Digital Image Processing

Lesson 8: Binary Image Processing

Master Course
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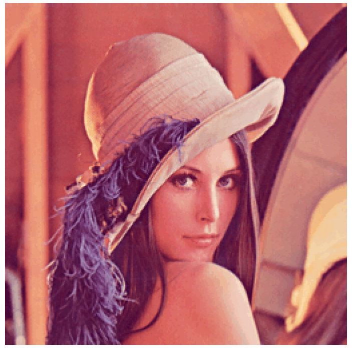
Binary Images

- Images whose pixels have two possible values (0 and 1) are called **binary images**
- Binary images are useful in several context
  - To visualize binary objects
    - Examples: bar-codes, QR codes, or text
  - To represent abstract shapes
    - Example: segmentation results
  - To provide digital print screens

Color to Grayscale Transformation

- Each channel of an RGB image can be interpreted as a grayscale image
- A simplistic approach for color to grayscale transformation consists in averaging the RGB channels





- However, to obtain a result that faithfully reflects the perception of the human visual system, the following linear combination of the RGB channels must be applied

$$0.299 \times R + 0.587 \times G + 0.114 \times B$$

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Binarization Methods

- Binarization of document images has specific requirements
  - The goal is to properly separate foreground and background pixels (ink vs. support)
- There are several sources of difficulty
  - Degradations of the support, faded ink
  - Unregular illumination
- Processing natural grayscale images requires specific technics: **dithering**

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Binarization Methods

- Many binarization methods have been proposed in the literature
  - Global thresholding
    - Otsu’s method: based on an optimal threshold
  - Local adaptive thresholding (using a shifting window)
    - Bernsen’s method: threshold as a function of local minimum and maximum
    - Niblack’s method: threshold determined as a function of the local histogram
    - Sauvala’s method: variant of Niblack’s method
    - More sophisticated methods take into account other parameters such as stroke width estimations

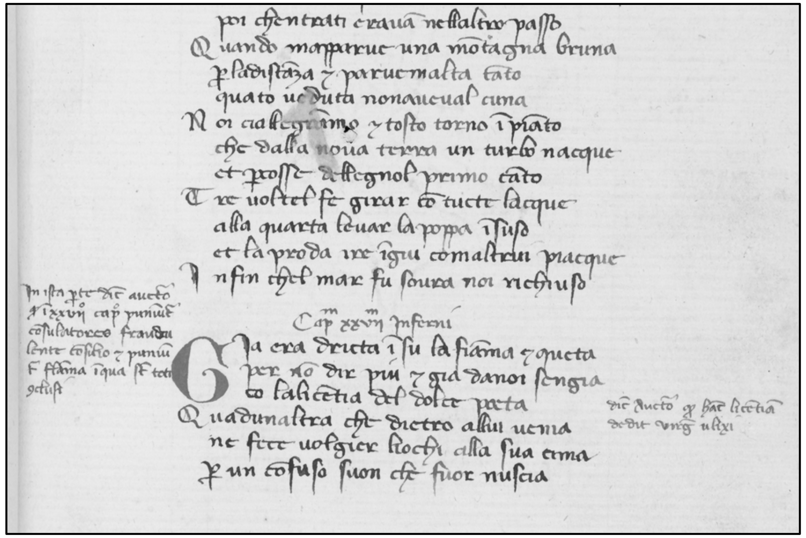
Otsu's Method

- The method chooses the threshold that minimizes the intra-class variance of the two supposed classes (foreground, background)

  $p_0\sigma_0^2 + p_1\sigma_1^2$ where p_i is the cumulative probability and σ_i^2 the variance of class i

- Equivalently one can also maximize the inter-class variance

  $p_0 p_1(\mu_0 - \mu_1)^2$ where μ_i is the mean of class i



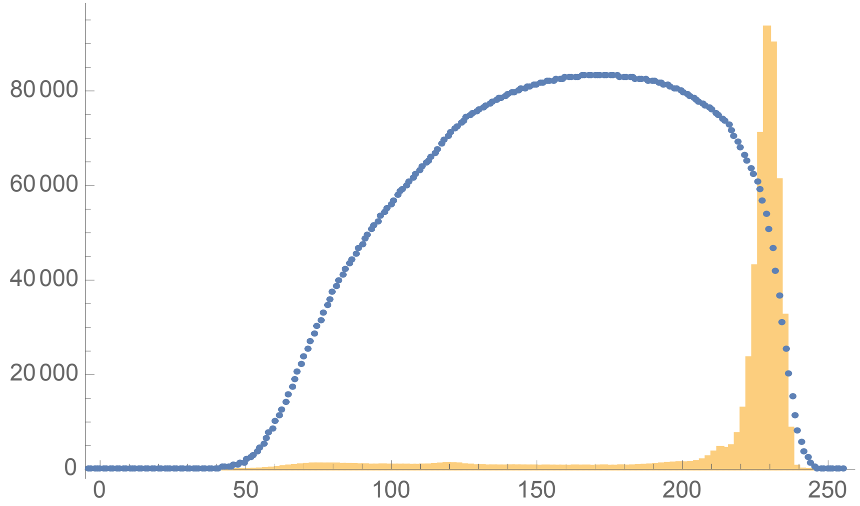


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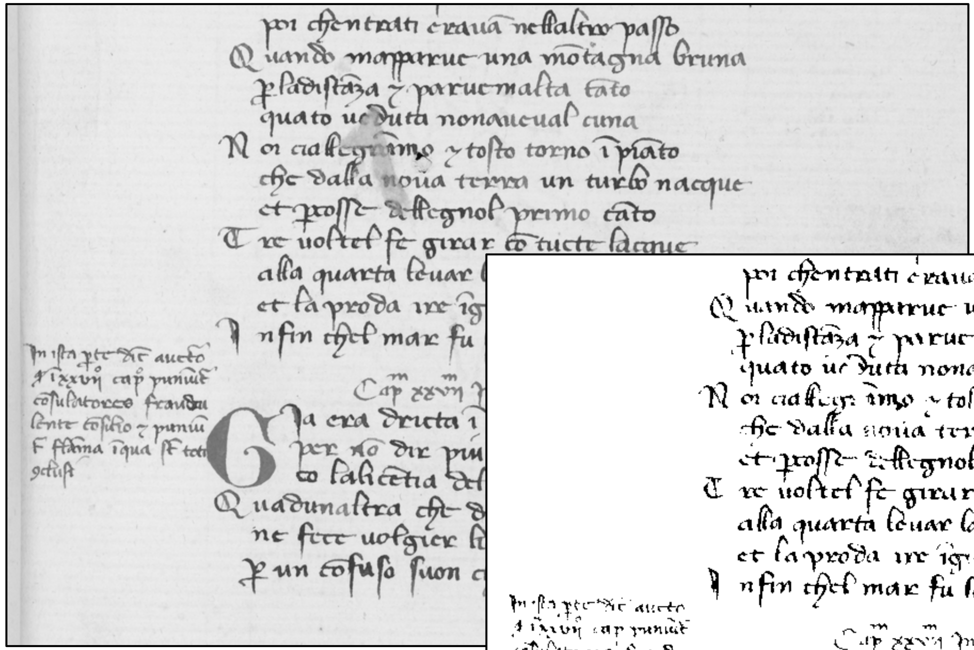
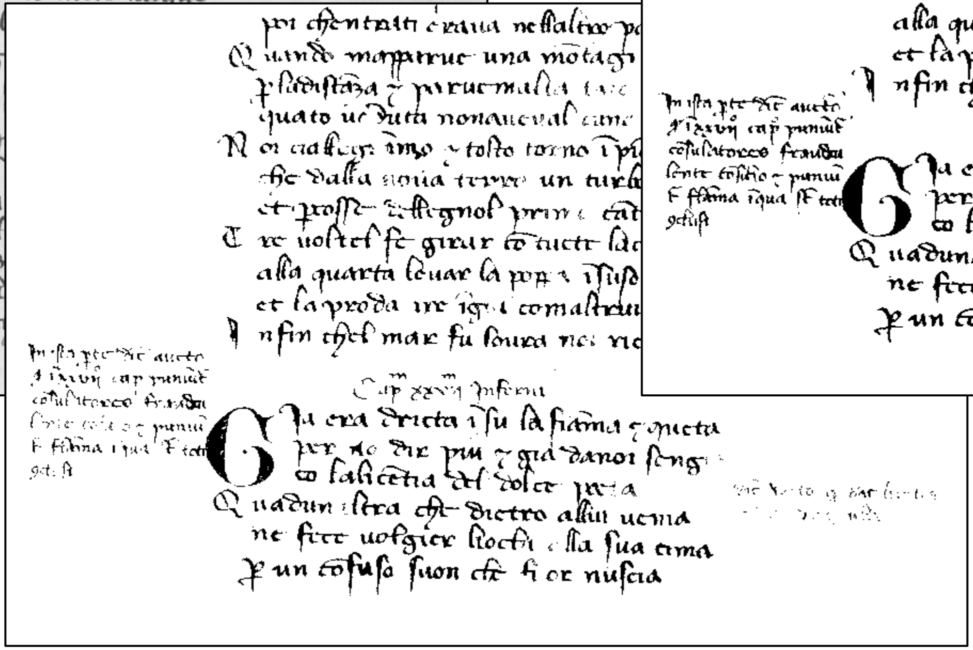
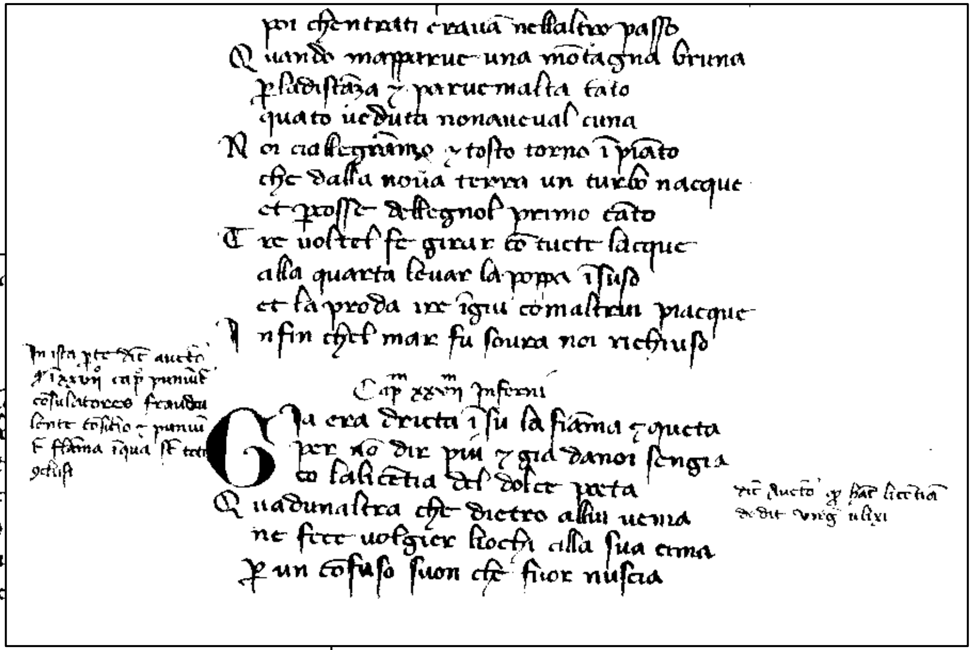
Illustration of Otsu binarization





