University of Tehran
College of Engineering
School of Electrical and Computer Engineering

Digital Communications Lab

Dr.Olfat

Prelab 3

Soroush Mesforush Mashhad

SN:810198472

Aban 01

Contents

Abstract

In this prelab we shall calculate the Spectrum of a signal by utilizing its correlation and comparing it with the built in MATLAB function.

In the second part we shall study the conversion of lowpass and bandpass signals to eachother.

1 Frequency Analysis

A way to calculate the discrete Fourier transform is by using the autocorrelation function.

$$\hat{r}[k] = \frac{1}{N}\sum_{n=k+1}^{N} x[n]x^*[n-k], \quad \hat{\varphi}(f) = \sum_{k=-(N-1)}^{N-1} \hat{r}[k]e^{-j2\pi fk}$$

Using the following equations, we've designed the following function.

```
function [X] = corr_spctrm(x, n_psd)
%Here we shall implement the fft exactly as depicted in the DSP with MATLAB
%help given in the lab instructions.
r_kpos = zeros(1,n_psd);
 for k=1:n_psd-1
    % We create Temp Column wise due to our preference to work with columns
    % in communications.
    Temp = zeros(n_psd-k,1);
    %Now we go on to create r for positive indiced values
    for n=k+1:n_psd
        Temp(n-k) = x(n)*conj(x(n-k));
    end
    r_kpos(k)=(1/n_psd)*sum(Temp(n-k));
 end
rforkzero=1/n_psd*x(n)*conj(x(n));
r=[rforkzero r_kpos];
X = fftshift(fft(r,n_psd));
end
```

Now we go on to test the function and check it with MATLAB's built in function for different f_0 frequencies.

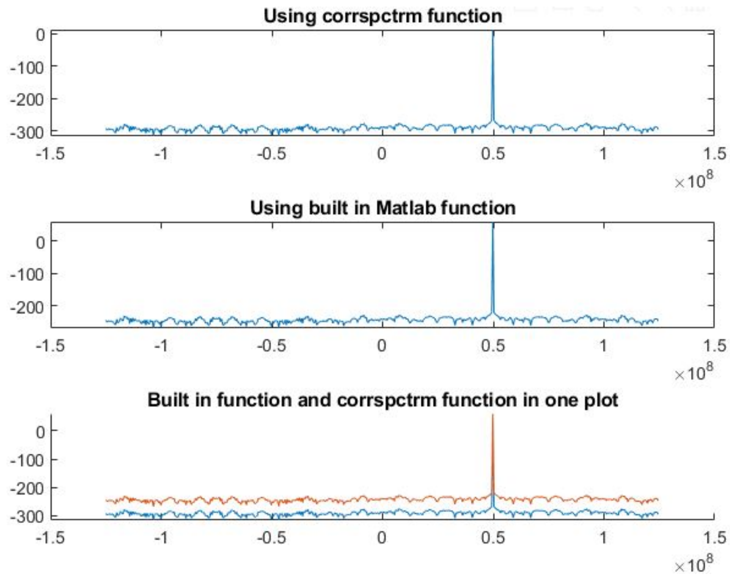

Figure 1: $f_0 = \frac{250\times 51}{256} MHz$

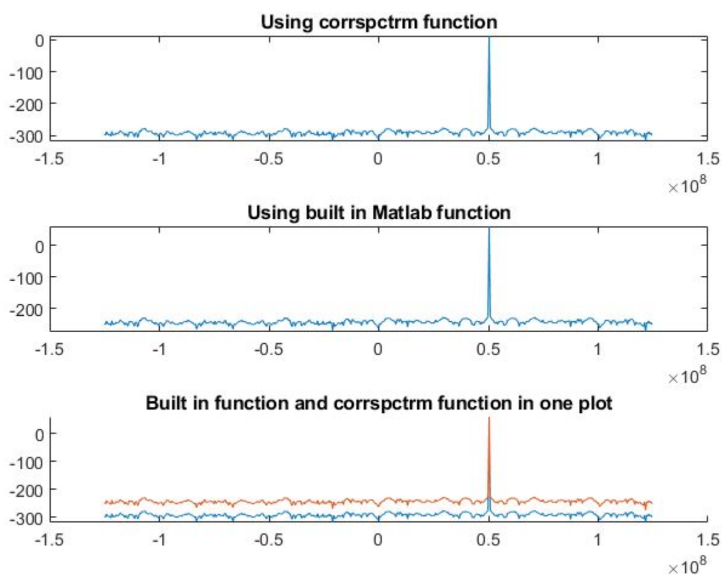

Figure 2: $f_0 = \frac{250\times 51.5}{256} MHz$

2 Conversion of Lowpass Equivalent of Bandpass signals

Here we shall study the conversion of lowpass equivalent of bandpass signals to eachother and find the output of the output low pass filter in terms of $x_q(t)$ and $x_i(t)$. With respect to the following form of the transmitter and receiver we have:

