START: 09.15

+ 1- Jacobi Iteration Method
2- Gauss- Seidel Iteration Method

Now

Gauss Seidel Iteration method is used in the solution of linear equation systems.

Example

$5x+2y+z=12$
$2x+3y+4z=20$
$-4x+5y+3z=15$

Step 1 Write the system of linear equation $Ax=b$ format

$$\begin{bmatrix} 5 & 2 & 1 \\ 2 & 3 & 4 \\ -4 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 15 \end{bmatrix} \quad 3_2 \leftarrow 5_3$$

Step 2 Write the largest absolute number in each colown in the diagonal

$$\begin{bmatrix} 5 & 2 & 1 \\ -4 & 5 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 20 \end{bmatrix}$$

Step 3 find x from equation 1
find y from " 2
find z from " 3

$5x+2y+z=12$ $\quad x=\frac{12-2y-z}{5}$
$-4x+5y+3z=15$ $\quad y=\frac{15+4x-3z}{5}$
$2x+3y+4z=20$ $\quad z=\frac{20-2x-3y}{4}$

The starting point is given to you the poem asking the question.
If the problem is yourself, you must determine the starting point.
Generally the (0,0,0) point is considered the starting point

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1. iteration

$x=\frac{12-2y-z}{5}$
$y=\frac{15+4x-3z}{5}$
$z=\frac{20-2x-3y}{4}$

$x=2.4$
$y=4.92$
$z=0.11$

$y=\frac{4(2.4)-3.0+15}{5}$
$z=\frac{20-2(2.4)-3(4.92)}{4}$

2. iteration

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2.4 \\ 4.92 \\ 0.11 \end{pmatrix}$$

$x=\frac{12-2(4.92)-0.11}{5}$
$y=\frac{4(0.41)-3(0.11)+5}{5}$
$z=\frac{20-2(0.41)-3(3.26)}{4}$

$x=0.41$
$y=3.26$
$z=2.35$

3. iterat

Example

$-x+y+7z=-6$
$4x-y-z=3$
$-2x+6y+z=9$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \\ 9 \end{pmatrix}$$

$$\begin{bmatrix} -1 & 1 & 7 \\ 4 & -1 & -1 \\ -2 & 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \\ 9 \end{bmatrix} \Longleftrightarrow$$

$$\begin{bmatrix} 4 & -1 & -1 \\ -2 & 6 & 1 \\ -1 & 1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ -6 \end{bmatrix}$$

$4x-y-z=3$ $\quad x=\frac{3+y+z}{4}$

$\begin{bmatrix} -1 & 1 & 7 \end{bmatrix} \begin{bmatrix} t \end{bmatrix} \begin{bmatrix} -6 \end{bmatrix}$

$4x - y - z = 3$

$-2x + 6y + z = 9$

$-x + y + 7z = -6$

$x = \frac{3 + y + z}{4}$

$y = \frac{9 + 2x - z}{6}$

$z = \frac{x + y - 6}{7}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x = 0.75$

$y = 1.75$

$z = -1$

2. iteration

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0,75 \\ 1.75 \\ -1 \end{pmatrix}$

$x = 0,938$

$y = 1,979$

$z = -1,006$

3. iteration

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0,938 \\ 1,979 \\ -1.006 \end{pmatrix}$

$x = 0.993$

$y = 1,999$

$z = -1.001$

	initial	1.iter	2.it	3.it	4.it	5.it	6	7	8
x	0	0.75	0,938	0,993	0,999	1.000	1	1	-
y	0	1,75	1,979	1,993	2.000	2.000	2	2	
z	0	-1	-1.006	-1.001	-1.000	-1.000	-1	-1	

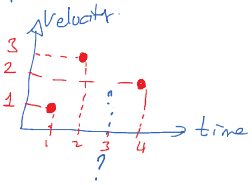
*. Interpolation

START: 10 15

INTERPOLATION

In some cases, some data may be missing from some data system.

