University of Tehran
College of Engineering
School of Electrical and Computer Engineering

Detection and Estimation Theory

Dr.Olfat

Autocorrelation-Based Decentralized Sequential Detection of OFDM Signals in Cognitive Radios

Explained by:
Soroush Mesforush Mashhad
SN:810198472

Tir 02

Contents

Abstract

In this report, we shall begin with the concept of cognitive radios, we shall learn what they are and how a phenomenon named spectrum sensing is used in such devices.

Next we shall take a look at the Orthogonal Frequency Division Multiplexing (OFDM) system. We shall delve into the realms of spectrum sensing where we shall deepen our study of the OFDM system.

In the next stage we shall take a look at the mathematical and statistical prerequisites such as the complex normal distribution.

After this we begin the first hypothesis testing problem, we define the needed parameters and coefficients such as the autocorrelation coefficient and continue are calculations accordingly. We go on to utilize the maximum likelihood estimator to complete the hypothesis testing format. In the end of this section we derive the Neyman-Pearson detector with precision.

Next we briefly study the effect of knowing and not knowing the cyclic prefix in the OFDM system.

Next we move on to the local detector in multipath, we study the concepts of a multipath channel and go on to perform another hypothesis testing analysis and derive the Neyman-Pearson detector accordingly.

Next we study sequential detection and the detectors and hypothesis testing problem accordingly. Finally we study concepts such as the number of secondary user statistics (NSUS) and derive ratios such as relative efficiency to compare sequential detection (SD) and fixed sample size (FSS) schemes.

In the final section simulations have been included and explained.

1 Introduction

First of all, we shall briefly explain what a **Cognitive Radio** actually is.

1.1 Cognitive Radio

A **Cognitive Radio** is a variant of wireless communication in which the transceiver can detect the communication channels it is okay to use and the ones that aren't. After detection, the transceiver goes on to use the vacant channels and avoids occupied channels to avoid signal interference.

Cognitive Radios optimize the available radio frequency spectrum and also minimize user interference, hence leading to enhanced **Quality of Service**.

A **Cognitive Radio** has two main networks depicted as below.

- **Primary Network**

  This network consists of the primary radio base station and users.

- **Secondary Network**

  This network shares the unused spectrum with the primary network accordingly.

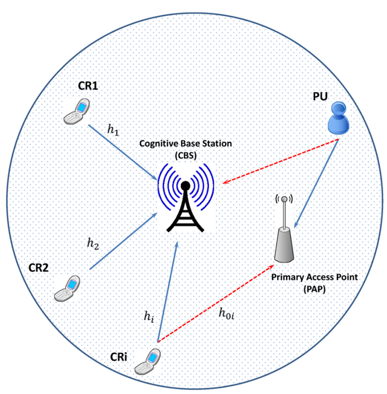

Figure 1: System Model of Cognitive Radio

1.2 Spectrum Sensing

Spectrum Sensing is defined to be the process of finding and indentifying which spectrums are not used by anyone an can be used to its fullest without causing harmful interference for other users.

In other words, sensing makes the radio aware of the environment which it is operating within, hence the radio can adjust its transmission according to the environment.[1]

Some of the transmission parameters of a **Cognitive Radio** are as follows.

- **Beam Pattern**
- **Power**
- **Modulation**
- **Frequency**

Spectrum sensing techniques are mostly based on **Cyclostationarity** and **Classical energy detection**.

1.2.1 Cyclostatinary specturm sensing

In a brief explanation, a cyclostationary process $X(t)$ is defined as follows.

$$m_X(t) = m_X(t + T_0), \quad R_X(t + \tau, t) = R_X(t + T_0 + \tau, t + T_0)$$

As for **Cyclostatinary specturm sensing**, this technique can be completed via the following block diagram.

[1] These parameters are adjusted dynamically, typically with learning methods these days

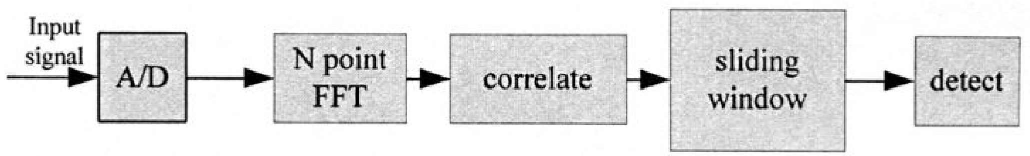

Figure 2: Block Diagram of Cyclostationary spectrum sensing

The sliding window can be different windows based on our need, some popular windows include **Hanning**,**Hamming**, **Blackman** windows, the most important window is the **Kaiser** window depicted below.

$$w(n) = \begin{cases} I_0\left(\beta\sqrt{1-\left[1-\frac{n}{\frac{q}{2}}\right]^2}\right), & 0 \leq n \leq q \\ 0, & otherwise \end{cases}$$

β is a parameter for optimizing the threshold between the main lobe width and the side lobe amplitudes.

1.2.2 Classical Energy Detection

In a brief explanation, this method is a very basic sensing technique in which we do not need any prior knowledge of the primary user, this gives us advantages as it reduces the computation complexity.

This detector compares measured energy with a threshold value and then decides whether a channel is occupied or not, a drawback in this method could be longer sensing time, this is to increase the **SNR**.

In some sources the decision metric for the energy detector is shown as below.

$$\xi_{ED} = \frac{1}{N}\sum_{k=0}^{N-1} yk^2$$

Nowadays, **OFDM**(Orthogonal Frequency Division Multiplexing) has become very popular in wideband systems. We have learned the advantages of this system in the **Wireless Communication** course, some applications are as follows.

- **WiMAX**
- **Long Term Evolution (LTE)**
- **Mobile Broadband Wireless Access(MBWA)**

A simple block diagram of an **OFDM** system is depicted below.

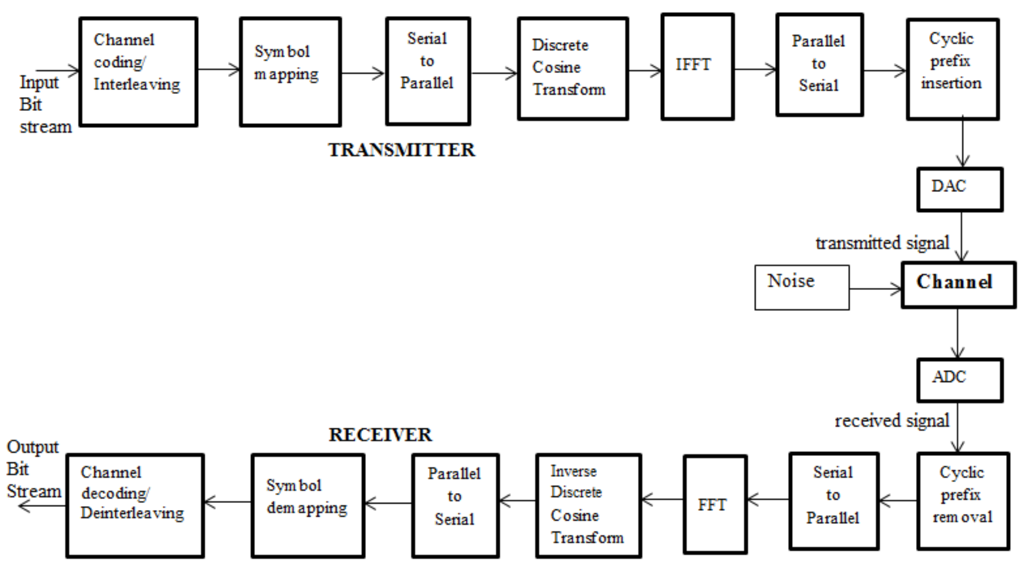

Figure 3: Block Diagram of OFDM System

We know that the **OFDM** system is becoming more popular day by day, so it is fair to make the assumption that many primary users shall be **OFDM** based, so we must carefully study the problem of detection in **OFDM** systems.

We know that we have a **Cyclic Prefix (CP)** in **OFDM** systems, this implies that the autocorrelation coefficients are **nonzero** at delays $\tau = \pm T_d$ in which T_d is the number if samples corresponding to useful symbol length in an **OFDM** block.

As implied in the paper, **Decentralized cooperative detection** is being more widely used and drawn attention recently, the benefits of this method are listed below.

- **Improvement of detector performance**
- **Alleviation of fading and shadowing effects**
- **Increase in coverage**
- **Use of simpler detectors**

It is also implied that in **decentralized detection**, a sequential procedure can be applied by the individual secondary users at the **Fusion Center(FC)**, inspired by this, the authors proposed a **Sequential Detection (SD)** scheme as depiced below.

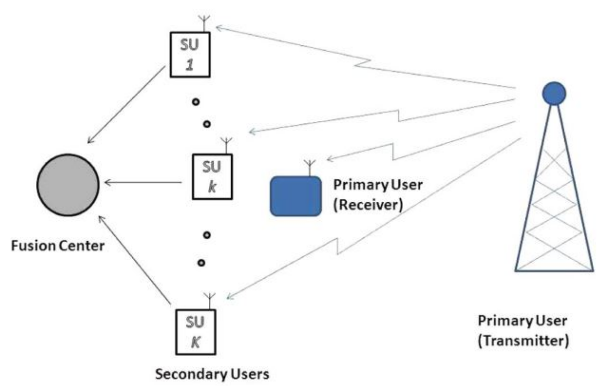

Figure 4: Spectrum Detecion Scheme

It is important to note that the **Fusion Center(FC)** has the final saying in cooperative detection.

The contributions of this paper have been listed without change:

1. For **CP-OFDM** based system, the autocorrelation coefficients corresponding to lags are shown to be the log likelihood ratio test (LLRT) statistic in the low signal-to-noise ratio (SNR) regime. The distributions of the corresponding local detector are established under both hypotheses for different channel conditions and the Neyman–Pearson detector is designed.

2. The effects of exploiting information related to CP on detection performance are shown through theory and simulation.

3. For autocorrelation coefficient based collaborative detection, a sequential detection scheme at the fusion center is proposed, where autocorrelation coefficient based log likelihood ratios (LLRs) from different secondary users are combined in a sequential manner at a FC in order to quickly detect the primary user reliably in the face of shadowing and fading.

4. The test statistic at the fusion center is derived. and the distributions of the test statistic at the FC under the two hypotheses are established then the performance of the proposed detector in the low SNR regime is analyzed.

5. Performance of the sequential detection scheme is compared to that of the fixed sample size test under different channel conditions. It is shown that the sequential detection scheme, on average, needs considerably fewer statistics to make a reliable decision at the fusion center

The organization and flow of the paper is as follows.

- Proposition of a detection scheme based on autocorrelation properties of **OFDM** signals for **AWGN** scenario.
- Discussion of effort of using the knowledge of the cyclic prefix on detector performance.
- Performance analysis of of the scheme in multipath scenario
- Proposition of sequential detection scheme in which each secondary user transmits an autocorrelation coeffiecient based decision statistic to the fusion center

2 Spectrum Sensing Based On Autocorrelation Coefficient

We have learned in **Digital Communications** and **Wireless Communications** that **OFDM** signals are actually a summation of narrowband subcarriers that are usually modulated via **QAM** or **PSK**, **QAM** is more frequently used, we know that the subcarriers must be orthogonal functions, we know that mathematically speaking, orthogonal functions have 0 inner product, or in an easier statement, they are statistically independent.

Normally, N subcarriers are combined to form an array of parallel signals, the modulated subcarriers can be used to support independent baseband signals but more typically they are combined.

