1. En la ecuación de transferencia de calor utilizada en clase

$$\frac{d}{dx}\left(\frac{d}{dx}T\right) = -Q$$

Modifique toda la formulación de la aplicación del MEF, considerando que Q ya no es constante, sino que corresponde a la siguiente función:

$$Q = Q(x) = x^2 - 3x$$

Mallado:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = -Q(x), \qquad k = kte$$

$$T = \begin{bmatrix} N_i & N_{i+1} \end{bmatrix} \begin{bmatrix} T_i \\ T_{i+1} \end{bmatrix}$$

$$\hat{T} \approx \mathbf{NT}, \qquad \mathbf{N}_{(\boldsymbol{x})}$$

Discretización:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = -Q(x)$$

$$\frac{d}{dx}\left(k\frac{d\hat{T}}{dx}\right) \approx -Q(x) \ \equiv \ \frac{d}{dx}\left(k\frac{d\mathbf{N}_{(x)}}{dx}\right)\mathbf{T} \approx -Q(x)$$

Residuos:

$$\frac{d}{dx}\left(k\frac{d\mathbf{N}_{(x)}}{dx}\right)\mathbf{T} + \mathrm{Q(x)} = \xi$$

Método de los residuos ponderados:

$$\int_{\Omega} \xi_i w_i \, d\Omega = 0$$

$$\int_{\Omega} w \left[\frac{d}{dx}\left(k\frac{d\mathbf{N}}{dx}\right)\mathbf{T} + \mathrm{Q(x)}\right] d\Omega = 0$$

$$\frac{d}{dx}[N_i \quad N_{i+1}] = \left[\frac{d}{dx}N_i \quad \frac{d}{dx}N_{i+1}\right]$$

$$\frac{d}{dx}N_i = \frac{-1}{x_{i+1} - x_i}$$

$$\frac{d}{dx}N_{i+1} = \frac{1}{x_{i+1} - x_i}$$

$$\int_{\Omega} w \left[\frac{d}{dx}\left[\frac{-k}{x_{i+1} - x_i} \quad \frac{k}{x_{i+1} - x_i}\right]\mathbf{T} + \mathrm{Q(x)}\right] d\Omega = 0$$

$$\int_{\Omega} \boldsymbol{w} \left[\frac{d}{dx}\left[\frac{-k}{x_{i+1} - x_i} \quad \frac{k}{x_{i+1} - x_i}\right]\mathbf{T} + \mathrm{Q(x)}\right] d\Omega = 0$$

$$\mathbf{w} = \begin{bmatrix} w_{x_i} \\ w_{x_i+1} \end{bmatrix}$$

$$\int_{\Omega} \begin{bmatrix} w_{x_i} \\ w_{x_i+1} \end{bmatrix} \left[\frac{d}{dx}\left[\frac{-k}{x_{i+1} - x_i} \quad \frac{k}{x_{i+1} - x_i}\right]\mathbf{T} + \mathrm{Q(x)}\right] d\Omega = 0$$

Método de Galerkin

$$W_i = N_i$$

Forma Fuerte:

$$\int_{x_i}^{x_i+1} \mathbf{N}^{\mathbf{T}} \left[\frac{d}{dx}\left[\frac{-k}{x_{i+1} - x_i} \quad \frac{k}{x_{i+1} - x_i}\right]\mathbf{T} + \mathrm{Q(x)}\right] dx = 0$$

Integración por partes:

$$\int_{x_i}^{x_i+1} \mathbf{N}^{\mathbf{T}} \frac{d}{dx}\left(\frac{-k}{x_{i+1}-x_i} \quad \frac{k}{x_{i+1}-x_i}\right) \mathbf{T}\, dx + \int_{x_i}^{x_i+1} \mathbf{N}^{\mathbf{T}} Q(x)\, dx = 0$$

$$\int u dv = uv - v \int du$$

$$u = \mathbf{N}^{\mathbf{T}}$$

$$du = \frac{d}{dx}\mathbf{N}^{\mathbf{T}} dx$$

$$dv = \ k\frac{d\mathbf{NT}}{dx}\mathrm{dx}$$

$$v = k\frac{d\mathbf{NT}}{dx}$$

$$\mathbf{N}^{\mathbf{T}} k\frac{d\mathbf{NT}}{dx} - \int \frac{d}{dx}\mathbf{N}^{\mathbf{T}}\, k\frac{d\mathbf{NT}}{dx}\ dx$$

