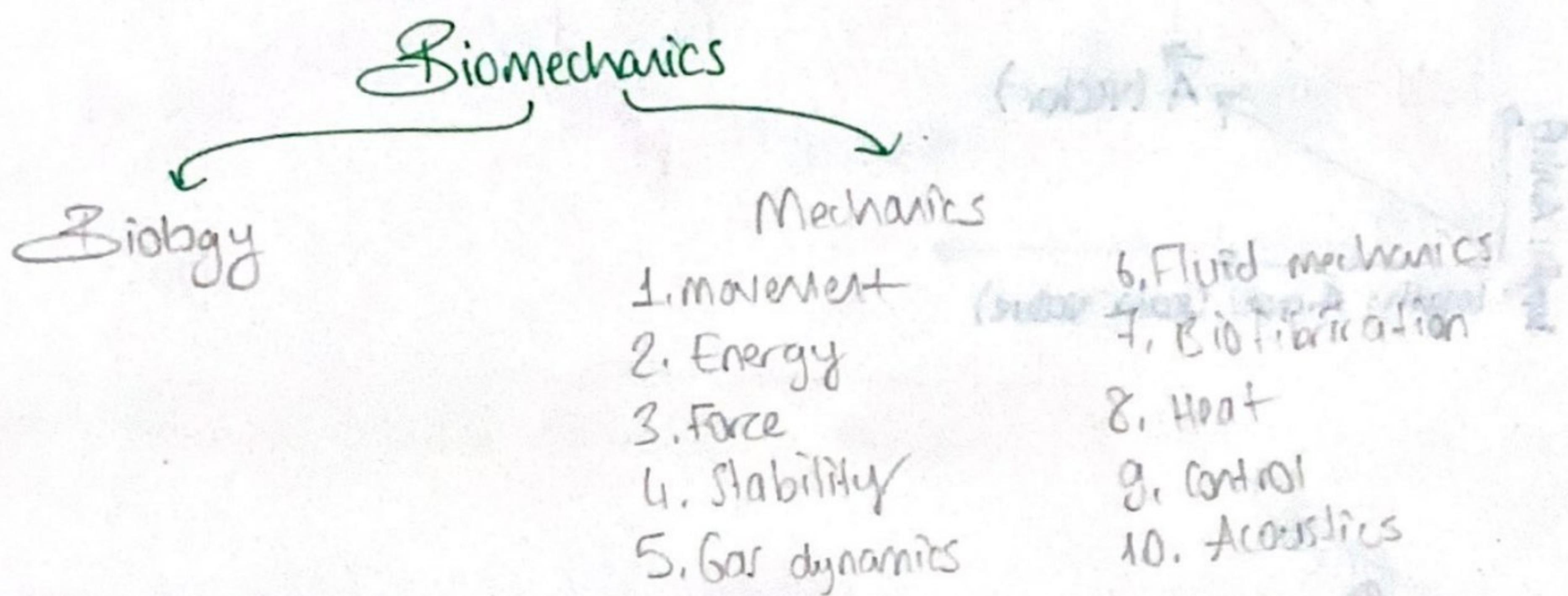


Biomechanics

Biomechanics has been defined as the study of the movement of all things. Forces acting on living things can create motion or be a healthy stimulus for growth and development or overload tissues, causing injury.



$$F = m \cdot \vec{a}$$

$\left\{ \begin{array}{l} \vec{a} \neq 0 \text{ dynamic system} \\ \vec{a} = 0 \text{ static system} \end{array} \right.$

$$\vec{a} = 0$$

$$\vec{F} = 0$$

$$\vec{v} = 0$$

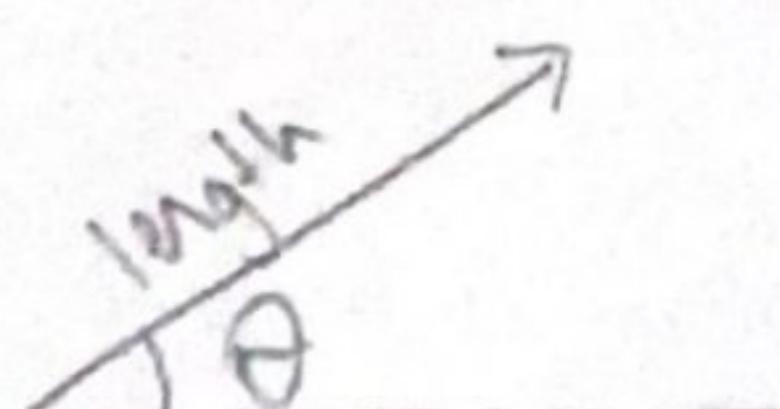
Fully static

$$\vec{v} \neq 0$$

Quadratic

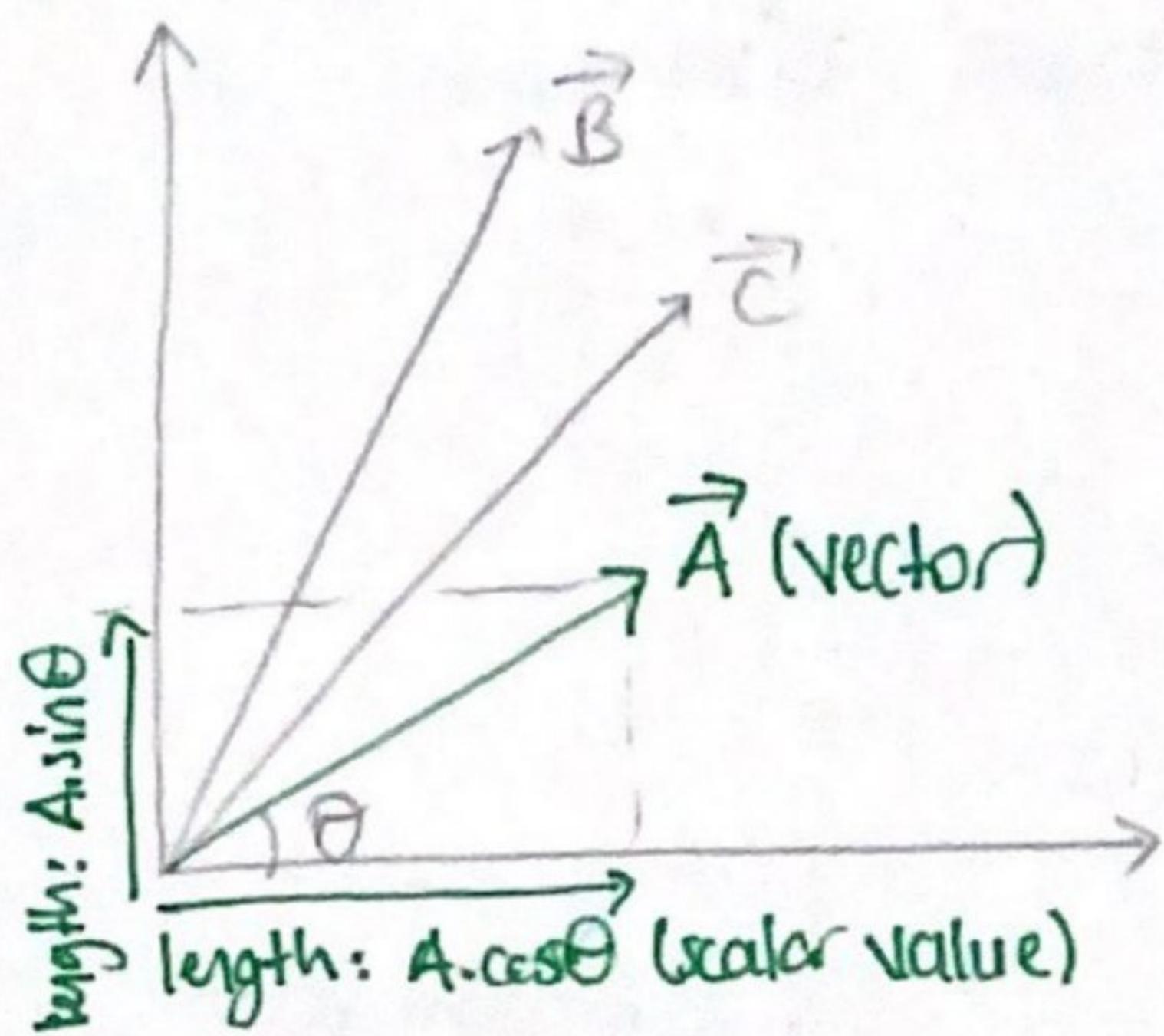
Scalar value: a. - indicates the size of the vector

Vectors: a(i) indicates the size and the direction of the vector.

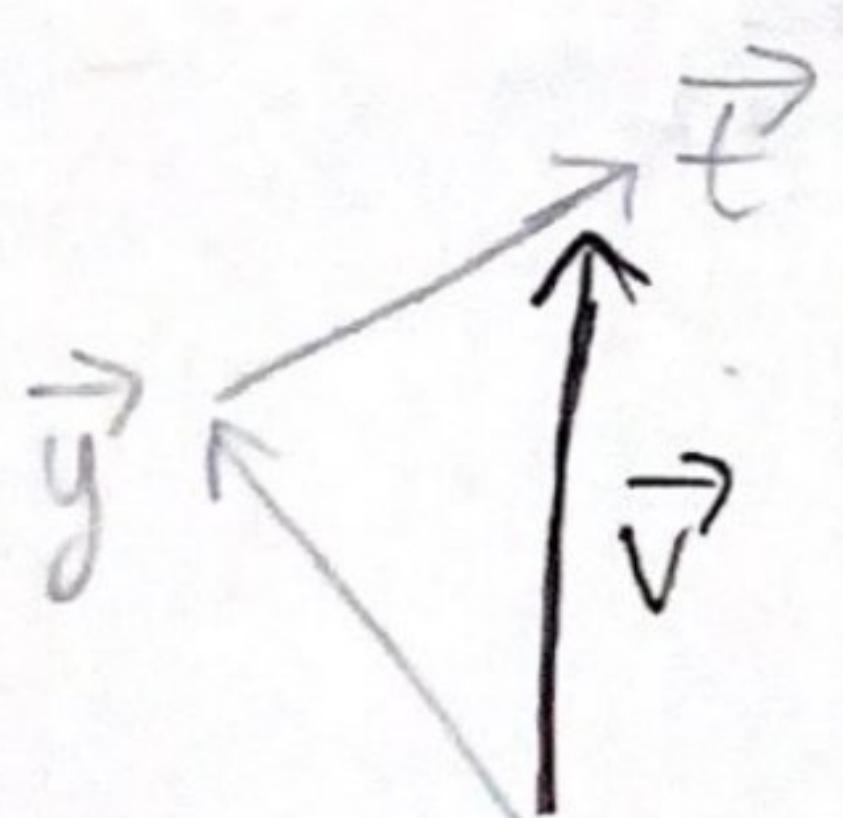


Units: 1.718.000 km → means far away
1.718.000 nm → means very close

Cartesian Coordinate System



$$\left. \begin{array}{l} |\mathbf{i}| = 1 \\ |\mathbf{j}| = 1 \\ |\mathbf{k}| = 1 \end{array} \right\} \text{Unit vector}$$



$$\vec{A} = A \cos \theta \mathbf{i} + A \sin \theta \mathbf{j}$$

Vector Vector Vector

Vector Sum up

$$\vec{A} = A \cos \theta \mathbf{i} + A \sin \theta \mathbf{j}$$

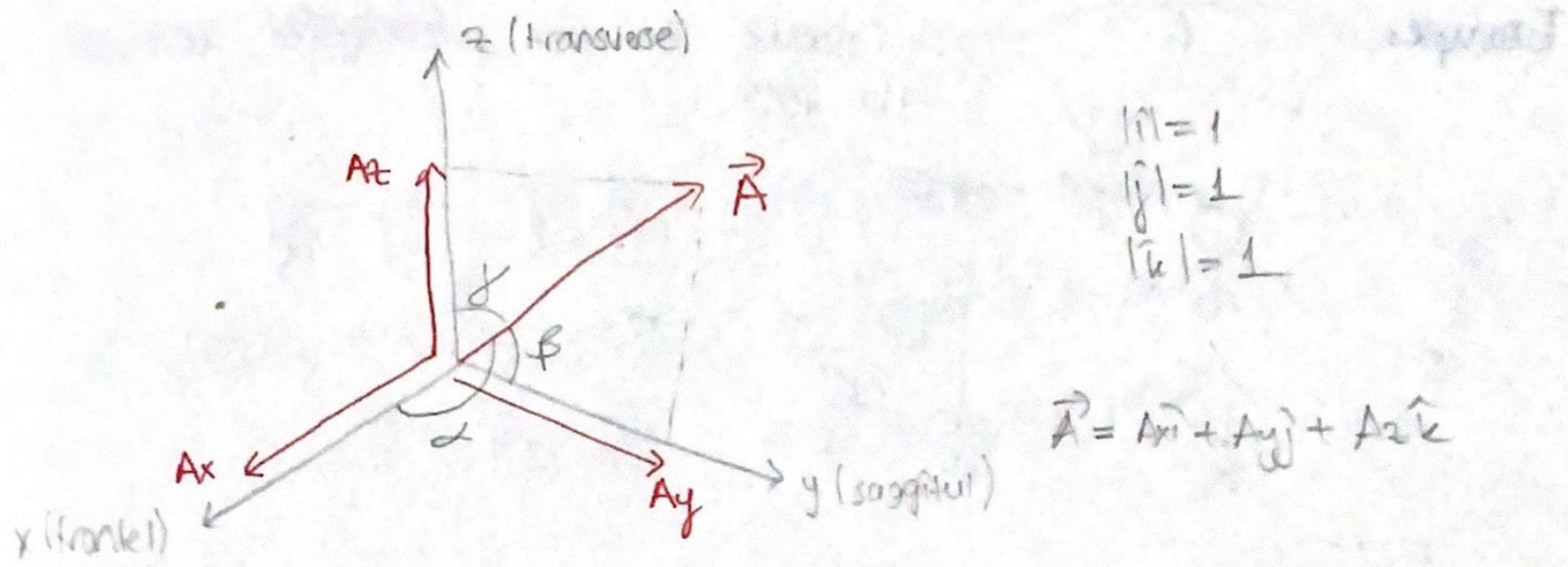
$$\vec{B} = B \cos \theta \mathbf{i} + B \sin \theta \mathbf{j}$$

$$\vec{A} + \vec{B} = (A \cos \theta + B \cos \theta) \mathbf{i} + (A \sin \theta + B \sin \theta) \mathbf{j}$$

C_x

C_y

$$\tan \alpha = \frac{C_y}{C_x}$$

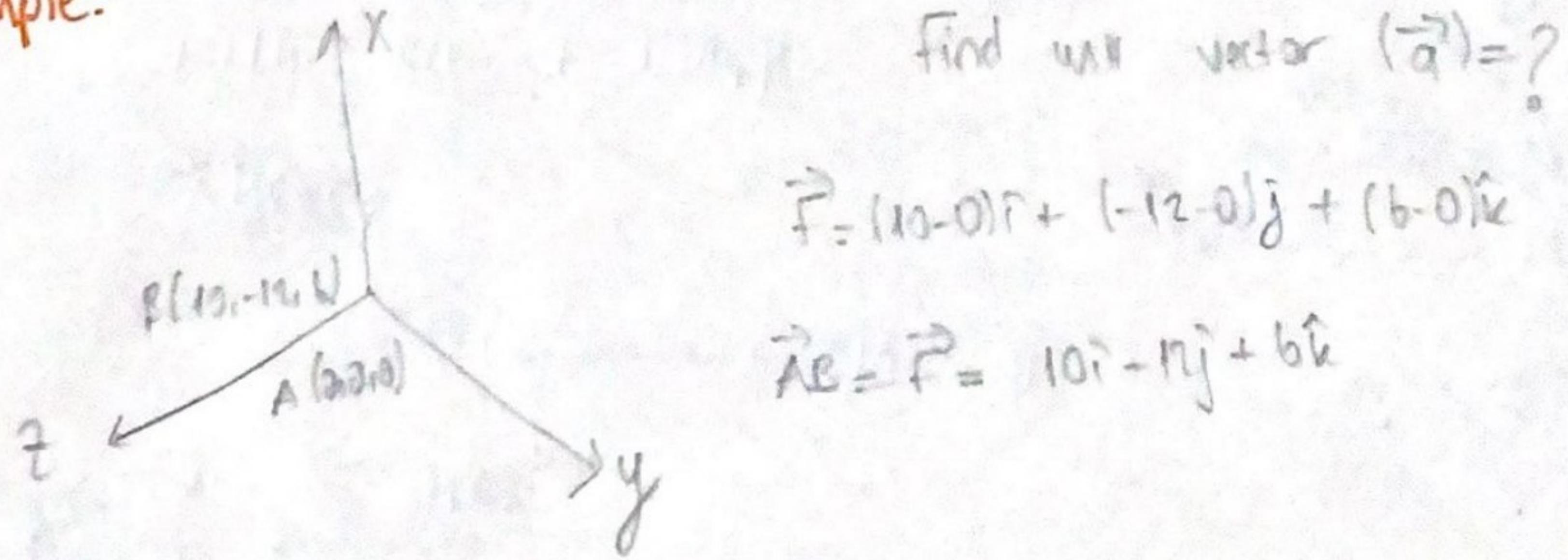


$$\text{length} = |\vec{A}| = \sqrt{(A \cos \alpha)^2 + (A \cos \beta)^2 + (A \cos \gamma)^2}$$

To find Unit Vector

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

Example:

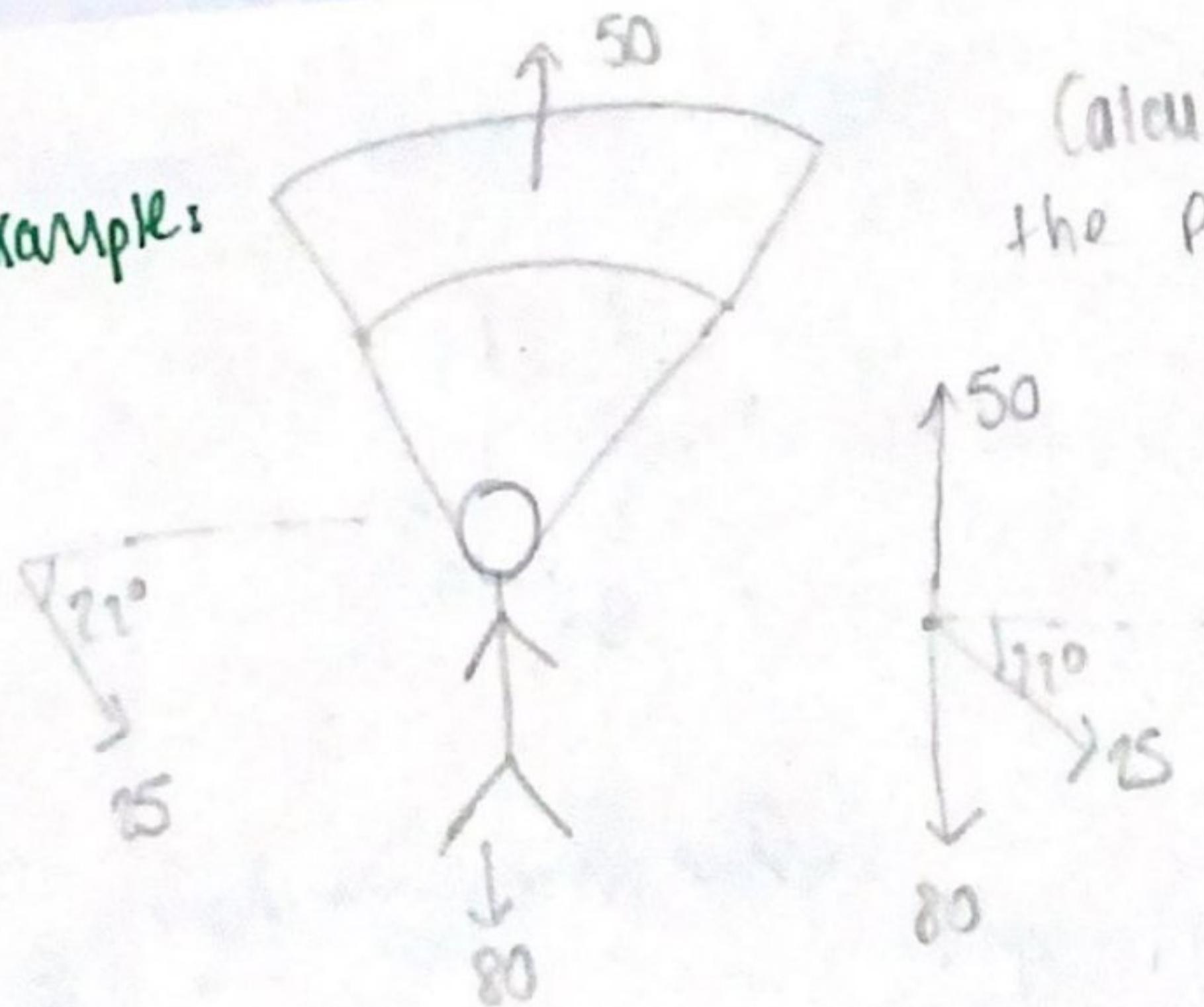


$$\vec{P} = (10-0)i + (-12-0)j + (6-0)k$$

$$\vec{AB} = \vec{P} = 10i - 12j + 6k$$

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|} \rightarrow \hat{P} = \frac{10i - 12j + 6k}{\sqrt{10^2 + (-12)^2 + 6^2}} = 0.598i - 0.717j + 0.498k$$

Example:



Calculate the total force applied to the person.

$$\vec{A} = 50\hat{j} \quad \vec{B} = -80\hat{j}$$

$$\vec{C} = 25\cos 21\hat{i} + 25(-\sin 21)\hat{j}$$

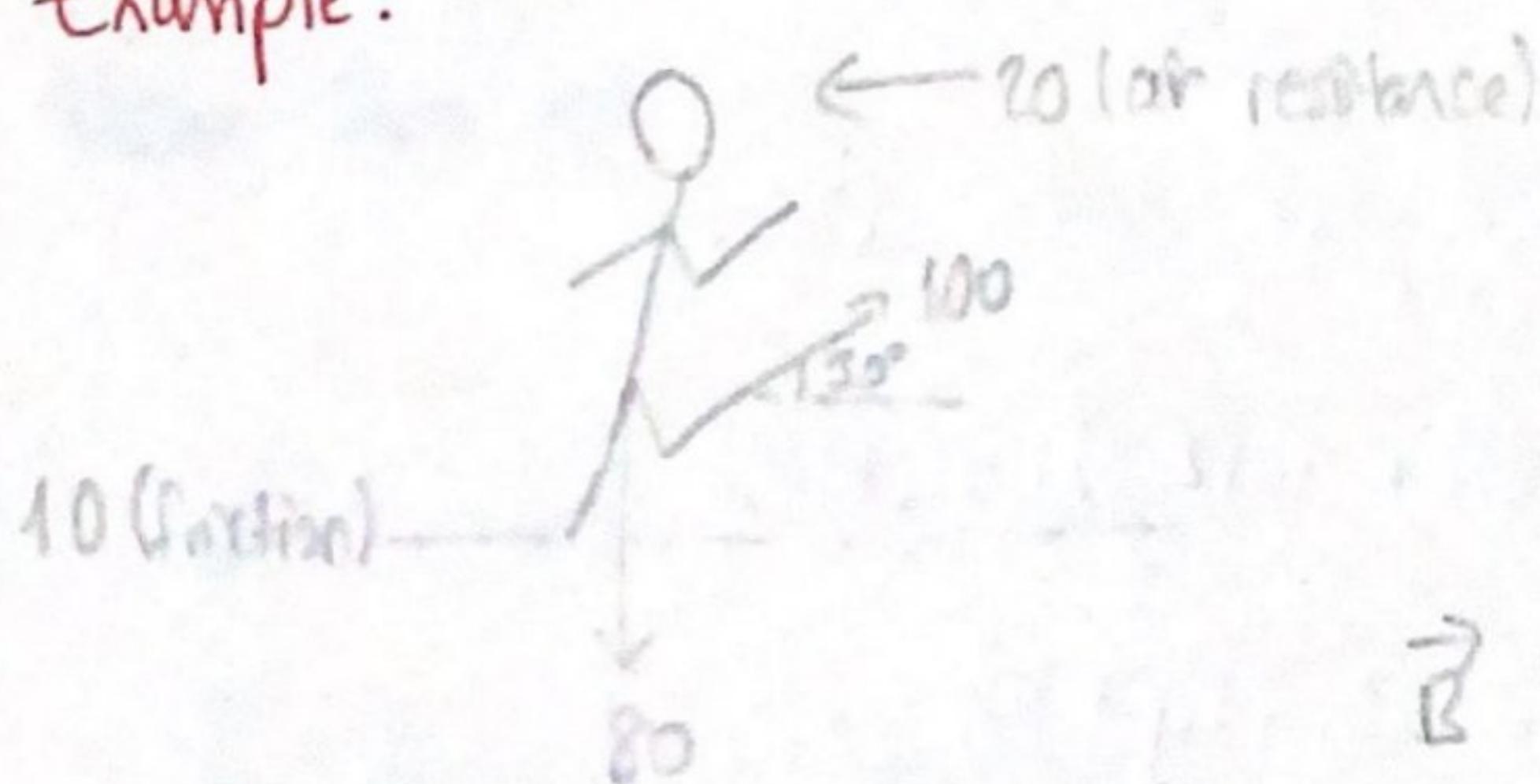
$$\vec{C} = 23.2\hat{i} - 9.3\hat{j}$$

$$\vec{A} + \vec{B} + \vec{C} = 23.2\hat{i} - 39.3\hat{j}$$

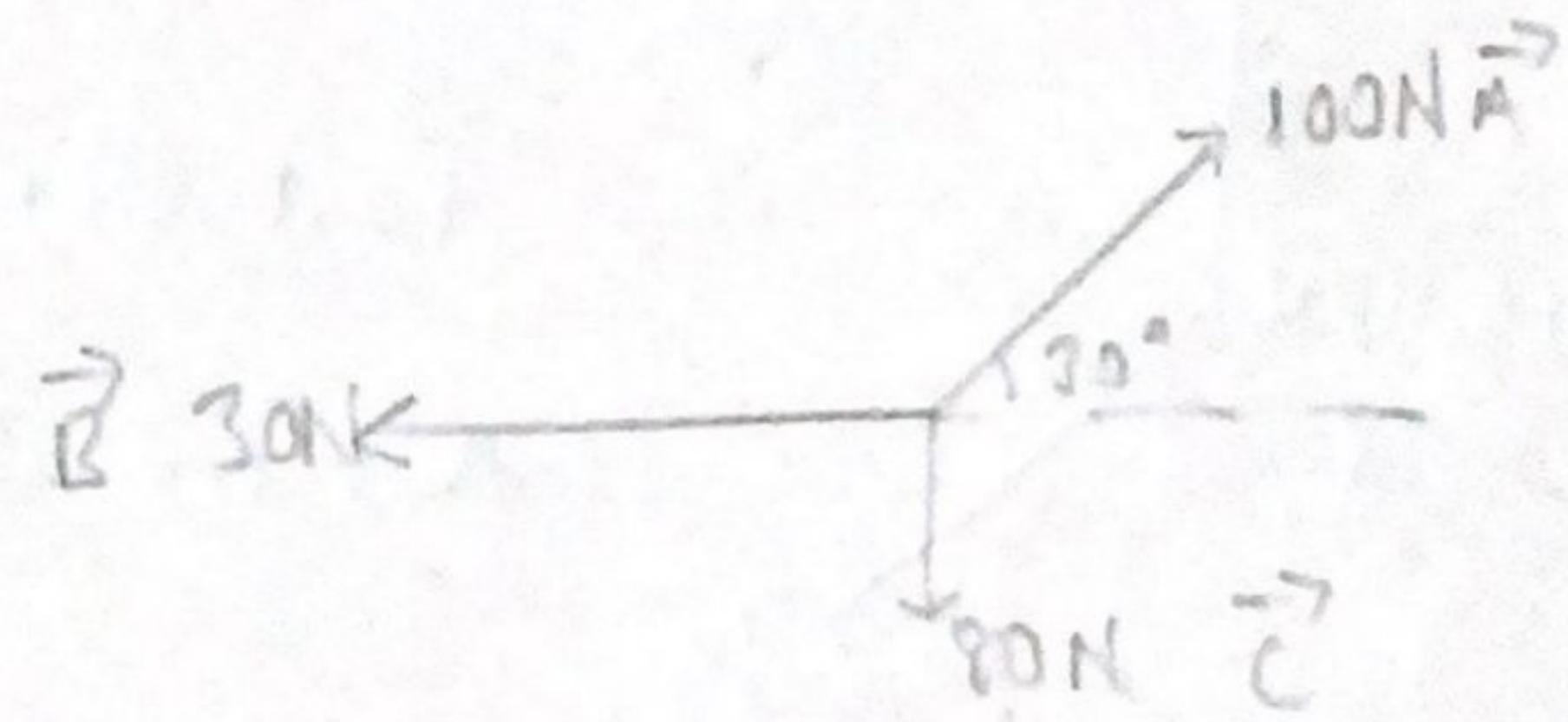
$$\vec{A} + \vec{B} + \vec{C} = |\vec{F}| = \sqrt{(23.2)^2 + (-39.3)^2} = 65.7$$

$$\tan \alpha = \frac{-39.3}{23.2} \Rightarrow \alpha = -59.5$$

Example:



