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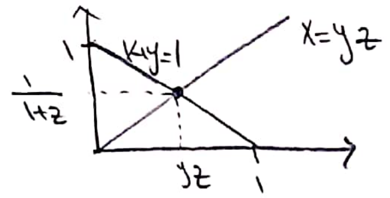
آمار و احتمال مهندسی
تکلیف سری سوم

تاریخ تحویل :

4-

$$f_{XY}(x,y)=\begin{cases}24xy & x\geq 0,\ y\geq 0,\ x+y\leq 1\\ 0 & \text{o.w}\end{cases}$$



$Z=\frac{X}{Y}$ $F_Z(z)=\Pr\{Z\leq z\}=\Pr\{\frac{X}{Y}\leq z\}=\Pr\{X\leq Yz,\ Y\geq 0\}$

$$F_Z(z)=\int_0^{\frac{1}{1+z}}\int_0^{yz}24xy\,dx\,dy \xrightarrow[z]{\text{مشتق نسبت به}} f_Z(z)=\int_0^{\frac{1}{1+z}}y\cdot 24(yz)y\,dy \quad (z\geq 0)$$

$$=\int_0^{\frac{1}{1+z}}24y^3z\,dy=\frac{6z}{(1+z)^4}\quad (z\geq 0)\rightarrow f_Z(z)=\frac{6z}{(1+z)^4}u(z)\ \text{all } z$$

6- الف)

$$\begin{matrix}Z=X+Y\\ W=X\end{matrix}\rightarrow J(x,y)=\det\begin{bmatrix}1&0\\1&1\end{bmatrix}=+1$$

$$\begin{matrix}z=x+y\rightarrow y=z-x\\ w=x\end{matrix}\qquad f_{WZ}(w,z)=f_{XZ}(x,z)=f_{XY}(z-x,x)$$

$$f_{XY}(x,y)=f_X(x)f_Y(y)=\lambda^2e^{-\lambda(x+y)}\rightarrow f_{XZ}(x,z)=\lambda^2e^{-\lambda z}\quad z\geq x\geq 0$$

$$f_Z(z)=f_X(x)*f_Y(y)=\int_{-\infty}^{\infty}f_X(z-y)f_Y(y)dy\Rightarrow f_Z(z)=\int_0^z\lambda^2e^{-\lambda z}dy=\lambda^2ze^{-\lambda z}\quad z\geq 0$$

ب)

$$\rightarrow f_Z(z)=\lambda^2ze^{-\lambda z}u(z)$$

$$F_W(w)=\Pr\{W\leq w\}=\Pr\{\frac{X}{Y}\leq w\}=\Pr\{Y\geq\frac{X}{w}\}=\int_0^{\infty}\int_{\frac{x}{w}}^{\infty}e^{-\lambda(x+y)}dy\,dx$$

$$=\frac{w}{w+1}\rightarrow f_W(w)=\frac{w+1-w}{(w+1)^2}=\frac{1}{(w+1)^2}\quad w\geq 0\rightarrow f_W(w)=\frac{u(w)}{(1+w)^2}$$

$$\begin{matrix}Z=X+Y\rightarrow x=z-y\\ W=\frac{X}{Y}\rightarrow y=\frac{x}{w}\end{matrix}\qquad J=\begin{vmatrix}1&1\\ \frac{1}{y}&-\frac{x}{y^2}\end{vmatrix}\rightarrow |J|=\frac{x+y}{y^2}\qquad f_{ZW}(z,w)=\frac{f_{XY}(z-y,\frac{x}{w})}{\left|\frac{x+y}{y^2}\right|}$$

$$=\frac{\lambda^2z}{(1+w)^2}e^{-\lambda z}\quad z\geq x\geq 0\rightarrow f_{ZW}(z,w)=f_Z(z)f_W(w)\rightarrow$$ مستقل اند.

تاریخ تحویل :

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ب)

$Z = \min(X,Y) \quad W = \max(X,Y) - \min(X,Y) \quad V = \max(X,Y)$

$$F_{ZV}(z,v) = P_r\{\overbrace{\min(X,Y) \leq z}^{A}, \overbrace{\max(X,Y) \leq v}^{B}\}$$

$$P(A \cap B) = P(B) - P(B \cap \bar{A}) = P_r\{\max(X,Y) \leq v\} - P_r\{\max(X,Y) \leq v, \min(X,Y) > z\}$$

$$= P_r\{X \leq v, Y \leq v\} - P_r\{z < X \leq v, z < Y \leq w\} = F_{XY}(v,v) - F_{XY}(v,v) + F_{XY}(z,v)$$

$$+ F_{XY}(v,z) + F_{XY}(z,z) \longrightarrow F_{ZV}(z,v) = \begin{cases} F_{XY}(z,v) + F_{XY}(v,z) - F_{XY}(z,z) & z \leq v \\ F_{XY}(v,v) & z > v \end{cases}$$

