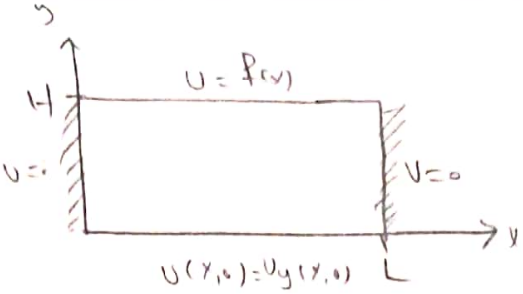
۱-

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0 \\ u(0,y) = u(L,y) = 0 \\ u(x,0) = u_y(x,0) \;,\; u(x,H) = f(x) \end{cases}$$



در معادله لاپلاس مستطیلی به ازای $k^2=0$ پاسخ ها بی ارزش اند.

$$u(x,y) = X(x)\,Y(y)$$

$$\to X''Y + Y''X = 0 \xrightarrow{\div XY} \frac{X''}{X} + \frac{Y''}{Y} = 0 \to \frac{X''}{X} = \frac{-Y''}{Y} = -k^2$$

$$\to \frac{X''}{X} = -k^2 \to X'' + k^2 X = 0 \to X = A\cos kx + B\sin kx$$

اعمال شرایط مرزی: $u(0,y)=0 \to X(0)=0 \to A=0 \quad u(L,y)=0 \to X(L)=0 \quad B\sin kL = 0$

$$\to kL = m\pi \to k = \frac{m\pi}{L} \quad m = 1, 2, 3, \ldots$$

$$\frac{Y''}{Y} = k^2 \to Y'' - k^2 Y = 0 \longrightarrow Y(y) = C\sinh k(H-y) + D\sinh ky$$

$$\xrightarrow{k=\frac{m\pi}{L}} Y(y) = C\sinh \frac{m\pi}{L}(H-y) + D\sinh \frac{m\pi}{L}y$$

$$u_m(x,y) = \left[B_m \sinh\left(\frac{m\pi}{L}y\right) + A_m \sinh\left(\frac{m\pi}{L}(H-y)\right)\right] \sin\frac{m\pi}{L}x$$

$$\to u(x,y) = \sum_{m=1}^{\infty} \left[B_m \sinh\left(\frac{m\pi}{L}y\right) + A_m \sinh\left(\frac{m\pi}{L}(H-y)\right)\right] \sin\frac{m\pi}{L}x$$

$$\xrightarrow{\text{شرایط مرزی}} u(x,H) = f(x) \to \sum_{m=1}^{\infty} \underbrace{B_m \sinh\frac{m\pi}{L}y}_{\text{ضریب سری فوریه سینوسی تابع } f(x)} \sin\frac{m\pi}{L}x = f(x)$$

$$\to B_n = \frac{2}{L\sinh\left(\frac{m\pi H}{L}\right)} \int_0^L f(x) \sin\frac{n\pi}{L}x$$

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$$U(x,0)=\frac{\partial U}{\partial y}(x,0)\longrightarrow \sum_{m=1}^{\infty} A_m \sinh\frac{m\pi}{L}H \sin\frac{m\pi}{L}x=\sum_{m=1}^{\infty}\frac{m\pi}{L}\left(B_m-A_m\sinh\frac{m\pi}{L}H\right)\sin\frac{m\pi}{L}x$$

$$\longrightarrow \sum_{m=1}^{\infty}\left(1+\frac{m\pi}{L}\right)A_m\sinh\frac{m\pi}{L}H\sin\frac{m\pi}{L}x=\sum_{m=1}^{\infty}\frac{m\pi}{L}B_m\sin\frac{m\pi}{L}x$$

طرف راست تساوی بالا عبارتی معلوم است (چون B_m را در صفحه قبل بدست آوردیم) پس می‌توان آن را مثل سری $g(x)$ در نظر گرفت

$$\longrightarrow A_m=\frac{2}{L\sinh\left(\frac{m\pi}{L}H\right)}\times\frac{1}{1+\frac{m\pi}{L}}\int_0^L g(x)\sin\frac{m\pi}{L}x\,dx$$

$$\longrightarrow U(x,y)=\sum_{m=1}^{\infty}\left[B_m\sinh\left(\frac{m\pi}{L}y\right)+A_m\sinh\left(\frac{m\pi}{L}(H-y)\right)\right]\sin\frac{m\pi}{L}x$$

