# Introduction to Deep Learning for Computer Vision

Adhyayan '23 - ACA Summer School
Department of Computer Science and Engineering
Indian Institute of Technology Kanpur

Lecture 6

# Unsupervised Learning: Learning without Labels!

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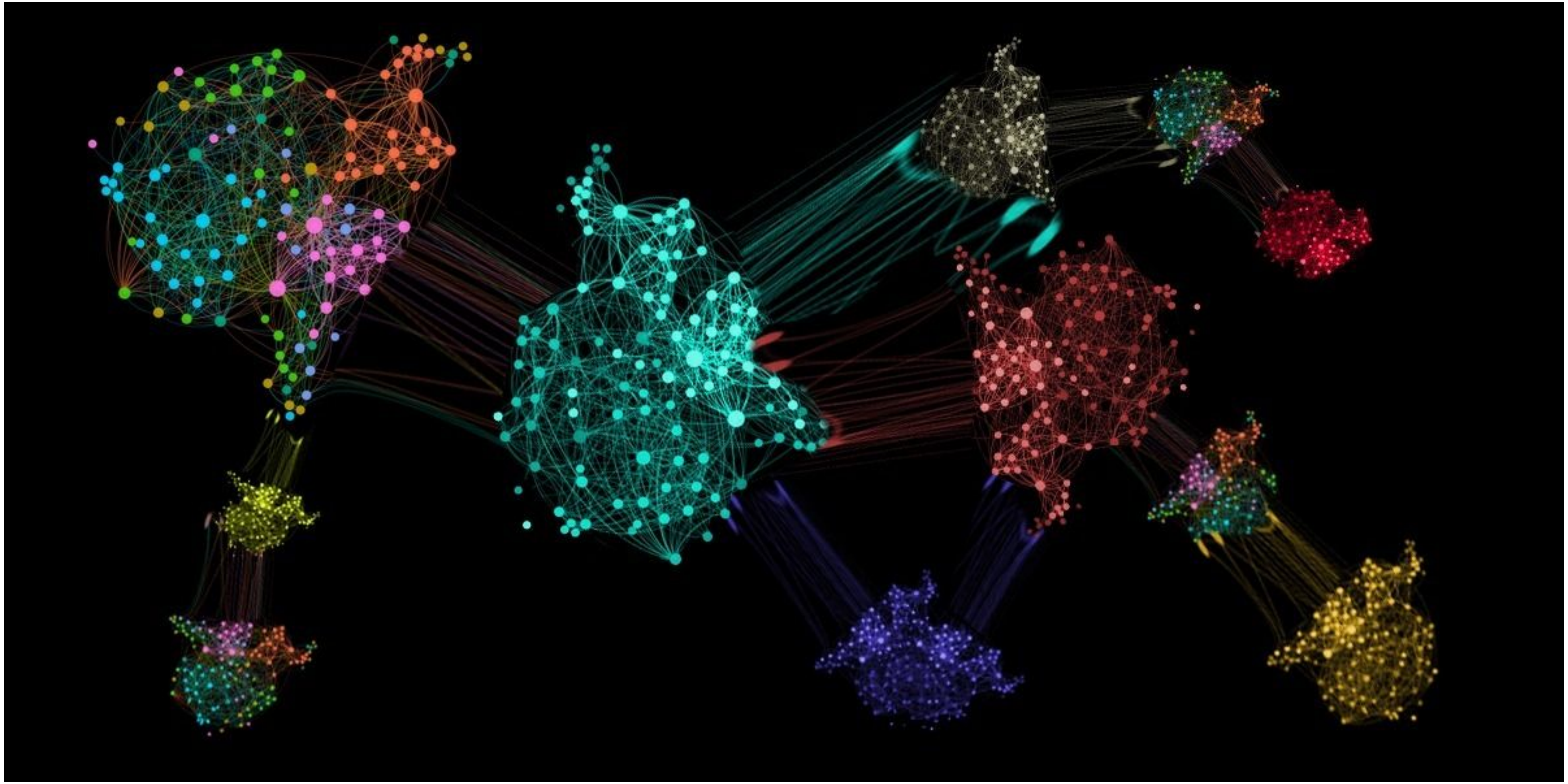
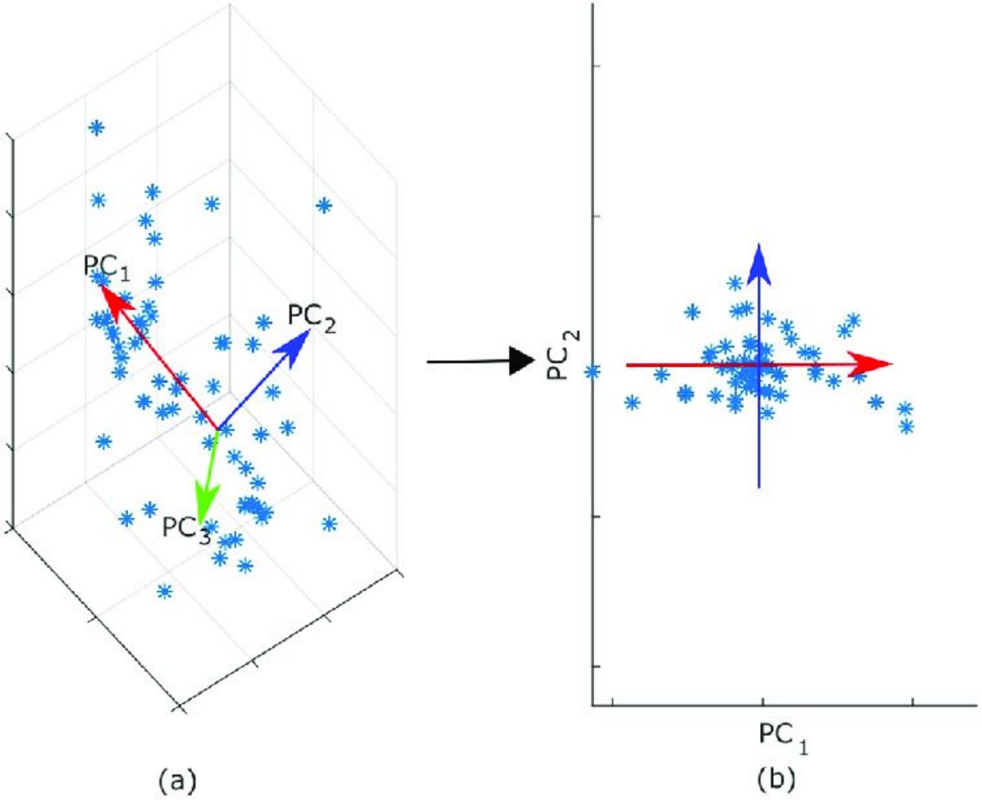
  - Handling Unlabeled or Scarce Labeled Data

