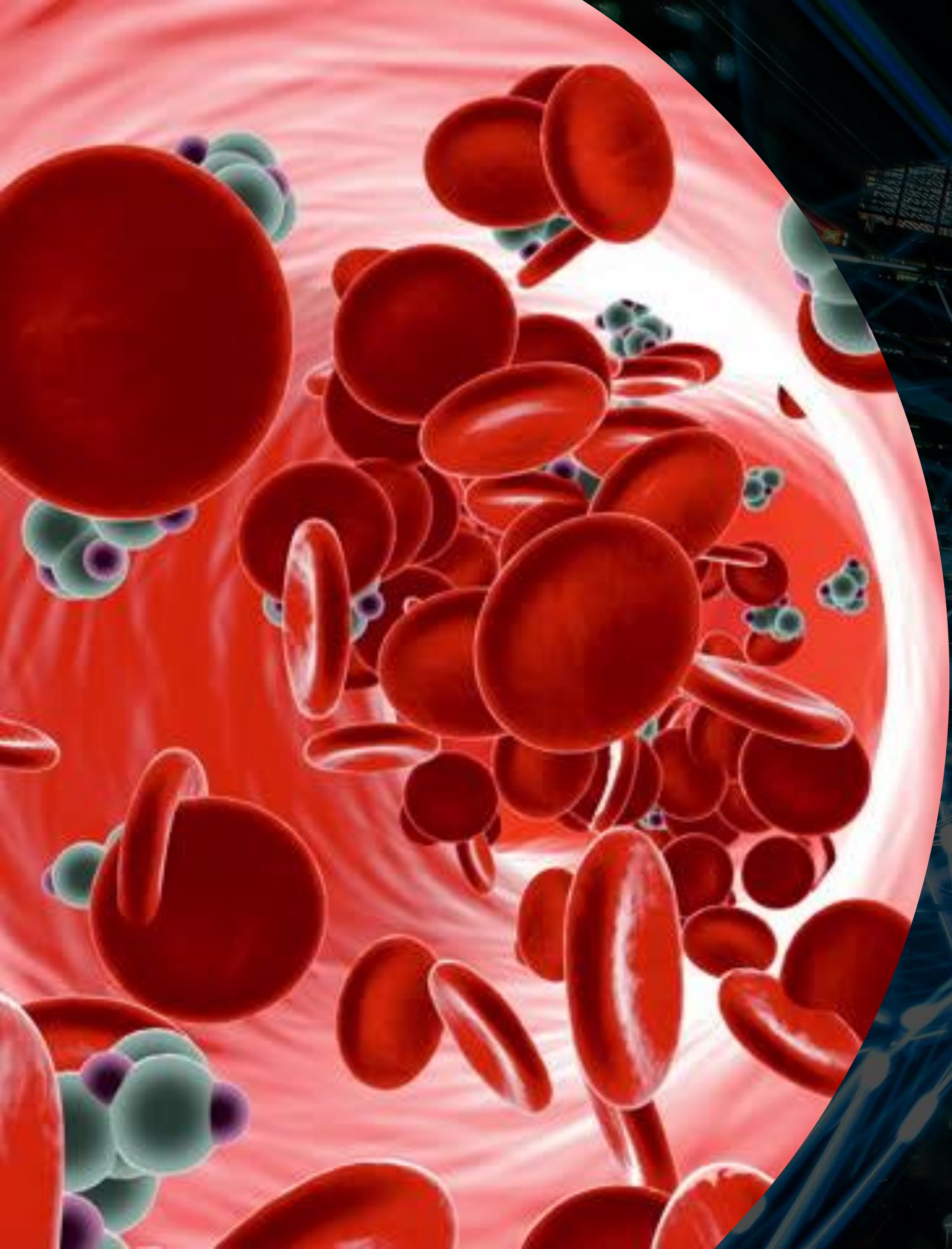


Physics Informed Neural Networks for Fluid Dynamics

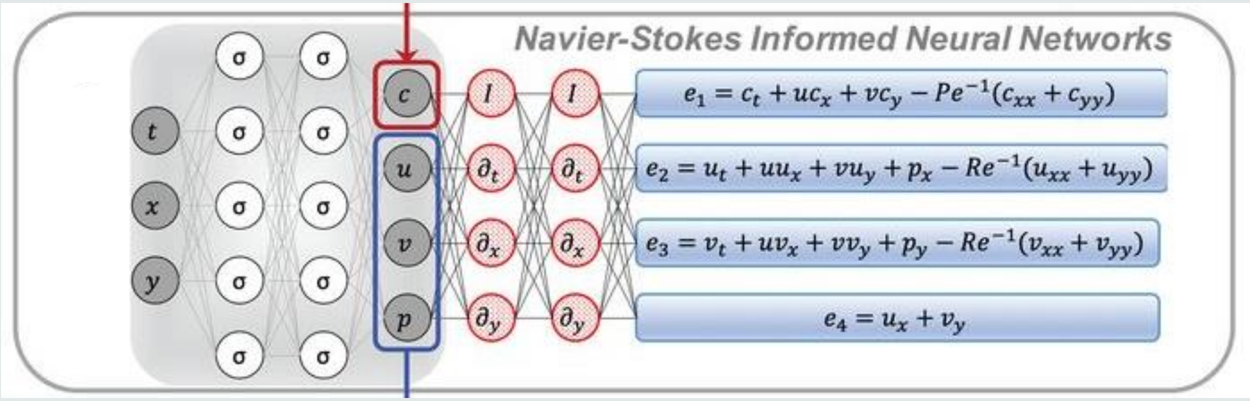
NAPDE Course Project

Tutors: Stefano Pagani, Francesco Regazzoni
Students: Giulia Mescolini, Luca Sosta

The idea

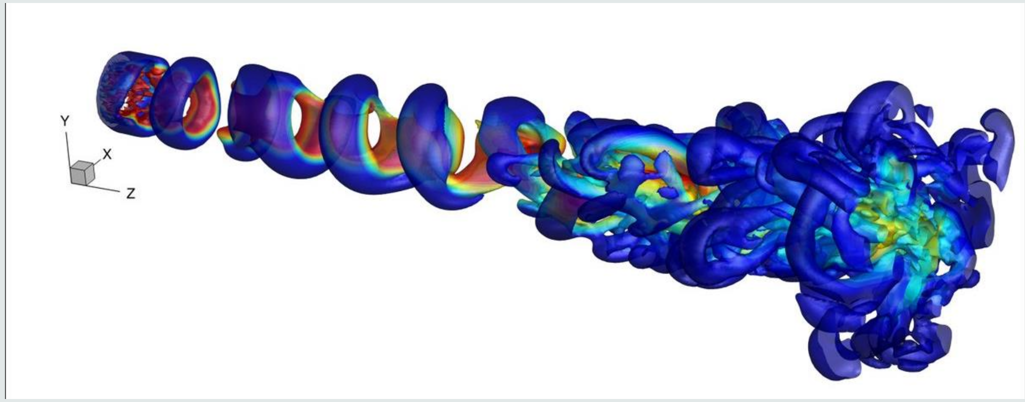
In the Fluid Dynamics framework we often meet problems involving **PDEs** which cannot be solved analitically.

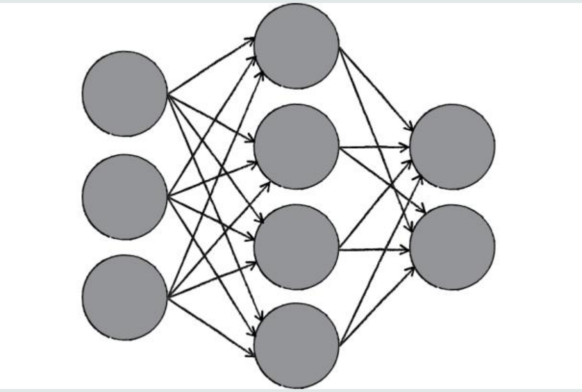
In fact, in this field many applications require to reconstruct **numerically** the behavior of **pressure** and **velocity**, starting from partial measures.



What is a PINN?

- PINNs are Neural Networks trained to solve supervised learning tasks while respecting any given **law of physics** described by general **nonlinear partial differential equations**.





