$\mathcal{L}[e^{+j\omega t}] = \mathcal{L}\{\cos\omega t + j\sin\omega t\} = \mathcal{L}\{\cos\omega t\}$

$+ j\,\mathcal{L}\{\sin\omega t\} = \frac{1}{s-j\omega}\,\frac{s+j\omega}{s+j\omega} = \underbrace{\frac{s}{s^2+\omega^2} + j\,\frac{\omega}{s^2+\omega^2}}_{\cos\omega t + j\sin\omega t}$

what is the laplace of $\cos\omega t = \mathcal{L}\left\{\frac{e^{j\omega t} + e^{-j\omega}}{2}\right\}$

'' '' '' '' '' $\sin\omega t = \frac{1}{2}\mathcal{L}(e^{j\omega t} - e^{-j\omega})$

$= \frac{1}{2j}\{\mathcal{L}[e^{j\omega t}] - \mathcal{L}[e^{-j\omega t}]\}$

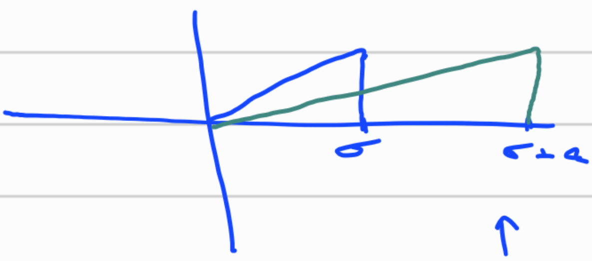
$\frac{1}{2j}\left\{\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right\} = \frac{1}{2j}\,\frac{s+j\omega-(s+j\omega)}{s^2+\omega^2} =$

$\frac{1}{2j}\,\frac{2j\omega}{s^2+\omega^2}$

$\mathcal{L}[e^{-at} f(t)] = \int_0^\infty e^{at} f(t)\, e^{-st}\, dt$

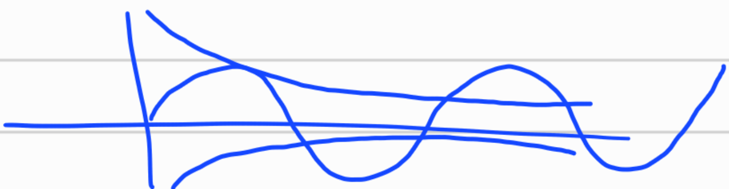
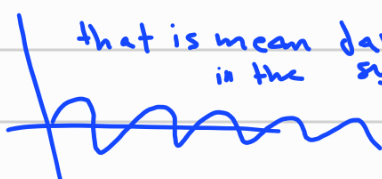
Ex: $e^{s-a} = \int_0^\infty e^{(s-a)t} f(t) = f(s-a)$

↳ this is the shift of laplace domain



$\mathcal{L}\{f(t)\} = f(s) \qquad s = \sigma + j\omega$

$f(t) = \sin\omega t\, e^{-at}$ a damped sinesoul

 = 

that is mean damper in the system

تذکر :

$$\mathcal{L}\{f'(t)\} = \int_0^\infty \underset{\dot v}{\dot x(t)}\ \underset{u}{e^{-st}}\, dt$$

$$\frac{d}{dt} u(t)\, v(t) = u\dot v + v\dot u$$

$$\int d(uv) = \int u\dot v + \int v\dot u$$

$$uv = \int u\dot v + \int v\dot u$$

$$\int_0^\infty u\dot v = uv\Big|_0^\infty - \int_0^\infty v\dot u$$

$$\dot V = \dot x\, dt \longrightarrow V = x(t)$$

$$u = e^{-st} \longrightarrow \dot u = -s e^{-st}$$

$$\int \dot x e^{-st}\, dt = e^{-st}\Big|_0^\infty + s\cdot\int_0^\infty x e^{-st}\, dt$$

$$= e^{-s\infty} x(\infty) - e^{s0} x(0) + sX(s) = sX(s) - x_0$$

$\ddot X$

$$x(t=0) = x_0$$

$$\dot x(t=0) = v(t=0) = v_0$$

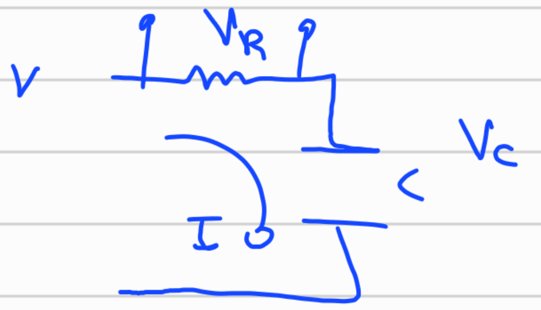
$$\mathcal{L}\left\{\frac{d}{dt}\dot x\right\} = s\mathcal{L}\{\dot x - \dot x(0)\} - \dot x$$