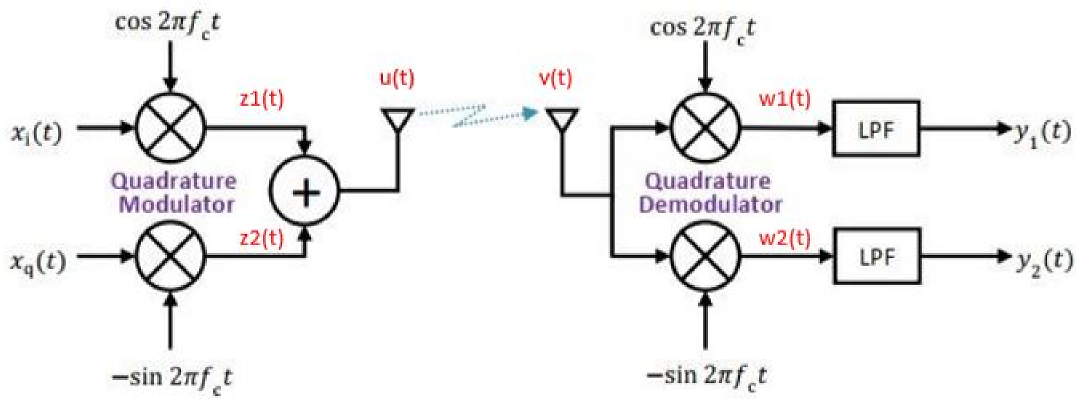

Figure 3: Transmitter & Receiver structure

$$z_1(t) = x_i(t)\cos(2\pi f_c t), \quad z_2(t) = -x_q(t)\sin(2\pi f_c t)$$
$$u(t) = z_1(t) + z_2(t) = x_i(t)\cos(2\pi f_c t) - x_q(t)\sin(2\pi f_c t)$$

Now we transmit the signal, hence we have

$$v(t) = u(t) = x_i(t)\cos(2\pi f_c t) - x_q(t)\sin(2\pi f_c t)$$
$$w_1(t) = v(t)\cos(2\pi f_c t) = x_i(t)\cos^2(2\pi f_c t) - x_q(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$
$$w_2(t) = v(t)\cos(2\pi f_c t) = -x_i(t)\sin(2\pi f_c t)\cos(2\pi f_c t) + x_q(t)\sin^2(2\pi f_c t)$$

We can simplify $w_1(t)$ and $w_2(t)$ with trigonometric equalities.

$$w_1(t) = \frac{1}{2}x_i(t) + \frac{1}{2}x_i(t)\cos(4\pi f_c t) - \frac{1}{2}x_q(t)\sin(4\pi f_c t)$$
$$w_2(t) = \frac{1}{2}x_q(t) - \frac{1}{2}x_q(t)\cos(4\pi f_c t) - \frac{1}{2}x_i(t)\sin(4\pi f_c t)$$

As we know, the Fourier transform of $\cos(4\pi f_c t)$ and $\sin(4\pi f_c t)$ are as follows:

$$c(t) = \cos(4\pi f_c t) \xrightarrow{\mathcal{F}} \mathcal{C}(f) = \frac{1}{2}\delta(f - 2f_c) + \frac{1}{2}\delta(f + 2f_c)$$
$$s(t) = \sin(4\pi f_c t) \xrightarrow{\mathcal{F}} \mathcal{S}(f) = \frac{1}{2j}\delta(f - 2f_c) - \frac{1}{2j}\delta(f + 2f_c)$$

So it is easily deduced that the terms which are multiplied in $\cos(4\pi f_c t)$ and $\sin(4\pi f_c t)$ shall be discarded after passing through the LPF, so in the end we have:

$$y_1(t) = \frac{1}{2}x_i(t), \quad y_2(t) = \frac{1}{2}x_q(t)$$