Calculate the total force applied to the athlete.



$$\vec{B} = -30\hat{i}$$

$$\vec{C} = -80\hat{j}$$

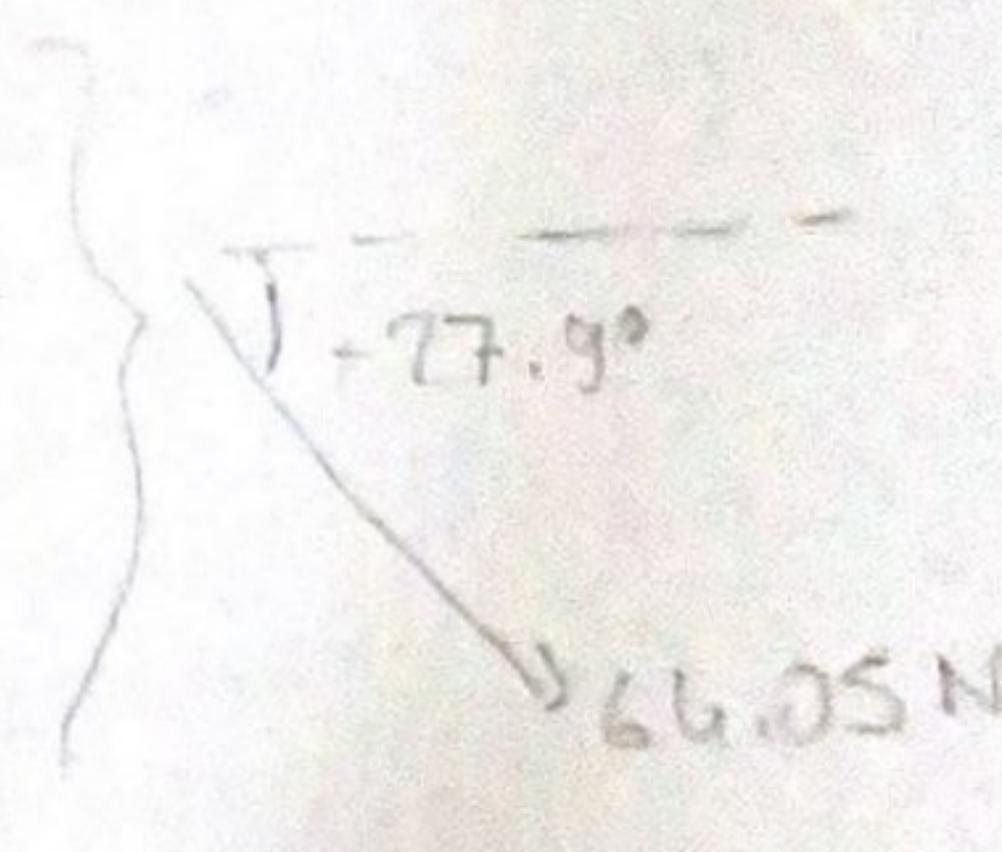
$$\vec{A} = 100\cos 30\hat{i} + 100\sin 30\hat{j}$$

$$\vec{A} = 86.6\hat{i} + 50\hat{j}$$

$$\vec{A} + \vec{B} + \vec{C} = 56.6\hat{i} - 30\hat{j}$$

$$\text{The magnitude} = \sqrt{(56.6)^2 + (-30)^2} = 64.05 \text{ N}$$

$$\tan \alpha = \frac{-30}{56.6} \Rightarrow -27.9 = \alpha$$



Vector Product :

II. Way :

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = +i[A_y B_z - A_z B_y] - j[A_x B_z - A_z B_x] + k[A_x B_y - A_y B_x]$$

$$\left. \begin{array}{l} C_x = A_y B_z - A_z B_y \\ C_y = A_x B_z - A_z B_x \\ C_z = A_x B_y - A_y B_x \end{array} \right\} \vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

Example: $\vec{A} = 2\hat{i} + 4\hat{j}$ $\vec{B} = 2\hat{j} + 3\hat{k}$

a) Find $\vec{C} = \vec{A} \times \vec{B}$

b) Prove that \vec{C} is perpendicular to \vec{A} .

a) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 0 & 2 & 3 \end{vmatrix} = \frac{(12-0)\hat{i}}{(6-0)\hat{j}} = 12\hat{i} - 6\hat{j} + 0\hat{k} = \vec{C}$

b) $\vec{A} \cdot \vec{C} = A \cdot C \cdot \cos\theta$

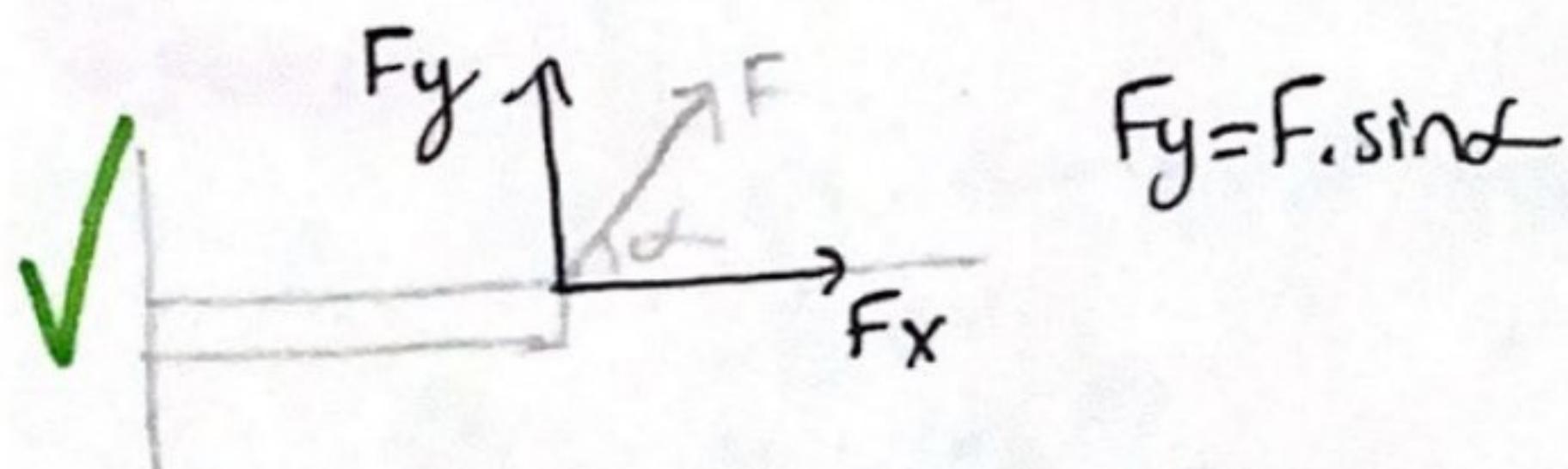
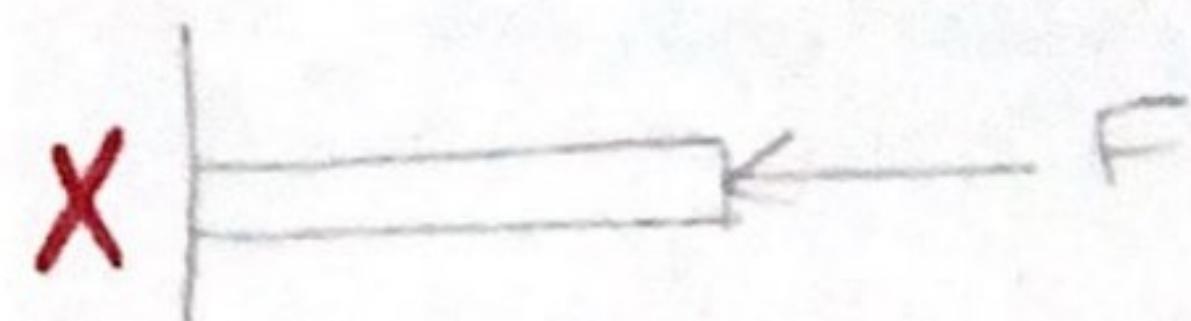
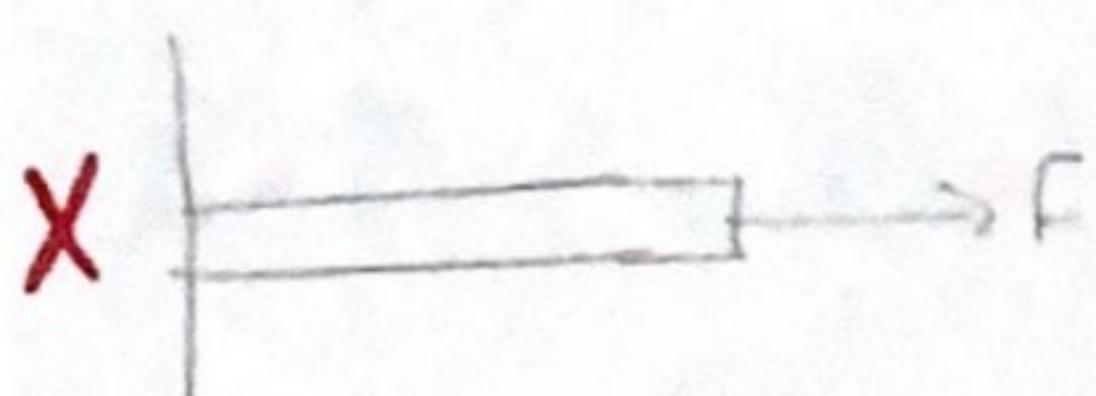
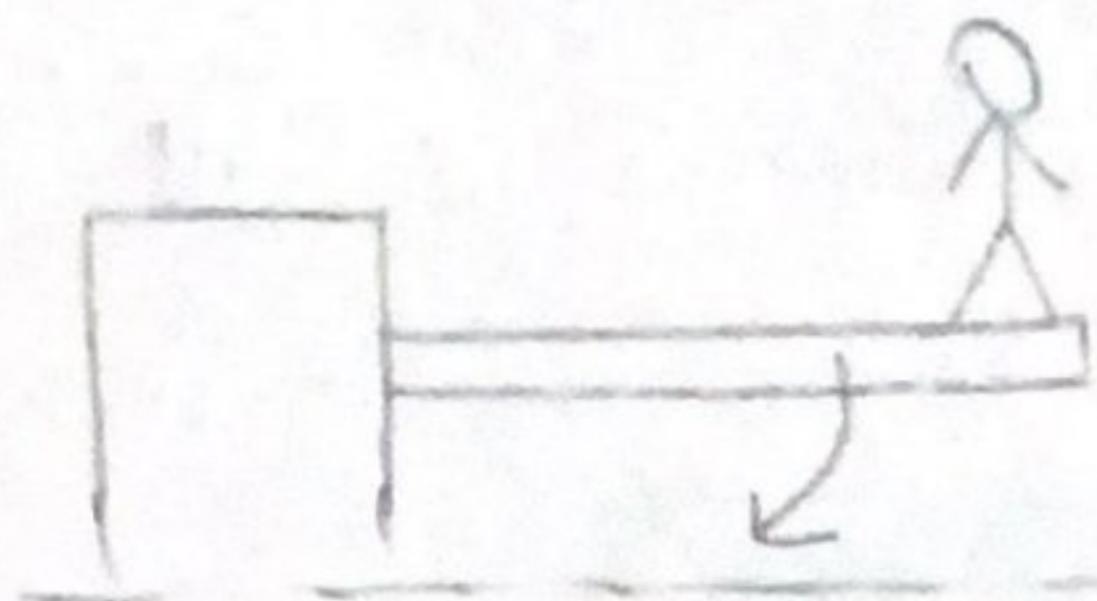
$$\vec{A} \cdot \vec{C} = (2\hat{i} + 4\hat{j}) \cdot (12\hat{i} - 6\hat{j} + 0\hat{k})$$

$$24 - 24 - 0 = 0$$

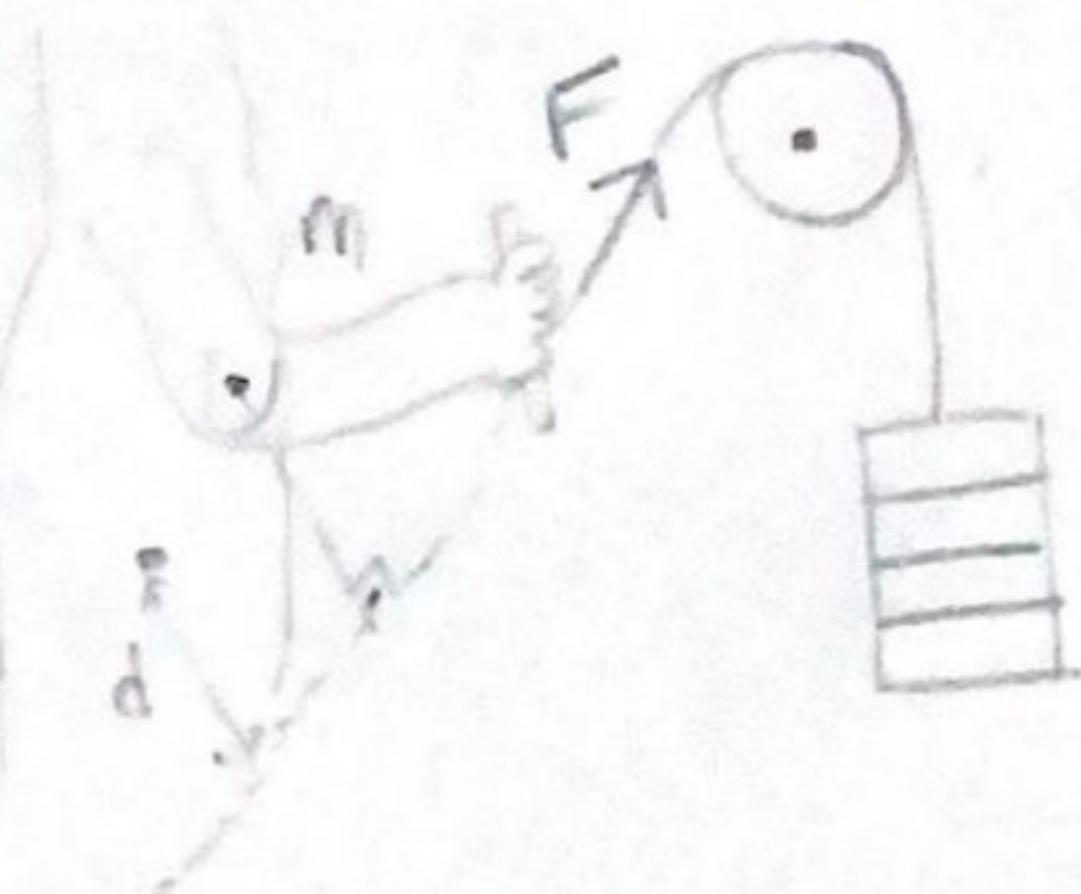
$$\cos\theta = 0$$

$\Rightarrow 90^\circ$ that's mean \vec{C} is perpendicular to \vec{A} .

moment; A force applied to an object can translate, rotate and/or deform the object. If one stands on the free end of a diving board, the board will bend what bends the board is the moment of the body weight about the fixed end of the board.



Magnitude of moment

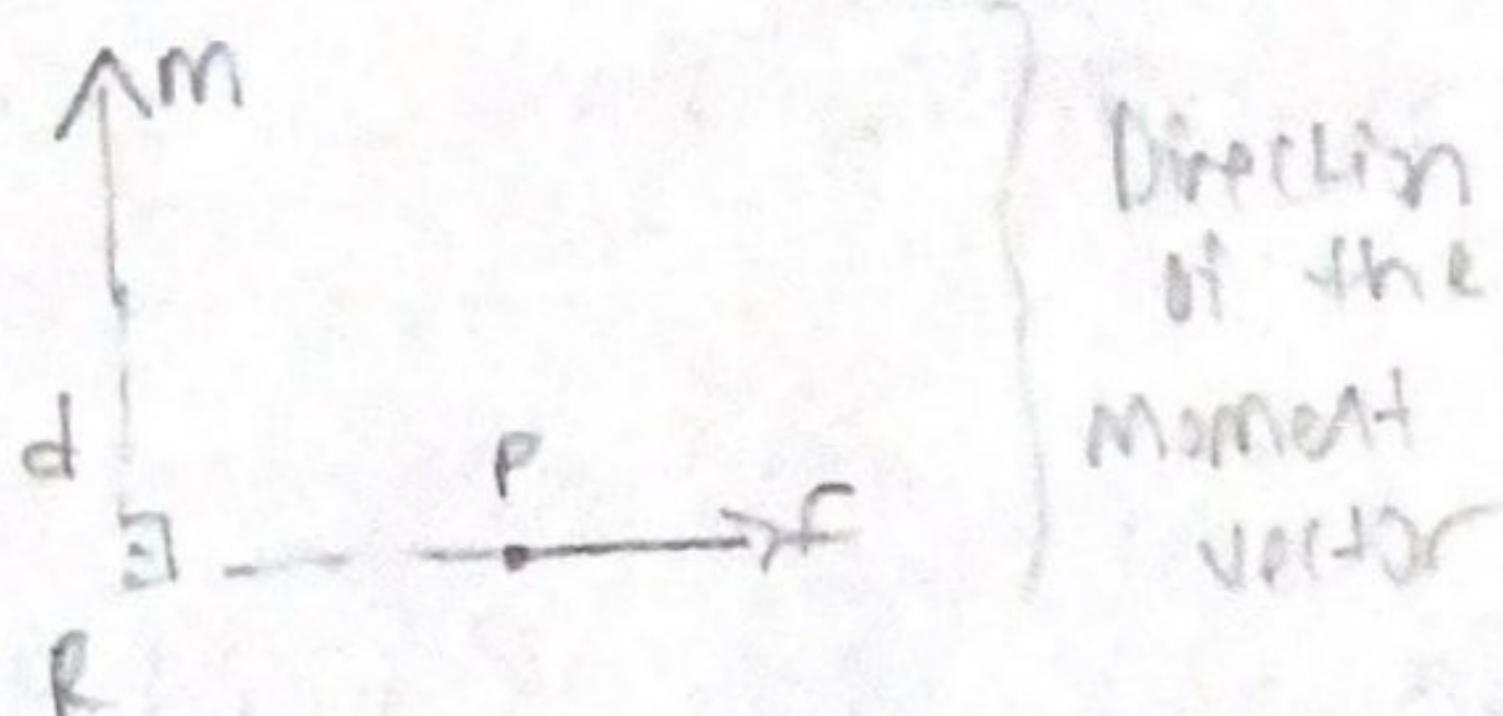


$$m = F \cdot d$$

$$m \perp d$$

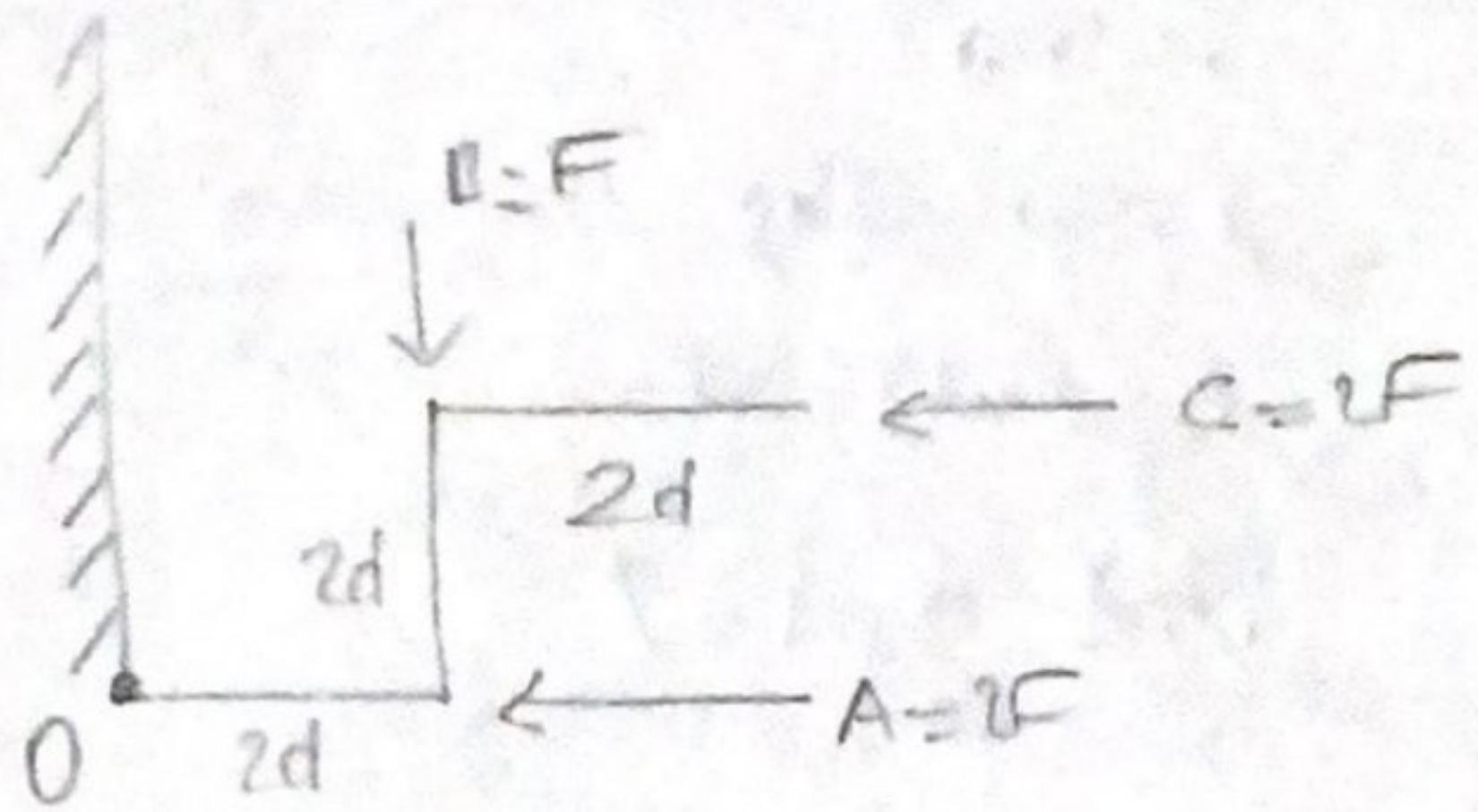
$$m \neq F$$

The magnitude of the moment of a force about a point is equal to the magnitude of the force times the length of the shortest distance between the point and the line of action of the force.