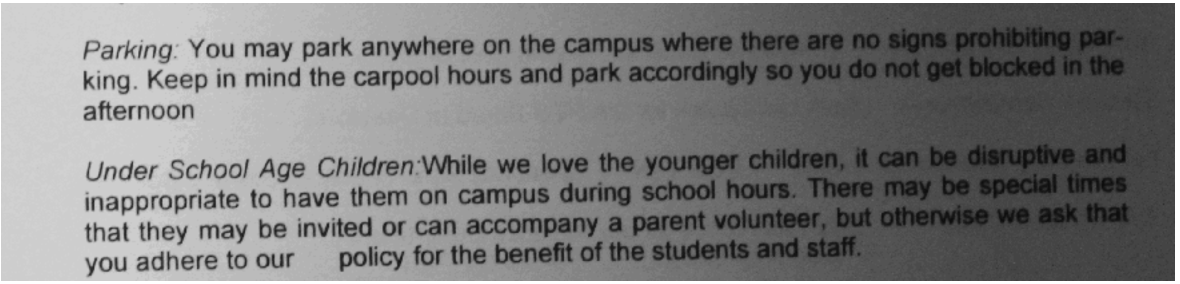
Standard threshold



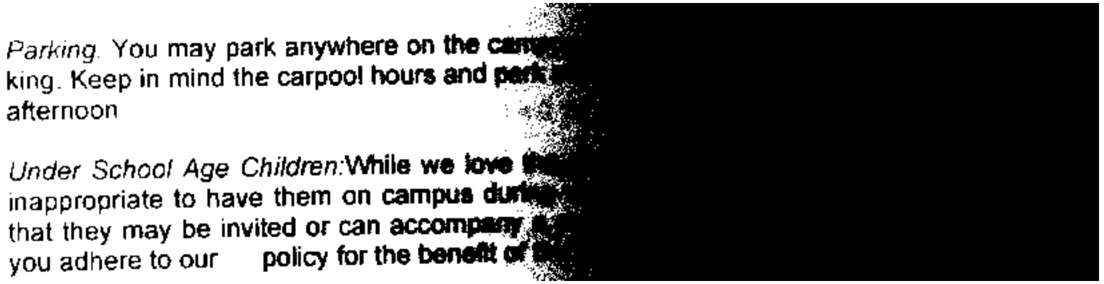
Otsu threshold

Problem with Global Thresholding

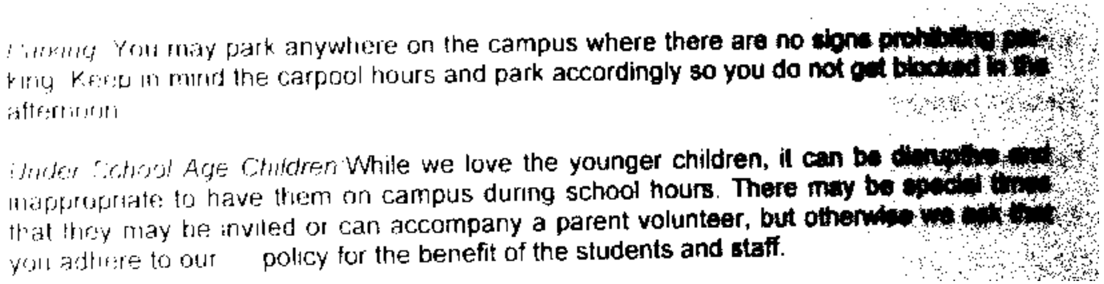
- Original



- Otsu's Method



- Lower threshold (0.25)



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Local Thresholding

- Local thresholding consists in computing the threshold $T_{x,y}$ for pixel (x, y) is obtained as a function of pixel values in a neighborhood
  - Shape and size of the neighborhood are parameters

- The Bernsen's method uses

$$T(x, y) = \frac{I_{min} + I_{max}}{2}$$

  - where I_{min} and I_{max} are respectively the minimum and the maximum gray level value of the pixels in the neighborhood

Niblack's method

- Niblack's method is using a local threshold $T_{x,y}$ estimated in the neighborhood of the pixel x,y

  $$T_{x,y} = \mu_{x,y} - k * \sigma_{x,y}$$

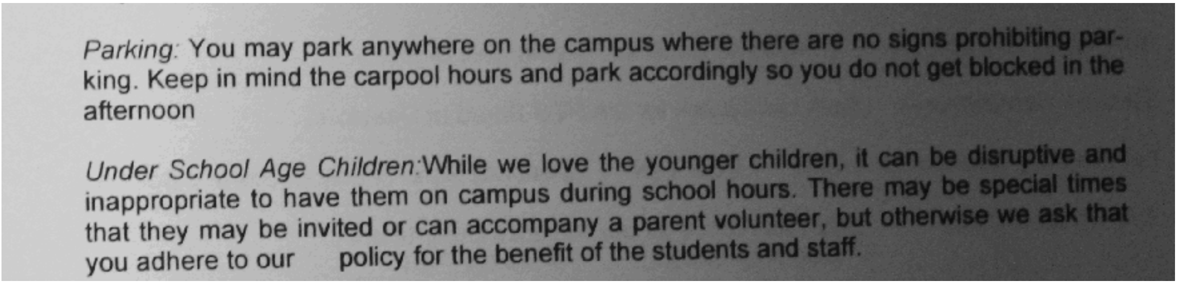
  where

  - $\mu_{x,y}$ and $\sigma_{x,y}$ represent respectively the mean and standard deviation of grey levels in the neighborhood
  - k is a constant between 0 and 1 (suggested value 0.2)
  - The neighborhood consists of a square window, which size is chosen appropriately
    - It should include several characters (usually between 20 and 100 pixels)

- Niblack's method assumes the window to include some text
  - In case of only background, the result might be noisy!
  - A preliminary analysis can avoid this case

Illustrations with Niblack's Method

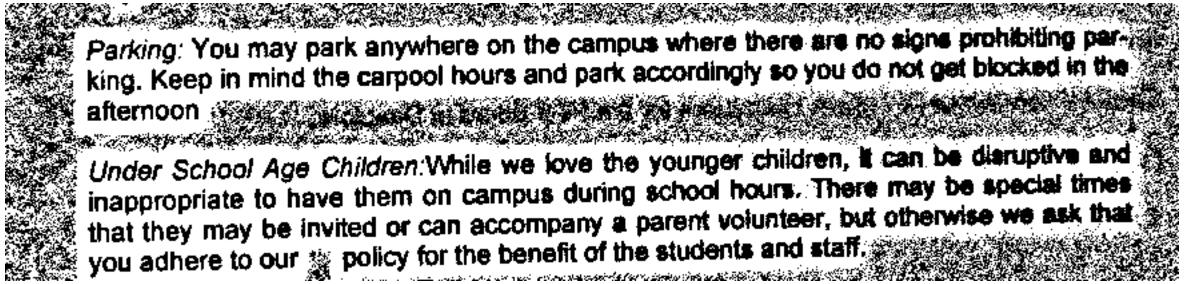
- Original



Parking: You may park anywhere on the campus where there are no signs prohibiting par-
king. Keep in mind the carpool hours and park accordingly so you do not get blocked in the
afternoon

Under School Age Children: While we love the younger children, it can be disruptive and
inappropriate to have them on campus during school hours. There may be special times
that they may be invited or can accompany a parent volunteer, but otherwise we ask that
you adhere to our policy for the benefit of the students and staff.