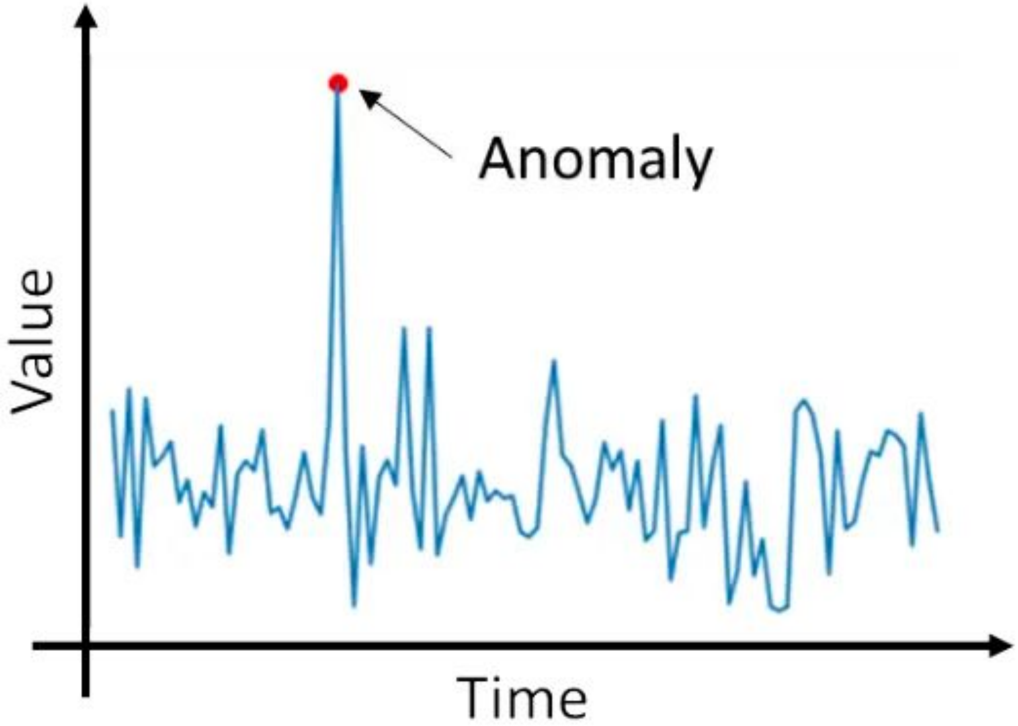
# Applications: Clustering



# Applications: Dimensionality Reduction



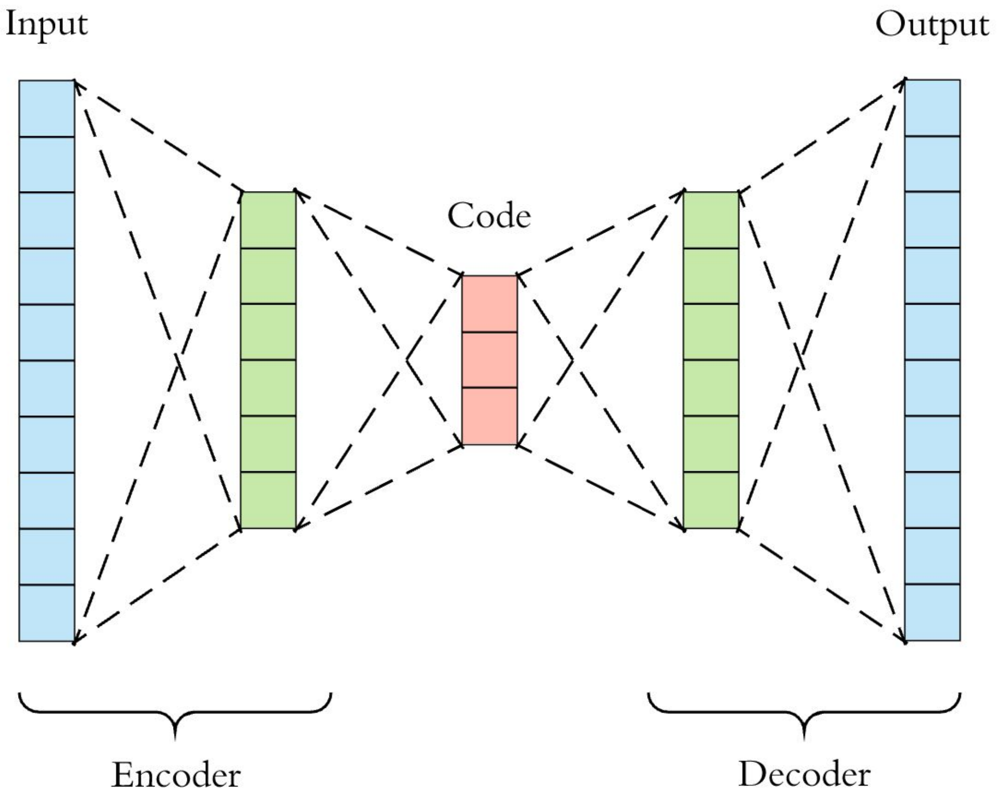
# Applications: Anomaly Detection



