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ریاضیات مهندسی
تکلیف سری چهارم

تاریخ تحویل: ۹۶/8/24

1- الف) خطی است - همگن است - مرتبه 2 است $\qquad \dfrac{\partial u}{\partial t} - 4\dfrac{\partial^2 u}{\partial x^2} = 0$

متغیر وابسته: u $\qquad$ متغیرهای مستقل: x, t

ب) خطی است - همگن است - مرتبه 3 است $\qquad x^2 \dfrac{\partial^3 R}{\partial y^3} - y^3 \dfrac{\partial^2 R}{\partial x^2} = 0$

متغیر وابسته: R $\qquad$ متغیرها مستقل: y, x

ج) غیرخطی است - همگن است - متغیرها مستقل: u, v $\qquad \left(\dfrac{\partial z}{\partial u}\right)^2 + \left(\dfrac{\partial z}{\partial v}\right)^2 = 1$

متغیر وابسته: z

2- الف) $\dfrac{\partial u}{\partial x} + 3\dfrac{\partial u}{\partial t} = 0$ $\qquad$ حدس جواب $u(x,t) = e^{ax+bt} \rightarrow \dfrac{\partial u}{\partial x} = a e^{ax+bt}$

$\dfrac{\partial u}{\partial t} = b e^{ax+bt}$

$$\xrightarrow[\text{PDE}]{\text{جایگذاری در}} a e^{ax+bt} + 3b e^{ax+bt} = 0 \rightarrow (a+3b)\underbrace{e^{ax+bt}}_{\neq 0} = 0 \rightarrow a+3b=0$$

$$\rightarrow a = -3b \rightarrow u(x,t) = e^{-3bx+bt} = e^{b(t-3x)} \rightarrow u(x,t) = \alpha e^{b(t-3x)} = F(t-3x)$$

ب) فرم F سازگار است $\rightarrow$ $u(x,t) = F(t-3x) \xrightarrow{u(0,t) = \sin t} u(0,t) = F(t) = 4\sin t$

$$\xrightarrow{t \to (t-3x)} u(x,t) = F(t-3x) = 4\sin(t-3x) \xrightarrow{\text{تست جواب}} \frac{\partial}{\partial x}(4\sin(t-3x)) + 3\frac{\partial}{\partial t}(4\sin(t-3x))$$

$$= -12\cos(t-3x) + 12\cos(t-3x) = 0 \checkmark$$

3- $x u_x - y u_y = 0 \rightarrow u(x,y) = X(x)Y(y)$ $\qquad \dfrac{\partial u}{\partial x} = X'Y$ $\qquad \dfrac{\partial u}{\partial y} = XY'$

$$\rightarrow xX'Y - yY'X = 0 \xrightarrow{\div XY} x\frac{X'}{X} - y\frac{Y'}{Y} = 0 \rightarrow x\frac{X'}{X} = y\frac{Y'}{Y} = cte \begin{cases} k^2 & (a \\ 0 & (b \\ -k^2 & (c \end{cases}$$

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تمرین سری دوم

تاریخ تحویل: ۹۹/۸/۲۴

a) $x\dfrac{X'}{X}=k^2 \rightarrow X'-\dfrac{k^2}{x}X=0 \quad \mu_{(x)}=e^{\int -\frac{k^2}{x}dx}=x^{-k^2}$

$\rightarrow X=\alpha x^{+k^2}$, $y\dfrac{Y'}{Y}=k^2 \rightarrow Y'-\dfrac{k^2}{y}Y=0 \rightarrow \mu_{(y)}=e^{\int -\frac{k^2}{y}dy}=y^{-k^2}$

$\rightarrow Y=\beta y^{+k^2} \rightarrow U=X_{(x)}Y_{(y)}=c\,x^{+k^2}y^{+k^2}$

b) $x\dfrac{X'}{X}=0 \rightarrow X'=0 \rightarrow X=\alpha \qquad y\dfrac{Y'}{Y}=0 \rightarrow Y'=0 \rightarrow Y=\beta$

$\rightarrow U=XY=\alpha\beta=c$ — چون ارزش خاصی ندارد

c) $x\dfrac{X'}{X}=-k^2 \rightarrow X'+\dfrac{k^2}{x}X=0 \quad \mu_{(x)}=e^{\int\frac{k^2}{x}dx}=x^{k^2} \rightarrow X=\alpha x^{-k^2}$