$$\mathcal{L}\left\{\frac{d}{dt} y\right\} = \mathcal{L}\{\dot y\} = sY(s) - y_0$$

$$= s\{\mathcal{L}\{\dot x\}\} - y_0 = s\mathcal{L}[\dot x] - \dot x_0$$

$$= s[sX(s) - sx_0 - \dot x_0]$$

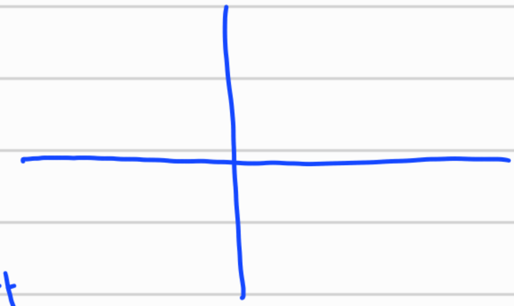
$$\mathcal{L}\{\sin\omega t\} = -\sin\omega t = \frac{d}{dt}\cos\omega t$$



$$V_{tot} = V_c + V_R$$

$V_{ext} - iR + q\frac{1}{c}$

$\dot{q} = -\frac{1}{RC}q + \frac{1}{R}X_{ext}$

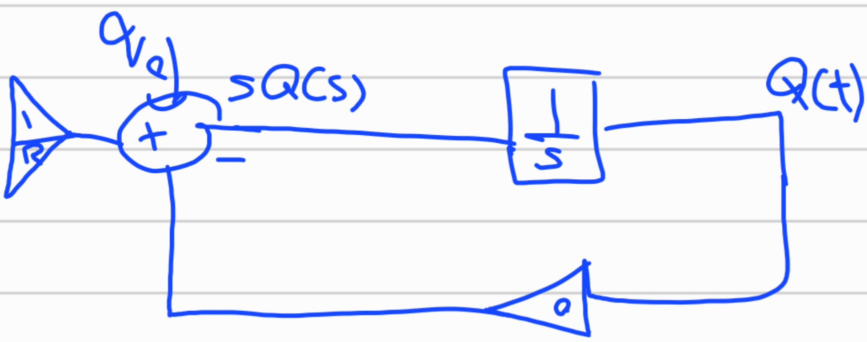


