University of Tehran
College of Engineering
School of Electrical and Computer Engineering

Digital Signal Processing

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Computer Assignment 1

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Ordibehesht 01

Abstract

In this project we start with decomposing a discrete signal into odd and even components, then we study the effect of the DTFT on these components and the original signal.

In the next part we study the effects of aliasing.

Next we go on to obtain the frequency response of a difference equation with two possible methods.

In the next section we are given an LTI system and we go on to plot the zero-pole plots and plot the magnitude and phase of the frequency response, and in the end we find the impulse response.

Next we get familiar with audio processing, in a series of tasks, we change the sampling rate, perform upsampling and downsampling, plot the spectrums and so on.

In the last part we get familiar with image processing, we are given two photos and proceed to study the effect of swapping their phase and magnitude.

1 Question 1

1.1 Part A

We know that the odd and even part of discrete signals are defined as follows

$$x_o[n] = \frac{x[n] - x[-n]}{2}, \quad x_e[n] = \frac{x[n] + x[-n]}{2}$$

Hence we implement the code accordingly and go on to plot the functions.

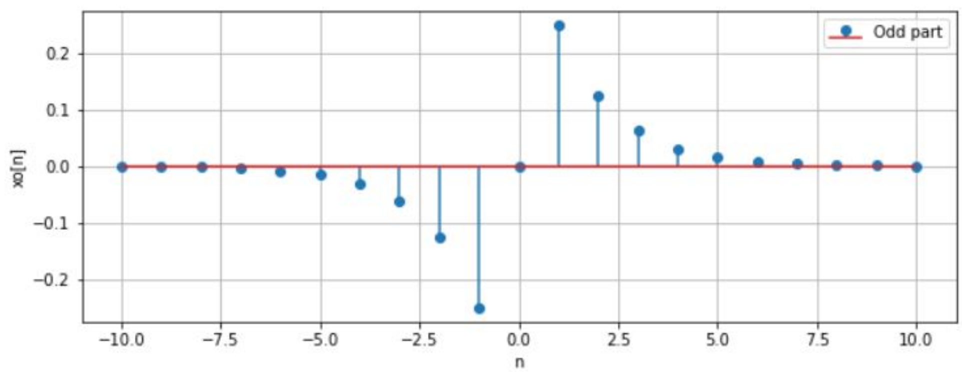

Figure 1: The Odd part

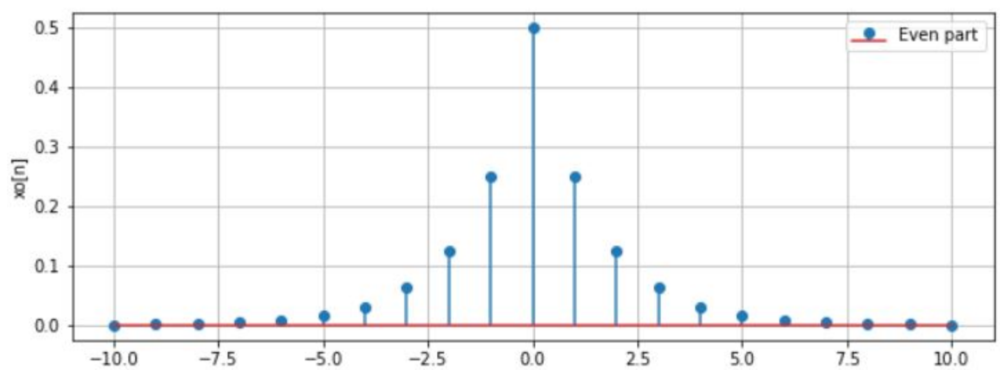

Figure 2: The Even part

1.2 Part B

First we shall plot the fft without applying fft shift as follows.

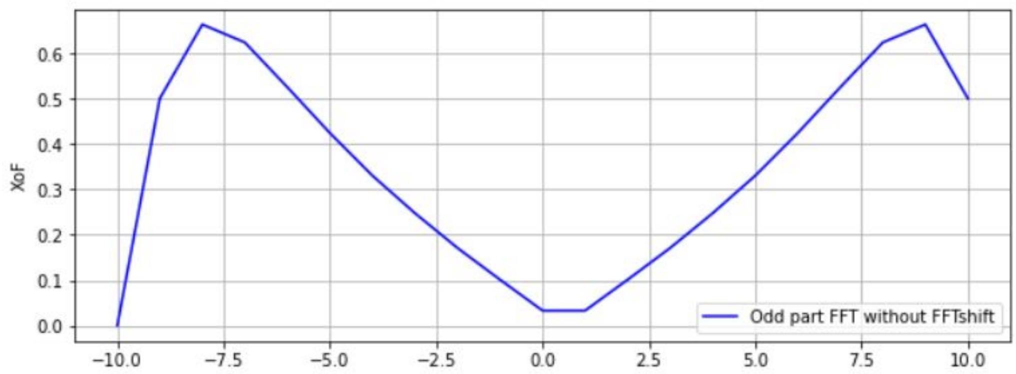

Figure 3: The fft of the odd part without fftshift

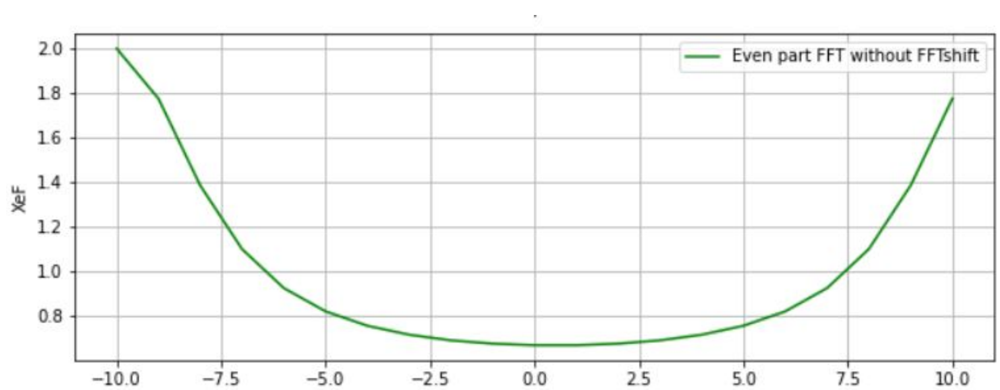

Figure 4: The fft of the even part without fftshift

Now we shall go on to plot the fft after applying the fftshift.

