سروش مس فروش مشهد
شماره دانشجویی: 810198472

ریاضیات مهندسی
تمرین سری دوم

تاریخ تحویل: 99/8/13

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$$f(x)=\frac{1}{|x|+\pi} \qquad T=2\pi \qquad f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx=\int_{-\pi}^{0}\frac{1}{\pi-x}dx+\int_{0}^{\pi}\frac{1}{\pi+x}dx=-\ln(\pi-x)\Big|_{-\pi}^{0}+\ln(\pi+x)\Big|_{0}^{\pi}=2\ln 2$$

$$a_0=\frac{\ln 2}{\pi}$$

چیزی که صورت سوال می‌گوید مضرب آن را دارد نه کُل آن. تابع زوج است.

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{1}{|x|+\pi}\cos nx\,dx \longrightarrow a_n=\frac{2}{\pi}\int_{0}^{\pi}\frac{1}{x+\pi}\cos nx\,dx$$

$$I=\int_{0}^{\pi}\Big(\frac{1}{\pi+x}\sum_{n=1}^{\infty}\cos nx\Big)dx=\underbrace{\int_{0}^{\pi}\frac{1}{\pi+x}\cos x\,dx}_{\frac{\pi}{2}a_1}+\underbrace{\int_{0}^{\pi}\frac{1}{\pi+x}\cos 2x\,dx}_{\frac{\pi}{2}a_2}+\underbrace{\int_{0}^{\pi}\frac{1}{\pi+x}\cos 3x\,dx}_{\frac{\pi}{2}a_3}+\dots$$

$$\longrightarrow I=\frac{\pi}{2}(a_1+a_2+\dots)=\frac{\pi}{2}\sum_{n=1}^{\infty}a_n$$

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$$f(x)=\frac{\pi-x}{2} \quad 0<x<\pi \qquad a_0=\frac{1}{\pi}\int_{0}^{\pi}\Big(\frac{\pi-x}{2}\Big)dx=\frac{1}{\pi}\Big(\frac{\pi^2}{2}-\frac{\pi^2}{4}\Big)=\frac{\pi}{4}$$

$$a_n=\frac{2}{\pi}\int_{0}^{\pi}\Big(\frac{\pi-x}{2}\Big)\cos 2nx\,dx=\frac{1}{\pi}\int_{0}^{\pi}(\pi-x)\cos 2nx\,dx=\int_{0}^{\pi}\cos 2nx-\frac{1}{\pi}\int_{0}^{\pi}x\cos 2nx$$

$$\int_{0}^{\pi}x\cos 2nx\,dx \longrightarrow x=u \rightarrow dx=du, \quad \cos 2nx\,dx=dv \rightarrow v=\frac{1}{2n}\sin 2nx \longrightarrow \int_{0}^{\pi}x\cos 2nx\,dx=-\frac{1}{2n}\int_{0}^{\pi}\sin 2nx$$

$$=-\frac{1}{2n}\Big(-\frac{1}{2n}\cos 2nx\Big|_{0}^{\pi}\Big)=0 \longrightarrow a_n=0$$

$$b_n=\frac{1}{\pi}\int_{0}^{\pi}(\pi-x)\sin 2nx\,dx=-\frac{1}{\pi}\int_{0}^{\pi}(x-\pi)\sin 2nx\,dx \qquad u=x-\pi \rightarrow du=dx$$

$$\sin 2nx\,dx=dv \rightarrow v=-\frac{1}{2n}\cos 2nx$$

$$=-\frac{(x-\pi)\cos 2nx}{2n}\Big|_{0}^{\pi}+\int_{0}^{\pi}\frac{\cos 2nx}{2n}dx \rightarrow b_n=\frac{(x-\pi)\cos 2nx}{2n\pi}\Big|_{0}^{\pi}=\frac{1}{2n}$$

$$f(x)=\frac{\pi-x}{2}=\frac{\pi}{4}+\sum_{n=1}^{\infty}\frac{\sin 2nx}{2n} \quad \xrightarrow{x=1} \quad \frac{\pi-1}{2}=\frac{\pi}{4}+\underbrace{\Big(\frac{\sin 2}{2}+\frac{\sin 4}{4}+\frac{\sin 6}{6}+\dots\Big)}_{B}$$

از قبل داریم: $\overbrace{\Big(\sin 1+\frac{\sin 3}{3}+\frac{\sin 5}{5}+\dots\Big)}^{A}=\frac{\pi}{4}$ $\qquad \sum_{n=1}^{\infty}\frac{\sin n}{n}=A+B$

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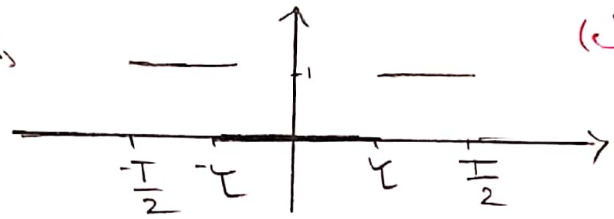
$$B=\frac{\pi-1}{2}-\frac{\pi}{4}=\frac{\pi}{4}-\frac{1}{2}\qquad A=\frac{\pi}{4}\ \rightarrow\ \sum_{n=1}^{\infty}\frac{\sin n}{n}=A+B=\frac{\pi-1}{2}$$