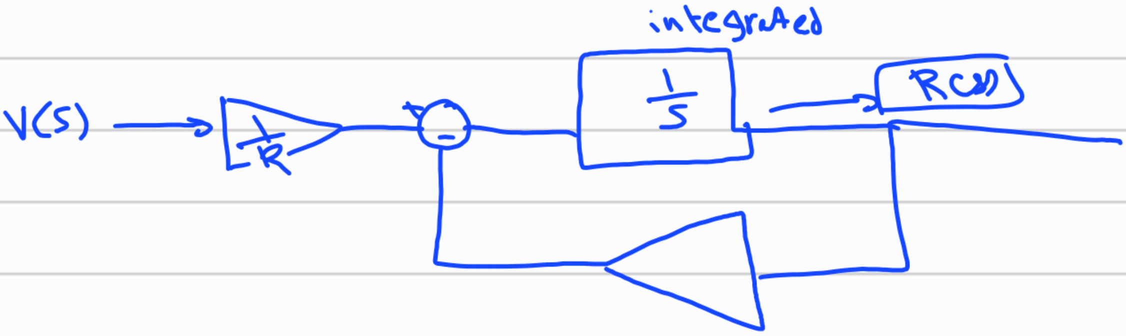
$sQ(s) - q_0 = \frac{1}{c}$

$\dot{q} = -\frac{1}{RC}q + R\ V_{ext}$

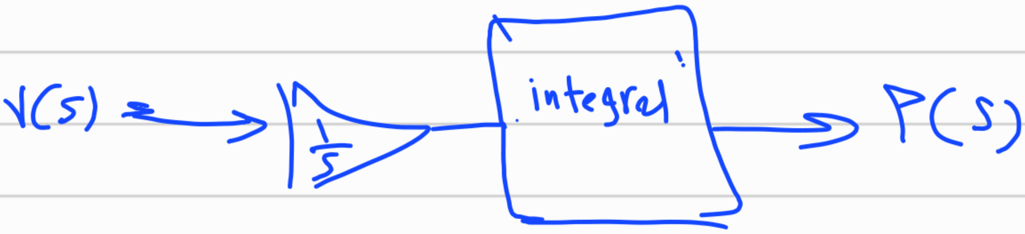
$sQ(s) - q_0 = \sigma Q(s) + \frac{V}{R_3}$ constant

$sQ(s) = \sigma Q(s) + N(t) + q_0$





close loop system



$$\dot{q} = -\sigma q + \frac{V_{ext}}{R}$$

$$sQ(s) - q_0 = -\sigma Q(s) + \frac{V_{ext}(s)}{R}$$

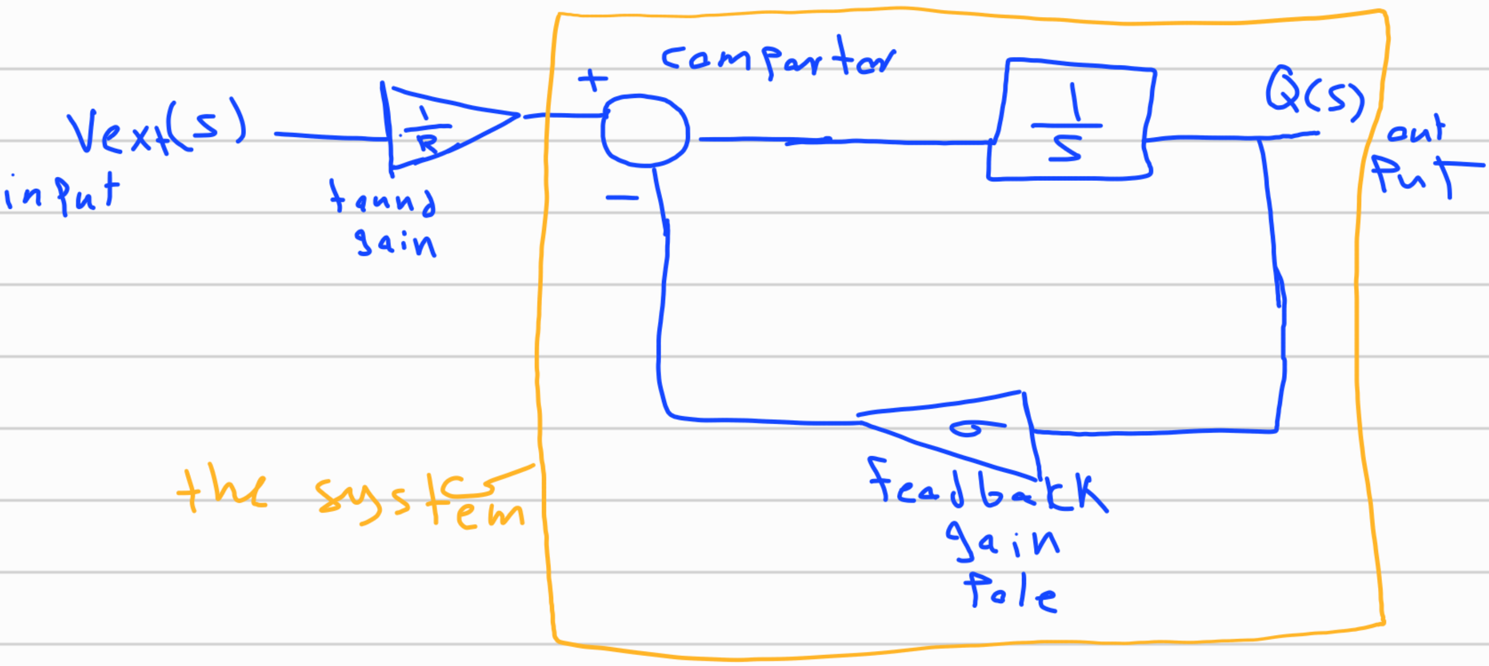
$$sQ(s) = -\sigma Q(t) + \frac{V_{ext}(s)}{R}$$