Forma débil:

$$\left.\mathbf{N}^{\mathbf{T}} k\frac{d\mathbf{NT}}{dx}\right|_{\Gamma} - \int_{x_i}^{x_{i+1}} \frac{d}{dx}\mathbf{N}^{\mathbf{T}}\, k\frac{d\mathbf{NT}}{dx}\ dx + \int_{x_i}^{x_i+1} \mathbf{N}^{\mathbf{T}} Q(x)\, dx = 0$$

$$\frac{d}{dx}\mathbf{N}^{\mathbf{T}} = \frac{d}{dx}\begin{bmatrix} N_i \\ N_{i+1} \end{bmatrix} = \begin{bmatrix} \dfrac{dN_i}{dx} \\ \dfrac{dN_{i+1}}{dx} \end{bmatrix} = \frac{1}{x_{i+1}-x_i}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\frac{d}{dx}\mathbf{N} = \ \frac{1}{x_{i+1}-x_i}\begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$\frac{d}{dx}\mathbf{N}^{\mathbf{T}}\frac{d}{dx}\mathbf{N} = \frac{1}{(x_{i+1}-x_i)^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k\int_{x_i}^{x_{i+1}} \frac{1}{(x_{i+1}-x_i)^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\ \mathbf{T}\mathrm{dx}$$

$$\frac{k}{(x_{i+1}-x_i)^2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}(x_{i+1}-x_i)\,\mathbf{T} = \frac{k}{(x_{i+1}-x_i)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_i\\ T_{i+1}\end{bmatrix}$$

$$\int_{x_i}^{x_i+1}\mathbf{N^T}Q(x)\,dx = -\int_{x_i}^{x_i+1}\begin{bmatrix}N_i\\ N_{i+1}\end{bmatrix}(x^2-3x)\,dx = \int_{x_i}^{x_i+1}\begin{bmatrix}N_i(x^2-3x)\\ N_{i+1}(x^2-3x)\end{bmatrix}dx$$

$$= \begin{bmatrix}\int_{x_i}^{x_{i+1}}\frac{x_{i+1}-x}{x_{i+1}-x_i}(x^2-3x)\\ \int_{x_i}^{x_{i+1}}\frac{x-x_i}{x_{i+1}-x_i}(x^2-3x)\end{bmatrix} = \begin{bmatrix}-\frac{(x_i-x_{i+1})\left(3({x^2}_i)+2(x_i)(x_{i+1}-6)+(x_{i+1}-6)(x_{i+1})\right)}{12}\\ \frac{(x_i-x_{i+1})\left(({x^2}_i)+2(x_i)(x_{i+1}-3)+3(x_{i+1}-4)(x_{i+1})\right)}{12}\end{bmatrix}$$

$$\frac{k}{(x_{i+1}-x_i)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_i\\ T_{i+1}\end{bmatrix} = -\begin{bmatrix}-\frac{(x_i-x_{i+1})\left(3({x^2}_i)+2(x_i)(x_{i+1}-6)+(x_{i+1}-6)(x_{i+1})\right)}{12}\\ \frac{(x_i-x_{i+1})\left(({x^2}_i)+2(x_i)(x_{i+1}-3)+3(x_{i+1}-4)(x_{i+1})\right)}{12}\end{bmatrix} + \mathbf{N^T}k\frac{d\mathbf{NT}}{dx}\Bigg|$$

$$\frac{k}{(x_{i+1}-x_i)}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}T_i\\ T_{i+1}\end{bmatrix} = -\frac{(x_i-x_{i+1})}{12}\begin{bmatrix}-\left(3({x^2}_i)+2(x_i)(x_{i+1}-6)+(x_{i+1}-6)(x_{i+1})\right)\\ \left(({x^2}_i)+2(x_i)(x_{i+1}-3)+3(x_{i+1}-4)(x_{i+1})\right)\end{bmatrix} + \mathbf{N^T}k\frac{d\mathbf{NT}}{dx}\Bigg|$$

$$\boldsymbol{kT} = \boldsymbol{b}$$

Para cada elemento i:

$$\boldsymbol{k} = \begin{bmatrix}k_{\boldsymbol{i1}} & k_{\boldsymbol{i2}}\\ k_{\boldsymbol{i3}} & k_{\boldsymbol{i4}}\end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix}b_{i1}\\ b_{i2}\end{bmatrix}$$

Ensamblaje:

Elemento	$\boldsymbol{i}$	$\boldsymbol{i+1}$
1	1	2
2	2	3
3	3	4

$$\begin{bmatrix} k_{11} & k_{12} & 0 & 0 \\ k_{13} & k_{14}+k_{21} & k_{22} & 0 \\ 0 & k_{23} & k_{24}+k_{31} & k_{32} \\ 0 & 0 & k_{33} & k_{34} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{12}+b_{21} \\ b_{22}+b_{31} \\ b_{32} \end{bmatrix}$$

$$\boldsymbol{KT} = \boldsymbol{B}$$

Condiciones de contorno:

Condiciones de Neumann: $\frac{dT}{dx} = T_0$ en $\Gamma_N, \Gamma_N \subseteq \Gamma$

$$\mathbf{N^T} k \frac{d\mathbf{NT}}{dx}\bigg|_{\boldsymbol{\Gamma}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} k_{11} & k_{12} & 0 & 0 \\ k_{13} & k_{14}+k_{21} & k_{22} & 0 \\ 0 & k_{23} & k_{24}+k_{31} & k_{32} \\ 0 & 0 & k_{33} & k_{34} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{12}+b_{21} \\ b_{22}+b_{31} \\ b_{32} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

Condiciones de Dirichlet: $T = T_0$ en $\Gamma_D, \Gamma_D \subseteq \Gamma$

$$\begin{bmatrix} 10 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$