Autocorrelation-Based Decentralized Sequential Detection of OFDM Signals in Cognitive Radios

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***Abstract*—This paper introduces a simple and computationally efficient spectrum sensing scheme for Orthogonal Frequency Division Multiplexing (OFDM) based primary user signal using its autocorrelation coefficient. Further, it is shown that the log likelihood ratio test (LLRT) statistic is the maximum likelihood estimate of the autocorrelation coefficient in the low signal-to-noise ratio (SNR) regime. Performance of the local detector is studied for the additive white Gaussian noise (AWGN) and multipath channels using theoretical analysis. Obtained results are verified in simulation. The performance of the local detector in the face of shadowing is studied by simulations. A sequential detection (SD) scheme where many secondary users cooperate to detect the same primary user is proposed. User cooperation provides diversity gains as well as facilitates using simpler local detectors. The sequential detection reduces the delay and the amount of data needed in identification of the underutilized spectrum. The decision statistics from individual detectors are combined at the fusion center (FC). The statistical properties of the decision statistics are established. The performance of the scheme is studied through theory and validated by simulations. A comparison of the SD scheme with the Neyman–Pearson fixed sample size (FSS) test for the same false alarm and missed detection probabilities is also carried out.**

***Index Terms*—Autocorrelation coefficient, cooperative detection, cyclic prefix, sequential detection, spectrum sensing.**

I. Introduction

SPECTRUM sensing is the process of identifying the unused spectrum that may be exploited efficiently without causing harmful interference to the existing incumbent (primary) users. Sensing provides awareness of the radio operating environment. Consequently, a cognitive radio may adjust its transmission parameters such as power, modulation, beampattern, and frequency dynamically. This facilitates using the spectrum in an agile manner while satisfying the constraints on interference. Spectrum sensing techniques have been proposed based on cyclostationarity [1], [2] and classical energy detection [3], [4], for example.

Orthogonal Frequency Division Multiplexing (OFDM) has developed into a popular scheme for wideband digital wireless communication. It is used in various applications such as digital television, audio broadcasting, wireless networking and broadband internet access. Among OFDM systems are [5]: IEEE 802.11a/g Wireless LANs, IEEE 802.16 or WiMAX Wireless MANs, IEEE 802.20 or Mobile Broadband Wireless Access (MBWA) systems, Long Term Evolution (LTE) of 3G, for example. Moreover, it is going to be a key technology for various future broadband wireless communication systems. Therefore, it is fair to assume that many of the primary users will be OFDM based. Hence, the problem of detecting OFDM signals is very relevant.

Correlation based techniques are commonly used for hypothesis testing in various fields like biometry [6], psychology [7], radar and communications. In wireless communications, it has been used in spread spectrum systems, synchronization [8], [9] and estimation of various channel parameters such as carrier frequency offset and Doppler spread [10]. The presence of a cyclic prefix (CP) gives OFDM signals the following well-known, convenient property. The autocorrelation coefficients are nonzero at delays $\tau = \pm T_d$, where T_d is the number of samples corresponding to useful symbol length in an OFDM block. In this paper, we exploit this property to design detectors for CP-OFDM based primary users in cognitive radios.

Decentralized cooperative detection has attracted lot of attention recently [11]–[17]. Among the benefits of cooperative detection of primary users in cognitive radio networks are: improvement of the overall detector performance, mitigation of the effects of shadowing and fading, and increase in coverage [18]. Simpler detectors may be used as well. For cooperative detection, sequential detection (SD) scheme [12], [19]–[22] offers the possibility of making highly reliable decisions without unnecessary delay. In decentralized detection, a sequential procedure can be applied by individual secondary users or at the fusion center (FC) [12]. Motivated by this, we propose a decentralized SD scheme at the FC. Fig. 1 illustrates the target scenario for the issues addressed in this paper.

The contributions of this paper are as follows.

1) For CP-OFDM based system, the autocorrelation coefficients corresponding to lags $\tau = \pm T_d$ are shown to be the log likelihood ratio test (LLRT) statistic in the low signal-to-noise ratio (SNR) regime. The distributions of the corresponding local detector are established under both hypotheses for different channel conditions and the Neyman–Pearson detector is designed. The simulation results presented validate the theoretical analysis. The proposed scheme does not make any assumptions related to the CP.

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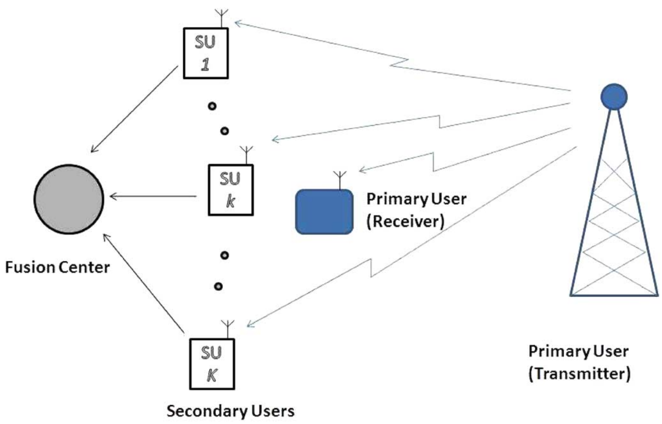


Fig. 1. Secondary users are detecting CP-OFDM based primary user transmission. In cooperative detection, multiple secondary users transmit their decision statistics (such as likelihood ratios) to the FC, which makes the final decision.

2) The effects of exploiting information related to CP on detection performance are shown through theory and simulation.
3) For autocorrelation coefficient based collaborative detection, a SD scheme at the FC is proposed, where autocorrelation coefficient based log likelihood ratios (LLRs) from different secondary users are combined in a sequential manner at a FC in order to quickly detect the primary user reliably in the face of shadowing and fading.
4) The test statistic at the FC center is derived. We establish the distributions of the test statistic at the FC under the two hypotheses and analyze the performance of the proposed detector in the low SNR regime. Results of extensive simulation match with the analysis.
5) Performance of the SD scheme is compared to that of the fixed sample size (FSS) test under different channel conditions. It is shown that the SD scheme, on average, needs considerably fewer statistics to make a reliable decision at the FC.

Two issues are worth noting here. First, the very low SNR regime is relevant as a cognitive radio should be able to detect very weak primary signals in order to avoid causing them interference. Second, correlation properties of CP have been used in many other tasks like synchronization [9], Doppler spread estimation [10], etc., and detection of primary user in cognitive radio is just one of the tasks. In [23], an autocorrelation based detector for Digital Video Broadcasting Terrestrial (DVB-T) has been presented. In this case, absolute value of autocorrelation over a window of T_c samples is evaluated. Here T_c is number of samples corresponding to CP in an OFDM block. Next, $T_d - 1$ number of similar statistics are evaluated by sliding the window by one sample each time. The decision statistic for an OFDM block is then chosen to be the maximum of these T_d statistics. In [24], an autocorrelation-based scheme has been proposed for collaborative spectrum sensing of OFDM based signals in cognitive radios. Although an autocorrelation value based local detector for CP-OFDM has been presented in the prior work [23], [24], here we have proposed an autocorrelation coefficient based detector and established the optimality of the detector in the low SNR regime. Moreover, we do not make any assumption on knowledge of length of the CP for the local detector, while

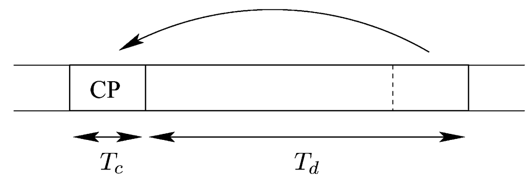


Fig. 2. An OFDM Block. A CP of T_c symbols is copied in front of the data block.

the scheme in [23] assumes T_c to be known. Also we have established the distributions of decision statistic under alternate hypotheses and have designed a detector which gives good detection performance under the constraint of constant false alarm rate for different channel conditions.