# Applications: Generative Modelling

# Unsupervised Deep Learning!

# Autoencoders



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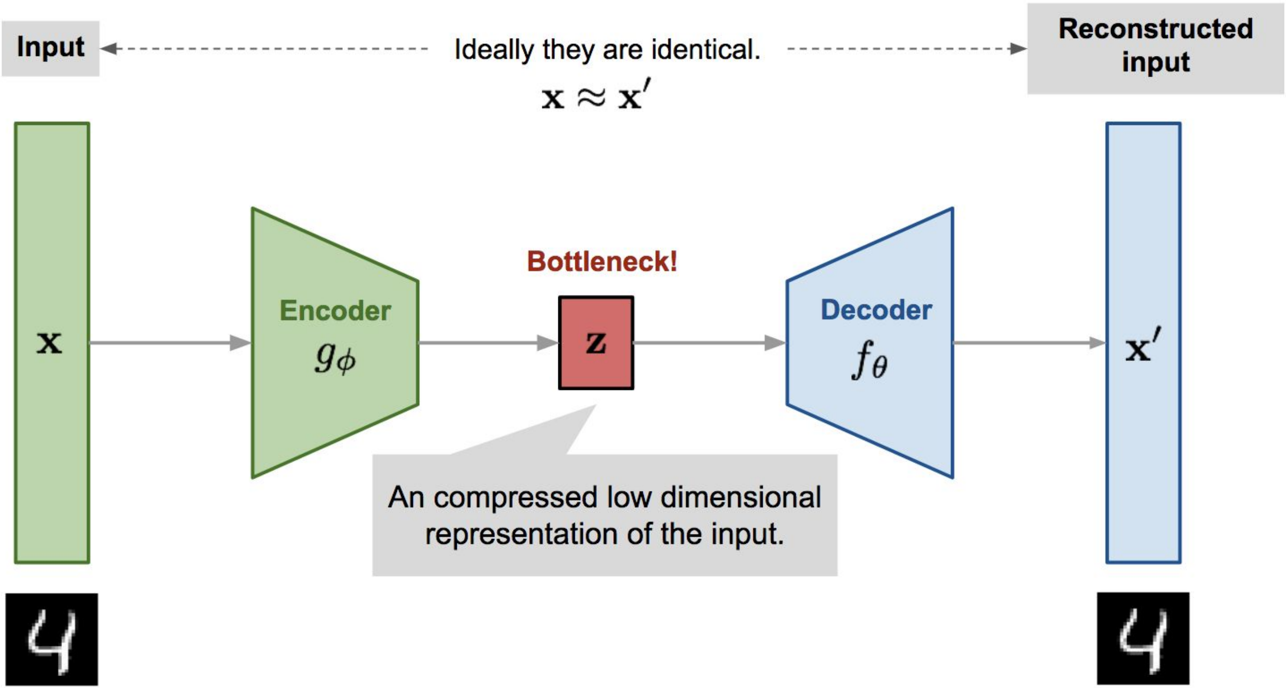