$$f_{ZV}(z,v) = \frac{\partial^2}{\partial z \partial v} F_{ZV}(z,v) = \begin{cases} f_{XY}(z,v) + f_{XY}(v,z) & z \leq v \\ 0 & z > v \end{cases}$$

$$\longrightarrow f_{ZV}(z,v) = \begin{cases} 2\lambda^2 e^{-\lambda(z+v)} & 0 \leq z \leq v \\ 0 & z > v \end{cases}$$

$$\begin{matrix} Z = V - W \\ W = V - Z \end{matrix} \Longrightarrow f_{ZW}(z,v) = f_{ZV}(v-w, w+z) = f_{ZV}(z, w+z)$$

$$= 2\lambda^2 e^{-\lambda(z+w+z)} = 2\lambda^2 e^{-\lambda(2z+w)} \quad (z>0, w>0)$$

$$f_Z(z) = \int_0^\infty 2\lambda^2 e^{-\lambda(2z+w)} dw = 2\lambda e^{-\lambda(2z)} \longrightarrow f_Z(z) \sim E(2\lambda)$$

$$f_W(w) = \int_0^\infty 2\lambda^2 e^{-\lambda(2z+w)} dz = \lambda e^{-\lambda w}$$

$$f_{ZW}(z,w) = f_Z(z) f_W(w) \longrightarrow \text{مستقل}$$

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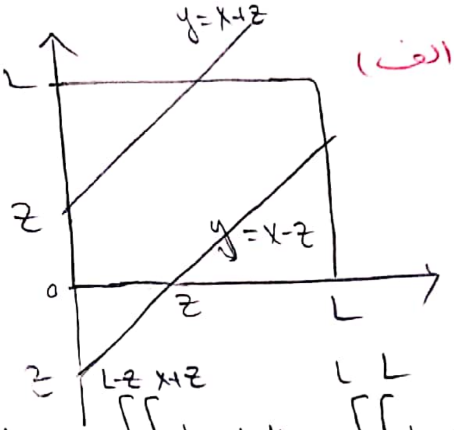
تاریخ تحویل:

2 - الف)

X: مکان آمبولانس

Y: مکان حادثه

$F_Z = \Pr\{|X-Y| \leq z\}$



$|X-y| \leq z \rightarrow y-z \leq x \leq y+z$

$$\Pr\{|X-Y|\leq z\} = \int_0^{L-z}\int_y^{y+z} \frac{1}{L^2}\,dx\,dy + \int_{L-z}^{L}\int_y^{L} \frac{1}{L^2}\,dx\,dy + \int_0^{L-z}\int_x^{x+z} \frac{1}{L^2}\,dy\,dx + \int_{L-z}^{L}\int_x^{L} \frac{1}{L^2}\,dy\,dx$$

$$= \frac{1}{L^2}\left(\int_0^{L-z}\int_y^{y+z} dx\,dy + \int_{L-z}^{L}\int_y^{L} dx\,dy + \int_0^{L-z}\int_x^{x+z} dy\,dx + \int_{L-z}^{L}\int_x^{L} dy\,dx \right)$$

$$\rightarrow F_Z(z) = \frac{2zL - z^2}{L^2} \rightarrow f_Z(z) = \frac{dF_Z(z)}{dz} = \frac{2L-2z}{L^2} \rightarrow f_Z(z) = \begin{cases} \frac{2(L-z)}{L^2} & 0 \leq z \leq L \\ 0 & \text{o.w} \end{cases}$$

ب)

$$E\{Z\} = \frac{1}{L^2}\int_0^L 2Lz - 2z^2\,dz = \frac{1}{L^2}\left(Lz^2 - \frac{2z^3}{3}\Big|_0^L\right) = \frac{L}{3}$$

$$V = \frac{S}{t} \rightarrow t = \frac{L}{3V}$$

10 - الف)

$R = \sqrt{x^2+y^2}$ $\quad \Phi = \tan^{-1}\left(\frac{Y}{X}\right)$ $\quad$ (قطبی) مختصات : $x = r\cos\varphi$ $\quad y = r\sin\varphi$

$$\begin{cases} R = \sqrt{x^2+y^2} \rightarrow x = \sqrt{R^2-y^2} \\ \Phi = \tan^{-1}\left(\frac{Y}{X}\right) \quad y = x\tan\varphi \end{cases} \rightarrow J = \det\begin{bmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{bmatrix}$$

$$J = \frac{x^2+y^2}{r^3} = \frac{1}{r} \rightarrow f_{R\Phi}(r,\varphi) = \frac{f_{XY}(\sqrt{r^2-y^2}, \sqrt{r^2-y^2}\tan\varphi)}{\frac{1}{r}}$$

$$\rightarrow f_{R\Phi}(r,\varphi) = \frac{f_{XY}(x,y)}{\frac{1}{r}} \xrightarrow[y=r\sin\varphi]{x=r\cos\varphi} f_{R\Phi}(r,\varphi) = r\, f_{XY}(r\cos\varphi, r\sin\varphi)$$

