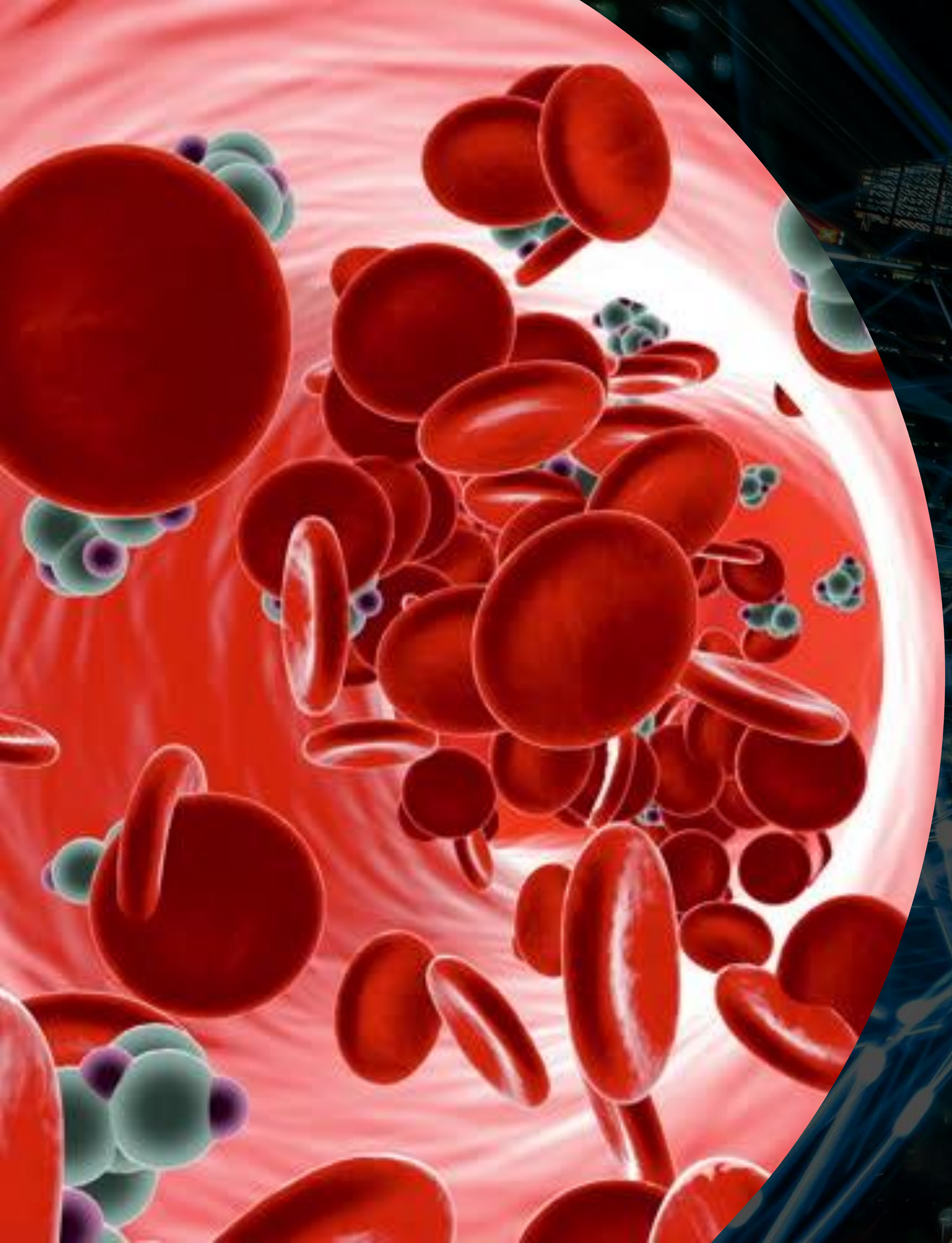

# *Physics Informed Neural Networks for Fluid Dynamics*

**NAPDE Course Project**


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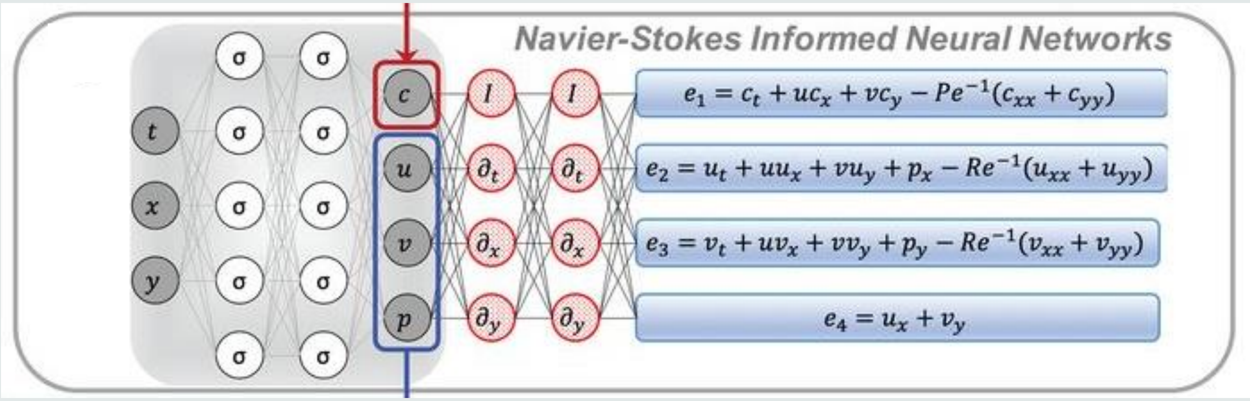
# The idea

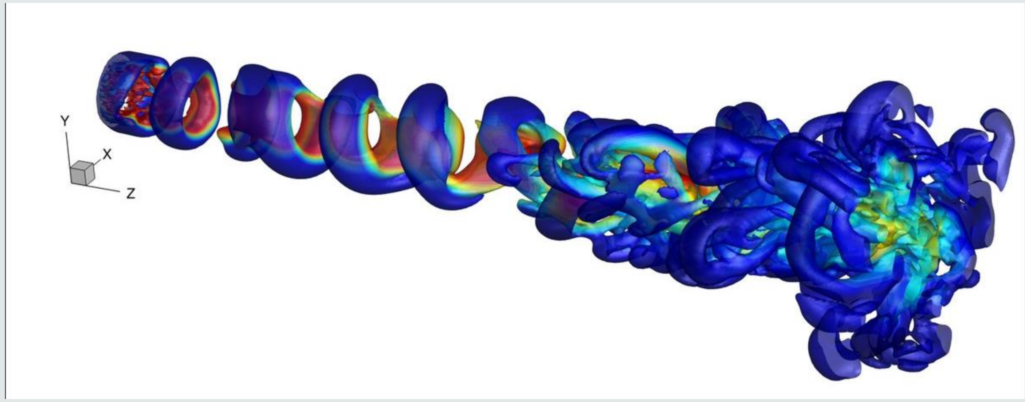
In the Fluid Dynamics framework we often meet problems involving **PDEs** which cannot be solved analitically.

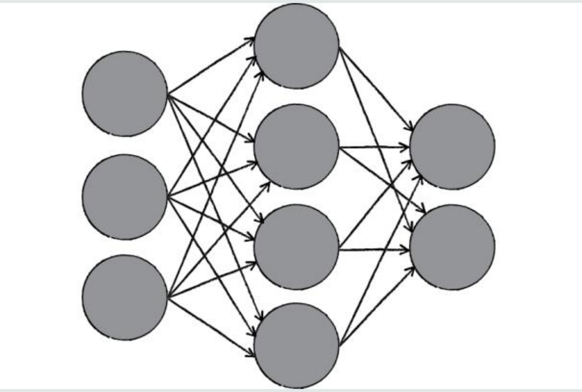
In fact, in this field many applications require to reconstruct **numerically** the behavior of **pressure** and **velocity**, starting from partial measures.



# *What is a PINN?*

- PINNs are Neural Networks trained to solve supervised learning tasks while respecting any given **law of physics** described by general **nonlinear partial differential equations**.