- For our project, the reference library for the automatic differentiation is **nisaba**, a Python Library built on the top of Tensorflow.

nisaba

A simple test case

- We started from the **Colliding Flows** benchmark case, whose analytical solution is known:

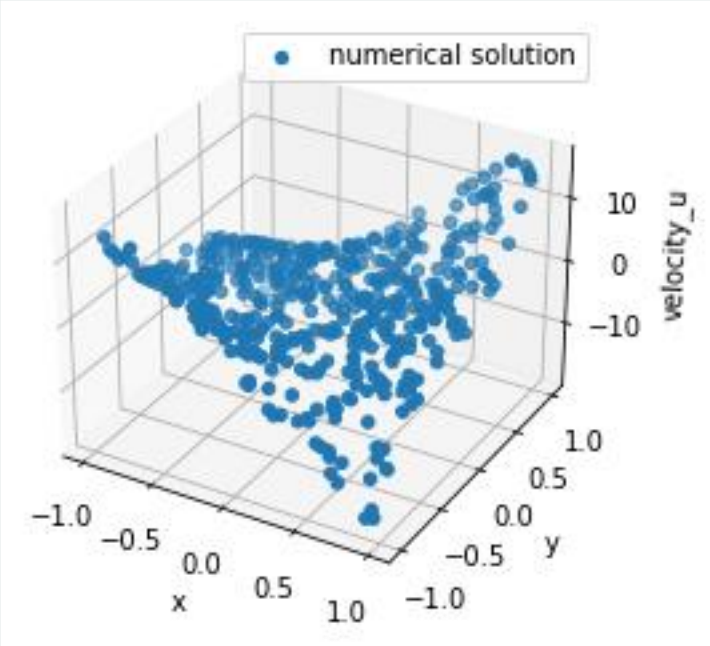
Given $\Omega = (-1,1) \times (-1,1)$, find $(\mathbf{u}, p)$ s.t.

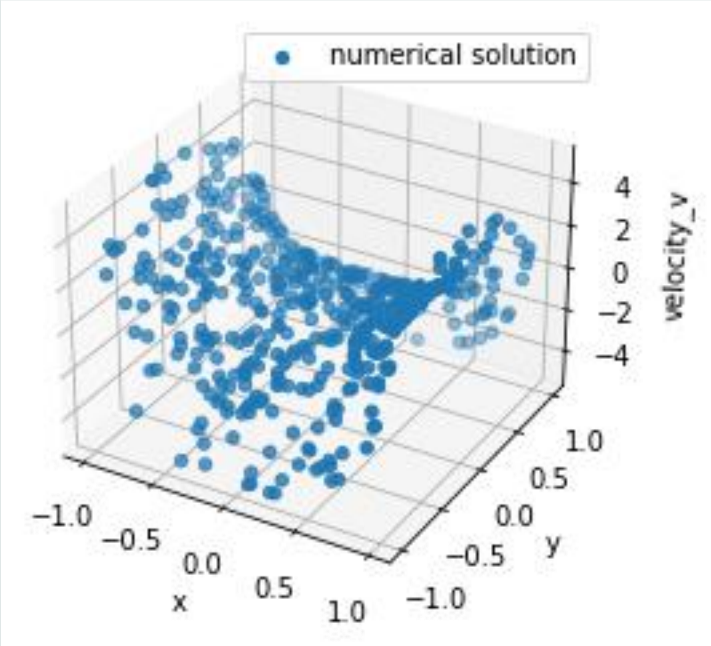
$$\begin{cases} -\Delta \mathbf{u} + \nabla p = \mathbf{f} = 0 & \text{in } \Omega \\ \text{div } \mathbf{u} = 0 & \text{in } \Omega \\ \mathbf{u} = \mathbf{g} = (20xy^3, 5x^4 - 5y^4)^T & \text{on } \partial\Omega \end{cases}$$