Direction of the moment vector

Example:



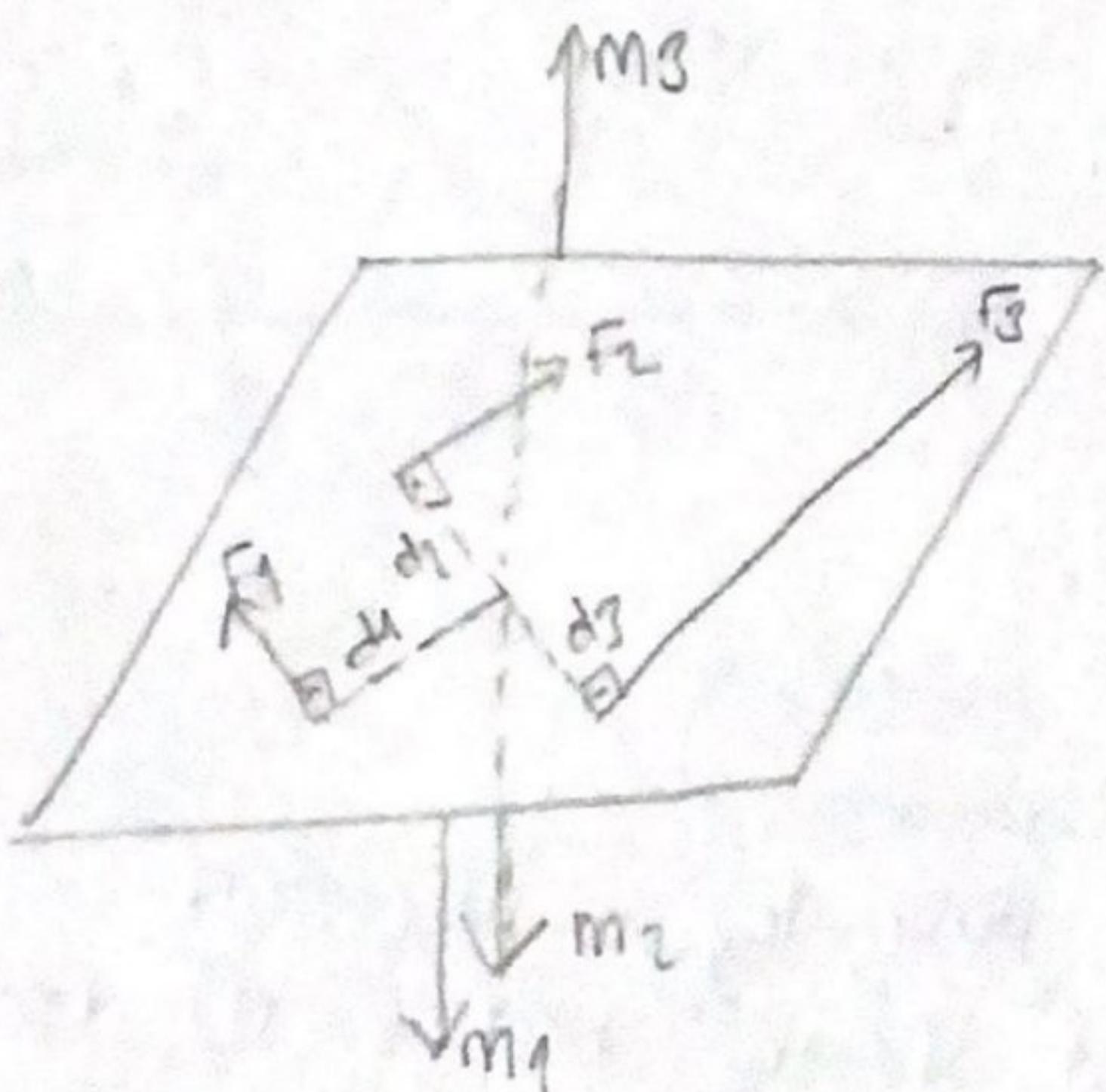
$M_A, M_B, M_C = ?$: signs?

$$M_A = 0$$

$$M_B = 2d \cdot F = 2Fd$$

$$M_C = 2F \cdot 2d = 4Fd$$

Net moment: When there is more force applied on a body the net moment can be calculated by considering the vector sum of the moments of all forces.



$$M_1 = d_1 \cdot F_1$$

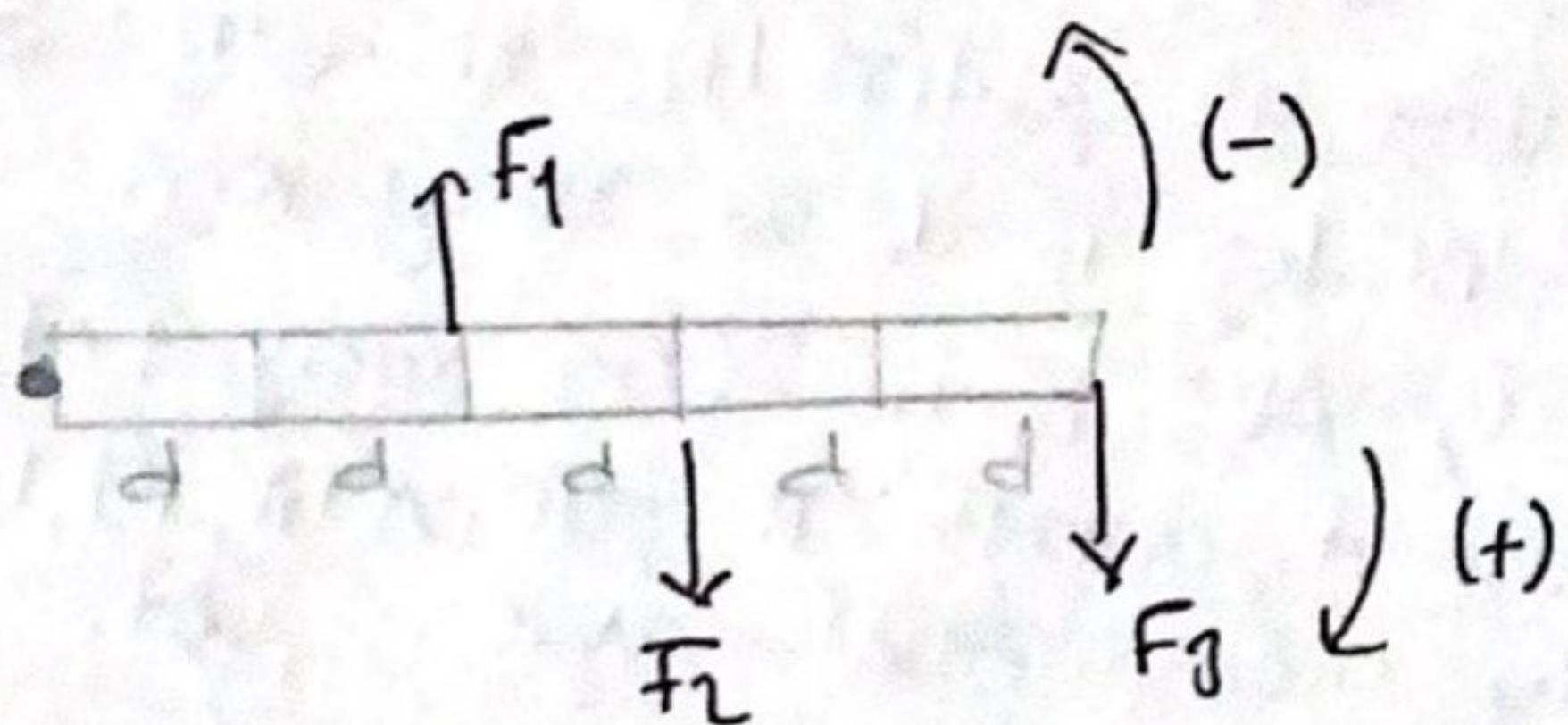
$$M_2 = d_2 \cdot F_2$$

$$M_3 = d_3 \cdot F_3$$

$$M_{\text{net}} = M_1 + M_2 + M_3$$

$$M_{\text{net}} = M_1 + M_2 - M_3$$

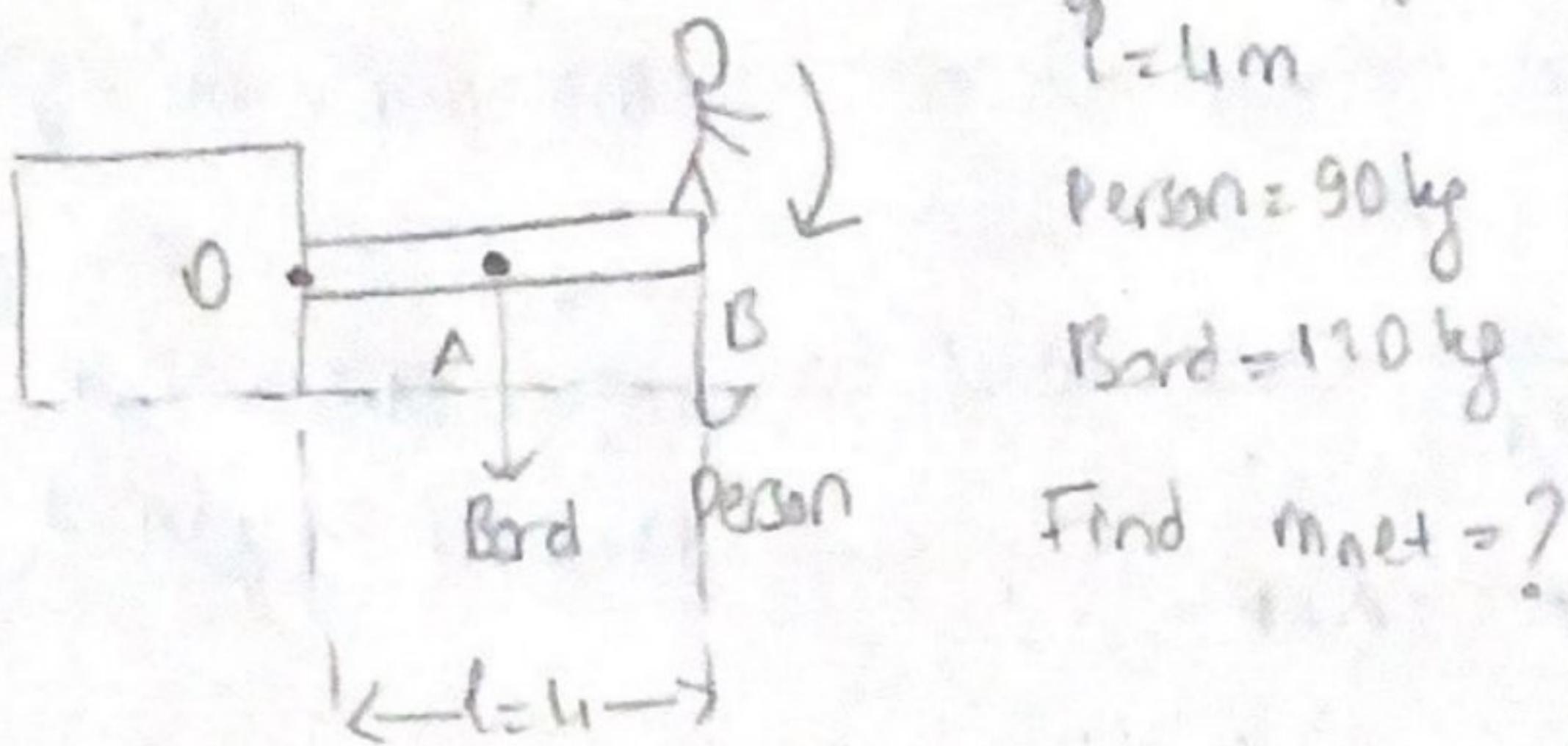
Example:



$$\Sigma m = -F_1 \cdot 2d + 3 \cdot F_2 \cdot d + F_3 \cdot 5d$$

If resultant moment is (+), then the bar turns to the positive side. It is also correct for vice versa.

Example :



$$g = 98 \text{ m/g}^2$$

$$l = 4\text{m}$$

$$\text{Person} = 90 \text{ kg}$$

$$\text{Board} = 110 \text{ kg}$$

Find $M_{\text{net}} = ?$

$$l \leftarrow l = 4 \rightarrow$$

$$W_1 = m_1 \cdot g = 90 \text{ kg} (98 \text{ m/g}^2) = 882 \text{ N}$$

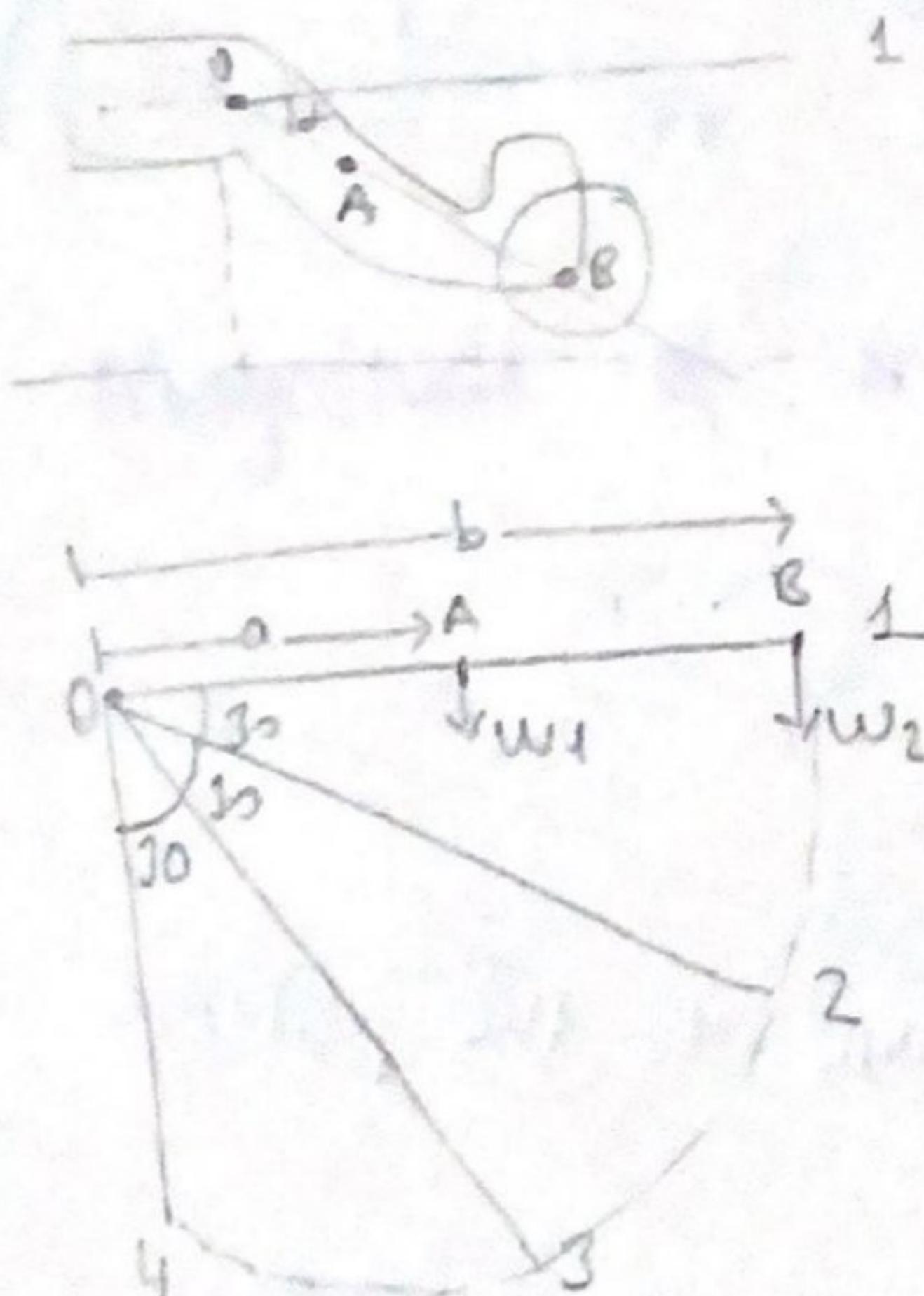
$$W_2 = m_2 \cdot g = 110 \text{ kg} (98 \text{ m/g}^2) = 1176 \text{ N}$$

$$M_1 = l \cdot W_1 = 4\text{m} \cdot 882 \text{ N} = 3528 \text{ Nm}$$

$$M_2 = \frac{l}{2} \cdot W_2 = 2\text{m} \cdot 1176 \text{ N} = 2352 \text{ Nm}$$

$$M_{\text{net}} = M_1 + M_2 = 5880 \text{ Nm}$$

Example:



Athlete's lower leg is $W_1 = 50\text{N}$

The weight of the boot is $W_2 = 100\text{N}$

When at position (1) the center of gravity of the leg is 20 cm away from knee point at O. Also B is 50cm away. Determine the resultant moment at position (1), (2) and (3). Now the moment-angle plot.

$$M_1 = a \cdot W_1 + b \cdot W_2 = 10 \cdot 50 \cdot 0.50 + (0.50 \cdot 100) = 60 \text{ Nm}$$

$$d_1 = a \cdot \cos \alpha$$

$$d_2 = b \cdot \cos \beta$$

$$M_2 = d_1 \cdot W_1 + d_2 \cdot W_2$$

$$= 0.50 \cdot 50 + 0.50 \cdot 100$$

$$= (0.50 + 0.50) \cdot 50 =$$

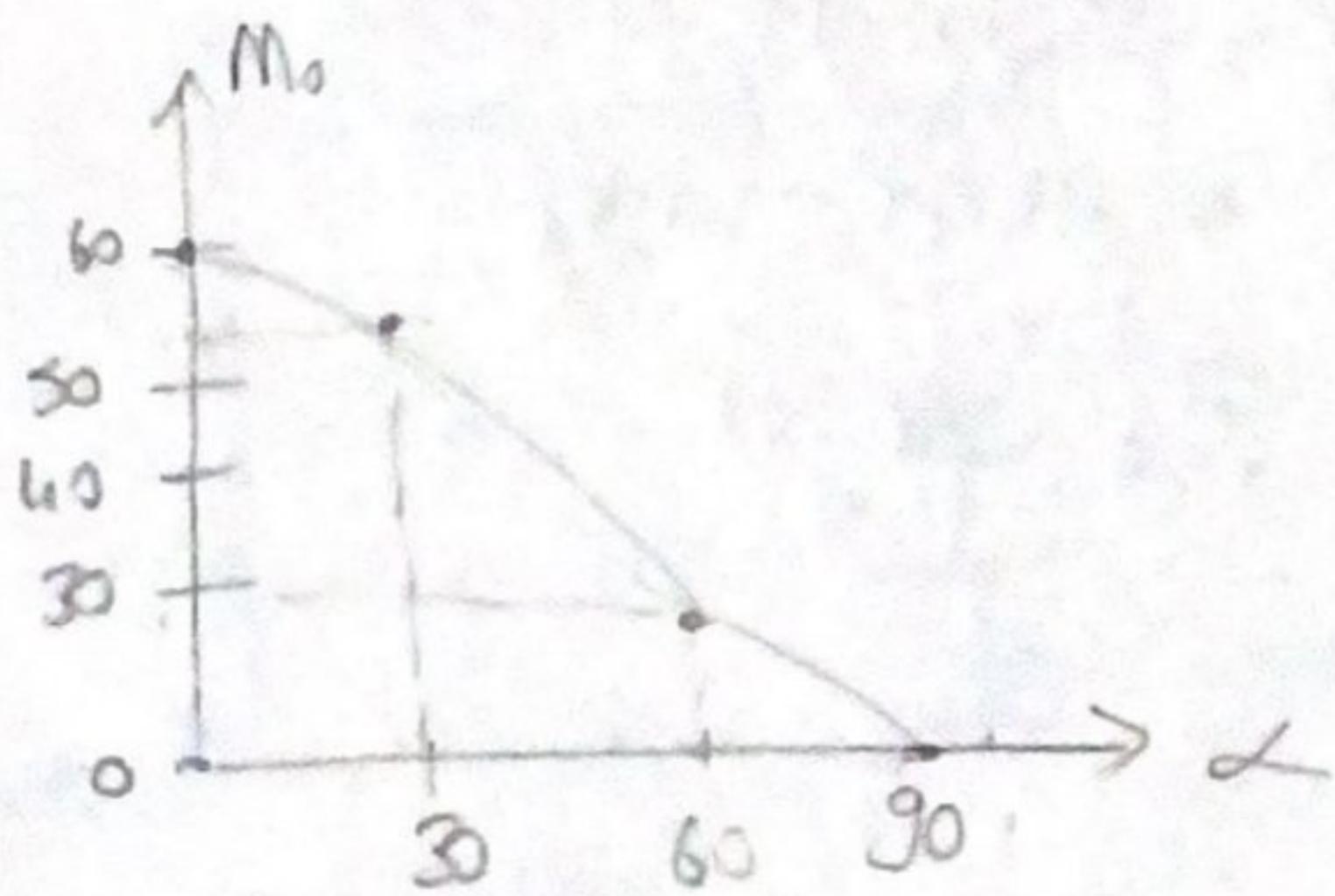
$$M_o = 60 \cdot \cos \alpha$$

For position 1 $\alpha = 0^\circ, M_o = 60 \text{ Nm}$

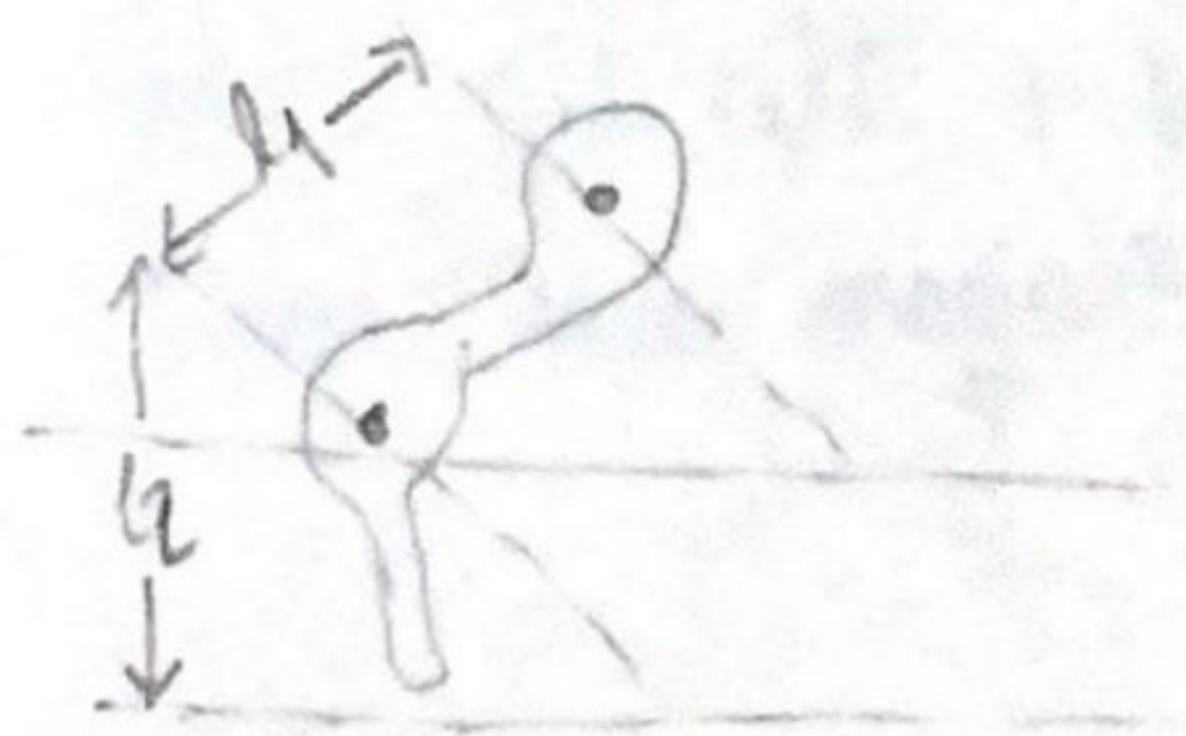
2 $\alpha = 30^\circ, M_o = 52 \text{ Nm}$