This paper is organized as follows. In Section II, a detection scheme based on the well-known autocorrelation properties of OFDM signals is introduced for an additive white Gaussian noise (AWGN) scenario. Section III discusses the effect of using the knowledge of CP on detector performance. Next, performance analysis of the scheme in a multipath scenario is presented in Section IV. A Sequential Detection (SD) scheme in which each secondary user transmits an autocorrelation coefficient-based decision statistic to the FC is proposed in Section V. The distributions of the decision statistics are established. Moreover, theoretical comparison of the scheme with the FSS test is also presented. Simulations in Section VI demonstrate the highly reliable performance of the local autocorrelation coefficient based detector, the gain obtained in cooperative detection, and the efficiency of the SD scheme. Finally, Section VII concludes the paper.

II. Spectrum Sensing Based on Autocorrelation Coefficient

An OFDM signal consists of sum of narrowband subcarriers that are typically modulated by using phase shift keying (PSK) or quadrature amplitude keying (QAM). Without loss of generality, we assume sampling factor as 1. Therefore, T_d also represents number of subcarriers for the OFDM system. Now an OFDM signal is constructed by feeding T_d symbols to an Inverse Fast Fourier Transform (IFFT) through serial to parallel conversion. If $C(0), C(1), \ldots, C(T_d-1)$ are T_d complex QAM (or PSK) symbols, then the outputs of the IFFT are

$$c(t) = \frac{1}{\sqrt{T_d}} \sum_{f=0}^{T_d-1} C(f) e^{\frac{j2\pi tf}{T_d}}, \quad t = 0, \ldots, T_d - 1 \tag{1}$$

where t is a discrete time index, and f is a discrete frequency index. Thus, T_d also denotes the number of symbols in an OFDM data block. They are then converted to a serial stream by parallel to serial conversion. Let T_c be the number of symbols in the CP, then T_c symbols $c(T_d - T_c), \ldots, c(T_d - 1)$ are added in front of the block as the CP to form an OFDM block $[c(T_d - T_c), \ldots, c(T_d - 1), c(0), c(1), \ldots, c(T_d - 1)]$ as shown in Fig. 2. A transmitted OFDM frame may contain several such blocks. Let us denote the symbols of the transmitted OFDM frame by $s(t)$ for convenience.

Let H_0 be the null hypothesis, i.e., an OFDM based primary user is absent, and H_1 be the alternate hypothesis, i.e., an

OFDM-based primary user is active. Therefore, the hypothesis testing problem may be written as

$$\begin{aligned} H_0 &: x(t) = w(t) \\ H_1 &: x(t) = s(t) + w(t) \end{aligned} \tag{2}$$

where $x(t)$ is the received complex OFDM signal, and $w(t)$ is complex circular additive white Gaussian noise. Invoking the Central Limit Theorem (CLT) under the assumption of sufficiently large IFFT size, we have

$$\begin{aligned} s(t) &\sim \mathcal{N}_c\left(0, \sigma_s^2\right) \\ \text{and} \quad w(t) &\sim \mathcal{N}_c\left(0, \sigma_w^2\right) \end{aligned} \tag{3}$$

where $\mathcal{N}_c(.)$ denotes the Gaussian distribution for a complex random variable. Therefore, for different hypotheses we have

$$\begin{aligned} H_0 &: x(t) \sim \mathcal{N}_c\left(0, \sigma_w^2\right) \\ H_1 &: x(t) \sim \mathcal{N}_c\left(0, \sigma_s^2 + \sigma_w^2\right). \end{aligned} \tag{4}$$

Considering $x(t) = x_r(t) + jx_i(t)$ to be a circularly symmetric Gaussian random variable, we have for real and imaginary parts of x:

$$\begin{aligned} x_r(t) &\sim \mathcal{N}_r\left(0, \sigma_x^2/2\right) \\ \text{and} \quad x_i(t) &\sim \mathcal{N}_r\left(0, \sigma_x^2/2\right) \end{aligned} \tag{5}$$

where $\mathcal{N}_r(.)$ denotes the Gaussian distribution for a real random variable. For a CP-OFDM signal, the values of the autocorrelation coefficient $\rho(\tau) = E[x(t)x^*(t+\tau)]/E[x(t)x^*(t)]$ for lags $\tau = \pm T_d$ under the two hypotheses are

$$\begin{aligned} H_0 &: \rho(\pm T_d) = 0 \\ H_1 &: \rho(\pm T_d) = \rho_1 \end{aligned} \tag{6}$$

where

$$\begin{aligned} \rho_1 &= \frac{T_c}{T_d + T_c} \frac{\sigma_s^2}{\sigma_s^2 + \sigma_w^2} \\ &= \frac{T_c}{T_d + T_c} \frac{\text{SNR}}{1 + \text{SNR}}. \end{aligned} \tag{7}$$

Appendix gives a general proof of $\rho_2 = (T_c/(T_d + T_c))(\delta\sigma_s^2/(\delta\sigma_s^2 + \sigma_w^2))$ for multipath channel of order P and tap coefficients $h(l), l = 0, \ldots, P-1$ with $\delta = \sum_{l=0}^{P-1} E[|h(l)|^2]$. For AWGN scenario, $\delta = 1$ gives $\rho_1 = \rho_2$ which completes proof for (7). Here $\text{SNR} \triangleq (\sigma_s^2/\sigma_w^2)$ is the SNR (power ratio). Note that $0 < \rho_1 < 1$. Also for a real valued coefficient, $\rho(\tau) = \rho(-\tau)$. Hence, without loss of generality, we will consider only $\tau = T_d$ and denote $\rho(T_d)$ by ρ for convenience.

Our aim is to devise a detection scheme based on the above property. The observations over several OFDM symbols, i.e., $x(0), \ldots, x(M + T_d - 1)$ where $M \gg T_d$ are used. We can form two real random vectors from the observations such that $\mathbf{z_1} = [x_r(0)\, x_i(0)\, x_r(1)\, x_i(1) \ldots\, x_r(M-1)\, x_i(M-1)]$ and $\mathbf{z_2} = [x_r(T_d)\, x_i(T_d)\, x_r(T_d+1)\, x_i(T_d+1) \ldots\, x_r(M+T_d-1)\, x_i(M+T_d-1)]$. Here $x_r(t)$ and $x_i(t)$ are the real and the imaginary parts of $x(t)$. Due to the circular symmetry assumption, the zero mean random variables $x_r(t)$ and $x_i(t)$ are independent and identically distributed (i.i.d). Now the random variables $z_1(t)$ and $z_2(t)$, which are t^{th} component of the random vectors $\mathbf{z_1}$ and $\mathbf{z_2}$ respectively, are jointly Gaussian under both hypotheses with probability densities [6] given by

$$f\left(z_1(t), z_2(t) | H_0\right) = \frac{\exp\left\{-\frac{1}{2}\left[\frac{z_1^2(t) + z_2^2(t)}{\sigma_0^2}\right]\right\}}{2\pi\sigma_0^2}$$