- Niblack (w = 51, k = 0)



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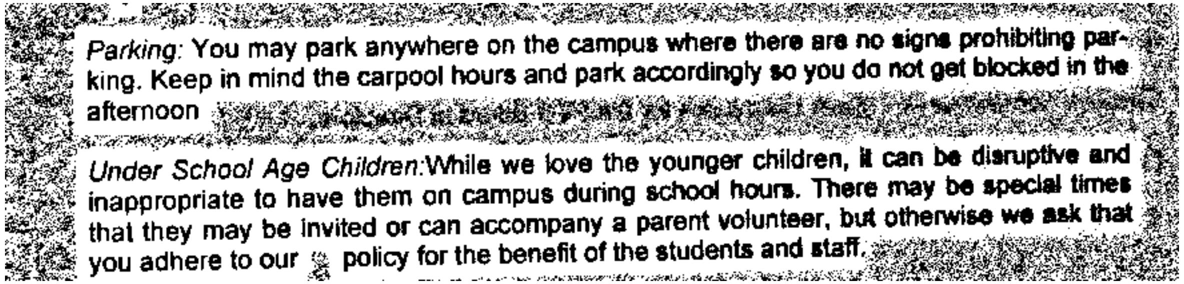
- Niblack (w = 51, k = 0.2)



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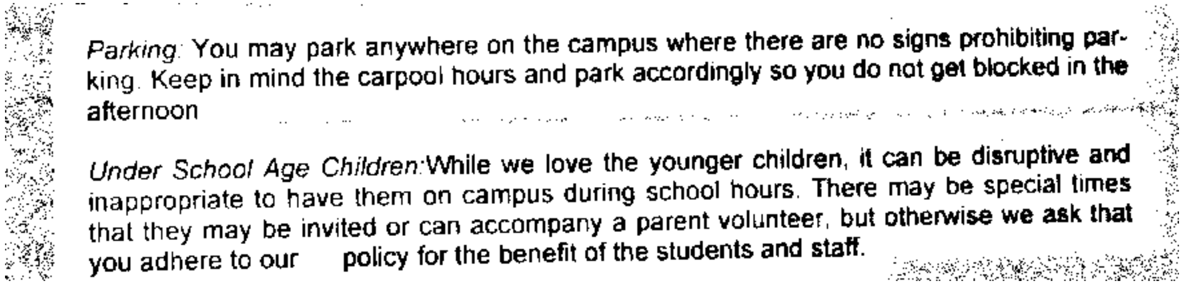
- Niblack (w = 121, k = 1)



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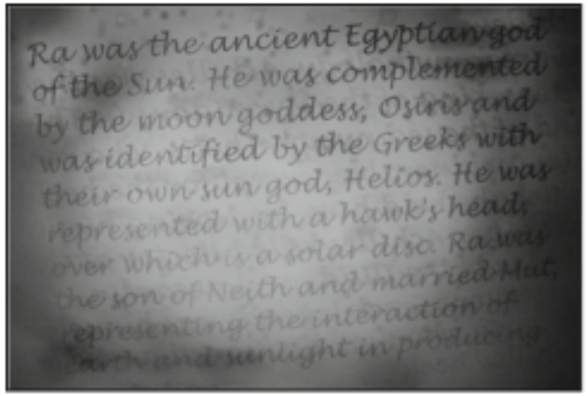
Sauvola's method

- Sauvola et al. have proposed a variant of local thresholding that uses

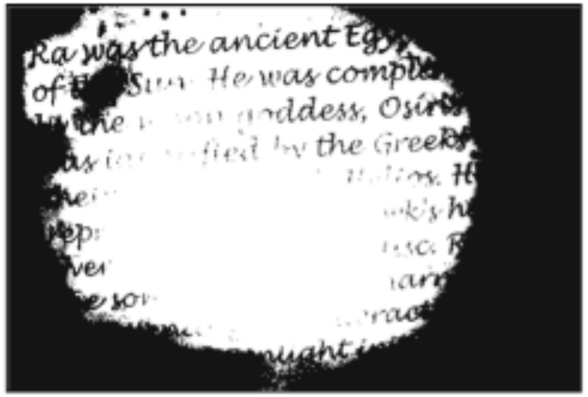
  $$T_{x,y} = \mu_{x,y}(1 + k(\frac{\sigma_{x,y}}{R} - 1))$$

  where
  - R is the median gray value (R = 0.5)
    - For 8-bit grayscale R = 128
  - k is a constant between 0 and 1 (suggested value 0.5)

Comparison of Binarization Methods

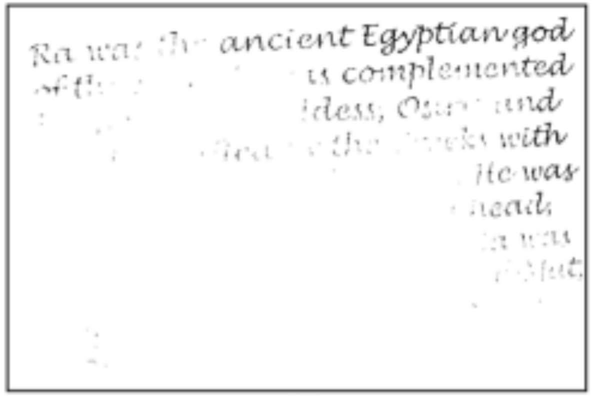


Original Image

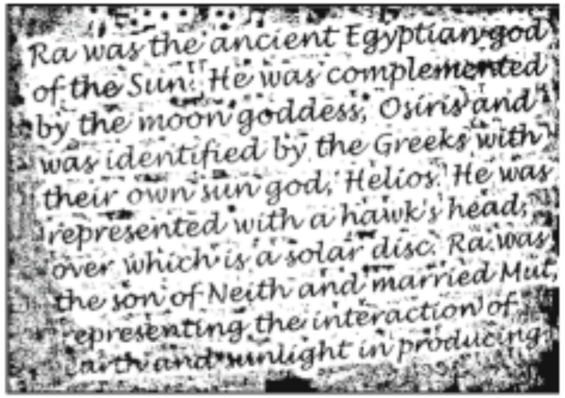


Otsu

Niblack



Sauvola

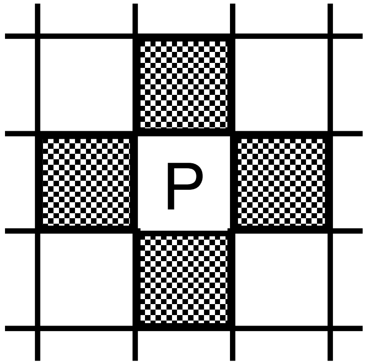


Bernsen

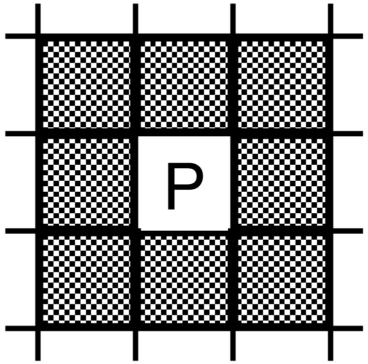
source : Handbook of Document Image Processing and Recognition, p. 89

Neighborhood

- Two type of neighborhoods may be defined on a squared grid:
  - Two pixels are said to be 4-neighbors if they share an edge
  - Two pixels are said to be 8-neighbors if they share an edge or a corner



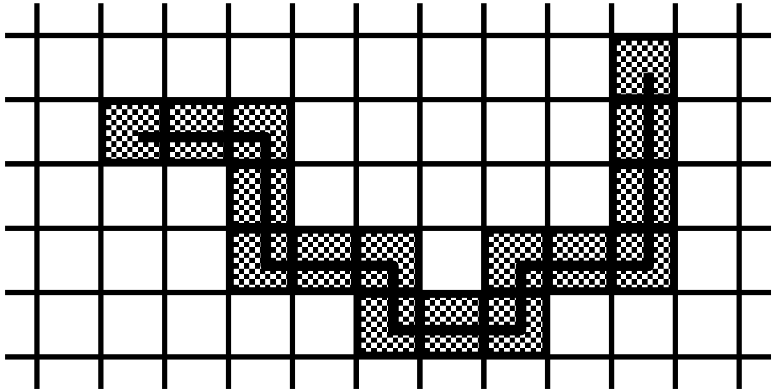
4-adjacency

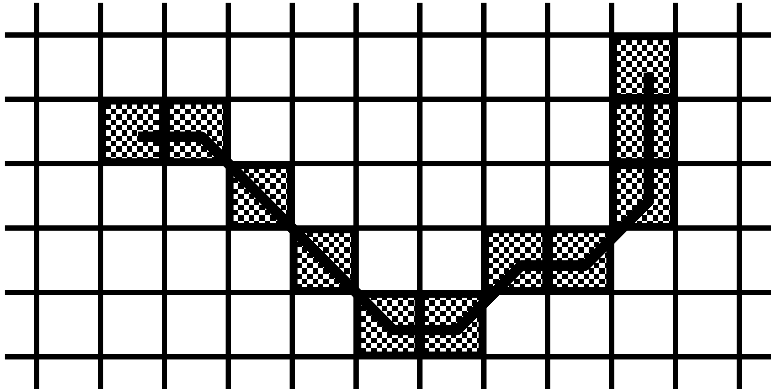


8-adjacency

Connected Paths

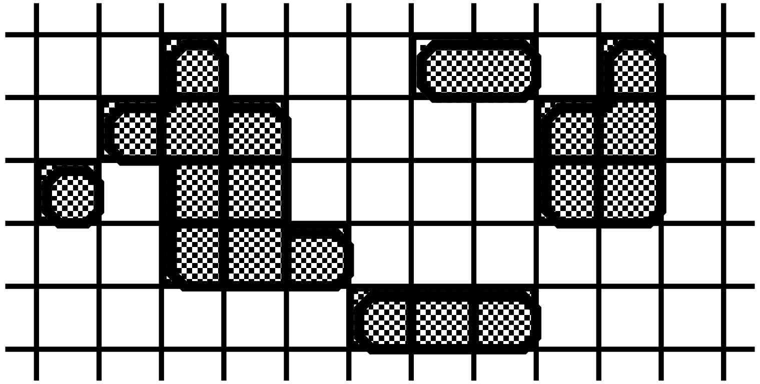
- A sequence of pixels p_1, p_2,..., p_n is called a **4-connected path** if all successive pairs of pixels p_i, p_{i+1} (for i = 1,…,n-1) are 4-neighbors
- A sequence of pixels p_1, p_2,..., p_n is called a **8-connected path** if all successive pairs of pixels p_i, p_{i+1} (for i = 1,…,n-1) are 8-neighbors


4-connected path

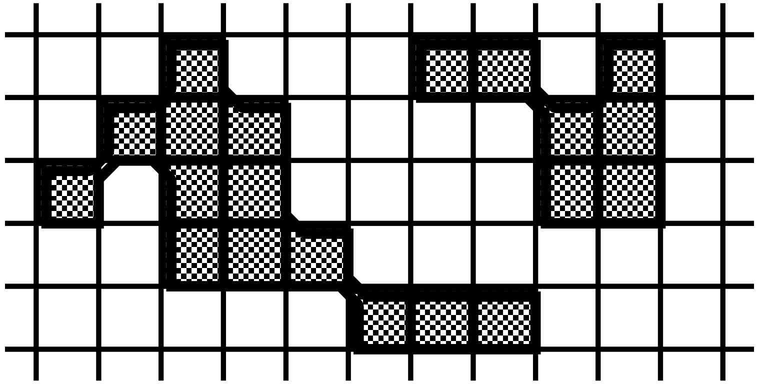

8-connected path

Connected Components

- A set of pixels S is called a **4-component** if and only if for each pair of pixels p, p' $\in$ S, there is a 4-path $p_1, p_2, \ldots, p_n$ with $p_1 = p$ and $p_n = p'$ where all $p_i \in S$
- A set of pixels S is called a **8-component** if and only if for each pair of pixels p, p' $\in$ S, there is a 8-path $p_1, p_2, \ldots, p_n$ with $p_1 = p$ and $p_n = p'$ where all $p_i \in S$