3 $\alpha = 60^\circ, M_o = 30 \text{ Nm}$

4 $\alpha = 90^\circ, M_o = 0 \text{ Nm}$

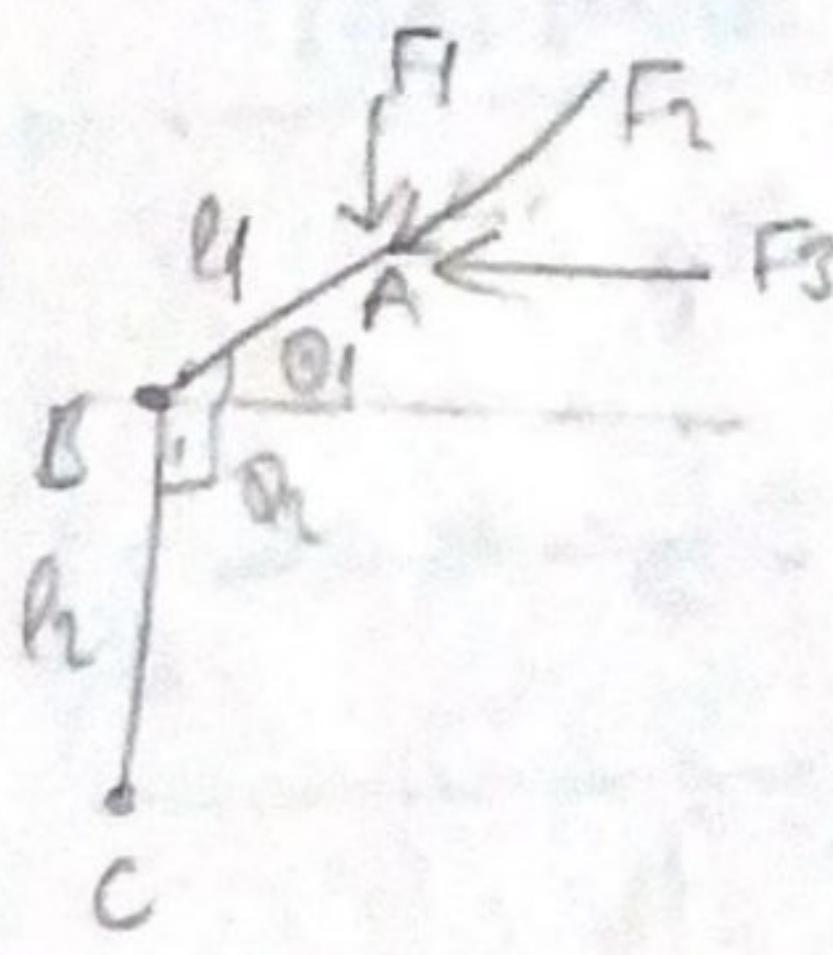


Examples



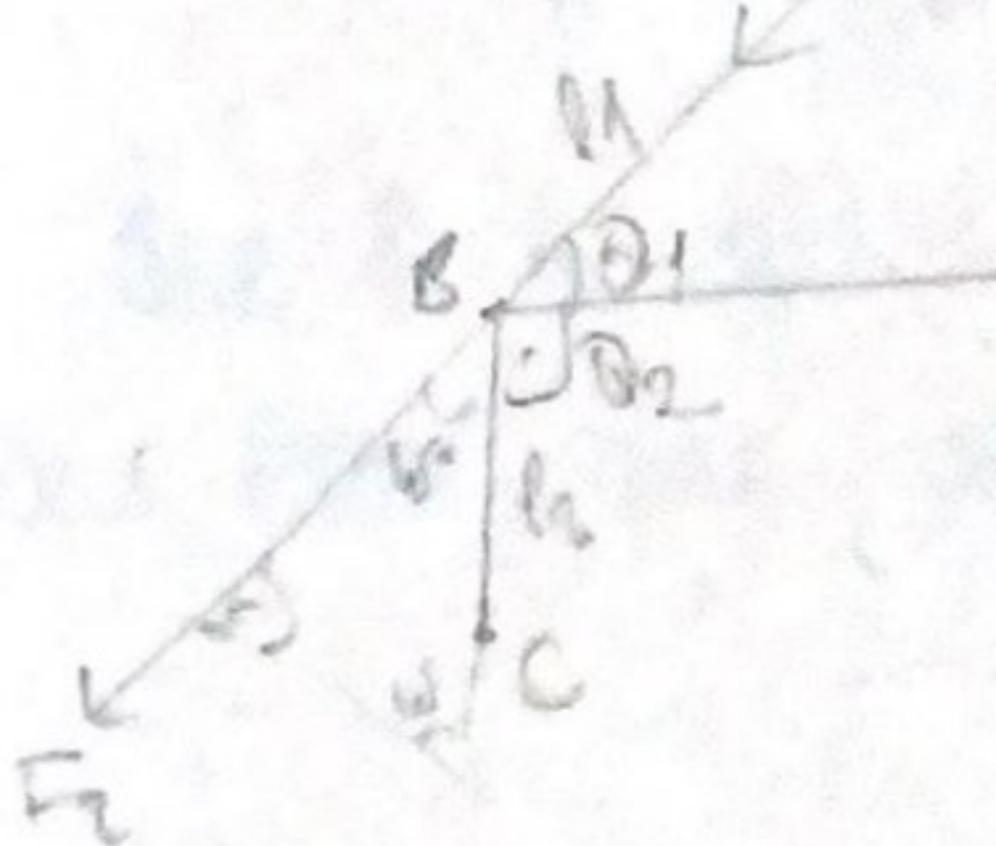
$$\begin{aligned} l_1 &= 50 \text{ mm} \\ l_2 &= 100 \text{ mm} \\ \alpha_1 &= 45^\circ \\ \alpha_2 &= 90^\circ \end{aligned}$$

Assume that when standing symmetrically on both feet, a joint reaction force of $F = 600 \text{ N}$ is acting at the femoral head due to the body weight of the patient. Consider three different modes of action for the applied force, which are shown.



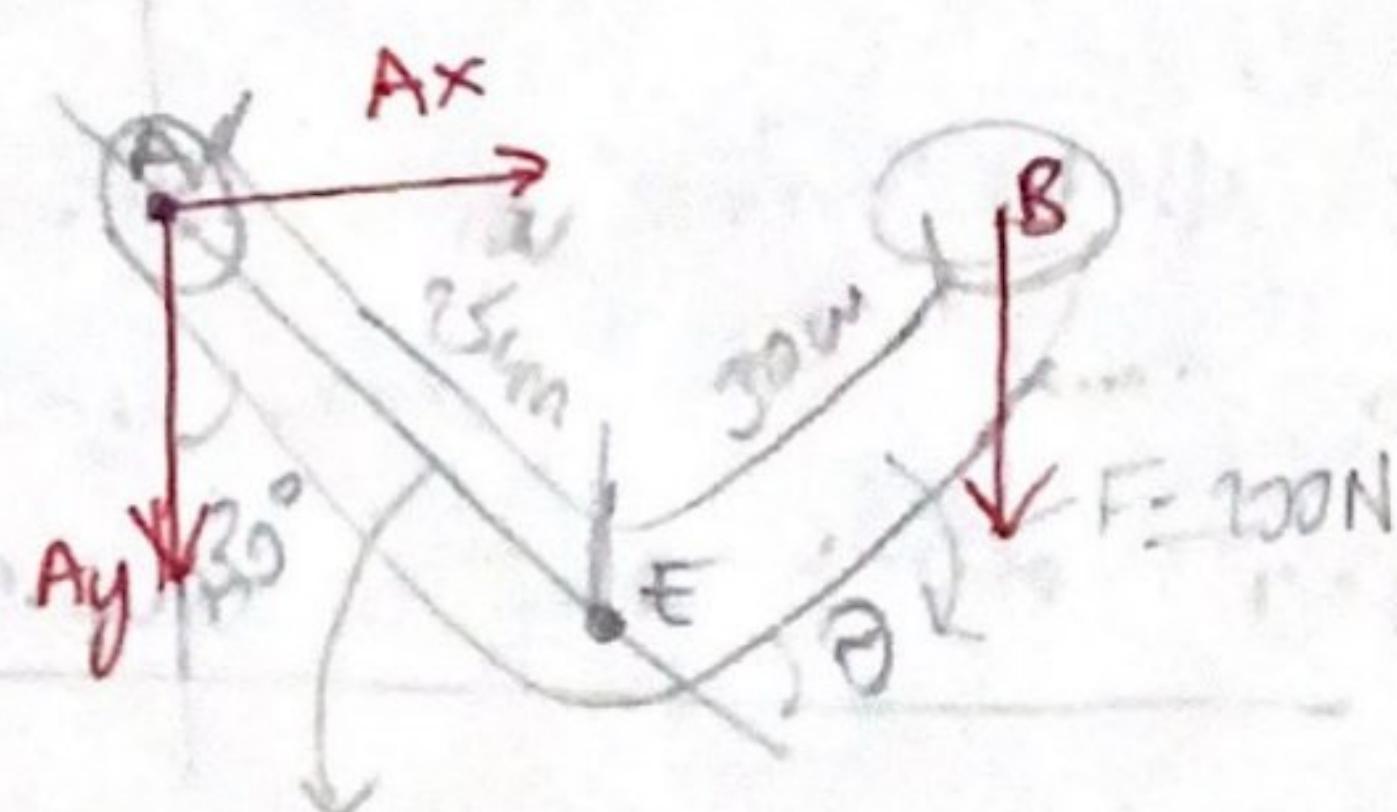
$$\begin{aligned} d_1 &= l_1 \cos \alpha_1 = 50 \cos 45^\circ = 35 \text{ mm} \\ M_B &= m \cdot g \cdot d_1 \cdot g = (2023) \cdot 1000 = 10 \text{ Nm} \end{aligned}$$

$$\begin{aligned} d_2 &= l_2 \cos \alpha_2 = 100 \cos 90^\circ = 0 \text{ mm} \\ M_C &= d_2 \cdot F = 0 \cdot 600 = 0 \text{ Nm} \end{aligned}$$



$$\begin{aligned} d_1 &= l_1 \sin \alpha_1 = 50 \sin 45^\circ = 35 \text{ mm} \\ F_3 &= d_1 \cdot F = 35 \cdot 600 = 21000 \text{ N} \\ M_B &= d_1 \cdot F = 35 \cdot 600 = 21000 \text{ Nm} \\ M_C &= d_2 \cdot F = 100 \cdot 600 = 60000 \text{ Nm} \end{aligned}$$

Example:



A: Acrenion

B: Epicondyle

$\theta = 0^\circ, 60^\circ$ and 90° epicondyle
and acrenion what is the
moment of $|\vec{AE}| = ?$

$$g = 10 \text{ m/s}^2$$

$$|\vec{AE}| = 25 \sin 30^\circ - 25 \cos 30^\circ \hat{j}$$
$$= 42.5 \hat{i} - 21.6 \hat{j}$$
$$\vec{r} = -200 \hat{j} \text{ N}$$

$$\theta = 0^\circ$$

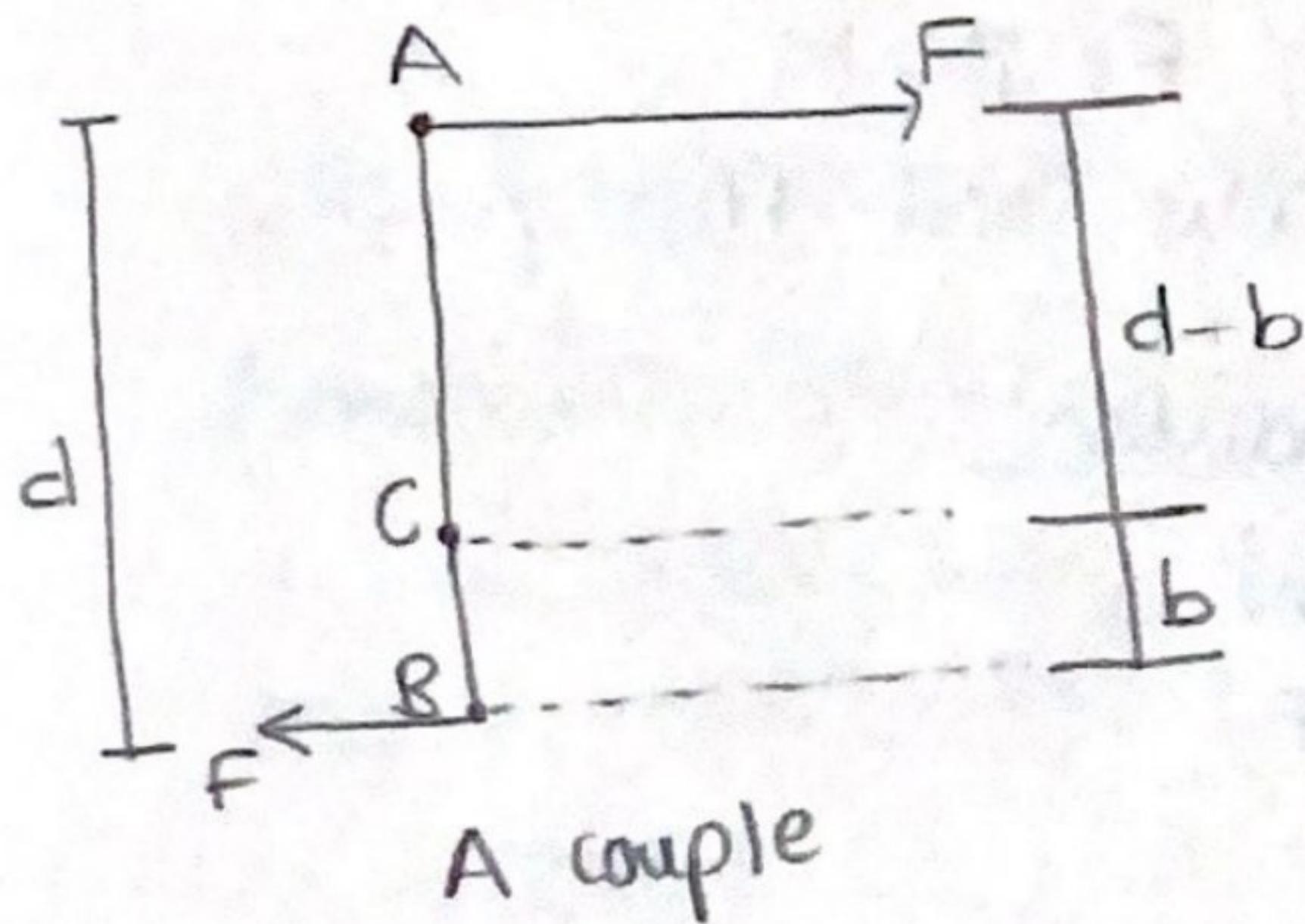
$$\vec{EB} = 30 \cos 0^\circ \hat{i} + 30 \sin 0^\circ \hat{j}$$
$$= 30 \hat{i} \text{ m}$$

$$\vec{m}_E = \vec{EB} \times \vec{F} = 30 \hat{i} \times -200 \hat{j}$$
$$= -6000 \hat{k} \text{ N m}$$
$$= -60 \hat{k} \text{ N m}$$

$$MA = (\vec{AE} + \vec{EB}) \times \vec{F}$$
$$= (25 \hat{i} - 21.6 \hat{j}) \times -200 \hat{j} + 30 \hat{i} \times -200 \hat{j}$$
$$= -8500 \hat{k} \text{ N m} = -85 \text{ k N m}$$

The Couple and Couple Moment

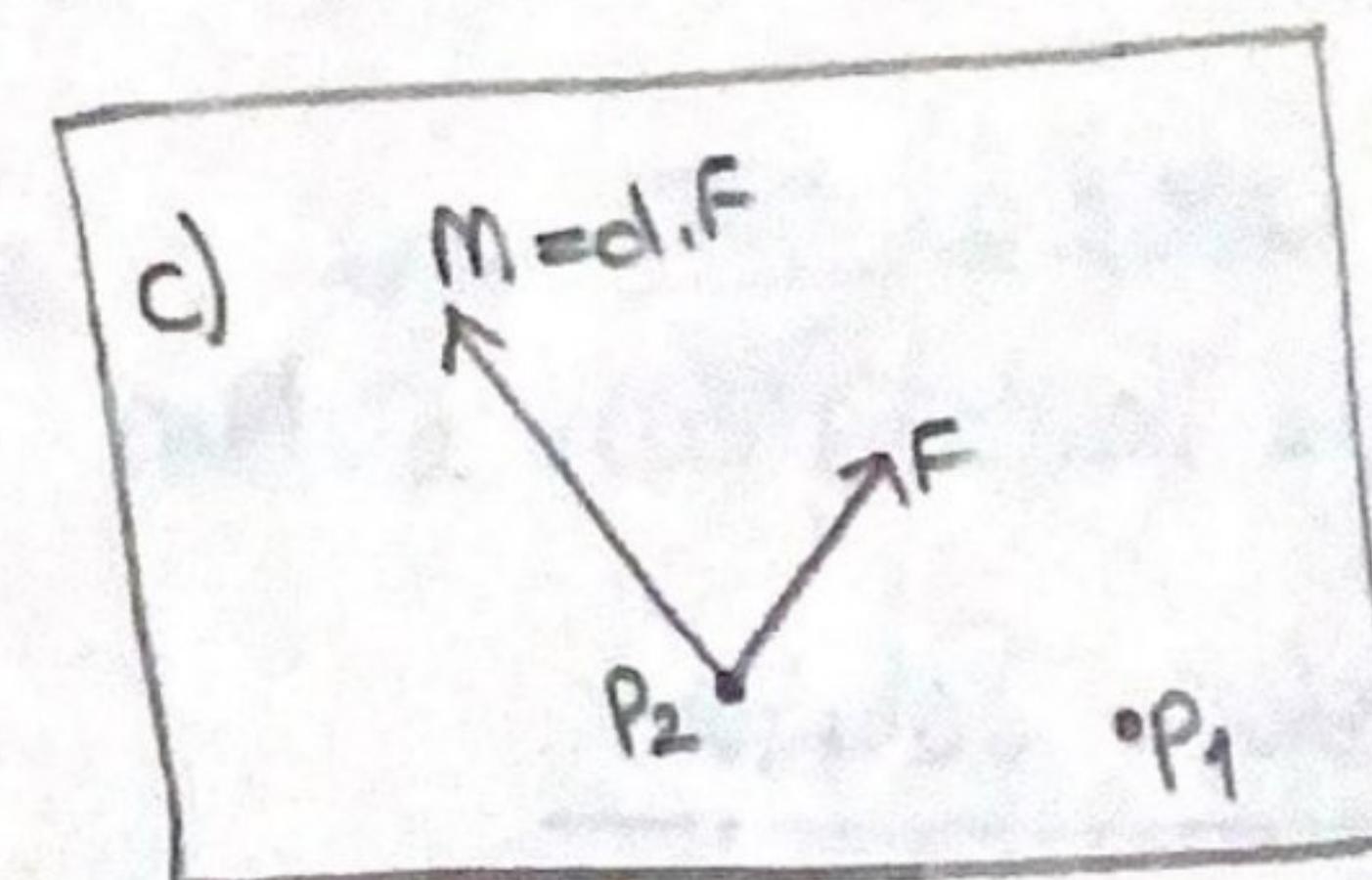
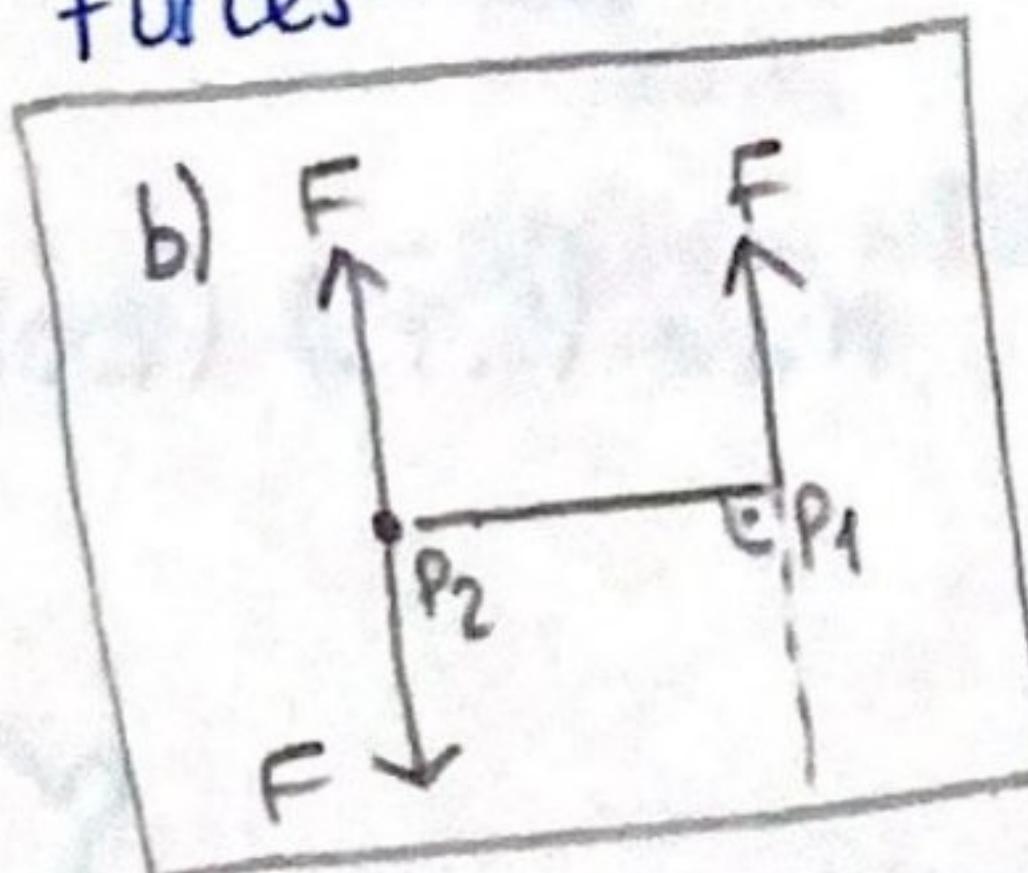
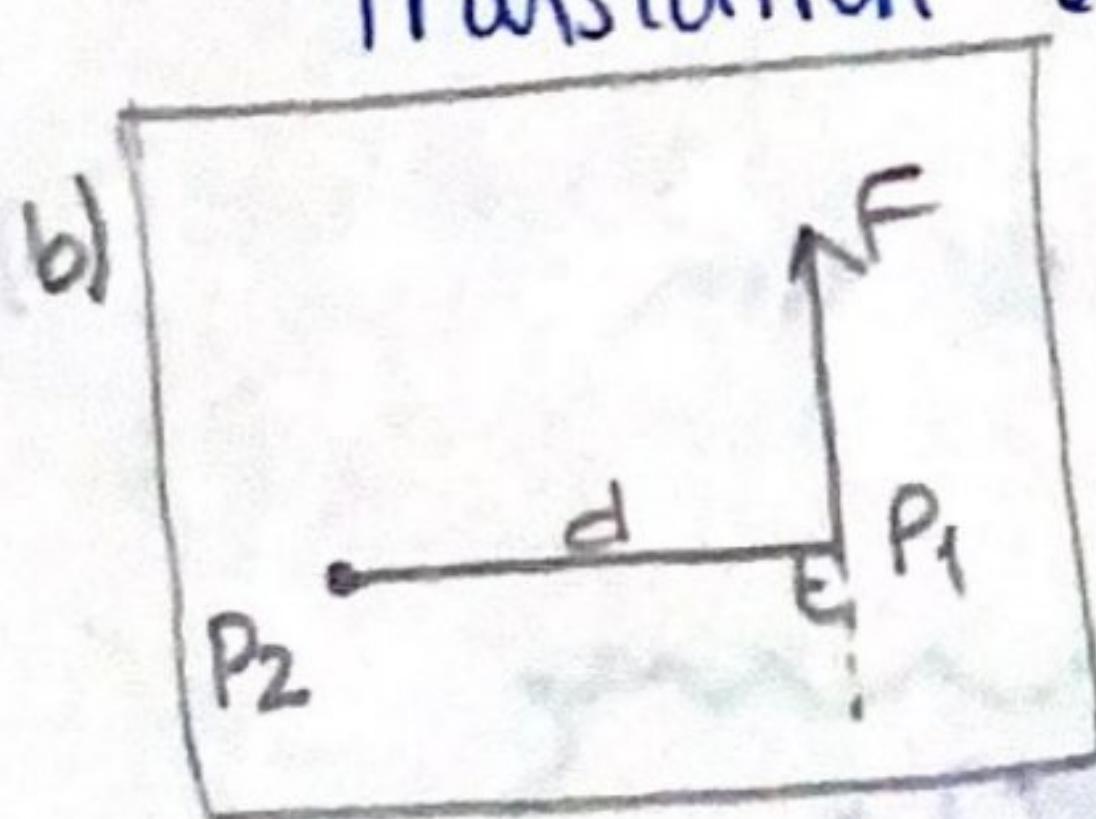
A special arrangement of forces that is of importance is called a couple, which is fixed by two parallel forces with equal magnitude and opposite directions on a rigid body, the couple has a pure rotational effect. The rotational effect of a couple is quantified with couple-moment.



$m = (d-b)F + b.F = dF$
Then the magnitude of the couple moment is ;

$$m = d \cdot F$$

Translation of Forces



Translation of a force from point P₁ to P₂

One force system in fig a, three force system in fig b, one force one couple moment system in fig c are equals.

