Midterm 2 Practice for COMP6321 Fall 2019

The questions in this practice midterm are suggestive only of the *style* and *difficulty* of questions that will be asked on on the real midterm. The length and the particular course content evaluated will be different. Not all topics are covered by this practice midterm.

Q1. [5 marks total] You are given a very large historical data set $\{(\mathbf{x}_i, y_i)\}_{i=1}^{N}$ of residential property sales. It has the following features $\mathbf{x}_i$ and targets y_i:

- x_1: $\text{residence\_type} \in \{\text{condo}, \text{house}\}$, the type of property;
- x_2: $\text{num\_bedroom} \in \{0, 1, 2, 3, 4\}$, the number of bedrooms;
- x_3: $\text{num\_bathroom} \in \{1, 2, 3\}$, the number of bathrooms;
- x_4: $\text{year\_of\_sale} \in \{1980, 1981, \ldots, 2018, 2019\}$, the year the property was sold; and
- y: $\text{sale\_price} \in \mathbb{R}_{\geq 0}$, the price at which the property actually sold.

You are building a system to predict a property's potential sale price in 2020, *i.e.* where for a property with current features $\{x_1, x_2, x_3\}$ you set $x_4 = 2020$ and predict its extrapolated price.

a) [3 marks] Suppose you applied the *sklearn.model_selection.train_test_split* function to create a training set for your model and a held-out test set for evaluating it. Would this test set provide an *over*estimate or an *under*estimate of your model's squared error for the intended use? Explain.

> It would ***under*estimate** squared error. The model will be used to *extrapolate* one year into the future. However, when such a model is evaluated on test cases that are not strictly in the future, it can instead interpolate rather than extrapolate. Interpolation is 'easier'. The *train_test_split* function does a random split, so its test set does not evaluate extrapolation ability.

b) [2 marks] Propose a training and testing split that, if used, would provide a more accurate estimate of your model's test-time accuracy.

> Split the data so that cases with $\text{year\_of\_sale} \in \{1980, \ldots, 2018\}$ are used for training and cases with $\text{year\_of\_sale} = 2019$ is used for testing. The estimated test accuracy will be of extrapolating 1 year into the future, matching the intended use.
>
> [Interested students can see that this is a special case of the *sklearn.model_selection.TimeSeriesSplit* cross validator.]

Q2. [8 marks] Given a data set $(\mathbf{X}, \mathbf{y})$, use scikit-learn to do the following:

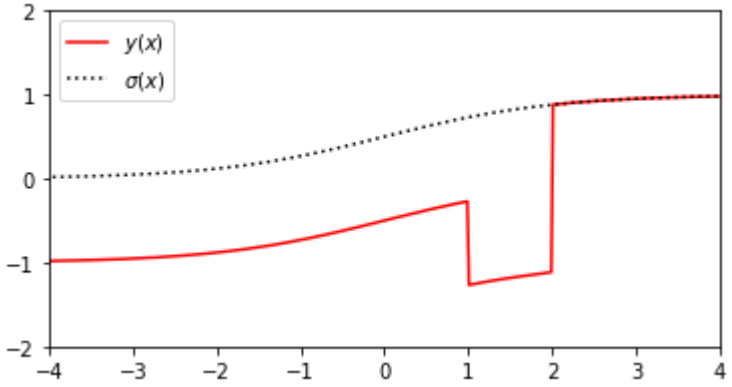
- Split $(\mathbf{X}, \mathbf{y})$ into 75% training and 25% testing, and normalize the features.
- Train a *GradientBoostingClassifier* using random hyperparameter search with
  - 5-fold cross validation,
  - the maximum tree depth (for weak learners) sampled uniformly from range $\{1, \ldots, 10\}$,
  - the gradient boosting learning rate sampled logarithmically from the interval $[0.01, 10]$, and
  - 30 total hyperparameter settings evaluated.
- Print the training and testing accuracy of your classifier.