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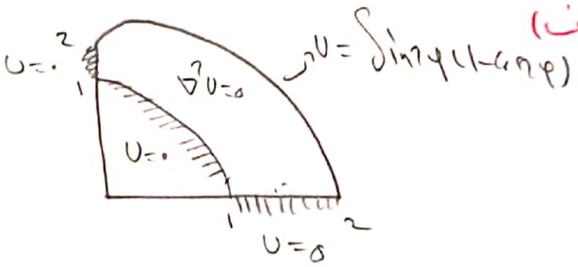
2 - الف)

$$\begin{cases}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial\varphi^2}=0\\ U(1,\varphi)=0 \quad U(2,\varphi)=\sin 2\varphi(1-\cos 2\varphi)\\ U(r,0)=0 \quad U(r,\pi/2)=0\end{cases}$$



$$U(r,\varphi)=R(r)\Phi(\varphi)\longrightarrow \frac{1}{r}\frac{\partial}{\partial r}(rR'\Phi)+\frac{1}{r^2}R\Phi''=0$$

$$\longrightarrow \frac{-r(rR')'}{R}=\frac{\Phi''}{\Phi}=cte=\begin{cases}k^2\\0\\-k^2\end{cases}$$

حالت k^2: $\frac{\Phi''}{\Phi}=k^2 \Rightarrow \Phi''-k^2\Phi=0 \longrightarrow \Phi(\varphi)=Ae^{k\varphi}+Be^{-k\varphi}$

$\longrightarrow U(r,0)=0\longrightarrow\Phi(0)=0\longrightarrow A=0 \quad U(r,\pi/2)=0\longrightarrow\Phi(\pi/2)\longrightarrow B\underbrace{e^{-\frac{\pi}{2}k}}_{\neq 0}=0 \quad B=0$

$\longrightarrow \Phi(\varphi)=0$ پاسخ بی‌ارزش

حالت 0: $\frac{\Phi''}{\Phi}=0\longrightarrow\Phi''=0\longrightarrow\Phi(\varphi)=A\varphi+\varphi_0 \longrightarrow \Phi(0)=0\longrightarrow\varphi_0=0,\ \Phi(\pi/2)=0$

$\longrightarrow A\pi/2=0\longrightarrow A=0\longrightarrow\Phi(\varphi)=0$ پاسخ بی‌ارزش

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حالت $-k^2$:

$$\frac{\phi''}{\phi} = -k^2 \to \phi'' + k^2\phi = 0 \to \phi(\varphi) = A\cos k\varphi + B\sin k\varphi$$

$$\to \phi(0)=0 \to A=0 \qquad \phi(\pi/2)=0 \to B\sin\frac{k\pi}{2}=0 \to \begin{cases} B=0 \to \text{بی‌ارزش} \\ \frac{k\pi}{2}=n\pi \to k=2n \end{cases}$$

$$\to \phi_n(\varphi) = B\sin 2n\varphi$$

$$\frac{-r(rR')'}{R} = -k^2 \to r^2R'' + rR' - k^2R = 0 \xrightarrow{k=2n} r^2R'' + rR' - 4n^2R$$ بسل نیست!!

$$R(r) = r^\alpha \to r^2\alpha(\alpha-1)r^{\alpha-2} + r\alpha r^{\alpha-1} - 4n^2r^\alpha = 0 \to (\alpha(\alpha-1)+\alpha-4n^2)r^\alpha = 0$$

$$\to \alpha^2 - 4n^2 = 0 \to \alpha^2 = 4n^2 \to \alpha = \pm 2n \to R(r) = C_n r^{2n} + D_n r^{-2n}$$

$R(1)=0,\ C_n=-D_n$ چون که با اساساً یک مساله معادلات است پس با شرط آن جلو می‌رویم.