and

$$f\left(z_1(t), z_2(t) | H_1\right) = \frac{\exp\left\{-\frac{1}{2(1-\rho_1^2)}\left[\frac{z_1^2(t) - 2\rho_1 z_1(t) z_2(t) + z_2^2(t)}{\sigma_1^2}\right]\right\}}{2\pi\sigma_1^2\sqrt{1-\rho_1^2}} \tag{8}$$

where $\sigma_1^2 = (\sigma_s^2 + \sigma_w^2)/2$, and $\sigma_0^2 = \sigma_w^2/2$. The likelihood ratio (LR) for the hypothesis test (6) is given as

$$\begin{aligned} \Lambda &= \prod_{t=0}^{2M-1} \frac{f\left(z_1(t), z_2(t)|H_1\right)}{f\left(z_1(t), z_2(t)|H_0\right)} \\ &= \frac{\sigma_0^{2M}}{\sigma_1^{2M}\left(1-\rho_1^2\right)^M} \exp\left\{ -\frac{1}{2}\left(\frac{1}{(1-\rho_1^2)\sigma_1^2} - \frac{1}{\sigma_0^2}\right) \cdot \sum_{t=0}^{2M-1}\left(z_1^2(t) + z_2^2(t)\right) + \frac{\rho_1 \sum_{t=0}^{2M-1} z_1(t) z_2(t)}{(1-\rho_1^2)\sigma_1^2} \right\}. \end{aligned} \tag{9}$$

Since $z_1(t)$ and $z_2(t)$ are identically distributed random variables, $E[z_1^2(t)] = E[z_2^2(t)]$. Let us define $\hat{\sigma}_z^2$ as the maximum likelihood estimate of $E[z_1^2(t)]$ based on vectors $\mathbf{z_1}$ and $\mathbf{z_2}$. Then $\hat{\sigma}_z^2$ [6] is given by

$$\hat{\sigma}_z^2 = \frac{1}{4M} \sum_{t=0}^{2M-1} \left(z_1^2(t) + z_2^2(t)\right). \tag{10}$$

Equivalently $\hat{\sigma}_z^2$ can be rewritten in terms of the observations $[x_r(0), x_i(0), \ldots, x_r(M+T_d-1), x_i(M+T_d-1)]$ as

$$\begin{aligned} \hat{\sigma}_z^2 &= \frac{1}{2(M+T_d)} \sum_{t=0}^{M+T_d-1} x_r^2(t) + x_i^2(t) \\ &= \frac{1}{2(M+T_d)} \sum_{t=0}^{M+T_d-1} |x(t)|^2. \end{aligned}$$

Using the approximation $\hat{\sigma}_z^2 \approx \sigma_1^2 \approx \sigma_0^2$ for the low SNR case, and substituting the value of $\hat{\sigma}_z^2$ from (10) in (9), we get

$$\begin{aligned} \Lambda &= \frac{\exp\left\{-2M\left[\frac{1}{(1-\rho_1^2)} - 1\right] + \frac{\rho_1 \sum_{t=0}^{2M-1} z_1(t) z_2(t)}{(1-\rho_1^2)\hat{\sigma}_z^2}\right\}}{\left(1-\rho_1^2\right)^M} \\ &= \frac{\exp\left\{-2M\left[\frac{\rho_1^2}{(1-\rho_1^2)}\right] + \frac{2M\rho_1\hat{\rho}_{ML}}{(1-\rho_1^2)}\right\}}{\left(1-\rho_1^2\right)^M} \end{aligned} \tag{11}$$

where $\hat{\rho}_{ML}$ is the maximum likelihood estimate of ρ from the vectors $\mathbf{z_1}$ and $\mathbf{z_2}$ [6] given by

$$\hat{\rho}_{ML} = \frac{\frac{1}{2M}\sum_{t=0}^{2M-1} z_1(t)z_2(t)}{\hat{\sigma}_z^2}. \tag{12}$$

Equivalently $\hat{\rho}_{ML}$ can be rewritten in terms of the observations $[x_r(0), x_i(0), \ldots, x_r(M+T_d-1), x_i(M+T_d-1)]$ as

$$\hat{\rho}_{ML} = \frac{\frac{1}{2M}\sum_{t=0}^{M-1} x_r(t)x_r(t+T_d) + x_i(t)x_i(t+T_d)}{\hat{\sigma}_z^2} = \frac{\frac{1}{M}\sum_{t=0}^{M-1} \Re\left\{x(t)x^*(t+T_d)\right\}}{\hat{\sigma}_z^2}.$$

Here $\Re\{.\}$ denotes the real part of a complex number. As the distributions are exponential in our case, it is convenient to use the LLR instead of the LR. The LLR is given by

$$\begin{aligned} L &= \log(\Lambda) \\ &= -M\log\left(1-\rho_1^2\right) + 2M\frac{\rho_1(\hat{\rho}_{ML}-\rho_1)}{1-\rho_1^2} \end{aligned} \tag{13}$$

where $\log(.)$ denotes the natural logarithm. Therefore, when using the LLRT, we decide the alternative hypothesis for the present hypothesis test if $L > \eta_1$, where η_1 is the threshold of the detector. Equivalently, we decide H_1 if

$$\hat{\rho}_{ML} > \frac{\left(1-\rho_1^2\right)}{2M\rho_1}\left(\eta_1 + M\log\left(1-\rho_1^2\right)\right) + \rho_1 \triangleq \eta_l. \tag{14}$$

Thus $\hat{\rho}_{ML}$ is the required LLRT statistic for the hypothesis test. The probability density of $\hat{\rho}_{ML}$ under H_0 based on $2M$ real symbols [6] is

$$f_{2M}(\hat{\rho}_{ML}|H_0) = \frac{\Gamma\left[\frac{1}{2}(2M-1)\right]}{\Gamma\left[\frac{1}{2}(2M-2)\right]\sqrt{\pi}}\left(1-\hat{\rho}_{ML}^2\right)^{(2M-4)/2}. \tag{15}$$

Theorem 1: If $\hat{\rho}_{ML}$ is the symbol correlation coefficient for $2M$ symbols from a real valued Gaussian distribution with correlation ρ, then $\sqrt{2M}(\hat{\rho}_{ML}-\rho)/(1-\rho^2)$ is asymptotically distributed according to $\mathcal{N}_r(0,1)$ [6].

