Decision Trees
Besluitnemingsbome

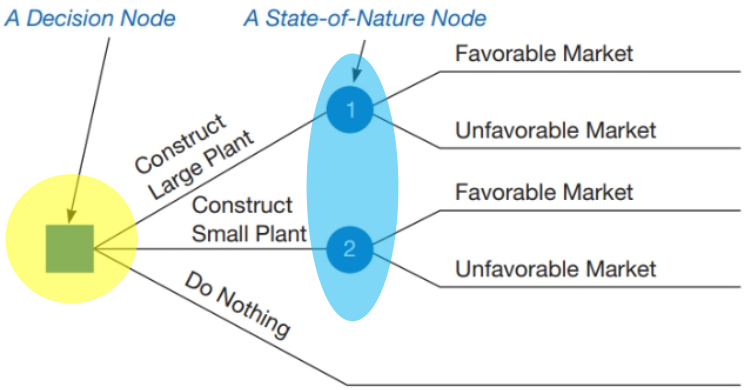
Enige probleem in 'n besluitnemingstabel kan grafies as 'n besluitnemingsboom voorgestel word

- *Beste wanneer opeenlopende besluite gemaak moet word*
- *Alle besluitnemingsbome bevat besluitnemingsnodes en uitkomstenodes*
- *By besluitnemingsnodes kan verskeie alternatiewe gekies word*
- *By uitkomstenodes sal een toestand plaasvind*

Any problem that can be presented in a decision table can be graphically represented in a decision tree

- Most beneficial when a sequence of decisions must be made
- All decision trees contain decision nodes and state-of-nature nodes
- At decision nodes one of several alternatives may be chosen
- At state-of-nature nodes one state of nature will occur

	STATE OF NATURE	
	FAVORABLE MARKET	UNFAVORABLE MARKET
ALTERNATIVE	($)	($)
Construct a large plant	200,000	−180,000
Construct a small plant	100,000	−20,000
Do nothing	0	0



Five steps of decision tree analysis
Vyf stappe van besluitnemingsboom-analise

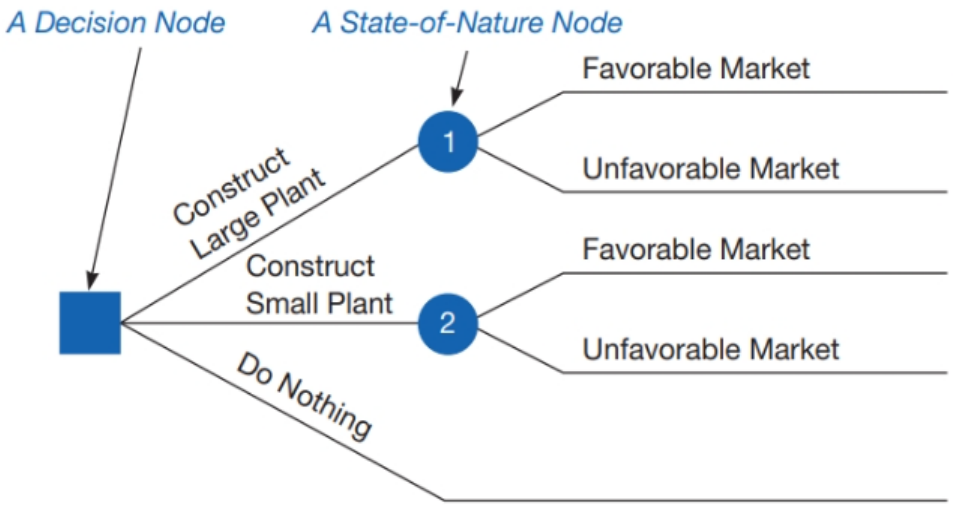
1. *Definieer die probleem*
2. *Teken die besluitnemingsboom*
3. *Ken waarskynlikhede aan die uitkomste toe*
4. *Bepaal die opbrengste vir elke kombinasie van alternatiewe en uitkomste*
5. *Los die probleem op deur verwagte geldelike waardes (EMV) vir elke uitkoms te bereken*

1. Define the problem
2. Draw the decision tree
3. Assign probabilities to the states of nature
4. Estimate payoffs for each possible combination of alternatives and states of nature
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node

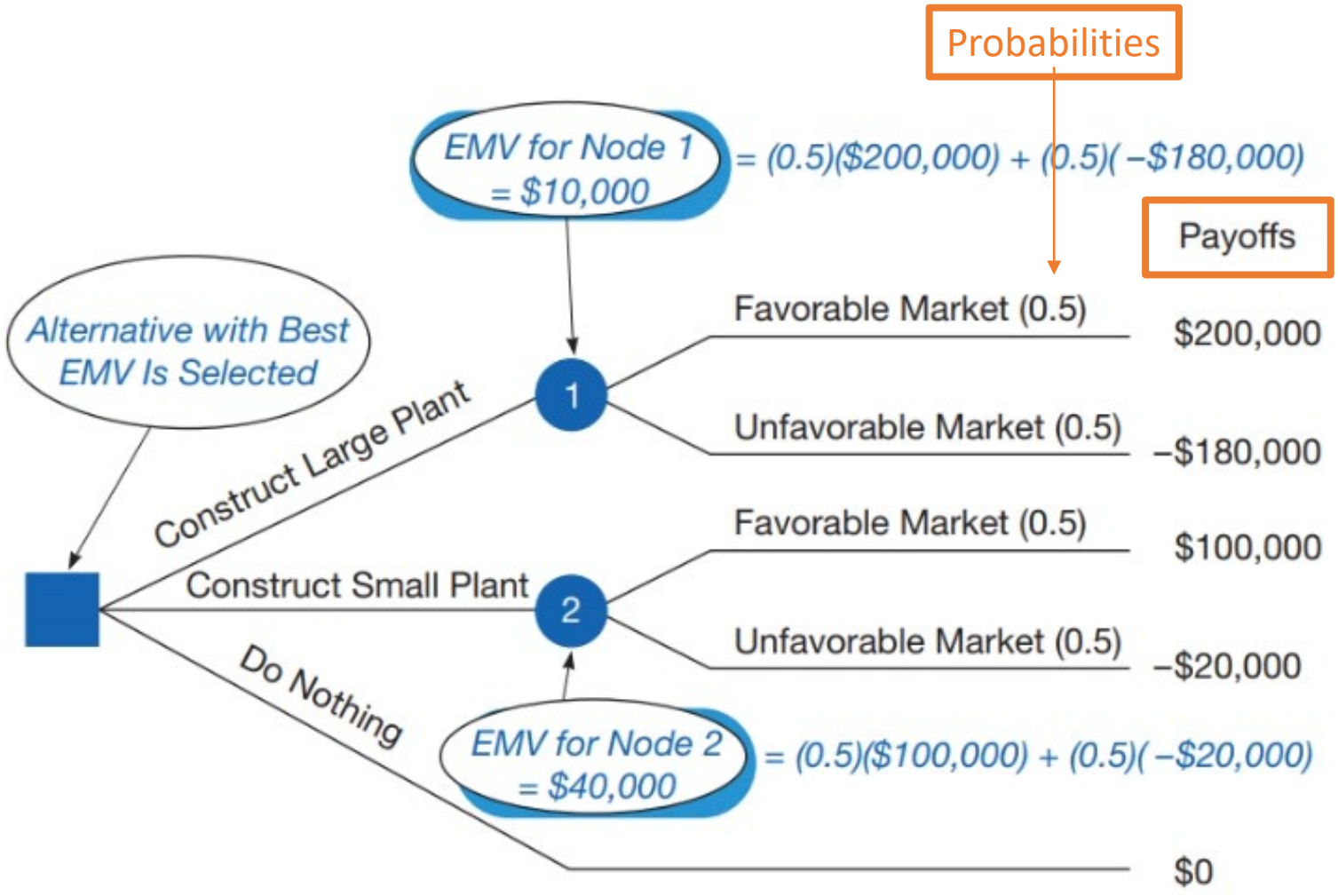
Structure of a decision tree
Struktuur van 'n besluitnemingsboom

