Exam 12 August 2015, questions

Basiswiskunde (Technische Universiteit Eindhoven)

TECHNISCHE UNIVERSITEIT EINDHOVEN
Faculteit Wiskunde en Informatica

Exam Basiswiskunde (2DL00)
on 12 August 2015, 18.00-21.00 hour.

The results of the problems have to be formulated in a clear way and also readable written on the paper(s).

The use of a computer, graphical calculator, or a programmable calculator is NOT ALLOWED.
A calculator which has for every letter out of the alphabet a key or a combination of keys is NOT ALLOWED.
The use of a book NOT ALLOWED.
During the exam, you DON'T HAVE A MOBILE TELEPHONE in your direct neighbourhood.
The order in which questions will be resolved is entirely free.
This exam consists of **9** problems.

Note about natural logarithm:
With: ln is meant the same as: log, so $\ln(x) = \log(x)$, and $\exp(x) = e^{(x)}$. Further is meant with $\arctan(x) = \tan^{-1}(x)$ the inverse of the function $\tan(x)$.

1. Solve the inequality: $1 + 2\sqrt{(1-x)} \geq -x + 2$.

2. Sketch in the flat plane the set of point (x, y) that satisfy:

   - the inequality: $x^2 + y^2 \leq 2y - 2x$

   and

   - the inequality: $0 \leq y - x - 2$.

   So the points (x, y), that are asked to sketch,
   have to satisfy both inequalities.

   See next page!

3. Consider the function f, defined by $f(x) = -\sqrt{(x - 2)}$ for $x \geq 2$.

   a. Show that the given function f is one-to-one.

   b. Determine the domain $\mathrm{D}(f^{(-1)})$ and the range $\mathrm{R}(f^{(-1)})$ of the inverse function $f^{(-1)}$.

   c. Determine the inverse function $f^{(-1)}$.

4. Consider the curve K, given by the equation:

$$e^{(x^2 + y^2)} - e^{(2x - 2y)} = 0.$$

   and the point $P = (0, -2)$ on the curve K.

   a. Determine a linear equation of the tangent line to the curve K at the point P.

   b. Calculate a linear equation of the line through the point P perpendicular to the tangent line out of part (a).

5. Consider the function f, defined by $f(x) = \sqrt{(x - 1)}$ for $x \geq 1$. The linearisation, the Taylor polynomial of order 1 around $a = 2$, is denoted by $L(x)$.

   a. Determine the linearisation $L(x)$.

   b. Give, using L, an approximation for $f(2.04)$.

   c. Give an expression for the error $E(2.04)$, using $f^{(2)}$ such that $f(2.04) = L(2.04) + E(2.04)$. Is the error $E(2.04)$ positive or negative? ($f^{(2)}$: the second order derivative of f)

6. Calculate

$$\lim_{x \to 0} \frac{(1 + x^2) - (1 + x^2)^2 + \log(1 + x^2)}{6x(x - \sin(x))}.$$

See next page!

7. Let $c \in \mathbb{R}$ be a constant.

   a. Show that
   $$\arctan(e^{(x)}) + \arctan(e^{(-x)}) = c.$$

   b. Determine c.

8. Evaluate the following integrals

   a.
   $$\int_{-2}^{2} \frac{|\, y \,|}{1 + y^2} \, \mathrm{d}y$$

   b.
   $$\int_{-1}^{1} \frac{(y^5 + y^4 + y^3 + y^2 + y + 1)}{(y^2 + 1)} \, \mathrm{d}y$$

9. Simplify the expression
   $$\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_{\exp(-x)}^{1} \frac{t}{1 + \log(t)} \mathrm{d}t\right)$$
   without calculating the integral.

   No further questions!

The following number of points can be earned for each part of the problems:

1 : 3	4 a : 3	6 : 4	9 : 3
2 : 3	4 b : 2	7 a : 3	
3 a : 2	5 a : 2	7 b : 1	
3 b : 2	5 b : 2	8 a : 3	
3 c : 2	5 c : 2	8 b : 3	

The result of this exam is decided by dividing the total of the scored points by 4 and will be rounded to one decimal.