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Digital Communications

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Computer Assignment 3

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Abstract

In this assignment, we start with designing a function to create the Raised-Cosine pulse(exactly like the previous assignment), then we shall attempt to send our symbols with the help of the 4-PAM modulation, we do this as instructed in the assignment description, we also pay attention that the calculated error probability here, is the error probability of each symbol.

In the next section we attempt to detect the symbols with MAP and ML detectors, then we proceed to calculate the error probability for each of these scenarios and in the end we plot the error probability in the scale of the SNR_{dB} and compare them together.

1 Creation and transmission of symbols with 4-PAM

We have completed this part by following the steps instructed in the CA assignment step by step with precision.

2 Detecting symbols with MAP and ML detectors

As instructed we calculate the thresholds and error probabilities for each detector respectively.

2.1 Maximum Likelihood

In this case the probabilities are assumed to be the same and equal to $\frac{1}{M}$ hence we can go forward as follows.

2.1.1 Error Probability

$$P_{E_{ML}} = \frac{1}{M}(Pr\left[A_m \neq A | A_m = A\right] + Pr\left[A_m \neq B | A_m = B\right] + Pr\left[A_m \neq C | A_m = C\right]$$
$$+ Pr\left[A_m \neq D | A_m = D\right])$$

As we know we can write $Y(t_m) = A_m + n(t_m)$ so we proceed:

$$P_{E_{ML}} = \frac{1}{4}(Pr\left[n(t_m) > \Delta_1 + 3\right] + Pr\left[n(t_m) < \Delta_1 + 1 \textit{ or } n(t_m) > \Delta_2 + 1\right]$$
$$+ Pr\left[n(t_m) < \Delta_2 - 1 \textit{ or } n(t_m) > \Delta_3 - 1\right] + Pr\left[n(t_m) < \Delta_3 - 3\right])$$
$$\longrightarrow P_{E_{ML}} = \frac{1}{4}(Q\left(\frac{\Delta_1 + 3}{N_0}\right) + Q\left(\frac{-\Delta_1 - 1}{N_0}\right) + Q\left(\frac{\Delta_2 + 1}{N_0}\right) + Q\left(\frac{-\Delta_2 + 1}{N_0}\right)$$
$$+ Q\left(\frac{\Delta_3 - 1}{N_0}\right) + Q\left(\frac{-\Delta_3 + 1}{N_0}\right))$$

2.1.2 Thresholds

Here we attempt to calculate the needed thresholds.

$$\frac{1}{\sqrt{2\pi N_0}}e^{\frac{(\Delta_1+3)^2}{2N_0}} = \frac{1}{\sqrt{2\pi N_0}}e^{\frac{(\Delta_1+1)^2}{2N_0}}$$
$$\frac{1}{\sqrt{2\pi N_0}}e^{\frac{(\Delta_2+1)^2}{2N_0}} = \frac{1}{\sqrt{2\pi N_0}}e^{\frac{(\Delta_2-1)^2}{2N_0}}$$
$$\frac{1}{\sqrt{2\pi N_0}}e^{\frac{(\Delta_3-3)^2}{2N_0}} = \frac{1}{\sqrt{2\pi N_0}}e^{\frac{(\Delta_3-1)^2}{2N_0}}$$

If we proceed to solve the above equations we shall obtain the following results.

$$\Delta_1 = -2, \quad \Delta_2 = 0, \quad \Delta_3 = 2$$

Which was predictable due to symmetry.

2.2 Maximum A Posteriori

In this case the probabilities are not the same we assume them to be P_A, P_B, P_C, P_D hence we can go forward as follows.

2.2.1 Error Probability

$$P_{E_{MAP}} = P_A Pr\left[A_m \neq A | A_m = A\right] + P_B Pr\left[A_m \neq B | A_m = B\right] + P_C Pr\left[A_m \neq C | A_m = C\right]$$
$$+ P_D Pr\left[A_m \neq D | A_m = D\right]$$

As we know we can write $Y(t_m) = A_m + n(t_m)$ so we proceed:

$$P_{E_{MAP}} = P_A Pr\left[n(t_m) > \Delta_1 + 3\right] + P_B Pr\left[n(t_m) < \Delta_1 + 1 \text{ } or \text{ } n(t_m) > \Delta_2 + 1\right]$$
$$+ P_C Pr\left[n(t_m) < \Delta_2 - 1 \text{ } or \text{ } n(t_m) > \Delta_3 - 1\right] + P_D Pr\left[n(t_m) < \Delta_3 - 3\right]$$
$$\longrightarrow P_{E_{MAP}} = P_A Q\left(\frac{\Delta_1 + 3}{N_0}\right) + P_B\left(Q\left(\frac{-\Delta_1 - 1}{N_0}\right) + Q\left(\frac{\Delta_2 + 1}{N_0}\right)\right) + P_C Q\left(\frac{-\Delta_2 + 1}{N_0}\right)$$
$$+ P_C Q\left(\frac{\Delta_3 - 1}{N_0}\right) + P_D Q\left(\frac{-\Delta_3 + 1}{N_0}\right)$$

2.2.2 Thresholds

Here we attempt to calculate the needed thresholds.

$$\frac{P_A}{\sqrt{2\pi N_0}} e^{\frac{(\Delta_1+3)^2}{2N_0}} = \frac{P_B}{\sqrt{2\pi N_0}} e^{\frac{(\Delta_1+1)^2}{2N_0}}$$
$$\frac{P_B}{\sqrt{2\pi N_0}} e^{\frac{(\Delta_2+1)^2}{2N_0}} = \frac{P_C}{\sqrt{2\pi N_0}} e^{\frac{(\Delta_2-1)^2}{2N_0}}$$
$$\frac{P_C}{\sqrt{2\pi N_0}} e^{\frac{(\Delta_3-3)^2}{2N_0}} = \frac{P_D}{\sqrt{2\pi N_0}} e^{\frac{(\Delta_3-1)^2}{2N_0}}$$

If we proceed to solve the above equations we shall obtain the following results.

$$\Delta_1 = -2 - \frac{N_0}{2} \ln \frac{P_B}{P_A}, \quad \Delta_2 = 0, \quad \Delta_3 = 2 + \frac{N_0}{2} \ln \frac{P_C}{P_D}$$

3 ML and MAP error probability comparison

Due to the analysis that we performed we expect the MAP detector to be better in case of probability error, to verify this, using the *semilogy* command in MATLAB we plot the following graph.

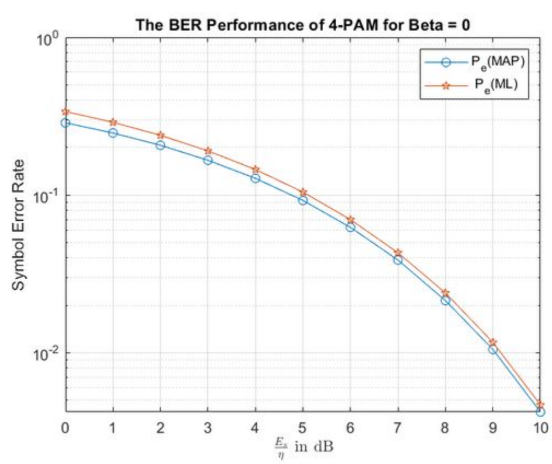

Figure 1: Error probability plot

As we expected we can conclude the MAP detector in general has a lower error probability compared to the ML detector, this probability decreases when we increase the SNR, it is also interesting and expected to see that the space between the ML error probability and MAP probability error decreases when we increase the SNR and if we continue for a long enough interval they shall converge.