- *Bome begin van links na regs*
- *Bome verteenwoordig besluite en uitkomstes in sekwensiële volgorde*
- *Vierkante verteenwoordig besluitnemingsnodes*
- *Sirkels verteenwoordig uitkomstes*
- *Vertakkings verbind besluitnemingsnodes met uitkomstes*

- Trees start from left to right
- Trees represent decisions and outcomes in sequential order
- Squares represent decision nodes
- Circles represent states of nature nodes
- Lines or branches connect the decisions nodes and the states of nature



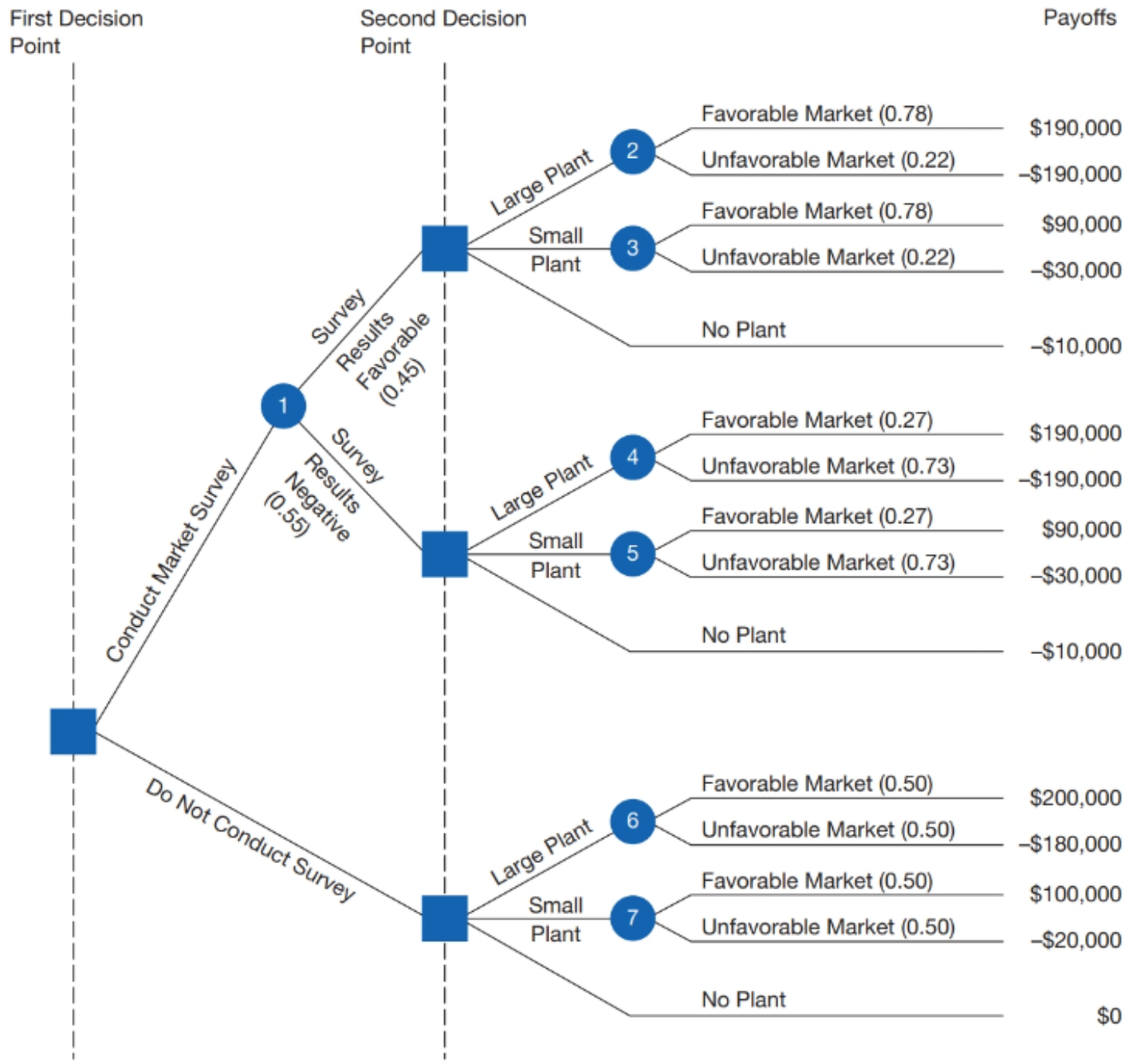
Example: Thompson Lumber Co.

