

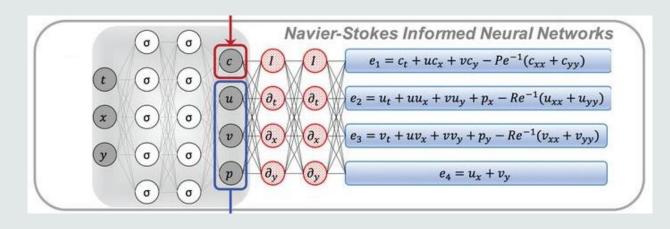




In the Fluid Dynamics framework we often meet problems involving **PDEs** which cannot be solved analitically.

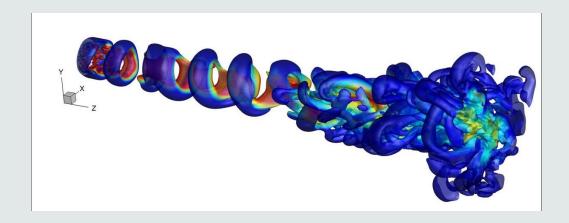


In fact, in this field many applications require to reconstruct **numerically** the behavior of **pressure** and **velocity**, starting from partial measures.

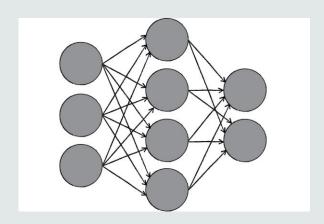


## What is a PINN?

• PINNs are Neural Networks trained to solve supervised learning tasks while respecting any given law of physics described by general nonlinear partial differential equations.



 For our project, the reference library for the automatic differentiation is **nisaba**, a Python Library built on the top of Tensorflow.





### A simple test case

• We started from the **Colliding Flows** benchmark case, whose analytical solution is known:

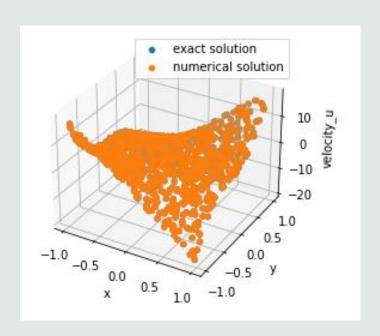
Given 
$$\Omega = (-1,1) \times (-1,1)$$
, find  $(\mathbf{u}, p)$  s.t.

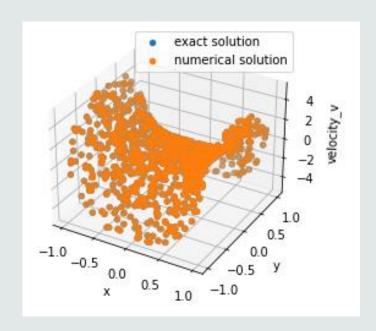
$$\begin{cases}
-\Delta \mathbf{u} + \nabla p = \mathbf{f} = 0 & \text{in } \Omega \\
\text{div } \mathbf{u} = 0 & \text{in } \Omega \\
\mathbf{u} = \mathbf{g} = (20xy^3, 5x^4 - 5y^4)^T \text{ on } \partial\Omega
\end{cases}$$

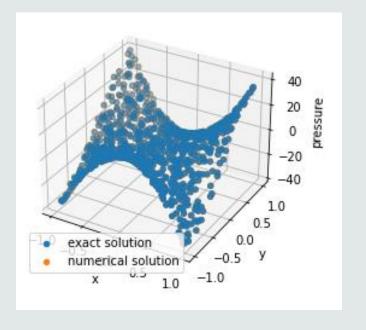
#### **CHALLENGE:**

 Train the PINN only with data on the BCs for the velocities and by constraining the mean value of the pressure!