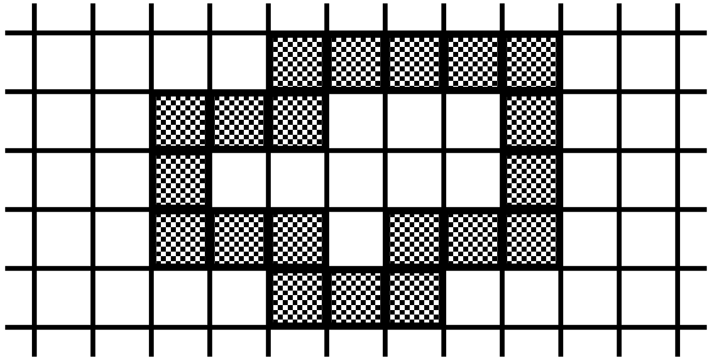
Five 4-connected components



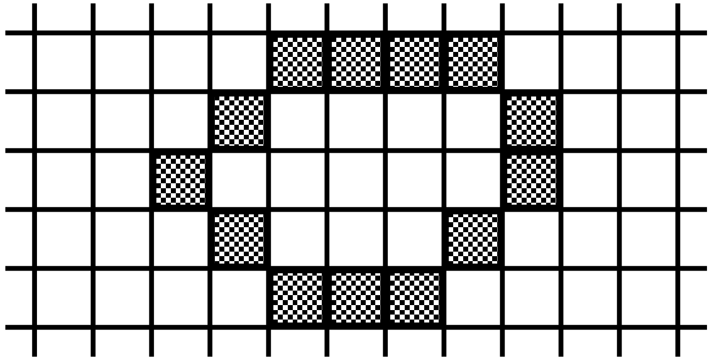
Two 8-connected components

Closed Curves

- A sequence of pixels p_1, p_2,..., p_n is called a **4-connected curve** if and only if each p_i has exactly two 4-neighbors belonging to the sequence
- A sequence of pixels p_1, p_2,..., p_n is called a **8-connected curve** if and only if each p_i has exactly two 8-neighbors belonging to the sequence



4-connected curve



8-connected curve

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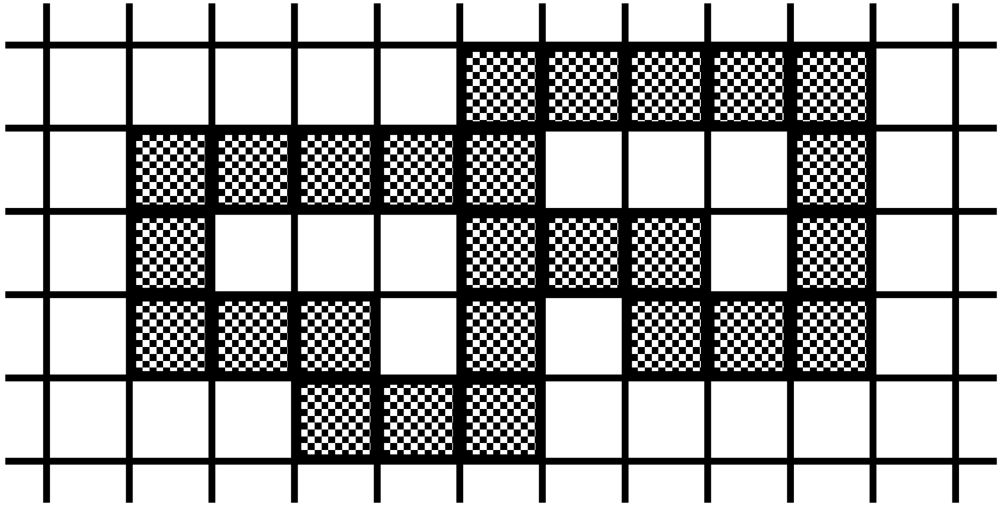
Comment on Closed Curves

- The previous definition prevents a closed curve to have more than two k-neighbors (k=4,8)



Example of a 4-connected component
that is **not** a closed 4-connected curve

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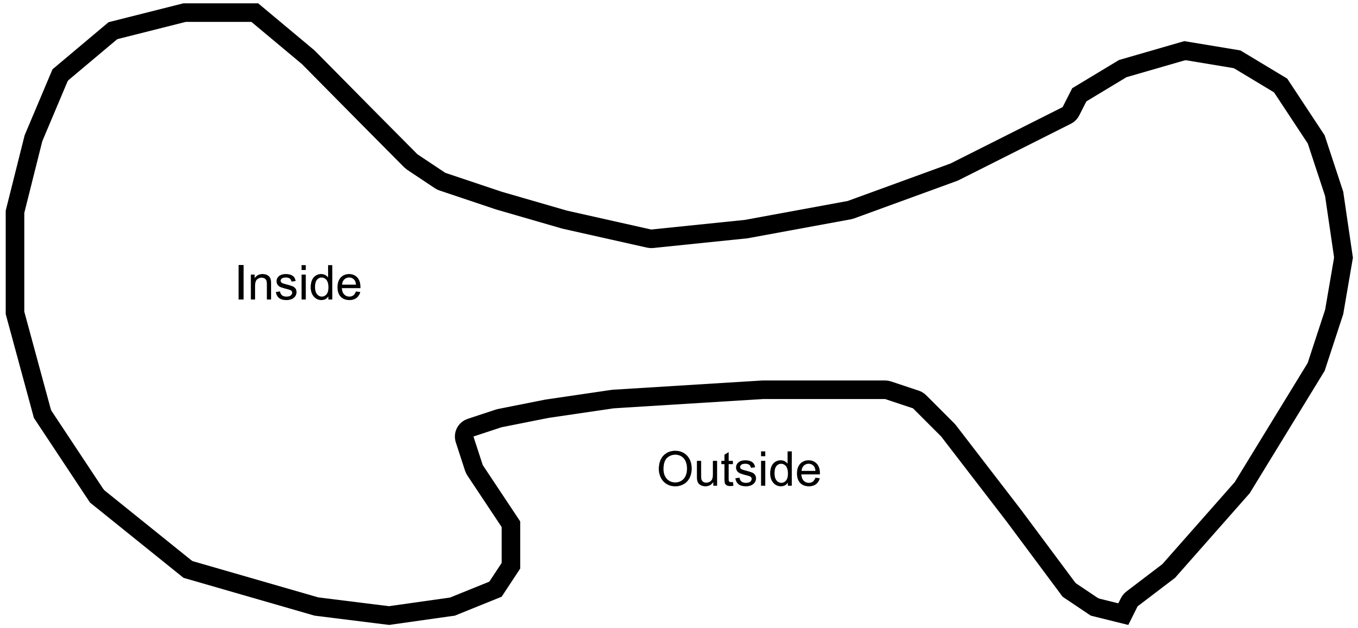
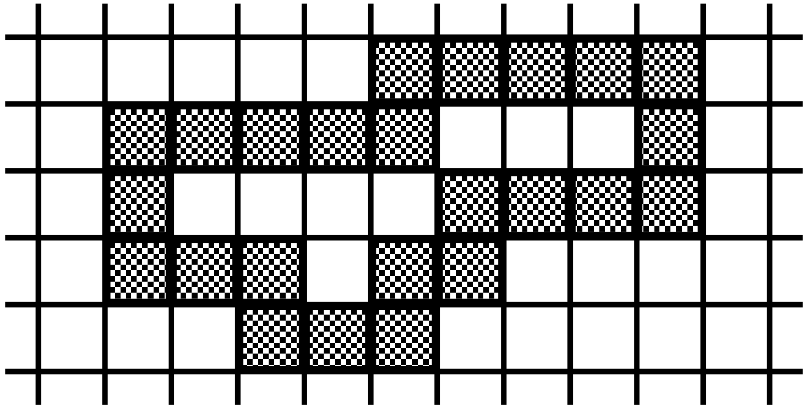
Jordan's Theorem in the Continuous Case

- In the 2D Euclidean (continuous) space
  - A **closed curve is separating** all remaining points into two distinct connected regions called **outside** and **inside**

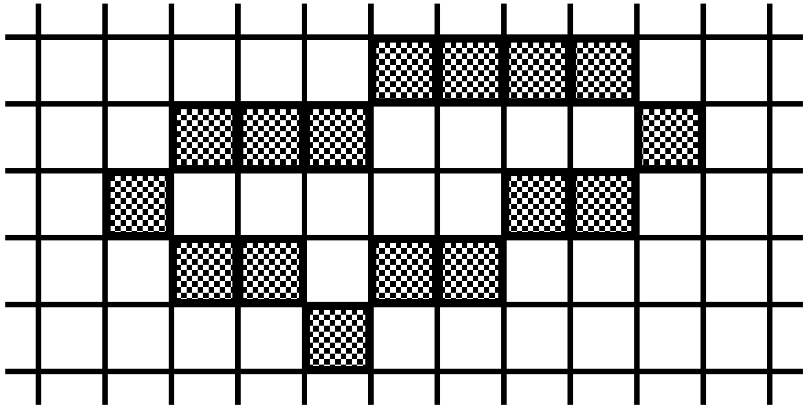


Jordan's Paradoxes in the Discrete Case

- A closed 4-connected curve may divide the remaining space into more than two 4-connected components
- The complementary space of a closed 8-connected curve consists in most cases of a single 8-connected component



A 4-conneted closed curve separating three 4-connected white components



A 8-conneted closed curve without any separation (single 8-connected white component