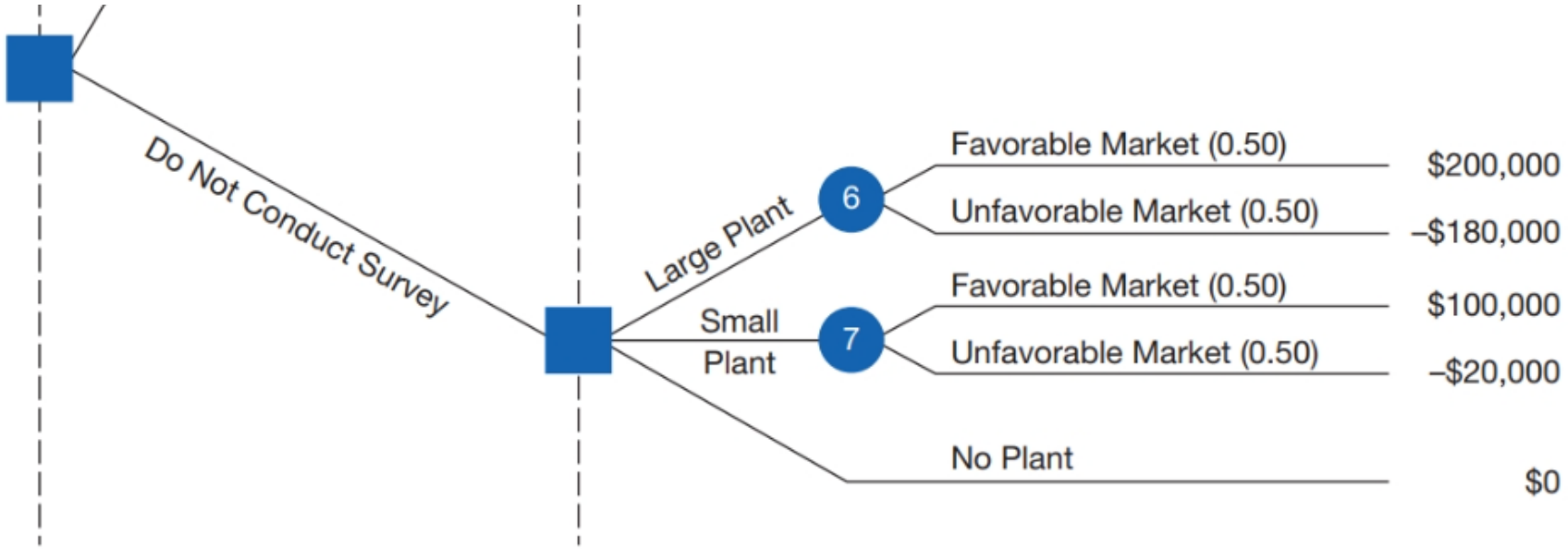


Sequential decisions
Opeenlopende besluite

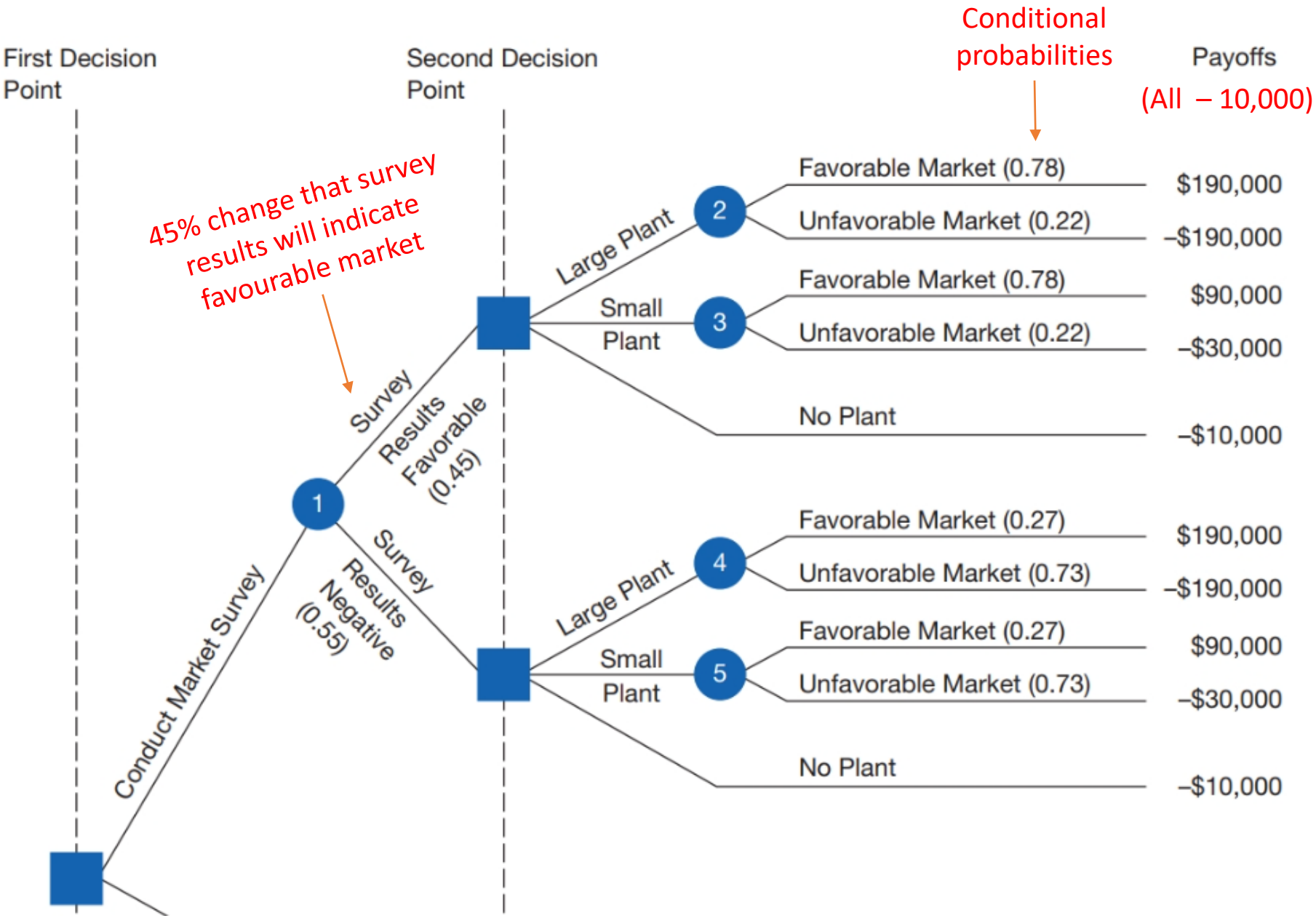
- Kom ons sê dat Jabar Thompson twee besluite het om te neem, met die tweede besluit afhanklik van die uitkoms van die eerste.
- Voordat hy besluit om 'n nuwe aanleg te bou, het Jabar die opsie om 'n bemarkingsnavorsingsopname teen 'n koste van $10 000 te doen.
- Die inligting van hierdie opname kan Jabar help om te besluit of hy 'n groot aanleg of 'n klein aanleg wil bou of glad nie.
- Jabar besef dat so 'n markopname nie perfekte inligting sal verskaf nie, maar dit kan tog nogal baie help.