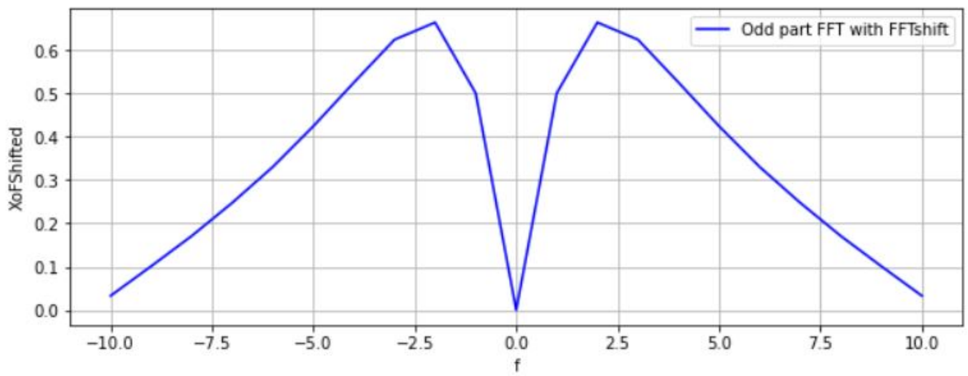

Figure 5: The fft of the odd part with fftshift

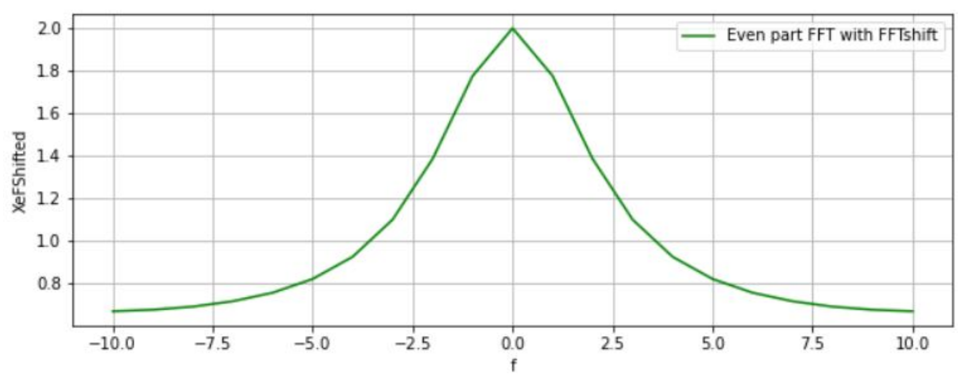

Figure 6: The fft of the even part with fftshift

1.2.1 Brief explanation

We have an unwanted shift which must be resolved by using the fftshift function. The fftshift function rearranges the output of fft by moving the zero-frequency component to the center of the array,hence the visualization shall become more understandable(whilst plotting of course.)

1.3 Part C

Here it is obvious that:

$$x[n] = x_o[n] + x_e[n] \xrightarrow{DTFT} \mathscr{X}(e^{j\omega}) = \mathscr{X}_o(e^{j\omega}) + \mathscr{X}_e(e^{j\omega})$$

Now by using the concept of the second rate error we comprehend that the error can be defined as:

$$Err = \frac{(\mathscr{X}(e^{j\omega}) - (\mathscr{X}_o(e^{j\omega}) + \mathscr{X}_e(e^{j\omega})))^2}{Len}$$

The plot is as follows.

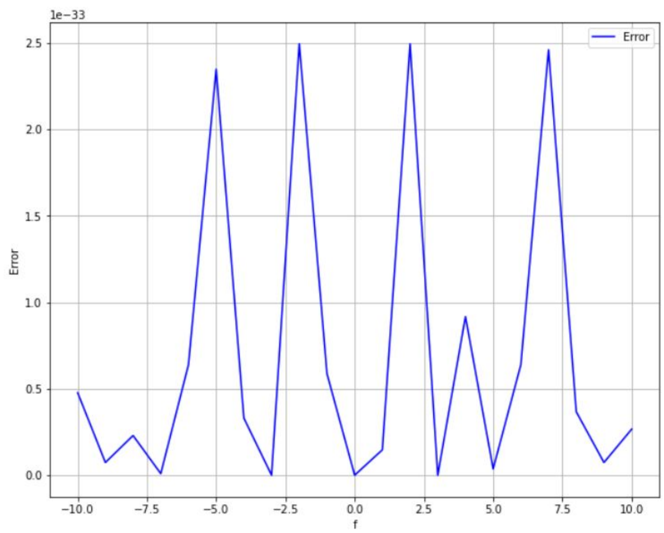

Figure 7: Absolute value of error plot

As we can see in the plot the amount of the error is very small which verifies our analysis. so we can conclude that the equation below holds.

$$\mathscr{X}(e^{j\omega}) = \mathscr{X}_o(e^{j\omega}) + \mathscr{X}_e(e^{j\omega})$$

The slight amount of error that we obtain is because of the algorithm used to perform the fft with numpy, it is not an analytical approach and it is considered a numeric approach, hence the slight error occurs.

1.4 Part D

We know that if we perform even-odd decomposition of a discrete signal, the following terms are true.

$$\mathscr{X}_e(e^{j\omega}) = \Re(\mathscr{X}(e^{j\omega})), \quad \mathscr{X}_o(e^{j\omega}) = j\Im(\mathscr{X}(e^{j\omega}))$$

By applying the same method as part C, we proceed to plot the corresponding graphs and verify our analysis.

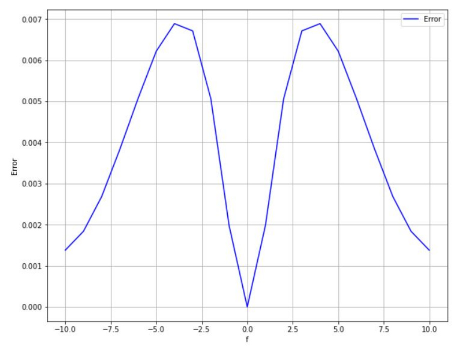

Figure 8: Absolute value of error of even part

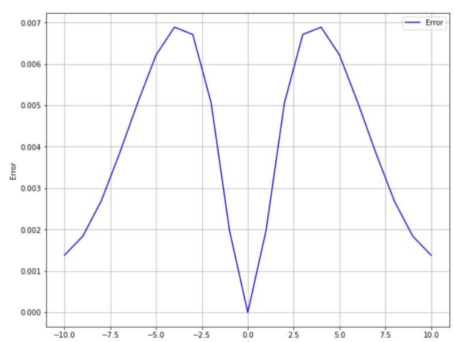

Figure 9: Absolute value of error of odd part

Due to the low amount of the error in both cases, we can conclude that our analysis holds.

