Assignment 5

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1 Fourier transform

1.1 On annex 1, we will find different images numbered from 1 to 8 and different Fourier Transform results numbered from "a" to "h". Associate each image with its Fourier transform.

1-f:
Figure 1 shows a monkey's face, the colour change is smooth in most areas, so the corresponding Fourier transform plot is brighter in the central area; at the same time, the monkey's nose and the upper part of the monkey are the places where the colour change is strong, and there are some textures in specific directions, and there are lines perpendicular to each other with these textures in the Fourier transform plot, but not many.

2-d:
In Figure 2, you can see that this is a very regular looking like a floor of triangular planks put together. In this case, the triangles are acute triangles with three sides repeated over and over again. The three sides are orientated perpendicular to the three lines in the Fourier transform diagram.

3-h, 4-e:
Figure 3 shows a square house with horizontal eaves and vertical walls, so the Fourier transform plot has horizontal and vertical lines. Also, the eaves of the attic window are a symmetrical triangle, so the Fourier transform plot has symmetrical two lines each perpendicular to the eaves in two directions. Finally, on the right side of the original diagram, there is a slope down to the right and the line is long, so there are lines perpendicular to this line on the Fourier transform diagram as well.
Figure 4 is a rotated version of Figure 3, and you can see that the direction of the perpendicular line of the symmetrical triangle has changed, as well as the direction of the perpendicular line of the slope has rotated with it.

5-c:
Figure 5 is a vertical strip of watermelon rind, so the horizontal frequency is high on the Fourier transform plot.

6-b:
Figure 6 is a manuscript where there is no great pattern in the direction of the lines of the text, so there are no very significant lines on the Fourier transform diagram. There are some vertical or horizontal creases in the paper, which may make for some horizontal and vertical lines on the Fourier transform diagram.

7-a:
Figure 7 shows a ceiling made from a combination of multiple polygons. Finding the direction of each of the sides of these polygons summarises which directions occur multiple times, which corresponds to figure e.

8-g:
Figure 8 is a zebra, although it has regular stripes, but because it poses the action so that the direction of the stripes is not uniform, there are many different directions, but not all directions, so the Fourier transform map has some bright shiny spots; the middle is brighter because there are some lawns and other objects in the figure that change colour slowly, making the central area of the Fourier transform map brighter.

2 Hadamard transform

2.1 elow, we provided a 4*4 pixels image. We ask you to compute the Hadamard transform which is defined by the following matrix H. More details in the course.

Given a 4×4 image matrix:

$$\text{Image} = \begin{bmatrix} 7 & 3 & 1 & 5 \\ 7 & 9 & 3 & 1 \\ 3 & 5 & 1 & 3 \\ 7 & 9 & 9 & 3 \end{bmatrix}$$

and the Hadamard matrix:

$$H = \frac{1}{2} \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$

The Hadamard transform $H_{\text{transformed}}$ of the image is computed as:

$$H_{transformed} = H * Image * H^T = \begin{bmatrix} 19 & 6 & 0 & -1 \\ -1 & 2 & 0 & 3 \\ -5 & -2 & -2 & 5 \\ 3 & -2 & 2 & 1 \end{bmatrix}$$

The inverse Hadamard transform to retrieve the original image is:

$$H_{original} = H^T * H_{transformed} * H = \begin{bmatrix} 7 & 3 & 1 & 5 \\ 7 & 9 & 3 & 1 \\ 3 & 5 & 1 & 3 \\ 7 & 9 & 9 & 3 \end{bmatrix}$$

which is same like original image.