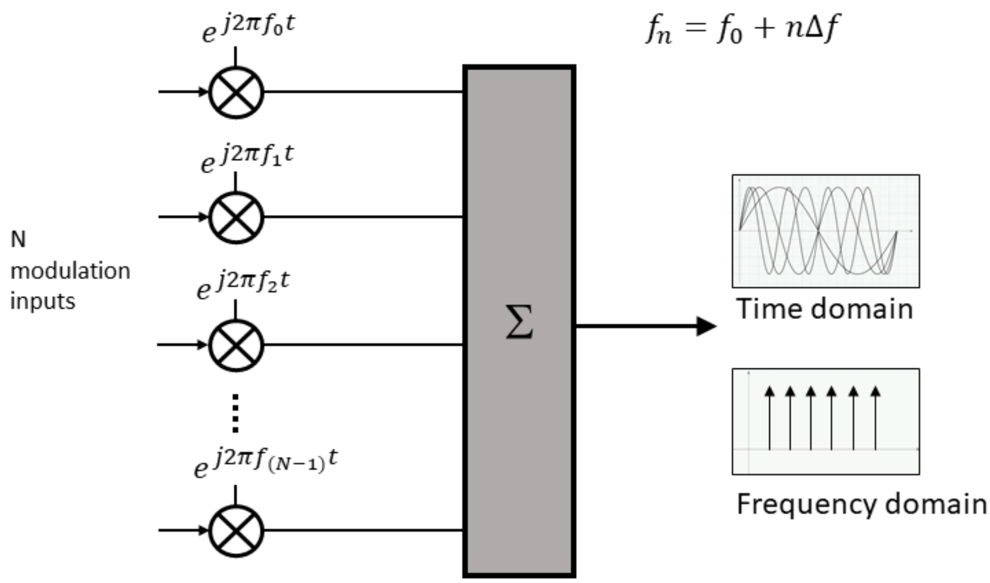

Figure 5: OFDM modulator signal summation

It is assumed that we have a unit sampling factor, therefore T_d is a representative of the number of subcarriers in the **OFDM** system.

We know that in an **OFDM** system, the signal is constructed by feeding T_d symbols through an **IFFT** block.

$$c(t) = \frac{1}{\sqrt{T_d}} \sum_{f=0}^{T_d-1} C(f) e^{\frac{\mathrm{j}2\pi t f}{T_d}}, \quad t = 0, \ldots, T_d - 1$$

t : Discrete time index, f : Discrete frequency index

So we can deduce that T_d also represents the number of data in an **OFDM** block.

After this the cyclic prefix is added to the front of the block (T_C symbols) to create the **OFDM** block.

$$\textbf{Symbols to add}: \; [c(T_d - T_c), \ldots, c(T_d - 1)]\,,$$
$$\textbf{OFDM block}: \; [c(T_d - T_c), \ldots, c(T_d - 1), c(0), c(1), \ldots, c(T_d - 1)]$$

It is important ot note that a **Transmitted Frame** may contain a lot of these blocks, we shall show the symbols of an **OFDM** with $s(t)$ from now on.

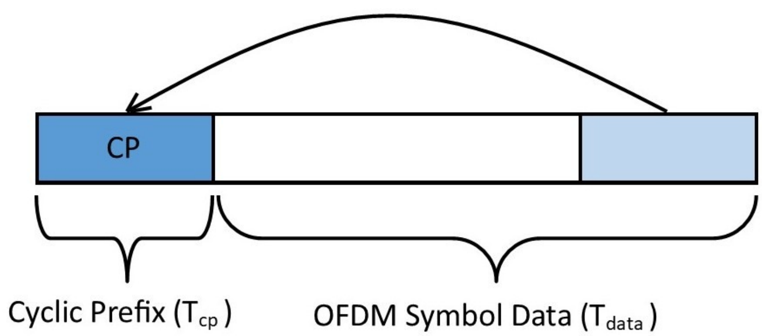

Figure 6: Cyclic Prefix in single OFDM block

From here on forward, we shall enter the mathematics of the problem. Before we start, we shall define a version of the Gaussian distribution which shall come in handy in due course.

2.1 Complex Normal Distribution

In a generalized form, the complex Gaussian distribution is defined as follows.

$$\mu = \mathbb{E}\{\mathbf{Z}\}, \quad \Gamma = \mathbb{E}\{(\mathbf{Z} - \mu)(\mathbf{Z} - \mu)^H\}, \quad C = \mathbb{E}\{(\mathbf{Z} - \mu)(\mathbf{Z} - \mu)^T\},$$
$$P = \bar{\Gamma} - C^H \Gamma^{-1} C$$

Here the location parameter μ is a n-dimensional complex vector; the covariance matrix Γ is Hermitian and n.n.d and the pseudo-covariance matrix C is symmetric. The complex normal random vector $\mathbf{Z}$ is denoted as below.

$$\mathbf{Z} \sim \mathcal{N}_c(\mu, \Gamma, C),$$

The probability density function of this distribution can be shown as below.

$$f(z) = \frac{1}{\pi^n \sqrt{\det(\Gamma)\det(P)}} \exp\left\{ -\frac{1}{2} \left((\bar{z} - \bar{\mu})^T, (z - \mu)^T \right) \begin{pmatrix} \Gamma & C \\ \bar{C} & \bar{\Gamma} \end{pmatrix}^{-1} \begin{pmatrix} z - \mu \\ \bar{z} - \bar{\mu} \end{pmatrix} \right\}$$

2.1.1 Circularly symmetric case

We know that a complex random vector $\mathbf{Z}$ is circularly symmetric if for every deterministic $\varphi \in [-\pi, \pi)$ the distribution of $e^{\mathrm{j}\varphi}\mathbf{Z}$ equals the distribution of $\mathbf{Z}$.

Now if the complex random variable, has a distribution in which data is symmetrically distributed with no skew, in other words if it is central normal then we shall have the following.

$$C = 0 \longrightarrow \mathbf{Z} \sim \mathcal{N}_c(\mu, \Gamma)$$

The probability density function shall be as follows.

$$f(z) = \frac{1}{\pi^n \det(\Gamma)} \exp\{-(z - \mu)^H \Gamma^{-1} (z - \mu)\}$$

2.2 Central Limit Theorem

We know that $X_1, X_2, \ldots, X_n$ are random variables with $\mathbb{E}(X_i) = m_i$ and $Var(X_i) = \sigma_i^2 < \infty$, if we define $a = \lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} \sum_{i=1}^n m_i$ and $b = \lim\limits_{n\to\infty} \frac{1}{n} \sum_{i=1}^n \sigma_i^2$, it can be proven that $\mathbf{Z}_n = \frac{\sum_{i=1}^n X_i}{\sqrt{n}}$ converges to $\mathbf{Z}$ in distribution where $\mathbf{Z}$ is defined as below.

$$\mathbf{Z}_n \xrightarrow{d} \mathbf{Z}, \quad \mathbf{Z} \sim \mathcal{N}(a, b)$$

After stating these preliminary definitions, we go back to formulating the problem.

We define a hypothesis testing problem as follows.

$$H_0 : x(t) = w(t)$$
$$H_1 : x(t) = s(t) + w(t)$$

It is important to note that $x(t)$ is the received signal which has a complex format and $w(t)$ is a complex circular additive white Gaussian noise

The null hypothesis is when we have no primary user and the other hypothesis is when a primary user is active.

Logically, we must find the distribution of $x(t)$ under each hypothesis, by using the central limit theorem as explained above we can deduce that if we have a sufficient amount of samples we can use the Gaussian distribution as our approximation.

In order to have enough samples we need to assume that T_d or in other words the IFFT is large enough to satisfy the central limit theorem, we also assume that $x(t)$ is circularly symmetric for brevity and to abstain from extremely complicated probability density functions.

$$w(t) \sim \mathcal{N}_c(0, \sigma_w^2), \quad s(t) \sim \mathcal{N}_c(0, \sigma_s^2)$$

Now we can easily find the distribution for the different hypothesis.

$$H_0 : x(t) \sim \mathcal{N}_c(0, \sigma_s^2)$$
$$H_1 : x(t) \sim \mathcal{N}_c(0, \sigma_s^2 + \sigma_w^2)$$

As we explained before, $x(t)$ is circularly symmetric so it consists of real and imaginary parts, both of these parts are Gaussian random variables.

$$x(t) = x_r(t) + \mathrm{j}x_i(t), \quad x_i(t) \& x_r(t) \sim \mathcal{N}(0, \frac{\sigma_x^2}{2})$$

We shall define the autocorrelation coefficient for an **OFDM** signal as follows.

$$\rho(\tau) = \frac{\mathbb{E}\{x(t)x^\star(t+\tau)\}}{\mathbb{E}\{x(t)x^\star(t)\}}$$

In the above formula τ are the lags, we go on to calculate $\rho(\pm T_d)$ for both hypothesis.

$$H_0 : x(t) = w(t) \longrightarrow \rho(\pm T_d) = \mathbb{E}\{x(t)x^\star(t+\tau)\} = \mathbb{E}\{w(t)w^\star(t \pm T_d)\} = 0$$

$$H_1 : x(t) = s(t) + w(t), \quad \mathbb{E}\{s(t)s^\star(t)\} = \sigma_s^2 \quad \mathbb{E}\{s(t)s^\star(t+T_d)\} = \frac{T_c}{T_c + T_d}\sigma_s^2,$$

$$\mathbb{E}\{x(t)x^\star(t+T_d)|H_1\} = \mathbb{E}\{(s(t)+w(t))\,(s^\star(t+T_d)+w^\star(t+T_d))\}$$

$$= \mathbb{E}\{s(t)s^\star(t+T_d)\} + \mathbb{E}\{s(t)w^\star(t+T_d)\} + \mathbb{E}\{w(t)s^\star(t+T_d)\} + \mathbb{E}\{w(t)w^\star(t+T_d)\}$$

$$= \mathbb{E}\{s(t)s^\star(t+T_d)\} = \frac{T_c}{T_c+T_d}\sigma_s^2$$

$$\mathbb{E}\{x(t)x^\star(t)|H_1\} = \mathbb{E}\{(s(t)+w(t))\,(s^\star(t)+w^\star(t))\}$$

$$= \mathbb{E}\{s(t)s^\star(t)\} + \mathbb{E}\{s(t)w^\star(t)\} + \mathbb{E}\{w(t)s^\star(t)\} + \mathbb{E}\{w(t)w^\star(t)\}$$

$$= \mathbb{E}\{s(t)s^\star(t)\} + \mathbb{E}\{w(t)w^\star(t)\} = \sigma_s^2 + \sigma_w^2$$

After this hefty calculations we have:

$$\rho(\pm T_d) = \frac{\mathbb{E}\{x(t)x^\star(t \pm T_d)\}}{\mathbb{E}\{x(t)x^\star(t)\}} = \frac{\frac{T_c}{T_c+T_d}\sigma_s^2}{\sigma_s^2+\sigma_w^2} = \frac{T_c}{T_d+T_c}\frac{\sigma_s^2}{\sigma_s^2+\sigma_w^2}$$

It is important to note that we have used the fact that $\rho(\tau)$ is an even function in this scenario. If we go on to define the signal to noise ratio as $SNR = \frac{\sigma_s^2}{\sigma_w^2}$ we can simplify the autocorrelation coefficient as follows.

$$\rho(\pm T_d) = \frac{T_c}{T_d+T_c}\frac{\text{SNR}}{1+\text{SNR}}$$

Due to the fact that we implied that $\rho(tau)$ is an even function, we shall consider the $\rho(T_d)$ scenario from now on without any problems. Also for brevity, we shall show $\rho(T_d) = \rho$ from now on.

We aim to devise a scheme based on everything discussed until now. Our observations are several **OFDM** symbols as below.

$$x(0), \ldots, x(M+T_d-1), \quad M >> T_d,$$
$$\mathbf{z_1} = [x_r(0), x_i(0), x_r(1), x_i(1), \ldots, x_r(M-1), x_i(M-1)]$$
$$\mathbf{z_2} = [x_r(T_d), x_i(T_d), x_r(T_d+1), x_i(T_d+1), \ldots, x_r(M+T_d-1), x_i(M+T_d-1)]$$

As discussed before, $x_r(t)$ and $x_i(t)$ are circularly symmetric this means they are i.i.d. Now we shall study the t^{th} component of the random vectors $\mathbf{z_1}$ and $\mathbf{z_2}$ respectively, we shall do this under both hypothesis.