2 Question 2

2.1 Part A

The function to create a sine wave has been designed accordingly.

2.2 Part B

The function to perform the DTFT has been designed accordingly.

2.3 Part C

We have written the aliasing function as instructed.

2.4 Part D

The plots are as follow.

2.4.1 f=100

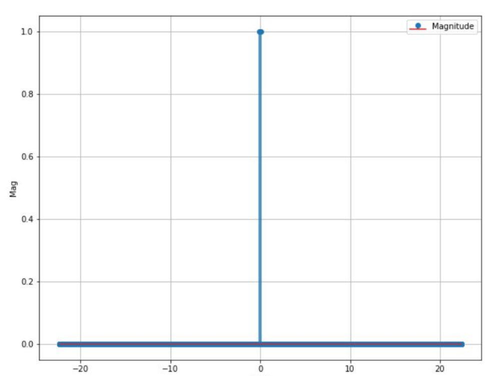

Figure 10: The Spectrum

2.4.2 f=1000

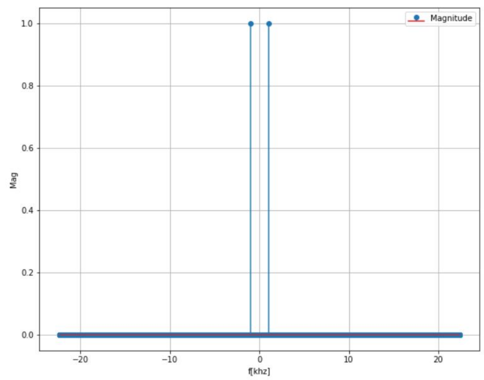

Figure 11: The Spectrum

2.4.3 f=10000

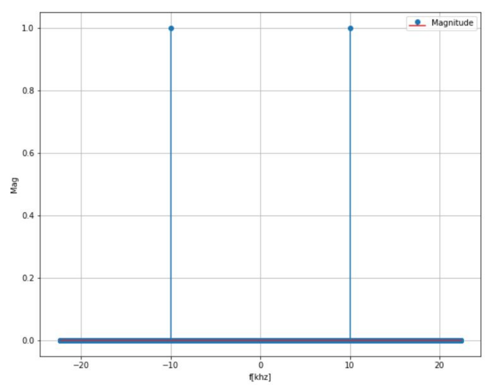

Figure 12: The Spectrum

2.4.4 f=100000

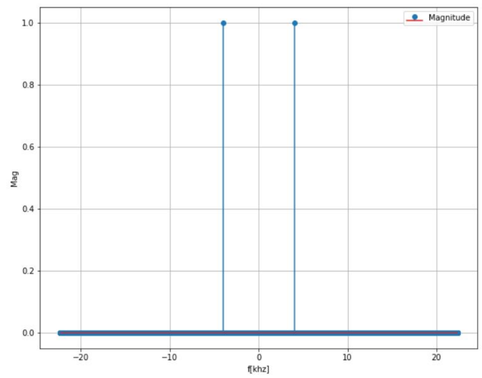

Figure 13: The Spectrum

2.4.5 f=108000

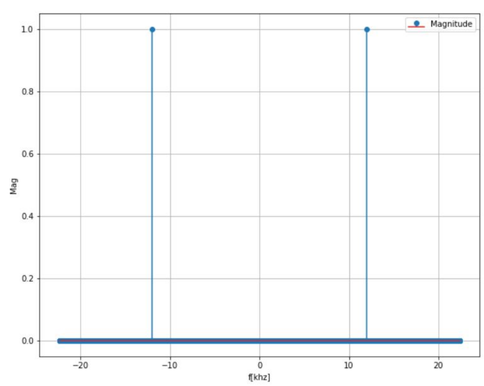

Figure 14: The Spectrum

The DTFT of a sine wave is two impulses which can be seen in each scenario accordingly, the difference is in the shift amount we have in each case. [1]

[1] Since I wasn't sure in the last case 108000 was intended or 100000 I've included both of them.

3 Question 3

3.1 Part A

In this part we attempt to find the frequency response of the following difference equation.

$$y[n] - 0.5y[n-1] + 0.25y[n-2] = x[n] + 2x[n-1] + x[n-3]$$

The plots are as follows.

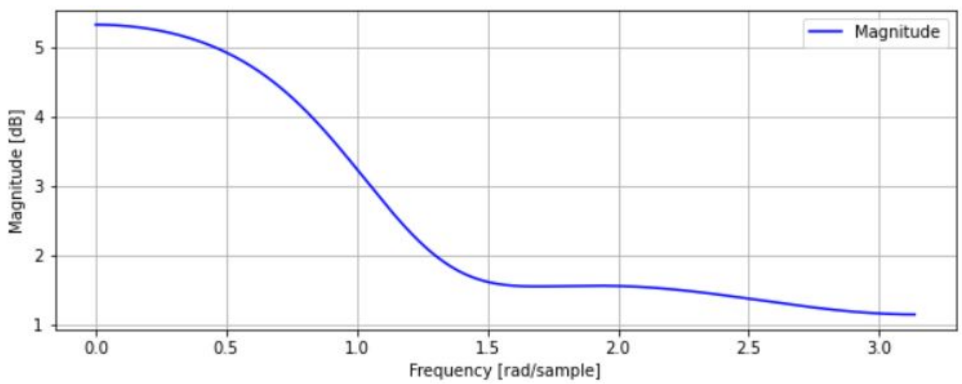

Figure 15: The Magnitude

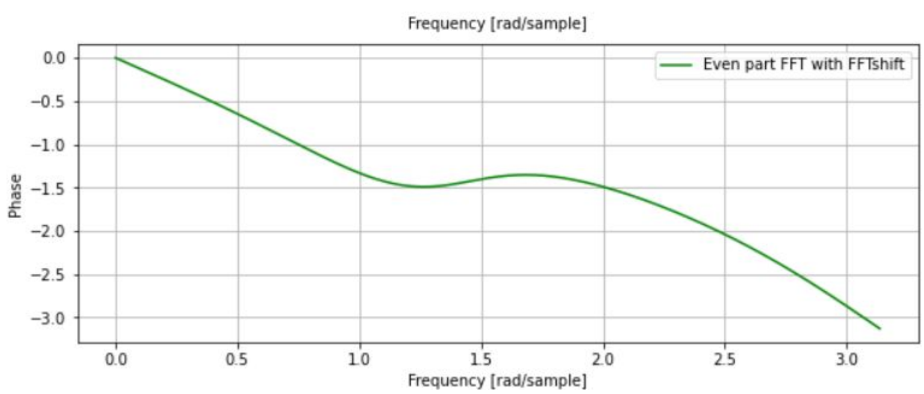

Figure 16: The Phase

3.2 Part B

Here we find the impulse response in the time domain with a for loop and then perform the FFT on it, the plots are as follows.