$$\mathcal{L}\{V_{ext}\, u(t)\} = \frac{V_{ext}}{s}$$

$$\mathcal{L}[V_{ext}\, \delta(t)] = V_{ext}$$

$$\mathcal{L}[V_{ext}\, r(t)] = \frac{V_{ext}}{s^2}$$

$$\boxed{-\omega \sin\omega t = \frac{d}{dt}\cos\omega t}$$



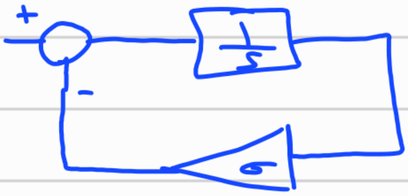
$$\frac{1}{s}\left[\frac{V(s)}{R} - \sigma Q(s)\right] = Q(s)$$

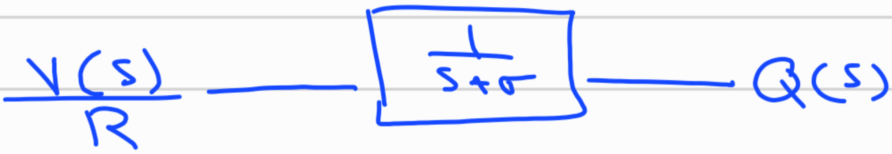
$$sQ(s) + \sigma Q(s) = (s+\sigma)Q(s) = \frac{V(s)}{R}$$

$$Q(s) = \boxed{\frac{1}{s+\sigma}}\,\frac{V(s)}{R}$$

that became my system

$$\frac{1}{s+\sigma} \equiv$$

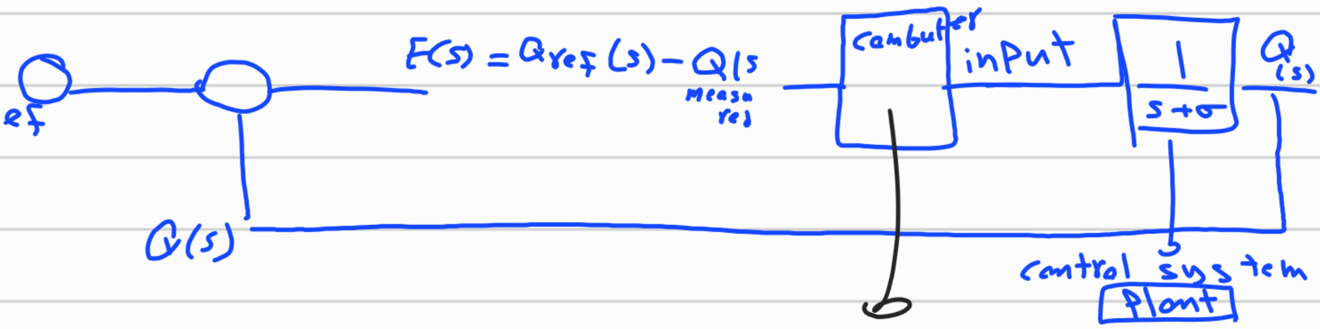




oppen loop canling

system that response to the input

I put my refrence → r)



my cambuter gives the nesserg reffrence to the input → so my error will be zero it keeps my output constant

that called servo signal

$$m\ddot{x} + b\dot{x} = I_{input}$$

system in laplace domain $\longrightarrow m[s^2 x(s) - s x_0 - \dot{x}_0] + b x(s) = I(s)$

the second order represent $\longrightarrow (ms^2 + bs)X(s) = \underbrace{I(s)}_{\text{externenly excited}} + s x_0 + V_0$

