COMPUTER ASSIGMENT

1. A simple model of a vehicle moving in one dimension is given by

$$\begin{bmatrix} s_1(t+1) \\ s_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.95 \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t), \qquad t = 0, 1, 2, \ldots$$

$s_1(t)$ is the position at time t, $s_2(t)$ is the velocity at time t, and $u(t)$ is the actuator input. Roughly speaking, the equations state that the actuator input affects the velocity, which in turn affects the position. The coefficient 0.95 means that the velocity decays by 5% in one sample period (for example, because of friction), if no actuator signal is applied. We assume that the vehicle is initially at rest at position 0 : $s_1(0) = s_2(0) = 0$. We will solve the minimum energy optimal control problem: for a given time horizon N, choose inputs $u(0), \ldots, u(N-1)$ so as to minimize the total energy consumed, which we assume is given by

$$E = \sum_{t=0}^{N-1} u(t)^2$$

In addition, the input sequence must satisfy the constraint $s_1(N) = 10, s_2(N) = 0$. Your task therefore is to bring the vehicle to the final position $s_1(N) = 10$ with final velocity $s_2(N) = 0$, as efficiently as possible.

a) Formulate the minimum energy optimal control problem as a least norm problem

$$\text{minimize}\|x\|^2 \qquad \text{subject to} Cx = d.$$

Clearly state what the variables x, and the problem data C and d are.

b) Solve the problem for $N = 30$. Plot the optimal $u(t)$, the resulting position $s_1(t)$, and velocity $s_2(t)$.

c) Solve the problem for $N = 2, 3, ..., 29$. For each N calculate the energy E consumed by the optimal input sequence. Plot E versus N. (The plot looks best if you use a logarithmic scale for E)

d) Suppose we allow the final position to deviate from 10. However, if $s_1(N) \neq 10$, we have to pay a penalty, equal to $(s_1(N) - 10)^2$. The problem is to find the input sequence that minimizes the sum of the energy E consumed by the input and the terminal position penalty,

$$\sum_{t=0}^{N-1} u(t)^2 + (s_1(N) - 10)^2,$$

subject to the constraint $s_2(N) = 0$.

Formulate this problem as a least norm problem, and solve it for $N = 30$. Plot the optimal input signals $u(t)$, the resulting position $s_1(t)$ and the resulting velocity $s_2(t)$.

2. Two vehicles are moving along a straight line. For the first vehicle we use the same model as in the previous question i.e

$$\begin{bmatrix} s_1(t+1) \\ s_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.95 \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t), \qquad t = 0, 1, 2, \ldots$$

assuming that the vehicle is initially at rest at position $0 : s_1(0) = s_2(0) = 0$. The model for the second vehicle is

$$\begin{bmatrix} p_1(t+1) \\ p_2(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} v(t), \qquad t = 0, 1, 2, \ldots$$

$p_1(t)$ is the position at time $t, p_2(t)$ is the velocity at time t, and $v(t)$ is the actuator input. We assume that the second vehicle is initially at rest at position $1 : p_1(0) = 1, p_2(0) = 0$. Formulate the following problem as a least norm problem, and solve it . Find the control inputs $u(0), u(1), \ldots, u(19)$ and $v(0), v(1), \ldots, v(19)$ that minimize the total energy