$$R(r) = C_n(r^{2n} - r^{-2n}) \qquad u(r,\varphi) = \sum_{n=1}^{\infty} b_n\sin 2n\varphi\, r^{2n} - b_n\sin 2n\varphi\, r^{-2n}$$

$$u(2,\varphi) = \sin 2\varphi - \sin 2\varphi\cos 2\varphi = \sin 2\varphi - \frac{1}{2}\sin 4\varphi$$

$$\to \sum_{n=1}^{\infty} b_n\sin 2n\varphi(2^{2n} - 2^{-2n}) = \sin 2\varphi - \frac{1}{2}\sin 4\varphi$$

$$n=1 \quad b_1\sin 2\varphi(2^2 - \frac{1}{2^2}) = \sin 2\varphi \to b_1 = \frac{4}{15} \qquad n=2 \to b_2\sin 4\varphi(2^4 - \frac{1}{2^4}) = \frac{\sin 4\varphi}{2}$$

$$\to b_2(2^4 - \frac{1}{2^4}) = \frac{-1}{2} \to b_2 = \frac{-1}{2(2^4 - \frac{1}{2^4})} = \frac{-8}{255}$$

$$\to u(r,\varphi) = \frac{4}{15}(r^2 - \frac{1}{r^2})\sin 2\varphi - \frac{8}{255}(r^4 - \frac{1}{r^4})\sin 4\varphi$$

(به ازای سایر n ها صفر می‌شود)

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ب) به طوری که u نسبت به φ متناوب باشد.

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}=0 \\ u(r,\pi)=u(r,-\pi) \\ u_\varphi(r,\pi)=u_\varphi(r,-\pi) \\ u(1,\varphi)=c_1 \quad u(2,\varphi)=2c_1 \end{cases}$$

$u(r,\varphi)=R(r)\Phi(\varphi)$

$\to \frac{-r(rR')'}{R}=\frac{\Phi''}{\Phi}=\begin{cases} k^2 \\ 0 \\ -k^2 \end{cases}$

حالت k^2: $\frac{\Phi''}{\Phi}=k^2 \to \Phi=Ae^{k\varphi}+Be^{-k\varphi}$ $\quad u(r,\pi)=u(r,-\pi)\to \Phi(\pi)=\Phi(-\pi)$

$\to Ae^{k\pi}+Be^{-k\pi}=Ae^{-k\pi}+Be^{k\pi}\to A=B\to \Phi(\varphi)=2A\cosh k\varphi$

$u_\varphi(r,\pi)=u_\varphi(r,-\pi)\to (e^{k\pi}-e^{-k\pi})A=-(e^{-k\pi}-e^{-k\pi})A\to A=0 \xrightarrow{} $ بی ارزش

حالت 0: $\Phi''=0\to\Phi(\varphi)=A\varphi+\varphi_0$ $\quad \Phi(\pi)=\Phi(-\pi)\to A\pi+\varphi_0=-A\pi\to A=0$

$\Phi(\varphi)=\varphi_0$. شرط دوم u_φ به صورت بدیهی ارضا شد $\qquad \frac{r(rR')'}{R}=0\to rR'=c\to R'=\frac{c}{r}\to R(r)=c\ln r+D$

$\to u(r,\varphi)=a_0+b_0\ln r$

حالت $-k^2$: $\frac{\Phi''}{\Phi}=-k^2\to\Phi''+k^2\Phi=0\to\Phi(\varphi)=A\cos k\varphi+B\sin k\varphi$

$\Phi(\pi)=\Phi(-\pi)\to A\cos k\pi+B\sin k\pi=A\cos k\pi-B\sin k\pi\to B\sin k\pi=0\begin{cases} B=0 & \text{بی ارزش} \\ k=n & \checkmark \end{cases}$

$u_\varphi(r,\pi)=u_\varphi(-r,\pi)\to -Ak\sin k\pi+Bk\cos k\pi=Ak\sin k\pi+Bk\cos k\pi\to Ak\sin k\pi=0\begin{cases} A=0 & \text{بی ارزش} \\ k=n \end{cases}$

$\to \Phi_n(\varphi)=A_n\cos n\varphi+B_n\sin n\varphi$

$\frac{-r(rR')'}{R}=-n^2\to r^2R''+rR'-n^2R=0$. بسل $\qquad R(r)=r^\alpha$

$\to r^2\alpha(\alpha-1)r^{\alpha-2}+\alpha r^{\alpha-1}-n^2r^\alpha=0\to(\alpha^2-n^2)r^\alpha=0\quad \alpha=\pm n\to R(r)=C_nr^n+D_nr^{-n}$

$u(r,\varphi)=a_0+b_0\ln r+\sum^{\infty}(a_n\cos n\varphi+b_n\sin n\varphi)r^n+(c_n\cos n\varphi+d_n\sin n\varphi)r^{-n}$ مسأله محدود است.