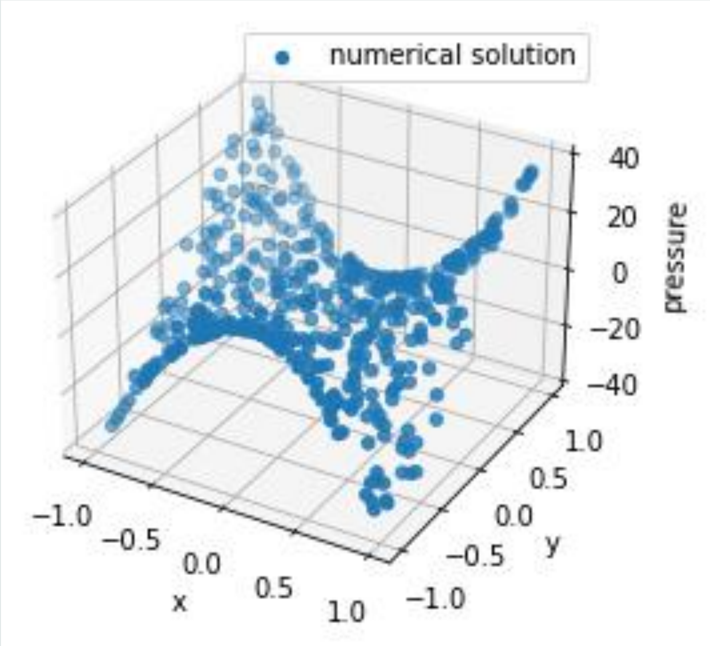
CHALLENGE:

- Train the PINN only with **data on the BCs** for the velocities and by constraining the **mean value** of the pressure!

