

The project

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1 introduction to the approaches

We use 2 approaches to solve this problem. The first one is iteration method and the second one is lp-based method. In iteration method, we update our answer, and continue it until it does not change considerably. In the lp-based method, we use some linear inequalities to limit our answer, and then, find the answer by a linear solver.

2 Iteration method for question 1

In this method, we consider an array v of length 110 that for $0 \leq i \leq 99$, $v[i] = 0$, and for $100 \leq i \leq 105$, $v[i] = i$. Then, in each step, for $0 \leq i \leq 99$, we update $v[i]$ by formula $v[i] = \max(i, (x[i+3] + x[i+5] + x[i+6] + 3 * x[0]) / 6)$, when $x[] = v[]$ at the beginning of the step. The amount $(x[i+3] + x[i+5] + x[i+6] + 3 * x[0]) / 6$ refers to the estimated profit we will have if continue to the game when we have i dollars. We do this algorithm until the amount of update of $v[]$ becomes very small. In other words, we do this algorithm until $\sum |x[i] - v[i]| \leq \epsilon$, that ϵ is a very small number.

3 Iteration method for question 2

The iterative method for the second question is like this method for the first equation, with some trivial differences. We consider an array v of length 125 that for $0 \leq i \leq 99$, $v[i] = 0$, and for $100 \leq i \leq 115$, $v[i] = i$. Then, in each step, for $0 \leq i \leq 99$, we update $v[i]$ by formula $v[i] = \max(i, (4 * x[i+4] + 2 * x[i+9] + x[i+6] + 9 * x[0]) / 16)$, when $x[] = v[]$ at the beginning of the step. We do this algorithm until the amount of update of $v[]$ becomes very small.

4 lp-based method for the question 1

As we said before, we use inequalities in this method to find the answer. Again, we define an array v of length 110 that all of its cells are 0. We want that each cell of v has its largest amount. In other words, we want that each cell cannot be updated (like the first section). Therefore, for each $0 \leq i \leq 99$, we should have this inequality: $v[i] \geq (v[i+3] + v[i+5] + v[i+6] + 3 * v[0]) / 6$. We add all of these inequalities to a linear model and then find the maximum answer for $v[0]$. For the second question, the inequalities are $v[i] \geq (4 * v[i+4] + 2 * v[i+9] + v[i+6] + 9 * v[0]) / 16$.