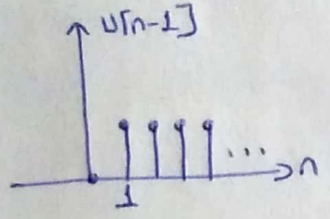
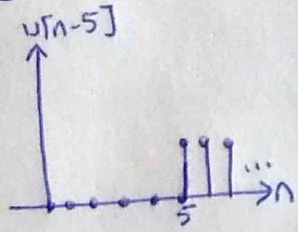
BME 3161

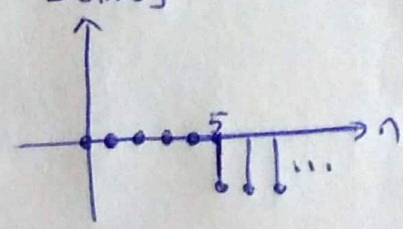
BIOSIGNAL PROCESSING – HW1

SOLUTIONS

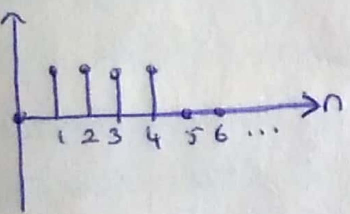
Q1- a) $h[n] = u[n-1] - u[n-5]$



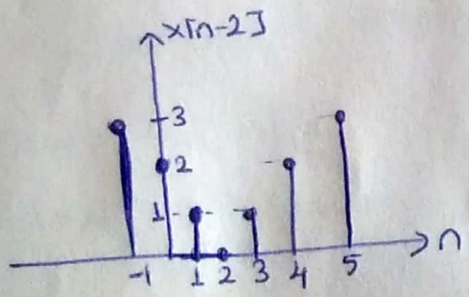


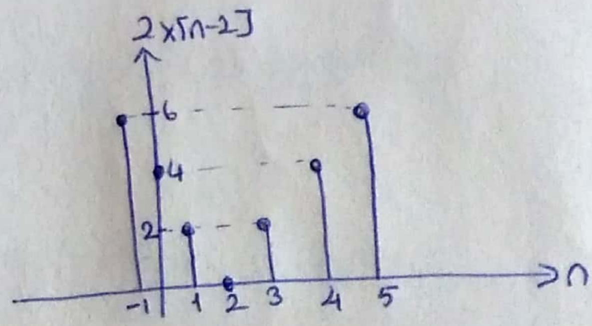


$h[n] = u[n-1] - u[n-5]$



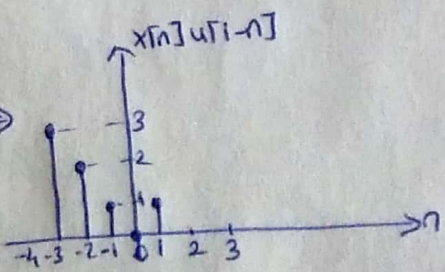
b) $2x[n-2]$



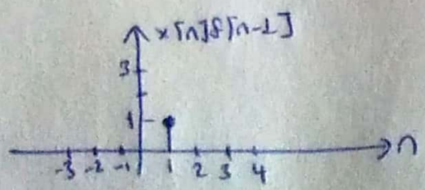


c) $x[n]u[1-n]$

$$u[1-n] = \begin{cases} 1 & n \leq 1 \\ 0 & n > 1 \end{cases} \Rightarrow$$



d) $$x[n]\delta[n-1] = x[1]\delta[n-1] = \delta[n-1] = \begin{cases} 1 & n=1 \\ 0 & n \neq 1 \end{cases}$$



Q2 $y[n] = x[n] + n\,x[n+1]$

a) Linearity

- $x_1[n] \rightarrow y_1[n]$

  $y_1[n] = x_1[n] + n\,x_1[n+1]$

- $x_2[n] \rightarrow y_2[n]$

  $y_2[n] = x_2[n] + n\,x_1[n+1]$

- $x_3[n] = c_1 x_1[n] + c_2 x_2[n] \rightarrow y_3[n]$

$y_3[n] = (c_1 x_1[n] + c_2 x_2[n]) + n\,(c_1 x_1[n+1] + c_2 x_2[n+1])$

$y_3[n] = c_1 y_1[n] + c_2 y_2[n] \Rightarrow$ LINEAR

b) Time-invariance

$x_1[n] \rightarrow y_1[n] = x_1[n] + n\,x_1[n+1]$

$x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = x_1[n-n_0] + n\,x_1[n+1-n_0]$

$y_1[n-n_0] = x_1[n-n_0] + n\,x_1[n+1-n_0]$

$y_1[n-n_0] = y_2[n] \Rightarrow$ TIME VARIANT

c) Casual / noncausal

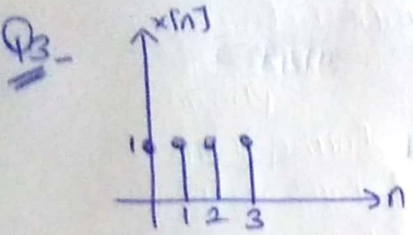
The system output depend on future values $\Rightarrow$ NONCAUSAL
($x[n+1]$)