Using Theorem 1, under the two hypotheses, we have

$$\begin{aligned} H_0 &: \lim_{M\to\infty} \sqrt{2M}\hat{\rho}_{ML} \xrightarrow{d} \mathcal{N}_r(0,1) \\ H_1 &: \lim_{M\to\infty} \sqrt{2M}\hat{\rho}_{ML} \xrightarrow{d} \mathcal{N}_r\left(\sqrt{2M}\rho_1, \left(1-\rho_1^2\right)^2\right) \end{aligned} \tag{16}$$

where $\rho_1 = (T_c/(T_d+T_c))(\sigma_s^2/(\sigma_s^2+\sigma_w^2))$, and '$\xrightarrow{d}$' denotes convergence in distribution. Using these distributions, we can approximate the distribution of the test statistic for sufficiently large M as

$$\begin{aligned} H_0 &: \hat{\rho}_{ML} \sim \mathcal{N}_r\left(0, \frac{1}{2M}\right) \\ H_1 &: \hat{\rho}_{ML} \sim \mathcal{N}_r\left(\rho_1, \frac{(1-\rho_1^2)^2}{2M}\right). \end{aligned} \tag{17}$$

This is a classical detection problem with test statistic $\hat{\rho}_{ML}$. Different detection strategies like Neyman–Pearson, Min-Max, Bayes, etc., can be used depending on the prior information available and error constraints to be satisfied. The threshold for the detection scheme will depend on the detection strategy used. Here, we have considered the Neyman–Pearson detector to satisfy a constant false alarm (CFAR) rate constraint at the local detector. For a Gaussian random variable $r \sim \mathcal{N}_r(\mu_r, \sigma_r^2)$, we have [25]

$$P(r > \eta_r) = \frac{1}{2}\mathrm{erfc}\left(\frac{\eta_r - \mu_r}{\sqrt{2}\sigma_r}\right) \tag{18}$$

where erfc(.) is the complementary error function. Using (18), the false alarm probability P_{fa} is given by

$$P_{fa} = P(\hat{\rho}_{ML} > \eta_l|H_0) = \frac{1}{2}\mathrm{erfc}(\sqrt{M}\cdot\eta_l). \tag{19}$$

Thus the threshold at the local detector can be calculated as

$$\eta_l = \frac{1}{\sqrt{M}}\cdot\mathrm{erfc}^{-1}(2P_{fa}). \tag{20}$$

Similarly, the probability of detection P_d is given by

$$P_d = P(\hat{\rho}_{ML} > \eta_l|H_1) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{M}\cdot\frac{\eta_l-\rho_1}{1-\rho_1^2}\right). \tag{21}$$

This scheme requires knowledge of T_d. This is a reasonable assumption on primary user waveforms as this information is specified in standards. Even if the exact value is not known, we can detect for different values of T_d from the possible few options.

III. Effect of Knowledge of CP

In Section II, we have assumed a very conservative view that we do not have any knowledge of the CP duration in samples (T_c), the range of values it can take and the synchronization information (position of the CP in an OFDM block). This is valid for the primary user systems in which depending on the parameters like cell size, multipath channel order, etc., the CP length changes adaptively. However, in practice, they may be typically known. For example, in case of WLAN, $T_c = T_d/4$. If we have some knowledge of the CP, the performance can be improved. In this section, we consider the best case where we assume that the CP length, and synchronization are known.

The maximum likelihood estimate of the autocorrelation coefficient (12) can be rewritten as

$$\hat{\rho}_{ml} = \frac{\frac{1}{M}\sum_{t=0}^{M-1}\Re\left\{x(t)x^*(t+T_d)\right\}}{\hat{\sigma}_z^2} = \frac{\frac{1}{M}\sum_{n=0}^{N_s-1}\sum_{t=0}^{T_s-1}\Re\left\{x(nT_s+t)x^*(nT_s+t+T_d)\right\}}{\hat{\sigma}_z^2}$$

where $N_s = M/T_s$ is the number of OFDM blocks over which autocorrelation coefficient is estimated and $T_s = T_c + T_d$ is the number of symbols in an OFDM block. Thus, in absence of information related to the CP, the numerator is obtained by summing the product over a sliding window of length $T_c + T_d$ for an OFDM block and again adding these sums for N_s blocks. In case where the length of CP and synchronization is known, the length of the sliding window can be reduced from T_s to T_c. Hence the autocorrelation coefficient estimate in this case is given by

$$\hat{\rho}_c = \frac{\sum_{n=0}^{N_s-1} \sum_{t \in CP} \Re\left\{x(nT_s+t)x^*(nT_s+t+T_d)\right\}}{M_1\hat{\sigma}_z^2} \tag{22}$$

where $M_1 = T_cN_s$. The distributions of this estimate under different hypotheses for sufficiently large M_1 are

$$\begin{aligned} H_0 &: \hat{\rho}_c \sim \mathcal{N}_r\left(0, \frac{1}{2M_1}\right) \\ H_1 &: \hat{\rho}_c \sim \mathcal{N}_r\left(\rho_c, \frac{\left(1-\rho_c^2\right)^2}{2M_1}\right) \end{aligned} \tag{23}$$

where $\rho_c = \mathrm{SNR}/(\mathrm{SNR}+1)$. Using these distributions, the Neyman–Pearson detector can be designed as given in (19)–(21) with ρ_1 replaced by ρ_c and M replaced by M_1. Thus it can be seen that the mean of the estimate has increased by a factor of T_s/T_c under H_1, which increases the detection probability. The variance of the coefficient estimate increases by the same amount under H_0, while the increase is smaller under H_1. The effect of increase in variance is to reduce the detection probability. However, the effect of increase in mean is stronger than that of increase in variance, resulting in an overall increase in detection performance.

The assumption of knowing T_c is reasonable for the wireless standards that have only a few allowed values for CP length. For example, DVB-T has 4 different allowed CP lengths while DVB-T2 has 6 different allowed CP lengths. However, assuming perfect synchronization may not be valid. In such case, the detection scheme in [23] can be used which gives slightly degraded performance as compared to the one discussed in this section. Also the distribution of the test statistic is no longer Gaussian. For further analysis in the rest of the paper, without any loss of generality, we have considered the detector in Section II for convenience.

IV. Performance of Local Detector in Multipath

Consider a multipath channel of order P. Let us denote the channel tap coefficients by $h(l)$, for $l = 0, 1, \ldots, P-1$. Channel taps are assumed to be independent of each other, of the transmitted data $s(t)$, and of the noise $w(t)$. Under the alternative hypothesis H_1, the received signal in the multipath scenario is

$$\begin{aligned} x(t) &= s(t) * h(t) + w(t) \\ &= \sum_{l=0}^{P-1} h(l)s(t-l) + w(t), \end{aligned}$$

where '$*$' denotes convolution. The autocorrelation coefficient $\rho = E[x(t)x^*(t+T_d)]/E[x(t)x^*(t)]$ under the two hypothesis is given by

$$\begin{aligned} H_0 &: \rho = 0 \\ H_1 &: \rho = \rho_2 \end{aligned} \tag{24}$$

where $\rho_2 = E[x(t)x^*(t+T_d)|H_1]/E[x(t)x^*(t)|H_1] = (T_c/(T_c+T_d))(\delta\sigma_s^2/(\delta\sigma_s^2+\sigma_w^2))$ and $\delta = \sum_{l=0}^{P-1} E[|h(l)|^2]$. See the Appendix for a proof. Moreover, $0 < \rho_2 < 1$. Assuming that $s(t)$, $h(t)$ and $w(t)$ are circularly symmetric complex Gaussian random variables, $x(t) = s(t) * h(t) + w(t)$ is also a circularly symmetric complex Gaussian random variable. Similar to the AWGN case, the LLRT statistic for the hypothesis testing problem (24) in the multipath scenario is given by $\hat{\rho}_{ML}$ of (12). Using Theorem 1, the asymptotic distributions of $\hat{\rho}_{ML}$ under the two hypotheses are

$$\begin{aligned} H_0 &: \hat{\rho}_{ML} \sim \mathcal{N}_r\left(0, \frac{1}{2M}\right) \\ H_1 &: \hat{\rho}_{ML} \sim \mathcal{N}_r\left(\rho_2, \frac{\left(1-\rho_2^2\right)^2}{2M}\right). \end{aligned} \tag{25}$$

Using these distributions, the Neyman–Pearson detector can be designed as given in (19)–(21) with ρ_1 replaced by ρ_2. Assuming for the multipath case that SNR in power ratio is defined as $\mathrm{SNR} = \delta\sigma_s^2/\sigma_w^2$, it is clear that the performance of the scheme remains unchanged in the multipath channel as compared to the AWGN channel for a given SNR. Hence for analysis of the cooperative sequential detection case, we will consider AWGN channel without any loss of generality.

