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Prelab 2

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Mehr 01

Contents

Abstract

In this prelab we shall study the concepts of convolution, implementing convolution with matrix, the concept of the discrete Fourier transform and the concept of correlation.

First we implement normal convolution, then we go on to perform convolution with matrices then we study the concepts of the discrete Fourier transform and correlation.

1 Convolution

Here we write a function $conv_m(x, h)$ which performs convolution, we do this with paying attention to the discrete convolution formula.

$$y[n] = \sum_{k=-\infty}^{k=\infty} x[k]h[n-k]$$

The code is as follows.

```
function Y = conv_m(x, h)
%First of all we need to form the X and H vectors, we must pay attention to
%swap the lengths of h and x when initializing X and H
    X = [x, zeros(1, length(h))];
    H = [h, zeros(1, length(x))];
    %For brevity we define leny
    leny=length(x) + length(h) - 1;
    Y = zeros(1,leny);
    %Here we go on to perform the convolution as it is defined.
    for i = 1 : leny
        for j = 1 : length(x)
            if (i - j + 1 > 0)
                Y(i) = Y(i) + X (j) * H(i - j + 1);
            end
        end
    end
end
```

Figure 1: Convolution code

2 Matrician Convolution(Bonus)

2.1 Part A

Has been completed in the CSII_Lab_Prelab2_SoroushMesforushMashhad_810198472 file, the output is as follows:

```
The convolution using conv_m.m is as follows:

     6    19    40    70   100    94    76    45
```

2.2 Part B

Here we want to form the H matrix in a way to satisfy the following equation:

$$\bar{y} = H\bar{x}$$

where $\bar{y}\&\bar{x}$ are filled with the samples of $x[n]$ and $y[n]$ column wise.

To explain the procedure from which we calculate H, I would like to direct your attention to the following:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k], \quad y[1] = h[1] \times x[1], \quad y[2] = h[2] \times x[2], \dots,$$

So, due to the symmetry and definition of convolution we have:

$$\bar{y} = \begin{pmatrix} h[1] & 0 & 0 & \dots & 0 \\ h[2] & h[1] & & \vdots & \vdots \\ h[3] & h[2] & \dots & 0 & 0 \\ \vdots & h[3] & \dots & h[1] & 0 \\ h[m-1] & \vdots & \ddots & h[2] & h[1] \\ h[m] & h[m-1] & & \vdots & h[2] \\ 0 & h[m] & \ddots & h[m-2] & \vdots \\ 0 & 0 & \dots & h[m-1] & h[m-2] \\ \vdots & \vdots & & h[m] & h[m-1] \\ 0 & 0 & 0 & \dots & h[m] \end{pmatrix} \bar{x}$$

So in the case of our problem which is an 8×5 matrix H is defined as:

$$H = \begin{pmatrix} 6 & 0 & 0 & 0 & 0 \\ 7 & 6 & 0 & 0 & 0 \\ 8 & 7 & 6 & 0 & 0 \\ 9 & 8 & 7 & 6 & 0 \\ 0 & 9 & 8 & 7 & 6 \\ 0 & 0 & 9 & 8 & 7 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix}$$

After writing the code and printing the transpose of $\bar{y}$ in MATLAB we get the following result:

```
The convolution result using the matrix form is :
      6    19    40    70   100    94    76    45
```

2.3 Part C

The H matrix has been described in the previous page, to be sure We've also included it here.

$$\bar{y} = \begin{pmatrix} h[1] & 0 & 0 & \dots & 0 \\ h[2] & h[1] & & \vdots & \vdots \\ h[3] & h[2] & \dots & 0 & 0 \\ \vdots & h[3] & \dots & h[1] & 0 \\ h[m-1] & \vdots & \ddots & h[2] & h[1] \\ h[m] & h[m-1] & & \vdots & h[2] \\ 0 & h[m] & \ddots & h[m-2] & \vdots \\ 0 & 0 & \dots & h[m-1] & h[m-2] \\ \vdots & \vdots & & h[m] & h[m-1] \\ 0 & 0 & 0 & \dots & h[m] \end{pmatrix} \bar{x}$$

In our special case, where H is a 8×5 matrix, it can be observed that the elements in the matrix are constant whilst moving diagonally, this is a simple definition of the **Toeplitz** matrix.

2.3.1 Toeplitz Matrix

A **Toeplitz** matrix also named a **Diagonal - Constant** matrix, is a matrix in which each descending diagonal from left to right is constant. This matrix was named after Otto Toeplitz who was a German mathematician working in functional analysis.

In a **Toeplitz** matrix, the element h_{ij} is equal to h_{i+kj+k}, this equality denotes that in a systematic view, we have time invariance.

It can be easily seen that the first row of the **Toeplitz** matrix contains h_{11} and $N_x - 1$ zeros.