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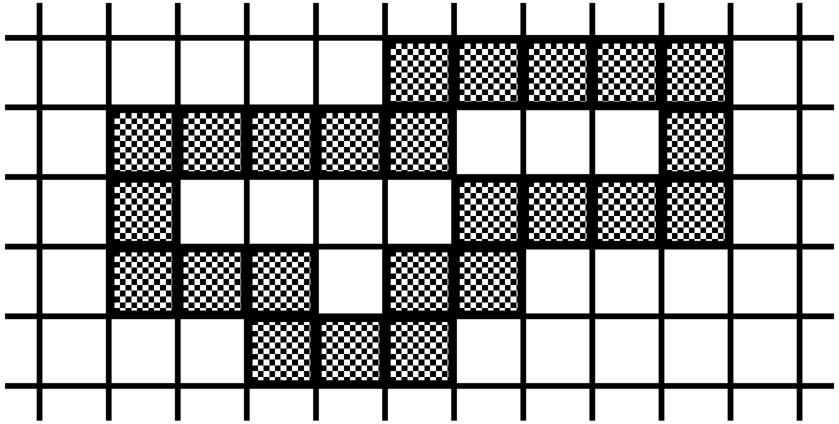
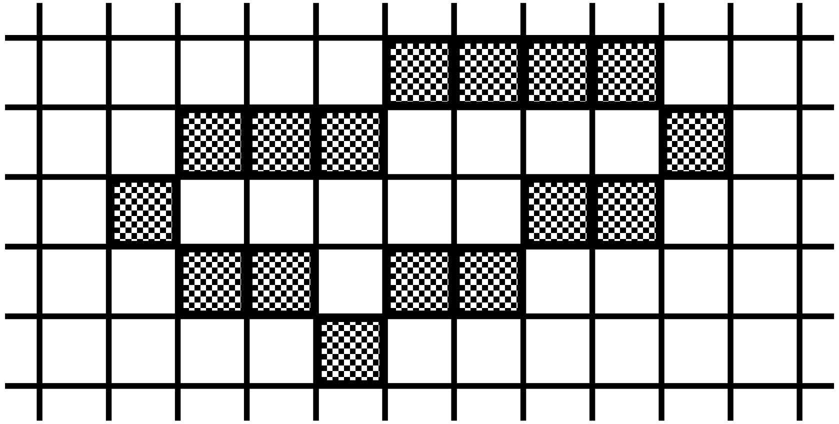
Jordan's Theorem in the Discrete Space

- The complementary part of a 4-connected closed curve is divided into two 8-connected components
- The complementary part of a 8-connected closed curve is divided into two 4-connected components



A 4-conneted closed curve separating two 8-connected white components



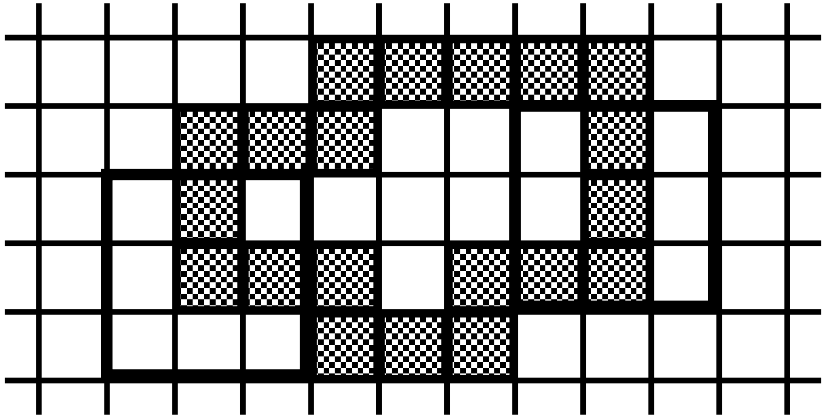
A 8-conneted closed curve separating two 4-connected white components

Inside / Outside and Curve Crossings

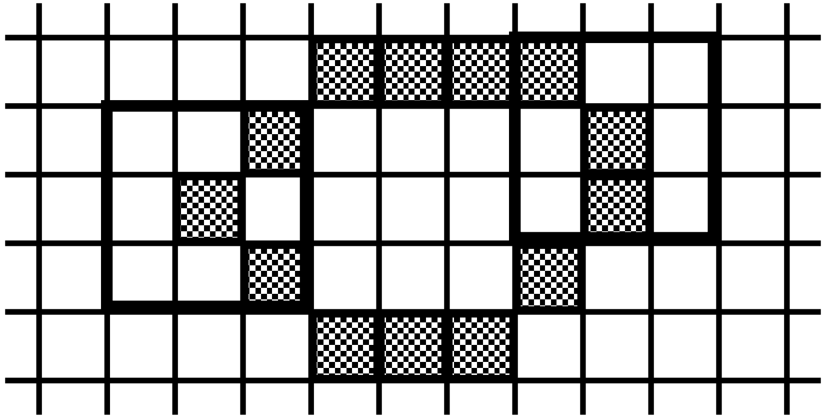
- According to the Jordan's theorem, a closed curve splits the remaining pixel set into an inner and an outer region
  - If the closed curve is 4-connected, each 8-path linking a pixel of the inner region with a pixel of the outer region intersects the curve at least on one pixel
  - If the closed curve is 8-connected, each 4-path linking a pixel of the inner region with a pixel of the outer region intersects the curve at least on one pixel

Local characterization

- In a 8-neighborhood of a closed 4-connected curve, there is at least one pixel belonging to the inner region and one pixel belonging to the outer region (both being considered as 8-connected components)

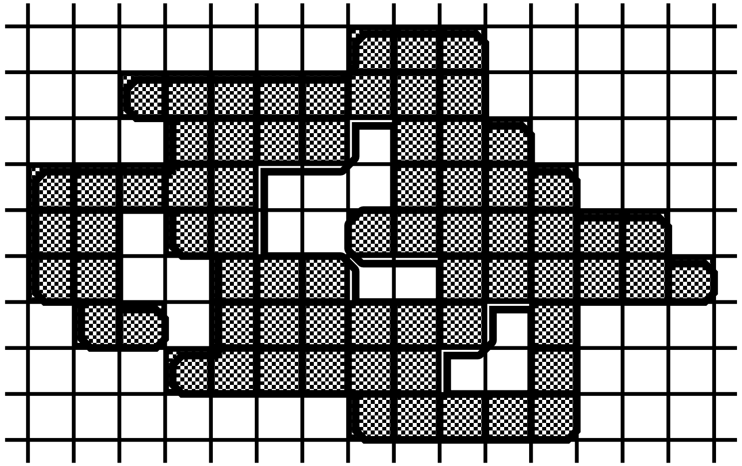


- In a 4-neighborhood of a closed 8-connected curve, there is at least one pixel belonging to the inner region and one pixel belonging to the outer region (both being considered as 4-connected components)

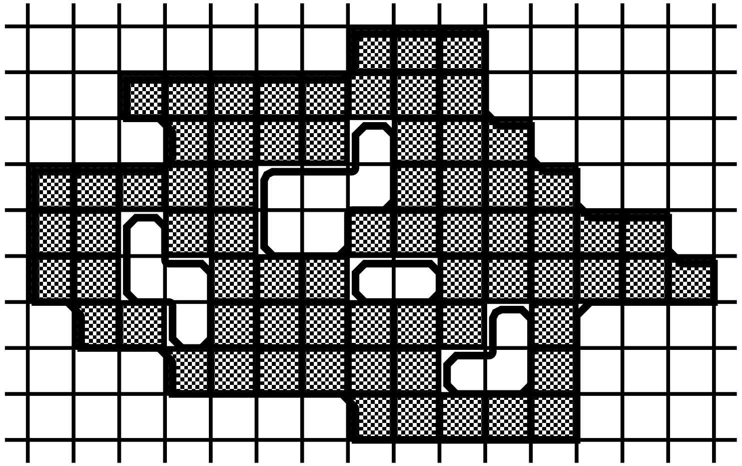


Background, Objects and Holes

- When considering a finite k-connected components (for k =4,8) on the infinite plane
  - The component is called **object**,
  - The complementary connected component surrounding the object is called **background**
  - The remaining complementary connected components inside the object are **holes**



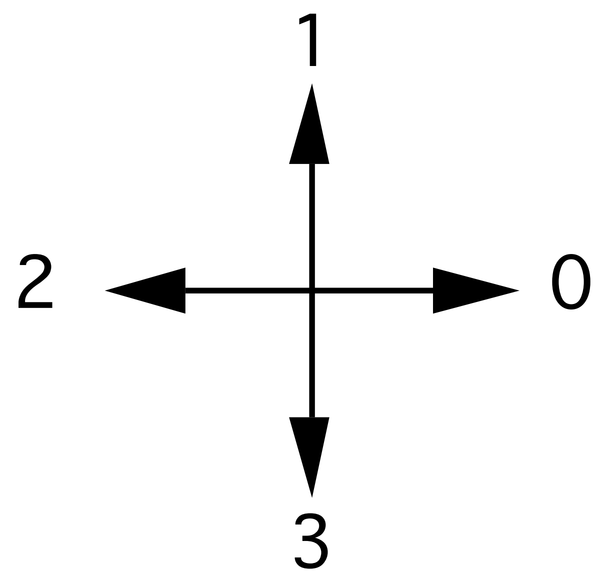
A 4-conneted component
with two 8-connected holes