Answer by completing the script below. (Scipy/sklearn documentation will be provided on last page of exam.)

```
import scipy
from sklearn.ensemble import GradientBoostingClassifier
from sklearn.model_selection import RandomizedSearchCV, train_test_split
from sklearn.preprocessing import StandardScaler
X, y = load_dataset()

# Your code below. Aim for 8-12 lines.


X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.25)

scaler = StandardScaler().fit(X_train)
X_train = scaler.transform(X_train)
X_test  = scaler.transform(X_test)

params = {
    'max_depth'     : scipy.stats.randint(1, 11),
    'learning_rate' : scipy.stats.reciprocal(0.01, 10.),
}

cv = RandomizedSearchCV(GradientBoostingClassifier(), params, cv=5, n_iter=30)
cv.fit(X_train, y_train)

print(cv.best_estimator_.score(X_train, y_train))
print(cv.best_estimator_.score(X_test, y_test))
```

Q3. [16 marks total] This question is about *neural networks*.

a) [3 marks] Give a sigmoidal neural network, including coefficients, that would closely approximation the function $y(x)$ shown below. Use as few hidden units as possible. The sigmoid $\sigma(x)$ is shown for reference.



The neural network would require three sigmoidal hidden units, taking the form

$$
\begin{aligned}
z_1 &= \sigma(w_1 x + b_1) \\
z_2 &= \sigma(w_2 x + b_2) \\
z_3 &= \sigma(w_3 x + b_3) \\
y &= w_4 z_1 + w_5 z_2 + w_6 z_3 + b_4
\end{aligned}
$$

By using a large first layer weight, such as $w = 1000$, we can achieve a step function with a sigmoid. A good approximation of the red curve is:

$$
\begin{aligned}
w_1 &= 1 \qquad & b_1 &= 0 \\
w_2 &= 1000 \qquad & b_2 &= -1000 \\
w_3 &= 1000 \qquad & b_3 &= -2000 \\
w_4 &= 1 \qquad & w_5 &= -1 \qquad & w_6 &= 2 \qquad & b_4 &= -1
\end{aligned}
$$

b) [4 marks] Consider the neural network below for a 3-dimensional input $\boldsymbol{x}$:

$$
\begin{aligned}
a_1 &= w_1 x_1 + w_2 x_2 + w_3 \\
a_2 &= w_1 x_2 + w_2 x_3 + w_3 \\
z_1 &= \tanh(a_1) \\
z_2 &= \tanh(a_2) \\
y &= w_4 z_1 + w_5 z_2 + w_6
\end{aligned}
$$

Suppose we compute the squared error loss $\ell = \frac{1}{2}(y-t)^2$ with respect to a target t. Derive the gradient $\nabla_{\boldsymbol{w}} \ell$ with respect to $\boldsymbol{w} = \begin{bmatrix} w_1 & \cdots & w_6 \end{bmatrix}^T$. Note that $\tanh'(a) = 1 - \tanh(a)^2$. Show your steps.

Working backwards from w_6 we have:

$$
\begin{aligned}
\frac{\partial \ell}{\partial w_6} &= \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial w_6} = (y-t) \\
\frac{\partial \ell}{\partial w_5} &= \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial w_5} = (y-t) z_2 \\
\frac{\partial \ell}{\partial w_4} &= \frac{\partial \ell}{\partial y}\frac{\partial y}{\partial w_4} = (y-t) z_3 \\
\frac{\partial \ell}{\partial w_3} &= \frac{\partial \ell}{\partial y}\left(\frac{\partial y}{\partial z_1}\frac{\partial z_1}{\partial a_1}\frac{\partial a_1}{\partial w_3} + \frac{\partial y}{\partial z_2}\frac{\partial z_2}{\partial a_2}\frac{\partial a_2}{\partial w_3}\right) = (y-t)\big(w_4(1-z_1^2) + w_5(1-z_2^2)\big) \\
\frac{\partial \ell}{\partial w_2} &= \frac{\partial \ell}{\partial y}\left(\frac{\partial y}{\partial z_1}\frac{\partial z_1}{\partial a_1}\frac{\partial a_1}{\partial w_2} + \frac{\partial y}{\partial z_2}\frac{\partial z_2}{\partial a_2}\frac{\partial a_2}{\partial w_2}\right) = (y-t)\big(w_4(1-z_1^2)x_2 + w_5(1-z_2^2)x_3\big) \\
\frac{\partial \ell}{\partial w_1} &= \frac{\partial \ell}{\partial y}\left(\frac{\partial y}{\partial z_1}\frac{\partial z_1}{\partial a_1}\frac{\partial a_1}{\partial w_1} + \frac{\partial y}{\partial z_2}\frac{\partial z_2}{\partial a_2}\frac{\partial a_2}{\partial w_1}\right) = (y-t)\big(w_4(1-z_1^2)x_1 + w_5(1-z_2^2)x_2\big)
\end{aligned}
$$

These give the six components of $\nabla_{\boldsymbol{w}} \ell$.

c) [3 marks] Bishop uses the symbols δ_k to and δ_j to represent the "errors" that are computed during the backpropagation algorithm. What is the formula for each "error" computed in your answer to (b), and how many times is each quantity re-used when computing the gradient?