$z_1(t)$ & $z_2(t)$ $\longrightarrow$ Jointly Gaussian

$$H_0 : f\left(z_1(t), z_2(t)\right) = \frac{1}{2\pi\sigma_0^2} \exp\left\{ -\frac{1}{2} \left[\frac{z_1^2(t) + z_2^2(t)}{\sigma_0^2} \right] \right\}$$
$$H_1 : f\left(z_1(t), z_2(t)\right) = \frac{1}{2\pi\sigma_1^2\sqrt{1-\rho_1^2}} \exp\left\{ -\frac{1}{2(1-\rho_1^2)} \left[\frac{z_1^2(t) - 2\rho_1 z_1(t) z_2(t) + z_2^2(t)}{\sigma_1^2} \right] \right\}$$
$$\sigma_w^2 = 2\sigma_0^2, \quad 2\sigma_1^2 = \sigma_s^2 + \sigma_w^2$$

As we learned in the **Detection and Estimation Theory** course for a number of i.i.d measurements such as $X_1, \ldots X_n$the likelihood ratio is defined as follows.

$$L(\underline{\mathbf{x}}) = \frac{p_1(\underline{\mathbf{x}})}{p_0(\underline{\mathbf{x}})} = \prod_{i=0}^{N} \frac{p_1(x)}{p_0(x)}$$

For this scenario we shall define the likelihood function as Λ as follows.

$$\Lambda = \prod_{t=0}^{2M-1} \frac{f\left(z_1(t), z_2(t)|H_1\right)}{f\left(z_1(t), z_2(t)|H_0\right)}$$
$$\Lambda = \prod_{t=0}^{2M-1} \frac{\frac{1}{2\pi\sigma_1^2\sqrt{1-\rho_1^2}} \exp\left\{ -\frac{1}{2(1-\rho_1^2)} \left[\frac{z_1^2(t) - 2\rho_1 z_1(t) z_2(t) + z_2^2(t)}{\sigma_1^2} \right] \right\}}{\frac{1}{2\pi\sigma_0^2} \exp\left\{ -\frac{1}{2} \left[\frac{z_1^2(t) + z_2^2(t)}{\sigma_0^2} \right] \right\}}$$

$$\Lambda = \frac{\sigma_0^{2M}}{\sigma_1^{2M}(1-\rho_1)^M} \exp\left\{ -\frac{1}{2}\left(\frac{1}{(1-\rho_1^2)\sigma_1^2} - \frac{1}{\sigma_0^2}\right) \sum_{t=0}^{2M-1} \left(z_1^2(t) + z_2^2(t)\right) + \frac{\rho_1 \sum_{t=0}^{2M-1} z_1(t) z_2(t)}{(1-\rho_1^2)\sigma_1^2} \right\}$$

It is important to note that the upper bound of the product is set to $2M-1$ because the vectors $\mathbf{z_1}$ and $\mathbf{z_2}$ both have $2M$ elements.

2.3 Maximum Likelihood Estimator

We have learned that this method is used in statistics to estimate parameters of a assumed distribution after observing some data, we do this by maximizing a likelihood function so that, under the assumed statistical model, the observed data is most probable.

$$\begin{aligned} &\text{Likelihood Function}: \mathcal{L}_n(\theta, \mathbf{y}), \quad \theta = [\theta_1, \theta_2, \dots, \theta_k]^T \\ &\hat{\theta} = \underset{\theta \in \Theta}{argmax} \mathcal{L}_n(\theta, \mathbf{y}) \end{aligned}$$

We know that for a Gaussian random variable such as $X \sim \mathcal{N}(\mu, \sigma^2)$ we have the following estimators for mean and variance.

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Having given this introduction about the ML estimator, due to the fact that $z_1(t)$ and $z_2(t)$ are both zero mean Gaussian distributions, the ML estimator for $\mathbb{E}\{z_i^2(t)\}$ which is obviously an estimate for the variance is as follows.

$$\hat{\sigma_z}^2 = \frac{1}{4M} \sum_{t=0}^{2M-1} \left(z_1^2(t) + z_2^2(t)\right)$$

This estimate comes from the vectors $\mathbf{z_1}$ and $\mathbf{z_2}$, they have $4M$ elements summed up hence the $\frac{1}{4M}$ in the summation.

It is also possible to write this estimate in term of the observations vector as below.

Observation Vector $[x_r(0), x_i(0), \ldots, x_r(M+T_d-1), x_i(M+T_d-1)]$

$$\hat{\sigma_z}^2 = \frac{1}{2(M+T_d)} \sum_{t=0}^{M+T_d-1} x_r^2(t) + x_i^2(t) = \frac{1}{2(M+T_d)} \sum_{t=0}^{M+T_d-1} |x(t)|^2$$

If the SNR value is low, we can approximate $\hat{\sigma_z}^2 \approx \sigma_0^2 \approx \sigma_1^2$

The likelihood function shall be as follows.

$$\Lambda = \frac{\exp\left\{-2M\left[\frac{1}{(1-\rho_1^2)} - 1\right] + \frac{\rho_1 \sum_{t=0}^{2M-1} z_1(t) z_2(t)}{(1-\rho_1^2)\hat{\sigma_z^2}}\right\}}{(1-\rho_1^2)^M}$$

$$\longrightarrow \Lambda = \frac{\exp\left\{-2M\left[\frac{\rho_1^2}{(1-\rho_1^2)} + \frac{2M\rho_1\hat{\rho}_{ML}}{(1-\rho_1^2)}\right]\right\}}{(1-\rho_1^2)^M}$$

It is important to note that $\hat{\rho}_{ML}$ is the maximum likelihood estimate of ρ from the $\mathbf{z_1}$ and $\mathbf{z_2}$ vectors.

$$\hat{\rho}_{ML} = \frac{\frac{1}{2M}\sum_{t=0}^{2M-1} z_1(t) z_2(t)}{\hat{\sigma_z}^2}$$

Evidently, we can write the ML estimate of ρ using the vector $\mathbf{z_2}$ as follows.

$$\hat{\rho}_{ML} = \frac{\frac{1}{M}\sum_{t=0}^{M-1} x_r(t)x_r(t+T_d) + x_i(t)x_i(t+T_d)}{\hat{\sigma_z}^2}$$

$$\hat{\rho}_{ML} = \frac{\frac{1}{M}\sum_{t=0}^{M-1} \Re\left[x(t)x^\star(t+T_d)\right]}{\hat{\sigma_z}^2}$$

In this paper we are working with Gaussian distributions, therefore using the log-likelihood function is very reasonable.

$$L = \ln(\Lambda) = \ln\left(\frac{\exp\left\{-2M\left[\frac{\rho_1^2}{(1-\rho_1^2)} + \frac{2M\rho_1\hat{\rho}_{ML}}{(1-\rho_1^2)}\right]\right\}}{(1-\rho_1^2)^M}\right)$$
$$L = -M\ln\left(1-\rho_1^2\right) - 2M\left[\frac{\rho_1^2}{(1-\rho_1^2)} + \frac{2M\rho_1\hat{\rho}_{ML}}{(1-\rho_1^2)}\right]$$
$$L = -M\ln\left(1-\rho_1^2\right) + 2M\frac{\rho_1(\hat{\rho}_{ML}-\rho_1)}{1-\rho_1^2}$$

It is important to note that what we have written is for the H_1 hypothesis, we know that in hypothesis testing problems we look for a threshold for the problem for example in this case with log-likelihood function we have:

$$L \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

In the case we shall attempt to simplify the hypothesis testing problem by keeping $\hat{\rho}_{ML}$ on one side and the other parameters on the other.

$$-M\ln\left(1-\rho_1^2\right) + 2M\frac{\rho_1(\hat{\rho}_{ML}-\rho_1)}{1-\rho_1^2} > \eta$$
$$\longrightarrow 2M\frac{\rho_1(\hat{\rho}_{ML}-\rho_1)}{1-\rho_1^2} > \eta + M\ln\left(1-\rho_1^2\right)$$
$$\longrightarrow (\hat{\rho}_{ML}-\rho_1) > \frac{1-\rho_1^2}{2M\rho_1}\left(\eta + M\ln\left(1-\rho_1^2\right)\right)$$
$$\longrightarrow \hat{\rho}_{ML} > \frac{1-\rho_1^2}{2M\rho_1}\left(\eta + M\ln\left(1-\rho_1^2\right)\right) + \rho_1$$

For brevity we shall make the following definition.

$$\eta_l \triangleq \frac{1-\rho_1^2}{2M\rho_1}\left(\eta + M\ln\left(1-\rho_1^2\right)\right) + \rho_1, \quad \longrightarrow \hat{\rho}_{ML} > \eta_l$$

After this we go on to denote the probability density function of $\hat{\rho}_{ML}$ under the null hypothesis based on 2M real symbols as follows.

$$f_{2M}(\hat{\rho}_{ML}|H_0) = \frac{\Gamma\left[\frac{1}{2}(2M-1)\right]}{\Gamma\left[\frac{1}{2}(2m-2)\right]}(1-\hat{\rho}_{ML}^2)^{\frac{2M-4}{2}}$$

Where $\Gamma(.)$ is the Gamma function defined as below.

$$\Gamma(n) = (n-1)!, \quad n \in \mathbb{N}^+$$
$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt, \quad \Re(z) > 0$$

2.4 Theorem

If $\hat{\rho}_{ML}$ is the symbol correlation coefficient for $2M$ symbols from a real valued Gaussian distribution with correlation ρ , then $\frac{\sqrt{2M}(\hat{\rho}_{ML}-\rho)}{(1-\rho^2)}$ is asymptotically distributed according to $\mathcal{N}_r(0,1)$.[2]

If I plug in the theorem above to our hypothesis we shall have the following.

$$H_0 : \lim_{M\to\infty} \sqrt{2M}\hat{\rho}_{ML} \xrightarrow{d} \mathcal{N}_r(0,1)$$
$$H_1 : \lim_{M\to\infty} \sqrt{2M}\hat{\rho}_{ML} \xrightarrow{d} \mathcal{N}_r(\sqrt{2M}\rho_1, (1-\rho_1^2)^2)$$
$$\text{"}\xrightarrow{d}\text{"} : \text{Convergence in distribution}$$

We've calculated ρ_1 before in the previous parts, its value is as follows.

$$\rho_1 = \frac{T_c}{T_d+T_c}\frac{\sigma_s^2}{\sigma_s^2+\sigma_w^2}$$

We shall also assume that M is sufficiently large so the theorem holds, we shall easily have the following.

$$H_0 : \hat{\rho}_{ML} \sim \mathcal{N}\left(0, \frac{1}{2M}\right)$$
$$H_1 : \hat{\rho}_{ML} \sim \mathcal{N}_r\left(\rho_1, \frac{(1-\rho_1^2)^2}{2M}\right)$$

[2] This theorem can be proved utilizing the Asymptotic normality in An Introduction to Multivariate Statistical Analysis T.W. Anderson

2.5 Derivation of Neyman-Pearson Detector

What we have written is a detection problem with two hypothesis, we shall derive the Neyman-Pearson detector for this scenario.

