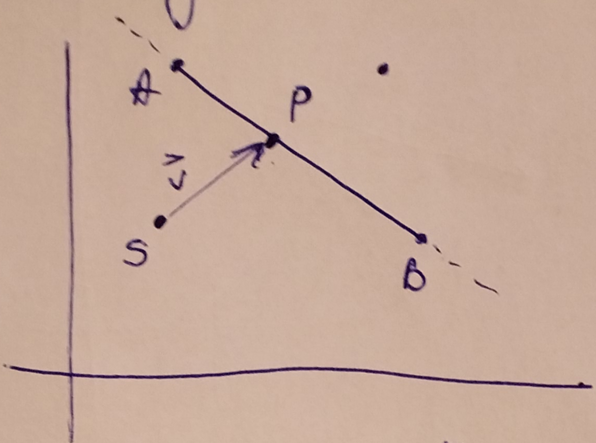
Line segment with line segment intersection



We first rewrite them as parametric equations

$$\left.\begin{aligned} f_1 &= (B-A)t_1 + A \\ f_2 &= \vec{v}t_2 + S \end{aligned}\right\} \quad \begin{aligned} &\vec{AB}t_1 + A \\ &\vec{v}t_2 + S \end{aligned}$$

these expand to:

$$\begin{bmatrix} AB_x \\ AB_y \end{bmatrix} t_1 + \begin{pmatrix} A_x \\ A_y \end{pmatrix} \quad \Bigg| \quad \begin{bmatrix} v_x \\ v_y \end{bmatrix} t_2 + \begin{bmatrix} S_x \\ S_y \end{bmatrix}$$

To find an intersection, we set them equal to eachother

$$\underbrace{\begin{bmatrix} AB_x \\ AB_y \end{bmatrix}}_{\vec{AB}} t_1 + \underbrace{\begin{pmatrix} A_x \\ A_y \end{pmatrix}}_{A} = \underbrace{\begin{bmatrix} v_x \\ v_y \end{bmatrix}}_{\vec{v}} t_2 + \underbrace{\begin{bmatrix} S_x \\ S_y \end{bmatrix}}_{S}$$

$$\vec{AB}t_1 - \vec{v}t_2 = S - A$$

$$\underbrace{\begin{bmatrix} AB_x & v_x \\ AB_y & v_y \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} t_1 \\ -t_2 \end{bmatrix}}_{t} = \vec{AS}$$

we inverse the matrix in order to solve for t

$$\begin{bmatrix} t_1 \\ -t_2 \end{bmatrix} = M^{-1}\vec{AS}$$

M^{-1} in 2D is

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{AB_x v_y - AB_y v_x}\begin{bmatrix} v_y & -v_x \\ -AB_y & AB_x \end{bmatrix}\begin{bmatrix} AS_x \\ AS_y \end{bmatrix} = \begin{bmatrix} t_1 \\ -t_2 \end{bmatrix}$$

$$\frac{1}{AB_x v_y - AB_y v_x}\begin{bmatrix} (v_y)(AS_x) - (v_x)(AS_y) \\ (-AB_y)(AS_x) + (AB_x)(AS_y) \end{bmatrix} = \begin{bmatrix} t_1 \\ -t_2 \end{bmatrix}$$

$$\Rightarrow t_1 = \frac{(v_y)(AS_x) - (v_x)(AS_y)}{(AB_x)(v_y) - (AB_y)(v_x)}$$

To get point of intersection; we substitute (t_2 easier)

$$-t_2 = \frac{(-AB_y)(AS_x) + (AB_x)(AS_y)}{(AB_x)(v_y) - (AB_y)(v_x)}$$

$$P = \vec{v}\,(-t_2) + S$$