$y\dfrac{Y'}{Y}=-k^2 \rightarrow Y'+\dfrac{k^2}{y}Y=0 \quad \mu_{(y)}=e^{\int\frac{k^2}{y}dy}=y^{k^2} \rightarrow Y=\beta y^{-k^2}$

$\rightarrow U(x,y)=\alpha\beta\, x^{-k^2}y^{-k^2} \xrightarrow[\alpha\beta=c]{} c\,x^{-k^2}y^{-k^2}$

C2-4

$$\begin{cases}\dfrac{\partial^2 U}{\partial x^2}-\dfrac{1}{k}\dfrac{\partial U}{\partial t} \\ U(0,t)=U(1,t)=100 \\ U(x,0)=0\end{cases}$$

جهت سهولت $\dfrac{1}{k}=\dfrac{1}{c^2}$

$$\begin{cases}\dfrac{\partial^2 w}{\partial x^2}-\dfrac{1}{c^2}\dfrac{\partial w}{\partial t}+\dfrac{\partial^2 U_s}{\partial x^2}=0 \\ w(0,t)+U_s(0)=100 \\ w(1,t)+U_s(1)=100 \\ w(x,0)+U_s(x)=f(x)=0\end{cases}$$

تفکیک $\longrightarrow$

$$\begin{cases}\dfrac{\partial^2 w}{\partial x^2}-\dfrac{1}{c^2}\dfrac{\partial w}{\partial t}=0 \\ w(0,t)=0 \\ w(1,t)=0 \\ w(x,0)=0-U_s(x)\end{cases}$$

$$\begin{cases}\dfrac{\partial^2 U_s}{\partial x^2}=0 \rightarrow U_s=Ax+B \\ U_s(0)=100 \rightarrow B=100 \\ U_s(1)=100 \rightarrow A=0\end{cases} \rightarrow U_s(x)=100$$

$$\rightarrow \begin{cases}\dfrac{\partial^2 w}{\partial x^2}-\dfrac{1}{c^2}\dfrac{\partial w}{\partial t} \\ w(0,t)=0 \\ w(1,t)=0 \\ w(x,0)=-100\end{cases}$$

$$w(x,t)=\sum_{n=1} D_n e^{-\left(\frac{n\pi}{c}\right)^2}\sin\left(\frac{n\pi}{1}x\right)$$

$$D_n=2\int_0^1 -100\sin(n\pi x)\,dx=\frac{200}{n\pi}\cos(n\pi x)\Big|_0^1=\frac{200\left((-1)^n-1\right)}{n\pi}$$

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$$D_n = \begin{cases} 0 & \text{زوج } n \\ \dfrac{-400}{n\pi} & \text{فرد } n \end{cases}$$

$$u(x,t) = w(x,t) + u_s(x)$$

$$\longrightarrow u(x,t) = \sum_{\text{فرد } n} \frac{-400}{n\pi} e^{-n^2\pi^2 k} \sin(n\pi x) + 100$$

6- معادله غذار اسکر ممکن است!

$$\frac{\partial u}{\partial t} = a^2 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right)$$

$$u(1,t) = 0, \quad u(r,0) = f(r)$$

$$\frac{1}{a^2}\frac{\partial u}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right) \qquad w(r,t) = r\,u(r,t)$$

$$\longrightarrow u(r,t) = \frac{1}{r} w(r,t) \longrightarrow \frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial w}{\partial t}$$

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial w}{\partial r} - \frac{1}{r^2} w \longrightarrow \frac{1}{a^2}\frac{1}{r}\frac{\partial w}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r} - w\right)$$

$$\longrightarrow \frac{1}{a^2 r}\frac{\partial w}{\partial t} = \frac{1}{r^2}\left(\frac{\partial w}{\partial r} + r\frac{\partial^2 w}{\partial r^2} - \frac{\partial w}{\partial r}\right) \longrightarrow \frac{1}{a^2 r}\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} \times \frac{1}{r}$$

$$\longrightarrow \frac{1}{a^2}\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} \qquad w(r,t) = R(r)T(t) \longrightarrow \frac{\partial w}{\partial t} = RT' \quad \frac{\partial^2 w}{\partial r^2} = R''T$$