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$U(1,\varphi)=c_1 \to a_0+\sum_{n=1}^{\infty} a_n\cos n\varphi + b_n\sin n\varphi + c_n\cos n\varphi + d_n\sin n\varphi = c_1 \to \sum_{n=1}^{\infty}(a_n+c_n)\cos n\varphi$
$+(b_n+d_n)\sin n\varphi \quad + a_0 = c_1$

$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} c_1\,d\varphi \Rightarrow a_0=c_1 \qquad a_n+c_n=\frac{1}{\pi}\int_{-\pi}^{\pi} c_1\cos n\varphi\,d\varphi=0 \qquad a_n+c_n=0$

$b_n+d_n=\frac{1}{\pi}\int_{-\pi}^{\pi} c_1\sin n\varphi\,d\varphi=0 \qquad b_n+d_n=0$

$U(e,\varphi)=2c_1 \longrightarrow a_0+b_0+\sum_{n=1}^{\infty}(a_n e^{n}+c_n e^{-n})\cos n\varphi+(b_n e^{n}+d_n e^{-n})\sin n\varphi=2c_1$

$\to a_0+b_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} 2c_1\,d\varphi = 2c_1 \xrightarrow{a_0=c_1} b_0=a_0=c_1$

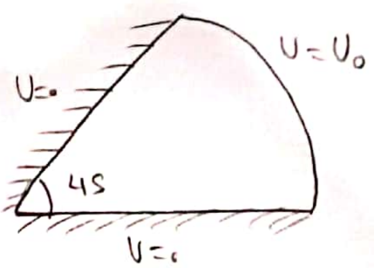
$a_n e^{n}+c_n e^{-n}=\frac{1}{\pi}\int_{-\pi}^{\pi} 2c_1\cos n\varphi=0 \to a_n e^{n}+c_n e^{-n}=0 \xrightarrow{c_n=-a_n} a_n(\underbrace{e^{n}-e^{-n}}_{\neq 0})=0$

$a_n=0, \quad b_n e^{n}+d_n e^{-n}=\frac{1}{\pi}\int_{-\pi}^{\pi} 2c_1\sin n\varphi\,d\varphi=0 \quad b_n e^{n}+d_n e^{-n}=0 \xrightarrow{d_n=-b_n} b_n(\underbrace{e^{n}-e^{-n}}_{\neq 0})=0$

$b_n=0 \longrightarrow \begin{cases} a_0=b_0=c_1 \\ a_n=b_n=c_n=d_n \end{cases} \to U(r,\varphi)=c_1+c_1\ln r$

3- ب)

$$\begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2}=0 \\ U(r,0)=U(r,\frac{\pi}{4})=0 \\ U(a,\varphi)=U_0 \end{cases}$$



$U(r,\varphi)=R(r)\Phi(\varphi) \longrightarrow \frac{1}{r}\frac{\partial}{\partial r}(rR'\Phi)+\frac{1}{r^2}R\Phi''=0 \to \frac{-r(rR')'}{R}=\frac{\Phi''}{\Phi}=\begin{cases}k^2\\0\\-k^2\end{cases}$

حالت k^2: $\frac{\Phi''}{\Phi}=k^2 \to \Phi''-k^2\Phi=0 \to \Phi=Ae^{k\varphi}+Be^{-k\varphi} \to U(r,0)=0 \to \Phi(0)=0$

$\to A=0 \quad U(r,\frac{\pi}{4})=0 \to B\underbrace{e^{-\frac{\pi}{4}k}}_{\neq 0}=0 \quad B=0 \to \Phi(\varphi)=0$ بی‌ارزش

حالت 0: $\Phi''=0 \to \Phi=A\varphi+\varphi_0 \to \Phi(0)=0 \to \varphi_0=0 \quad \Phi(\frac{\pi}{4})=0 \to A\frac{\pi}{4}=0 \to A=0 \to \Phi(\varphi)=0$ بی‌ارزش

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حالت ۲: $\frac{\Phi''}{\Phi} = -k^2 \rightarrow \Phi'' + k^2\Phi = 0 \quad \Phi(\varphi) = A\cos k\varphi + B\sin k\varphi \quad \Phi(0)=0 \rightarrow A=0$

$\Phi(\frac{\pi}{4}) = 0 \rightarrow B\sin\frac{k\pi}{4} = 0 \rightarrow \begin{cases} B=0 & \text{نکات} \\ \frac{k\pi}{4} = n\pi \rightarrow k = 4n \end{cases}$