Image: https://lilianweng.github.io/posts/2018-08-12-vae/

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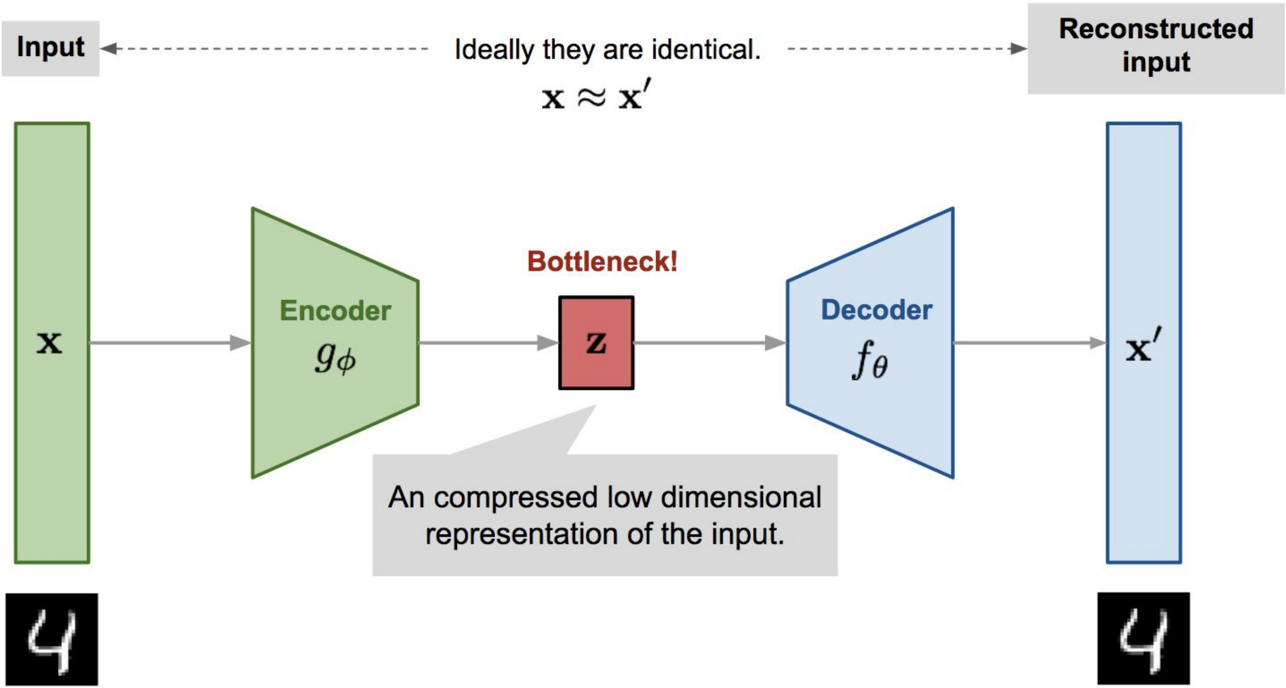


$$L_{\text{AE}}(\theta, \phi) = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}^{(i)} - f_\theta(g_\phi(\mathbf{x}^{(i)})))^2$$