The process of finding these missing data is called interpolation.



$5x + 6y + 6z = 10$

$2x + 4y + 2z = 12$

$3x + 4y + 5z$

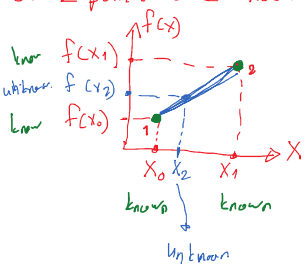
① Linear interpolation method → Easy

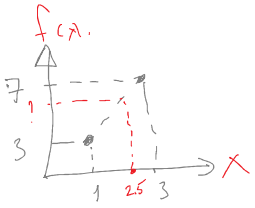
② Quadratic " "

③ Langrange interpolation method

Linear Interpolation method

It is a method used only if the coordinates of 2 point are know.

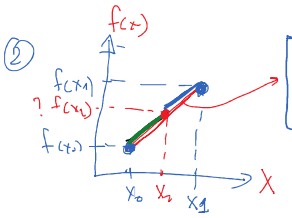




① Write linear equation

$y - y_0 = m(x - x_0)$

$$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

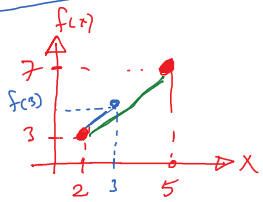
② 

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

Same slope method

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_2) - f(x_0)}{x_2 - x_0} \quad *$$

Q:



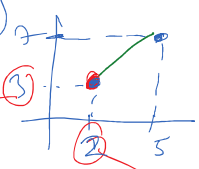
f(3) = ?

2. Method Slope = Slope

$$\frac{7-3}{5-2} = \frac{f(3)-3}{3-2}$$

$$f(3) = \frac{13}{4} = 4.33$$

1. method



$$m = \frac{7-3}{5-2} = \frac{4}{3}$$

$$y - y_0 = m(x - x_0)$$

$$y - 3 = \frac{4}{3}(x - 2)$$

$$y = \frac{4}{3}x - \frac{8}{3} + 3$$

$$y = \frac{4}{3}x + \frac{1}{3}$$

$$f(x) = \frac{4}{3}x + \frac{1}{3}$$

$$? \quad f(3) = \frac{4}{3} \cdot 3 + \frac{1}{3} = \frac{13}{3} = 4.33$$

Ex:

X	3	4	(4.5)	5	6
f(x) = log(x)	0.477	0.602	(0.653)	0.700	0.778

a) find log(4.5) using x=3 and x=6 point

b) find log(4.5) using x=4 and x=5 point

c) find relative error (a)

d) " " " (b)

log(4.5) = 0.653

a) $\frac{f(6) - f(3)}{6 - 3} = \frac{f(4.5) - f(3)}{4.5 - 3}$

$$\frac{f(6)-f(3)}{6-3} = \frac{f(4.5)-f(3)}{4.5-3}$$

$$\frac{0.778-0.477}{3} = \frac{f(4.5)-0.477}{1.5}$$

$f(4.5) = 0.6275$

b) $$\frac{f(5)-f(4)}{5-4} = \frac{f(4.5)-f(4)}{4.5-4}$$

$f(4.5) = 0.651$

c) $$\frac{|0.653-0.6275|}{|0.653|} = 0.04$$ %4

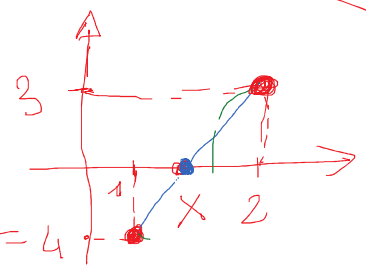
d) $$\frac{|0.653-0.651|}{|0.653|} = 0.003$$ %0.3

