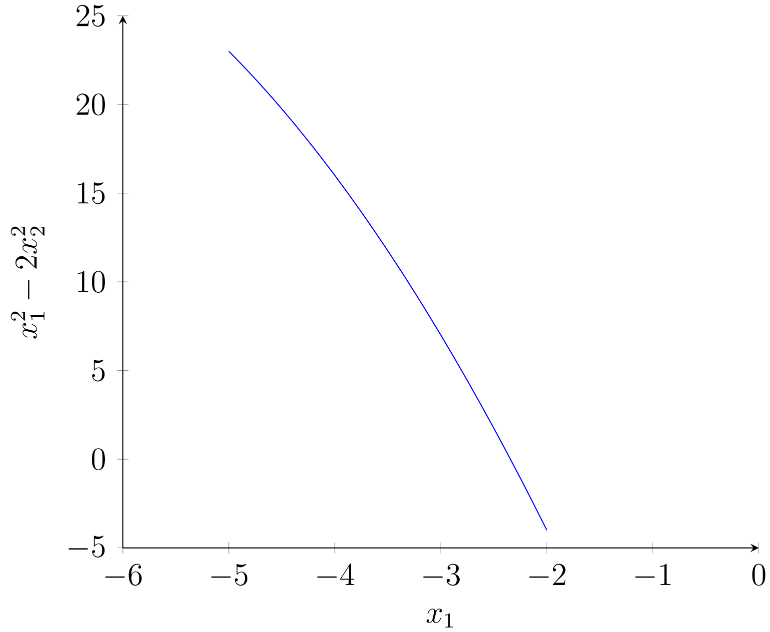
KKT Condition

$$\begin{aligned} \text{minimize } & x_1^2 - 2x_2^2 \\ \text{subject to } & (x_1+4)^2 - 2 \leq x_2 \\ & x_1 - x_2 + 4 = 0 \\ & x_1 \geq -10 \end{aligned}$$

Sketch



Graphically, the primal solution x^* looks to be $x^* = (-2, 2)$.

KTT conditions

$$\mathcal{L}\left(x_1, x_2, \lambda_1, \lambda_2, \nu\right) = \left(x_1^2 - 2x_2^2\right) + \lambda_1\left((x_1+4)^2 - 2 - x_2\right) + \lambda_2\left(-10 - x_1\right) + \nu\left(x_1 - x_2 + 4\right)$$

The vanishing gradient is

$$\nabla_x \mathcal{L}\left(x, \lambda, \nu\right) = \begin{pmatrix} 2x_1 \\ -4x_2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2(x_1+4) \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Using the primal solution found above we get

$$\nabla_x \mathcal{L}\left(x^*, \lambda, \nu\right) = \begin{pmatrix} -4 + \lambda_1 4 - \lambda_2 + \nu \\ -8 - \lambda_1 - \nu \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

which is satisfied for $(\lambda_1, \lambda_2, \nu) = (a, 12 - 3a, -8 - a)$. As we need $\lambda^* \succeq 0$, $a \in [0, 4]$ which gives us suitable condition for x^*.