$$X(s) = \frac{1}{ms^2 + bs} I(s)$$

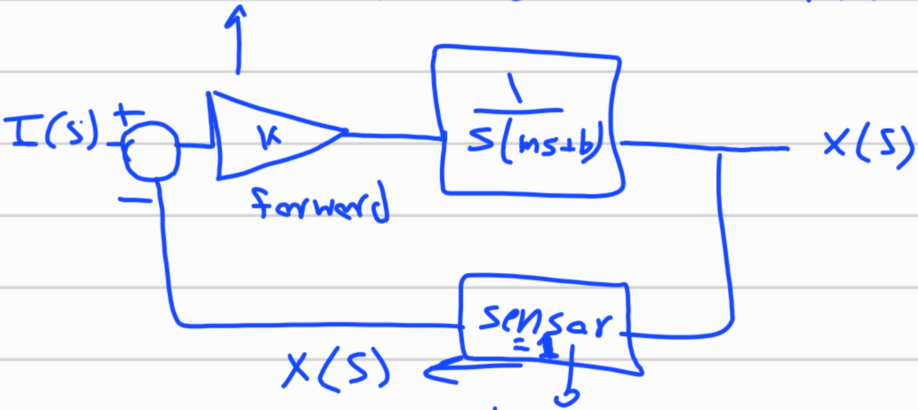
$$I(s) \longrightarrow \boxed{\frac{1}{ms^2+bs}} \longrightarrow X(s)$$

$$I(s) \longrightarrow \boxed{\frac{1}{s(ms+b)}} \longrightarrow X(s)$$

$s=0 \quad s = -\frac{b}{m}$

two plants

if you want to control system
you will put appropriate input
control your system



it is mesure
unit feedback gain

$$K\left[I(s) - X(s)\right]\left[\frac{1}{s(ms+b)}\right] = X(s)$$

$$K\,I(s) - K\,X(s) = X(s) \quad s(ms+s) \quad + \quad K\,x(s) = K\,I(s) = X(s)$$

$$[K + s(ms+b)]$$

$$X(s) = \frac{K}{s(ms+b)+K} I(s)$$

$$X(s) = \frac{K}{ms^2 + bs + K} I(s)$$

$$X(s) = \frac{K/m}{s^2 + \frac{b}{m}s + \frac{K}{m}} I(s).$$

Parametrization

1) $\omega_n^2 = \frac{K}{m}$

2) $\zeta = \frac{b}{2\omega_n m}$

$$X(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} I(s)$$

this is second polynomim

2^{nd} order

2^{nd} dgree poly

$$I(s) \longrightarrow \boxed{\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}} \longrightarrow X(s)$$

OLTF

fundemental Rule of algebra "Any n^{th} degree Polynomial can be expresed as the Product of n 1^{st} degre Polynomials

G(s) = $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{\omega_n^2}{(s+\sigma_1)(s+\sigma_2)} = \frac{\omega_n^2}{s^2+\sigma_2 s+\sigma_1 s+\sigma_1\sigma_2}$

OLTF
open loop

$$= \frac{\omega_n^2}{s^2+(\sigma_1+\sigma_2)s+\sigma_1\sigma_2}$$

$\sigma = \frac{K}{b}$ but now we have two of σ which σ_1 and σ_2

$$2\zeta\omega_n = \sigma_1 + \sigma_2$$

زتا

$$\omega_n^2 = \sigma_1 \sigma_2$$

so $\Rightarrow$ $$G(s) = \frac{\sigma_1 \sigma_2}{s^2 + (\sigma_1 + \sigma_2)s + \sigma_1 \sigma_2}$$

σ_1, σ_2 both be real

or σ_1, σ_2 both be complex

$$s^2 + (\sigma_1 + \sigma_2)s + \sigma_1\sigma_2 = (s+\sigma_1)(s+\sigma_2)$$

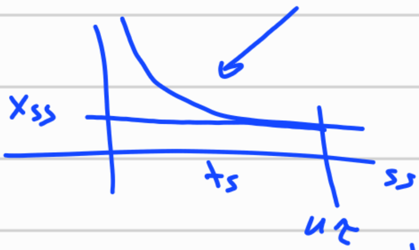
$\sigma_1 = ?$ $\quad$ $\sigma_2 = ?$ $\quad$ $(s+\sigma_1) \to 0$, $(s+\sigma_2) \to 0$

$s + \sigma_1 = 0 \qquad s + \sigma_2 = 0$

$s_1 = -\sigma_1 \qquad s_2 = -\sigma_2$

$$\sigma = \frac{K}{b} = \frac{1}{\tau}$$

Poles of the 2th order dynamic system are the solution of the Polynomial in the denominator of the OLTF



$$\frac{1}{s+\sigma}$$

$$s + \sigma = 0$$

$$\frac{\sigma_1\sigma_2}{(s+\sigma_1)(s+\sigma_2)} = \frac{A}{s+\sigma_1} + \frac{B}{s+\sigma_2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \longrightarrow s_1, s_2 = ?$$

$$m\ddot{x} + b\dot{x} + Kx = 0$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = 0$$

$$s^2 X(s) + \frac{b}{m} s X(s) + \frac{K}{m} X(s) = 0$$

$$\underbrace{\left(s^2 + s\frac{b}{m} + \frac{K}{m}\right)}_{0} X(s) = 0$$

$$X(s) \neq 0$$

$$\frac{b}{m} = 2\zeta\omega_n$$

$$\frac{K}{m} = \omega_n^2$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \longrightarrow s_1, s_2 = ?$$