## The results

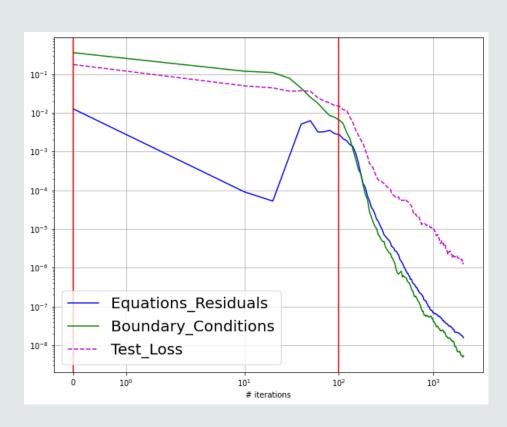




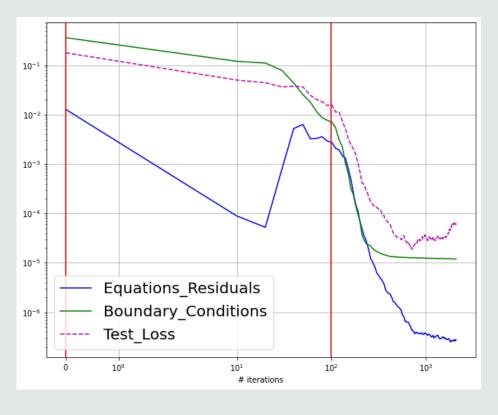


2000 epochs, **no noise** on data

# The results



2000 epochs, **no noise** on data

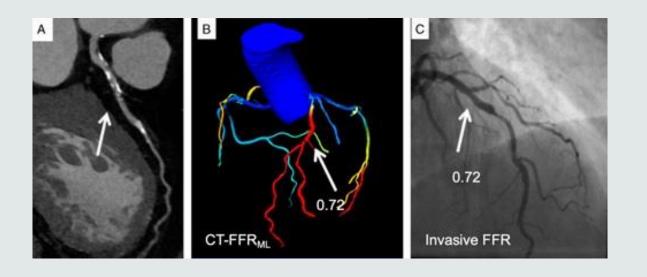


2000 epochs, **with noise** (percentuale???) on data



Our next step is to apply this strategy to a test case more complex and closer to reality, such as the **Coronary Flow Assessment**, fundamental to evaluate **coronary artery disease**.

In this case, there is no hope for an analytical solution!





### Work Plan

• To train our Neural Network, we plan to exploit two types of information:

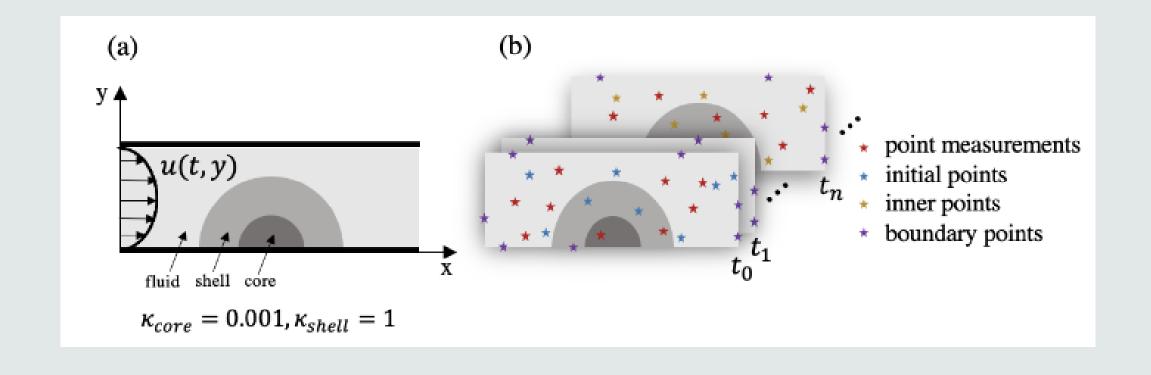
#### **Geometrical Data**

(eg: branch length, vessel radius...)



#### **Data on velocity**

(provided by medical imaging)



### Bibliography

- Hidden fluid mechanics: Learning velocity and pressure fields from flow visualizations - Maziar Raissi; Alireza Yazdani; George Em Karniadakis
- Machine Learning and Deep Neural Networks
   Applications in Coronary Flow Assessment Christian Tesche; Hunter N.Gray
- Physics-informed neural networks (PINNs) for fluid mechanics: A review - Shengze Cai; Zhiping Mao; Zhicheng Wang; Minglang Yin; George Em Karniadakis