4 - (به سوال 3 در انتها پاسخ دادم)

الف)

$$f(x)=\begin{cases}0 & -\tau<x<\tau\\ 1 & |\tau|<x<|\frac{T}{2}|\end{cases}\ \xrightarrow{\text{رسم}}$$



$$C_n=\frac{1}{T}\int_{-T/2}^{T/2}f(x)\,e^{-jn\frac{2\pi}{T}x}\,dx$$

$$\rightarrow C_n=\frac{1}{T}\left(\int_{-\frac{T}{2}}^{-\tau}e^{-jn\frac{2\pi}{T}x}\,dx+\int_{\tau}^{\frac{T}{2}}e^{-jn\frac{2\pi}{T}x}\,dx\right)$$

$$=\frac{1}{T}\left(\frac{-1}{jn\frac{2\pi}{T}}\left(e^{-jn\frac{2\pi}{T}x}\Big|_{-\frac{T}{2}}^{-\tau}+e^{-jn\frac{2\pi}{T}x}\Big|_{\tau}^{\frac{T}{2}}\right)\right)$$

$$=\frac{-1}{jn2\pi}\left(e^{jn\frac{2\pi}{T}\tau}-e^{jn\pi}+e^{-jn\pi}-e^{-jn\frac{2\pi}{T}\tau}\right)$$

$$=\frac{-1}{jn2\pi}\left(-2j\sin n\pi+2j\sin\frac{2\pi}{T}n\tau\right)=-\frac{2j\sin(\frac{2\pi}{T}n\tau)}{jn2\pi}$$

$$\rightarrow C_n=\frac{-\sin(\frac{2\pi}{T}n\tau)}{n\pi}\ \xrightarrow{\frac{2\pi}{T}=\omega_0}\ f(x)=\sum_{n=-\infty}^{\infty}\frac{-\sin(\omega_0 n\tau)}{n\pi}e^{jn\omega_0 x}$$

و ثابت dc همان c_0 است که از لوپیتال دادن $n\to 0$ در c_n حاصل می‌شود

$$C_0=\lim_{n\to0}C_n=\lim_{n\to0}\frac{-\sin(\frac{2\pi}{T}n\tau)}{n\pi}=\lim_{n\to0}\frac{-\frac{2\pi}{T}n\tau}{n\pi}=\frac{-2\tau}{T}$$

ب)

$$\text{Parseval}:\ \frac{1}{T}\int_{-T/2}^{T/2}f^2(t)\,dt=\sum_{n=-\infty}^{\infty}|C_n|^2$$

$$\frac{1}{T}\int_{-T/2}^{T/2}f^2(t)\,dt=\frac{1}{T}\left(\int_{-T/2}^{-\tau}1^2\,dt+\int_{\tau}^{T/2}1^2\,dt\right)=\frac{1}{T}\left(T/2-\tau+T/2-\tau\right)=1-\frac{2\tau}{T}$$

$$\rightarrow\sum_{n=-\infty}^{\infty}|C_n|^2=1-\frac{2\tau}{T}$$

رابطه پارسوال در واقع بیانگر اصل بقای انرژی است. با از پارسوال نتیجه می‌گیریم که انرژی سیگنال پیوسته را از روی انرژی سیگنال در یک پریود برابر با مجموع انرژی جملات هارمونیک

5 - الف)

$f(x) = e^{a|x|} \rightarrow C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{a|x|} \cdot e^{-jnx} dx$

$= \frac{1}{2\pi} \left(\int_{-\pi}^{0} e^{(-a-jn)x} dx + \int_{0}^{\pi} e^{(a-jn)x} dx \right) \qquad \int_{-\pi}^{0} e^{(-a-jn)x} dx = \frac{1}{-a-jn} e^{(-a-jn)} \Big|_{-\pi}^{0}$

$= \frac{1}{-a-jn} (1 - e^{a\pi + jn\pi}) \qquad \int_{0}^{\pi} e^{(a-jn)x} dx = \frac{1}{a-jn} (e^{a\pi - jn\pi} - 1)$

$\rightarrow C_n = \frac{1}{2\pi} \left(\frac{e^{a\pi} \cdot e^{jn\pi} - 1}{a+jn} + \frac{e^{a\pi} \cdot e^{-jn\pi} - 1}{a-jn} \right) \qquad e^{jn\pi} = e^{-jn\pi} = (-1)^n$

$\rightarrow C_n = \frac{1}{2\pi} \left(\frac{(-1)^n e^{a\pi} - 1}{a+jn} + \frac{(-1)^n e^{a\pi} - 1}{a-jn} - \frac{1}{a+jn} - \frac{1}{a-jn} \right)$

$\rightarrow C_n = \frac{1}{2\pi} \left(\frac{(-1)^n 2a e^{\pi a}}{a^2+n^2} - \frac{2a}{a^2+n^2} \right) \rightarrow C_n = \frac{(-1)^n a e^{a\pi} - a}{\pi (a^2+n^2)}$

