



# **MAT1320-Linear Algebra**

## **Lecture Notes**

Sarrus' Rule, Finding Inverse Matrices Using Adjoint Matrices

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## Sarrus' Rule

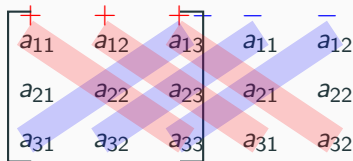
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## Sarrus' Rule

Let  $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$  be a square matrix of order 3. Then the determinant of  $\mathbf{A}$  can be computed as follows:

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This method is called **Sarrus' Rule**.

# Sarrus' Rule

## Example

By using Sarrus' Rule, find the determinant of the matrix

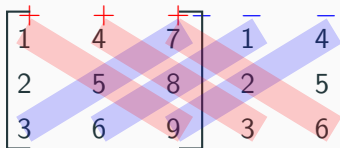
$$\mathbf{A} = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}.$$

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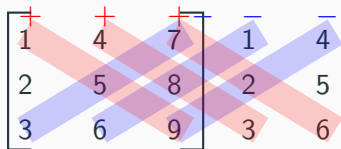


# Sarrus' Rule

## Example

By using Sarrus' Rule, find the determinant of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}.$$



$$\Rightarrow (1 \cdot 5 \cdot 9 + 4 \cdot 8 \cdot 3 + 7 \cdot 2 \cdot 6) - (7 \cdot 5 \cdot 3 + 1 \cdot 8 \cdot 6 + 4 \cdot 2 \cdot 9)$$

$$= 45 + 96 + 84 - 105 - 48 - 72 = 0$$



## **Finding Inverse Matrices Using Adjoint Matrices**

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## Finding Inverse Matrices Using Adjoint Matrices

Let  $A = [a_{ij}]$  be an  $n \times n$  matrix over a field  $K$  and let  $A_{ij}$  denote the cofactor of  $a_{ij}$ .

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### Theorem

*Let  $A$  be any square matrix. Then*

$$A(\text{adj}A) = (\text{adj}A)A = |A|I$$

*where  $I$  is the identity matrix. Thus, if  $|A| \neq 0$ ,*

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

# Finding Inverse Matrices Using Adjoint Matrices

## Example

$$\text{Let } A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

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Let  $A = \begin{pmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ . The cofactors of the nine elements of  $A$  follow:

$$A_{11} = + \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4, \quad A_{12} = - \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} = 2, \quad A_{13} = + \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = -1$$

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$$A_{31} = + \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5, \quad A_{32} = - \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = -4, \quad A_{33} = + \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = 2$$

# Finding Inverse Matrices Using Adjoint Matrices

## Example

The transpose of the above matrix of cofactors yields the classical adjoint of  $A$ ; that is,

$$\text{adj}A = \begin{pmatrix} -4 & 2 & 5 \\ 2 & -1 & -4 \\ -1 & -1 & 2 \end{pmatrix}$$

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Thus,  $A$  does have an inverse, and, by Theorem 8.9 ,

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = -\frac{1}{3} \begin{pmatrix} -4 & 2 & 5 \\ 2 & -1 & -4 \\ -1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{3} & -\frac{2}{3} & -\frac{5}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$

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