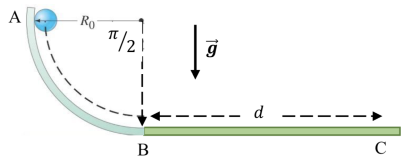
Group Number		Name		Type
List Number		Surname		A
Student ID		Signature		
E-mail				

ATTENTION: There is normally only one correct answer for each question and each correct answer is equal to 1 point. Only the answers on your answer sheet form will be evaluated. Please be sure that you have marked all of your answers on the answer sheet form by using a pencil (*not* pen).

Questions 1-2

A ball of mass m and radius r (moment of inertia $I_{cm} = \frac{2}{5}mr^2$) is placed on the inside of a frictionless circular track of radius R_0 as shown in the figure. It starts from rest at the vertical edge of the track, and since there is no friction, it slides down without rotation.



1. What will be the speed of its center of mass when it reaches the lowest point B of the track?

(a) 0 (b) $\sqrt{4g(R_0 - r)}$ (c) $\sqrt{4g(R_0 + r)}$ (d) $\sqrt{2g(R_0 + r)}$ (e) $\sqrt{2g(R_0 - r)}$

2. The horizontal section of the track starting from B is a surface with coefficient of kinetic friction μ_k. If the ball starts to roll without slipping after traveling a distance d, what is the expression for the coefficient of kinetic friction in terms of the given parameters?

(a) $\frac{12(R_0-r)}{49d}$ (b) $\frac{5(R_0-r)}{49d}$ (c) $\frac{5(R_0-r)}{64d}$ (d) $\frac{24(R_0-r)}{49d}$ (e) $\frac{3(R_0-r)}{8d}$

Questions 3-6

An object with mass m is initially at rest at the origin $x = 0$. At time $t = 0$ it starts to accelerate with a changing acceleration along the $+x$ direction. At time $t = T$ it is at the point $x = x_T$ and its speed is measured as $v(T) = v_T$.

3. How much work is done by the force to accelerate the object during the time interval T?

(a) $-\frac{1}{2}mv_T^2$ (b) 0 (c) $\frac{1}{2}mv_T^2$ (d) mv_T^2 (e) $-mv_T^2$

4. What is the average power supplied by the force during the time interval T?

(a) $\frac{2mv_T^2}{T}$ (b) $\frac{mv_T^2}{2T}$ (c) 0 (d) $\frac{mv_T^2}{4T}$ (e) $\frac{mv_T^2}{T}$

5. If the force accelerating the object is of the form $F(t) = F_0\left(1 - \frac{t}{T}\right)$ for $0 \le t \le T$, what is the power supplied by the force at $t = T$?

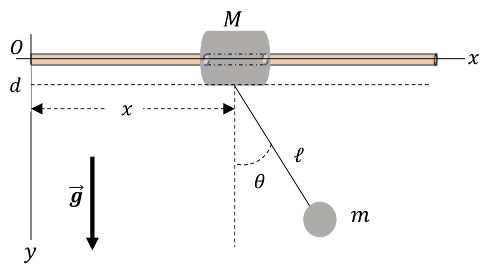
(a) $\frac{mv_T^2}{4T}$ (b) $\frac{mv_T^2}{2T}$ (c) 0 (d) $\frac{2mv_T^2}{T}$ (e) $\frac{mv_T^2}{T}$

6. Find the expressions for v_T and x_T in terms of $F, m,$ and T.

(a) $v_T = \frac{F_0T}{2m}$, $x_T = \frac{F_0T^2}{m}$ (b) $v_T = \frac{F_0T}{m}$, $x_T = \frac{F_0T^2}{2m}$ (c) $v_T = \frac{F_0T}{2m}$, $x_T = \frac{F_0T^2}{3m}$ (d) $v_T = \frac{F_0T}{2m}$, $x_T = \frac{F_0T^2}{2m}$
(e) $v_T = \frac{F_0T}{m}$, $x_T = \frac{F_0T^2}{3m}$

Questions 7-9

A cylinder of mass M is free to slide on a frictionless horizontal shaft passing through its axis. A ball of mass m is attached to the cylinder by a massless string of length ℓ. Initially, both the cylinder and the ball are at rest, with the center of the cylinder at a perpendicular distance x_0 from the y-axis, and the ball displaced by an angle $\theta = \pi/2$ to the right relative to the vertical. Use the coordinate system indicated in the figure and assume that the motion takes place on the xy- plane.