- For our project, the reference library for the automatic differentiation is **nisaba**, a Python Library built on the top of Tensorflow.

nisaba

# A simple test case

- We started from the **Colliding Flows** benchmark case, whose analytical solution is known:

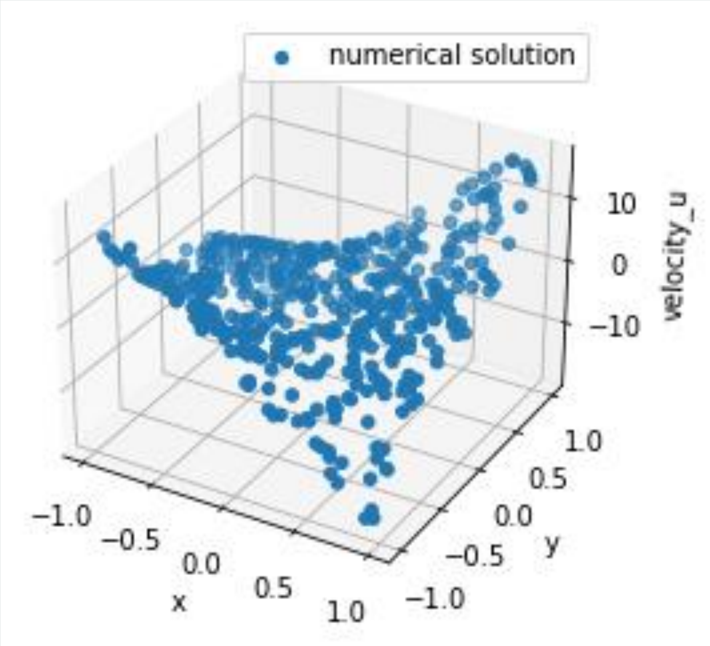
Given $\Omega = (-1,1) \times (-1,1)$, find $(\mathbf{u}, p)$ s.t.

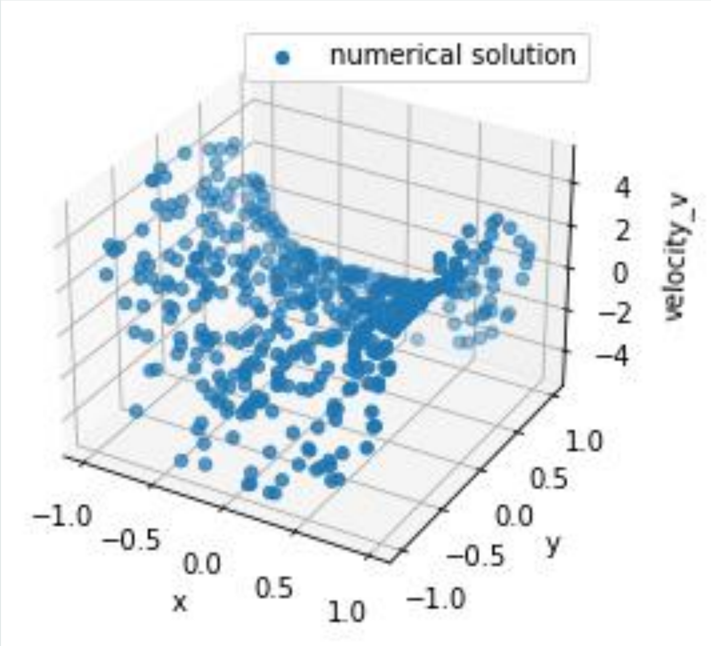
$$\begin{cases} -\Delta \mathbf{u} + \nabla p = \mathbf{f} = 0 & \text{in } \Omega \\ \text{div } \mathbf{u} = 0 & \text{in } \Omega \\ \mathbf{u} = \mathbf{g} = (20xy^3, 5x^4 - 5y^4)^T & \text{on } \partial\Omega \end{cases}$$