Example:

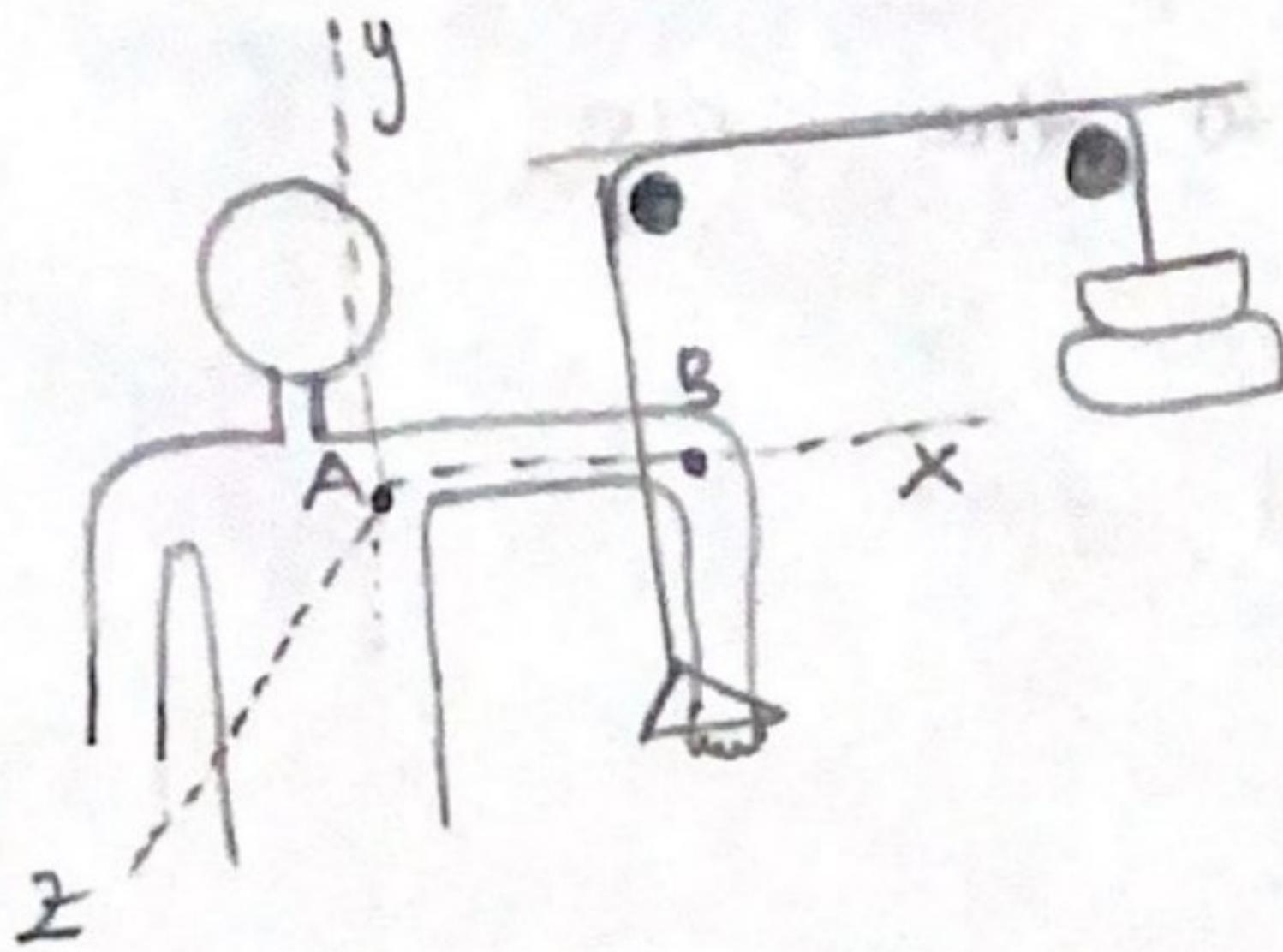
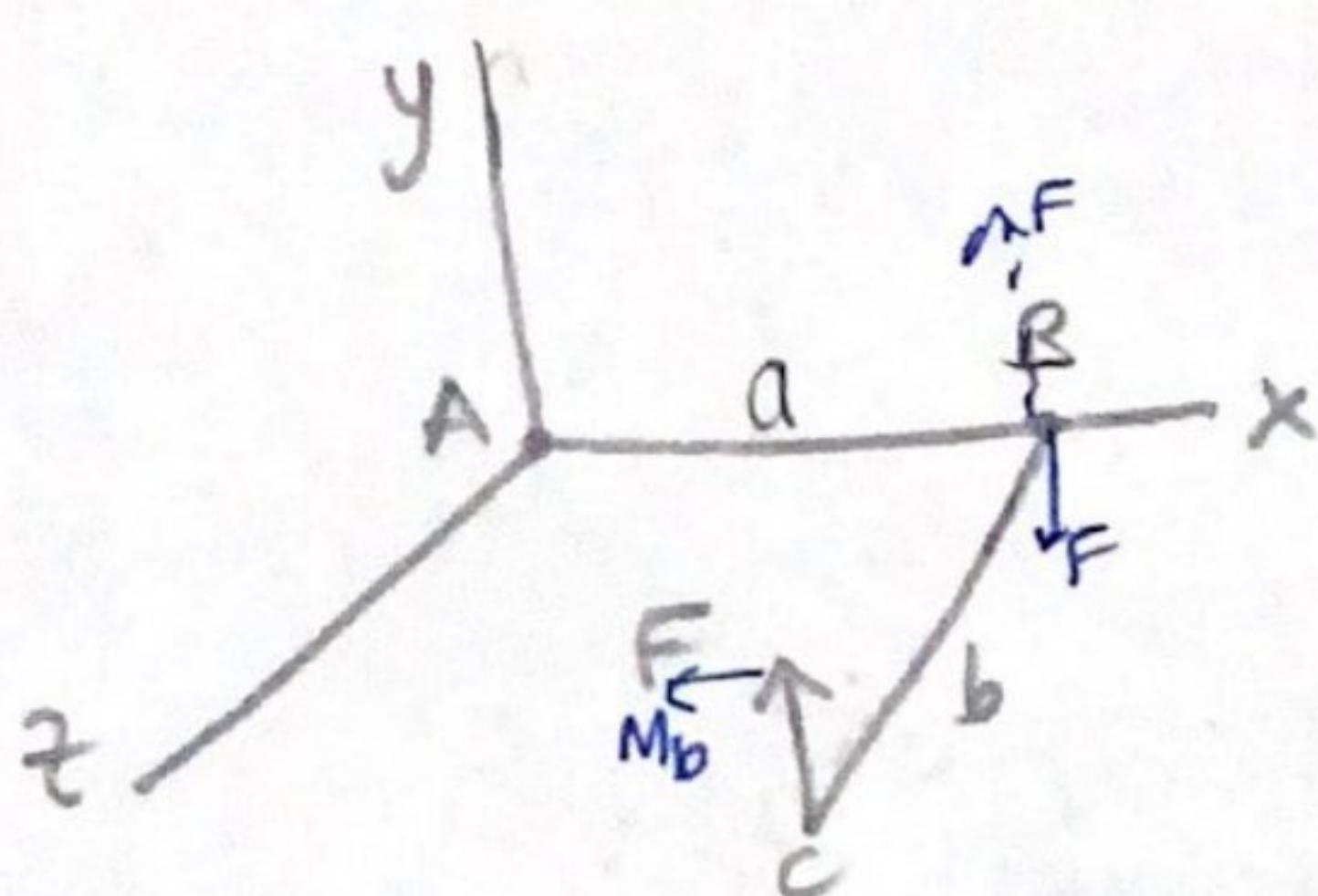


Figure illustrates a person using an exercise machine. The "L" shaped beam shown in fig b represents the left arm of the person. Points A and B represents to the the shoulder and...?

$a=15\text{ cm}$ $b=30\text{ cm}$ The magnitude of applied is $F=200\text{ N}$.

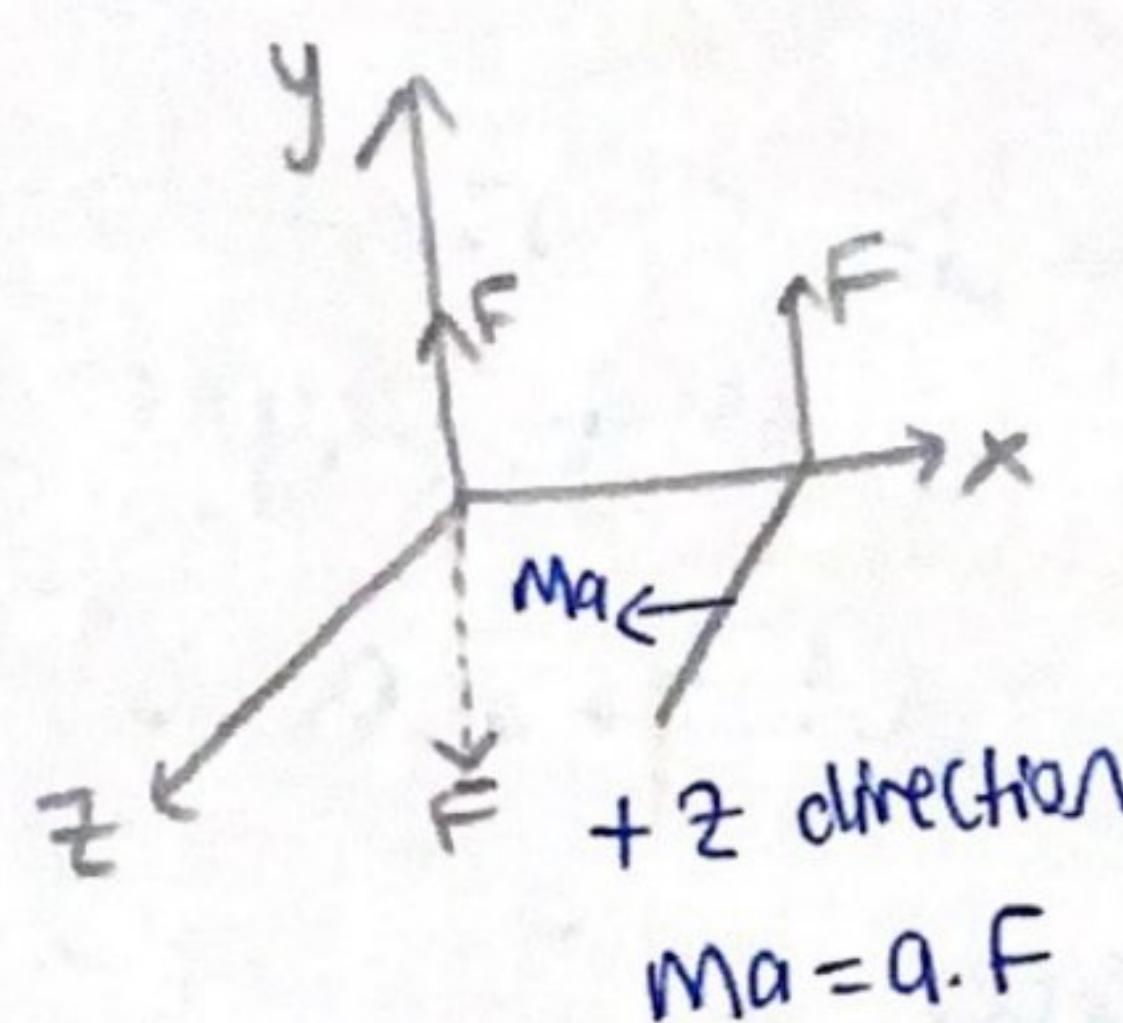
Determine M_{AB} and M_{BC} ?



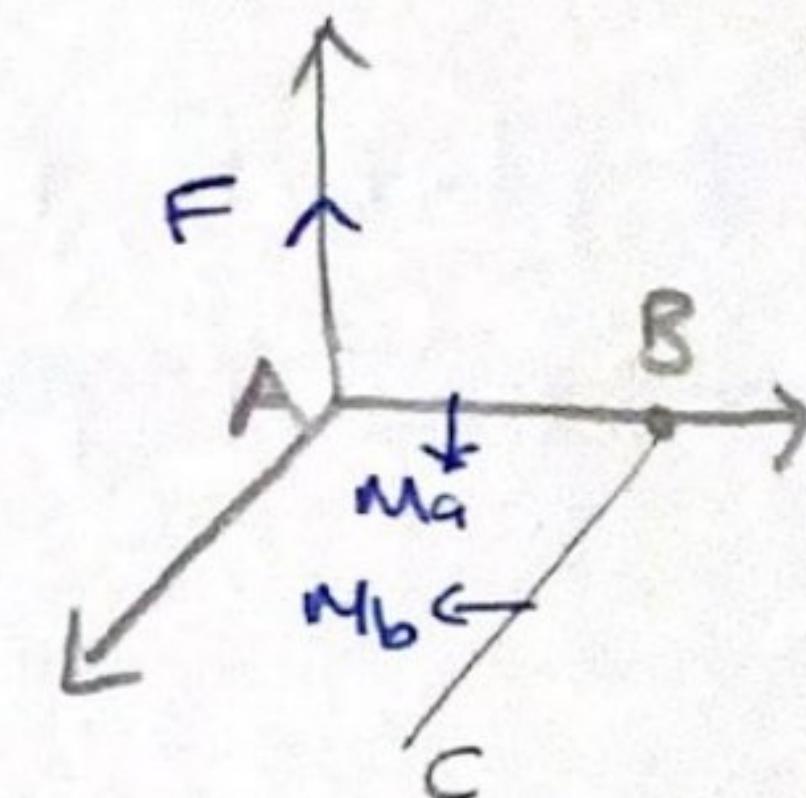
Freebody diagram

Scalar Solution
(-x direction)

$$M_B = b \cdot F$$



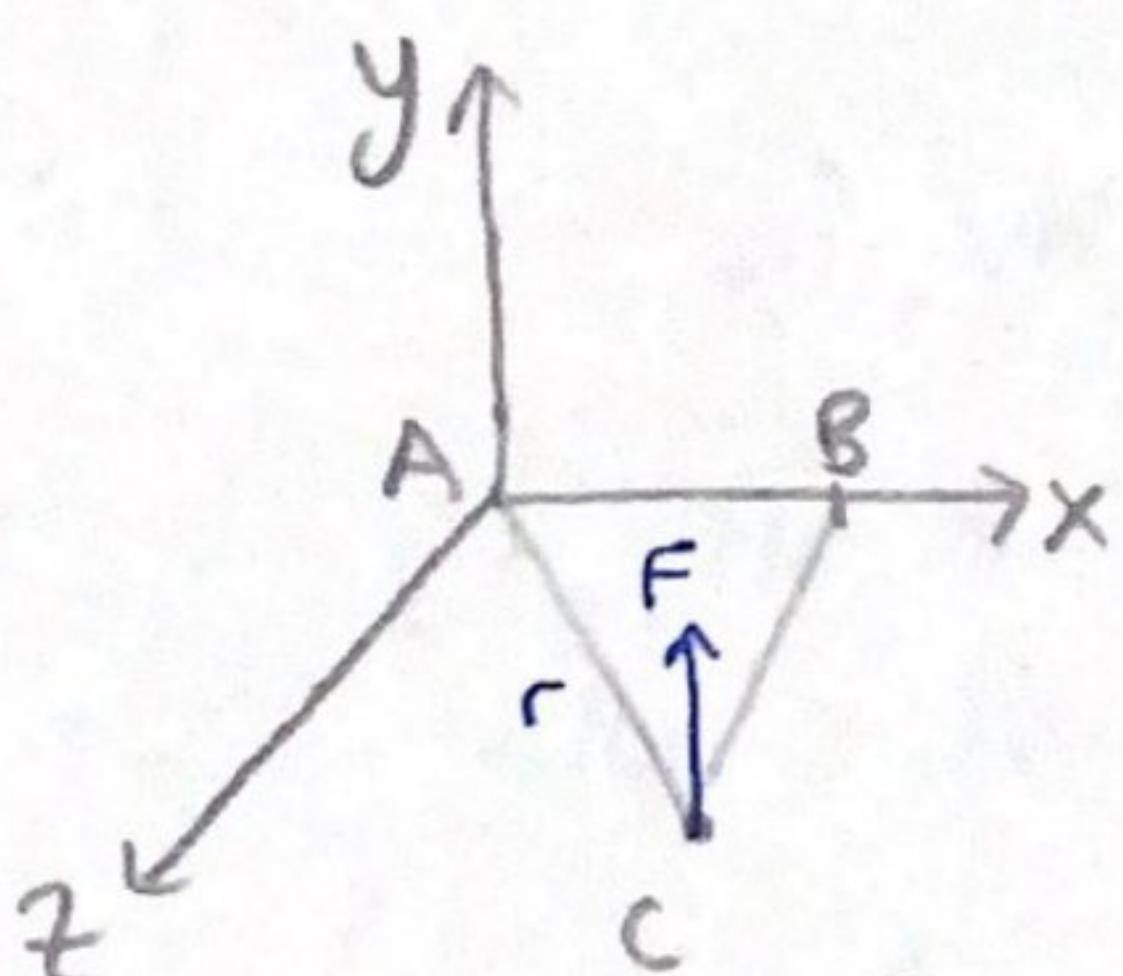
$$M_A = a \cdot F$$



$$M_B = (0, 30) \cdot (200) = 60 \text{ Nm}$$

$$M_A = (0, 15) \cdot (200) = 30 \text{ Nm}$$

Vector Solution



$$M_x = 60 \text{ Nm} \text{ (-x direction)}$$

$$M_y = 0$$

$$M_z = 50 \text{ Nm} \text{ (+z direction)}$$

$$\vec{r} = a\hat{i} + b\hat{k}$$

$$F = F\hat{j}$$

$$M = \vec{r} \times F$$

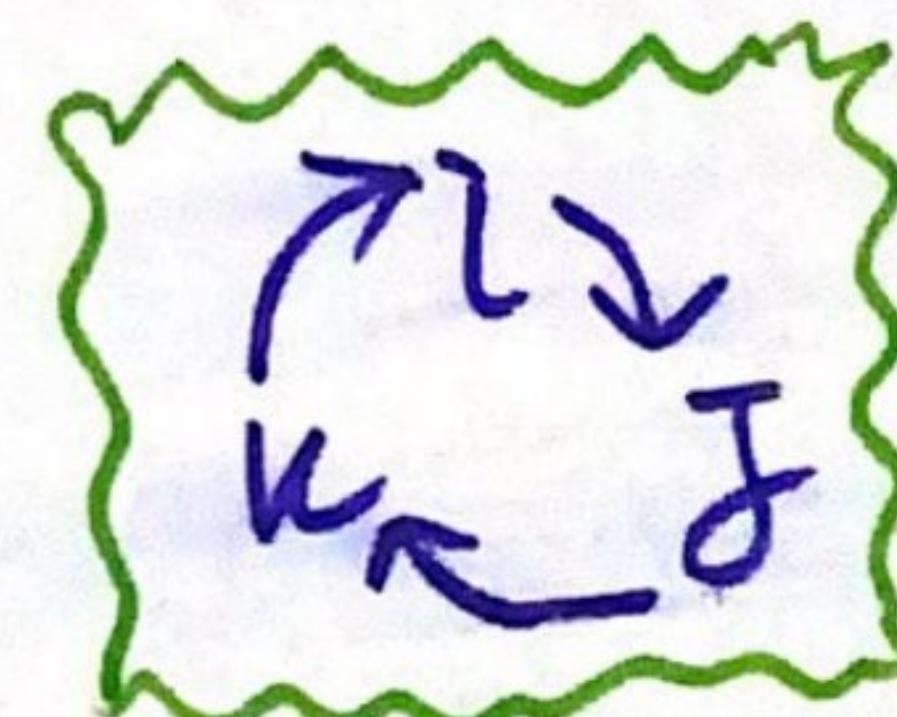
$$= (a\hat{i} + b\hat{k}) \cdot (F\hat{k})$$

$$= aF(\hat{i} \times \hat{j}) + bF(\hat{k} \times \hat{j})$$

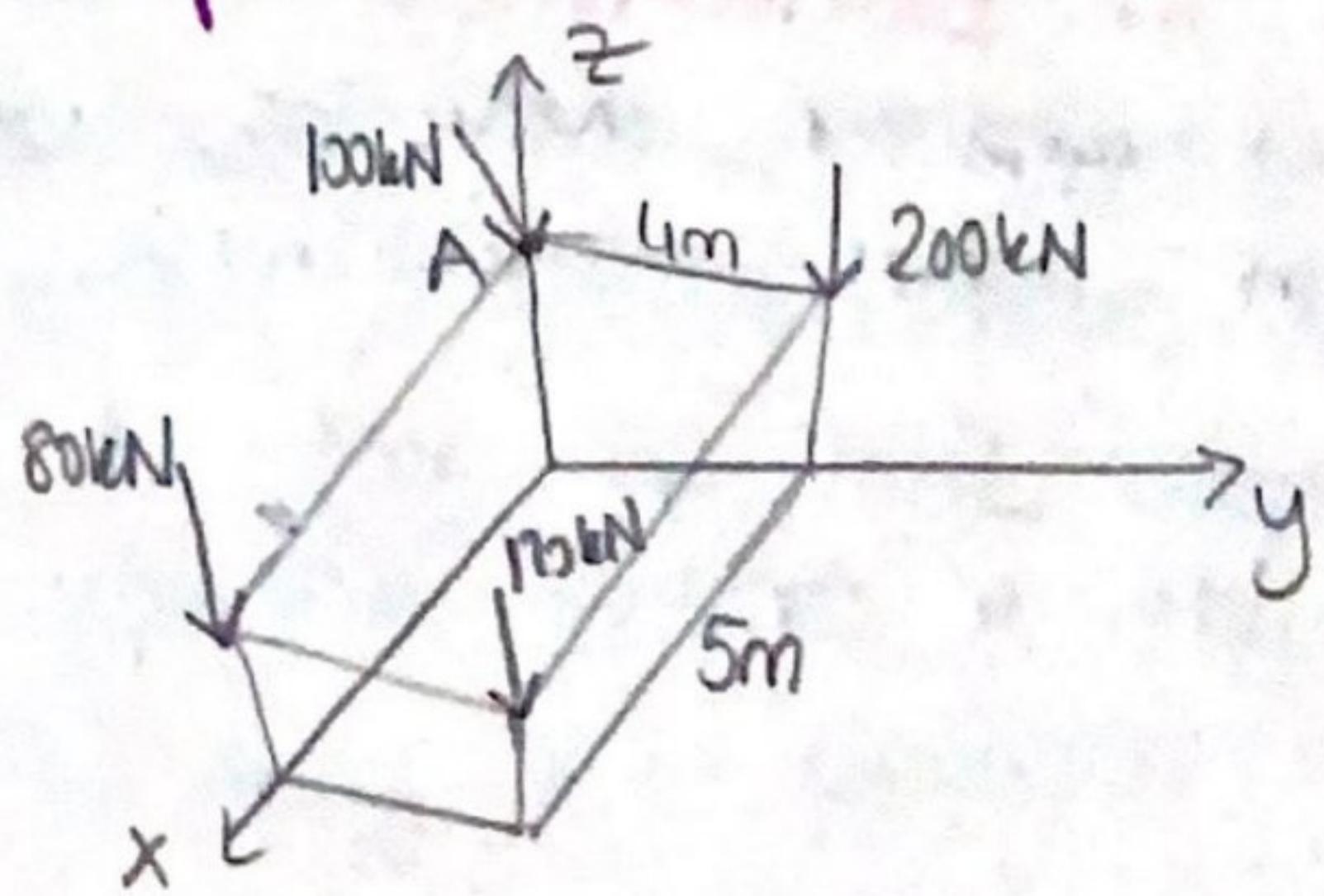
$$= aF\hat{k} - bF\hat{i}$$

$$= (0, 15)(200)\hat{k} - (0, 30)(200)\hat{i}$$

$$= 30\hat{k} - 60\hat{i}$$



Example:

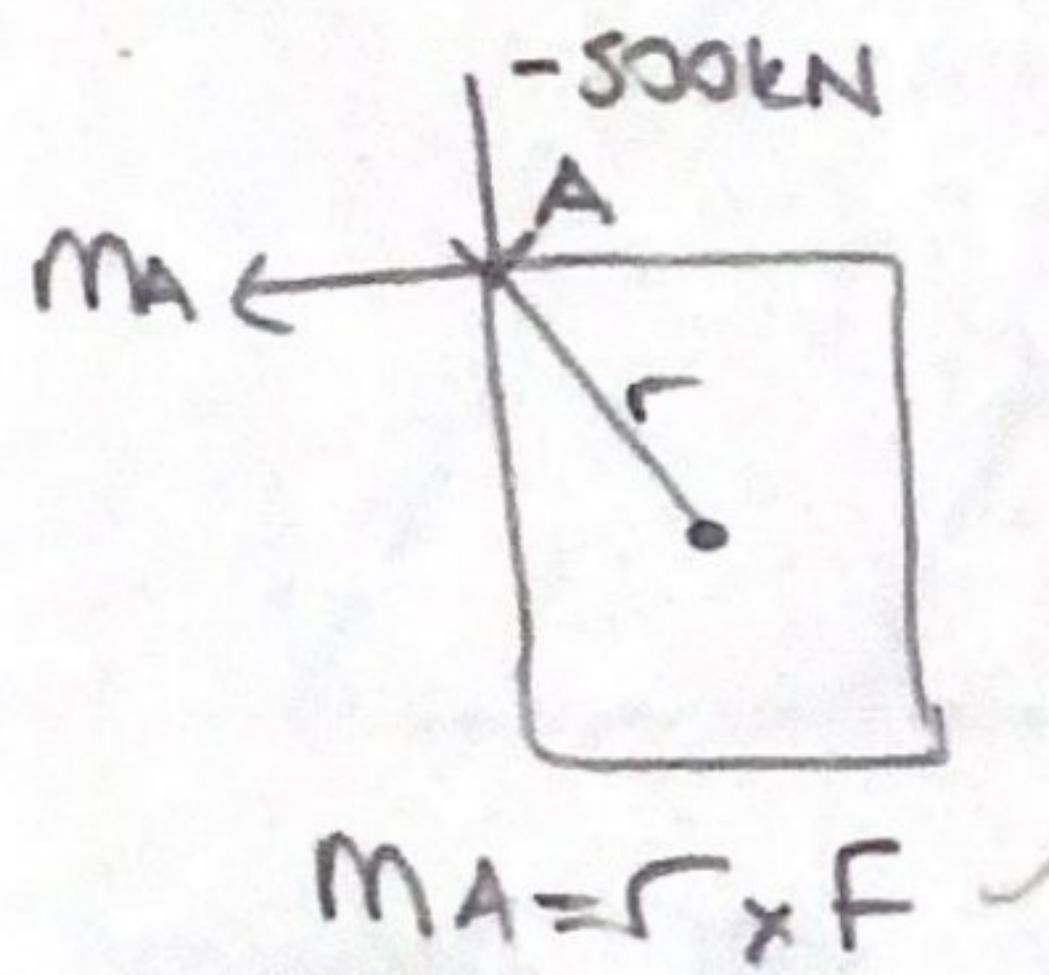


What is the center of loads applied to the board?