The results

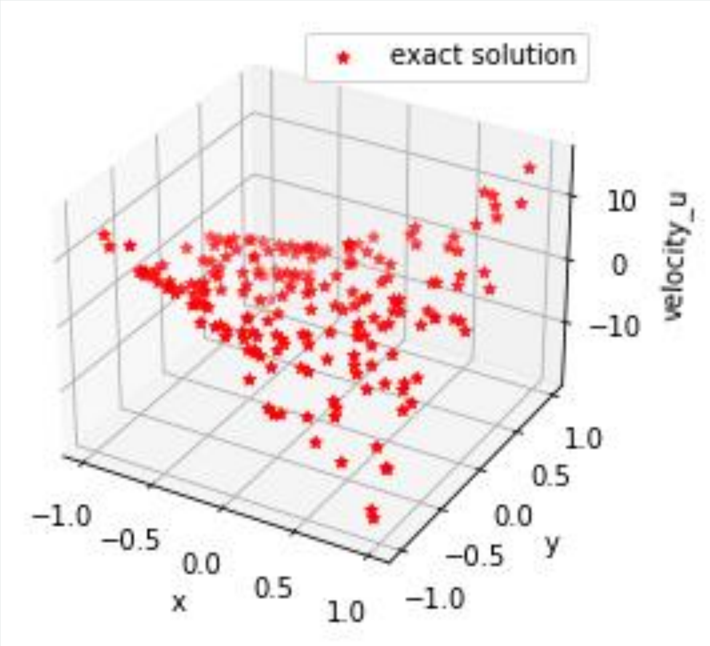


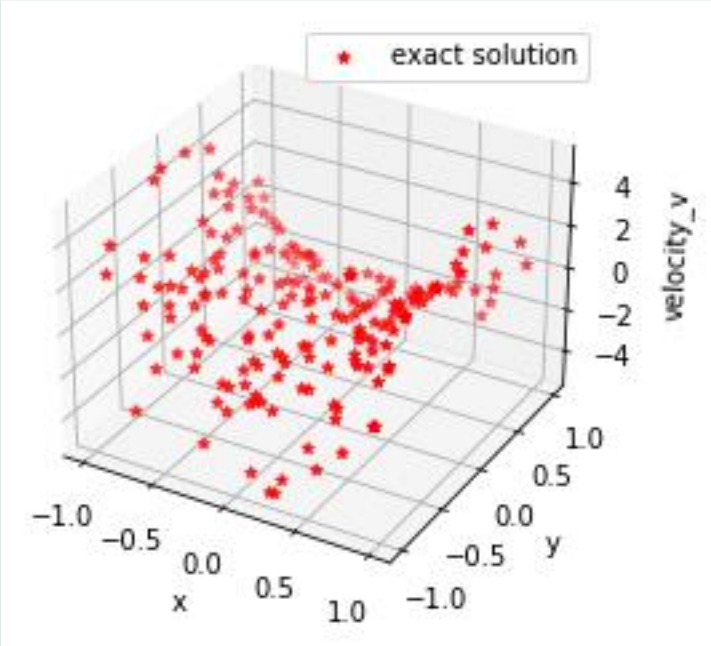


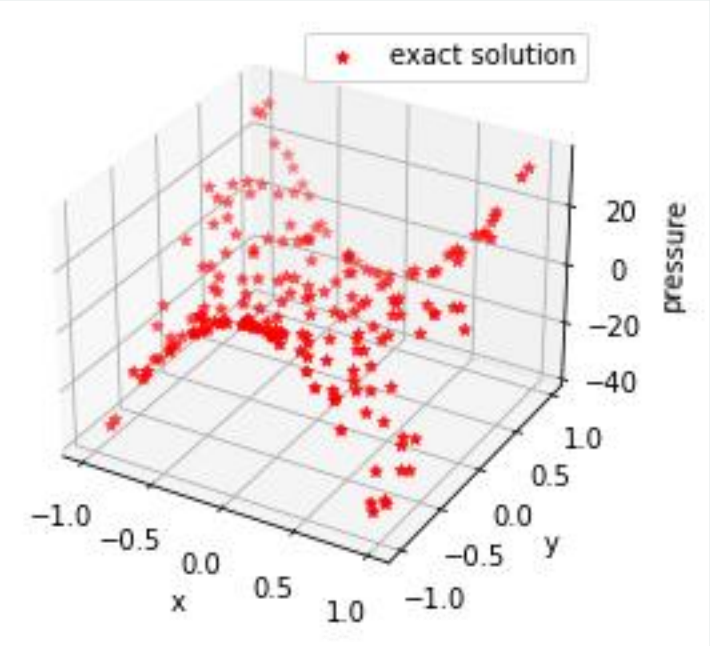


2000 epochs, **no noise** on data

The results

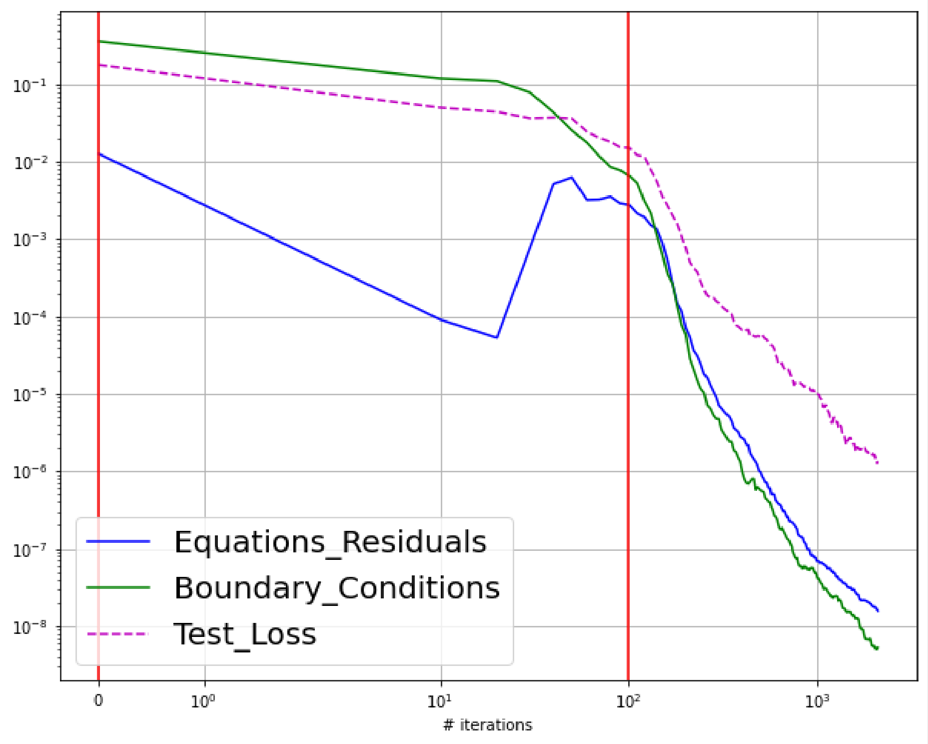




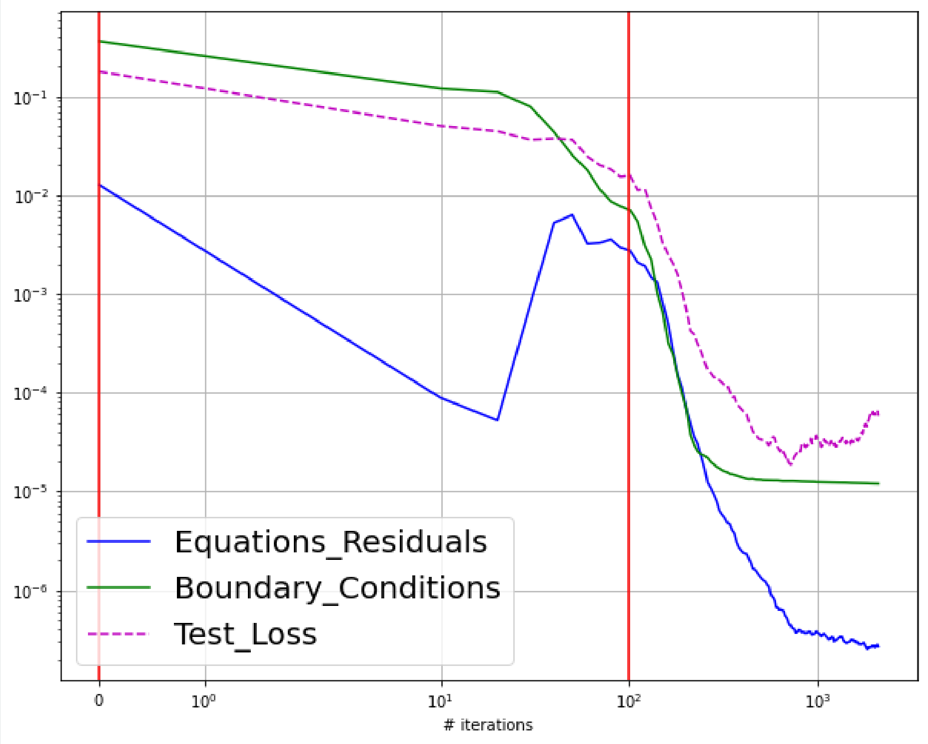


2000 epochs, **no noise** on data

The results



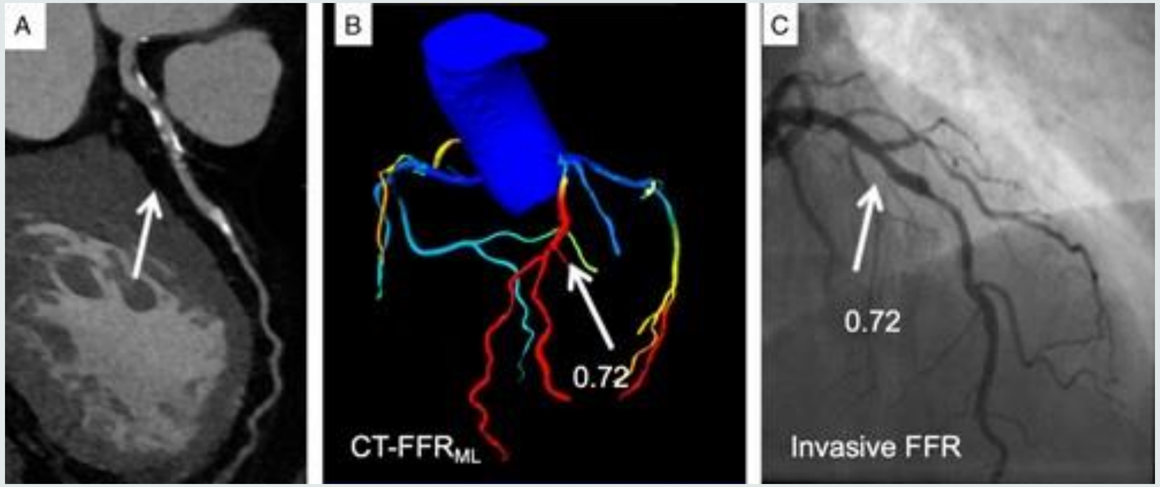
2000 epochs, **no noise** on data



2000 epochs, **with gaussian noise** $(\mu = 0, \sigma = 1)$ on data

Our prospect

Our next step is to apply this strategy to a test case more complex and closer to reality, such as the **Coronary Flow Assessment**, fundamental to evaluate **coronary artery disease**.
In this case, there is no hope for an analytical solution.