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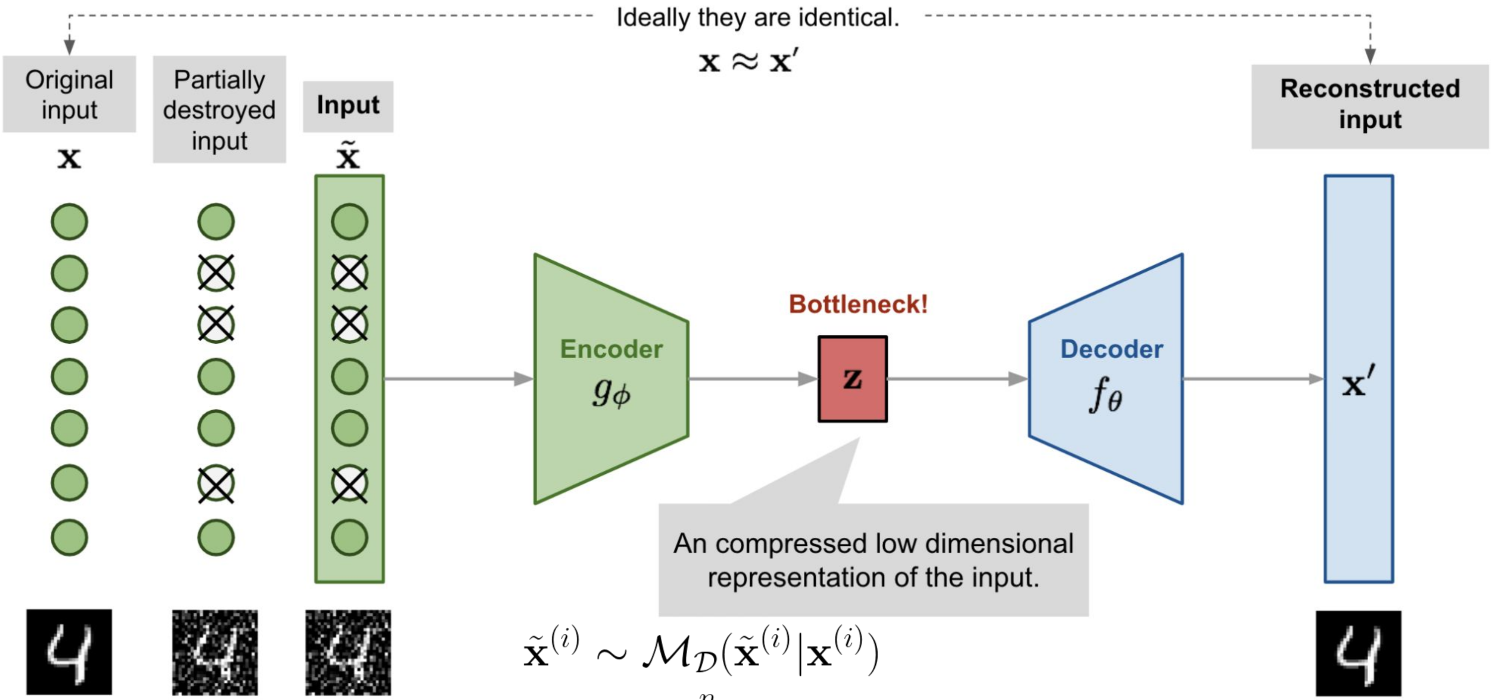
# Denoising Autoencoder




$$\tilde{\mathbf{x}}^{(i)} \sim \mathcal{M}_{\mathcal{D}}(\tilde{\mathbf{x}}^{(i)} | \mathbf{x}^{(i)})$$

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Not easy to compute.
Need to approximate!

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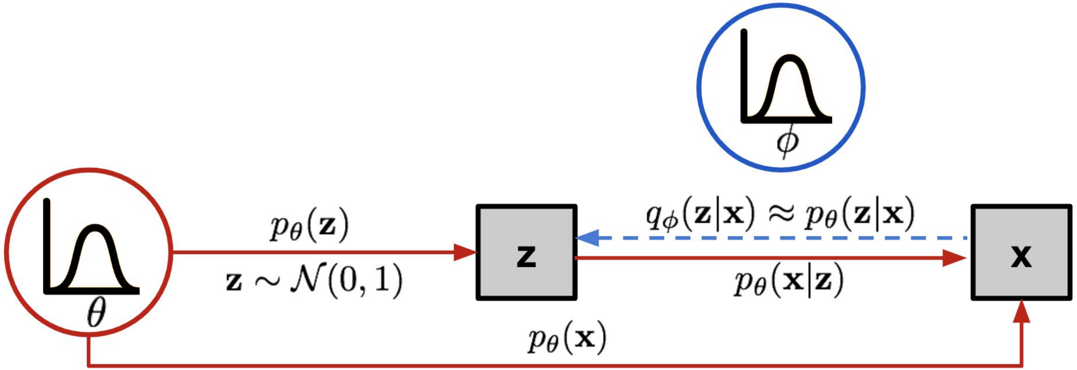


- Unfortunately it is not easy to compute $p_\theta(\mathbf{x}^{(i)})$
- **Better Idea**: Introduce a new approximation function to output what is a likely code given an input, $\mathbf{X}$ $q_\phi(\mathbf{z}|\mathbf{x})$, parameterized by $\phi$
- Now the structure resembles an Autoencoder:
  - $p_\theta(\mathbf{x}|\mathbf{z})$ is similar to the decoder $f_\theta(\mathbf{x}|\mathbf{z})$. Also known as *probabilistic decoder*.
  - $q_\phi(\mathbf{z}|\mathbf{x})$ is similar to the encoder $g_\phi(\mathbf{z}|\mathbf{x})$. Also known as *probabilistic encoder*.