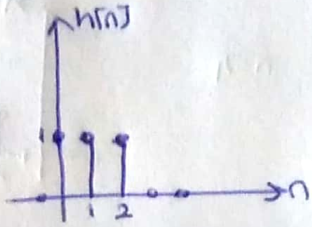
d) Stabl / Unstable

Bounded input produces an unbounded input $\Rightarrow$ UNSTABLE

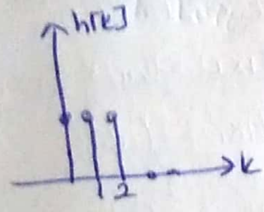
Scanned with CamScanner

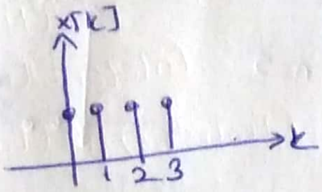
Q3-

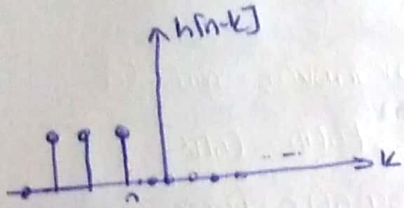




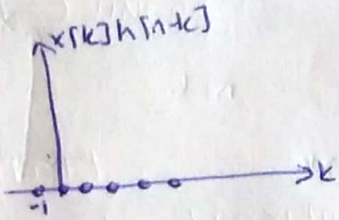
$y[n] = x[n] * h[n] = ?$

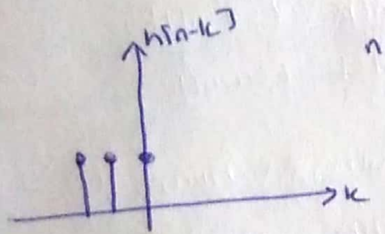




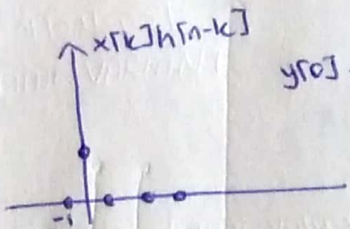


$n<0$

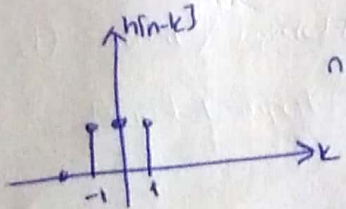




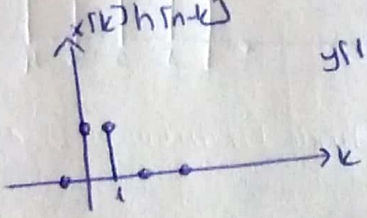
$n=0$



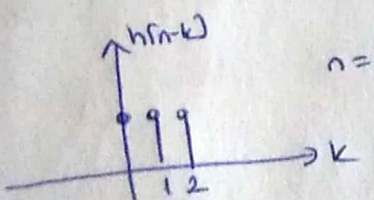
$y[0] = 1$



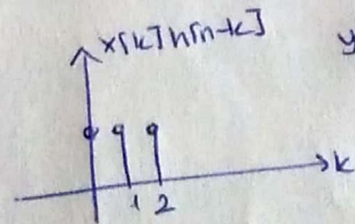
$n=1$



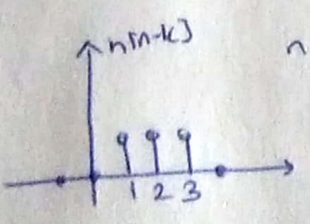
$y[1] = 2$



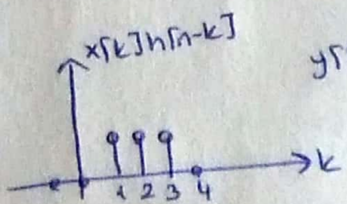
$n=2$



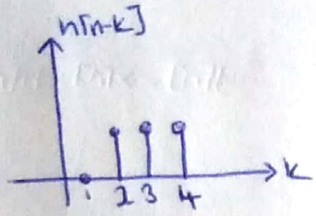
$y[2] = 3$



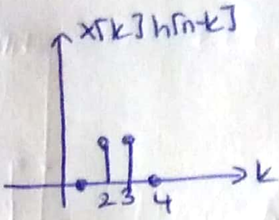
$n=3$



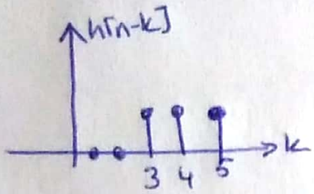
$y[3] = 3$



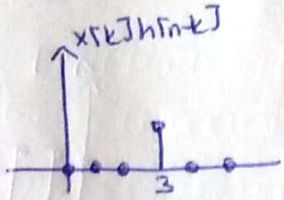
n=4



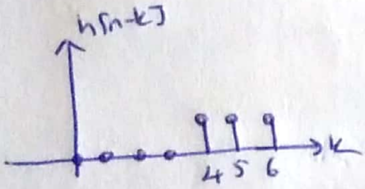
y[4]=2



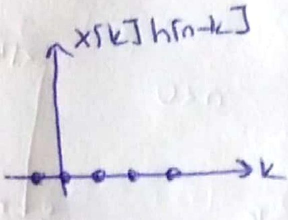
n=5

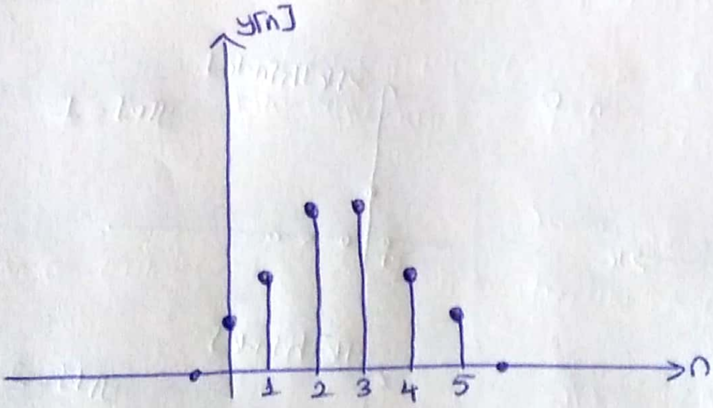


y[5]=1

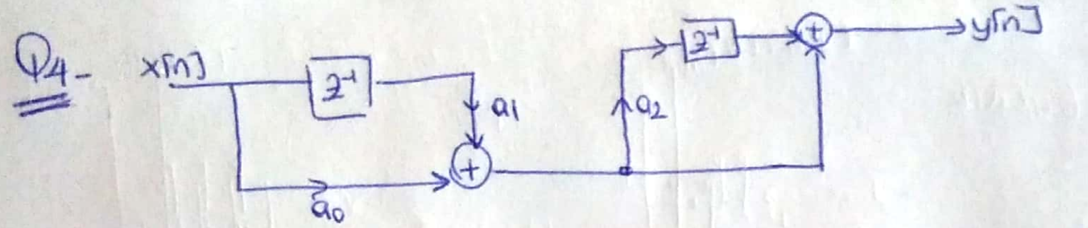


n=6





Q4-



a) $a_1 x[n-1] + a_0 x[n] + a_1 a_2 x[n-2] + a_0 a_2 x[n-1] = y[n]$