7. What is the initial x-coordinate of the center of mass of the system?

(a) $x_{cm} = x_0 + \frac{2M\ell}{M+m}$ (b) $x_{cm} = x_0 + \frac{2m\ell}{M+m}$ (c) $x_{cm} = x_0 + \frac{m\ell}{M+m}$ (d) $x_{cm} = x_0 + \frac{M\ell}{M+m}$ (e) $x_{cm} = x_0 + \frac{m\ell}{2(M+m)}$

8. If the ball is released from its initial position $(x_0 + \ell, d)$ with zero initial velocity, what will be its coordiates (x', y') when it is at the bottom of its swing, i.e., when $\theta = 0$?

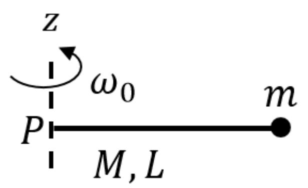
(a) $x' = x_0 + \frac{m\ell}{M+m}$, $y' = \ell + d$ (b) $x' = x_0 + \frac{M\ell}{M+m}$, $y' = \ell + 2d$ (c) $x' = x_0 + \frac{m\ell}{M+m}$, $y' = \ell + \frac{d}{2}$
(d) $x' = x_0 + \frac{2M\ell}{M+m}$, $y' = \ell + \frac{d}{2}$ (e) $x' = x_0 + \frac{2m\ell}{M+m}$, $y' = \ell + d$

9. Find the velocities of the ball v_B and the cylinder v_C when $\theta = 0$.

(a) $v_B = \sqrt{\frac{2Mg\ell}{M+m}}$, $v_C = \sqrt{\frac{2m^2g\ell}{M(M+m)}}$ (b) $v_B = \sqrt{\frac{Mg\ell}{M+m}}$, $v_C = \sqrt{\frac{m^2g\ell}{M(M+m)}}$ (c) $v_B = \sqrt{\frac{Mg\ell}{M+m}}$, $v_C = \sqrt{\frac{2m^2g\ell}{M(M+m)}}$
(d) $v_B = \sqrt{\frac{2mg\ell}{M+m}}$, $v_C = \sqrt{\frac{2M^2g\ell}{m(M+m)}}$ (e) $v_B = \sqrt{\frac{2Mg\ell}{M+m}}$, $v_C = \sqrt{\frac{2M^2g\ell}{m(M+m)}}$

Questions 10-12

A uniform rod of mass $M = 0.6\ kg$ and length $L = 1\ m$ with a point mass $m = 0.3\ kg$ attached to its free end is rotating with angular speed $\omega_0 = 10.0\ rad/s$ about the z-axis, as shown in the figure.



10. Find the rotational inertia of the system about point P in units of kgm^2. (For a uniform rod of mass M and length L, $I_{cm} = \frac{1}{12}ML^2$)
(a) 0.5 (b) 2.0 (c) 1.0 (d) 2.5 (e) 1.5

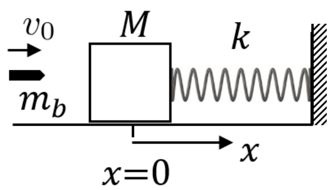
11. Another point mass $2m$ moving in the plane of rotation collides perpendicularly in the direction of the rotation of the rod and sticks to the rod at a distance $2L/3$ form point P with a linear speed $3\omega_0 L$. What is the angular momentum vector relative to point P just after the collision in units of kgm^2/s ?

(a) $19\hat{k}$ (b) $15\hat{k}$ (c) $21\hat{k}$ (d) $23\hat{k}$ (e) $17\hat{k}$

12. What is the angular speed of the system just after the collision in rad/s?
(a) $\frac{270}{17}$ (b) $\frac{510}{23}$ (c) $\frac{290}{13}$ (d) $\frac{410}{19}$ (e) $\frac{310}{29}$

Questions 13-16

A massless spring with spring constant k is attached at one end of a block of mass M that is at rest on a frictionless horizontal table. The other end of the spring is fixed to a wall. A bullet of mass m_b is fired into the block from the left with a speed v_0 and comes to rest in the block.