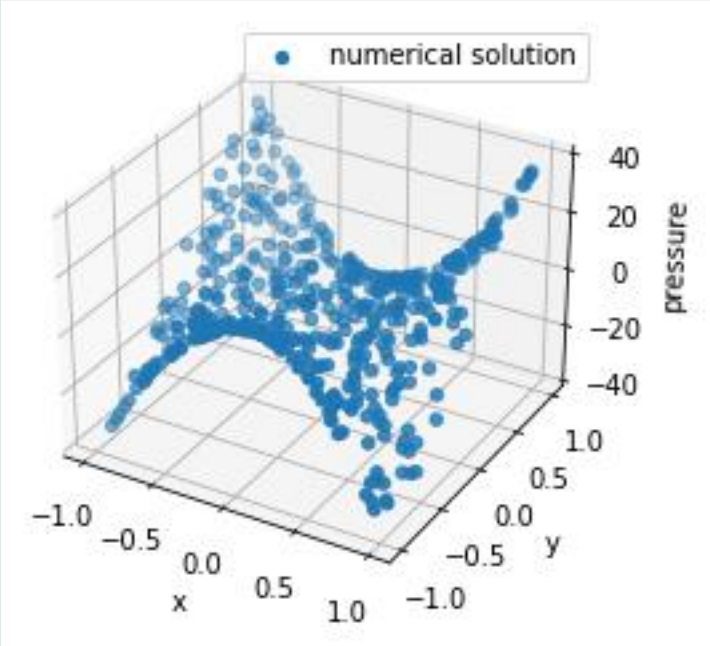
**CHALLENGE:**

- Train the PINN only with **data on the BCs** for the velocities and by constraining the **mean value** of the pressure!