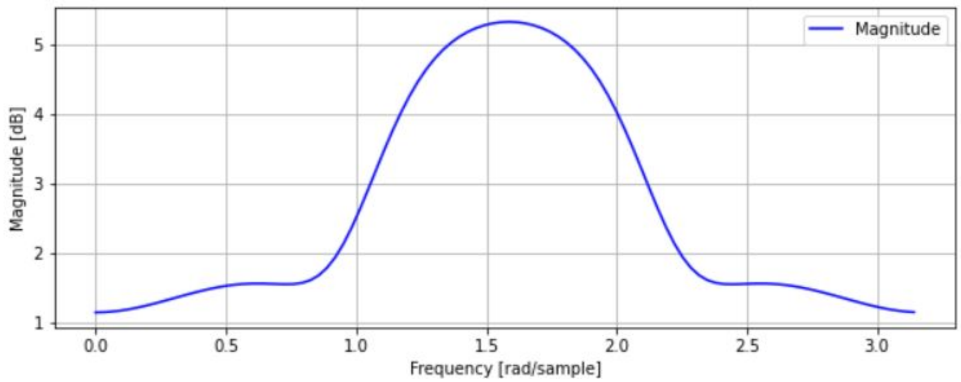

Figure 17: The Magnitude

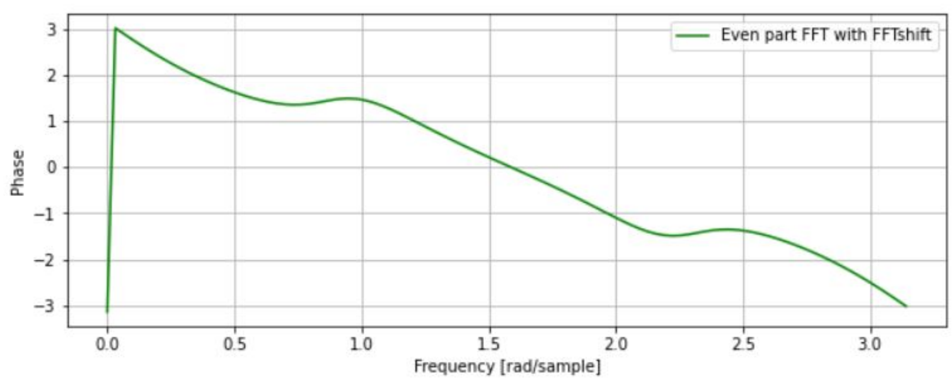

Figure 18: The Phase

The results are essentially the same as the magnitude is symmetrical and the phase is similar.

4 Question 4

In this part we are given a system defined as below:

$$\mathcal{H}(z) = \frac{1}{1 - 2r\cos(\omega_0) + r^2 z^{-2}}$$

4.1 Part A

Here we go on to calculate ω_0 and r so that the transfer function becomes stable.

We know the following

$$r^n \cos(\omega_0 n)u[n] \xrightarrow{\mathscr{Z}} \frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0) + r^2 z^{-2}}, \quad ROC : z > |r|$$

We comprehend that the denominator of each of the Z-transforms are the same, we also know that the numerator has nothing to do with obtaining the ROC, hence in this problem we must have.

$$|r| < 1, \quad 0 \leq \omega_0 \leq 2\pi$$

4.2 Part B

If we set $r = 0.5$ and $\omega_0 = \frac{\pi}{4}$ the system becomes as follows:

$$\mathcal{H}(z) = \frac{1}{1 + \frac{1}{4}z^{-2}}, \quad zeros : 0, 0 \quad poles : \pm\frac{j}{2}$$

The pole-zero plot is as follows:

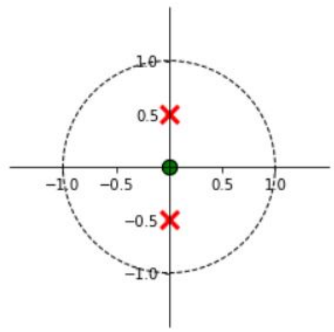

Figure 19: Pole-Zero plot

As we can see the poles and zeros are in correct positions.
Now we go on to plot the Magnitude and phase of $\mathscr{H}_e(e^{j\omega})$ accordingly.