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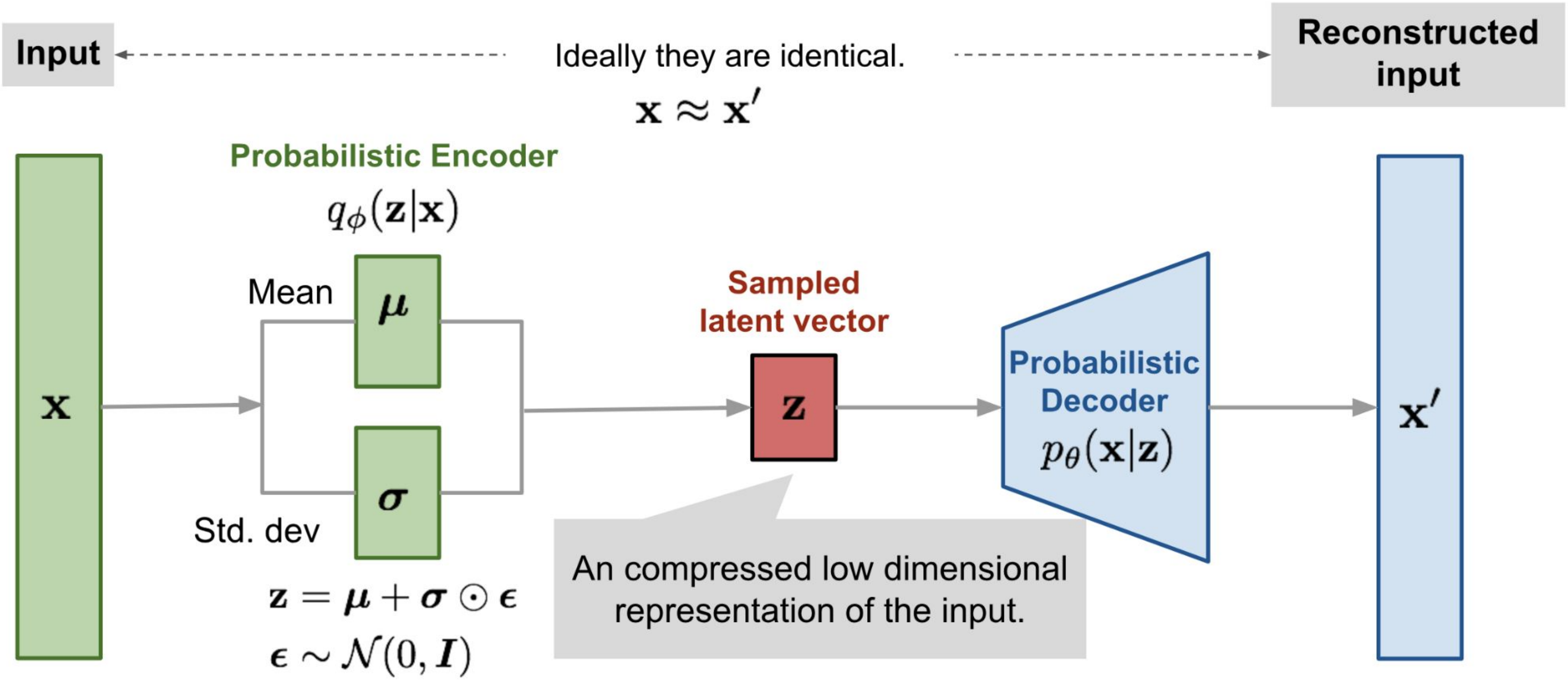


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$$D_{KL}(P\|Q) = \int_z P(z)\log\frac{P(z)}{Q(z)}dz$$

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- We want to minimize $D_{\mathrm{KL}}(q_\phi(\mathbf{z}|\mathbf{x})|p_\theta(\mathbf{z}|\mathbf{x}))$ with respect to *Φ*.
- Why use $D_{\mathrm{KL}}(q_\phi|p_\theta)$ (reverse KL) instead of $D_{\mathrm{KL}}(p_\theta|q_\phi)$ (forward KL)?
  Ans: [https://blog.evjang.com/2016/08/variational-bayes.html](https://blog.evjang.com/2016/08/variational-bayes.html)