بتا ↑

$$s_1, s_2 = -\frac{2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$

$$s_1, s_2 = -\zeta\omega_n \mp \frac{1}{2}\sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}$$

$$= -\zeta\omega_n \mp \omega_n\sqrt{(\zeta^2 - 1)}\ \cancel{\times} = -\zeta\omega_n \mp j\omega_d$$

$$s_{1,2} = -\sigma \mp j\omega_d$$

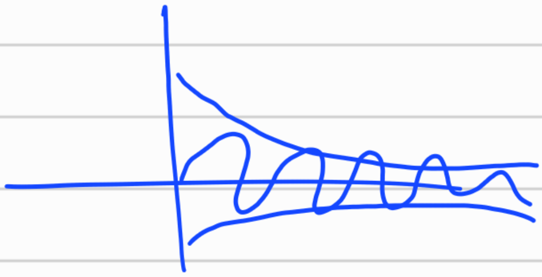
$\omega_n^2 = \frac{K}{m}$ natural freq

بتا $\delta = \frac{b}{2\omega_n m}$ damping coeff

$\omega_d = \omega_n\sqrt{1 - \zeta}$ damping freq

$\sigma = \underbrace{\zeta\omega_n}_{\text{ب}} = \frac{1}{\tau}$

أوميغا



$\omega_n^2 = \frac{K}{m}$ natural freq

زيتا $\zeta = \frac{b}{2\omega_n m}$ damping coeff

$\omega_d = \omega_n \sqrt{1-\zeta^2}$ damping freq

$\sigma = \zeta \omega_n$ pole of the exponential envelope

$\sigma_{1,2} = -\zeta \omega_n \mp j\omega_d$

$$G(s) = \frac{\sigma_1 \sigma_2}{s^2 + 2\zeta\omega_n s + \sigma_1\sigma_2} = \frac{\sigma_1 \sigma_2}{(s+\sigma_1)(s+\sigma_2)} = \frac{\sigma_1 \sigma_2}{(s+\sigma_1)(s+\sigma_2)}$$

$$= \frac{\sigma_1 \sigma_2}{[s+(-\zeta\omega_n + j\omega_d)][s+(-\zeta\omega_n - j\omega_d)]}$$

$$= \frac{\sigma_1 \sigma_2}{(s - \underbrace{\zeta}_{\text{زيتا}} \omega_n + j\omega_d)(s - \zeta\omega_n - j\omega_d)}$$

$$= \frac{\sigma_1 \sigma_2}{[(s - \zeta\omega_n) + j\omega_d][(s - \zeta\omega_n) - j\omega_d]}$$

Partial fraction

$$= \frac{A}{s - \zeta\omega_n + j\omega_d} + \frac{B}{s - \zeta\omega_n - j\omega_d} =$$

$$\frac{As - A\cancel{\zeta\omega_n} - A j\omega_d + Bs - B\cancel{\zeta\omega_n} + B j\omega_d}{[\quad][\quad]}$$

$$= \frac{(A+B)s - (B+A)\zeta\omega_n - (A+B)j\omega_d}{[\quad][\quad]}$$

$A = -B$

$\sigma_1 \sigma_2 =$ How in s domain

$$G(s) = \frac{\sigma_1 \sigma_2}{(s+\sigma_1)(s+\sigma_2)}$$

$\underbrace{m\ddot{x} + b\dot{x} + Kx}_{\text{homogeneous}} = \underbrace{\delta(t)}$ $x, \dot{x}(t=0) = 0$