$$\sum F_A = -100 - 200 - 80 - 150 = -500\hat{k}$$

$$\sum M_A = [0i \times (100\hat{i})] + [0j \times (100\hat{i})] + [(5i + 4j) \times (100\hat{i})] + [(5i) \times (80\hat{j})]$$
$$= -800\hat{i} + 600\hat{j} - 480\hat{i} + 400\hat{j}$$

$$M_A = -1280\hat{i} + 1000\hat{j} \text{ kNm}$$



$$M_A = r \times F$$

$$\vec{r} \times -500\hat{k} = -1280\hat{i} + 1000\hat{j}$$

$$\vec{r} = xi + yj$$

$$500x\hat{j} - 500y\hat{i} = -1280\hat{i} + 1000\hat{j}$$

$$500x = 1000$$

$$x = 2$$

$$-500y = -1280 \quad y = \frac{1280}{500} = 2.56$$

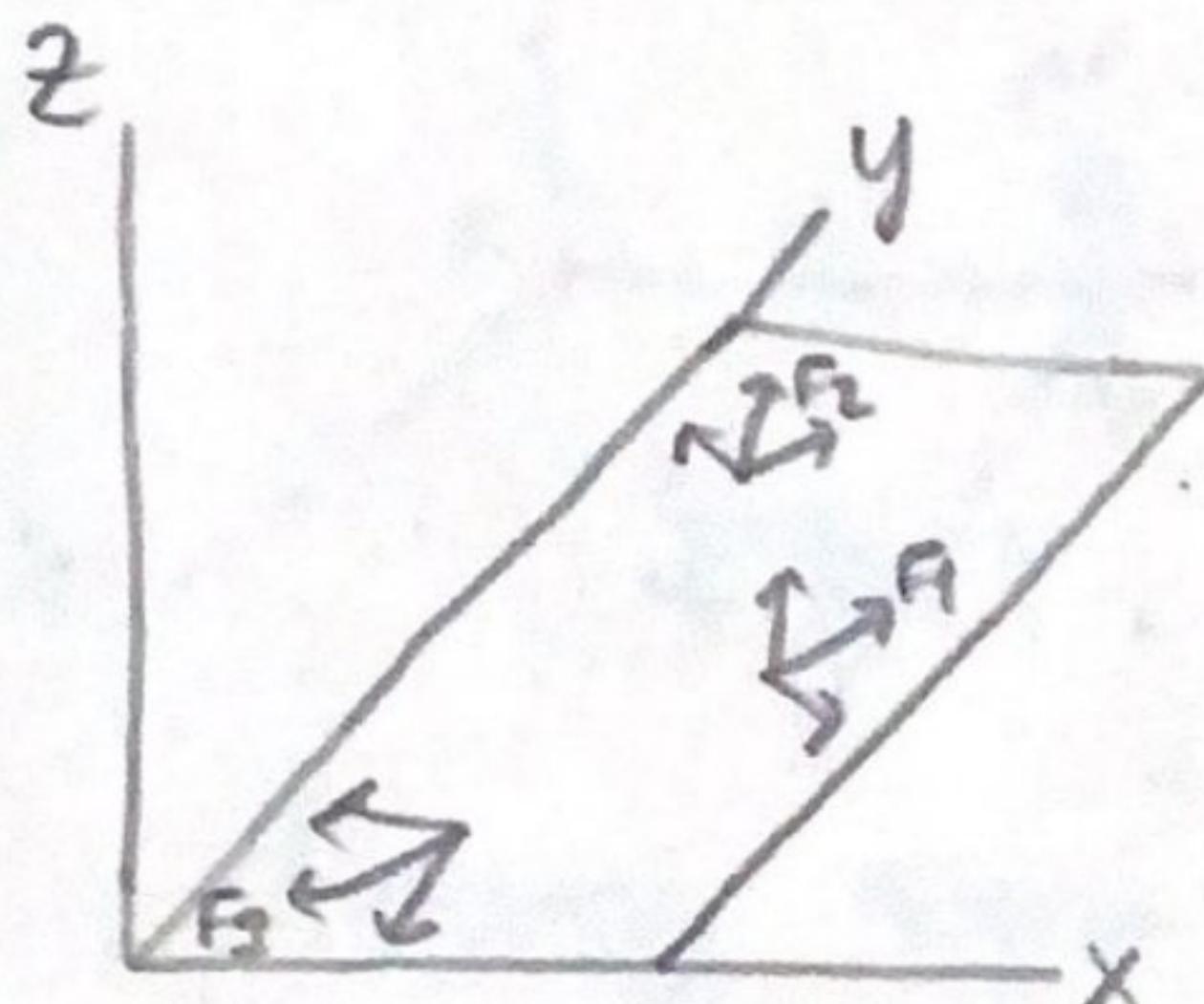
Statics: Systems in equilibrium in Biomechanics

Statics is an area within the field of applied mechanics that is concerned with the analysis of rigid bodies in equilibrium.

In mechanics, the term equilibrium implies that the body of concern is either at rest or moving with a constant velocity.

conditions of equilibrium

When the acceleration is zero, the body is said to be in equilibrium. If the velocity is zero as well then the body is in static equilibrium.



A coplanar force system in the xy plane

$$F_1 = F_{1x}\hat{i} + F_{1y}\hat{j}$$

$$F_2 = F_{2x}\hat{i} + F_{2y}\hat{j}$$

$$F_3 = -F_{3x}\hat{i} - F_{3y}\hat{j}$$

$$\sum F = 0 = (F_{1x} - F_{2x} - F_{3x})\hat{i} + (F_{1y} + F_{2y} + F_{3y})\hat{j}$$

$$F_{1x} - F_{2x} - F_{3x} = 0$$

$$F_{1y} + F_{2y} + F_{3y} = 0$$

Two conditions that need to be satisfied for equilibrium.

For translational equilibrium,

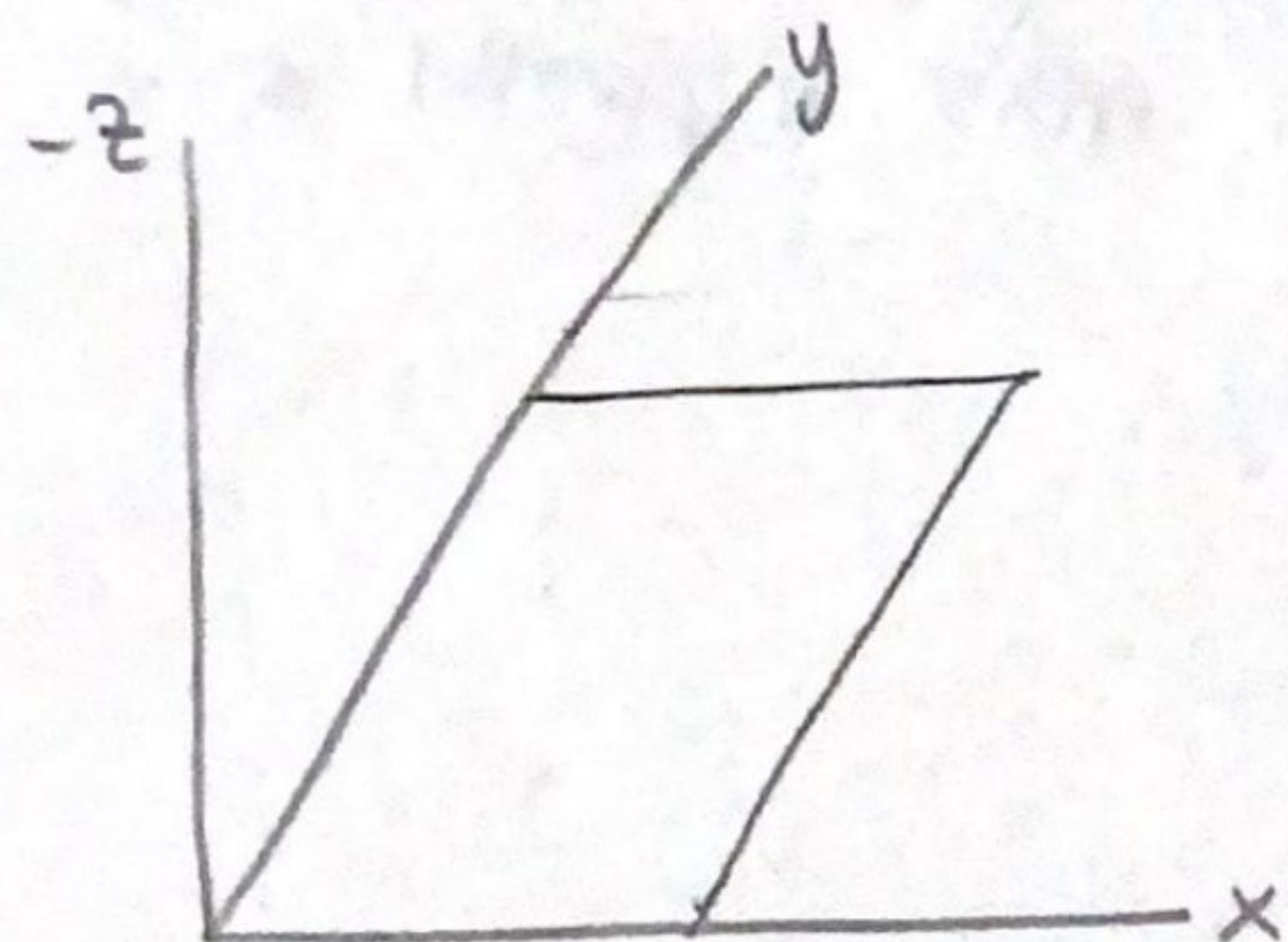
If the net force (vector sum of all forces) acting on it is zero.

$$\sum F = 0$$

The body is in rotational equilibrium,

If the net moment (vector sum of all moments) acting on it is zero.

$$\sum M = 0$$



Moments due to coplanar forces

$$M_1 = d_1 \cdot F_1 \cdot \hat{k}$$

$$M_2 = d_2 \cdot F_2 \cdot \hat{k}$$

$$M_3 = -d_3 \cdot F_3 \cdot \hat{k}$$

$$\sum M = M_1 + M_2 + M_3 - (d_1 F_1 - d_2 F_2 - d_3 F_3) \hat{k} = 0$$

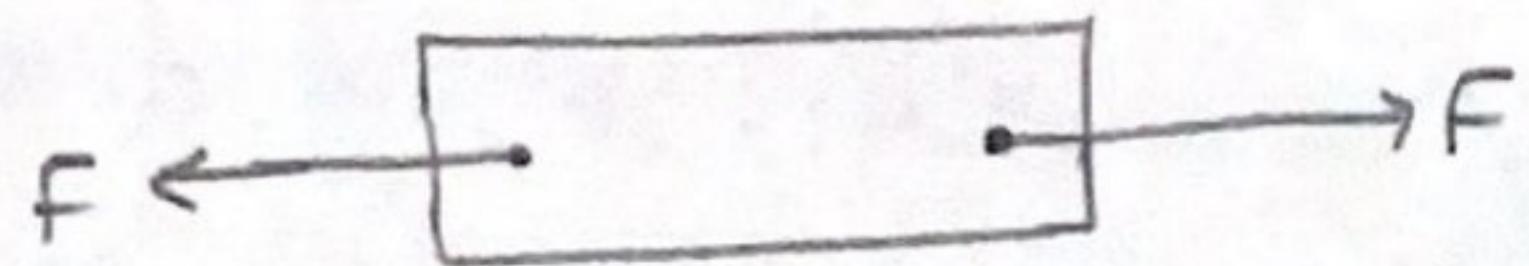
$$d_1 F_1 - d_2 F_2 - d_3 F_3 = 0$$

for three dimensions

$$\left\{ \begin{array}{l} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{array} \right.$$

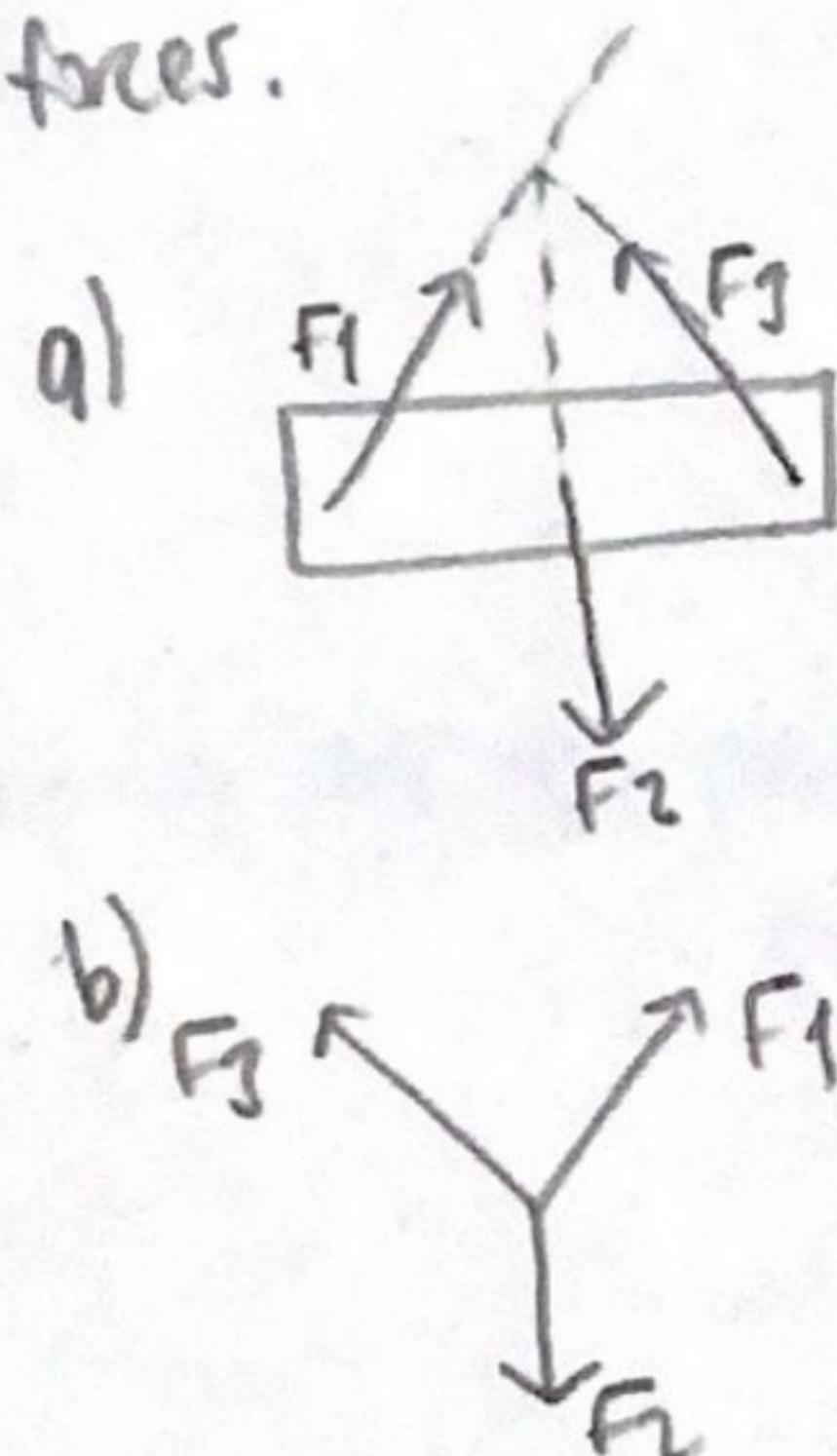
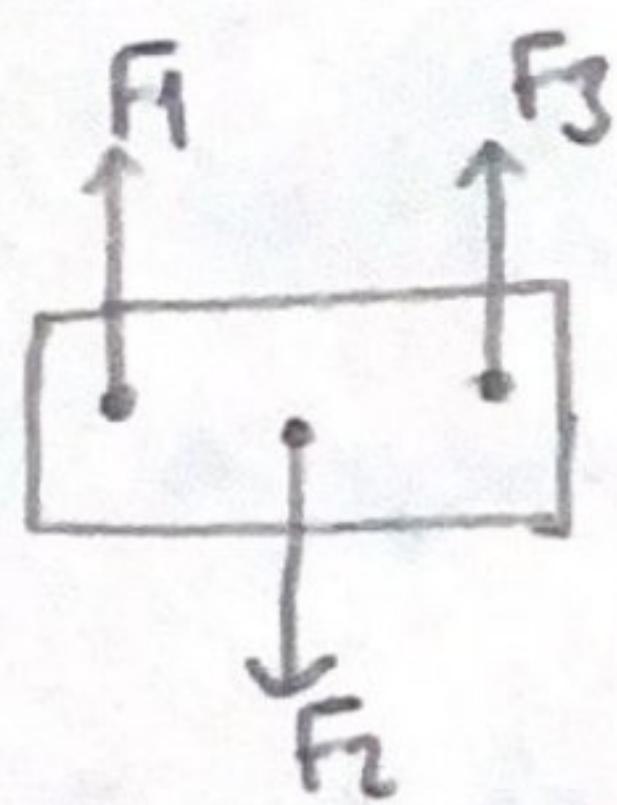
Two force systems;

If there are only two forces acting on a body then the body will be in equilibrium if and only if the forces are collinear, have an equal magnitude and act in opposite directions.

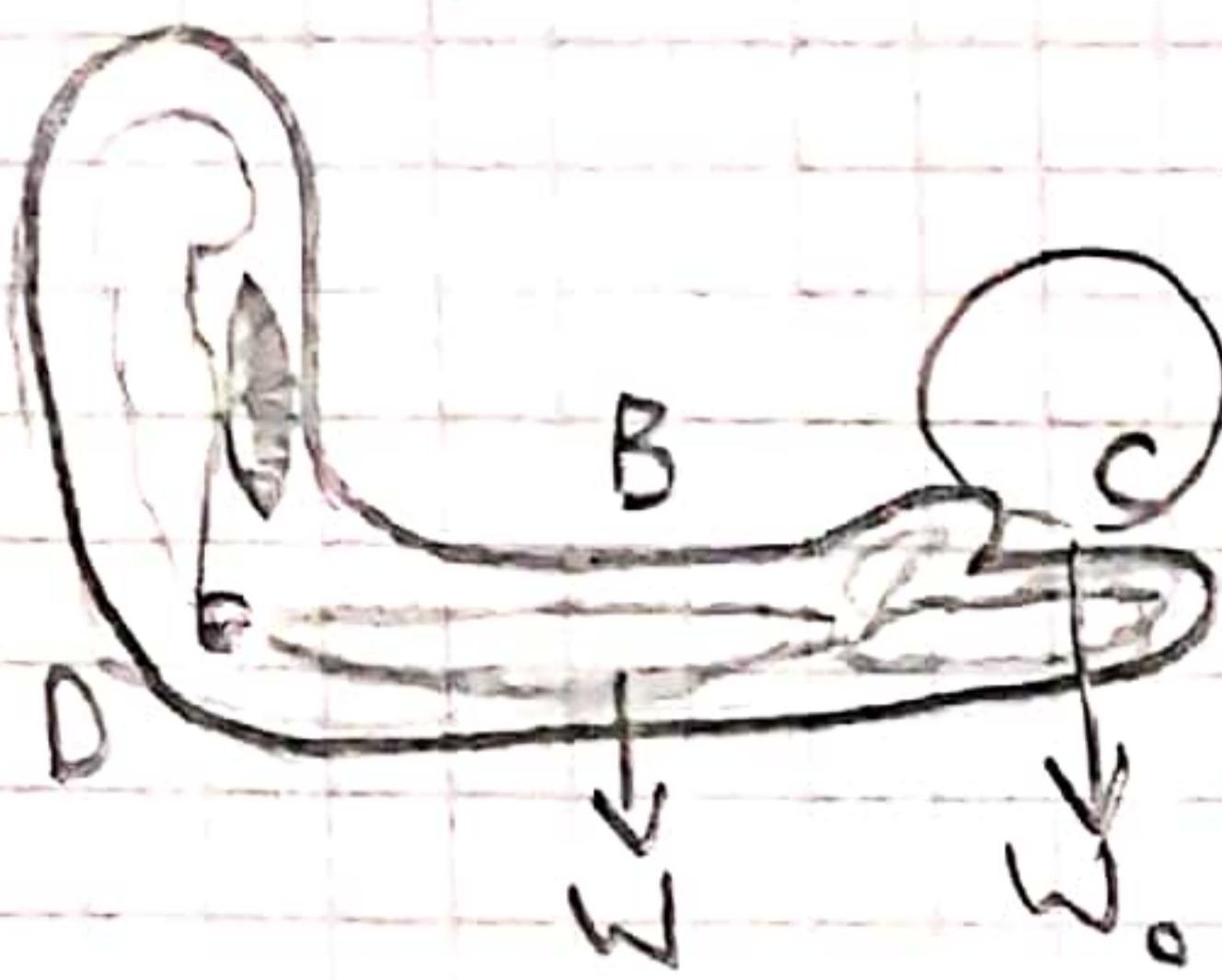


Three force systems;

If there are only three forces acting on a body and if the body is in equilibrium, then the forces must form either a parallel or a concu ... system of forces.

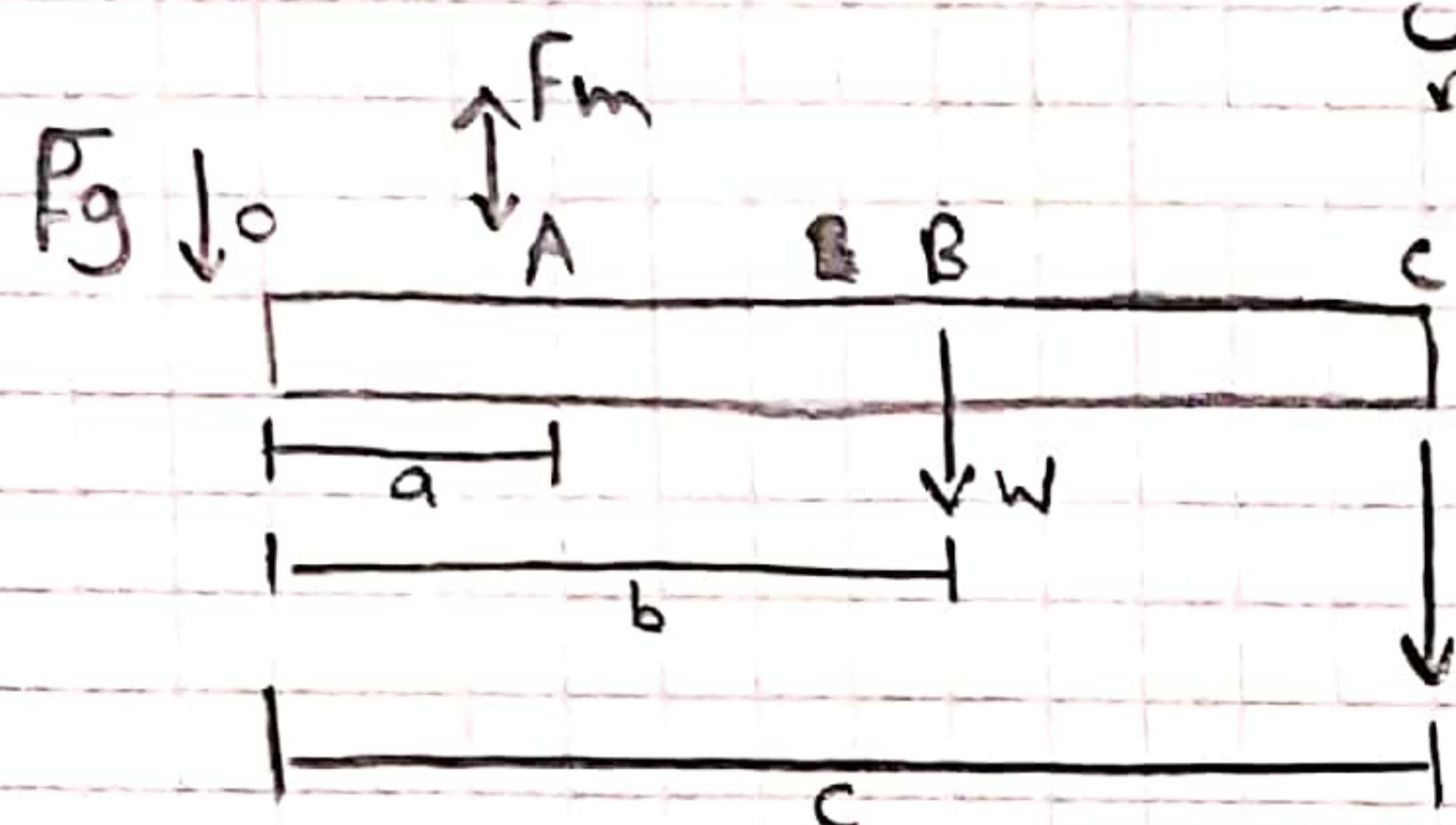


Application of statics To Biomechanics



Consider the arm shown in the fig. the elbow is flexed to a right angle and an object is held in the hand.