$$H_0 : \hat{\rho}_{ML} \sim \mathcal{N}\left(0, \frac{1}{2M}\right)$$
$$H_1 : \hat{\rho}_{ML} \sim \mathcal{N}_r\left(\rho_1, \frac{(1-\rho_1^2)^2}{2M}\right)$$

$$L(\hat{\rho}_{ML}) = \frac{p_1(\hat{\rho}_{ML})}{p_0(\hat{\rho}_{ML})} = \frac{\frac{1}{\sqrt{2\pi\frac{(1-\rho_1^2)^2}{2M}}}\exp\left\{-\frac{(\hat{\rho}_{ML}-\rho_1)^2}{2\frac{(1-\rho_1^2)^2}{2M}}\right\}}{\frac{1}{\sqrt{2\pi\frac{1}{2M}}}\exp\left\{-\frac{\hat{\rho}_{ML}^2}{2\frac{1}{2M}}\right\}} = \frac{1}{(1-\rho_1^2)}\exp\left\{\frac{\hat{\rho}_{ML}^2}{2\frac{1}{2M}} - \frac{(\hat{\rho}_{ML}-\rho_1)^2}{2\frac{(1-\rho_1^2)^2}{2M}}\right\}$$

$$L(\hat{\rho}_{ML}) = \frac{1}{(1-\rho_1^2)}\exp\left\{M\left(\hat{\rho}_{ML}^2 - \frac{(\hat{\rho}_{ML}-\rho_1)^2}{(1-\rho_1^2)^2}\right)\right\}$$
$$= \frac{1}{(1-\rho_1^2)}\exp\left\{M\left(\frac{\hat{\rho}_{ML}^2 - 2\hat{\rho}_{ML}^2\rho_1^2 + \hat{\rho}_{ML}^2\rho_1^4 - \hat{\rho}_{ML}^2 + 2\hat{\rho}_{ML}\rho_1 - \rho_1^2}{(1-\rho_1^2)^2}\right)\right\}$$

$$L(\rho_{ML}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta \longrightarrow \ln\left(L(\rho_{ML})\right) \underset{H_0}{\overset{H_1}{\gtrless}} \ln(\eta) \longrightarrow \ln\left(L(\rho_{ML})\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta'$$

$$\ln\left(L(\rho_{ML})\right) = -\ln\left((1-\rho_1^2)\right) + \frac{M}{(1-\rho_1^2)^2}\left((\rho_1^4 - 2\rho_1^2)\hat{\rho}_{ML}^2 + 2\rho_1\hat{\rho}_{ML} - \rho_1^2\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta'$$

$$(\rho_1^4 - 2\rho_1^2)\hat{\rho}_{ML}^2 + 2\rho_1\hat{\rho}_{ML} - \rho_1^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta'' \longrightarrow (\rho_1^4 - 2\rho_1^2)\hat{\rho}_{ML}^2 + 2\rho_1\hat{\rho}_{ML} - \rho_1^2 - \eta'' \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$$\hat{\rho}_{ML} = \frac{-2\rho_1 \pm \sqrt{4(\rho_1^2 + \eta'')(\rho_1^4 - 2\rho_1^2)}}{\rho_1^2(\rho_1^2 - 2)} = \tau, \quad \delta_{NP}(\hat{\rho}_{ML}) = \begin{cases} 1, & |\hat{\rho}_{ML}| > \tau \\ 0, & -\tau < |\hat{\rho}_{ML}| < \tau \end{cases}$$

It is evident that the autocorrelation coefficient is always positive, hence we can conclude.

$$\hat{\rho}_{ML} = \frac{-2\rho_1 + \sqrt{4(\rho_1^2 + \eta'')(\rho_1^4 - 2\rho_1^2)}}{\rho_1^2(\rho_1^2 - 2)} = \tau, \quad \delta_{NP}(\hat{\rho}_{ML}) = \begin{cases} 1, & \hat{\rho}_{ML} > \tau \\ 0, & 0 < \hat{\rho}_{ML} < \tau \end{cases}$$

This shall result in the exact same result as the threshold which we derived in the previous parts, if we substitute η'' for its value as shown above, we shall have the following.

$$\hat{\rho}_{ML} = \eta_l = \frac{1-\rho_1^2}{2M\rho_1}\left(\eta + M\ln\left(1-\rho_1^2\right)\right) + \rho_1, \quad \longrightarrow \hat{\rho}_{ML} > \eta_l$$

So the hypothesis test shall become as follows.

$$\hat{\rho}_{ML} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_l$$

Now we shall go on to evaluate the false alarm probability to change the form of η_l.

$$P_{fa} = P(H_1|H_0) = P\left(\hat{\rho}_{ML} > \eta_l|H_0\right) = \int_{\eta_l}^{\infty} \frac{1}{\sqrt{2\pi\frac{1}{2M}}}\exp\left\{-\frac{\hat{\rho}_{ML}^2}{2\frac{1}{2M}}\right\}d\hat{\rho}_{ML}$$

$$P_{fa} = 1-\phi\left(\sqrt{2M}\eta_l\right) = Q\left(\sqrt{2M}\eta_l\right) = \frac{1}{2}\left(1-\text{erf}\left(\sqrt{M}\eta_l\right)\right) = \frac{1}{2}\text{erfc}\left(\sqrt{M}\eta_l\right)$$

$$\eta_l = \frac{1}{\sqrt{M}}\text{erfc}^{-1}\left(2P_{fa}\right) \longrightarrow \hat{\rho}_{ML} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{\sqrt{M}}\text{erfc}^{-1}\left(2P_{fa}\right)$$

Now we shall calculate the probability of detection.

$$P_D = P(H_1|H_1) = P(\hat{\rho}_{ML} > \eta_l|H_1) = \int_{\eta_l}^{\infty} \frac{1}{\sqrt{2\pi\frac{(1-\rho_1^2)^2}{2M}}}\exp\left\{-\frac{(\hat{\rho}_{ML}-\rho_1)^2}{2\frac{(1-\rho_1^2)^2}{2M}}\right\}d\hat{\rho}_{ML}$$

$$P_D = 1-\phi\left(\sqrt{2M}\frac{\eta_l-\rho_1}{1-\rho_1^2}\right) = Q\left(\sqrt{2M}\frac{\eta_l-\rho_1}{1-\rho_1^2}\right) = \frac{1}{2}\left(1-\text{erf}\left(\sqrt{M}\frac{\eta_l-\rho_1}{1-\rho_1^2}\right)\right)$$
$$= \frac{1}{2}\text{erfc}\left(\sqrt{M}\frac{\eta_l-\rho_1}{1-\rho_1^2}\right)$$

It is important to note that the following transformations between the ϕ function,Q-function and the error and complementary error function hold.

$$\phi(z) = 1-Q(z), \quad \text{erf}(z) = 1-\text{erfc}(z)$$
$$Q(z) = \frac{1}{2}\left[1-\text{erf}\left(\frac{z}{\sqrt{2}}\right)\right], \quad Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

This scheme needs knowledge about T_d which is the number of data in an **OFDM** block, this assumption is reasonable for primary users because this information is given and specified in standards, even if we don't have the exact value of T_d we can detect it between the options we have.

3 Effect of Knowledge of CP

Until now, we have assumed that we know nothing about the cyclic prefix duration in samples which was T_c, we assumed we have no idea about which amounts it can take and the synchronization information which is the position of the cyclic prefix in the **OFDM** block, typically it's located at the beginning of the block but it doesn't have to be this way.

This assumption is correct in primary user systems in which the **CP** changes adaptively depending on cell size, multipath channel order, etc...

In practice we usually have knowledge about T_c, for example in **WLAN** it is known that $T_c = \frac{T_d}{4}$.

In this part, we shall assume that we know the length of the cyclic prefix and the synchronization, this is the best case and the performance of the system shall be increased admirably.

In the previous section we derived the maximum likelihood estimate of the autocorrelation coefficient as follows.

$$\hat{\rho}_{ML} = \frac{\frac{1}{M}\sum_{t=0}^{M-1} \Re\left[x(t)x^{\star}(t+T_d)\right]}{\hat{\sigma_z}^2}$$

We shall attempt to rewrite this expression based on our data, we define $N_s = \frac{M}{T_s}$ as the number of **OFDM** blocks over which we have derived the ML estimate for the autocorrelation coefficient, and $T_s = T_d + T_c$ is the number of symbols in an **OFDM** block.

$$\hat{\rho}_{ML} = \frac{\frac{1}{M}\sum_{n=0}^{N_s-1}\sum_{t=0}^{T_s-1}\Re\{x(nT_s+t)x^*(nT_s+t+T_d)\}}{\hat{\sigma_z}^2}$$

It is evident that the double summation in the formula implies summing $\Re\{x(nT_s+t)x^*(nT_s+t+T_d)\}$ over a sliding window with the length of T_s for each of the **OFDM** blocks.

It is logical to assume that if we know the length of the cyclic prefix and synchronization we can change the size of the second summation from T_s to T_d so we have the following.

$$\hat{\rho}_c = \frac{\frac{1}{M}\sum_{n=0}^{N_s-1}\sum_{t\in CP}\Re\{x(nT_s+t)x^*(nT_s+t+T_d)\}}{M_1\hat{\sigma_z}^2}$$
$$M_1 = T_c N_s$$

Similar to our deductions in the previous section, if M_1 is sufficiently large enough we shall have the following distributions for different hypothesis.

$$H_0 : \hat{\rho}_c \sim \mathcal{N}_r\left(0, \frac{1}{2M_1}\right)$$
$$H_1 : \hat{\rho}_c \sim \mathcal{N}_r\left(\rho_c, \frac{(1-\rho_c^2)^2}{2M_1}\right)$$

We shall calculate the value of ρ_c by getting inspiration from ρ_1.

$$\rho_c = \frac{T_c}{T_c}\frac{\text{SNR}}{1+\text{SNR}} = \frac{\text{SNR}}{1+\text{SNR}}$$

Now we shall go on to derive the **Neyman-Pearson** detector for this scenario.

We don't need to perform calculations as we did for the previous section because they are exactly the same, we just need to make the following adjustments.

$$\rho_1 \to \rho_c, \quad M \to M_1$$

Now the values for the false alarm and detection probability are as follows.

$$P_{fa} = P(H_1|H_0) = P\left(\hat{\rho}_{ML} > \eta_l|H_0\right) = \int_{\eta_l}^{\infty} \frac{1}{\sqrt{2\pi\frac{1}{2M_1}}} \exp\left\{-\frac{\hat{\rho}_{ML}^2}{2\frac{1}{2M_1}}\right\} d\hat{\rho}_{ML}$$

$$P_{fa} = 1-\phi\left(\sqrt{2M_1}\eta_l\right) = Q\left(\sqrt{2M_1}\eta_l\right) = \frac{1}{2}\left(1-\text{erf}\left(\sqrt{M_1}\eta_l\right)\right) = \frac{1}{2}\text{erfc}\left(\sqrt{M_1}\eta_l\right)$$

$$\eta_l = \frac{1}{\sqrt{M_1}}\text{erfc}^{-1}\left(2P_{fa}\right) \longrightarrow \hat{\rho}_{ML} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{\sqrt{M_1}}\text{erfc}^{-1}\left(2P_{fa}\right)$$

$$P_D = P(H_1|H_1) = P(\hat{\rho}_{ML} > \eta_l|H_1) = \int_{\eta_l}^{\infty} \frac{1}{\sqrt{2\pi\frac{(1-\rho_c^2)^2}{2M_1}}} \exp\left\{-\frac{(\hat{\rho}_{ML}-\rho_c)^2}{2\frac{(1-\rho_c^2)^2}{2M_1}}\right\} d\hat{\rho}_{ML}$$

$$P_D = 1-\phi\left(\sqrt{2M_1}\frac{\eta_l-\rho_c}{1-\rho_c^2}\right) = Q\left(\sqrt{2M_1}\frac{\eta_l-\rho_c}{1-\rho_c^2}\right) = \frac{1}{2}\left(1-\text{erf}\left(\sqrt{M_1}\frac{\eta_l-\rho_c}{1-\rho_c^2}\right)\right)$$

$$= \frac{1}{2}\text{erfc}\left(\sqrt{M_1}\frac{\eta_l-\rho_c}{1-\rho_c^2}\right)$$

It is obvious that the mean of the estimate has grown by a factor of $\frac{T_c+T_d}{T_c}$ under the hypothesis H_1, this fact increases the detection probability due to the form of the detection probability.

By checking the variance we can see that it grows by the same amount as the mean under the null hypothesis and by a slightly smaller amount under the H_1 hypothesis.

The effect of the growth of the mean is greater than the growth of the variance hence the detection probability shall increase in this scenario.

We assumed that T_c is known, this is reasonable in wireless standards because there are only a few acceptable values for the cyclic prefix, for example **DVB-T** which is short for **Digital Video Broadcasting - Terrestial** has four acceptable cyclic prefix amounts.