13. What is the speed of the block-bullet system immediately after the collision?

(a) $\frac{m_b}{m_b+M}v_0$ (b) $\sqrt{\frac{m_b}{m_b+M}}v_0$ (c) $\frac{m_b}{M}v_0$ (d) $\sqrt{\frac{m_b+M}{m_b}}v_0$ (e) $\frac{m_b+M}{m_b}v_0$

14. Find the amplitude of the resulting simple harmonic motion.

(a) $\sqrt{\frac{1}{k\, m_b}}(m_b + M)v_0$ (b) $\sqrt{\frac{1}{k(m_b+M)}}m_b v_0$ (c) $\sqrt{\frac{(m_b+M)}{m_b}}v_0$ (d) $\sqrt{\frac{1}{k\, M}}m_b v_0$ (e) $\sqrt{\frac{m_b}{(m_b+M)}}v_0$

15. How long does it take the block to first return to the position $x = 0$?

(a) $\frac{\pi}{2}\sqrt{\frac{m_b+M}{k}}$ (b) $2\pi\sqrt{\frac{m_b+M}{k}}$ (c) $\pi\sqrt{\frac{k}{m_b+M}}$ (d) $\frac{\pi}{4}\sqrt{\frac{m_b+M}{k}}$ (e) $\pi\sqrt{\frac{m_b+M}{k}}$

16. What is the maximum acceleration of the block?

(a) $\sqrt{\frac{k}{m_b+M}}v_0$ (b) $\sqrt{\frac{k\, m_b}{m_b+M}}v_0$ (c) $\sqrt{\frac{k\, m_b}{(m_b+M)^2}}v_0$ (d) $\sqrt{\frac{k\, m_b^2}{(m_b+M)^3}}v_0$ (e) $\sqrt{\frac{k\,(m_b+M))}{m_b^2}}v_0$

Questions 17-20

A small object of mass m is launched from the surface of the Earth with a speed of v_0 in a direction perpendicular to the Earths surface.

17. What is the total mechanical energy of the object at its starting point in terms of m, v_0, the radius of the Earth R, the mass of the Earth M, and the gravitational constant G ?

(a) $\frac{1}{2}mv_0^2$ (b) $\frac{1}{2}mv^2 + \frac{GMm}{R^2}$ (c) $\frac{1}{2}mv^2 - \frac{GMm}{R^2}$ (d) $\frac{1}{2}mv_0^2 + \frac{GMm}{R}$ (e) $\frac{1}{2}mv_0^2 - \frac{GMm}{R}$

18. Find an expression for the speed v of the object at a height $h = R$ (i.e., a distance $2R$ from Earth's center).

(a) $\sqrt{v_0^2 - \frac{GM}{2R}}$ (b) $\sqrt{v_0^2 - \frac{3GM}{R}}$ (c) $\sqrt{v_0^2 - \frac{2GM}{R}}$ (d) $\sqrt{v_0^2 - \frac{GM}{R}}$ (e) $\sqrt{v_0^2 - \frac{GM}{3R}}$

19. Now consider a different situation where the object is placed in a circular orbit at a height $h = R$ (i.e., a distance $2R$ from Earth's center). Find the speed the object needs to be in a circular orbit at that height.

(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{GM}{3R}}$ (c) $\sqrt{\frac{3GM}{R}}$ (d) $\sqrt{\frac{GM}{2R}}$ (e) $\sqrt{\frac{GM}{R}}$

20. Find the period of the object in this circular orbit at that height.

(a) $4\pi\sqrt{\frac{R^3}{2GM}}$ (b) $4\pi\sqrt{\frac{2R^3}{GM}}$ (c) $2\pi\sqrt{\frac{R^3}{2GM}}$ (d) $\pi\sqrt{\frac{R^3}{GM}}$ (e) $2\pi\sqrt{\frac{R^3}{GM}}$