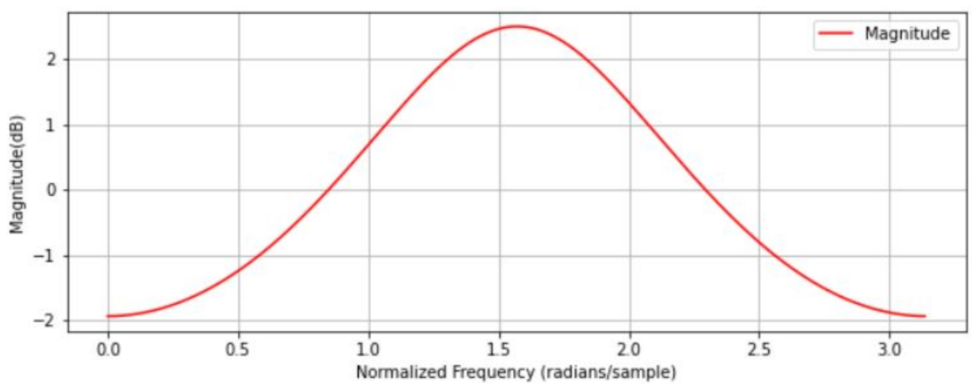

Figure 20: Magnitude

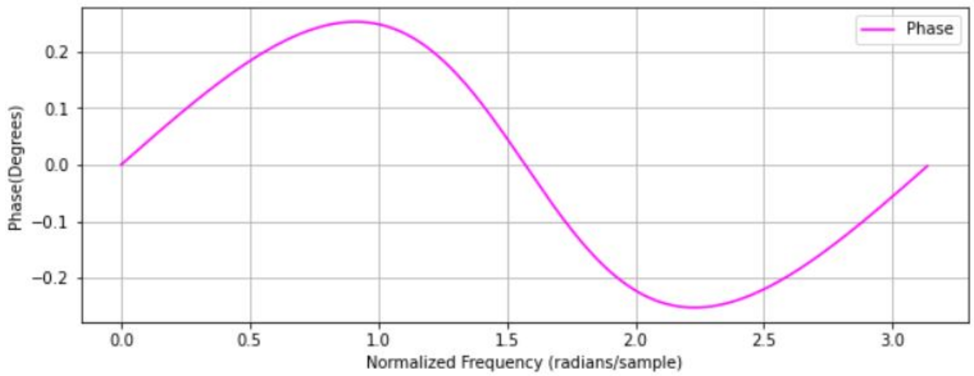

Figure 21: Frequency

4.3 Part C

Here we attempt to calculate the impulse response for $\omega_0 = \frac{\pi}{4}$ and $r = 0.5$, to do this we use the residuez command in python.

$$\mathcal{H}(z) = \frac{1}{1 - \frac{\sqrt{2}}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

We know that this system is stable due to the poles being inside the unit circle, and it is causal too, hence we conclude $h[n] = 0$, we also know that.

$$h[n] = \oint_C \mathcal{X}(z)z^{n-1}\,\mathrm{d}z = \sum_{n=-\infty}^{\infty} Res[\mathcal{X}(z)z^{n-1}]$$

If we calculate the residues and poles we have:

$$Poles : \frac{\sqrt{2}}{4} \pm j\frac{\sqrt{2}}{4}, \quad Residues : 0.5 \pm j0.5$$

If we proceed to calculate the inverse Z-transform we obtain the following:

$$h[n] = \left((0.5 - j0.5)(\frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4})^n + (0.5 + j0.5)(\frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4})^n \right) u[n]$$

$$\longrightarrow h[n] = \left(\frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} 0.5^n e^{j\frac{\pi}{4}n} + \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}} 0.5^n e^{-j\frac{\pi}{4}n} \right) u[n]$$

$$\longrightarrow h[n] = \left(\frac{1}{2^n\sqrt{2}} e^{j\frac{\pi}{4}(n-1)} + \frac{1}{2^n\sqrt{2}} e^{-j\frac{\pi}{4}(n-1)} \right) u[n]$$

$$\longrightarrow h[n] = \left(\frac{1}{2}\right)^n \sqrt{2}\cos\left(\frac{\pi}{4}(n-1)\right) u[n]$$

We continue to plot the impulse response.

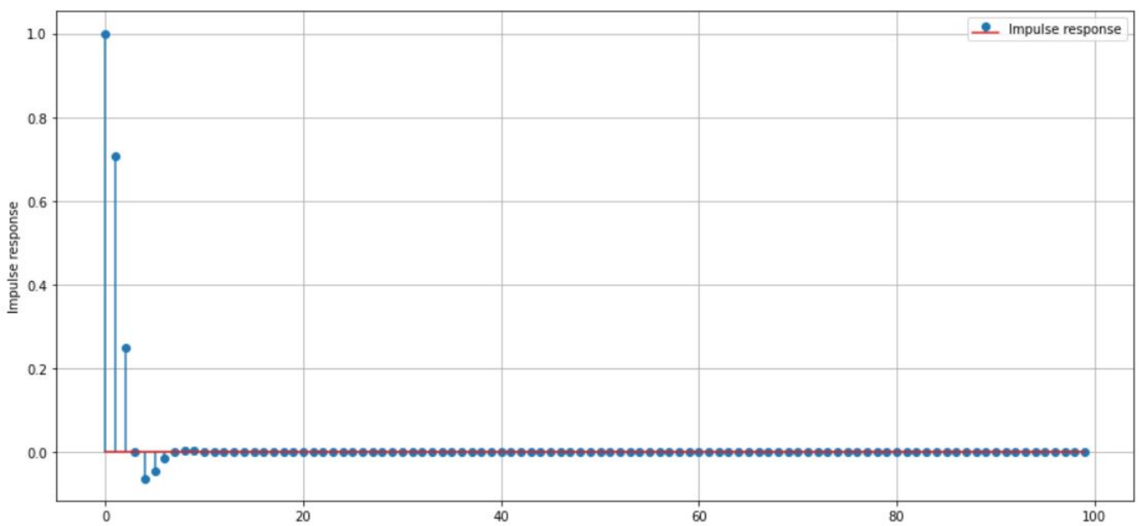

Figure 22: The impulse response

5 Question 5

5.1 Part A

We completed this part with the help of scipy as instructed.

$$f_s = 44100Hz$$

5.2 Part B

To do this, we sliced the audio array from 0 to $5 * f_s$ respectively.

5.3 Part C

We saved three new files with sampling rates f_s, $2f_s$ and $\frac{f_s}{2}$ respectively, some interesting changes occured which I have explained below.

It is obvious that when we sample at the normal rate, we shall get the sound at normal speed, when we increase the sample rate we literally increase the samples per second hence the voice becomes shorter and faster, and when we decrease the sampling rate the voice gets longer and slower.

5.4 Part D

Here we go on to plot the magnitude of the voice of part B, we pay attention that voice is a matrix with two columns, hence we can interpret this as two channels. so we plot the magnitude of the Fourier transform for each channel.