# Loss Function: ELBO

$$L_{\text{VAE}}(\theta, \phi) = -\mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} \log p_\theta(\mathbf{x}|\mathbf{z}) + D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x}) \| p_\theta(\mathbf{z}))$$

$$\theta^*, \phi^* = \arg\min_{\theta, \phi} L_{\text{VAE}}$$

Reconstruction Loss

Regularization

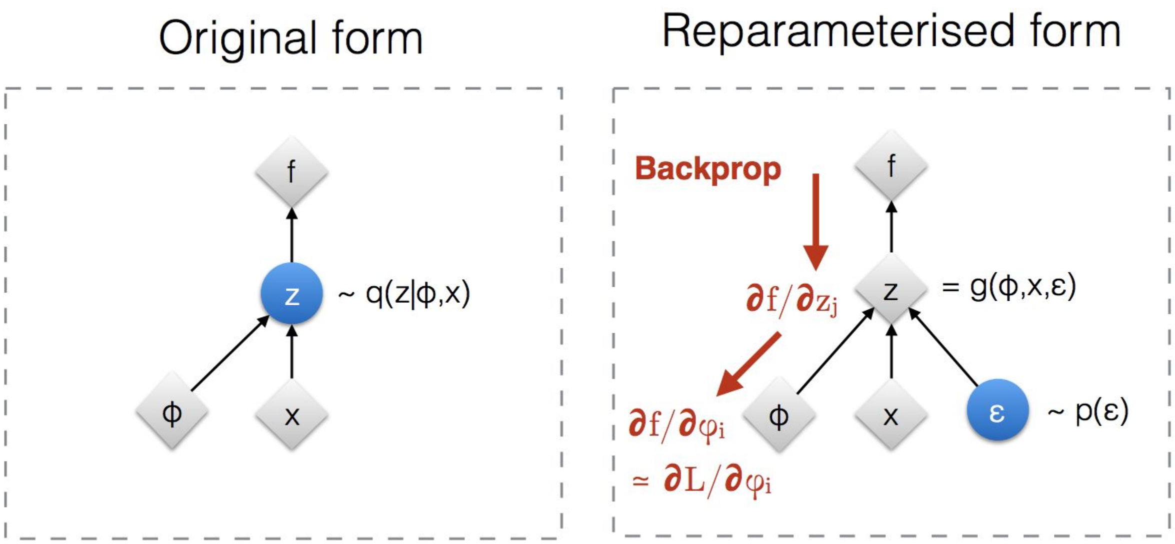
# Reparameterization Trick

- The expectation term in the loss function invokes generating samples from $\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})$
- Sampling is a stochastic process. Backpropagation is not possible.
- To make it trainable, the reparameterization trick is introduced:
  - It is often possible to express the random variable *z* as a deterministic variable $\mathbf{z} = \mathcal{T}_\phi(\mathbf{x}, \boldsymbol{\epsilon})$ where is an auxiliary independent random variable, and the transformation function $\mathcal{T}_\phi$ parameterized by *Φ* converts ε to z.

$$\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x}^{(i)}) = \mathcal{N}(\mathbf{z}; \boldsymbol{\mu}^{(i)}, \boldsymbol{\sigma}^{2(i)}\boldsymbol{I})$$

$$\mathbf{z} = \boldsymbol{\mu} + \boldsymbol{\sigma} \odot \boldsymbol{\epsilon}, \text{ where } \boldsymbol{\epsilon} \sim \mathcal{N}(0, \boldsymbol{I}) \quad ; \text{Reparameterization trick.}$$