- Let's say that Jabar Thompson has two decisions to make, with the second decision dependent on the outcome of the first.
- Before deciding about building a new plant, Jabar has the option of conducting a marketing research survey, at a cost of $10,000.
- The information from this survey could help Jabar decide whether to construct a large plant or a small plant or not to build at all.
- Jabar recognizes that such a market survey will not provide perfect information, but it may help quite a bit nevertheless.


First Decision Point
Second Decision Point
Payoffs
Conduct Market Survey
Do Not Conduct Survey
Survey Results Favorable (0.45)
Survey Results Negative (0.55)
Large Plant
Small Plant
No Plant
Favorable Market (0.78)
Unfavorable Market (0.22)
Favorable Market (0.27)
Unfavorable Market (0.73)
Favorable Market (0.50)
Unfavorable Market (0.50)
$190,000
–$190,000
$90,000
–$30,000
–$10,000
$200,000
–$180,000
$100,000
–$20,000
$0



Identical to the simpler decision tree



Remember: $10,000 market survey may not result in perfect or even reliable information

Calculating the EMVs

Given favorable survey results

EMV(node 2) = EMV(large plant|positive survey)

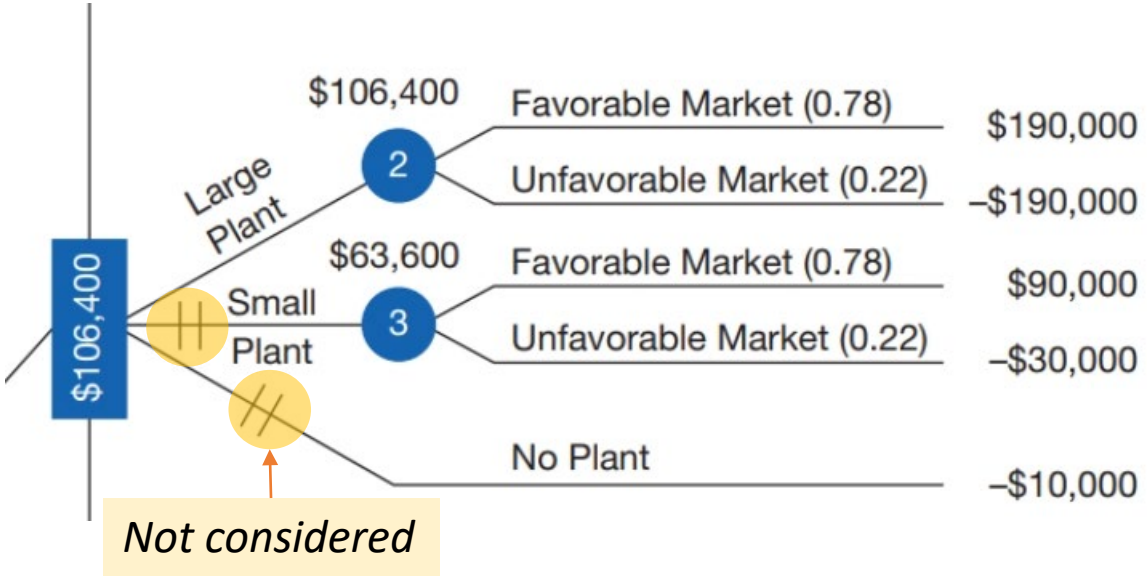
= (0.78)($190,000) + (0.22)(−$190,000) = $106,400

EMV(node 3) = EMV(small plant|positive survey)

= (0.78)($90,000) + (0.22)(−$30,000)

= $63,600

EMV for no plant = −$10,000



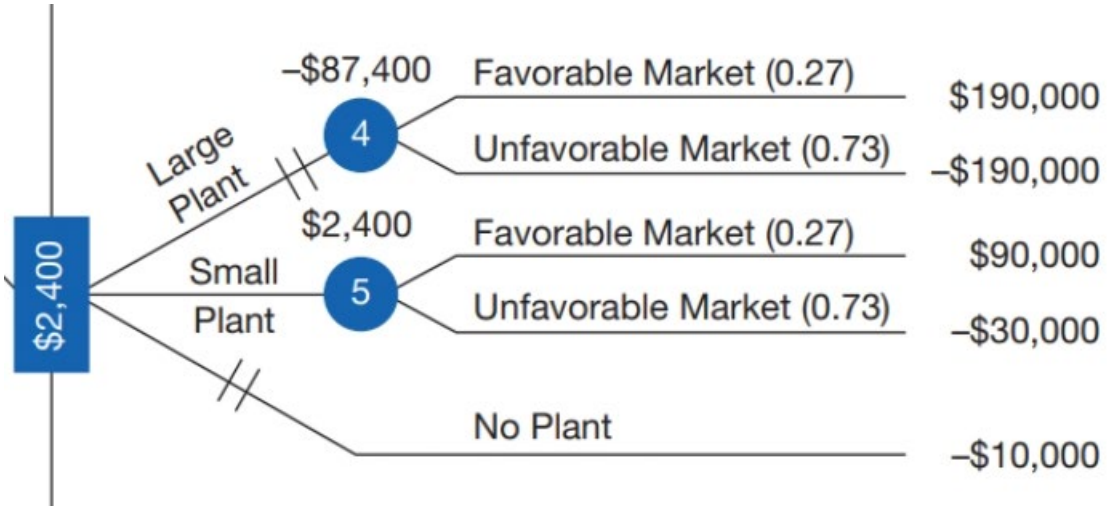
Calculating the EMVs

Given negative survey results

EMV(node 4) = EMV(large plant|negative survey)
= (0.27)($190,000) + (0.73)(−$190,000)
= −$87,400