We've also assumed perfect synchronization, this might not be valid for wireless systems, hence the detection probability shall have weaker performance compared to our analysis, the distribution shall no longer be Gaussian and shall have very complex distributions, to keep up with our analysis, we shall continue with the detector of the previous section.

4 Performance of Local Detector in Multipath

In this section we shall consider a multipath channel, before going further we shall explain the definition of a multipath channel.

4.1 Multipath channel

We learned in the **Wireless communications** course that if we have a multipath channel whilst transmitting, the output shall be as follows.

$$y(\tau) = \alpha_0 x(\tau) + \underbrace{\alpha_1 x(\tau - \tau_1) + \cdots + \alpha_N x(\tau - \tau_N)}_{\text{Intersymbol Interference}} + h(\tau)$$

It is quite simple to deduce that the impulse response of this channel is as follows.

$$\text{Static}: \ c(\tau) = \alpha_0 \delta(\tau) + \alpha_1 \delta(\tau - \tau_1) + \cdots + \alpha_N \delta(\tau - \tau_N)$$
$$\text{Dynamic}: \ c(\tau, t) = \alpha_0(t) \delta(\tau) + \alpha_1(t) \delta(\tau - \tau_1) + \cdots + \alpha_N(t) \delta(\tau - \tau_N)$$

It is important to note that the shape of the impulse response of a static multipath channel is something like the following shape.

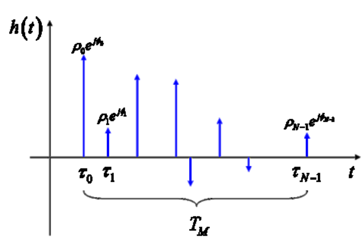

Figure 7: Static multipath channel impulse response

To see how fast a channel changes with time we use the concept of **Autocorrelation**. Furthermore we know that **Power Delay Profile (PDP)** gives the average power related to different delays in multipath channels, in this concept, we also introduce the following formulas.

$$R_c(\tau) = PDP, \quad \text{Average Delay Spread : } \mu_m = \frac{\int_0^\infty \tau R_c(\tau) d\tau}{\int_0^\infty R_c(\tau) d\tau}$$
$$\text{RMS Delay Spread : } \sigma_m = \sqrt{\frac{\int_0^\infty (\tau - \mu_m)^2 R_c(\tau) d\tau}{\int_0^\infty R_c(\tau) d\tau}}$$

In this paper we consider a multipath channel of order P (P is the number of multipath components), the channel taps are assumed independent from eachother for brevity, also they are independent of the transmitted data $s(t)$ and the noise $w(t)$.

$$h(.) \perp\!\!\!\perp s(.), \quad h(.) \perp\!\!\!\perp w(.)$$

Under the H_1 hypothesis the received signal $x(t)$ is obtained from a convolutional formula as follows.

$$x(t) = s(t) \star h(t) + w(t) = \sum_{l=0}^{P-1} h(l) s(t-l) + w(t)$$

If we go back to the hypothesis testing problem we designed before we have the following test.

$$H_0 : \rho = 0$$
$$H_1 : \rho = \rho_2$$

Now we shall go on to calculate ρ_2 as follows.

$$x(t) = s(t) \star h(t) + w(t) = \sum_{l=0}^{P-1} h(l) s(t-l) + w(t) = y(t) + w(t)$$

We shall define $y(t)$ as follows for brevity.

$$y(t) \triangleq \sum_{l=0}^{P-1} h(l)s(t-l)$$

As we remember the channel taps ($h(l)$) are independent of both noise and the signal hence we have the following.

$$\mathbb{E}\left[y(t)\right] = \mathbb{E}\left[\sum_{l=0}^{P-1} h(l)s(t-l)\right] = \sum_{l=0}^{P-1} \mathbb{E}\left[h(l)\right]\left[s(t-l)\right] = 0$$

Now we shall use our knowledge(from section 2) that $\mathbb{E}\{s(t)s^{\star}(t+T_d)\} = \frac{T_c}{T_c+T_d}\sigma_s^2$ and the independence between the channel taps and $s(t)$.

$$\mathbb{E}\left[y(t)y^{\star}(t+T_d)\right] = \mathbb{E}\left[\left(\sum_{l=0}^{P-1} h(l)s(t-l)\right)\left(\sum_{i=0}^{P-1} h^{\star}(i)s^{\star}(t+T_d-i)\right)\right]$$
$$= \sum_{l=0}^{P-1}\sum_{i=0}^{P-1} \mathbb{E}\left[h(l)s(t-l)h^{\star}(i)s^{\star}(t+T_d-i)\right] = \sum_{l=0}^{P-1} \mathbb{E}\left[h(l)h^{\star}(l)\right]\mathbb{E}\left[s(t-l)s^{\star}(t+T_d-l)\right]$$
$$= \delta\frac{T_c}{T_c+T_d}\sigma_s^2, \quad \delta \triangleq \sum_{l=0}^{P-1} \mathbb{E}\left[|h(l)|^2\right]$$

Once again we use our knowledge(from section 2) that $\mathbb{E}\{s(t)s^{\star}(t)\} = \sigma_s^2$ and the independence between the channel taps and $s(t)$.

$$\mathbb{E}\left[y(t)y^{\star}(t)\right] = \mathbb{E}\left[\left(\sum_{l=0}^{P-1} h(l)s(t-l)\right)\left(\sum_{i=0}^{P-1} h^{\star}(i)s^{\star}(t-i)\right)\right]$$
$$= \sum_{l=0}^{P-1}\sum_{i=0}^{P-1} \mathbb{E}\left[h(l)s(t-l)h^{\star}(i)s^{\star}(t-i)\right] = \sum_{l=0}^{P-1} \mathbb{E}\left[h(l)h^{\star}(l)\right]\mathbb{E}\left[s(t-l)s^{\star}(t-l)\right]$$
$$= \delta\sigma_s^2$$

Because the noise $w(t)$ is independent in different ts, we have the following.

$\mathbb{E}\left[x(t)x^{\star}(t+T_d)|H_1\right] = \mathbb{E}\left[(y(t)+w(t))(y^{*}(t+T_d)+w^{*}(t+T_d))\right] = \mathbb{E}\left[y(t)y^{\star}(t+T_d)\right] = \dfrac{T_c}{T_c+T_d}\delta\sigma_s^2$

Similarly we can imply that:

$$\mathbb{E}\left[x(t)x^{\star}(t)|H_1\right] = \mathbb{E}\left[(y(t)+w(t))(y^{*}(t)+w^{*}(t))\right] = \mathbb{E}\left[y(t)y^{\star}(t)\right] \\ + \mathbb{E}\left[w(t)w^{\star}(t)\right] = \delta\sigma_s^2 + \sigma_w^2$$

Now we can conclude our calculations and derive ρ_2 as follows.

$$\rho_2 == \frac{\mathbb{E}\left[x(t)x^{\star}(t+T_d)|H_1\right]}{\mathbb{E}\left[x(t)x^{\star}(t)|H_1\right]} = \frac{T_c}{T_d+T_c}\frac{\delta\sigma_s^2}{\delta\sigma_s^2+\sigma_w^2}$$

It is also evident that the autocorrelation coefficient is always a positive number less than 1.

We assume that $x(t), h(t)$ and $w(t)$ are circularly symmetric complex Gaussian random variables, so $x(t) = s(t) \star h(t) + w(t)$ shall also be circularly symmetric, the hypothesis testing problem shall be exactly like **Section 2** but with the following change.

$$H_0 : \hat{\rho}_{ML} \sim \mathcal{N}_r\left(0, \frac{1}{2M}\right) \\ H_1 : \hat{\rho}_{ML} \sim \mathcal{N}_r\left(\rho_2, \frac{(1-\rho_2^2)^2}{2M}\right)$$

The **Neyman-Pearson** detector shall be the same form as the previous one with slight adjustments.

$$\rho_1 \to \rho_r, \quad \text{SNR} = \frac{\delta\sigma_s^2}{\sigma_w^2}$$

Now the values for the false alarm and detection probability are as follows.

$$P_{fa} = P(H_1|H_0) = P\left(\hat{\rho}_{ML} > \eta_l|H_0\right) = \int_{\eta_l}^{\infty} \frac{1}{\sqrt{2\pi\frac{1}{2M}}} \exp\left\{-\frac{\hat{\rho}_{ML}^2}{2\frac{1}{2M}}\right\} d\hat{\rho}_{ML}$$

$$P_{fa} = 1 - \phi\left(\sqrt{2M}\eta_l\right) = Q\left(\sqrt{2M}\eta_l\right) = \frac{1}{2}\left(1 - \text{erf}\left(\sqrt{M}\eta_l\right)\right) = \frac{1}{2}\text{erfc}\left(\sqrt{M}\eta_l\right)$$

$$\eta_l = \frac{1}{\sqrt{M}}\text{erfc}^{-1}\left(2P_{fa}\right) \longrightarrow \hat{\rho}_{ML} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{\sqrt{M}}\text{erfc}^{-1}\left(2P_{fa}\right)$$

$$P_D = P(H_1|H_1) = P(\hat{\rho}_{ML} > \eta_l|H_1) = \int_{\eta_l}^{\infty} \frac{1}{\sqrt{2\pi\frac{(1-\rho_2^2)^2}{2M}}} \exp\left\{-\frac{(\hat{\rho}_{ML} - \rho_2)^2}{2\frac{(1-\rho_2^2)^2}{2M_1}}\right\} d\hat{\rho}_{ML}$$

$$P_D = 1 - \phi\left(\sqrt{2M}\frac{\eta_l - \rho_2}{1-\rho_2^2}\right) = Q\left(\sqrt{2M}\frac{\eta_l - \rho_2}{1-\rho_2^2}\right) = \frac{1}{2}\left(1 - \text{erf}\left(\sqrt{M}\frac{\eta_l - \rho_2}{1-\rho_2^2}\right)\right)$$
$$= \frac{1}{2}\text{erfc}\left(\sqrt{M}\frac{\eta_l - \rho_2}{1-\rho_2^2}\right)$$

It is quite obvious that the performance of this scheme is the same as the previous case which was an **AWGN** channel for a given signal to noise ration, hence we shall consider the **AWGN** channel for the rest of this paper.

5 Sequential Detection at FC

In this section a sequential detection scheme which is decentralized is proposed, the decision from the secondary users are combined in a sequential manner at the fusion center.

The fusion center is mostly depicted as a separate node but can be one of the secondary users, in our notation we denote the maximum likelihood estimate for the autocorrelation coefficient of the n^{th} secondary user by $\hat{\rho}_n$.

In the scheme proposed in this paper instead of a node making the final decision by its LLRT statistic ρ_n it sends it to the fusion center and the fusion center collects these statistics sequentially and each time after getting a statistic from a user it performs a hypothesis test and if the evidence is enough to make a reliable decision it shall proceed to do that and stops collecting further information otherwise it will keep on receiving data from the other secondary users and repeat the procedure.