$\Phi_n(\varphi) = B_n \sin 4n\varphi$

برای R داریم: $\frac{-r(rR')'}{R} = -16n^2 \rightarrow r^2R'' + rR' - 16n^2R = 0$

$R(r) = r^\alpha \quad r^2\alpha(\alpha-1)r^{\alpha-2} + r\alpha r^{\alpha-1} - 16n^2r^\alpha = 0 \rightarrow (\alpha^2 - 16n^2)r^\alpha = 0$

$\alpha = \pm 4n \quad R(r) = C_n r^{4n} + D_n r^{-4n}$

مسلماً D_n ما اینجا مسلماً صفر است چون $r=0$ جزء ناحیه حل است پس:

$U(r,\varphi) = \sum_{n=1}^{\infty} b_n \sin 4n\varphi \, r^{4n}$

$U(a,\varphi) = U_0 \rightarrow U_0 = \sum_{n=1}^{\infty} b_n \sin 4n\varphi \, a^{4n}$

حال U_0 را به سری فوریه بیان کنیم.

$U_0 = \sum_{n=1}^{\infty} A_n \sin \frac{2\pi n}{T}\varphi \qquad \frac{2\pi n}{T} = 4n \rightarrow T = \frac{\pi}{2} \qquad (A_n = b_n a^{4n})$

برای محاسبه ما باید U_0 را گسترش فرد دهیم

$\rightarrow b_n = \frac{4}{\pi a^{4n}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \underbrace{U_0 \sin 4n\varphi}_{\text{گسترش فرد داده شد}} d\varphi = \frac{8U_0}{\pi a^{4n}} \int_0^{\frac{\pi}{4}} \sin 4n\varphi \, d\varphi = \frac{8U_0}{\pi a^{4n}} \frac{1 - \cos\pi n}{4n}$

$= \frac{2U_0}{\pi a^{4n}} \frac{1-\cos\pi n}{n} \longrightarrow b_n = \begin{cases} \frac{4U_0}{n\pi a^{4n}} & n \text{ فرد} \\ 0 & n \text{ زوج} \end{cases}$

$\longrightarrow U(r,\varphi) = \sum_{n \text{ فرد}} \frac{4U_0}{n\pi a^{4n}} \sin 4n\varphi \, r^{4n}$

4- اگر معادله را نسبت به φ متقارن بگیریم:

$$\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial u}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial u}{\partial\theta}) = 0 \\ U(0,\theta) = \text{کراندار} \quad \text{پیوسته بودن} \\ \lim_{r\to\infty} U(r,\theta) = \text{کراندار} \quad \text{شرط مرزی} \\ U(a,\theta) = f(\theta) \\ 0 \le \theta \le \pi \quad \text{تفکیک} \end{cases}$$

$U(r,\theta) = R(r)\Theta(\theta) \rightarrow \Theta\left[\frac{1}{r^2}\frac{d}{dr}(r^2\frac{dR}{dr})\right]$

$= -R\left[\frac{1}{r^2\sin\theta}\frac{d}{d\theta}(\sin\theta\frac{d\Theta}{d\theta})\right]$

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$$\frac{\frac{1}{r^2}(r^2R')'}{R} = \frac{-\frac{1}{r^2\sin\theta}(\sin\theta\,\Theta')'}{\Theta} = K \begin{cases} + \\ 0 \\ - \end{cases}$$

معادله ۲: $r^2R'' + 2rR' - KR = 0$ معادله ۱: $\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) + K\Theta = 0$

$\rightarrow x = \cos\theta \quad \frac{d\Theta}{d\theta} = \frac{d\Theta}{dx}\cdot\frac{dx}{d\theta} = -\sin\theta\frac{d\Theta}{dx} \rightarrow \frac{d}{dx}\left(\sin^2\theta\frac{d\Theta}{dx}\right) + K\Theta = 0$

$\frac{d}{dx}\left((1-x^2)\frac{d\Theta}{dx}\right) + K\Theta = 0 \longrightarrow (1-x^2)\Theta'' - 2x\Theta' + K\Theta$ معادله لژاندر

$K = n(n+1) \longrightarrow (1-x^2)\Theta'' - 2x\Theta' + n(n+1)\Theta = 0$

$\Theta(x) = AP_n(x) + BQ_n(x)$

در معادله بالا $x = \pm 1$ نقاط تکین هستند، تا در رابطه تابع لژاندر نوع دوم که تابع لژاندر اول ندارند اما Θ در $(x = \pm 1)$ پس $B = 0$ چون $Q_n(x)$ شامل $\ln\left(\frac{1+x}{1-x}\right)$ است.

