29.03.2021

21 Mart 2021 Pazar 19:38

1. Bisection Method
2. False Position Method

START: 09:15

Need:
1. Notebook,
2. Calculator
3. Excel

Welcome Numerical Method

$f(x) = x^3 + 4x^2 - 10$

$f(x) = x^3 + 2x - 2$

$f(x) = x^3 - 2\sin x$

$f(x) = -2e^{-x}$

⋮

Nonlinear Equation in One Variable

$ax + b = 0$

Linear equat

$4x - 5 = 3$

Nonliear equation

$x^2 + 2x = 8$

$x^{1/2} - 6x = 0$

$2x - 6 = 0 \longrightarrow$ linear

$x^2 - 5x + 6 = 0 \longrightarrow$ nonlinear, it has analytics.

$x^5 + 3x^4 - 2x^3 + x^2 - 3 = 0 \longrightarrow$ nonlinear, it has not analy

1. Bisection method
2. False Position Method
3. Newton - Raphson "
4. Secant Method.

⋮

*. In numerical methods, the result is not found exactly. The results are approximate.

Bisection method

f is a continuous function defined on the interval [a,b] with f(a) and f(b) of opposite sign.

The intermediate Value Theorem implies that a number x_n exists in (a,b) with $f(x_n) = 0$

Example

$f(x) = x^3 + 4x^2 - 10$ has a root in [1, 2]

determine an approximation to the root.

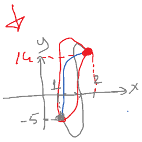
Step 1 : Continuous

Step 2 : Check root in [1,2]

$f(1) = -5$

$f(2) = 14$

$f(1) \cdot f(2) < 0$



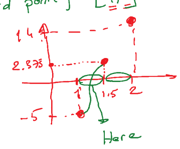
Step 3 Iteration (find mid point) [1,2]

1. Iteration

$x_1 = \frac{1+2}{2} = 1.5$

$f(x_1) = f(1.5) = 2.375$

~~Negative~~ Positive

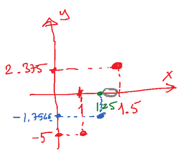


2. iteration $\boxed{[1, 1.5]}$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = f(1.25) = -1.7968...$

negatif

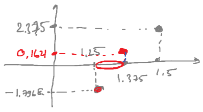


3. iteration $[1.25, 1.5]$ midpoint

$x_3 = \frac{1.25+1.5}{2} = 1.375$

$f(x_3) = f(1.375) = 0.1621$

positive



4. iteration $[1.25, 1.375]$

Well, when will we stop.

*. Stop Conditions for Bisection Method.

If the problem is our own problem

We identify this error.

If the problem is someone else's problem, the person determines the error.

We solve the question according to that error.

*. If the bisection algorithm is applied to a continuous function f(x) on an interval [a, b],

an approximate root will have been computed with error at most $\boxed{\frac{b-a}{2^n}}$

$$\boxed{\frac{b-a}{2^n} \leq \varepsilon}$$

ε : Error
n : iteration number

determine an approximation to the root that is accurate to at least within $\boxed{10^{-2}} = 0{,}01$

Continue problem

1. iteration $x_1 = 1.5$ $[1,2]$ (a, b)

$\boxed{\frac{b-a}{2^n}}$

check! $\frac{2-1}{2^1} = 0.5$ Check stop condition $0.5 > 10^{-2}$

continue

2. iteration $[1, 1.5]$ (a, b)
$x_2 = 1.25$

check

$$\frac{1.5-1}{2^2} = 0.125$$

$0{,}125 > 0{,}01$

Continue

3. iteration $[1.25, 1.5]$ (a, b)
$x_3 = 1.375$

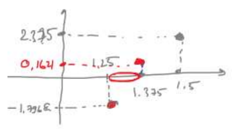
check

$$\frac{1.5-1.25}{2^3} = 0.03125$$

$0.03125 > \underline{0.01}$

continue

4. iteration $[1.25, 1.375]$ (a, b)



$$x_4 = \frac{1.25+1.375}{2} = \boxed{1.3125}$$

$x(4) = f(1.3125) = -0.84839$

Check

$\frac{b-a}{2^n}$ $\frac{1.375-1.25}{2^4} = 0.00781$

$0.00781 < 0.01$

$\boxed{\text{Stop}}$

$\boxed{\text{Ans; } 1.3125}$

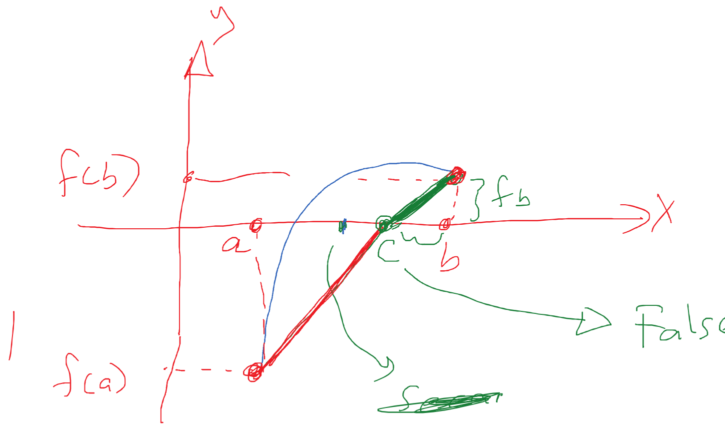
START : 10^{30}

False Position Method. (Regula falsi)

In the bisection method, a new x value in the middle of the intervals is used.