$\rightarrow f(x) = e^{a|x|} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n a e^{a\pi} - a}{\pi (a^2+n^2)} e^{jnx}$

ب)

$\pi e^{a|x|} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n a e^{a\pi} - a}{(a^2+n^2)} e^{jnx} \xrightarrow[a=1]{x=0} \pi = \sum_{n=-\infty}^{\infty} \frac{(-1)^n e^{\pi} - 1}{(n^2+1)}$

$\pi e^{|x|} = \sum_{n=-\infty}^{\infty} \underbrace{\frac{(-1)^n e^{\pi} - 1}{n^2+1}}_{C_n \text{ جدید}} e^{jnx}$

پارسوال : $\frac{1}{2\pi} \int_{-\pi}^{\pi} \pi^2 e^{2|x|} = \sum_{n=-\infty}^{\infty} \left[\frac{(-1)^n e^{\pi} - 1}{n^2+1} \right]^2$

$\rightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} \pi^2 e^{2|x|} dx = \frac{\pi}{2} \left(\int_{-\pi}^{0} e^{-2x} dx + \int_{0}^{\pi} e^{2x} dx \right) = \frac{\pi}{2} \left(-\frac{1}{2} e^{-2x} \Big|_{-\pi}^{0} + \frac{1}{2} e^{2x} \Big|_{0}^{\pi} \right)$

$= \frac{\pi}{2} \left(\frac{e^{2\pi} - 1}{2} + \frac{e^{2\pi} - 1}{2} \right) = \frac{\pi (e^{2\pi} - 1)}{2} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n e^{\pi} - 1}{(n^2+1)^2}$

سروش حسن فروش مشهد
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$$\sum_{n=-\infty}^{\infty}\left[\frac{(-1)^n e^{\pi}-1}{n^2+1}\right]^2 = (e^{\pi}-1)^2 + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n e^{\pi}-1}{n^2+1}\right]^2$$

$$\to \frac{\pi(e^{2\pi}-1)}{2} = (e^{\pi}-1)^2 + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n e^{\pi}-1}{n^2+1}\right]^2$$

$$\to \sum_{n=1}^{\infty}\left[\frac{(-1)^n e^{\pi}-1}{n^2+1}\right]^2 = \frac{\pi(e^{2\pi}-1)}{4} - \frac{(e^{\pi}-1)^2}{2}$$

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3 –

$e^{ax} \longrightarrow$ فوریه مختلط $\quad C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(a-jn)x}\,dx = \frac{1}{2\pi}\left(\frac{1}{a-jn}\, e^{(a-jn)x}\Big|_{-\pi}^{\pi}\right)$

$$= \frac{1}{2\pi}\left(\frac{1}{a-jn}\, e^{a\pi}\cdot e^{-jn\pi} - e^{-a\pi}\cdot e^{jn\pi}\right) \qquad e^{jn\pi} = \cos n\pi + i\sin n\pi = (-1)^n$$

$$e^{-jn\pi} = \cos n\pi - i\sin n\pi = (-1)^n$$

$$\to C_n = \frac{(-1)^n \sinh a\pi}{\pi(a-jn)} \qquad e^{ax} = \sum_{n=-\infty}^{\infty}\frac{(-1)^n \sinh a\pi}{\pi(a^2+n^2)}(a+jn)\,e^{jnx}$$

$$\xrightarrow{x=\pi} \pi e^{a\pi} = \sum_{n=-\infty}^{\infty}\frac{(-1)^{2n}\sinh a\pi}{(a^2+n^2)}(a+jn) \xrightarrow{(-1)^{2n}=1} \pi e^{a\pi} = \sum_{n=-\infty}^{\infty}\frac{\sinh a\pi}{a^2+n^2}(a+jn)$$

$$\underbrace{\sum_{n=-\infty}^{\infty}\frac{\sinh a\pi}{(a^2+n^2)}(a+jn)}_{\pi e^{a\pi}} = \frac{\sinh a\pi}{a} + 2a\sinh a\pi\sum_{n=1}^{\infty}\frac{1}{a^2+n^2}$$

$$\to \sum_{n=1}^{\infty}\frac{1}{a^2+n^2} = \frac{\pi e^{a\pi}}{2a\sinh a\pi} - \frac{1}{2a^2}$$

برای ساختن $\coth a\pi$ از ترتیب $a \to -a$ استفاده می‌کنم.

$$+\ \sum_{n=1}^{\infty}\frac{1}{a^2+n^2} = \frac{\pi e^{-a\pi}}{-2a\times -\sinh a\pi} - \frac{1}{2a^2}$$

$$2\sum_{n=1}^{\infty}\frac{1}{a^2+n^2} = \frac{\pi(e^{a\pi}+e^{-a\pi})}{2a\sinh a\pi} - \frac{1}{a^2} \longrightarrow \sum_{n=1}^{\infty}\frac{1}{a^2+n^2} = \frac{\pi\coth a\pi}{2a} - \frac{1}{2a^2}$$