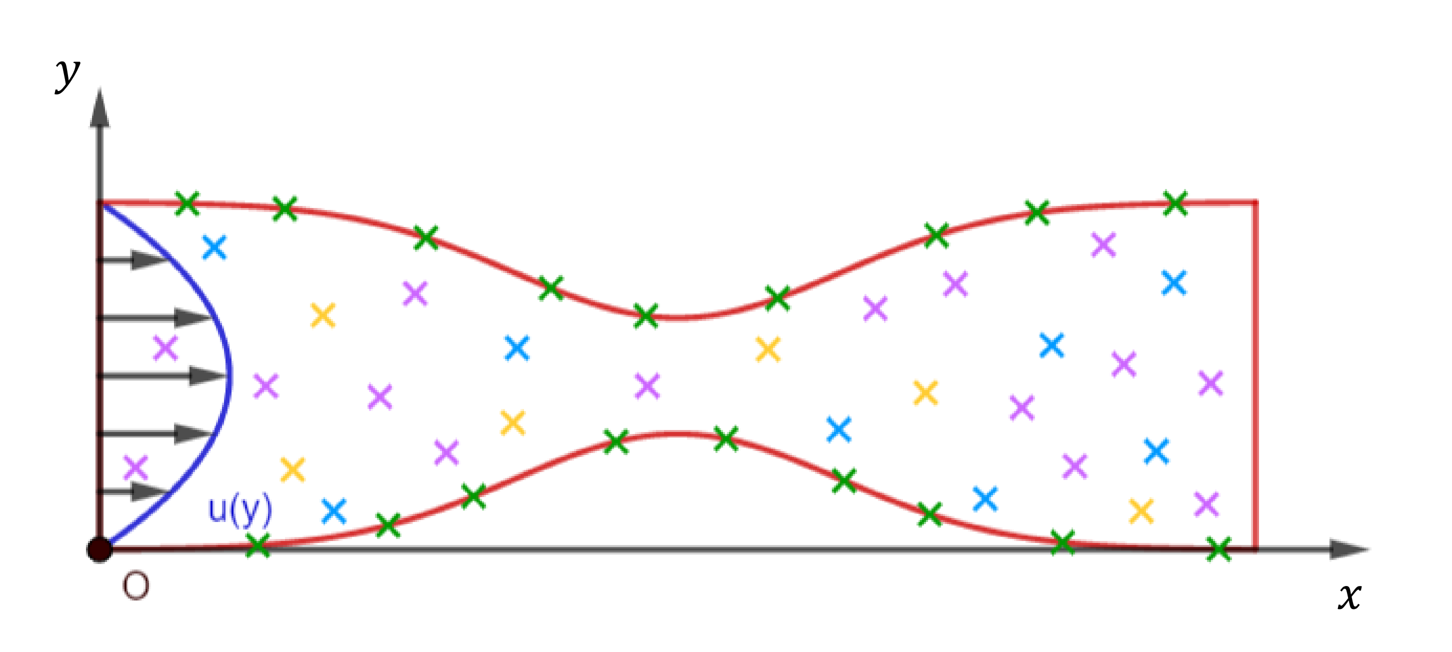
Work Plan

- To train our Neural Network, we plan to exploit two types of information:

Geometrical Data
(eg: branch length, vessel radius…)

+

Data on velocity
(provided by medical imaging)



Legend:
x Boundary Points
x Training Points
x Collocation Points
x Test Points

Bibliography

- ***Hidden fluid mechanics: Learning velocity and pressure fields from flow visualizations*** - Maziar Raissi; Alireza Yazdani; George Em Karniadakis

- ***Machine Learning and Deep Neural Networks Applications in Coronary Flow Assessment*** - Christian Tesche; Hunter N.Gray

- ***Physics-informed neural networks (PINNs) for fluid mechanics: A review*** - Shengze Cai; Zhiping Mao; Zhicheng Wang; Minglang Yin; George Em Karniadakis

Thank you for your attention!