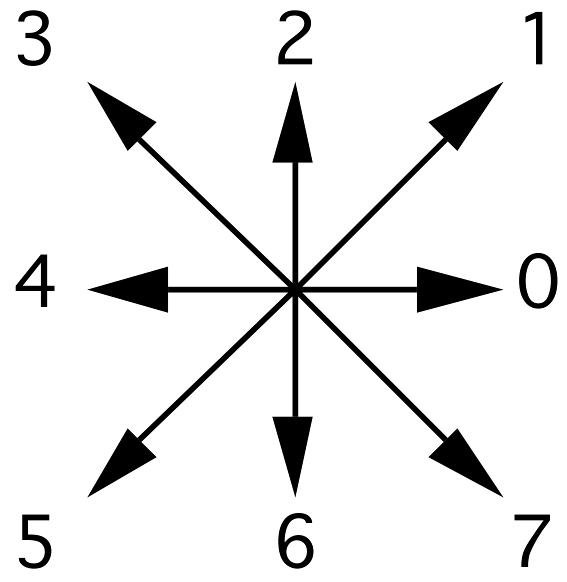


A 8-conneted component
with four 4-connected holes

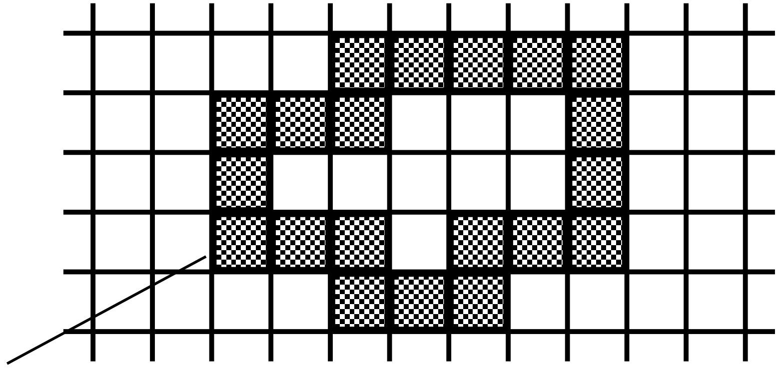
Freeman Chain Coding

- K-Paths (k=4,8) may be easily represented by the **Freeman chain coding**:
  - The sequence of pixels is encoded according to their relative positions

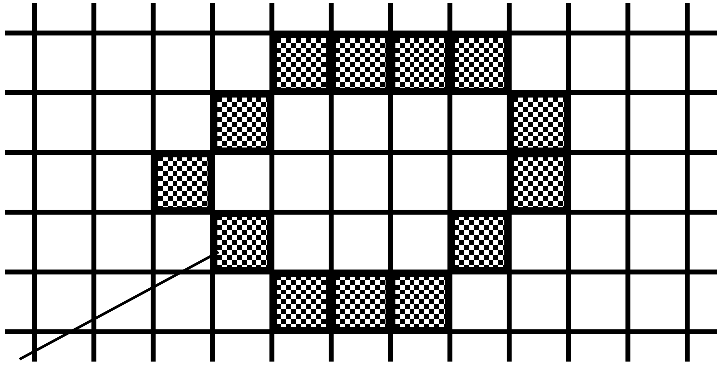




Examples of Freeman Chain coding



4-connected path coding:
0,0,3,0,0,1,0,0,1,1,
1,2,2,2,2,3,2,2,3,3



8-connected path coding:
7,0,0,1,1,2,3,4,4,4,5,5,7

- To locate the shape, the coordinates of the **start pixel** must be given too

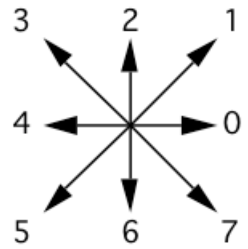
Connected Component Extraction Algorithm

- The following algorithm extracts k-connected (k=4,8) components of a binary image

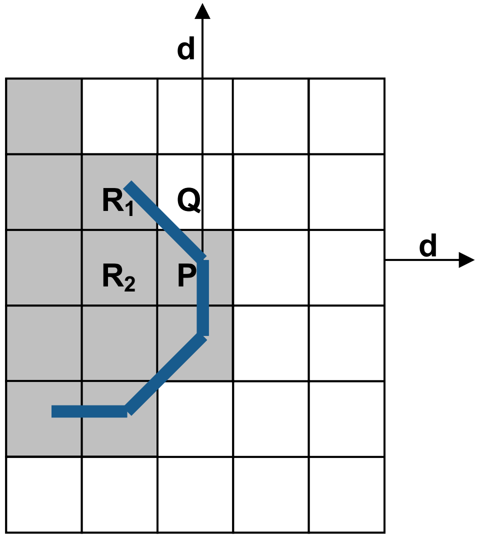
```
for each scan line l_y
    for each black run r in l_y
        if r on line l_y-1 there is no run that is k-adjacent to r
            create new component with r
        else if on line l_y-1 there is a run r′ that is k-adjacent to r
            add r to the component containing r′
        else if on line l_y-1 there are several runs r_i that are k-adjacent to r
            merge all components containing such a r_i
            add r to that component
```

Contour Following Algorithm

- This algorithm extracts the contour (8-connected, counter clockwise) of a 4-connected component




```
consider  P₀∈ S having a 4-neighbor Q₀∉ S
P ← P₀; Q ← Q₀; d ← direction of P to Q;
repeat
    let Rᵢ be neighbor of P in direction (d+i) mod 8
    if R₂∉ S then Q ← R₂; d ← (d+2) mod 8;
    else
        if R₁∉ S then P ← R₂ ; Q ← R₁;
        else P ← R₁; d ← (d–2) mod 8;
            add P to the contour
until P = P₀ and Q = Q₀
```



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Morphological Operators

- **Morphological operators** are local operations that transform a binary image according to a structuring element (binary mask)

- Basic morphological operators include
  - Erosion, that shrinks a connected component
  - Dilation, that enlarges a connected component

- Combined morphological operators include
  - Opening
  - Closing
  - "Hit and miss" operator

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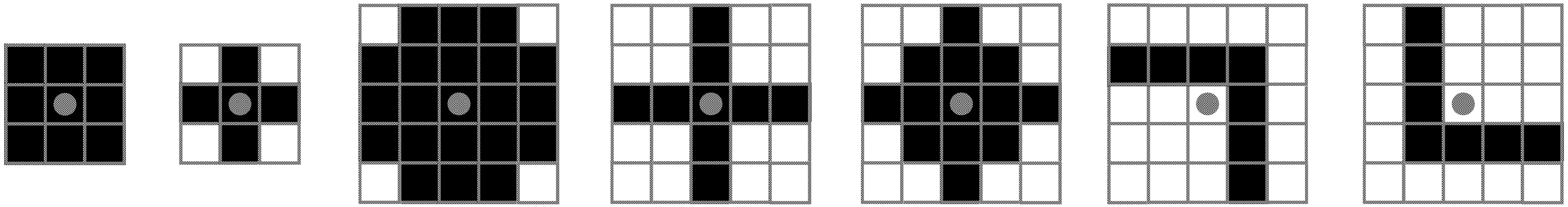
Structuring Elements

- A **structuring element** is a binary shape represented by a mask and an **anchor point**.



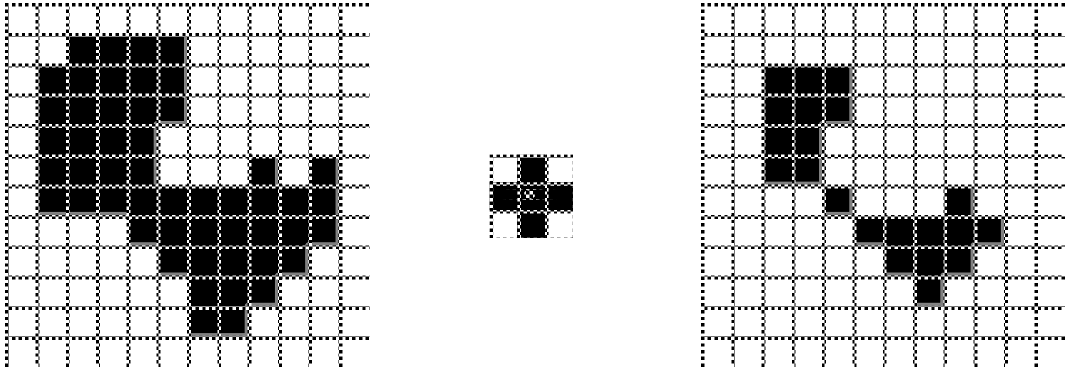
- Let p be a pixel of an image X and M a structuring element.
  - $\mathrm{M_p}$ represents the set of pixels in X corresponding to the black pixels of M when its anchor point is located at p

Erosion

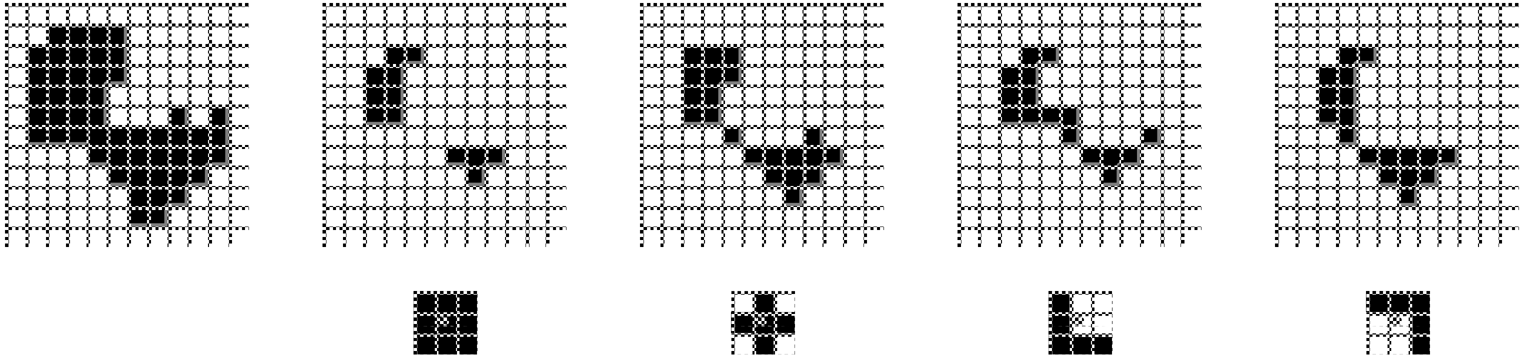
- Let X be a binary image and M a structuring element. The **erosion** of X by M is a binary image defined as