$$\frac{1}{a^2} RT' = R''T \xrightarrow{\div\, w = RT} \frac{1}{a^2}\frac{T'}{T} = \frac{R''}{R} = cte \qquad \frac{1}{a^2}\frac{T'}{T} = \frac{R''}{R} = -k^2$$

$$\longrightarrow R'' + k^2 R = 0 \longrightarrow r^2 + k^2 = 0 \longrightarrow r = \pm jk \longrightarrow R(r) = A\cos kr + B\sin kr$$

$$T' + k^2 a^2 T = 0 \longrightarrow \mu(t) = e^{\int k^2 a^2 dt} = e^{k^2 a^2 t} \longrightarrow T = Ce^{-k^2 a^2 t}$$

$$w(r,t) = (A\cos kr + B\sin kr)\,Ce^{-k^2 a^2 t} \longrightarrow u(r,t) = \left(\frac{A\cos kr + B\sin kr}{r}\right)Ce^{-k^2 a^2 t}$$

با توجه به این که $r=0$ در ناحیه پاسخ باید کراندار باشد پس طبیعتاً $A=0$ تا با ثابت مبهم $\frac{1}{0}$ برنخوریم.

$$\longrightarrow u(r,t) = \left(\frac{B\sin kr}{r}\right)Ce^{-k^2 a^2 t} \qquad u(1,t) = 0 \longrightarrow B\sin k = 0 \longrightarrow k = n\pi \quad n = 1,2,\dots$$

$$\longrightarrow u_n(r,t) = D\frac{\sin n\pi r}{r} e^{-n^2\pi^2 a^2 t}$$

$$\to U(r,t) = \sum_{n=1}^{\infty} \frac{D \sin n\pi r}{r} e^{-(n\pi c)^2 t}$$

-7

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - ku \\ u(0,t) = u(l,t) = 0 \\ u(x,0) = f(x) \quad u_t(x,0) = g(x) \end{cases}$$

$$U = X(x)\,T(t) \qquad \frac{\partial^2 u}{\partial t^2} = X T'' \qquad \frac{\partial^2 u}{\partial x^2} = X'' T$$

$$\to X T'' = c^2 X'' T - k X T \xrightarrow{\div U = XT} \frac{T''}{T} = c^2 \frac{X''}{X} - k = cte$$

$$\to \frac{1}{c^2}\frac{T''}{T} + \frac{k}{c^2} = \frac{X''}{X} = -p^2 \to X'' + p^2 X = 0 \qquad T'' + (c^2p^2 + k)T = 0$$

$$X'' + p^2 X = 0 \to r^2 + p^2 = 0 \to r = \pm jp \to X(x) = A \cos px + B \sin px$$

$$u(0,t) = 0 \to X(0) = 0 \to A = 0, \quad u(l,t) = 0 \to B \sin pl = 0 \to pl = n\pi$$

$$\to p = \frac{n\pi}{l} \quad n = 1,2,\dots \to X_n(x) = B_n \sin \frac{n\pi}{l} x$$

$$T'' + (c^2p^2 + k)T = 0 \to r^2 + c^2p^2 + k = 0 \to r = \pm j\sqrt{c^2p^2 + k}$$

$$\to T = C \cos(\sqrt{c^2p^2 + k}\; t) + D \sin(\sqrt{c^2p^2 + k}\; t)$$

چون p برحسب n متغیر است پس

$$T_n = C_n \cos\left(\sqrt{\frac{c^2n^2\pi^2}{l^2} + k}\; t\right) + D_n \sin\left(\sqrt{\frac{c^2n^2\pi^2}{l^2} + k}\; t\right)$$

$$\to U(x,t) = \sum_{n=1}^{\infty} \left[a_n \cos\left(\sqrt{\frac{c^2n^2\pi^2}{l^2} + k}\; t\right) + b_n \sin\left(\sqrt{\frac{c^2n^2\pi^2}{l^2} + k}\; t\right) \right] \sin \frac{n\pi}{l} x$$

چون شرایط مرزی زمانی صریح بیان نشده‌اند از این جلوتر نمی توانم کار را ادامه دهم و داشت لذا که:

$$B_n C_n = a_n \qquad B_n D_n = b_n$$