EMV(node 5) = EMV(small plant|negative survey)
= (0.27)($90,000) + (0.73)(−$30,000)
= $2,400

EMV for no plant = −$10,000



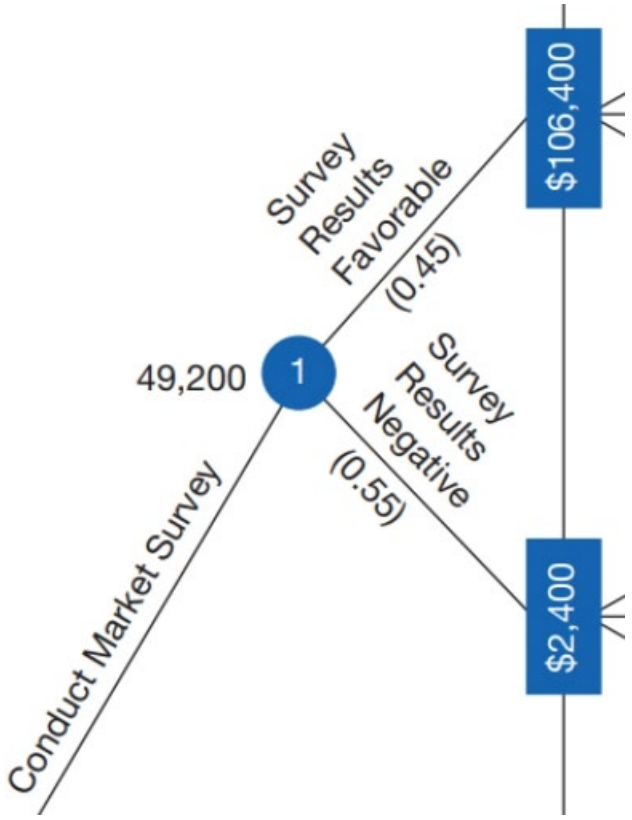
Calculating the EMVs

Expected value of the market survey

EMV(node 1) = EMV(conduct survey)

= (0.45)($106,400) + (0.55)($2,400)

= $47,880 + $1,320 = $49,200



Calculating the EMVs

Expected value no market survey

EMV(node 6) = EMV(large plant)

= (0.50)($200,000) + (0.50)(−$180,000)

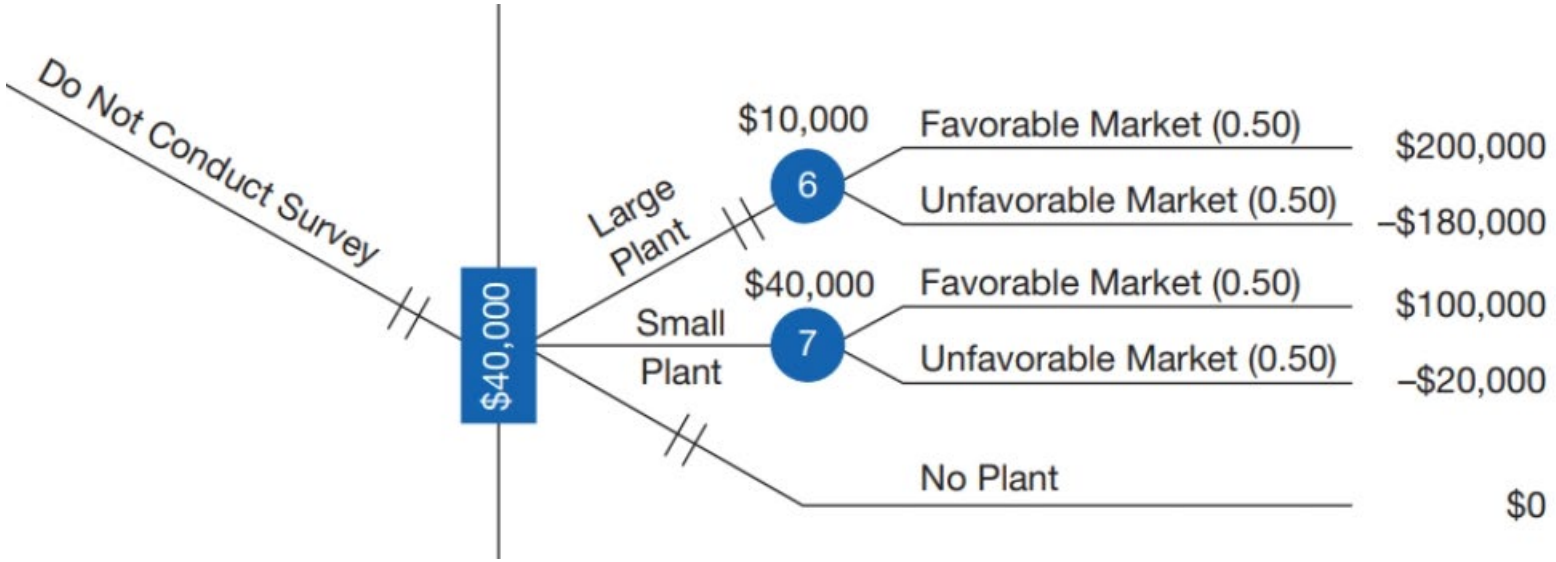
= $10,000

EMV(node 7) = EMV(small plant)