V. Sequential Detection at FC

In this section, we propose a cooperative and decentralized SD scheme, where soft decisions from several secondary users are combined sequentially at the FC. The FC may be a separate node or one of the secondary users. For convenience, we denote the maximum likelihood estimate of the autocorrelation coefficient $\hat{\rho}_{ML}$ at the n^{th} secondary user by $\hat{\rho}_n$. In our SD scheme, instead of a node making a final decision based on its LLRT statistic $\hat{\rho}_n$, it sends its LLR to the FC. The FC will collect these statistics (i.e., LLRs) sequentially. Each time after receiving a LLR from a user, it performs a hypothesis test. If the evidence is sufficient for making a decision at a specified reliability level, the FC stops collecting further information and makes a decision. Otherwise, it will acquire an additional decision statistic from another secondary user and repeat the procedure.

In terms of the LLRs, the SD scheme or the Sequential Probability Ratio Test (SPRT) [19] after receiving k statistics is

$$\begin{aligned} &\sum_{n=1}^{k} L_n \le \log B, \quad \text{Decide } H_0 \\ &\sum_{n=1}^{k} L_n \ge \log A, \quad \text{Decide } H_1 \\ &\text{Otherwise,} \quad \text{Take Next User's Statistics.} \end{aligned} \tag{26}$$

In the above expressions

$$\begin{aligned} A &= \frac{1-\beta}{P_{fa}}, \quad B = \frac{\beta}{1-P_{fa}}, \quad \text{and} \\ L_n &= -M\log\left(1-\rho_n^2\right) + 2M\frac{\rho_n(\hat{\rho}_n-\rho_n)}{1-\rho_n^2}. \end{aligned} \tag{27}$$

Here $\beta = 1 - P_d$ is the probability of missed detection. Note that the number of LLRs used to form the decision, i.e., $k = K_s$ will be a random variable.

Performance of a sequential detector can be expressed in terms of the average sample number (ASN). The ASN for a SD scheme is defined as the number of samples (i.e, the number of LLRs in our case) required on average for arriving at a decision under either hypotheses. Under the two hypotheses, the average numbers of samples for the SD when $\rho_n = \rho_1, \forall n$ [13] are

$$E[K_s|H_0] = \frac{P_{fa}\log A + (1-P_{fa})\log B}{E[L_n|H_0]}$$
$$\text{and}\quad E[K_s|H_1] = \frac{(1-\beta)\log A + \beta\log B}{E[L_n|H_1]}. \tag{28}$$

Here $E[L_n|H_0]$ and $E[L_n|H_1]$ can be computed from (27) and the distributions of $\hat{\rho}_n$, which are similar to (17) with $\hat{\rho}_{ML}$ replaced by $\hat{\rho}_n$. Therefore we have

$$E[L_n|H_0] = -M\log\left(1-\rho_1^2\right) - 2M\frac{\rho_1^2}{1-\rho_1^2}$$
$$E[L_n|H_1] = -M\log\left(1-\rho_1^2\right). \tag{29}$$

Now the ASN (or average number of secondary user statistics) for the SD is

$$K_m = \max\left\{E[K_s|H_0], E[K_s|H_1]\right\}. \tag{30}$$

Next, we compare the performance of the SD scheme with the Neyman–Pearson FSS test at the FC based on LLRs. Let K_f be the number of secondary user statistics required for the FSS test; then the FSS test at the FC is

$$\sum_{n=1}^{K_f} L_n < \eta_3, \qquad \text{Decide } H_0$$
$$\sum_{n=1}^{K_f} L_n \geq \eta_3, \qquad \text{Decide } H_1 \tag{31}$$

where η_3 is the threshold of the detector. Equivalently, we can use the test statistic $T_f = 2M\sum_{n=1}^{K_f}(\rho_n\hat{\rho}_n/(1-\rho_n^2))$ containing only the variable terms for convenience. The FSS test in this case is

$$T_f < \eta_f, \quad \text{Decide } H_0$$
$$T_f \geq \eta_f, \quad \text{Decide } H_1 \tag{32}$$

where $\eta_f \triangleq \eta_3 + M\sum_{n=1}^{K_f}\log(1-\rho_n^2) - 2M\sum_{n=1}^{K_f}(\rho_n^2/(1-\rho_n^2))$. Therefore, the distributions of the FSS test statistic T_f under the two hypotheses are

$$H_0 : \mathcal{T}_f \sim \mathcal{N}_r\left(0, \sigma_{f0}^2\right)$$
$$H_1 : \mathcal{T}_f \sim \mathcal{N}_r\left(m_f, \sigma_{f1}^2\right) \tag{33}$$

where

$$m_f = 2M\sum_{n=1}^{K_f}\frac{\rho_n^2}{(1-\rho_n^2)},$$
$$\sigma_{f0}^2 = 2M\sum_{n=1}^{K_f}\frac{\rho_n^2}{(1-\rho_n^2)^2},$$
$$\text{and}\quad \sigma_{f1}^2 = 2M\sum_{n=1}^{K_f}\rho_n^2. \tag{34}$$

The false alarm probability for the detector is

$$P_{fa} = P(T_f > \eta_f|H_0) = \frac{1}{2}\text{erfc}\left(\frac{\eta_f}{\sqrt{2}\sigma_{f0}}\right). \tag{35}$$

Therefore the threshold can be given in terms of the false alarm probability as

$$\eta_f = \sqrt{2}\cdot\sigma_{f0}\cdot\text{erfc}^{-1}(2P_{fa}) \tag{36}$$

where σ_{f0} is evaluated using (34) under the assumption that ρ_n are known for all secondary users for the FSS scheme. According to a definition similar to (7), ρ_n depends on T_c, T_d and SNR. Values of T_c and T_d are typically specified in standards. Also, SNR may be estimated using non-data-aided SNR estimation techniques [26]–[28]. Hence it is reasonable to assume that ρ_n are known for all values of $n = 1, \ldots, K_f$.

Now, the probability of detection P_d is

$$P_d = P(T_f > \eta_f|H_1) = \frac{1}{2}\text{erfc}\left(\frac{\eta_f - m_f}{\sqrt{2}\sigma_{f1}}\right). \tag{37}$$

Using (34), (36), (37), and $\rho_n = \rho_1, \forall n$, the number of secondary user statistics (NSUS) required for achieving a given P_{fa}, and P_d in the FSS case is

$$K_f = \frac{\left(\text{erfc}^{-1}(2P_{fa}) - \left(1-\rho_1^2\right)\text{erfc}^{-1}(2P_d)\right)^2}{M\rho_1^2}. \tag{38}$$

Fig. 3 shows a comparison of theoretical K_m and K_f for performance parameters $P_{fa} = 0.05$ and $\beta = 1 - P_d = 0.05$. It can be seen that average NSUS for SD is consistently and significantly lower than NSUS for FSS testing for the same performance parameters.