Point A is attachment of the biceps muscle and radius B is the center of gravity of forearm O designates the axis of rotation of elbow joint



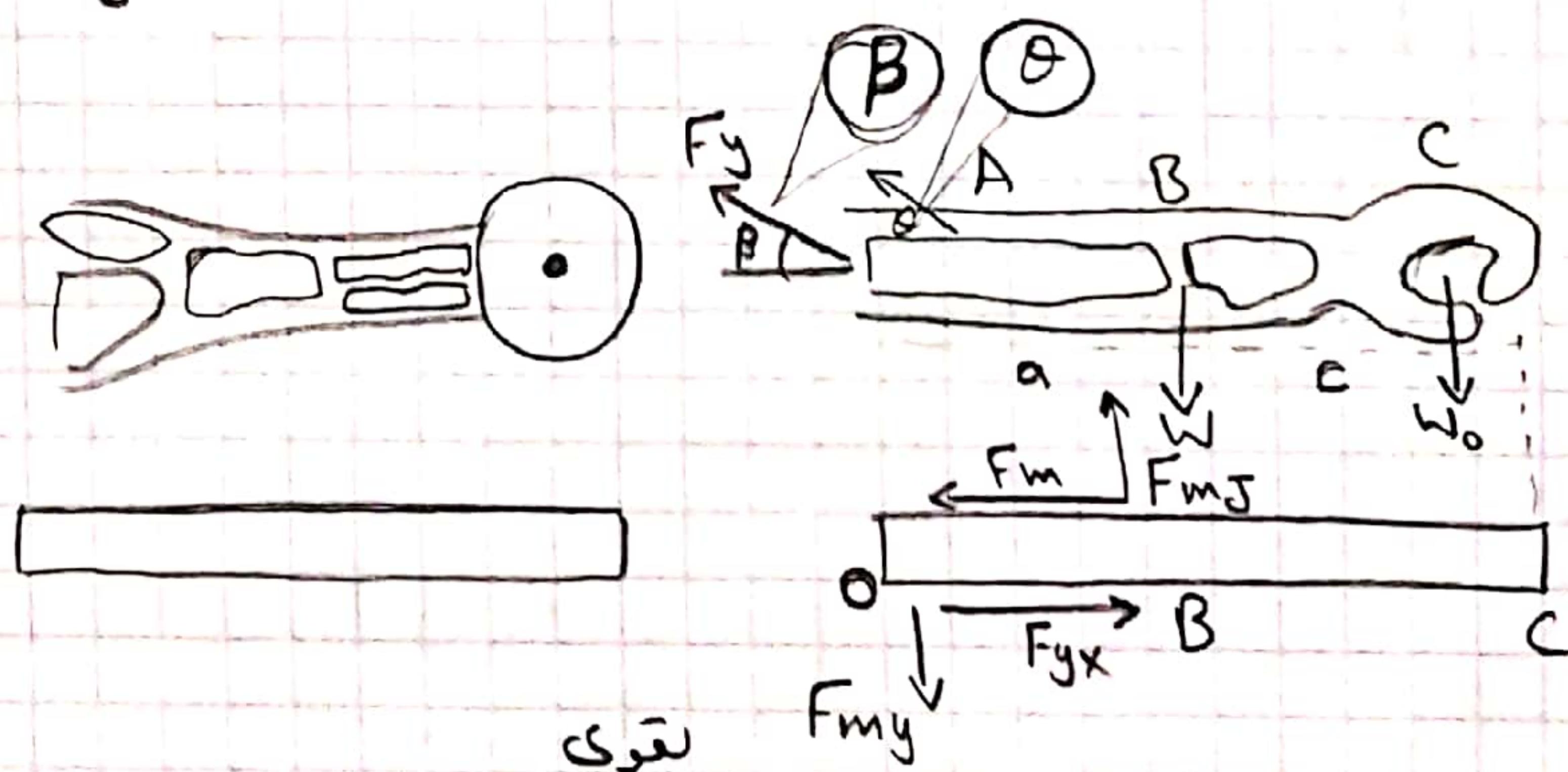
$$\sum M_O = 0, F_m = \frac{1}{a} (b \cdot W + c \cdot W_0)$$

$$\sum F_y = 0, F_J = F_m - W - W_0$$

Assume: $a = 4 \text{ cm}$, $b = 15 \text{ cm}$, $c = 35 \text{ cm}$, $W = 20 \text{ N}$
 ~~$W_0 = 80 \text{ N}$~~

$$F_m = \frac{1}{0.04} [(0, 15)(20) - (0, 35)(80)] = 775 \text{ N}$$

$$F_J = 775 - 20 - 80 = 675 \text{ N}$$



Consider a person strengthening shoulder muscles by dumbbell exercises. O corresponds to the axis of the shoulder joint, A is where deltoid muscle is attached to humerus. B center of gravity of dumbbell W weight of arm W_0 weight of dumbbell Fm is magnitude of tension of deltoid muscle, F_J is the joint reaction force at the shoulder

$$F_{mx} = F_m \cos(\theta) (-x)$$

$$F_{my} = F_m \sin(\theta) (+y)$$

$$F_{Jx} = F_m \cos(\beta) (+x)$$

$$F_{Jy} = F_m \sin(\beta) (-y)$$

$$\sum m_O = 0, a F_{my} - b W - c W_0 = 0$$

$$F_{my} = \frac{1}{a} (b W + c W_0)$$



$$F_m = \frac{F_m g}{\sin \theta}, \quad F_{mx} = F_m \cos \theta, \quad F_J = \sqrt{F_{Jx}^2 + F_{Jy}^2}$$

$$\sum F_x = 0, \quad F_{gx} = F_{mx}$$

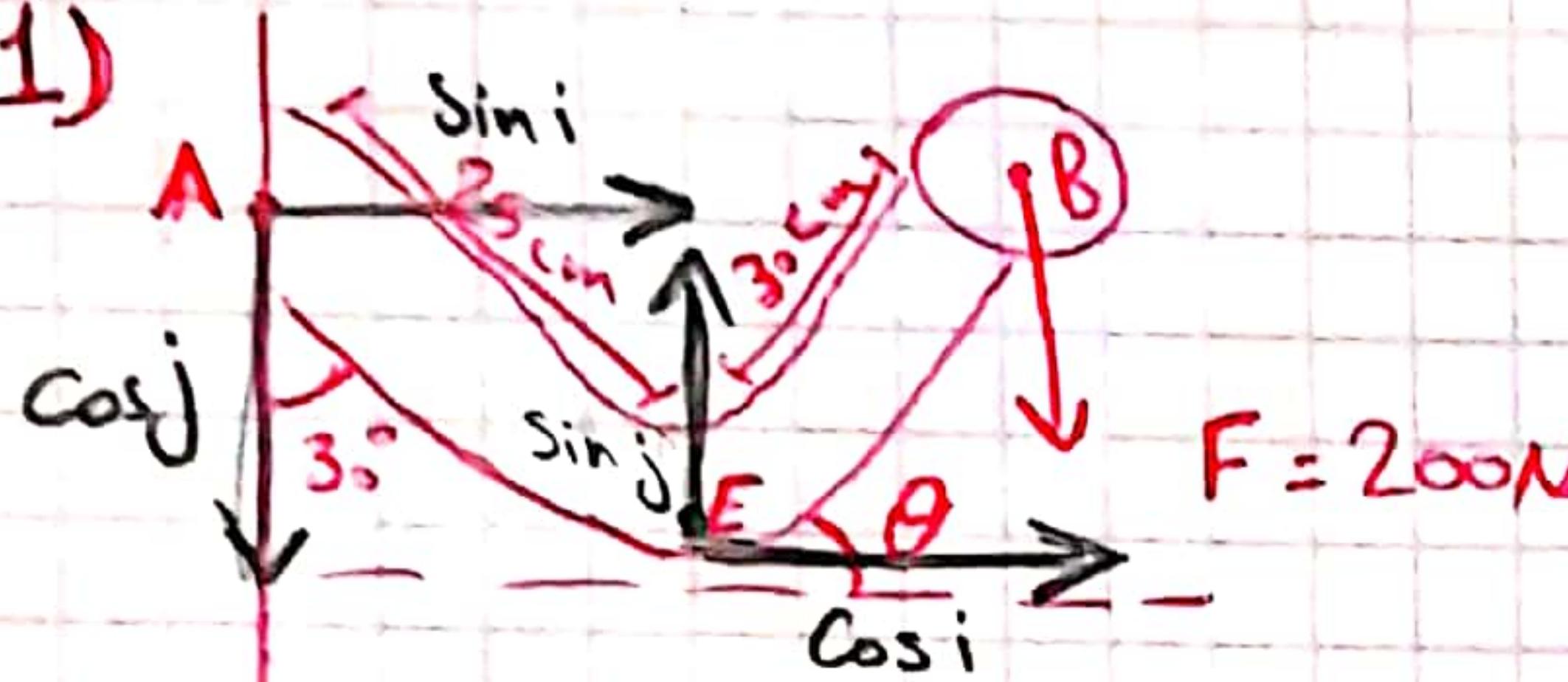
$$\sum F_y = 0$$

$$F_{Jy} = F_{my} - w - w_0$$

$$\beta = \tan^{-1} \left(\frac{F_{Jy}}{F_{Jx}} \right)$$

Quiz Questions:

1)



A = acromion

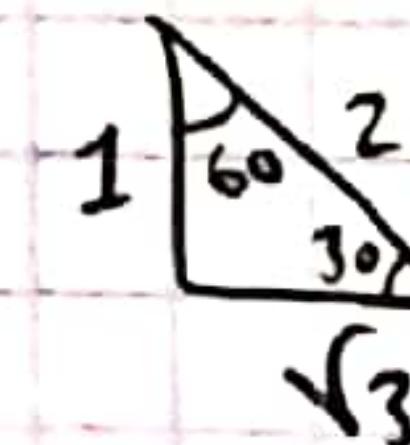
E = Epicondyle

$\theta = 0^\circ, 60^\circ, 90^\circ$ What is the moment of $|AE|$? $g = 10 \text{ m/s}^2$

A refers to Acromian and B refers to Epicondyle of arm. Figure shows that free body diagram of a arm which is picking up 200N weight.

$$|AE| = 25 \sin 30\hat{i} - 25 \cos 30\hat{j}$$

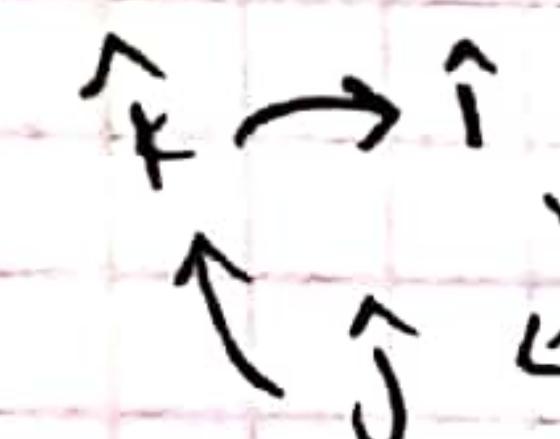
$$12,5\hat{i} - 21,65\hat{j}$$



$$\theta = 0$$

$$|EB| = 30 \cos 0\hat{i} + 30 \sin 0\hat{j} = 30\hat{i}$$

$$mE = 30\hat{i} \times (-200\hat{j}) = -6000\hat{k}$$



$$mA = (12,5\hat{i} - 21,65\hat{j}) \times (-200\hat{j})$$

$$42,5\hat{i} \times (-200\hat{j}) = -8500\hat{k}$$

Mid-term Q

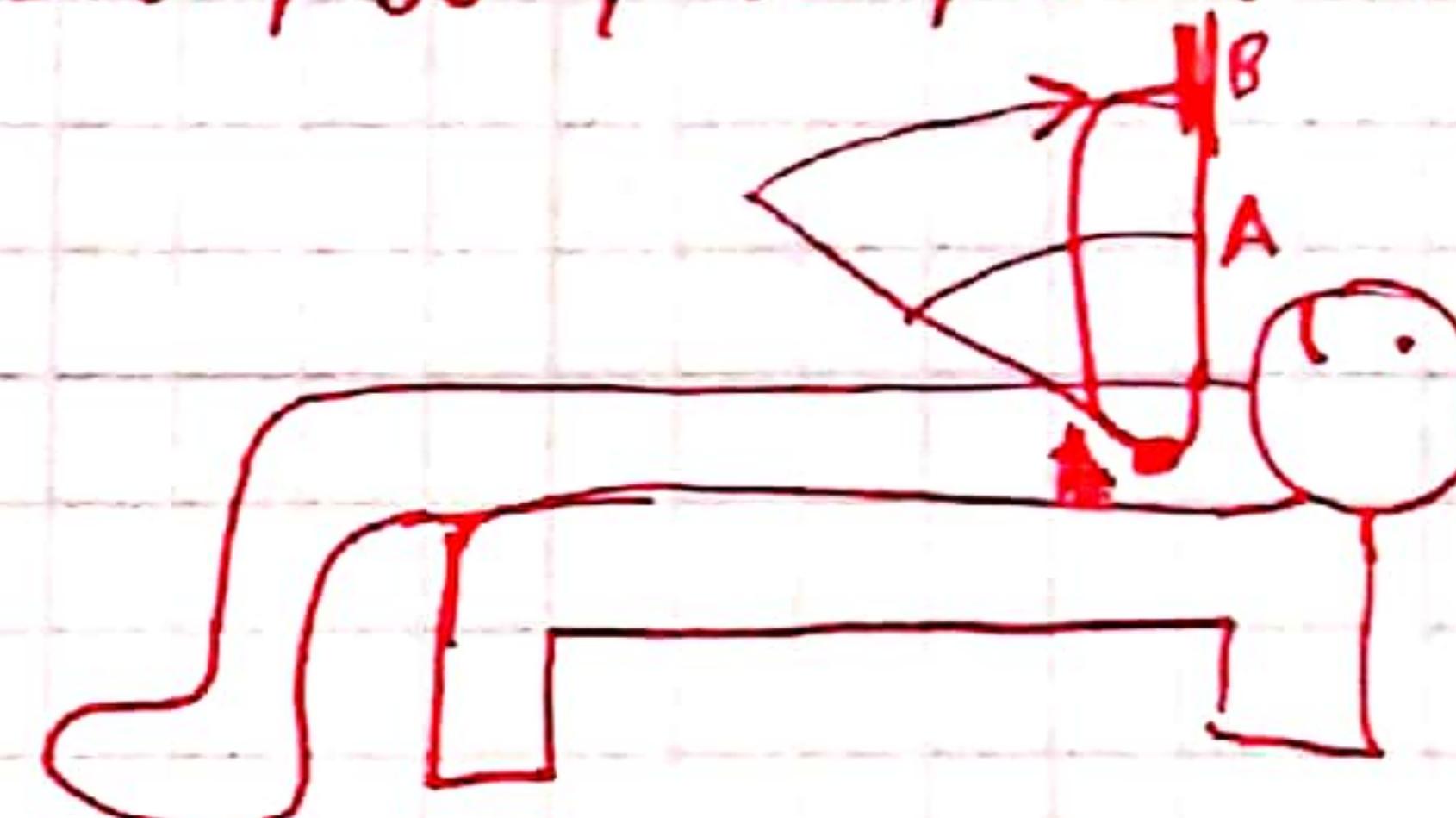
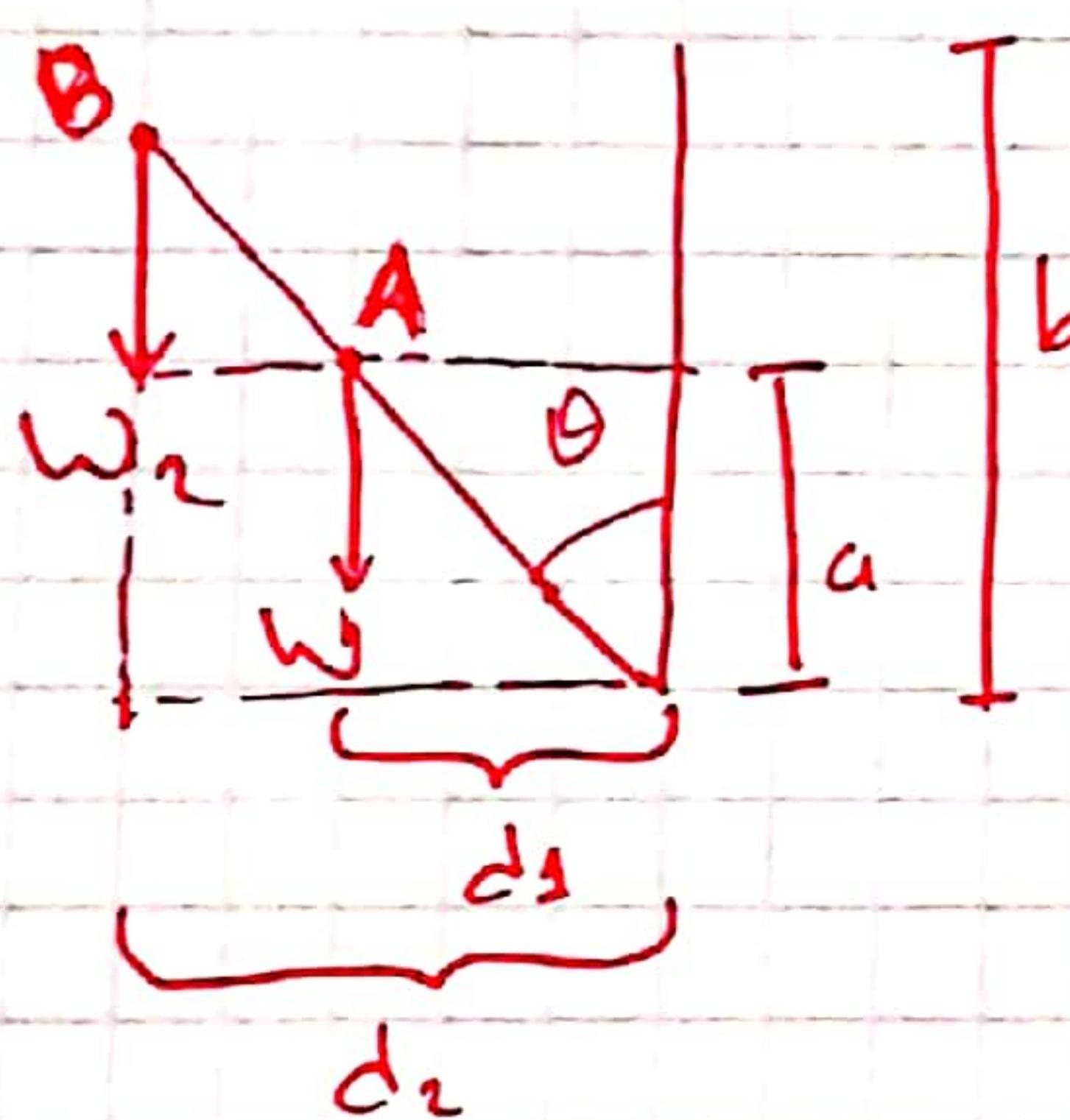
Quiz question:

the distance between OA is $a = 24\text{m}$
 the distance between O and B is $b = 60\text{m}$

Arm weight $W_1 = 50\text{N}$ and total weight of the dumbbell $W_2 = 300\text{N}$

Determine the Net amount due to W_1 and W_2 about shoulder joint as a function of θ , which is the angle the arm is making with the vertical

Calculate the net moment of $\theta = 0^\circ, 30^\circ, 45^\circ, 15^\circ, 60^\circ$



$$M_1 = d_1 W_1 = a W_1 \sin \theta = (24)(50)(\sin \theta)$$

$$= 1200 \sin \theta$$

because that's in m
↑ we keep it

$$M_2 = d_2 W_2 = b W_2 \sin \theta = (60)(300) \sin \theta = 18000 \sin \theta$$

$$M_o = M_1 + M_2 = 1200 \sin \theta + 18000 \sin \theta = 19200 \sin \theta$$

θ	$\sin \theta$	M_o
0°	0	0
30°	0,5	9600
45°	0,707	13574,4
15°	0,259	4972,8
60°	0,866	16627,2