We observe that the first column contains h in full followed by $N_y = N_x + N_h - 1$ zeros

2.4 Part D

As instructed, the code to perform convolution using the **Toeplitz** matrix method, the code is as follows.

```
function [y] = conv_tp(x, h,Toeplitz)
    %Here we create the Toeplitz matrix using the built in function.
    if (Toeplitz == 1)
        xCirculant = toeplitz([x(1) zeros(1, length(h) - 1) ], [x zeros(1, length(h) - 1) ]);
   %Here we create the Toeplitz matrix without the built in function.
    elseif (Toeplitz == 0)
        x = [x zeros(1, length(h) - 1)];
        xCirculant = gallery('circul',x);
        xCirculant = xCirculant(1:length(h),:);

    end
    %Here we perform the convolution.
     y = h * xCirculant;
end
```

Figure 2: Convolution with Toeplitz code

After testing the output is as follows.

```
The convoltion with utilizing toeplitz function is :
     6    31    47     6   -51    -5    41    18   -22    -3     8     2


The convoltion without utilizing toeplitz function is :
     6    31    47     6   -51    -5    41    18   -22    -3     8     2
```

3 Question 3

In this part we shall study the discrete Fourier transform, if $x[n]$ is our signal and X_N is a N - pointed vector of frequency samples and W_N is to be an $N \times N$ matrix we have:

$$x[n] = \begin{pmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{pmatrix}, \quad X_N = \begin{pmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{pmatrix}, \quad W_N = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W_N & W_N^2 & \dots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \dots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \dots & W_N^{(N-1)(N-1)} \end{pmatrix}$$

$W_N = e^{\frac{-j2\pi}{N}}$

With the following definitions the N-point discrete Fourier transform can be defined as such:

$$X_N = W_N x_N$$

Now we assume $N = 16$, we suppose that the elements of the 1st,2nd and 10th row of W_N to be coefficients of an FIR filter, we shall obtain the frequency response of these filters with MATLAB and plot the Fourier transform of these filters in dB scale in one plot.

3.1 MATLAB code for FIR filter

```
PointsN = 256^2
N = 16
%We need to create the WN matrix, to do so we use the ones matrix
WN = ones(N-1);
 %In the following loops we create WN matrix Accordingly.
 for i = 1 : N-1
      for j = 1 : N-1
          WN(i+1,j+1) = exp(-1j*2*pi/N)^(i*j);
      end
 end
 %Now we need to cut the needed rows for the FIR filters.
 FIR1Coef=WN(1,:);
 FIR2Coef=WN(2,:);
 FIR3Coef=WN(10,:);
 freqz(FIR1Coef,PointsN)
 title('Filter 1')
 freqz(FIR2Coef,PointsN)
 title('Filter 2')
 freqz(FIR3Coef,PointsN)
 title('Filter 3')
 %Now we go on to plot the filters in one plot, to do so we must take the
 %outputs of the freqz function
 [hFil1,wFil1] = freqz(FIR1Coef,PointsN);
 [hFil2,wFil2] = freqz(FIR2Coef,PointsN);
 [hFil3,wFil3] = freqz(FIR3Coef,PointsN);
 %Here we plot them in the logarithmic scale in one plot
  plot(wFil1,20*log10(abs(hFil1)))
  hold on
  plot(wFil2,20*log10(abs(hFil2)))
  plot(wFil3,20*log10(abs(hFil3)))
  grid on
```