# *The results*

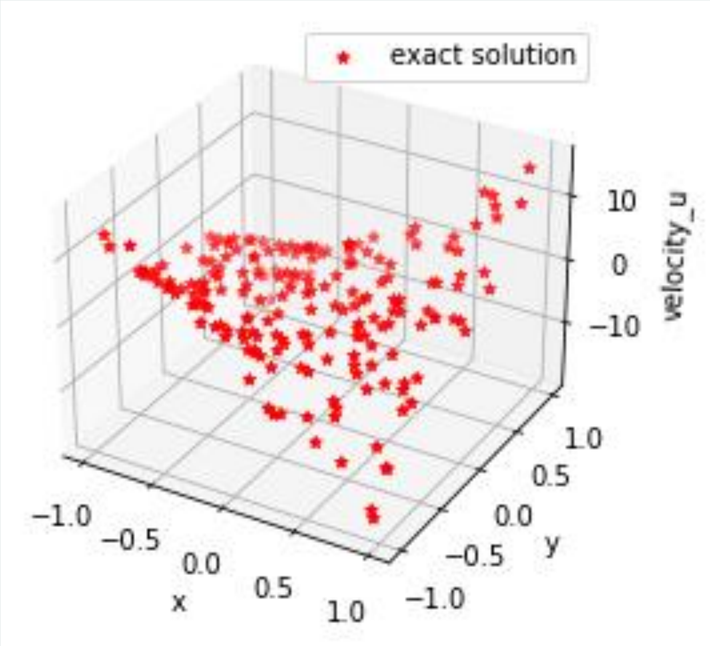


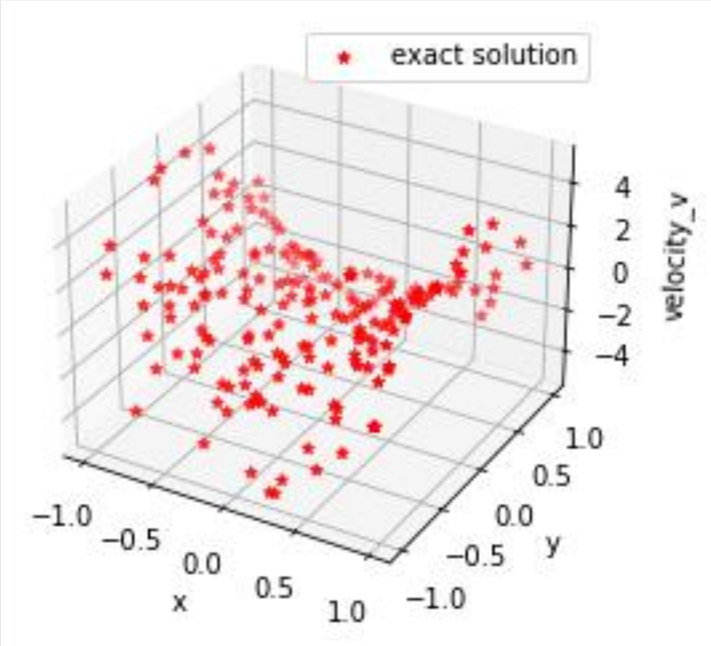



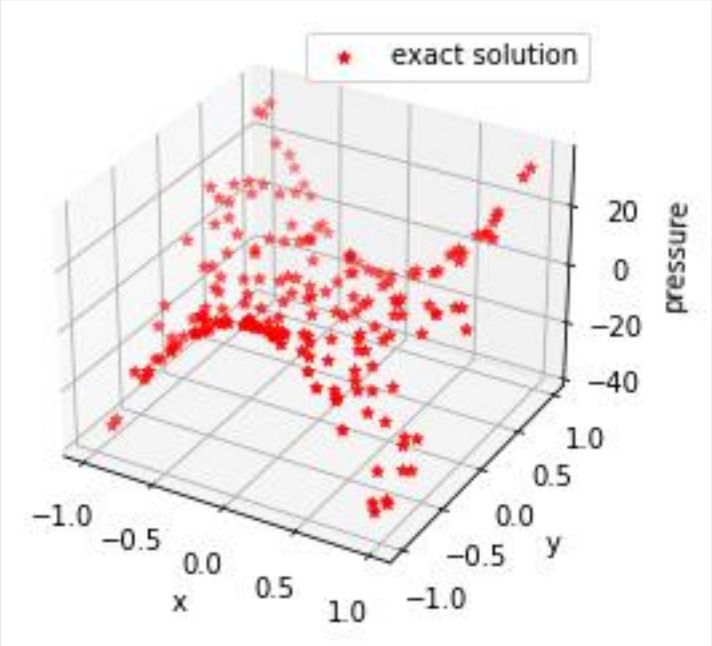




2000 epochs, **no noise** on data

# The results

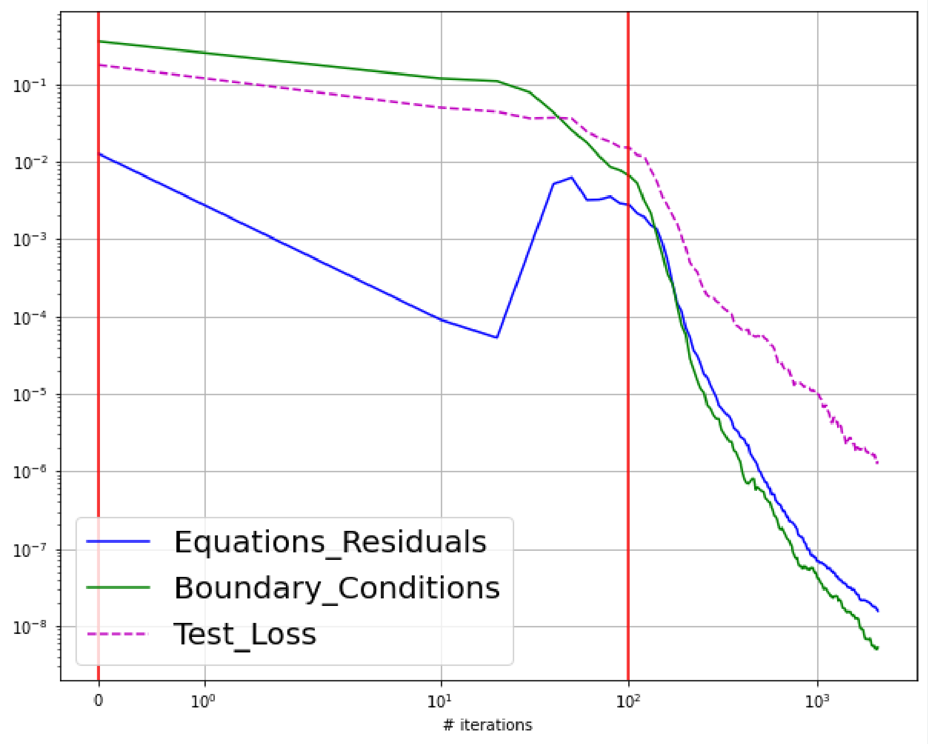


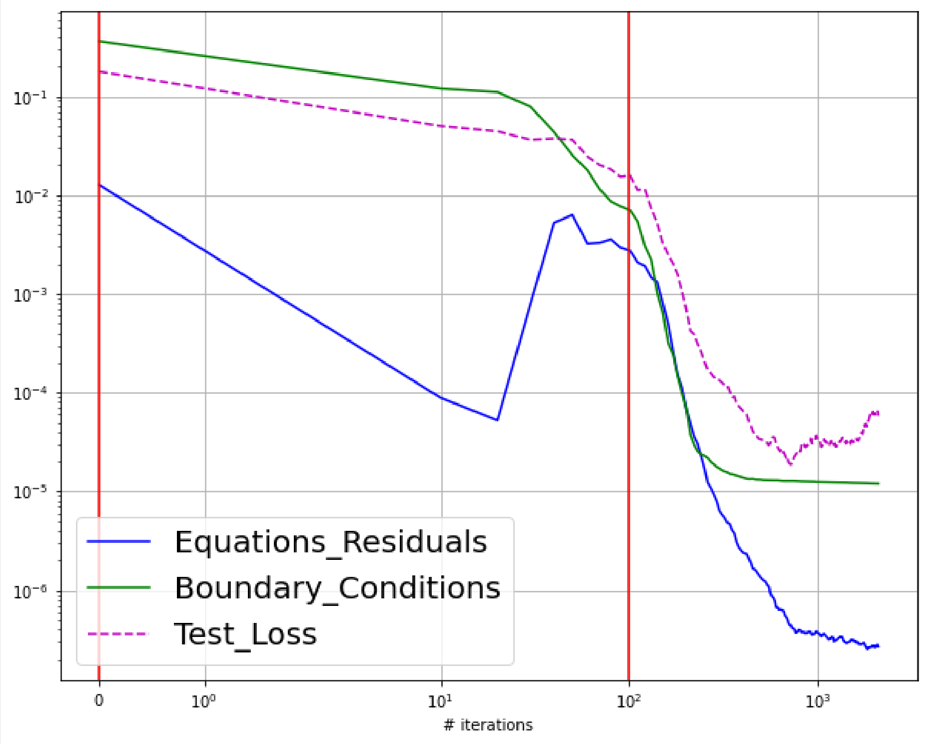




2000 epochs, **no noise** on data

# *The results*



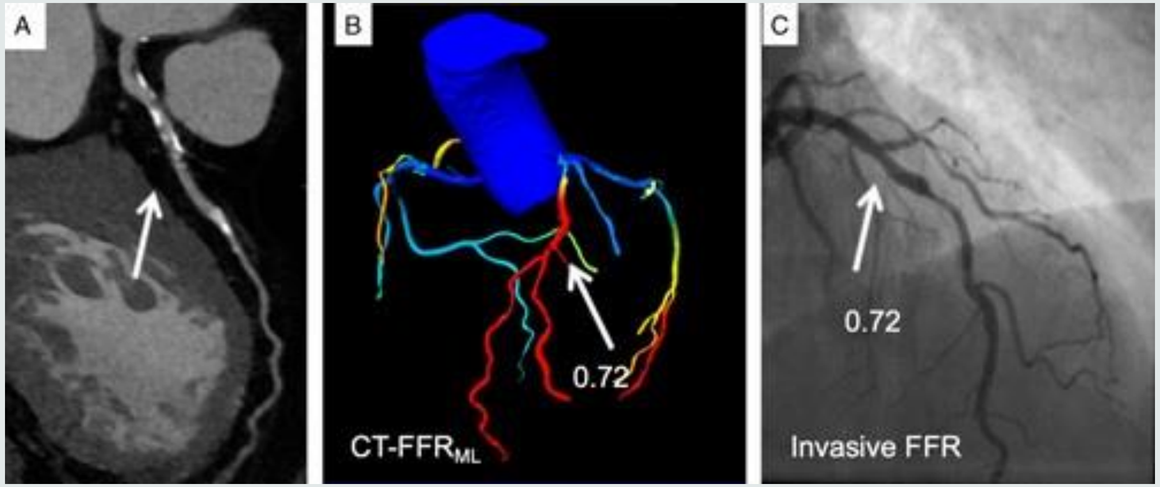

2000 epochs, **no noise** on data




2000 epochs, **with gaussian noise** ($\mu = 0, \sigma = 1$) on data

# Our prospect

Our next step is to apply this strategy to a test case more complex and closer to reality, such as the **Coronary Flow Assessment**, fundamental to evaluate **coronary artery disease**.
In this case, there is no hope for an analytical solution.