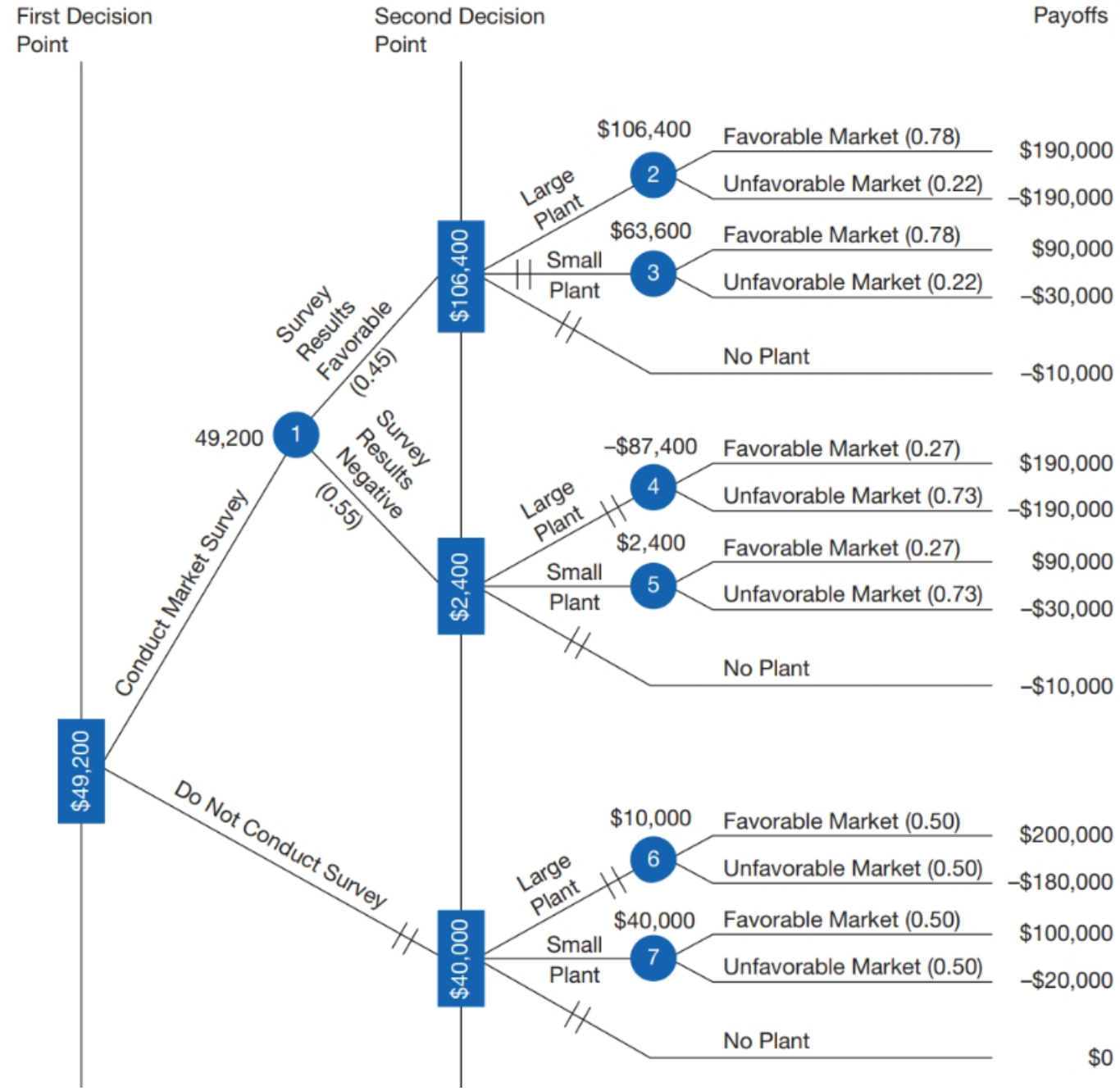
= (0.50)($100,000) + (0.50)(−$20,000)

= $40,000

EMV for no plant = $0



Thompson's Decision Tree with EMVs Shown



Expected Value of Sample Information (EVSI)

Verwagte waarde van steekproefinligting

- Thompson wil weet wat is die werklike waarde van die opname:
- Thompson wants to know the actual value of doing the survey:

Cost added since it was subtracted from all the payoffs before

EVSI = (EV with SI + cost) – (EV without SI)

= ($49,200 + $10,000) – $40,000

= $19,200

"the increase in expected value resulting from the sample information"

Since the survey costs only $10,000, it is indeed worthwhile

Efficiency of sample information
Doeltreffendheid van steekproef inligting

- *Baie moontlike tipes steekproefinligting beskikbaar*
- *Verskillende bronne kan geëvalueer word*

- Possibly many types of sample information available
- Different sources can be evaluated

While none of these sources of information are perfect, they can be evaluated by comparing the EVSI with the EVPI

Efficiency of sample information:

EVPI = EVwPI − Maximum EMV
= $100,000 − $40,000
= $60,000

$$\frac{\text{EVSI}}{\text{EVPI}}100\% = \frac{19{,}200}{60{,}000}100\% = 32\%$$

the market survey is only 32% as efficient as perfect information

Sensitivity analysis
Sensitiwiteitsanalise

- *Hoe sensitief is die besluite vir verandering in waarskynlikhede?*
- *Hoe sensitief is jou besluit vir die waarskynlikheid van ‘n gunstige opname?*
- *As die waarskynlikheid vir ‘n gunstige opname (p = 0.45) verander, sal jy dieselfde besluit maak?*
- *Hoeveel kan dit verander voordat jy ‘n ander besluit sou neem?*

- How sensitive are the decisions to changes in the probabilities?
- How sensitive is our decision to the probability of a favorable survey result?
- If the probability of a favorable result (p = 0.45) were to change, would we make the same decision?
- How much could it change before we would make a different decision?