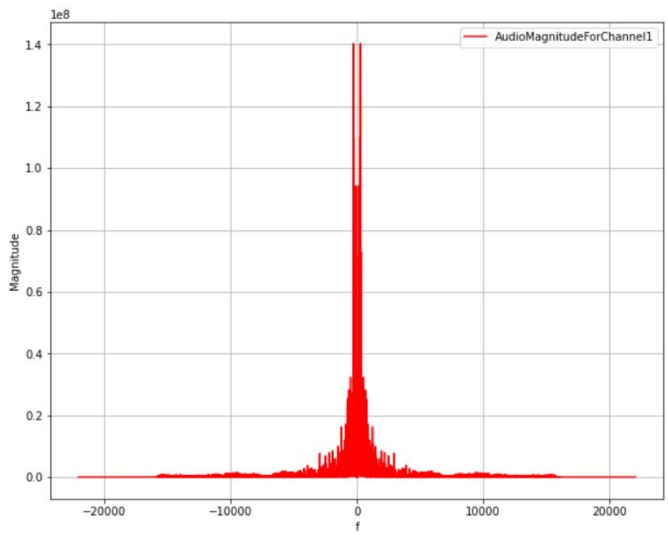

Figure 23: Magnitude of channel 1

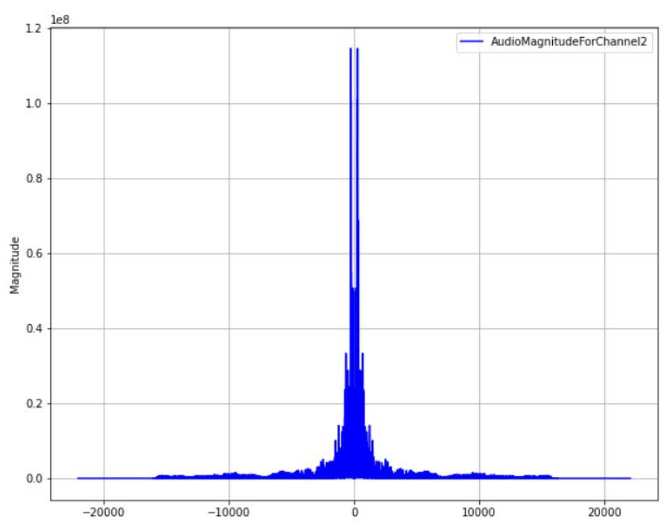

Figure 24: Magnitude of channel 2

5.5 Part E

We have implemented a function which performs compression or downsampling in other words.

$$o[n] = x[nM]$$

In the Fourier domain we expect the following

$$\mathscr{O}(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1}\mathscr{X}(e^{\frac{j(\omega-2\pi k)}{M}})$$

Due to the above formula we expect the magnitude to decrease by a factor of $\frac{1}{M}$ and the bandwidth to increase by a factor of M.

5.6 M=2

The plots are as follows.

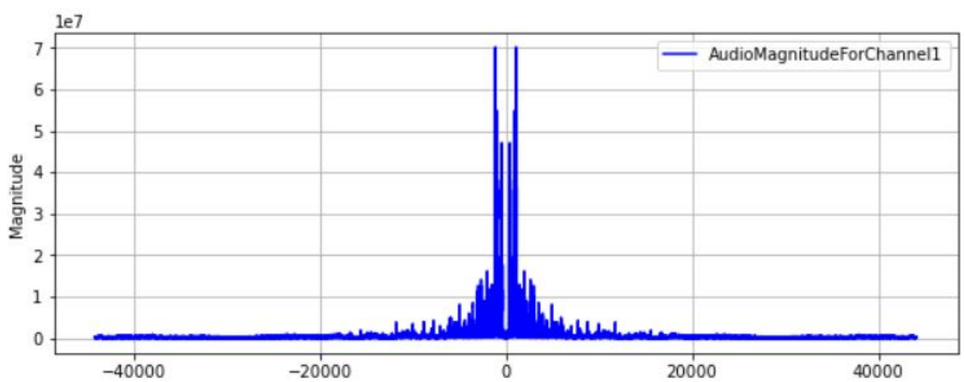

Figure 25: Magnitude of channel 1 for M=2

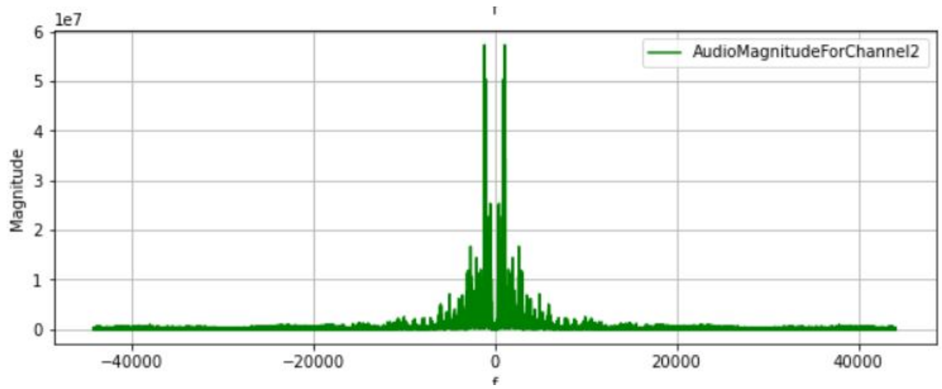

Figure 26: Magnitude of channel 2 for M=2

5.7 M=5

The plots are as follows.

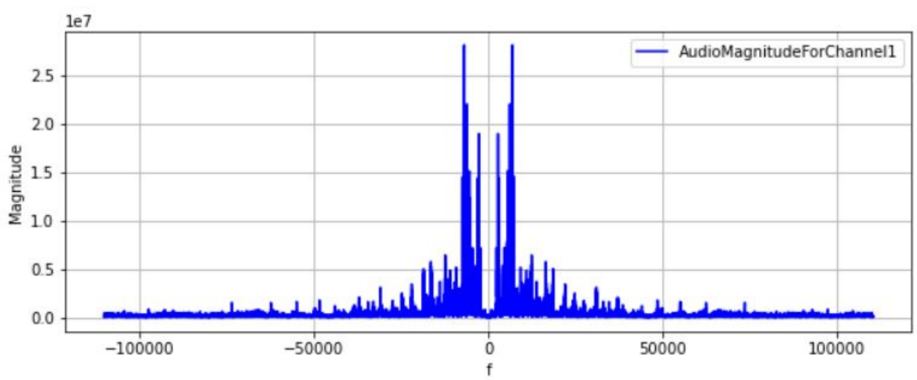

Figure 27: Magnitude of channel 1 for M=5

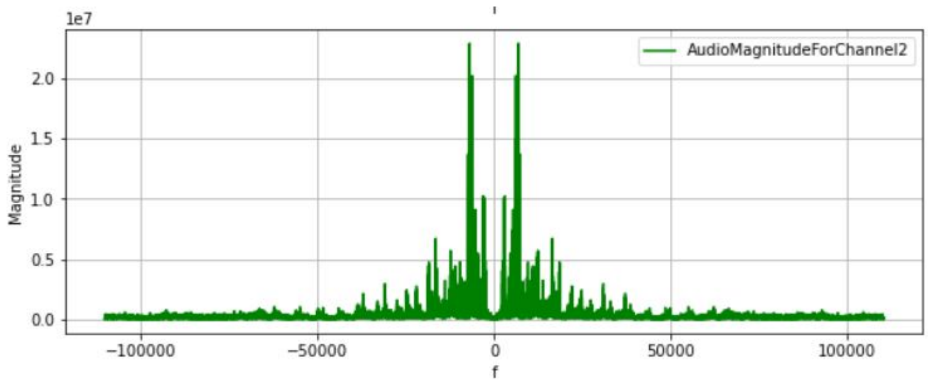

Figure 28: Magnitude of channel 2 for M=5

5.8 M=6

The plots are as follows.