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تاریخ تحویل: ۹۹/۹/۱۰

ب)

$$f_X(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma^2}} \qquad f_Y(y)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-y^2}{2\sigma^2}}$$

$$f_{XY}(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{(x^2+y^2)}{2\sigma^2}} \qquad f_{R\Phi}(r,\varphi)=r\,f_{XY}(r\cos\varphi, r\sin\varphi)$$

$$\rightarrow f_{R\Phi}(r,\varphi)=\frac{r}{2\pi\sigma^2}e^{-\frac{r^2}{2\sigma^2}} \qquad f_R(R)=\int_0^{2\pi}\frac{r}{2\pi\sigma^2}e^{\frac{-r^2}{2\sigma^2}}d\varphi$$

$$=\frac{r}{\sigma^2}e^{\frac{-r^2}{2\sigma^2}} \qquad f_\Phi(\varphi)=\int_0^{\infty}\frac{r}{2\pi\sigma^2}e^{\frac{-r^2}{2\sigma^2}}dr=\frac{-\sigma^2}{2\pi\sigma^2}e^{\frac{-r^2}{2\sigma^2}}\Big|_0^{\infty}$$

$$=\frac{1}{2\pi} \qquad f_\Phi(\varphi)f_R(r)=\frac{1}{2\pi}\times\frac{r}{\sigma^2}e^{\frac{-r^2}{2\sigma^2}}=\frac{r}{2\pi\sigma^2}e^{\frac{-r^2}{2\sigma^2}}=f_{R\Phi}(r,\varphi)$$

$\rightarrow R,\Phi$ مستقل $\qquad f_R(r)=\frac{r}{\sigma^2}e^{\frac{-r^2}{2\sigma^2}}u(r) \qquad f_\Phi(\varphi)=\frac{1}{2\pi}$

$$\Phi\sim U(0,2\pi)$$

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۱۴ - الف)

$$Z=X+Y\rightarrow E\{Z\}=E\{X\}+E\{Y\}=\eta_X+\eta_Y=0-1=-1$$

نرمال خواهد بود

$$\sigma_Z^2=\sigma_X^2+\sigma_Y^2-2\sigma_{XY} \qquad r_{XY}=-\frac{1}{2}=\frac{\sigma_{XY}}{\sigma_X\sigma_Y}=\frac{\sigma_{XY}}{2}$$

$$\sigma_{XY}=-1\rightarrow\sigma_Z^2=1+4-2=3$$

$$\rightarrow Z\sim N(-1,3)\longrightarrow P_r\{X+Y<-1\}=P_r\{Z<-1\}=F_Z(-1)$$

$$=G\left(\frac{-1-(-1)}{3}\right)=G(0)=\frac{1}{2}$$

ب) $Z=aX+Y \quad W=X+2Y$ شرط استقلال: $r_{ZW}=0\rightarrow E\{ZW\}=E\{Z\}E\{W\}$

$$E\{Z\}=aE\{X\}+E\{Y\}=-1 \qquad E\{W\}=E\{X\}+2E\{Y\}=-2 \qquad E\{Z\}E\{W\}=2$$

$$ZW=aX^2+(2a+1)XY+2Y^2\longrightarrow E\{ZW\}=aE\{X^2\}+(2a+1)E\{XY\}+2E\{Y^2\}$$

$$E\{X^2\}=\sigma_X^2+\eta_X^2=1 \qquad E\{Y^2\}=\sigma_Y^2+\eta_Y^2=5 \qquad E\{XY\}=\sigma_{XY}=-1$$

$$\longrightarrow a-1-2a+10=2\rightarrow -a=-7\longrightarrow a=7$$

-8

$$\left.\begin{array}{l} X+Y=n \\ Z=X-Y \end{array}\right\} \rightarrow 2X=Z+n \rightarrow Z=2X-n$$

$$P_Z(k)=\Pr\{Z=k\}=\Pr\{2X-n=k\}=\Pr\{X=\tfrac{n+k}{2}\}$$

$$\Pr\{X=\tfrac{n+k}{2}\}=\binom{n}{\frac{n+k}{2}}p^{\frac{n+k}{2}}(1-p)^{n-\frac{n+k}{2}}$$

$$\rightarrow P_Z(k)=\binom{n}{\frac{n+k}{2}}p^{\frac{n+k}{2}}(1-p)^{\frac{n-k}{2}}$$

با توجه به این که $2X=Z+n$ ، Z بعلاوه n زوج است ، پس باید: $k=-n,-n+2,\dots,n-2,n$

باشد تا آن عبارت زوج باشد.

$$E\{Z\}=E\{2X-n\}=2E\{X\}-n=2np-n=n(2p-1)$$

$$E\{Z^2\}=E\{(2X-n)^2\}=4E\{X^2\}+n^2-4nE\{X\} \qquad E\{X^2\}=np(1-p)$$

$$\rightarrow E\{Z^2\}=4np(1-p)+n^2-4n^2p \qquad \sigma_Z^2=E\{Z^2\}-\eta_Z^2$$