# Reparameterization Trick



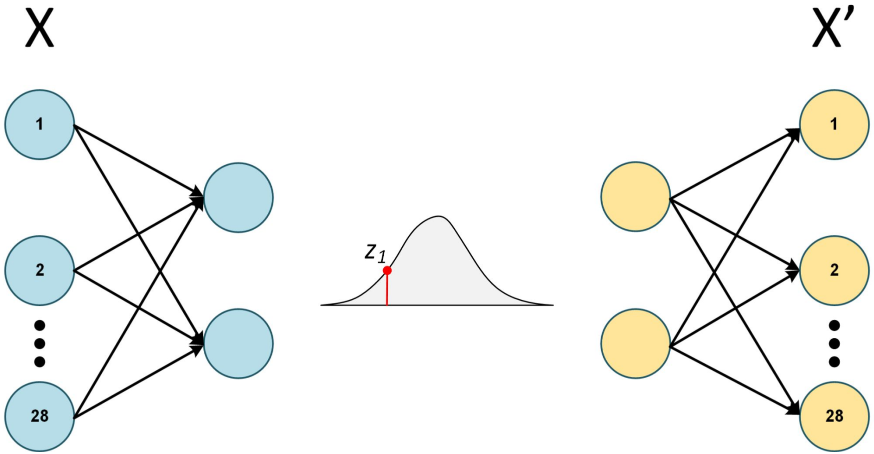

: Deterministic node

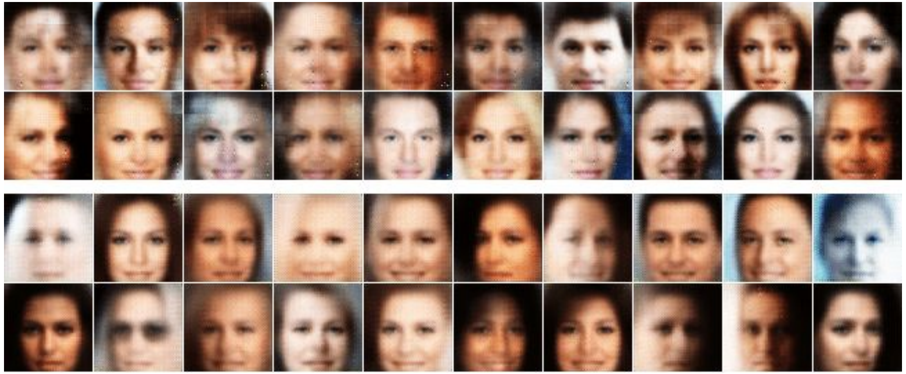
: Random node

[Kingma, 2013]
[Bengio, 2013]
[Kingma and Welling 2014]
[Rezende et al 2014]

Image: https://lilianweng.github.io/posts/2018-08-12-vae/

# Sample Generation using VAE



# β-VAE : Regularizing the Regularizer!

$$L_{\mathrm{BETA}}(\phi, \beta) = -\mathbb{E}_{\mathbf{z} \sim q_\phi(\mathbf{z}|\mathbf{x})} \log p_\theta(\mathbf{x}|\mathbf{z}) + \beta D_{\mathrm{KL}}(q_\phi(\mathbf{z}|\mathbf{x}) \| p_\theta(\mathbf{z}))$$

# Next Lecture: GANs, Diffusion Models!

# Expanding the KL Divergence

$$
\begin{aligned}
&D_{\mathrm{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}|\mathbf{x})) \\
&= \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{q_\phi(\mathbf{z}|\mathbf{x})}{p_\theta(\mathbf{z}|\mathbf{x})} d\mathbf{z} \\
&= \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{q_\phi(\mathbf{z}|\mathbf{x}) p_\theta(\mathbf{x})}{p_\theta(\mathbf{z}, \mathbf{x})} d\mathbf{z} && ;\ \text{Because } p(z|x)=p(z,x)/p(x) \\
&= \int q_\phi(\mathbf{z}|\mathbf{x}) \big( \log p_\theta(\mathbf{x}) + \log \frac{q_\phi(\mathbf{z}|\mathbf{x})}{p_\theta(\mathbf{z}, \mathbf{x})} \big) d\mathbf{z} \\
&= \log p_\theta(\mathbf{x}) + \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{q_\phi(\mathbf{z}|\mathbf{x})}{p_\theta(\mathbf{z}, \mathbf{x})} d\mathbf{z} && ;\ \text{Because } \int q(z|x)dz=1 \\
&= \log p_\theta(\mathbf{x}) + \int q_\phi(\mathbf{z}|\mathbf{x}) \log \frac{q_\phi(\mathbf{z}|\mathbf{x})}{p_\theta(\mathbf{x}|\mathbf{z}) p_\theta(\mathbf{z})} d\mathbf{z} && ;\ \text{Because } p(z,x)=p(x|z)p(z) \\
&= \log p_\theta(\mathbf{x}) + \mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}[\log \frac{q_\phi(\mathbf{z}|\mathbf{x})}{p_\theta(\mathbf{z})} - \log p_\theta(\mathbf{x}|\mathbf{z})] \\
&= \log p_\theta(\mathbf{x}) + D_{\mathrm{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z})) - \mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})} \log p_\theta(\mathbf{x}|\mathbf{z})
\end{aligned}
$$

# Deriving the Evidence Lower Bound

$$D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}|\mathbf{x})) = \log p_\theta(\mathbf{x}) + D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z})) - \mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}\log p_\theta(\mathbf{x}|\mathbf{z})$$

$$\log p_\theta(\mathbf{x}) - D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}|\mathbf{x})) = \mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}\log p_\theta(\mathbf{x}|\mathbf{z}) - D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}))$$

$$\begin{aligned} L_{\text{VAE}}(\theta, \phi) &= -\log p_\theta(\mathbf{x}) + D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}|\mathbf{x})) \\ &= -\mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})}\log p_\theta(\mathbf{x}|\mathbf{z}) + D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z})) \\ \theta^*, \phi^* &= \arg\min_{\theta,\phi} L_{\text{VAE}} \end{aligned}$$

$$-L_{\text{VAE}} = \log p_\theta(\mathbf{x}) - D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}|\mathbf{x})) \leq \log p_\theta(\mathbf{x})$$