→ non homogenc. external excitation

$x(t) = A e^{Bt}$

$\dot{x}(t) = AB e^{Bt}$

$\ddot{x}(t) = AB^2 e^{Bt}$

$mB^2 x(t) + bB x(t) + Kx(t) = 0$

$(mB^2 + bB + K)\, x(t) = 0$

$x \neq 0$

$mB^2 + bB + K = 0$

$$B_1, B_2 = -\frac{b}{m} \mp \sqrt{\frac{b^2}{m^2} - 4\frac{K}{m}}$$

$\frac{K}{m} = \omega_n^2$

$\frac{b}{2\omega_n m} = \zeta \Rightarrow \frac{b}{m} = 2\zeta\omega_n$

$$= -\zeta\omega_n \mp \frac{1}{2}\sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}$$

$$= -\zeta\omega_n \mp \omega_n\sqrt{\zeta^2 - 1}$$

$$= -\zeta\omega_n \mp j\omega_n\sqrt{1-\zeta^2} \Rightarrow B_{1,2} = -\zeta\omega_n \mp j\omega_d$$

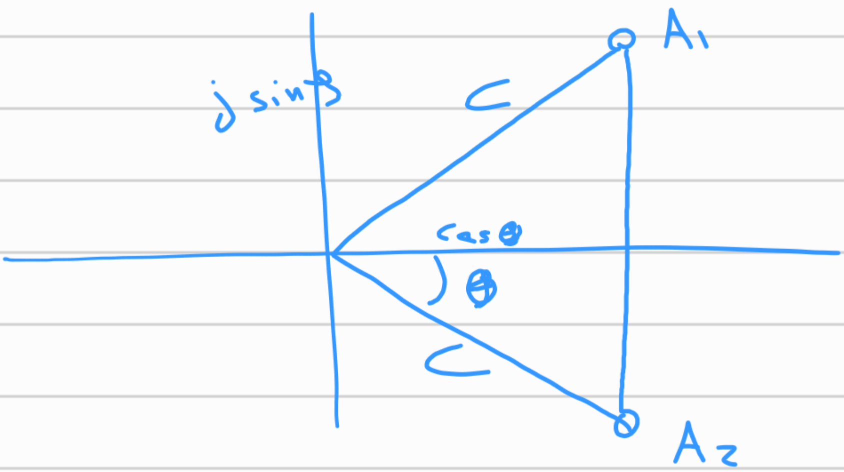
$$x_h(t) = A_1 e^{B_1 t} + A_2 e^{B_2 t}$$ A, B complex

$$x_h(t) = A_1 e^{(-\zeta\omega_n + j\omega_d)t} + A_2 e^{(-\zeta\omega_n - j\omega_d)t}$$ homo

$$= A_1 e^{-\zeta\omega_n t} e^{+j\omega_d t} + A_2 e^{-\zeta\omega_n t} e^{-j\omega_d t}$$

$$= e^{-\zeta\omega_n t}\left[A_1 e^{+j\omega_d t} + A_2 e^{-j\omega_d t}\right]$$

$$\sigma = -\zeta\omega_n$$



$A_1 = \cos\theta + j\sin y$

$A_2 = \cos\theta - j\sin$

$$e^{j\omega_d t} = \cos\omega_d t + j\sin\omega_d t$$

$$e^{-j\omega_d t} = \cos\omega_d t - j\sin\omega_d t$$

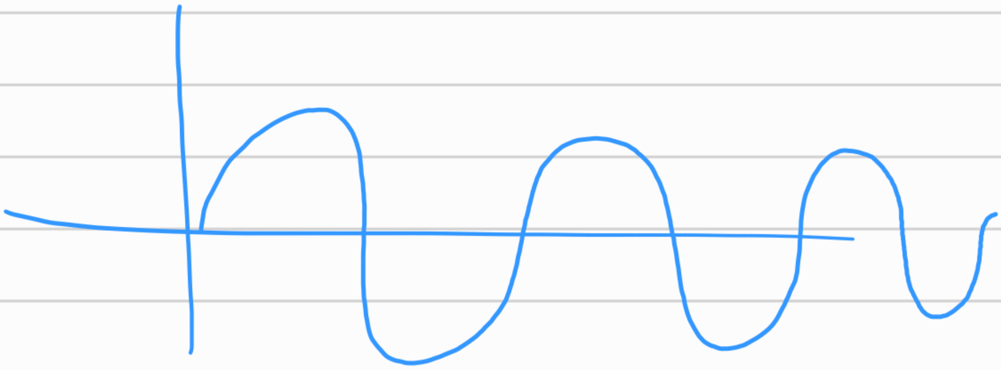
$$x_n(t) = e^{-\xi \omega_n t}\left[A_1(\cos \omega_d t + j\sin \omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)\right]$$