The performance of SD can also be compared to that of the FSS scheme in terms of Relative Efficiency (RE) in the NSUS (or number of LLRs) required to arrive at a final decision at the FC. The RE is computed as

$$\text{RE} = \frac{K_f}{K_m}. \tag{39}$$

The theoretical RE for the results shown in Fig. 3 is 2.042.

VI. Simulation Results

For all simulation cases, discrete time baseband processing is assumed. For each secondary user, the detection period is assumed to be 100 OFDM blocks. Complex symbols $C(f)$, which are the input to the IFFT at the transmitter, are chosen from a 16 QAM constellation normalized to unit energy. The size

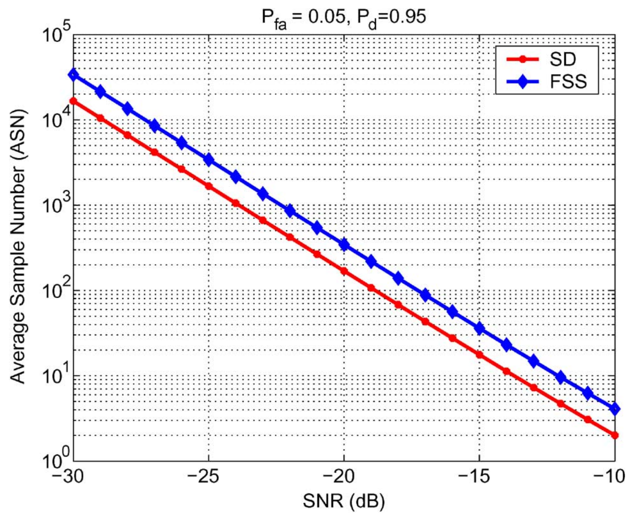


Fig. 3. Comparison of the theoretical average Number of Secondary User Statistics (NSUS), i.e., K_m for SD and the NSUS, i.e., K_f for FSS detection. The NSUS required for SD is significantly lower than that for FSS detection for the same error probabilities.

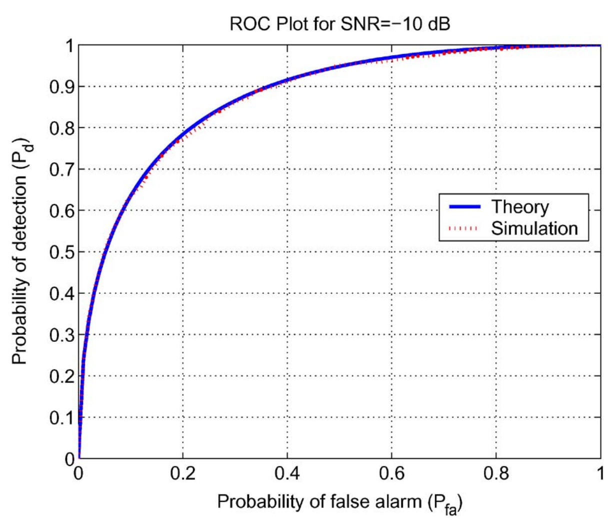


Fig. 4. Receiver Operating Characteristic (ROC) plots for a single user for $\mathrm{SNR} = -10$ dB. Theoretical analysis and simulation results are close.

of the IFFT is chosen to be 32. Therefore $T_d = 32$. The results are averaged over 1000 realizations. The CP is chosen as $T_c = T_d/4 = 8$. Therefore the number of samples for the autocorrelation estimate at the secondary user is $M = 100(T_d + T_c) = 4000$. This approximately corresponds to sensing time of 1 ms for an OFDM system having bandwidth of 5 MHz. The SNR in decibels is defined as $\mathrm{SNR(dB)} = 10\ \log_{10}(\mathrm{SNR}) = 10\ \log_{10}(\sigma_s^2/\sigma_w^2)$.

A. Performance of Local Detector

First we consider the performance of the proposed detector in Section II based on the autocorrelation coefficient test statistic $\hat{\rho}_{ML}$ in the case of a single secondary user trying to detect the primary user. Fig. 4 shows the Receiver Operating Characteristic (ROC) plot for simulated and theoretical cases for $\mathrm{SNR} = -10$ dB. Fig. 5 shows the plots for the probability

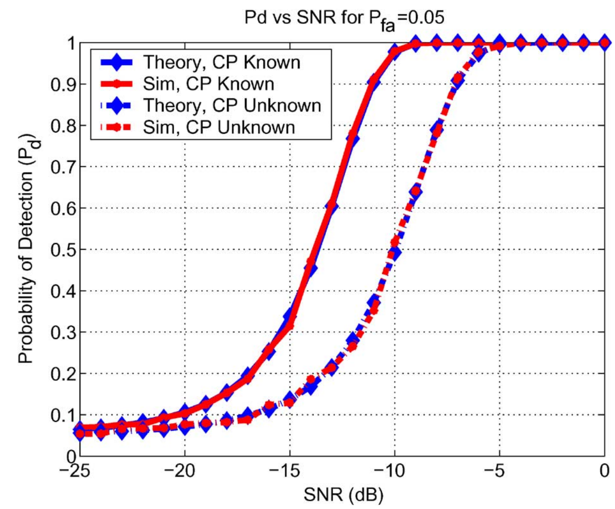


Fig. 5. Probability of detection P_d as a function of SNR for the single user case. Theoretical analysis and simulation results are close. Knowledge of the CP improves performance.

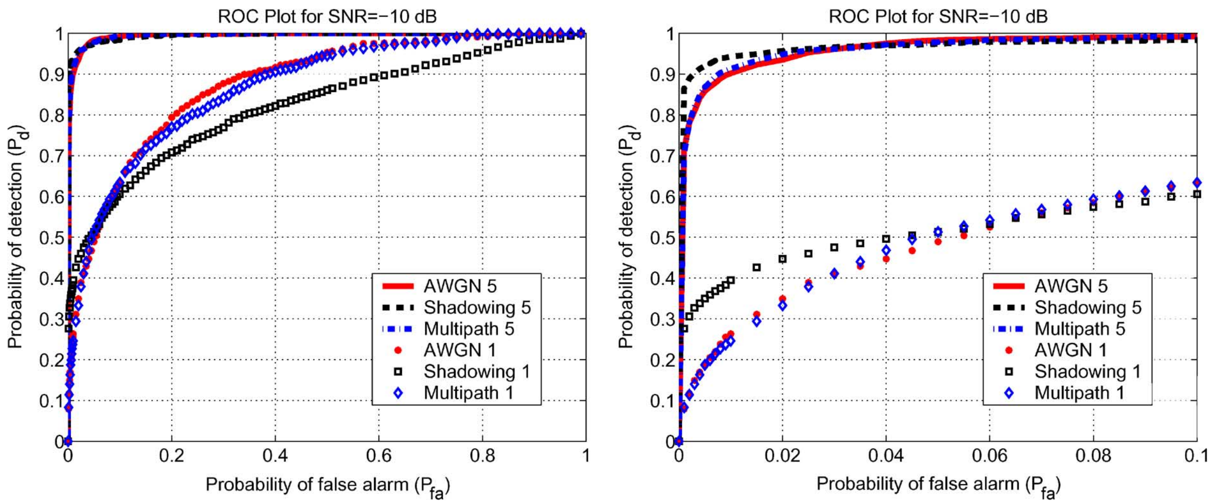
of detection P_d versus SNR based on theory and simulation. For these plots the probability of false alarm is $P_{fa} = 0.05$. Theoretical values of P_d for different SNRs for a given P_{fa} are calculated using (20) and (21). From the figures, it is clear that the theory and the simulation results are on par in the low SNR regime thus validating the theoretical analysis. The detector performance does not degrade at high SNR values even though the theoretical analysis is done for low SNR regime.