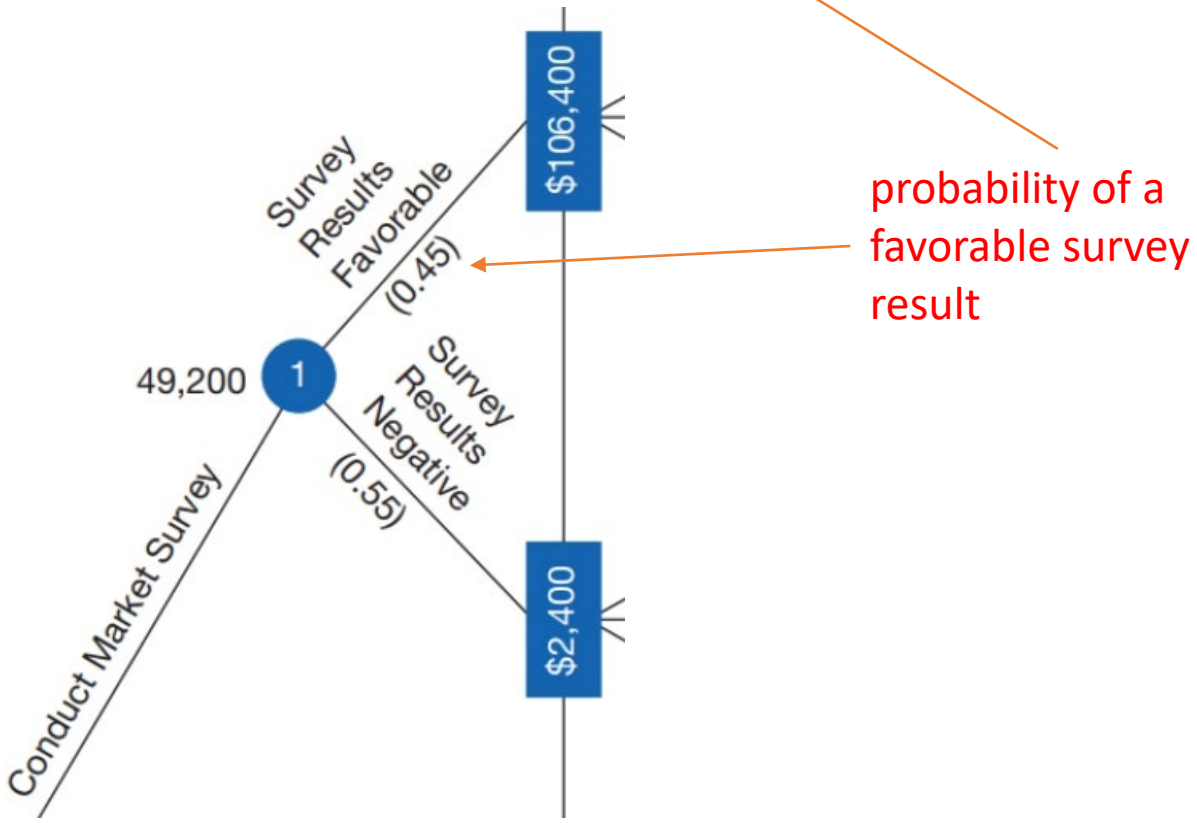
Calculating the expected value of the market survey

Expected value of the market survey

EMV(node 1) = EMV(conduct survey)

= (0.45)($106,400) + (0.55)($2,400)

= $47,880 + $1,320 = $49,200



probability of a favorable survey result

Sensitivity analysis

p = probability of a favorable survey result

$(1-p)$ = probability of a negative survey result

$$\text{EMV(node 1)} = (\$106{,}400)p + (\$2{,}400)(1-p)$$
$$= \$104{,}000p + \$2{,}400$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey

$$\$104{,}000p + \$2{,}400 = \$40{,}000$$

EMV of not conducting the survey

$$\$104{,}000p = \$37{,}600$$

$$p = \$37{,}600 \div \$104{,}000 = 0.36$$

If $p < 0.36$, do not conduct the survey

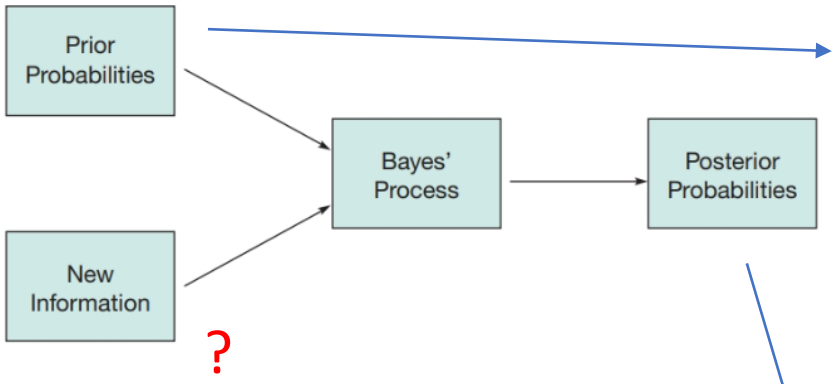
If $p > 0.36$, conduct the survey

Bayesian analysis
Bayiëse analise

- *Daar is baie maniere om waarskynlikheidsdata te kry*
  - *Ondervinding en intuïsie van bestuur*
  - *Historiese data*
  - *Bereken vanuit ander data met Bayes se teorie*
- *Bayes se teorie bevat aanvanklike beramings en inligting oor die akkuraatheid van die bronne*
- *Laat die hersiening van aanvanklike beramings toe op grond van nuwe inligting*

- Many ways of getting probability data
  - Management's experience and intuition
  - Historical data
  - Computed from other data using Bayes' theorem
- Bayes' theorem incorporates initial estimates and information about the accuracy of the sources
- Allows the revision of initial estimates based on new information

Calculating revised (posterior) probabilities
Berekening van hersiene waarskynlikhede



Prior probabilities *(Vooraf waarskynlikhede)*

$P(FM) = 0.50$

$P(UM) = 0.50$

Conditional probabilities *(voorwaardelike waarskynlikhede)*

$P(FM \mid \text{survey positive}) = 0.78$

$P(UM \mid \text{survey positive}) = 0.22$

$P(FM \mid \text{survey negative}) = 0.27$

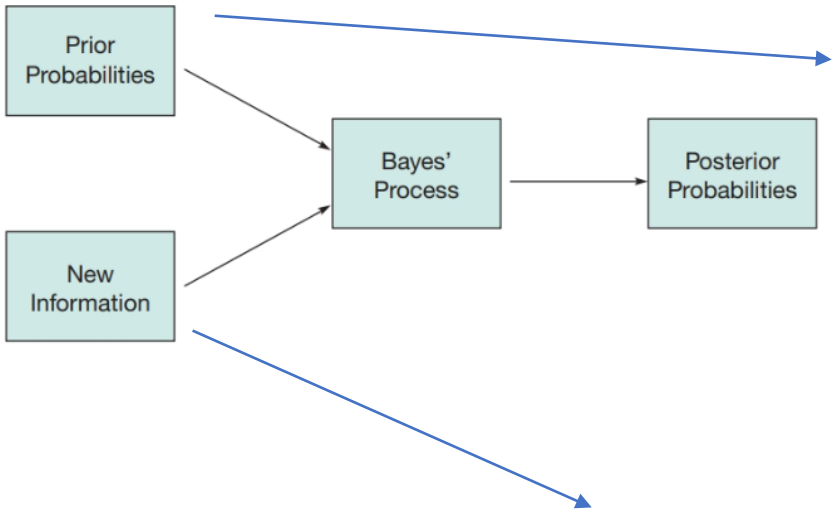
$P(UM \mid \text{survey negative}) = 0.73$

Calculating revised (posterior) probabilities
Berekening van hersiene waarskynlikhede