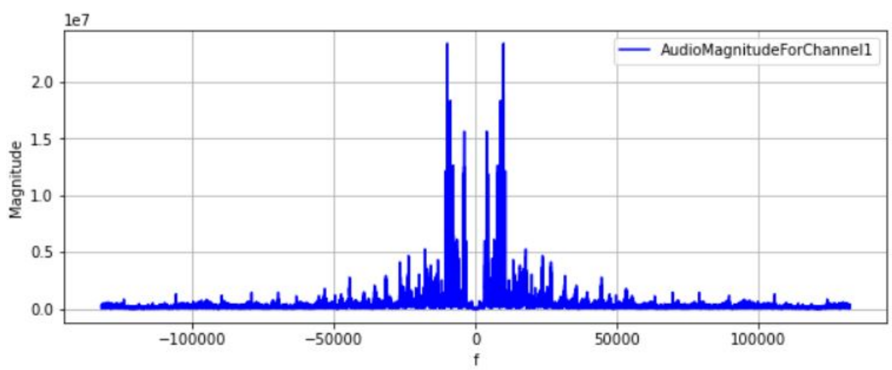

Figure 29: Magnitude of channel 1 for M=6

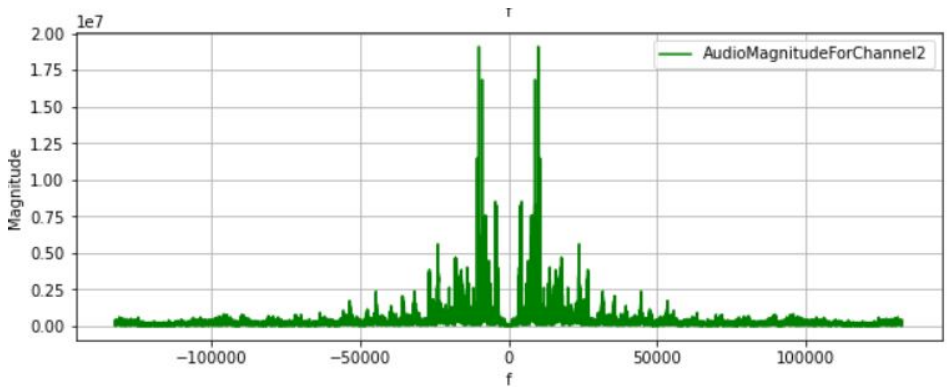

Figure 30: Magnitude of channel 2 for M=6

We see that our prediction and analysis were correct.

5.9 Part E

We have implemented a function which performs expansion or upsampling in other words.

Here we expect something somewhat reverse to what we had in the previous part, we expect the magnitude to increase by a factor of L and the bandwidth to decrease by a factor of $\frac{1}{L}$

5.10 L=2

The plots are as follows.

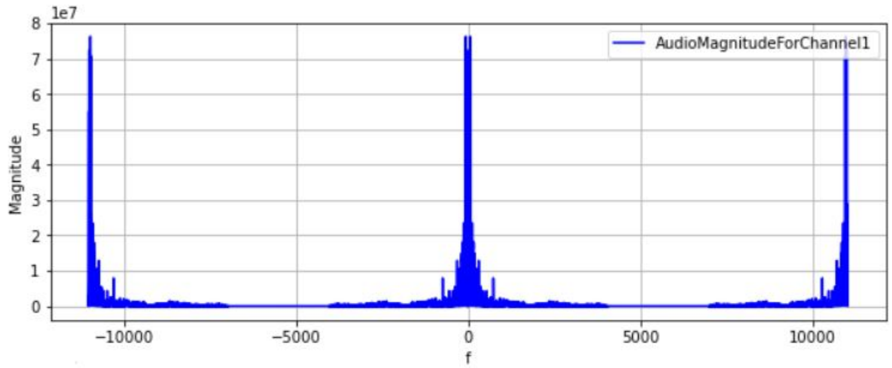

Figure 31: Magnitude of channel 1 for L=2

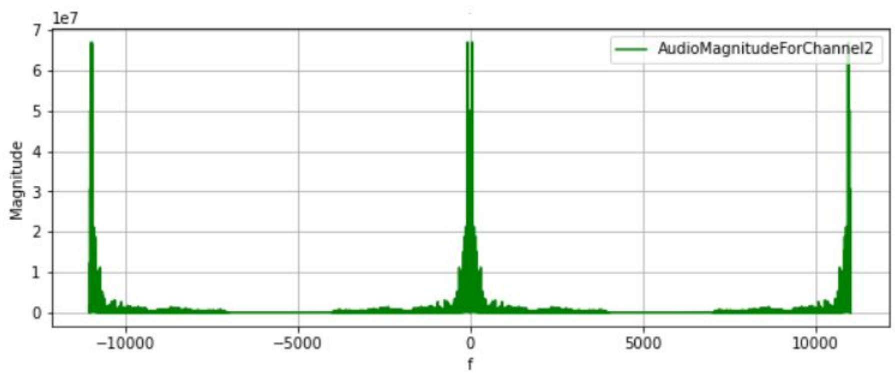

Figure 32: Magnitude of channel 2 for L=2

5.11 L=5

The plots are as follows.