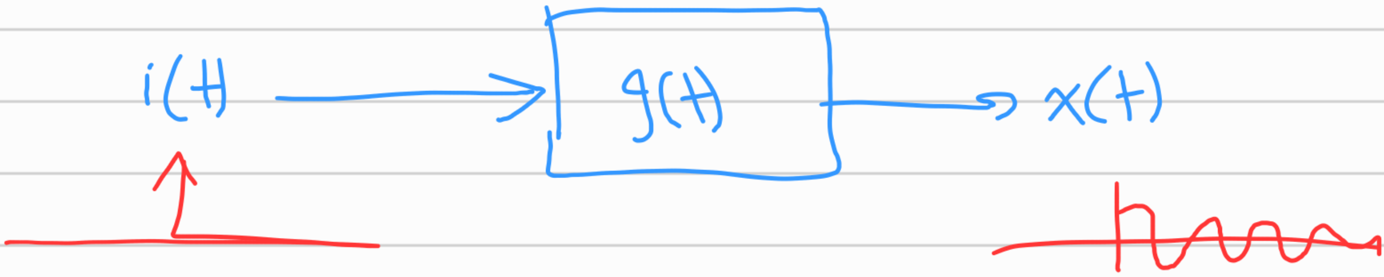
$$= e^{-\xi\omega_n t}\left[(A_1+A_2)\cos\omega_d t + j(A_1-A_2)\sin\omega_d t\right]$$

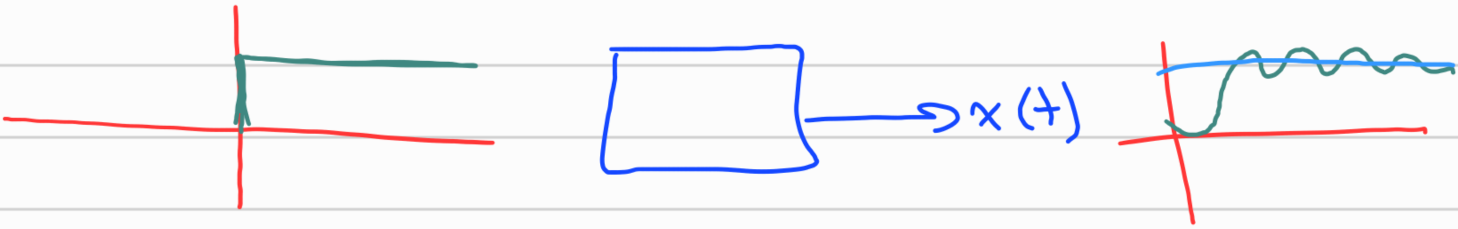
$$= C e^{-\xi\omega_n t}\left[2\cos\phi \cos\omega_d t + 2jj(\sin\phi)\sin\omega_d t)\right]$$

$$= C e^{-\xi\omega_n t}\left[\cos(\omega_d t + \phi)\right]$$

all that we try to explean about this







$$G(s) = \frac{\sigma_1 \sigma_2}{s^2 + (\sigma_1 \sigma_2)s + \sigma_1 \sigma_2} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$X(s) = G(s)\, I(s) \longrightarrow I(s) = \int 1 \quad \text{unit step}$$

$$X(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$= \frac{A}{s} + \frac{B}{s+\sigma_1} + \frac{C}{s+\sigma_2}$$

$\frac{1}{s}$ " "

$\frac{1}{s^2}$ " ramp

$\frac{1}{s+a}$ exponential

$\frac{s}{s^2+\omega^2}$ sin

$\frac{\omega}{s^2+\omega^2}$ cos

$$x(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t} \sin\omega_d t$$

impules · ramp

$$x(t) = 1 - \left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d t\right] e^{-\zeta\omega_d t}$$

unit step response

imagin that the input is exponetial it is not real input because of that we replaced in imagiry Part to be input

$$I(t) = e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$\mathrm{Re}\{e^{j\omega t}\} = \cos\omega t$$

eigen function of LTI system

$$f(t) \longrightarrow \boxed{\quad\quad} \longrightarrow f(t)\,\lambda$$

$\lambda \rightarrow$ eigen Value

$[A]\,\vec{v} = \lambda\,\vec{v}$

$\vec{v}$: eigen vector $[A]$

λ: eigen value $[A]$

Listen:

Denis freeman freq. domain

s(t)

u(t)