The error of the output is $\delta_k = (y-t)$. It is re-used 6 times.

The error of hidden unit z_1 is $\delta_{j_1} = \delta_k w_4 (1-z_1^2)$. It is re-used 3 times.

The error of hidden unit z_2 is $\delta_{j_2} = \delta_k w_5 (1-z_2^2)$. It is re-used 3 times.

d) [1 marks] Is the neural network in part (b) a convolutional neural network? You must explain your answer.

Yes. The first layer corresponds to a 1-dimensional convolution with filter size 2 (w_1 and w_2) and a bias term (w_3), with no padding of the input sequence (x_1, x_2, x_3).

e) [5 marks] You will use PyTorch to write a *build_convnet* function that can be used like this:

```
>>> convnet = build_convnet(32, 20, 5, 32, 3, 100, 4)
>>> x = torch.rand((5, 3, 20, 32))  # Create a batch of 5 random RGB images of size 32x20
>>> y = convnet(x)                  # Generate output activations
>>> torch.softmax(y, dim=1)         # Convert each row of outputs into class probabilities
tensor([[0.2271, 0.2611, 0.2132, 0.2987],
        [0.2312, 0.2519, 0.2127, 0.3042],
        [0.2188, 0.2627, 0.2169, 0.3017],
        [0.2242, 0.2588, 0.2092, 0.3078],
        [0.2299, 0.2538, 0.2166, 0.2998]])
```

Write PyTorch code to implement the *build_convnet* function below.

```
import torch

def build_convnet(in_width, in_height, num_filters, filter_size, pool_size, num_hidden, num_output):
    """
    Returns a convnet suitable for 4-way classification of images. Specifically:
    * one convolutional layer having num_filters of size filter_size, tanh activations, and
      accepting an RGB image of size in_width x in_height;
    * one max-pooling layer with pool_size regions of stride 1;
    * one fully-connected hidden layer with num_hidden units and rectified linear outputs; and
    * one fully-connected output layer with num_output outputs.
    """
    # Your code here


        return torch.nn.Sequential(
            torch.nn.Conv2d(in_channels=3, out_channels=num_filters,
                            kernel_size=filter_size, padding=filter_size//2),
            torch.nn.Tanh(),
            torch.nn.MaxPool2d(kernel_size=pool_size, stride=1, padding=pool_size//2),
            torch.nn.Flatten(),
            torch.nn.Linear(num_filters*in_width*in_height, num_hidden),
            torch.nn.ReLU(),
            torch.nn.Linear(num_hidden, num_output),
        )
```

The function documentation does not specify whether the convolutional and pooling layers have padding, so padding was used to simplify the calculation of the required number parameters for the first fully-connected layer.

Q4. [2 marks] Give the formula for the Kullbach-Leibler divergence $D_{\mathrm{KL}}(p(\boldsymbol{z}) \,||\, q(\boldsymbol{z}))$.

$$D_{\mathrm{KL}}(p(\boldsymbol{z}) \,||\, q(\boldsymbol{z})) = \int p(\boldsymbol{z}) \ln \frac{p(\boldsymbol{z})}{q(\boldsymbol{z})} \mathrm{d}\boldsymbol{z} \qquad \text{or} \qquad \int p(\boldsymbol{z})(\ln p(\boldsymbol{z}) - \ln q(\boldsymbol{z}))\mathrm{d}\boldsymbol{z}$$

Q5. [4 marks] You are given the following function for building and training a variational autoencoder:

```
def train_vae(X, num_layers, num_hidden, activation, beta_coefficient):
    """
    Returns a new VAE trained on trained on (N,D) matrix X.
    The encoder and decoder both have num_layers with num_hidden units per layer.
    Each layer calls activation(a) to compute its outputs, where 'a' is a tensor of activations.
    """
    ...
```

Can this function be used to perform principal component analysis on the first *num_components* of *X*? If not, explain why any such VAE is different from PCA. If yes, show how.

Yes it can. Call it as *train_vae(X, 1, num_components, lambda a: a, 0.0)*.
This will train a standard autoencoder (no KL term since $\beta = 0$) with 1 linear layer having *num_components* hidden units, which minimizes the same reconstruction loss as PCA.