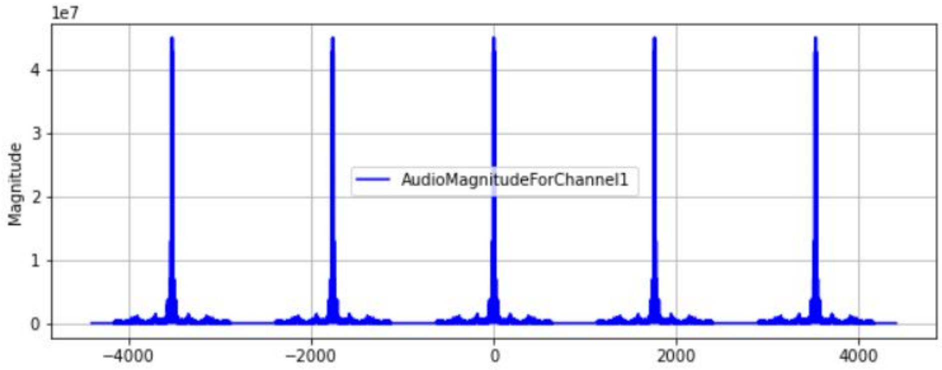

Figure 33: Magnitude of channel 1 for L=5

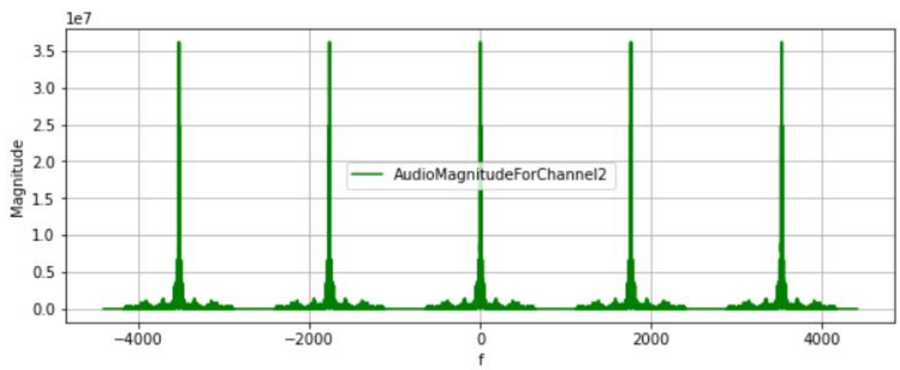

Figure 34: Magnitude of channel 2 for L=5

5.12 L=6

The plots are as follows.

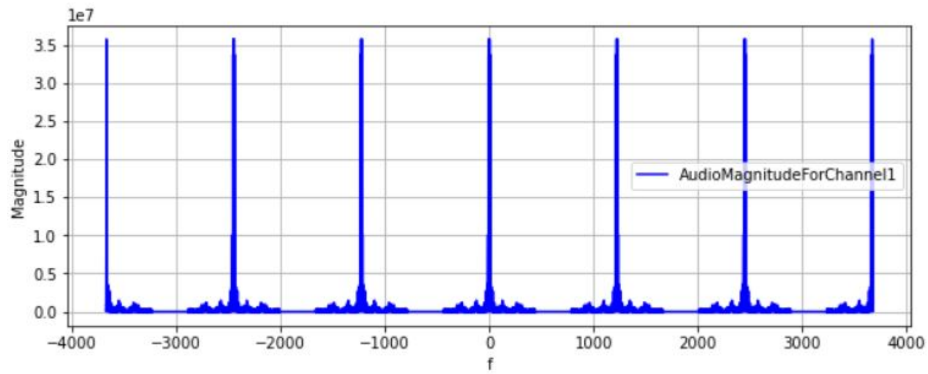

Figure 35: Magnitude of channel 1 for L=6

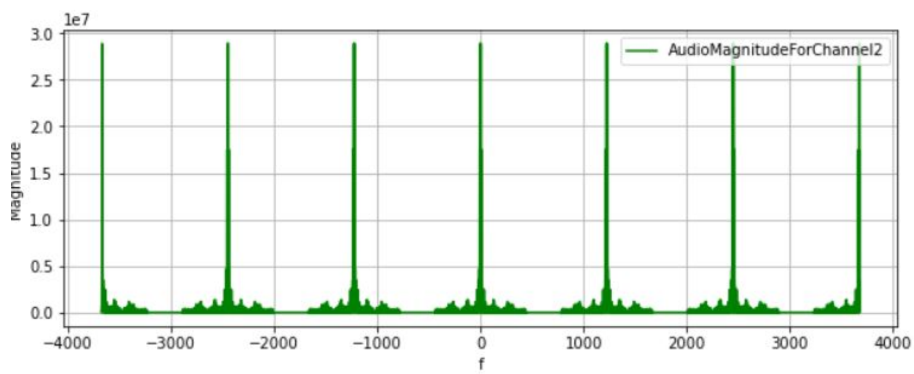

Figure 36: Magnitude of channel 2 for L=6

We see that our prediction and analysis were correct.

5.13 Part H

When we upsample a signal we inadvertantly add a hefty amount of zeros, an approach would be to perform interpolation, in other words we could replace the zeros with the average of the samples of before and after it, hence we increase the quality.(This part has not been implemented due to lack of time.)

6 Question 6

6.1 Part A

First we read the images with the proper tools, the photos are as follows.

Figure 37: The cat and horse

6.2 Part B

To obtain better results we grayscale the photos, the result is as follows.

Figure 38: The gray cat and horse

We then continue to save the magnitude and phase of the photos in the respective variables.

6.3 Part C

In the last part we first resize the cat photo to match the horse, the result is as follows:

Figure 39: The resized gray cat

Now we can proceed to change the magnitudes of the photos, after we do so the images shall become as such:

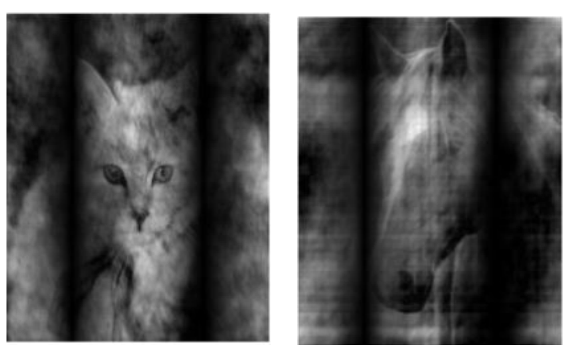

Figure 40: The gray cat and horse with swapped magnitudes

We see that the result is quite frightening! Yet the cat and horse are distinguishable, hence we go on to conclude that in photos, the phase is far more important than the magnitude, since by swapping the magnitudes of the cat and horse, the pictures are still recognizable. By changing the phase we essentially change a lot more of the data of the photo compared to when we change the magnitude, so the phase is a more important element.