b) $x[n] = \delta[n] \rightarrow h[n]$

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

$$a_1 \overset{0}{\cancel{\delta[n-1]}} + a_0 \delta[n] + a_1 a_2 \overset{0}{\cancel{\delta[n-2]}} + a_0 a_2 \overset{0}{\cancel{\delta[n-1]}} = h[n]$$

$$h[n] = a_0 \delta[n] = a_0 \quad (n=0)$$

Q5- a) $y[n] - 0{,}6y[n-1] + 0.08y[n-2] = x[n]$

b) $y_T[n] = y_c[n] + y_p[n]$

$y_c[n]$

$$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$$

$$\lambda = 0.4 \ ; \ 0.2$$

$$y_c[n] = \alpha_1 0.4^n + \alpha_2 0.2^n$$

$y_p[n]$

$$y_p[n] = \beta\delta[n]$$

$$y[n] = y_c[n] + y_p[n]$$

$$= \alpha_1 0.4^n + \alpha_2 0.2^n + \beta\delta[n]$$

$(x[n] = \delta[n])$

$n=0$

$y[0] - 0.6\overset{0}{\cancel{y[-1]}} + 0.08\overset{0}{\cancel{y[-2]}} = 1 \Rightarrow y[0] = 1$

$n=1$

$y[1] - 0.6y[0] + 0.08\overset{0}{\cancel{y[-1]}} = 0 \Rightarrow y[1] = 0.6$

$n=2$

$y[2] - 0.6y[1] + 0.08y[0] = 0 \Rightarrow y[2] = 0.28$

$\boxed{\alpha_1 + \alpha_2 + \beta = 1}$

$\boxed{\alpha_1 0.4 + \alpha_2 0.2 = 0.6}$

$\boxed{\alpha_1 (0.4)^2 + \alpha_2 (0.2)^2 = 0.28}$

3eq. 3unknows

$\alpha_1 = 2 \ ; \ \alpha_2 = -1 \ ; \ \beta = 0$

$$\boxed{h[n] = 2(0.4)^n - 1(0.2)^n \quad n \geq 0}$$