The **Sequential Probability Ratio Test(SPRT)** after receiving k statistics is as following.

$$\sum_{n=1}^{k} L_n \leq \log(B), \quad \text{Decide } H_0$$
$$\sum_{n=1}^{k} L_n \leq \log(A), \quad \text{Decide } H_1$$
$$\text{Otherwise,,} \quad \text{Take Next User's Statistics,}$$
$$A = \frac{1-\beta}{P_{fa}}, \quad B = \frac{\beta}{1-P_{fa}}, \quad L_n = -M\log(1-\rho_n^2) + 2M\frac{\rho_n(\hat{\rho}_n - \rho_n)}{1-\rho_n^2}$$

It is important to note that $\beta = 1 - P_d$ is the probability of missed detection, we must also pay attention that the number of LLRs used to form the decision shall be a random variable.

$$k = K_s$$

We can express the performance of a sequential detector in the terms of the **Average Sample Number (ASN)**. This value for our problem is the number of LLRs aka K_s. We shall calculate the average of this random variable under the two hypothesis.

$$H_0 : \mathbb{E}\left[K_s|H_0\right] = \frac{P_{fa}\log(A) + (1 - P_{fa}\log(B))}{\mathbb{E}\left[L_n|H_0\right]}$$
$$H_1 : \mathbb{E}\left[K_s|H_1\right] = \frac{(1-\beta)\log(A) + \beta\log(B)}{\mathbb{E}\left[L_n|H_1\right]}$$

We shall calculate the expected value of L_n under each hypothesis accordingly.

$$\mathbb{E}\left[L_n|H_0\right] = -M\log(1-\rho_1^2) - 2M\frac{\rho_1^2}{1-\rho_1^2}$$
$$\mathbb{E}\left[L_n|H_1\right] = -M\log(1-\rho_1^2)$$

We take the maximum of the calculated expected values as the ASN for the sequential detector.

$$K_m = \max\{\mathbb{E}\left[K_s|H_0\right], \mathbb{E}\left[K_s|H_1\right]\}$$

Now we shall compare the sequential detector with the Neyman-Pearson FSS test at the fusion center based on LLRs. We denote the number of secondary user statistics for the FSS test as K_f, the test itself is as follows.

$$\sum_{n=1}^{K_f} L_n < \eta_3, \quad \text{Decide } H_0$$
$$\sum_{n=1}^{K_f} L_n \geq \eta_3, \quad \text{Decide } H_1$$

Where η_3 is the threshold of our detector. Due to the fact that constant terms have no important effect in the Neyman-Pearson test, we can test the test statistic $T_f = 2M\sum_{n=1}^{K_f}\frac{\rho_n\hat{\rho}_n}{(1-\rho_n^2)}$. The test shall be as follows.

$$T_f < \eta_f, \quad \text{Decide } H_0$$
$$T_f \geq \eta_f, \quad \text{Decide } H_1$$
$$\eta_f \triangleq \eta_3 + M\sum_{b=1}^{K_f}\log(1-\rho_n^2) - 2M\sum_{n=1}^{K_f}\frac{\rho_n^2}{(1-\rho_n^2)}$$

The hypothesis testing problem shall be as follows.

$$H_0 : \mathcal{T}_f \sim \mathcal{N}_r(0, \sigma_{f0}^2)$$
$$H_1 : \mathcal{T}_f \sim \mathcal{N}_r(m_f, \sigma_{f1}^2)$$
$$m_f = 2M \sum_{n=1}^{K_f} \frac{\rho_n^2}{(1-\rho_n^2)}, \quad \sigma_{f0}^2 = 2M \sum_{n=1}^{K_f} \frac{\rho_n^2}{(1-\rho_n^2)^2}$$
$$\sigma_{f1}^2 = 2M \sum_{n=1}^{K_f} \rho_n^2$$

5.1 Derivation of Neyman-Pearson Detector

What we have written is a detection problem with two hypothesis, we shall derive the Neyman-Pearson detector for this scenario.

$$H_0 : \mathcal{T}_f \sim \mathcal{N}_r(0, \sigma_{f0}^2)$$
$$H_1 : \mathcal{T}_f \sim \mathcal{N}_r(m_f, \sigma_{f1}^2)$$
$$L(T_f) = \frac{p_1(T_f)}{p_0(T_f)} = \frac{\frac{1}{\sqrt{2\pi\sigma_{f1}^2}} \exp\left\{-\frac{(T_f - m_f)^2}{2\sigma_{f1}^2}\right\}}{\frac{1}{\sqrt{2\pi\sigma_{f0}^2}} \exp\left\{-\frac{T_f^2}{2\sigma_{f0}^2}\right\}} = \frac{\sigma_{f0}}{\sigma_{f1}} \exp\left\{\frac{T_f^2}{2\sigma_{f0}^2} - \frac{(T_f - m_f)^2}{2\sigma_{f1}^2}\right\}$$
$$L(T_f) = \frac{\sigma_{f0}}{\sigma_{f1}} \exp\left\{\frac{T_f^2}{2\sigma_{f0}^2} - \frac{(T_f - m_f)^2}{2\sigma_{f1}^2}\right\}$$
$$= \frac{\sigma_{f0}}{\sigma_{f1}} \exp\left\{\frac{2\sigma_{f1}^2 T_f^2 - 2\sigma_{f0}^2 T_f^2 + 4\sigma_{f0}^2 T_f m_f + 2\sigma_{f0}^2 m_f^2}{4\sigma_{f0}^2 \sigma_{f1}^2}\right\}$$
$$L(T_f) \underset{H_0}{\overset{H_1}{\gtrless}} \eta \longrightarrow \ln\left(L(T_f)\right) \underset{H_0}{\overset{H_1}{\gtrless}} \ln(\eta) \longrightarrow \ln\left(L(T_f)\right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta'$$
$$\ln\left(L(T_f)\right) = \ln\left(\frac{\sigma_{f0}}{\sigma_{f1}}\right) + (2\sigma_{f1}^2 - 2\sigma_{f0}^2)T_f^2 + 4\sigma_{f0}^2 T_f m_f + 2\sigma_{f0}^2 m_f^2 - 4\sigma_{f0}^2 \sigma_{f1}^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta'$$

$$(2\sigma_{f1}^2 - 2\sigma_{f0}^2)T_f^2 + 4\sigma_{f0}^2 T_f m_f \underset{H_0}{\overset{H_1}{\gtrless}} \eta'' \longrightarrow (2\sigma_{f1}^2 - 2\sigma_{f0}^2)T_f^2 + 4\sigma_{f0}^2 T_f m_f - \eta'' \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$$T_f = \frac{-4\sigma_{f0}^2 m_f \pm \sqrt{16\sigma_{f0}^4 m_f^2 + 8\eta''(\sigma_{f1}^2 - \sigma_{f0}^2)}}{4(\sigma_{f1}^2 - \sigma_{f0}^2)} = \tau, \quad \delta_{NP}(T_f) = \begin{cases} 1, & |T_f| > \tau \\ 0, & -\tau < |T_f| < \tau \end{cases}$$

It is evident that the T_f is always positive, hence we can conclude.

$$T_f = \frac{-4\sigma_{f0}^2 m_f + \sqrt{16\sigma_{f0}^4 m_f^2 + 8\eta''(\sigma_{f1}^2 - \sigma_{f0}^2)}}{4(\sigma_{f1}^2 - \sigma_{f0}^2)} = \tau, \quad \delta_{NP}(T_f) = \begin{cases} 1, & |T_f| > \tau \\ 0, & 0 < |T_f| < \tau \end{cases}$$

This shall result in the exact same result as the threshold which we derived in the previous parts(η_f), if we substitute η'' for its value as shown above, we shall have the following.

$$T_f = \eta_f = \eta_3 + M\sum_{b=1}^{K_f} \log(1-\rho_n^2) - 2M\sum_{n=1}^{K_f} \frac{\rho_n^2}{(1-\rho_n^2)} \longrightarrow T_f > \eta$$

So the hypothesis test shall become as follows.

$$T_f \underset{H_0}{\overset{H_1}{\gtrless}} \eta_f$$

Now we shall go on to evaluate the false alarm probability to change the form of η_f.

$$P_{fa} = P(H_1|H_0) = P\left(T_f > \eta_f|H_0\right) = \int_{\eta_f}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{f0}^2}} \exp\left\{-\frac{T_f^2}{2\sigma_{f0}^2}\right\} dT_f$$

$$P_{fa} = 1 - \phi\left(\frac{\eta_f}{\sigma_{f0}}\right) = Q\left(\frac{\eta_f}{\sigma_{f0}}\right) = \frac{1}{2}\left(1 - \operatorname{erf}\left(\frac{\eta_f}{\sqrt{2}\sigma_{f0}}\right)\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{\eta_f}{\sqrt{2}\sigma_{f0}}\right)$$

$$\eta_f = \sqrt{2}\sigma_{f0}\operatorname{erfc}^{-1}\left(2P_{fa}\right) \longrightarrow T_f \underset{H_0}{\overset{H_1}{\gtrless}} \sqrt{2}\sigma_{f0}\operatorname{erfc}^{-1}\left(2P_{fa}\right)$$

Now we shall calculate the probability of detection.

$$P_D = P(H_1|H_1) = P(T_f > \eta_f|H_1) = \int_{\eta_f}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{f1}^2}} \exp\left\{-\frac{(T_f - m_f)^2}{2\sigma_{f1}^2}\right\} dT_f$$

$$P_D = 1 - \phi\left(\frac{\eta_f - m_f}{\sigma_{f1}}\right) = Q\left(\frac{\eta_f - m_f}{\sigma_{f1}}\right) = \frac{1}{2}\left(1 - \text{erf}\left(\frac{\eta_f - m_f}{\sqrt{2}\sigma_{f1}}\right)\right)$$
$$= \frac{1}{2}\text{erfc}\left(\frac{\eta_f - m_f}{\sqrt{2}\sigma_{f1}}\right)$$

It is reasonable to assume that ρ_n is known for all of the secondary users in the FSS scheme, according to a definition similar to the one of ρ_1 we can understand that ρ_n is dependent on T_c and T_d and the SNR.

The values of T_c and T_d are specified in wireless standards and the SNR value can be estimated using techniques known as non-data-aided SNR estimation techniques.

Using the equations which we derived and $\rho_n = \rho_1, \forall n$, the number of secondary user statistics (**NSUS**) required for achieving a given P_{fa} and P_d and in the FSS case is derived as follows.

$$\sigma_{f1}^2 = 2M \sum_{n=1}^{K_f} \rho_n^2, \quad \eta_f = \sqrt{2}\sigma_{f0}\text{erfc}^{-1}\left(2P_{fa}\right), \quad P_d = \frac{1}{2}\text{erfc}\left(\frac{\eta_f - m_f}{\sqrt{2}\sigma_{f1}}\right)$$
$$\longrightarrow K_f = \frac{\left(\text{erfc}^{-1}(2P_{fa}) - (1-\rho_1^2)\text{erfc}^{-1}(2P_d)\right)^2}{M\rho_1^2}$$

We shall compare the theoretical K_m and K_f for performance parameters $P_{fa} = 0.05$ and $\beta = 1 - P_d = 0.05$ in due course and the results shall be compared, we expect that the average NSUS for SD to besignificantly lower than the NSUS for FSS testing for the same performance parameters.

The performance of **SD** can be compared to the **FSS** scheme in terms of **Relative Efficiency(RE)** in the NSUS (or number of LLRs) required to reach the final decision at the fusion center.

$$\text{RE} = \frac{K_f}{K_m}$$

6 Simulation Results

It is important to note that the following parameters were taken into notice whilst simulating.

$$T_d = 32, \quad T_c = 8, \quad M = 100(T_d + T_c) = 4000$$

The modulation used is **16-QAM**, and the energy is normalized to 1.

6.1 OFDM Implementation

In this explain, we shall explain how we implemented an OFDM system with MATLAB.

First of all we go on to create 16-QAM modulated data, to do so we create data 4 times the number of symbols we desire, then we go on te reshape it in into a $SymbolNum \times \log 2(ModNum)$ matrix. After this we create the 16-QAM constellation, then we go on to create the 16-QAM modulated data with the aid of this constellation. To bring our memory up to speed, the 16-QAM constellation is included below.

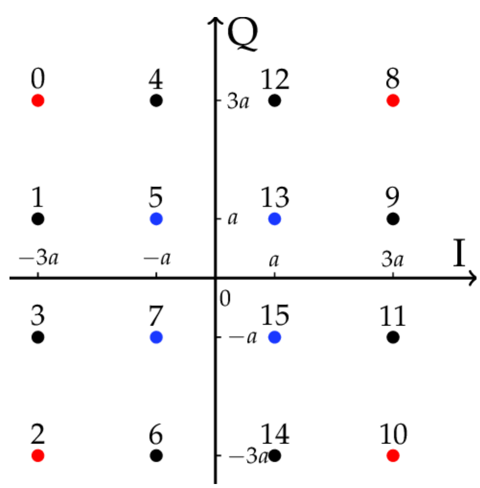

Figure 8: 16-QAM constellation

To create the OFDM vectors, we work with a matrix form. each column of the input matrix to the is an OFDM block. We implement the following block diagram step by step.