Figure 3: The code

3.2 Results

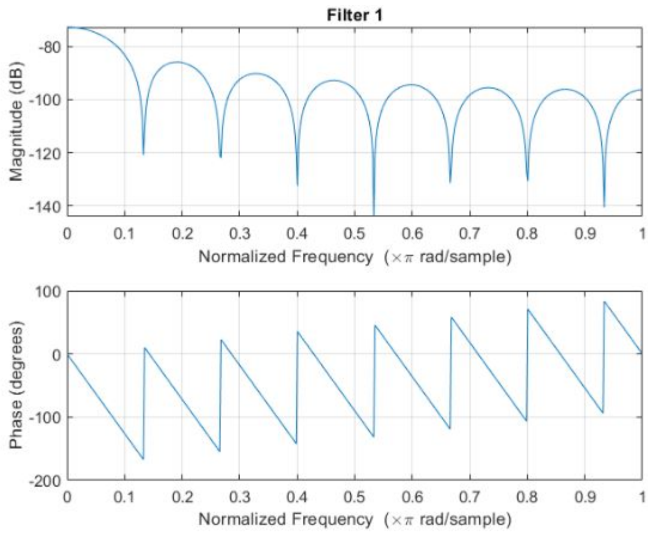

Figure 4: Filter 1 Magnitude and Phase

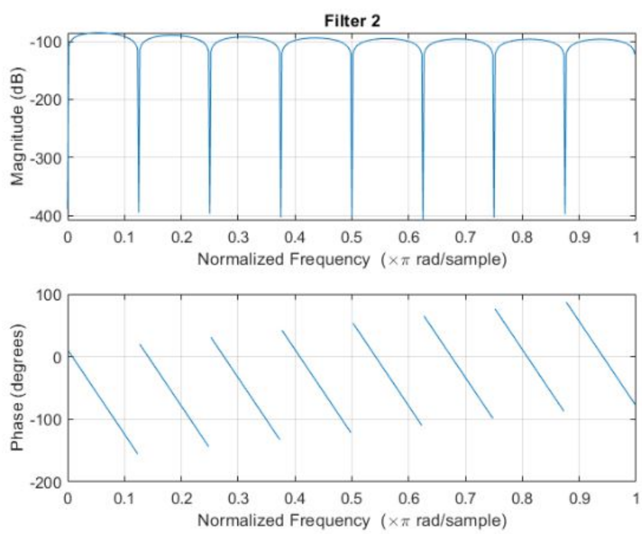

Figure 5: Filter 2 Magnitude and Phase

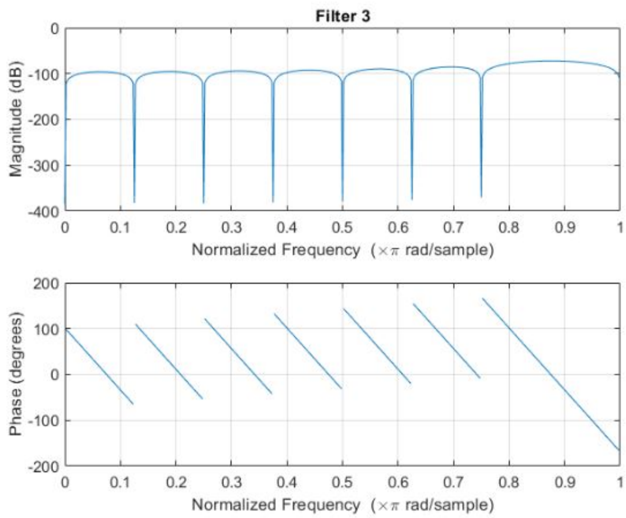

Figure 6: Filter 3 Magnitude and Phase

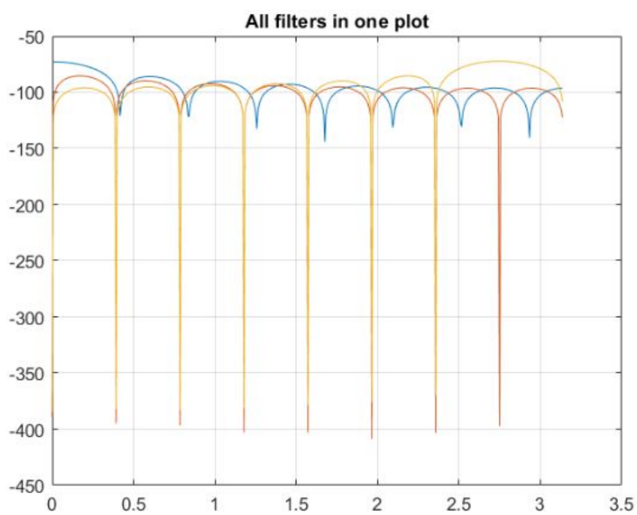

Figure 7: All filters in one plot

4 Question 4

In this part, first we define a communication signal with a linear digital modulation in baseband as follows.

$$s(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_s)$$

In the equation above, a_n are the symbols of the linear modulation, $p(t)$ is the shape of the pulse and T_s is the length of a symbol.

We presume that our sequence is WSS and iid with zero mean and a standard deviation of σ we shall go on to obtain its autocorrelation function and show that it is periodic with period T_s.

$$\mathbb{E}(a_k) = 0, \quad \mathbb{E}(a_k^2) - \mathbb{E}(a_k)^2 = \mathbb{E}(a_k^2)$$

Now we go on to utilize the definition of the autocorrelation function.

$$R_{XX}(t1, t2) = \mathbb{E}\left[X_{t_1}\bar{X}_{t_2}\right]$$

$$R(t+\tau, t) = \mathbb{E}\left(s(t+\tau)s(t)\right) = \mathbb{E}\left(\sum_{n=-\infty}^{\infty} a_n p(t+\tau-nT_s) \sum_{k=-\infty}^{\infty} a_k p(t-kT_s)\right)$$

$$= \mathbb{E}\left(\sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} a_n a_k p(t+\tau-nT_s)p(t-kT_s)\right) = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \mathbb{E}\left(a_n a_k\right) p(t+\tau-nT_s)p(t-kT_s)$$

$$= \sum_{n=-\infty}^{\infty} (a_n^2) p(t+\tau-nT_s)p(t-nT_s) = \sigma^2 \sum_{n=-\infty}^{\infty} p(t+\tau-nT_s)p(t-nT_s)$$

$$\xrightarrow{z(t)=p(t)p(t+\tau)} R(t, t+\tau) = \sigma^2 \sum_{n=-\infty}^{\infty} z(t-nT_s)$$

The above proves that the autocorrelation is periodic with period T_s.