$f(x) = x^3 + x^2 - 3x - 3$

1.57 1.57 1.5

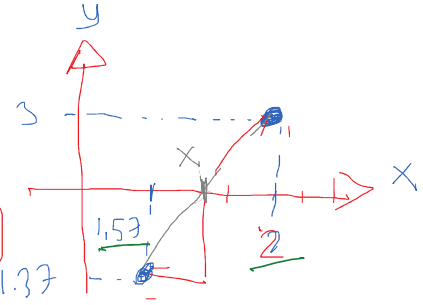
find root using linear interpolation method in interval $[1, 2]$

	x	f(x)
a	1	-4
b	2	3
c	1.57	-1.37
d	1.71	-0.24



$$\frac{3--4}{2-1} = \frac{0--4}{x-1}$$

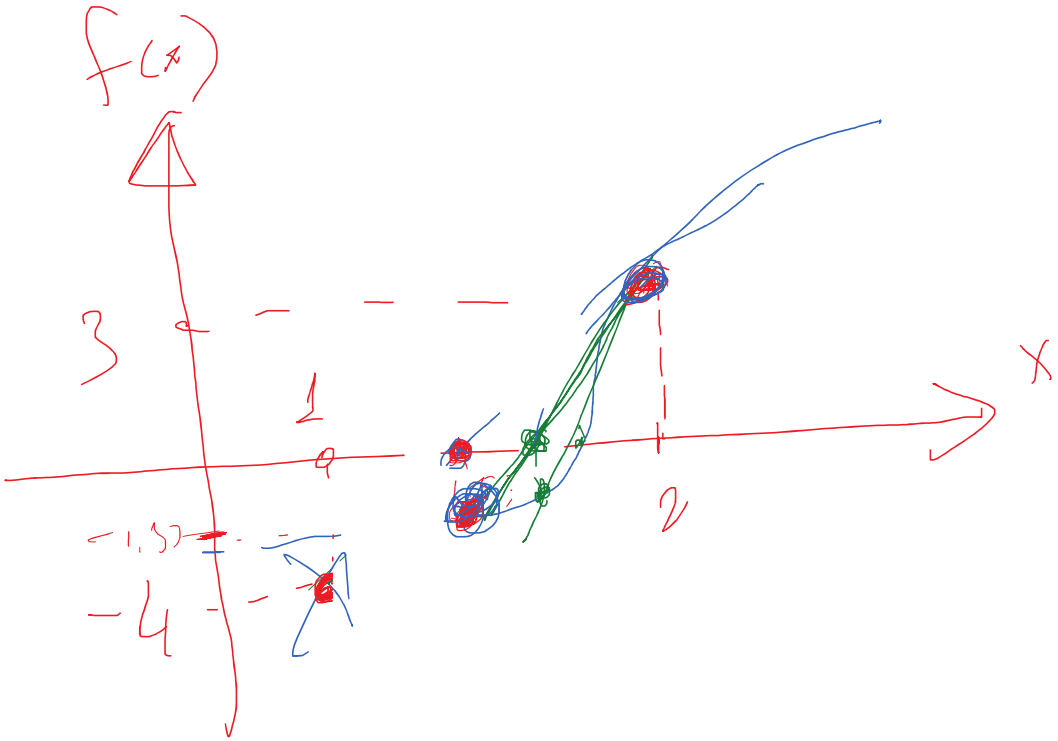
$x = 1.57$



1.7314	-0.006
1.73195	-0.00095
1.73203	-0.00015
	-0.00004

$$\frac{3--1.37}{} = \frac{0--1.37}{}$$

1,68 →



f

(0)

-(x1) -6

1.7320485 − 0.0004

$$\frac{3 - -1.37}{2 - 1.57} = \frac{0 - -1.37}{x - 1.57}$$

$$X = 1.71$$

Homework

(20) $f(x) = x \sin x - 1$ $[0, 2]$

find root

Using Linear interpolation meth