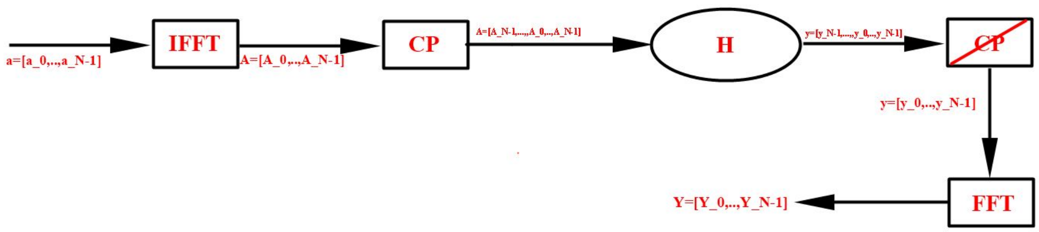

Figure 9: OFDM system

We pass the data matrix through the IFFT block then go on to add the cyclic prefix to it. We then go through the channel which in this case is an AWGN channel. On the other side we remove the cyclic prefix and perform IFFT on the data. After this we go on to calculate the probability of detection by comparing with a threshold. For this threshold in question, I used the threshold in the paper, but it didn't work appropriately at first, so I scaled it with the noise and added a shift to it to reach appropriate results.

6.2 Receiver Operator Characteristic for local detector

For the first part we simulate the ROC curve for the detector we discussed in section II, we can examine the plot which was done via **MATLAB** as follows.

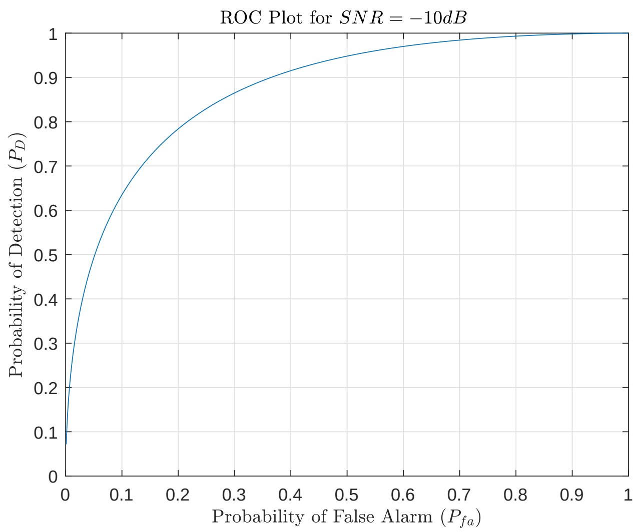

Figure 10: ROC for local detector

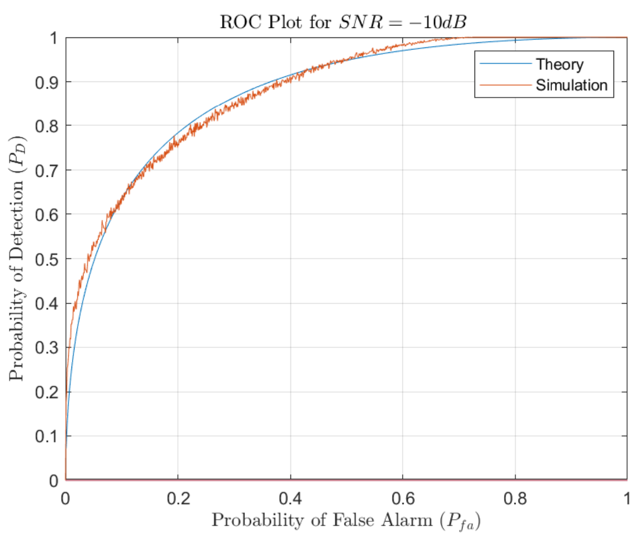

Figure 11: ROC for local detector

It is important to note that this plot is for a single user, as we can see our plot is concave which and for $P_{fa} = 1$ we have $P_D = 1$ so we can sure that we have acted correctly.

6.3 Probability of Detection for local detector with known and unknown cyclic prefix

In this part we shall simulate the detection probability for varying SNR values, we have assumed the probability of false alarm to be 0.05, we plot the curves using **MATLAB** and the following formula.

$$P_D = \frac{1}{2}\text{erfc}\left(\sqrt{M}\frac{\eta_l - \rho_1}{1-\rho_1^2}\right), \quad \eta_l = \frac{1}{\sqrt{M}}\text{erfc}^{-1}\left(2P_{fa}\right), \quad \rho_1 = \frac{T_c}{T_c + T_d}\frac{\text{SNR}}{1+\text{SNR}}$$

The results are as follows.

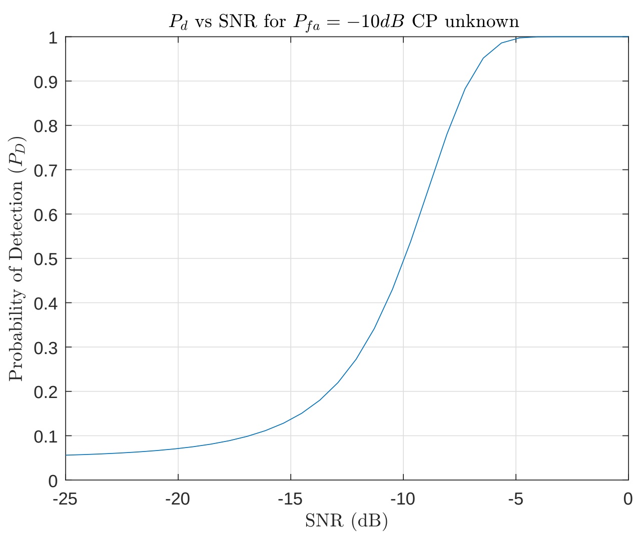

Figure 12: Probability of detection for different SNR values CP unknown

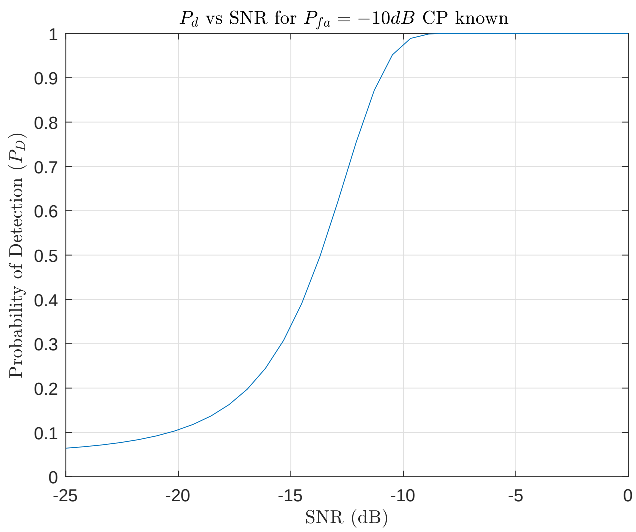

Figure 13: Probability of detection for different SNR values CP known

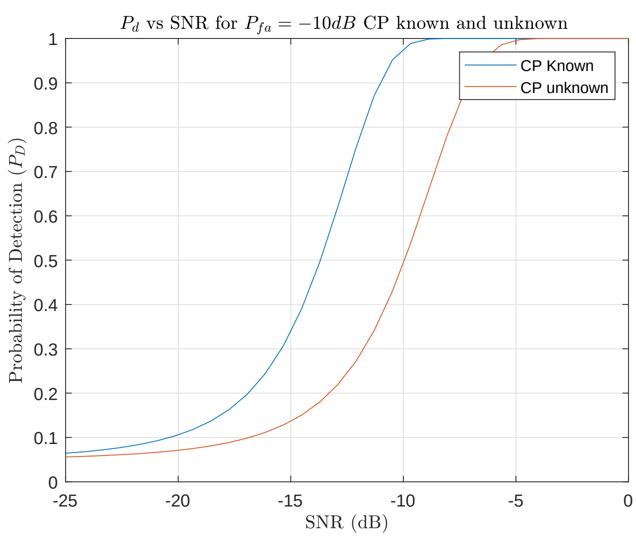

Figure 14: Probability of detection for different SNR values CP unknown and known

Before analyzing the plots, we shall explain what difference it makes to know the cyclic prefix and to have no knowledge about it.

6.3.1 Known cyclic prefix

When we talk about knowing the cyclic prefix, we generally talk about knowledge in the receiver, consider the following **OFDM** system.

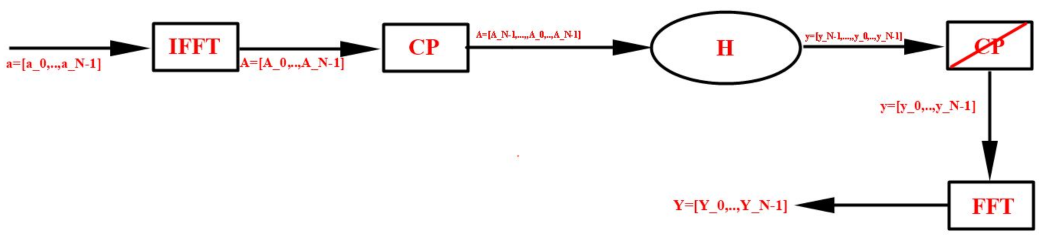

Figure 15: OFDM system

If the cyclic prefix remover block knows where to look find the cyclic prefix in the input vector then we say that the cyclic prefix is known to us.

The advantage of using a known cyclic prefix is that it allows for accurate channel estimation and efficient equalization. However, it requires additional overhead for transmitting the known cyclic prefix, reducing the overall data rate.

6.3.2 Unknown cyclic prefix

In some **OFDM** systems the receiver doesn't know where the cyclic prefix is located in the vector, so, logically it has to estimate the length and structure of the cyclic prefix, the estimation begins with estimating the length and then moving on to estimating the channel effects and finding a way to compensate it.

Estimating the cyclic prefix length can be done using different techniques, such as autocorrelation or pilot symbols. Once the cyclic prefix length is estimated, the receiver can get rid of the cyclic prefix and equalize the OFDM symbols based on the estimated channel using techniques such as zero forcing or MMSE equalizers.

This approach in **OFDM** systems is more flexible and allows for a higher data rate since no additional overhead is required for transmitting the known cyclic prefix. However, it adds complexity to the receiver's operations as it needs to estimate the cyclic prefix length and perform channel estimation and equalization based on that estimation.

Based on our explanation it is logical to presume that knowing the cyclic prefix shall increase the probability of detection, our simulation verifies this assumption.

It is important to note that the results obtained in this section is the best possible outcome for practical detectors and serves as an upper bound for them.

In the above plots, an **AWGN** channel is considered, other channels such as **Rayleigh** channels can be used, we shall discuss their effect in the next section.

6.4 Receiver Operator Characteristic for cooperative detection

We shall include the plot from the paper and analyze it thoroughly.

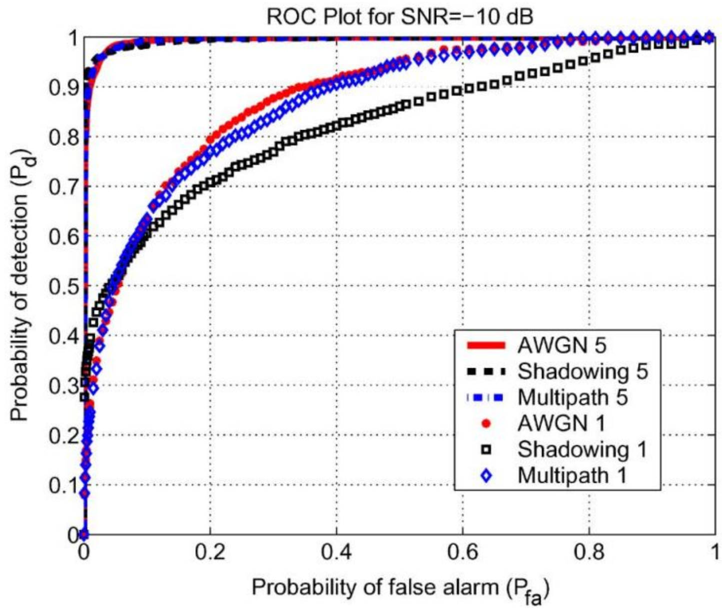

Figure 16: Receiver Operator Characteristic for cooperative detection

Before entering analysis, we shall explain the concepts of shadowing and multipath systems.