Fig. 5 also shows the effects of exploiting information regarding the CP length and its position on the detector performance. In presence of this additional information, the performance of the scheme improves considerably. Also note that this is the best performance one can get by using an autocorrelation coefficient based local detector for detecting a CP-OFDM based system. Hence this serves as an upper bound on the performance of the practical detectors where only partial or no information about the CP is available.

Now, we present the performance of the scheme in multipath and shadowing. For the multipath case, an exponentially decaying Rayleigh channel of order 6 is used with $\delta = 1$. For simulating shadowing effects, the SNR of the user is selected randomly from a Gaussian distribution with mean $\mathrm{SNR} = -10$ dB and standard deviation of 5 dB for each realization. The channel is considered stationary over the duration of an OFDM frame. Fig. 6(a) shows that the performance of the local detector based on the autocorrelation coefficient test statistic in a multipath channel is similar to that in the AWGN case. This is on par with the analysis. In the case of shadowing, there is a deterioration in the detection performance.

B. Performance for Cooperative Detection

Here, the performance of cooperative detection based on the autocorrelation coefficient in AWGN, multipath and shadowing scenarios is presented using (32)–(37). It is assumed that channels seen by different secondary users are independent. Fig. 6(a) and (b) demonstrates the gain in detection performance when nearby secondary users cooperate to detect the same primary



(a) (b)

Fig. 6. Comparison of collaborative detection gain over local detector performance in different channel scenarios. (a) Detector performs equally well in AWGN and multipath. Degradation in performance can be seen in case of shadowing. (b) Zoom of important portion of (a). Cooperative detection improves detector performance and mitigates shadowing effects.

user. It can also be seen that collaborative effort mitigates the shadowing effect because of spatial diversity.

From Fig. 6(a) and (b), it can be seen that the detection probability in shadowing is higher than that in AWGN for very low SNR values. However, this does not mean that there is performance gain in the presence of shadowing as compared to the AWGN case. The reason is explained next. Shadowing is a process of random attenuation in the received signals due to the presence of obstacles between the transmitter and the receiver. The effect of shadowing on detector performance is twofold:

- Since the attenuation due to obstacles is in addition to the attenuation due to path loss, the detection curves will shift to the left by the amount of mean attenuation (due to shadowing) in case of the AWGN+Shadowing as compared to only AWGN.
- The fluctuations will reduce the slope of the detection curve as compared to AWGN, i.e., the probability of detection will improve slowly with average SNR as compared to the case of AWGN.

Thus, for a given distance between transmitter and receiver, the average SNR in the case of AWGN will always be less than the average SNR in the case of AWGN+Shadowing. The curves shown in the above figures are for the same average SNR for the two channel conditions, which is possible only for different distances between transmitter and receiver (the receiver corresponding to the shadowing case is nearer to the transmitter as compared to the receiver in the AWGN case so that the average SNR is same in both cases). Hence the two curves are not comparable in terms of SNR gain for our simulation results. However the effect of shadowing on reduction in slope of the detection curve is valid and quite evident from the figures.

C. Comparison of SD and FSS Schemes

For the case of the SD scheme at the FC, the limit on false-alarm rate is set to $P_{fa} = 0.05$ while the limit on missed de-

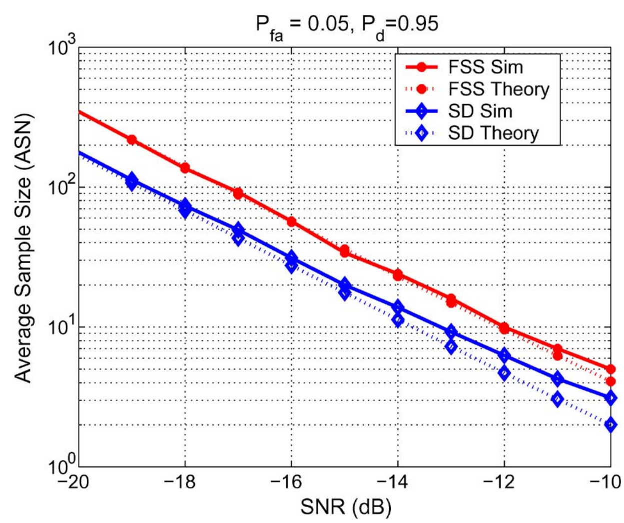


Fig. 7. Simulation and theoretical values of NSUS for FSS detection (K_f) are close. Although simulation values of average NSUS for SD (K_m) get closer to the theoretical values as average NSUS increase, SD uniformly gives significant savings as compared to FSS detection.

tection probability is $\beta = 1 - P_d = 0.05$. Thus the lower limit on the probability of detection P_d is 0.95. The SD scheme is truncated at a maximum number of secondary user statistics $K_{max} = 1000$. Note that K_{max} is chosen here such that it is at least three times greater than K_m in the SNR regime of interest. Truncating the SD ensures that the SD terminates and the choice of K_{max} ensures that the truncation has negligible effect on the error probabilities of the simulation. For the present simulations, it has been assumed that each user transmits one LLR per detection period. All parameters of the simulations regarding evaluation of LLRs at the secondary users are the same as in the case of a single user. Fig. 7 compares the theoretical and simulated values of average NSUS (K_m) for SD and NSUS

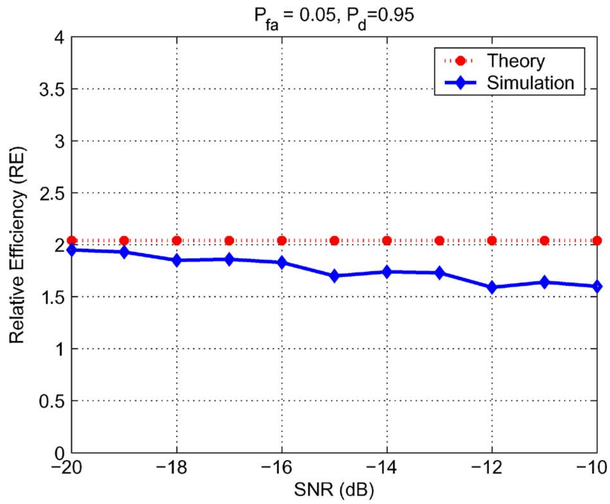


Fig. 8. Relative Efficiency (RE) of the SD with respect to the FSS detector for the same error probabilities. RE increase with increase in NSUS. RE values are significant around 1.53 even for the case of a few secondary users (<20).

TABLE I
RE VALUES FOR THE AWGN SCENARIO CORRESPONDING TO A FEW SECONDARY USERS (<20)

SNR (dB)	K_f	K_m	RE
-13	16	9.24	1.73
-12	10	6.28	1.59
-11	7	4.27	1.64
-10	5	3.12	1.60

(K_f) for FSS. It can be seen that simulation results are close to theoretical values. Note that average NSUS (K_m) given by (28) and (30) are asymptotic, i.e., $K_m \rightarrow \infty$. That explains why the curves for theory and simulation get close as average NSUS increases in the case of SD.