Rather than selecting the midpoint of each interval, False position method uses the

point where the secant lines the X-axis.



bisection meth same

$$slope = \frac{f(b)}{b-c} \qquad slope = \frac{-f(a)}{(c-a)}$$

$$\frac{f(b)}{b-c} \times \frac{-f(a)}{(c-a)} ,$$

$$f(b)(c-a) = -f(a)(b-c)$$

$$c f(b) - a f(b) = f(a) \cdot c - b f(a)$$

$$c(f(b) - f(a)) = a f(b) - b f(a)$$

$$\boxed{c = \frac{a(f(b)) - b(f(a))}{f(b) - f(a)}}$$

Exampl

Find root using False Positon methed

$$f(x) = x^3 - 7x^2 + 14x - 6 \qquad [0,1]$$

Step 1 . check continuous

Step 2 check root

$f(0) = -6$

$f(1) = 2$

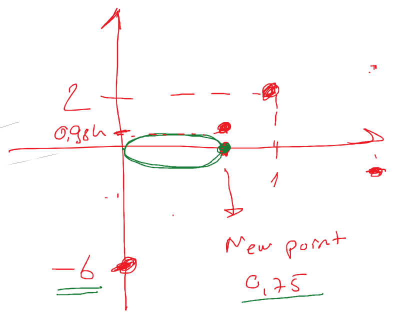
There is rout

Step 3 . Iteration . $[0,1]$

1. iteration

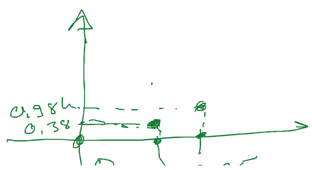
$X_1 = 0,75$

$f(x_1) = 0.984$



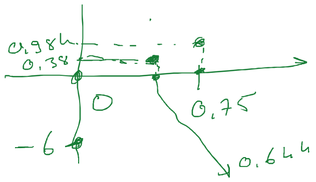
2. iteration $[0, 0.75]$

$X_2 = 0.644$



$x_2 = 0.644$

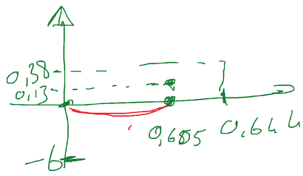
$f(x_2) = f(0.644) = 0.38$ → Positive



3. iteration $[0, 0.644]$

$x_3 = \cancel{0.608}$
$= 0.605$

$f(x_3) = f(0.605) = 0.13$ → positive



4. iteration $[0, 0.605]$

⋮

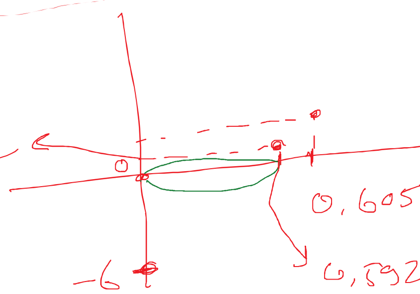
continue

find root that is accurate to at least within 10^{-2} (in this equation)

using $|x_n - x_{n-1}| < 10^{-2}$

4. iteration $[0, 0.605]$

$x_4 = 0.592$ $f(x_4) = f(0.592) = \cancel{0.042}$ 0.042 → positive



Check

$|x_4 - x_3| \overset{?}{\leq} 10^{-2}$

$|0.592 - 0.605| = 0.013$

→ continue

Stopping condition for False Position method.

x_n = The number x found from iteration

a_n : smallest number in interval (range)

b_n = largest number in it

$x_n - a_n < \text{Error}$

and

$b_n - x_n < \text{Error}$

and

$b_n - x_n < \text{Error}$

Extra stoping condition

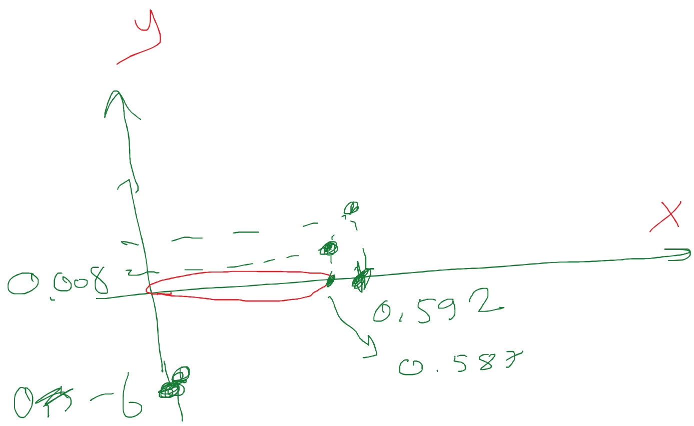
(You can use, bisection method and, False Position method)

$$\boxed{|x_n - x_{n-1}| < \text{Error}} \ *$$

5. iteration $[0, 0.592]$

$$x_5 = 0.587$$

$f(x_5) = 0.008$ → posit.



Check Stop

$$|x_5 - x_4| < 10^{-2}$$

$$|0.587 - 0.592| = 0.005$$

stop

$$\boxed{x_5 \cong 0.587}$$