$$X \ominus M = \{p | M_p \subset X\}$$

  - Example : binary image, structuring element and resulting erosion



  - More examples, with various structuring elements

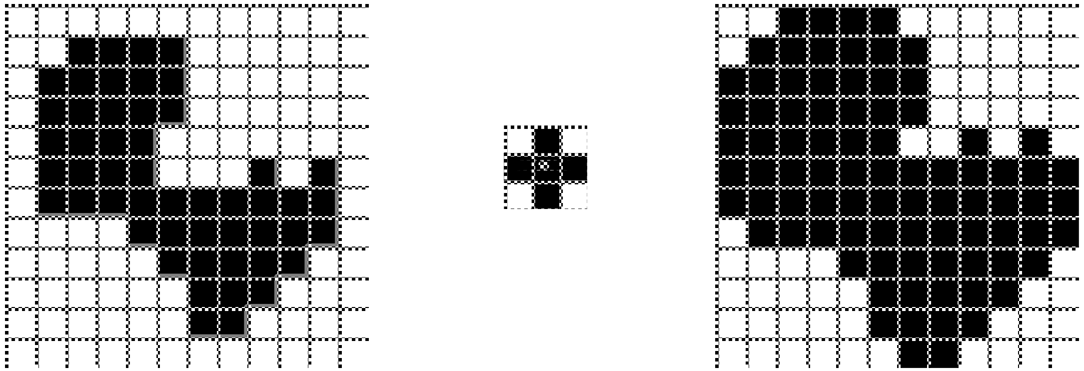


Dilation

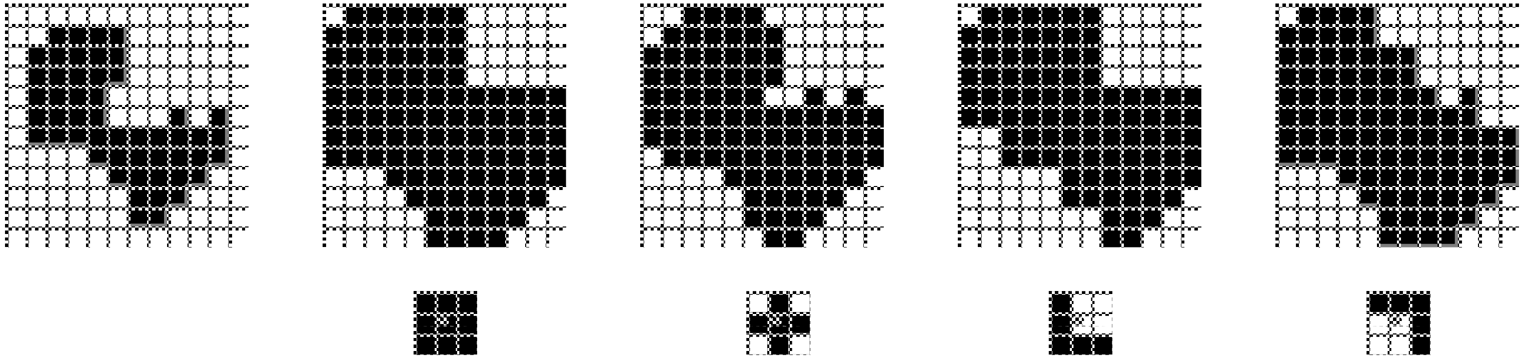
- Let X be a binary image and M a structuring element. The **dilation** of X by M is a binary image defined as

$$X \oplus M = \{p | M_p \cap X \neq \emptyset\}$$

  - Example : binary image, structuring element and resulting dilation



  - More examples, with various structuring elements



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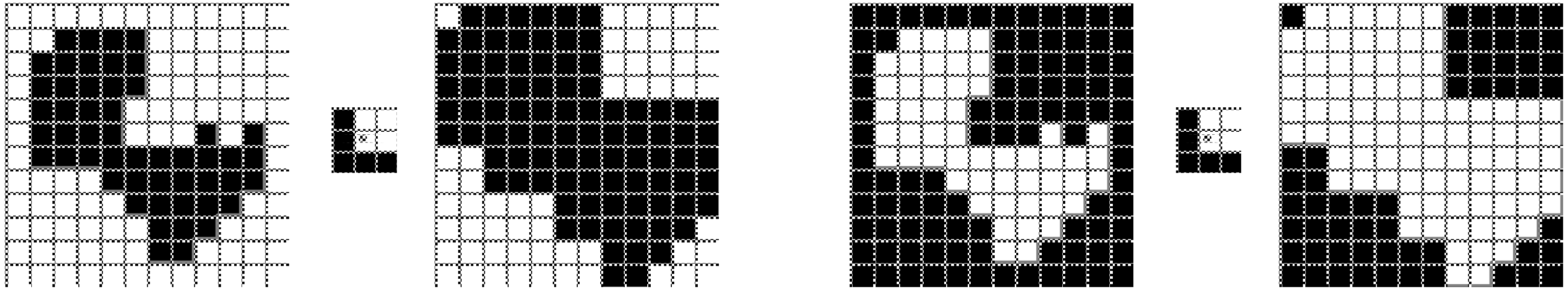
Duality of Erosion and Dilation

- Dilation and erosion are duals, i.e. they have the following properties

$$\bar{X} \ominus M = \overline{X \oplus M}$$
$$\bar{X} \oplus \mathrm{M} = \overline{X \ominus M}$$

where $\bar{X}$ represents the inverse of a binary image X.



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Opening and Closing

- Let X be a binary image and M a structuring element. The **opening** of X by M is a binary image defined as

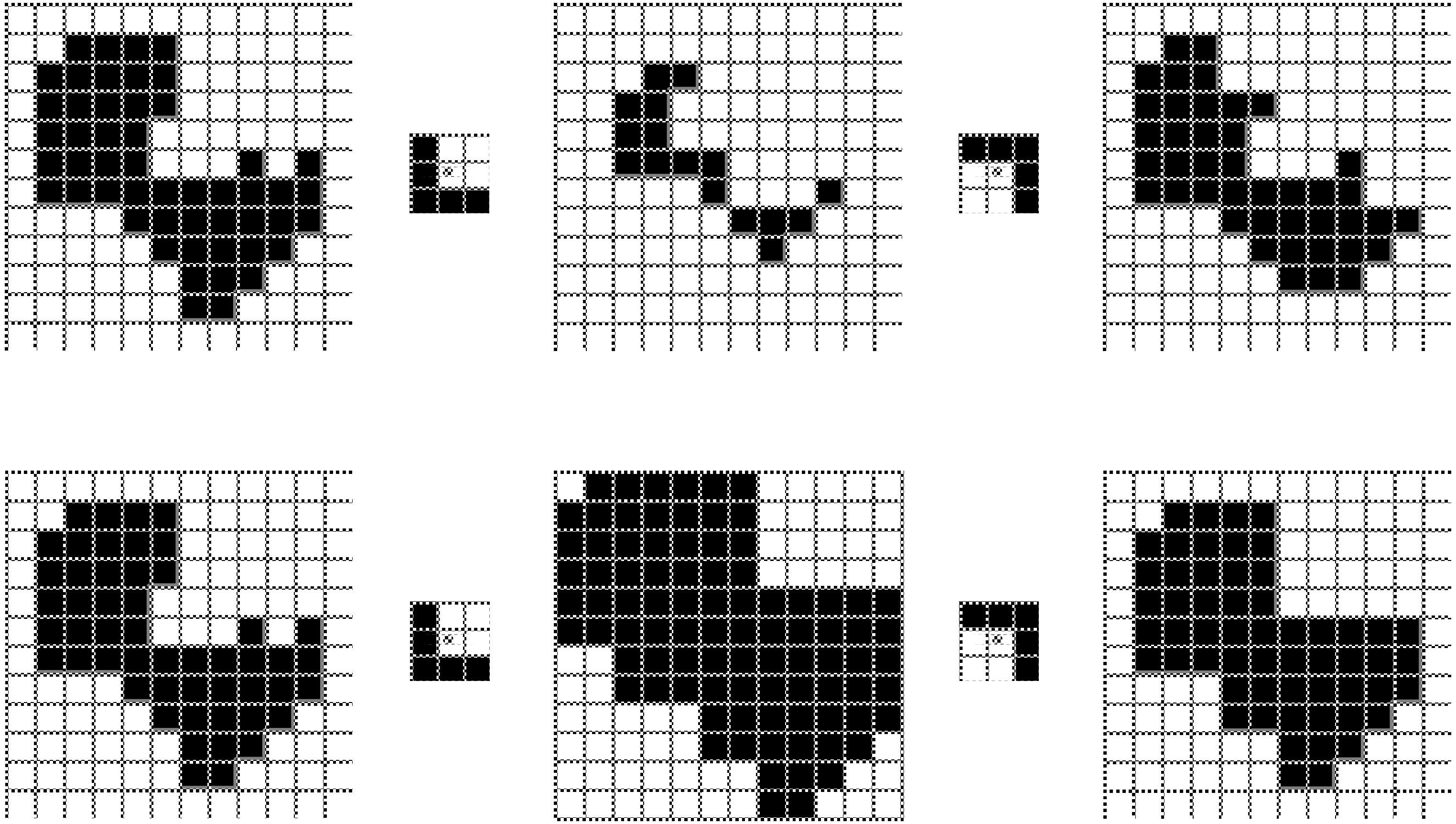
$$X \circ M = (X \ominus M) \oplus M^{-}$$

- Let X be a binary image and M a structuring element. The **closing** of X by M is a binary image defined as

$$X \bullet M = (X \oplus M) \ominus M^{-}$$

Where M^{-} represents the symmetric of M (with respect to the anchor point).