Fig. 8 shows the RE of the SD scheme as compared to the FSS detection in AWGN. Table I shows the corresponding values of SNR and RE obtained by using the SD for small NSUS instead of the FSS detection. The case of small NSUS correspond to a practical scenario of interest where very few number of users (<20) are cooperating. It can be seen that even for such a scenario, the SD gives significant savings. For example, at $\mathrm{SNR} = -10$ dB, only 3.12 secondary user statistics are required on average for the SD at the FC compared to 5 secondary user statistics in the case of FSS at the FC for the same detector performance, which amounts to an RE of around 1.60. As the number of secondary user statistics increase, RE value increases to the theoretical value of 2.04.

Table II shows simulation results of FSS and SD schemes in the shadowing scenario. The result show that the SD scheme has an RE of approximately 1.8 for different SNRs in the case of the shadowing, which is an impressive savings in the number of statistics required to make a crucial and reliable decision at the FC.

TABLE II
RE VALUES FROM SIMULATION IN THE SHADOWING SCENARIO

SNR (dB)	K_f	K_m	RE
-20	54	27.84	1.94
-19	38	20.44	1.86
-18	28	15.05	1.86
-17	22	11.46	1.82
-16	17	8.79	1.93
-15	13	6.73	1.93
-14	10	5.38	1.86
-13	8	4.25	1.88
-12	7	3.52	1.99
-11	5	2.98	1.68
-10	5	2.47	2.02

VII. CONCLUSION

In this paper, we have introduced a simple and computationally efficient spectrum sensing scheme based on the autocorrelation coefficient for CP-OFDM based primary user signals. The underlying assumptions made about the primary user signals are minimal. In the low SNR regime, the autocorrelation coefficient is shown to be the LLRT statistic. Theoretical analysis and simulations show that the local detector performs reliably under various channel conditions. Even though the performance analysis is carried out for low SNR values, performance remains good for high SNRs as well. Performance of the local detector remains unchanged in the multipath environment as compared to the AWGN case. The effects of CP knowledge on detector performance have also been studied. The local detector shows considerable improvement by exploiting complete CP information. This also serves as an upper bound for practical autocorrelation coefficient based local detectors.

Using theoretical analysis and simulations, considerable improvement in detection performance has been shown for the case of collaborative detection in AWGN and multipath. Theoretical analysis has been carried out for the AWGN case and validated by simulations. Based on theoretical analysis for a single detector in multipath, it is clearly seen that the performance of the scheme in a multipath scenario is the same as that of AWGN. This has been confirmed with simulations. Extensive simulations for the case of shadowing show that collaboration gives diversity gain by reducing the effects of shadowing on the detector performance.

A decentralized SD scheme is proposed to combine the soft decisions (autocorrelation-based LLRs) from the cooperating secondary users at the FC. In addition, performance of the proposed scheme has been analyzed. When compared to FSS detection, SD looks appealing, as shown by the theory and simulations in this paper. It gives significant saving in the number of secondary users statistics (or LLRs) required to make the final decision at the FC for the same performance defined by error probabilities, i.e., false alarm and missed detection. RE quantifies the savings in the amount of data needed to make the final decision by using the SD scheme instead of the FSS scheme. REs of approximately 2.0 for AWGN and 1.8 for shadowing scenarios can be obtained when there are large numbers of secondary user statistics. Also for a feasible scenario of a few secondary users collaborating, a notable RE of almost 1.6 can be observed in simulations. These REs demonstrate the significant reduction in the delay and the data needed in reliable decision making at the FC under different channel conditions.

APPENDIX

PROOF: $\rho_2 = (T_c/(T_d + T_c))(\delta\sigma_s^2/(\delta\sigma_s^2 + \sigma_w^2))$

Under the alternative hypothesis H_1, the received signal in case of multipath is

$$\begin{aligned} x(t) &= s(t) * h(t) + w(t) \\ &= \sum_{l=0}^{P-1} h(l)s(t-l) + w(t) \\ &= y(t) + w(t) \end{aligned} \tag{40}$$

where '$*$' denotes convolution and $y(t) \triangleq \sum_{l=0}^{P-1} h(l)s(t-l)$. Independence of channel taps $h(l)$ and $s(t)$ gives

$$\begin{aligned} E[y(t)] &= E\left[\sum_{l=0}^{P-1} h(l)s(t-l)\right] \\ &= \sum_{l=0}^{P-1} E[h(l)]\, E[s(t-l)] = 0. \end{aligned} \tag{41}$$

Next, using independence of channel taps $h(l)$ and $s(t)$ along with $E[s(t)s^*(t+T_d)] = (T_c/(T_c+T_d))\sigma_s^2$, we have

$$\begin{aligned} &E[y(t)y^*(t+T_d)] \\ &= E\left[\left(\sum_{l=0}^{P-1} h(l)s(t-l)\right)\left(\sum_{i=0}^{P-1} h^*(i)s^*(t+T_d-i)\right)\right] \\ &= \sum_{l=0}^{P-1}\sum_{i=0}^{P-1} E[h(l)s(t-l)h^*(i)s^*(t+T_d-i)] \\ &= \sum_{l=0}^{P-1} E[h(l)h^*(l)]\, E[s(t-l)s^*(t+T_d-l)] \\ &= \delta\frac{T_c}{T_c+T_d}\sigma_s^2 \end{aligned} \tag{42}$$

where $\delta = \sum_{l=0}^{P-1} E[|h(l)|^2]$ is used for convenience. Again, using independence of channel taps $h(l)$ and $s(t)$ along with $E[s(t)s^*(t)] = \sigma_s^2$, we have

$$\begin{aligned} E[y(t)y^*(t)] &= E\left[\left(\sum_{l=0}^{P-1} h(l)s(t-l)\right)\right. \\ &\qquad \left.\times\left(\sum_{i=0}^{P-1} h^*(i)s^*(t-i)\right)\right] \\ &= \sum_{l=0}^{P-1}\sum_{i=0}^{P-1} E[h(l)s(t-l)h^*(i)s^*(t-i)] \\ &= \sum_{l=0}^{P-1} E[h(l)h^*(l)]\, E[s(t-l)s^*(t-l)] \\ &= \delta\sigma_s^2. \end{aligned} \tag{43}$$

Independence of $w(t_1)$ and $w(t_2)$ for $t_1 \neq t_2$, and $y(t)$ and $w(t)$ gives

$$\begin{aligned} &E[x(t)x^*(t+T_d)|H_1] \\ &\quad = E[(y(t)+w(t))(y^*(t+T_d)+w^*(t+T_d))] \\ &\quad = E[y(t)y^*(t+T_d)] \\ &\quad = \frac{T_c}{T_c+T_d}\delta\sigma_s^2. \end{aligned} \tag{44}$$

Similarly,

$$\begin{aligned} E[x(t)x^*(t)|H_1] &= E[(y(t)+w(t))(y^*(t)+w^*(t))] \\ &= E[y(t)y^*(t)] + E[w(t)w^*(t)] \\ &= \delta\sigma_s^2 + \sigma_w^2. \end{aligned} \tag{45}$$

Substituting values of the expectation in (44) and (45) into the definition of autocorrelation coefficient in the multipath scenario ρ_2, we get

$$\rho_2 = \frac{E[x(t)x^*(t+T_d)|H_1]}{E[x(t)x^*(t)|H_1]} = \frac{T_c}{T_c+T_d}\frac{\delta\sigma_s^2}{\delta\sigma_s^2+\sigma_w^2}. \tag{46}$$

Hence the desired result is proved.

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