac a2 \le x' \le \frac a2 \quad -b/2 \le y' \le b/2$



$\vec N(\theta,\varphi) = \vec N_t(\theta,\varphi) + \hat R\,\cancel{N_R(\theta,\varphi)}$

در میدان دور حذف می‌شود.

$\vec N(\theta,\varphi) = \vec N_t(\theta,\varphi)$

پس با اعمال تقریب راه دور خواهیم داشت:

$\vec N(\theta,\varphi) = \vec N_t(\theta,\varphi) = \int_{V'} \vec J(\vec R')\, e^{jk\hat R\cdot\vec R'}\, dV'$

Sphere

$\hat R = \sin\theta\cos\varphi\,\hat x + \sin\theta\sin\varphi\,\hat y + \cos\theta\,\hat z \qquad \vec R' = x'\hat x + y'\hat y + z'\hat z$

$\rightarrow \vec N(\theta,\varphi) = \iiint_{-b/2,\,-a/2}^{b/2,\,a/2} K_0\,\delta(z')\,\hat x\, e^{jkx'\sin\theta\cos\varphi + jky'\sin\theta\sin\varphi}\, dx'dy'dz'$

$= \int_{-b/2}^{b/2}\int_{-a/2}^{a/2} K_0\hat x\, e^{jkx'\sin\theta\cos\varphi + jky'\sin\theta\sin\varphi}\, dx'dy' = K_0\hat x\left[\int_{-a/2}^{a/2} e^{jkx'\sin\theta\cos\varphi}dx' \cdot \right.$

$\left.\int_{-b/2}^{b/2} e^{jky'\sin\theta\sin\varphi}dy'\right] = K_0\hat x\left[\frac{1}{jk\sin\theta\cos\varphi}\, e^{jkx'\sin\theta\cos\varphi}\Big|_{-a/2}^{a/2}\right.$

$\left.\cdot\frac{1}{jk\sin\theta\sin\varphi}\, e^{jky'\sin\theta\sin\varphi}\Big|_{-b/2}^{b/2}\right] = K_0\hat x\left[\frac{1}{-k^2\sin^2\theta\cos\varphi\sin\varphi}\right][e^{jk\frac a2\sin\theta\cos\varphi}$

$- e^{-jka/2\sin\theta\cos\varphi}].[e^{jkb/2\sin\theta\sin\varphi} - e^{-jkb/2\sin\theta\sin\varphi}] \qquad \sin x = \frac{e^{jx}-e^{-jx}}{j2}$

$\vec N(\theta,\varphi) = \frac{+4K_0\hat x}{k^2\sin^2\theta\cos\varphi\sin\varphi}\sin\left(k\frac a2\sin\theta\cos\varphi\right)\sin\left(k\frac b2\sin\theta\sin\varphi\right)$

سروش مس‌فروش مشهد شماره دانشجویی: 810198472	نام درس: آنتن ۱ تکلیف سری اول	تاریخ تحویل: 15 / 12 / 01	6

برای بیان عبارت حاصل شده در مختصات کروی عنایت داریم که:

$$\hat{x} = \sin\theta \cos\varphi \hat{R} + \cos\theta \cos\varphi \theta - \sin\varphi \hat{\varphi}$$

$$\vec{N}(\theta,\varphi) = \frac{4K_0 \hat{x}}{K^2 \sin^2\theta \cos\varphi \sin\varphi} \sin(K a/2 \sin\theta \cos\varphi) \sin(K b/2 \sin\theta \sin\varphi)$$

جایگذاری $\hat{x}$، $\vec{N}(\theta,\varphi) = \sin(K\frac{a}{2} \sin\theta \cos\varphi) \sin(K\frac{b}{2} \sin\theta \sin\varphi) \Big[\frac{4K_0}{K^2 \sin\theta \sin\varphi} \hat{R}$

$$+ \frac{4K_0 \cos\theta}{K^2 \sin^2\theta \sin\varphi} \hat{\theta} - \frac{4K_0}{K^2 \sin^2\theta \cos\varphi} \hat{\varphi}\Big]$$

6- انجام شد.