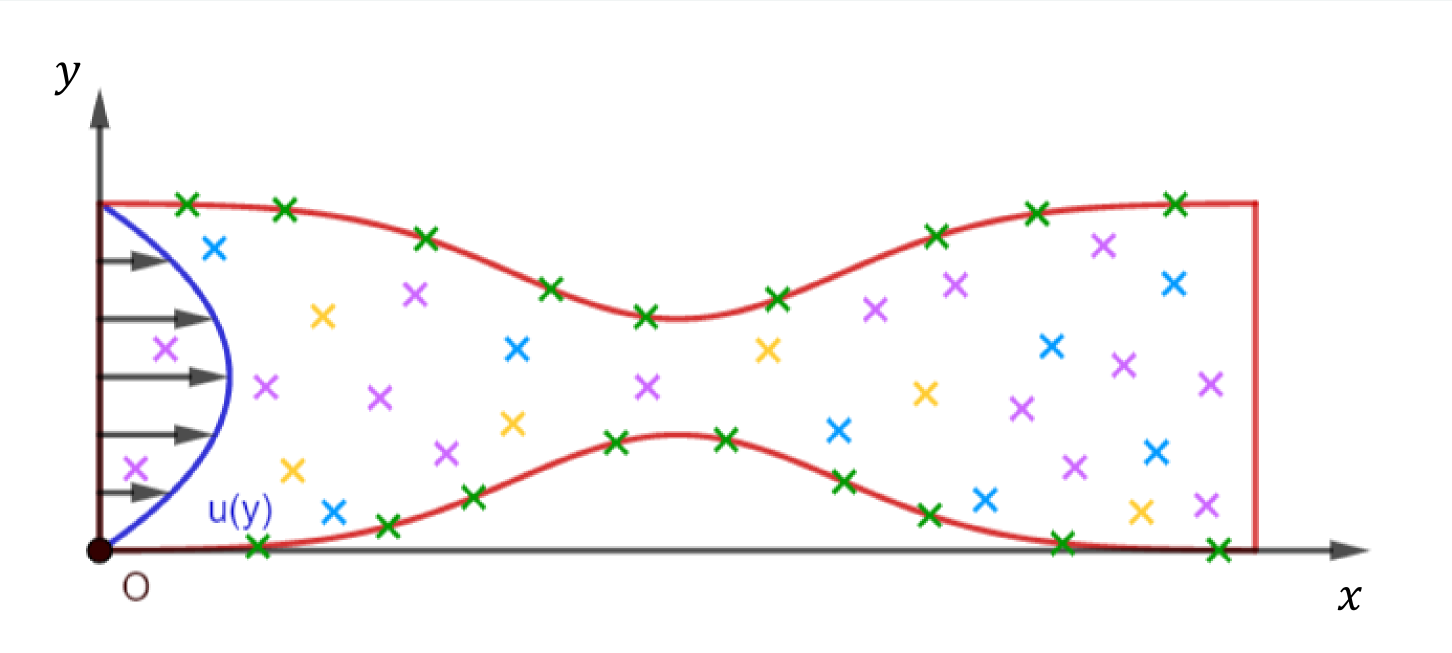
# Work Plan

- To train our Neural Network, we plan to exploit two types of information:

**Geometrical Data**
(eg: branch length, vessel radius…)

**+**

**Data on velocity**
(provided by medical imaging)




Legend:
x Boundary Points
x Training Points
x Collocation Points
x Test Points

# Bibliography


- ***Hidden fluid mechanics: Learning velocity and pressure fields from flow visualizations*** - Maziar Raissi; Alireza Yazdani; George Em Karniadakis

- ***Machine Learning and Deep Neural Networks Applications in Coronary Flow Assessment*** - Christian Tesche; Hunter N.Gray

- ***Physics-informed neural networks (PINNs) for fluid mechanics: A review*** - Shengze Cai; Zhiping Mao; Zhicheng Wang; Minglang Yin; George Em Karniadakis

# Thank you for your attention!