6.4.1 Shadowing

Shadowing, occurs due to the obstruction of the signal path by obstacles such as buildings, trees, towers rtc... It leads to signal attenuation or fluctuation in received signal strength at the receiver(in our case the **OFDM** receiver).

In other words, if there are obstacles in the signal path between the transmitter and receiver, we shall have the shadowing effect.

Shadowing is often modeled as a log-normal distribution the received signal power follows a logarithmic normal distribution. The mean value represents the average received signal power, while the standard deviation represents fluctuation in the received power.

$$\psi = \frac{P_t}{P_r}, \quad \psi_{dB} = 10\log_1 0(\psi), \quad \psi_{dB} \sim \mathcal{N}(\mu_{\psi_{dB}}, \sigma^2_{\psi_{dB}})$$

Here we add the simulated plot for AWGN and shadowing scenario.

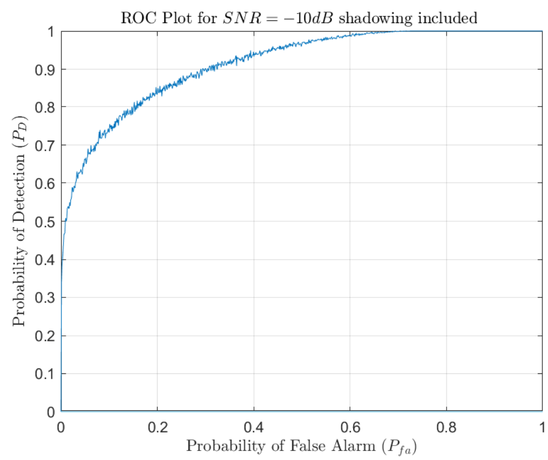

Figure 17: ROC for cooperative detection with shadowing

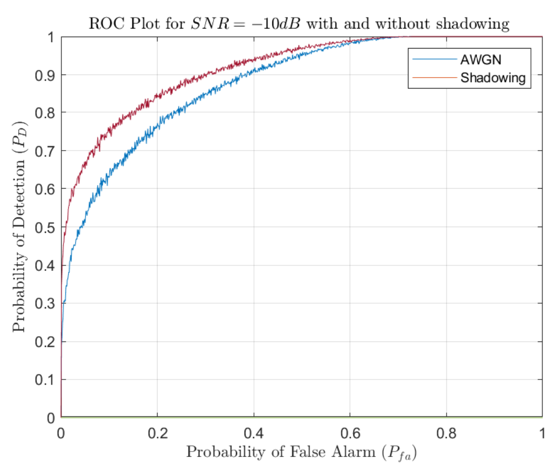

Figure 18: ROC for cooperative detection AWGN and shadowing

As we can see the probability of detection for shadowing scenario is higher in low SNR values. It is also important to not that to model shadowing we've included a coefficient in the channel of the OFDM system.

To overcome the nasty effects of shadowing, techniques such as diversity are used in wireless systems, other techniques such as power allocation, using repeaters and relays are also well known.

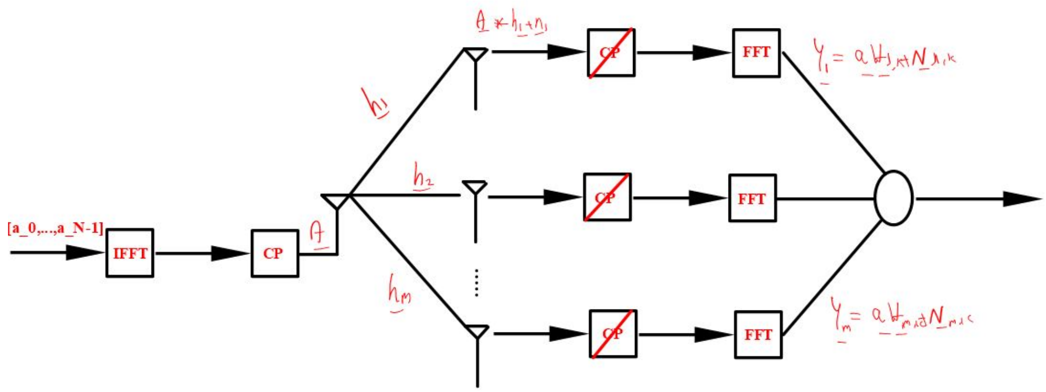

Figure 19: Diversity in OFDM system

6.4.2 Multipath

In multipath, multiple copies of the transmitted signal reach the receiver via different paths due to reflections, diffractions, and scattering of the signal by objects and the environment.

In an OFDM system, the transmitted signal is divided into multiple subcarriers, each of which is orthogonal to the others. These subcarriers are close in frequency and obtain different parts of the transmitted data. Each subcarrier experiences different delays and attenuations as they travel through different paths to reach the receiver.

The presence of multiple paths introduces the challenges of **ISI** and **Frequency Selective Fading** in **OFDM** systems.

Going back to the plot, we observe that for the shadowing scenario, the probability of detection is higher than the AWGN scenarion in low SNR values, this doesn't mean that shadowing is better but is simply due to the shift applied by the mean attenuation, also the fluctuations caused by the shadowing effect will reduce the slope of the probability of detection.

Due to the above reasoning, the SNR for AWGN will always be less than the case where shadowing is present, due to this fact, we can't compare these curves precisely in terms of SNR gain.

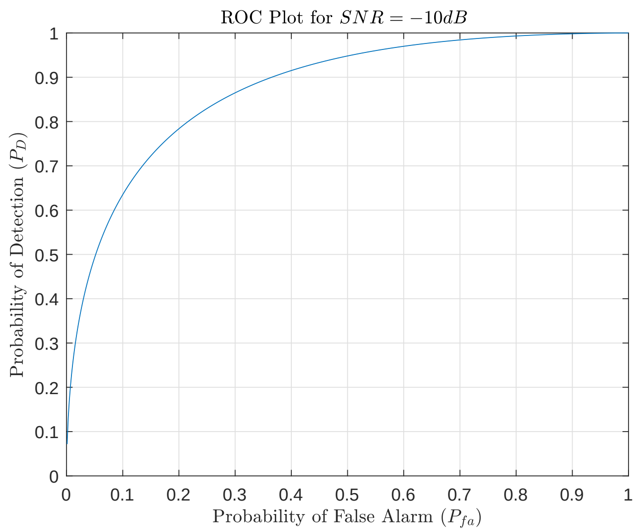

Figure 20: ROC for local detector

6.5 Comparison of SD and FSS schemes

In this section, the upper limit of the probability of false alarm is set to 0.05, hence the lower limit on the probability of detection is 0.95. Now we shall plot the number of secondary users for each of the FSS and SD schemes accordingly.

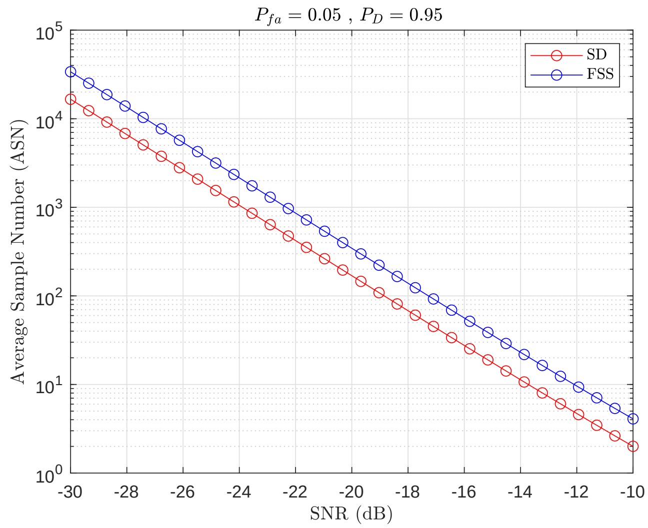

Figure 21: Comparison of average number of secondary users for SD and FSS detection

We have used the following formulas in the above plot.

$$K_f = \frac{\left(\text{erfc}^{-1}(2P_{fa}) - (1-\rho_1^2)\text{erfc}^{-1}(2P_d)\right)^2}{M\rho_1^2}, \quad K_m = \max\{\mathbb{E}\left[K_s|H_0\right], \mathbb{E}\left[K_s|H_1\right]\}$$

It is evident that the number of secondary users is significantly less in the SD scheme than the FSS scheme, it is important to note that it is assumed that one LLR is transmitted per detection period and that the results are asymptotic.

6.6 Relative efficiency of the SD with respect to the FSS detector

Here we have plotted the relative efficiency. This value is nearly static in theory and close to the value of 2.04, the plot is included accordingly.

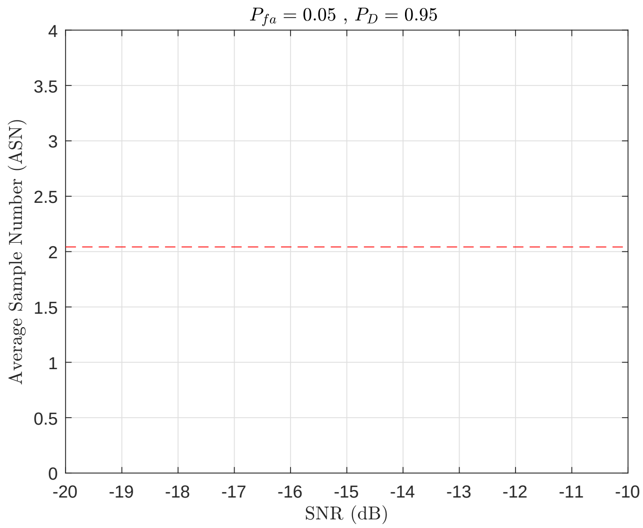

Figure 22: Relative efficiency of the SD with respect to the FSS detector

It is interesting to take notice that if we have the shadowing effect, the number of statistics needed to make a decision is reduced considerably.

7 Conclusion

In this paper we had a thorough examination of the given paper, we studied the concepts of the sensing scheme proposed which was based on autocorrelation coefficient for **CP-OFDM** based primary user signals.

We learned that in low SNR values the autocorrelation coefficient is a LLRT statistic, we performed theoretical analysis on the local detector and studied its reliability, we observed that the performance is acceptable in both low and high SNR values.

In the next phase, we studied the effects of knowing and not knowing the cyclic prefix in the detector, we saw considerable improvement for when we know the cyclic prefix information.

Next we studied the effect of different channels on the ROC curve and compared the different outcomes.

In the final part, a decentralized SD scheme is proposed, the performance was analyzed and compared with the FSS scheme. We learned that this scheme performs significant savings in the number of secondary users to make a final decision at the FC compared to the FSS scheme. We went on to calculate the relative efficiency and also discussed that if we have shadowing this parameter can be reduced significantly.

References

[1] David Tse, Pramod Viswanath, *Fundamentals of Wireless Communications, 1st edition, CUT Press, 2005*

[2] Andrea Goldsmith, *Wireless Communications, 1st edition, CUT Press, 2005*

[3] A. V. Dandawaté, G. B. Giannakis, *"Statistical tests for presence of cyclostationarity," IEEE Trans. Signal Process., vol. 42, pp.2355–2369, Sep. 1994*

[4] T. W. Anderson, *Introduction to Multivariate Statistical Analysis. New York: Wiley, 1958*

[5] Ali Olfat, *Detection and Estimation Theory, Lecture Notes, Spring 02*

[6] Ali Olfat, *Stochastic Processes, Lecture Notes, Fall 01*

[7] What is a cognitive radio? How does it work?