$\rightarrow \Theta(x) = AP_n(x) \Rightarrow \Theta(\theta) = AP_n(\cos\theta)$

معادله ۲: $r^2R'' + 2rR' - n(n+1)R = 0 \rightarrow$ Cauchy $\quad R(r) = r^\alpha$

$\rightarrow r^2\alpha(\alpha-1)r^{\alpha-2} + 2r\alpha r^{\alpha-1} - n(n+1)r^\alpha = 0 \rightarrow (\alpha(\alpha-1) + 2\alpha - n(n+1))r^\alpha = 0$

$\rightarrow \alpha^2 + \alpha - n(n+1) = 0 \rightarrow \begin{cases} \alpha = n \\ \alpha = -(n+1) \end{cases} \rightarrow R_n(r) = A_nr^n + \frac{B_n}{r^{n+1}}$

$$U(r,\theta) = \sum_{n=0}^{\infty}\left(A_nr^n + \frac{B_n}{r^{n+1}}\right)P_n(\cos\theta)$$

ابتدا مسئله داخلی را بررسی می کنیم.

$U(0,\theta) =$ کراندار $\rightarrow B_n = 0 \rightarrow U(r,\theta) = \sum_{n=0}^{\infty} A_nr^nP_n(\cos\theta)$

$U(a,\theta) = f(\theta)$, $f(\theta) = \sum_{n=0}^{\infty} A_na^nP_n(\cos\theta)$, می دانیم: $\int_{-1}^{1} P_n(x)P_m(x)\,dx = \begin{cases} 0 & n \neq m \\ \frac{2}{2n+1} & n = m \end{cases}$

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سروش عباس قربان
شماره دانشجویی: ۹۶۱۰۱۹۴۴۷۰

ریاضیات مهندسی
تکلیف سری ششم

تاریخ تحویل: ۹۹/۹/۱۵

$x = \cos\theta \to dx = -\sin\theta\, d\theta$ $\xrightarrow[\text{در انتگرال}]{\text{با توجه به}}$ $\int_0^\pi \sin\theta\, P_n(\cos\theta)\, P_m(\cos\theta)\, d\theta = \begin{cases} 0 & n \neq m \\ \frac{2}{2n+1} & n = m \end{cases}$

در رابطه بین r و a و $f(\theta)$ استفاده کرده و در $\sin\theta\, P_m(\cos\theta)$ ضرب می‌کنیم. انتگرال $\int_0^\pi$ می‌گیریم.

$$\int_0^\pi \sum_{n=0}^{\infty} A_n a^n P_n(\cos\theta) P_m(\cos\theta) \sin\theta\, d\theta = \int_0^\pi f(\theta) \sin\theta\, P_m(\cos\theta)\, d\theta$$

$$\longrightarrow A_n = \frac{2n+1}{2a^n} \int_0^\pi f(\theta) P_n(\cos\theta) \sin\theta\, d\theta \quad , \quad u(r,\theta) = \sum_{n=1}^{\infty} A_n a^n P_n(\cos\theta)$$

مسئله خارجی:

$$\lim_{r \to \infty} u(r,\theta) = 0 \xrightarrow[A_n = 0]{u(r,\theta) = \sum_{n=1}^{\infty} \left(A_n r^n + \frac{B_n}{r^{n+1}}\right) P_n(\cos\theta)} u(r,\theta) = \sum_{n=1}^{\infty} \frac{B_n}{r^{n+1}} P_n(\cos\theta)$$

$$u(a,\theta) = f(\theta) = \sum \frac{B_n}{a^{n+1}} P_n(\cos\theta)$$

مشابه بخش قبلی در ادامه داریم:

$$\int_0^\pi \sum_{n=0}^{\infty} \frac{B_n}{a^{n+1}} P_m(\cos\theta) \sin\theta\, d\theta = \int_0^\pi f(\theta) P_m(\cos\theta) \sin\theta\, d\theta$$

$$\longrightarrow B_n = \frac{2n+1}{2} \times a^{n+1} \int_0^\pi f(\theta) P_n(\cos\theta) \sin\theta\, d\theta \quad , \quad u(r,\theta) = \sum_{n=1}^{\infty} \frac{B_n}{r^{n+1}} P_n(\cos\theta)$$

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