Opening and Closing Examples Using Asymmetric Masks



Algebraic Properties of Opening and Closing

- Opening and closing are **duals** of each other

$$\bar{X} \circ M = \overline{X \bullet M} \qquad \bar{X} \bullet M = \overline{X \circ M}$$

- Opening is **anti-extensive** and closing is **extensive**

$$X \circ M \subset X \qquad X \subset X \bullet M$$

- Opening and closing are **monotonically increasing**

$$X \subset Y \Rightarrow (X \circ M) \subset (Y \circ M) \qquad X \subset Y \Rightarrow (X \bullet M) \subset (Y \bullet M)$$

- Opening and closing are **idempotent**

$$(X \circ M) \circ M = X \circ M \qquad (X \bullet M) \bullet M = X \bullet M$$

- Opening is characterized by

$$(X \circ M) = \bigcup \{M_p | M_p \subset \mathrm{X}\}$$

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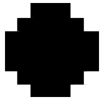
Visual Summary of Morphological Operations

Original

Structuring Element



Erosion :
$X \ominus M = \{p | M_p \subset X\}$

Opening
$X \circ M = (X \ominus M) \oplus M$

Dilation :
$X \oplus M = \{p | M_p \cap X \neq \emptyset\}$

Closing
$X \bullet M = (X \oplus M) \ominus M$

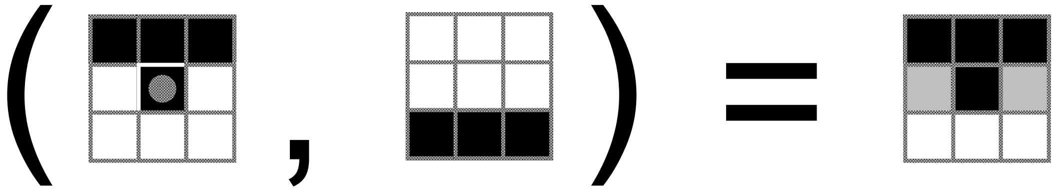
Hit and Miss Operator

- Let X be a binary image and $M=(M_1,M_0)$ a pair of structuring elements with the property $M_1 \cap M_0 = \emptyset$. The **hit and miss** operator is defined as follows

$$ham_{(M_0,M_1)}(X) = X \otimes (M_0, M_1) = (X \ominus M_1) \cap (\bar{X} \ominus M_0)$$

  - Erosion by M_1 selects foreground hits
  - Erosion by M_0 defines background misses

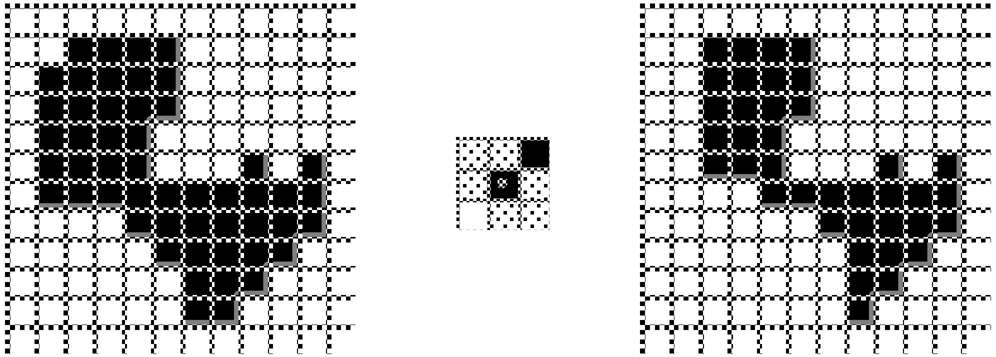
- **hit and miss** operator can be represented as ternary masks where:



Thinning

- Let X be a binary image and M a ternary mask whose anchor point is equal to 1. The **thinning** of X by M gives a binary image defined as

$$thin_M(X) = X - ham_M(X) = X \cap (ham_M(X))^C$$

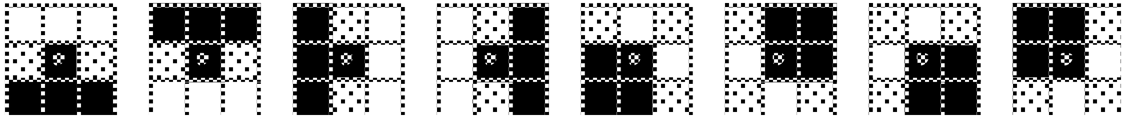


- The thinning of the left image X by the ternary mask M produces the right image $thin_M(X)$.

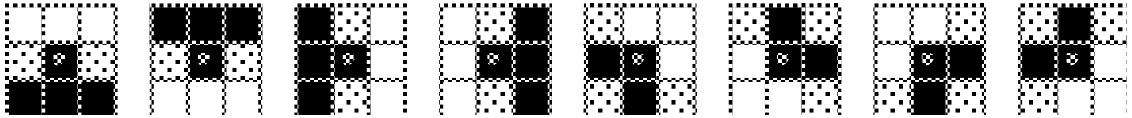
Homotopic Transformations

- A transformation is said to be **homotopic** if the connexity of all components is preserved, including wholes

- To **homotopic masks** used to erode a component, while preserving the connectivity
  - 4-connected components case:



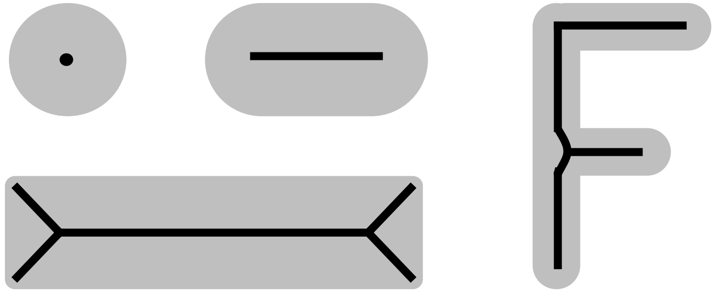
  - 8-connected components case:



Skeleton in Euclidean and Discrete Geometry

- Let X be a connected component in the Euclidean (continuous) space. The skeleton $Skel(X)$ of X is the set of all points being the canters of the maximal inscribed circles, that is

$$Skel(X) = \{ s \mid \exists x,y \in bound(X),\ x \neq y \text{ et } d(s,x)=\mathrm{d}(s,y) \}$$



- Unfortunately, there is no satisfying definition for the discrete space
- **Skeletonization** is achieved by iterative homotopic transforms converting a connected component X into skinny curves preserving the topology

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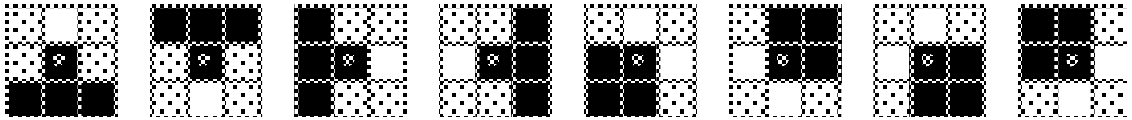
Skeletonization

- Computing the skeleton of a connected component X can be achieved with an iterative homotopic thinning, by means of the following ternary masks M_k (k=1, 2 ,..., 8):



Algorithm

$X_0 \leftarrow X$

<u>for</u> i = 0, 1, 2, ...

$$X_{i+1} \leftarrow thin_{M8}\,(thin_{M7}\,(\ldots\,(thin_{M2}\,(thin_{M1}\,(X_i)\,)\,\ldots)\,)$$

<u>repeat until</u> $X_{i+1} = X_i$

$Skel(X) \leftarrow X_i$

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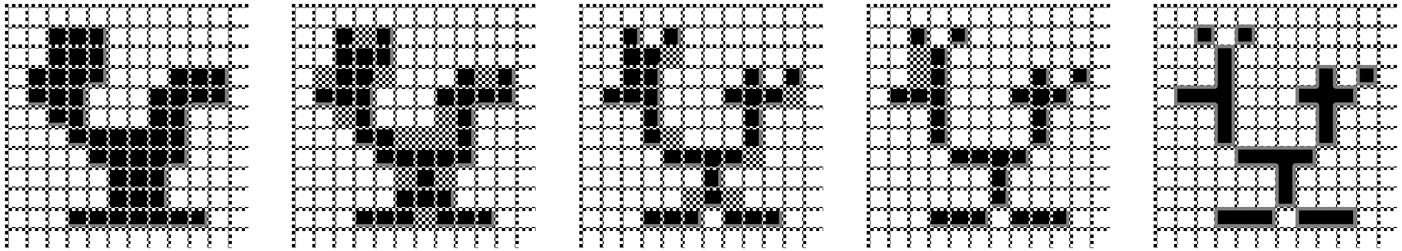
Skeleton Construction Steps

- Below, an illustration of the skeleton construction steps. The first image represents the initial connected component X, the second one results from the thinning of X by the masks M_1 to M_4, the next two images represent the iterations X_1, X_2, and finally the resulting skeleton, $Sk(X)$.



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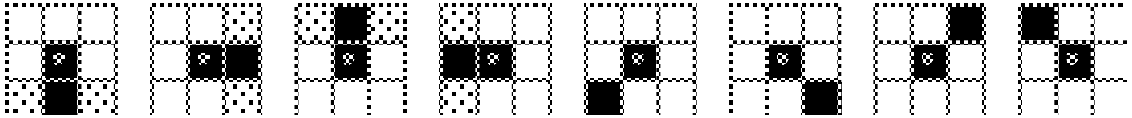
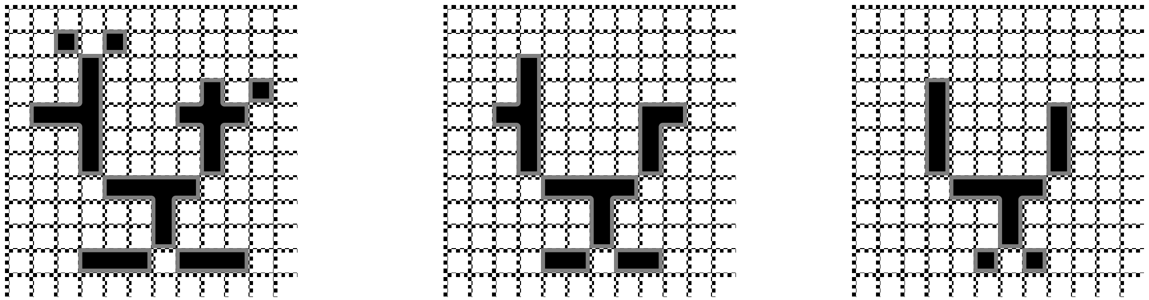
Pruning

- Skeletonization produces small noisy branches which are not considered to be part of the true skeleton structure.
- **Pruning** is a morphological operation which aims at removing those branches. It consists of an iterative process which removes terminal pixels by means of hit and miss operation using



- The illustration below shows two consecutive iterations of the pruning algorithm on a skeleton



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