Information from market research specialists
Indicates accuracy of the market survey considered:

	STATE OF NATURE	
RESULT OF SURVEY	**FAVORABLE MARKET (FM)**	**UNFAVORABLE MARKET (UM)**
Positive (predicts favorable market for product)	$P(\text{survey positive} \mid \text{FM}) = 0.70$	$P(\text{survey positive} \mid \text{UM}) = 0.20$
Negative (predicts unfavorable market for product)	$P(\text{survey negative} \mid \text{FM}) = 0.30$	$P(\text{survey negative} \mid \text{UM}) = 0.80$

Calculating revised (posterior) probabilities
Berekening van hersiene waarskynlikhede



Prior probabilities *(Vooraf waarskynlikhede)*

$P(\text{FM}) = 0.50$

$P(\text{UM}) = 0.50$

$$P(\text{survey positive} \mid \text{FM}) = 0.70 \quad P(\text{survey positive} \mid \text{UM}) = 0.20$$

$$P(\text{survey negative} \mid \text{FM}) = 0.30 \quad P(\text{survey negative} \mid \text{UM}) = 0.80$$

Calculating revised (posterior) probabilities
Berekening van hersiene waarskynlikhede

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid A') \times P(A')}$$

Waar

A, *B* = enige twee gebeure

A' = komplement van *A*

Where

A, *B* = any two events

A'= complement of *A*

Calculating revised (posterior) probabilities
Berekening van hersiene waarskynlikhede

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid A') \times P(A')}$$

A = FM
B = survey positive

$$P(\text{FM} \mid \text{survey positive}) = \frac{P(\text{survey positive} \mid \text{FM})P(\text{FM})}{P(\text{survey positive} \mid \text{FM})P(\text{FM}) + P(\text{survey positive} \mid \text{UM})P(\text{UM})}$$

$$= \frac{(0.70)(0.50)}{(0.70)(0.50) + (0.20)(0.50)} = \frac{0.35}{0.45} = 0.78$$

Probability of positive survey

$$P(\text{FM}) = 0.50 \qquad P(\text{survey positive} \mid \text{FM}) = 0.70 \qquad P(\text{survey positive} \mid \text{UM}) = 0.20$$

$$P(\text{UM}) = 0.50$$

$$P(\text{survey negative} \mid \text{FM}) = 0.30 \qquad P(\text{survey negative} \mid \text{UM}) = 0.80$$

Calculating revised (posterior) probabilities
Berekening van hersiene waarskynlikhede

Can also calculate using a probability table:

Probability Revisions Given a Positive Survey

STATE OF NATURE	CONDITIONAL PROBABILITY *P*(SURVEY POSITIVE | STATE OF NATURE)	PRIOR PROBABILITY	POSTERIOR PROBABILITY: JOINT PROBABILITY	POSTERIOR PROBABILITY: *P*(STATE OF NATURE | SURVEY POSITIVE)
FM	0.70	×0.50	= 0.35	0.35/0.45 = 0.78
UM	0.20	×0.50	= 0.10	0.10/0.45 = 0.22
		P(survey results positive)	= 0.45	1.00

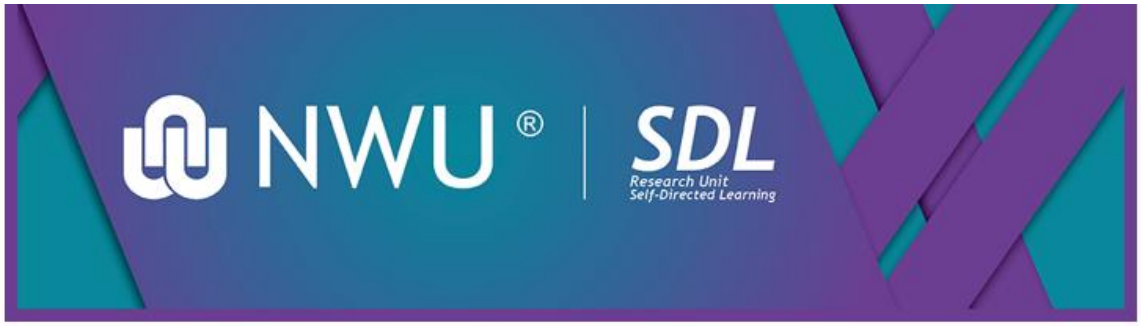
Probability Revisions Given a Negative Survey

STATE OF NATURE	CONDITIONAL PROBABILITY *P*(SURVEY NEGATIVE | STATE OF NATURE)	PRIOR PROBABILITY	POSTERIOR PROBABILITY: JOINT PROBABILITY	POSTERIOR PROBABILITY: *P*(STATE OF NATURE | SURVEY NEGATIVE)
FM	0.30	×0.50	= 0.15	0.15/0.55 = 0.27
UM	0.80	×0.50	= 0.40	0.40/0.55 = 0.73
		P(survey results negative)	= 0.55	1.00

Exercise: Solved problem 3-3 (p117)

Monica Britt has always enjoyed sailing small boats. Today, Monica is considering the possibility of starting a company to produce small sailboats for the recreational market. Unlike other mass-produced sailboats, however, these boats will be made specifically for children between the ages of 10 and 15. The boats will be of the highest quality and extremely stable, and the sail size will be reduced to prevent problems of capsizing.

Monica's basic decision is whether to build a large manufacturing facility, a small manufacturing facility, or no facility at all. With a favorable market, Monica can expect to make $90,000 from the large facility or $60,000 from the smaller facility. If the market is unfavorable, however, Monica estimates that the losses will be $30,000 with the large facility and $20,000 with the small facility. Because of the expense involved in developing the initial molds and acquiring the necessary equipment to produce fiberglass sailboats for young children, Monica has decided to conduct a pilot study to make sure that the market for the sailboats will be adequate. Monica estimates that the pilot study will cost $10,000. Furthermore, the pilot study can be either favorable or unfavorable. Monica estimates that the probability of a favorable market, given a favorable pilot study, is 0.8. The probability of an unfavorable market, given an unfavorable pilot study, is estimated to be 0.9. Monica feels that there is a 0.65 chance that the pilot study will be favorable. Of course, Monica could bypass the pilot study and simply decide whether to build a large plant, a small plant, or no facility at all. Without doing any testing in a pilot study, Monica estimates that the probability of a favorable market is 0.6. What do you recommend? Compute the EVSI.



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