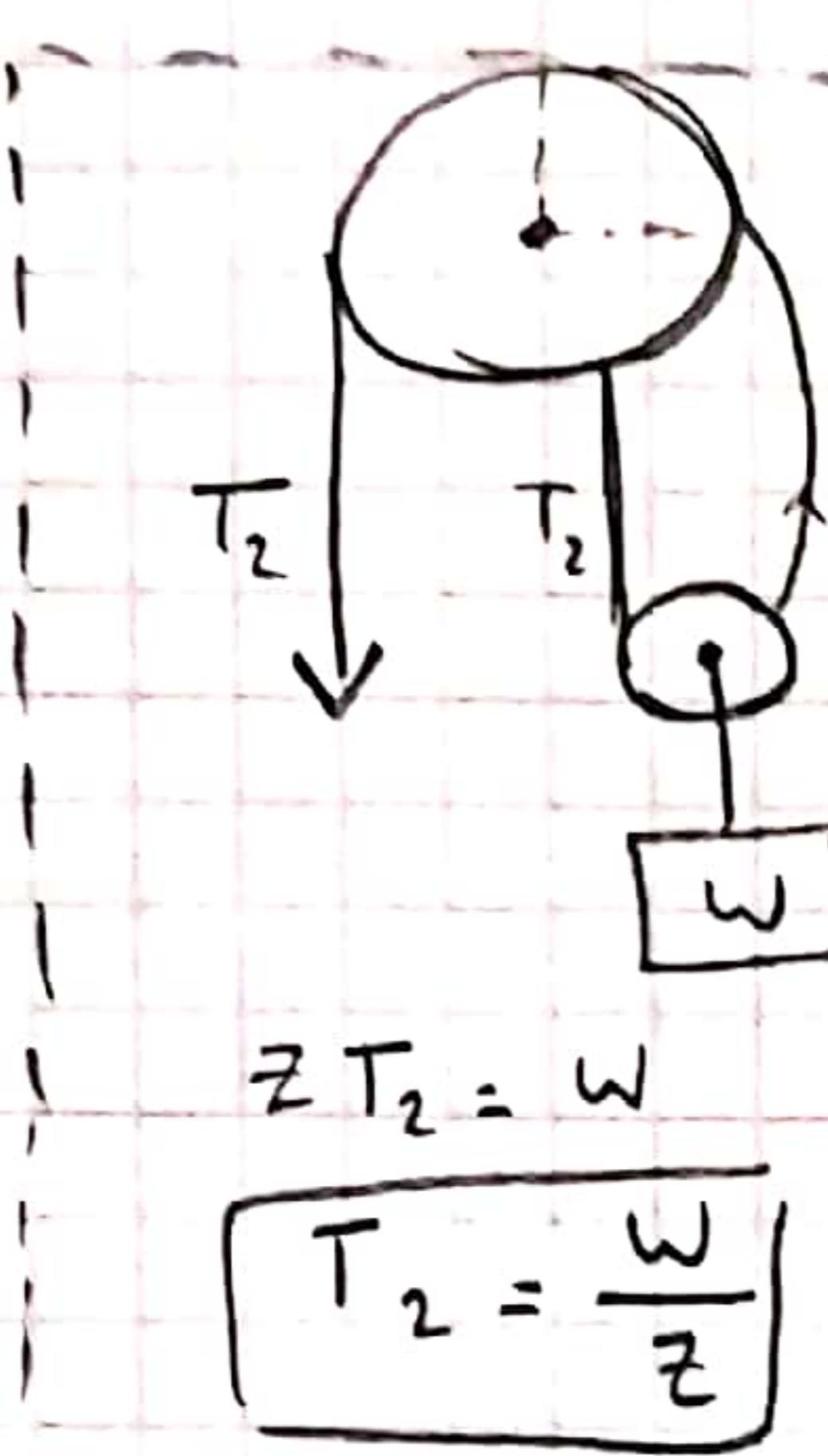
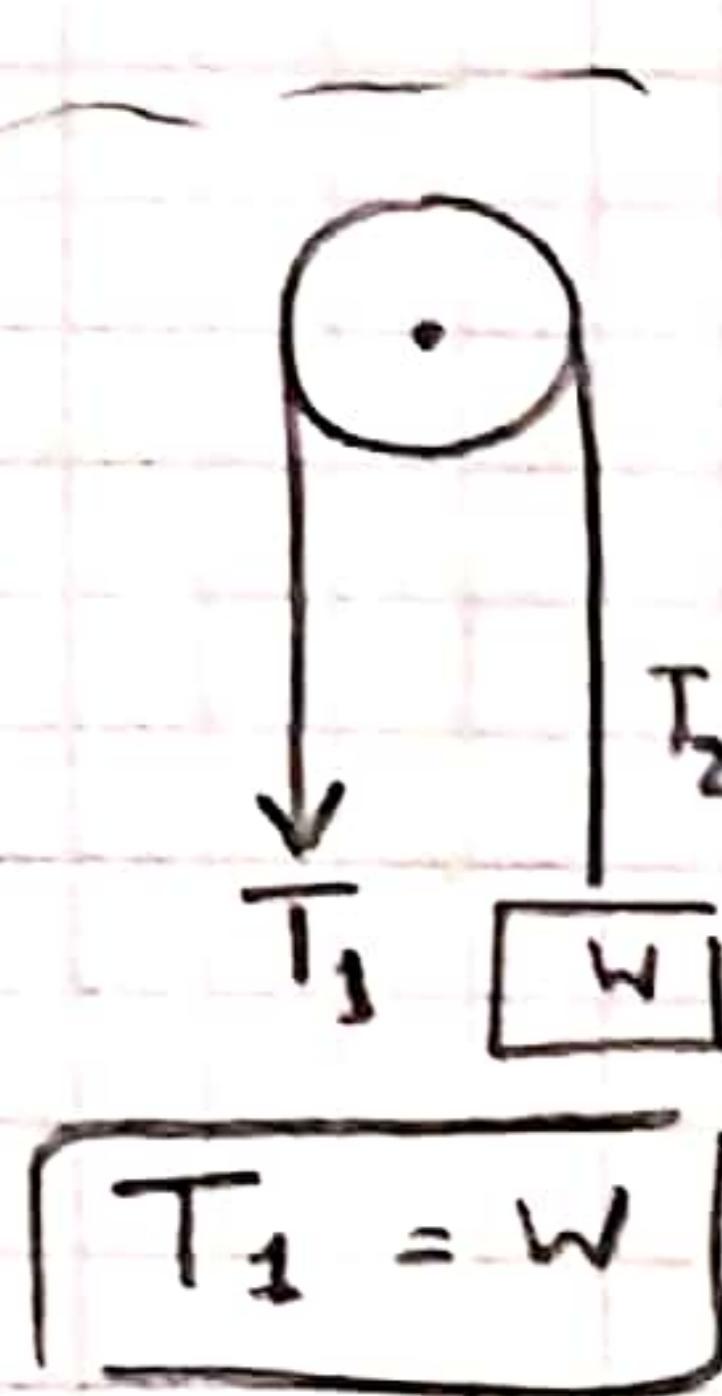
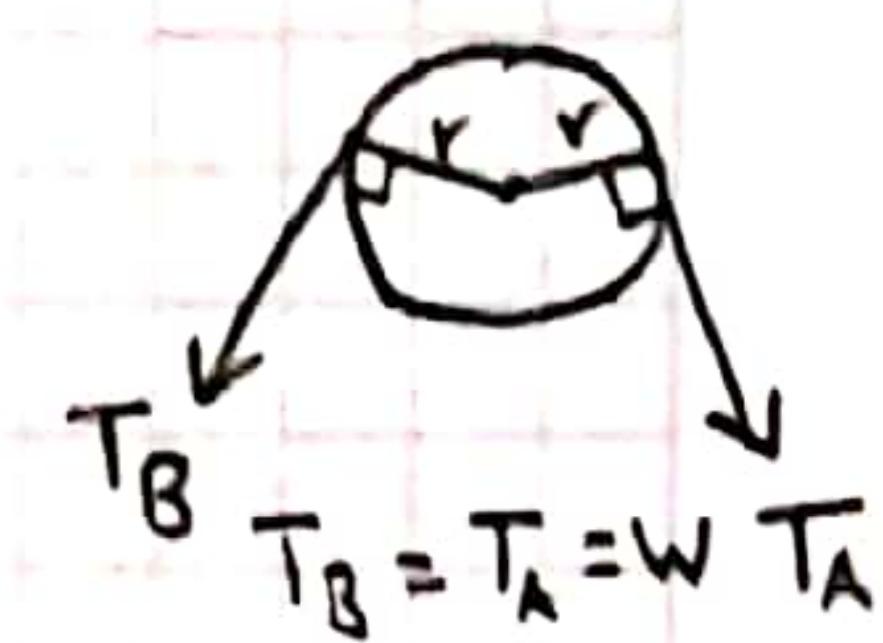
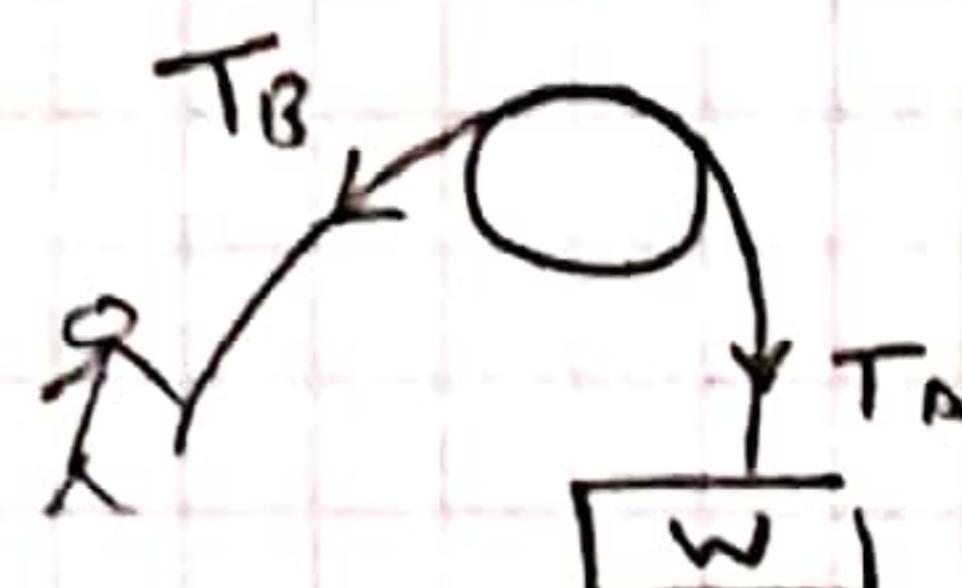
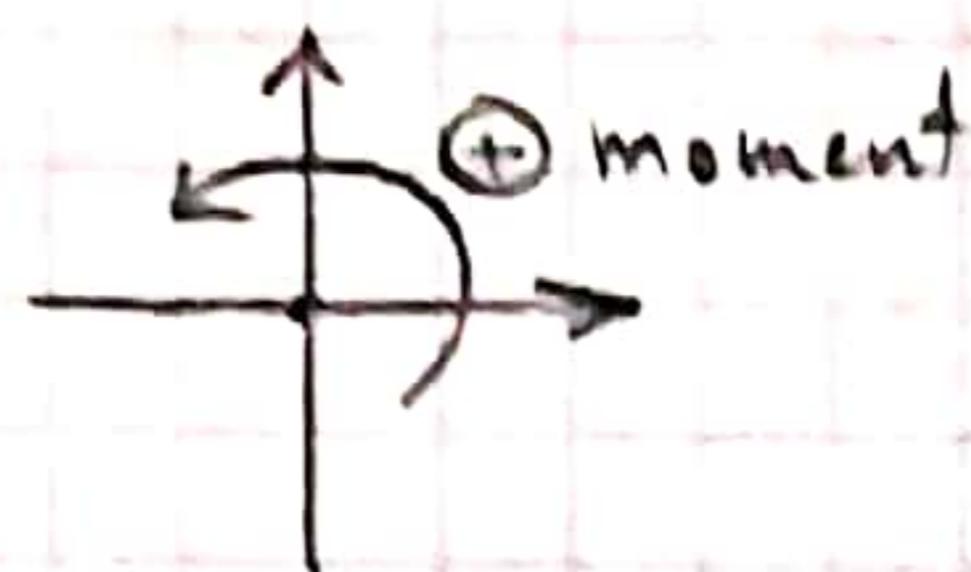
الكابل بكرة
Cable pulley arrangements

we commonly used to electric weight
and now applications in the design of patient rehabilitation

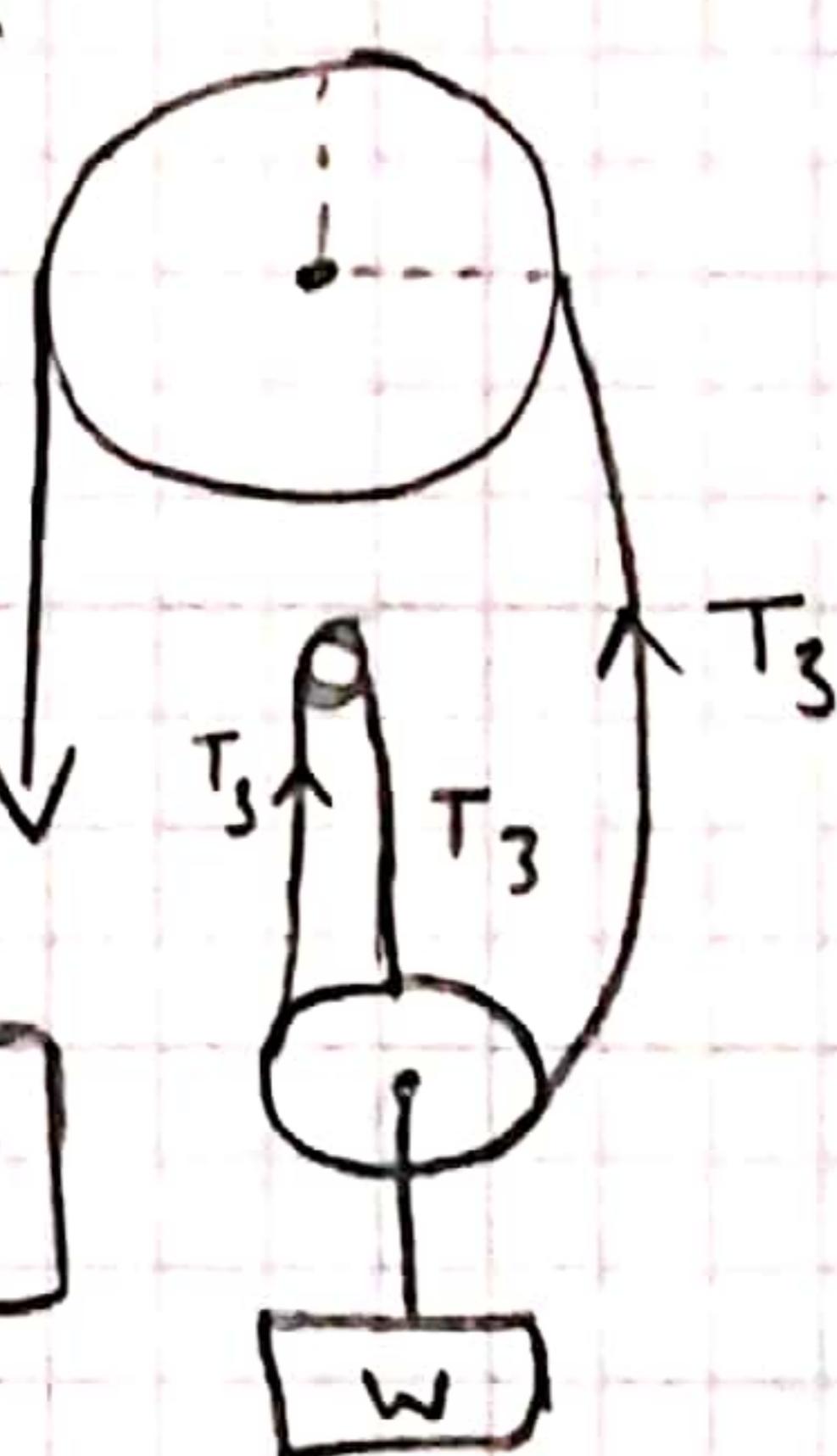
دبل بكرة

$$\sum m = 0$$

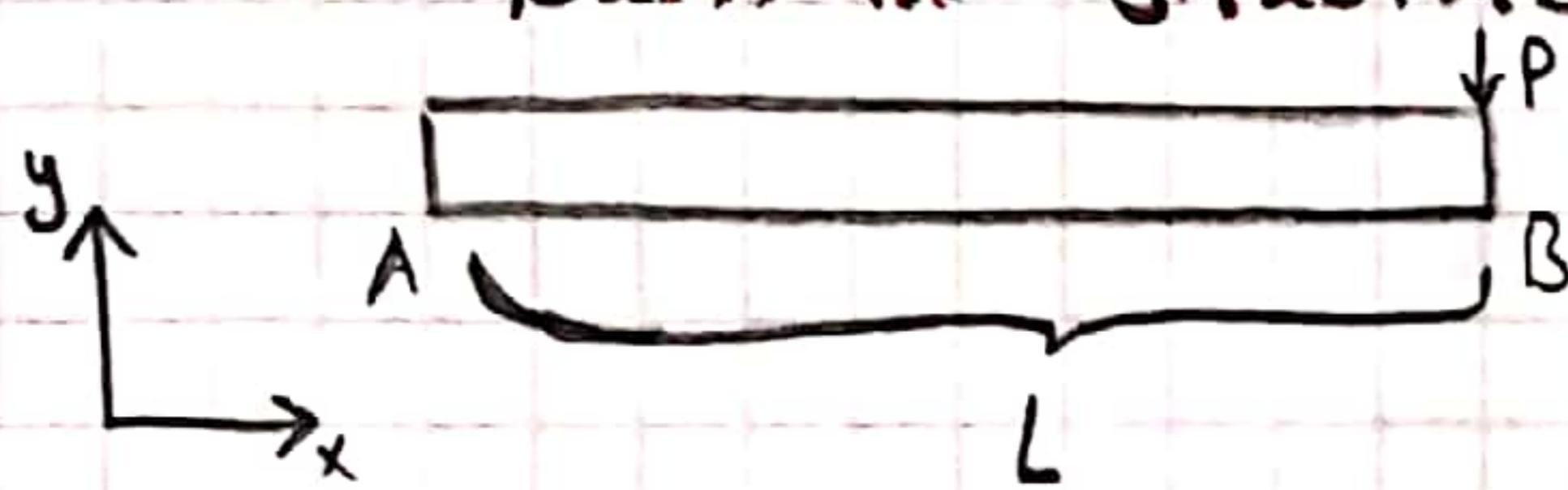
$$rT_B - rT_A$$



Additional direction
are suggested
not by prof.



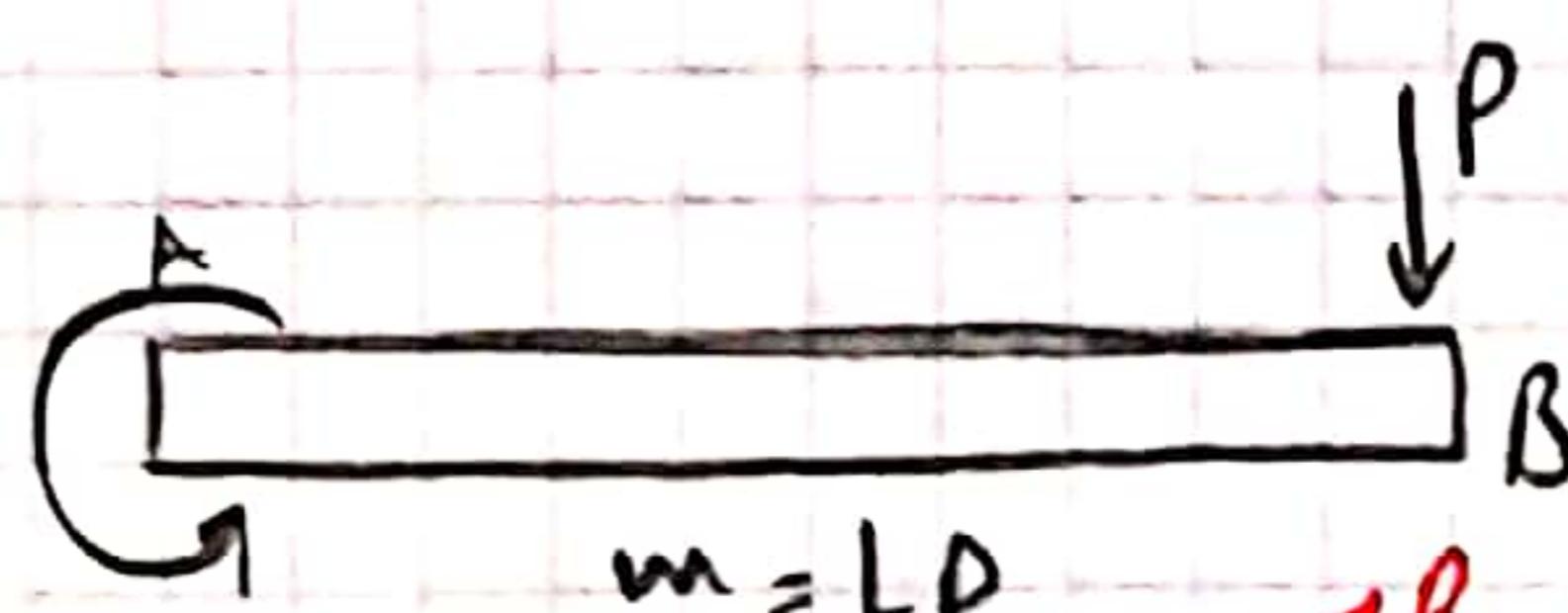
Built in structure



$$\sum F_y = 0$$

$$R_A - P = 0$$

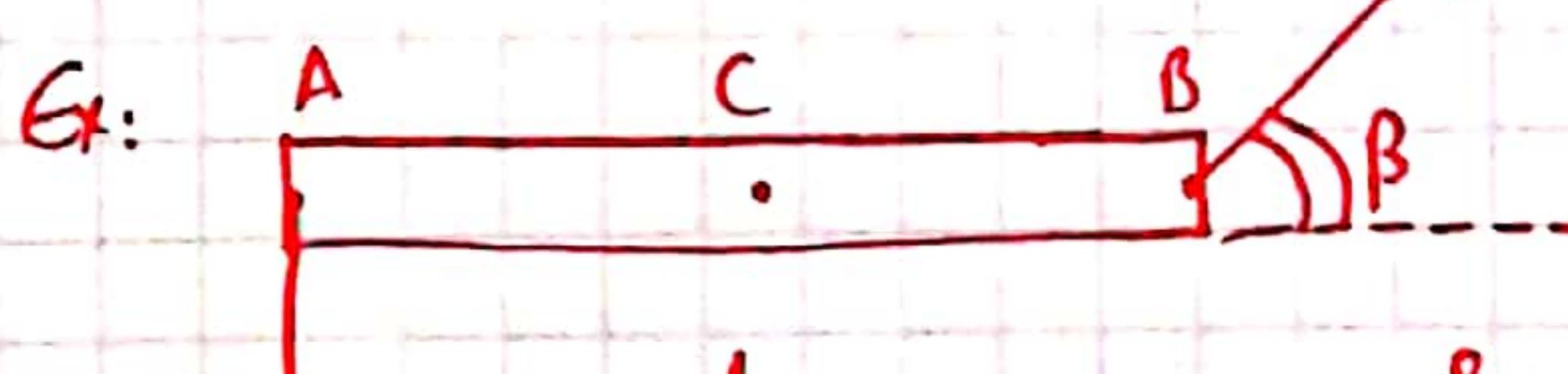
$$R_A = P$$



$$\sum m_A = 0$$

$$m - Lp = 0$$

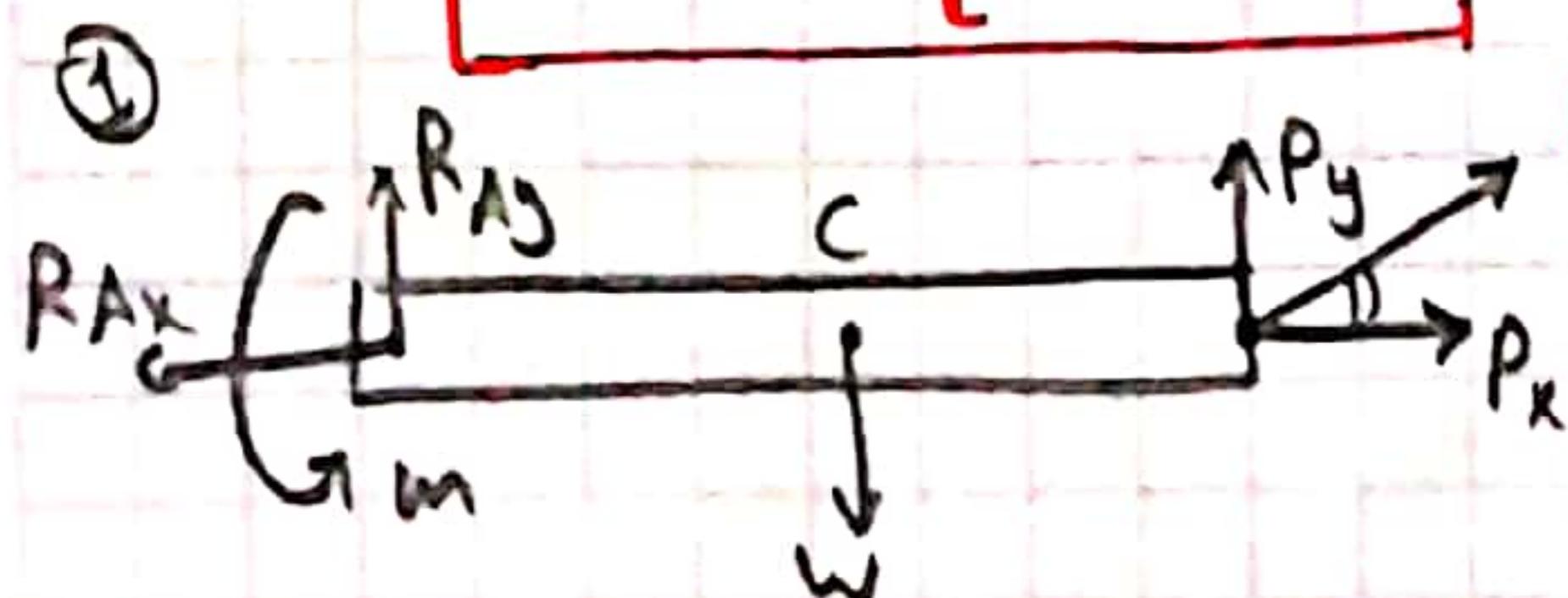
$$m = Lp$$



Consider beam in fig. the beam is fixed at a force that makes the angle $\beta = 30^\circ$
 $P = 100N$, C is center of gravity

Beam weight $50N$, $L = 2m$

Determine the reaction force generated at the fixed end



$$\sum F_y = 0, R_{Ay} - W + P_y = 0$$

$$R_{Ay} = 50 - 50 = 0$$

$$\text{at a point } \sum m_A = 0 \Rightarrow m - \frac{L}{2}W + Lp_y = 0$$

$$m = \frac{L}{2}W - Lp_y = -50$$

At point $\sum F = 0$

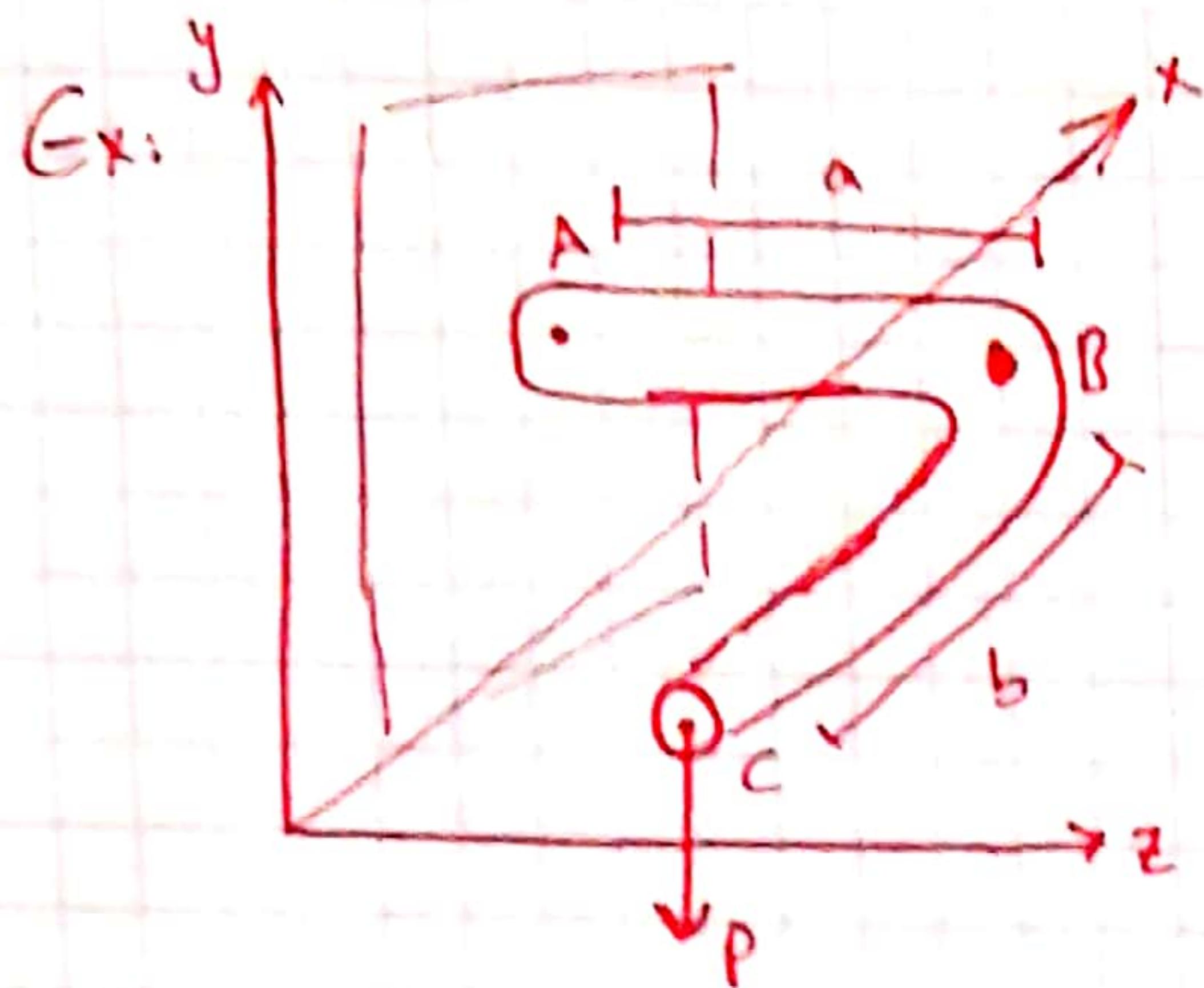
$$\sum F_x = 0, P_x - R_{Ax} = 0$$

$$P_y = P \sin 30$$

$$P_y = 50N$$

$$P_x = 86,6N$$

Midterm Q



Consider the shaped beam in Fig. The beam is welded to a wall at A

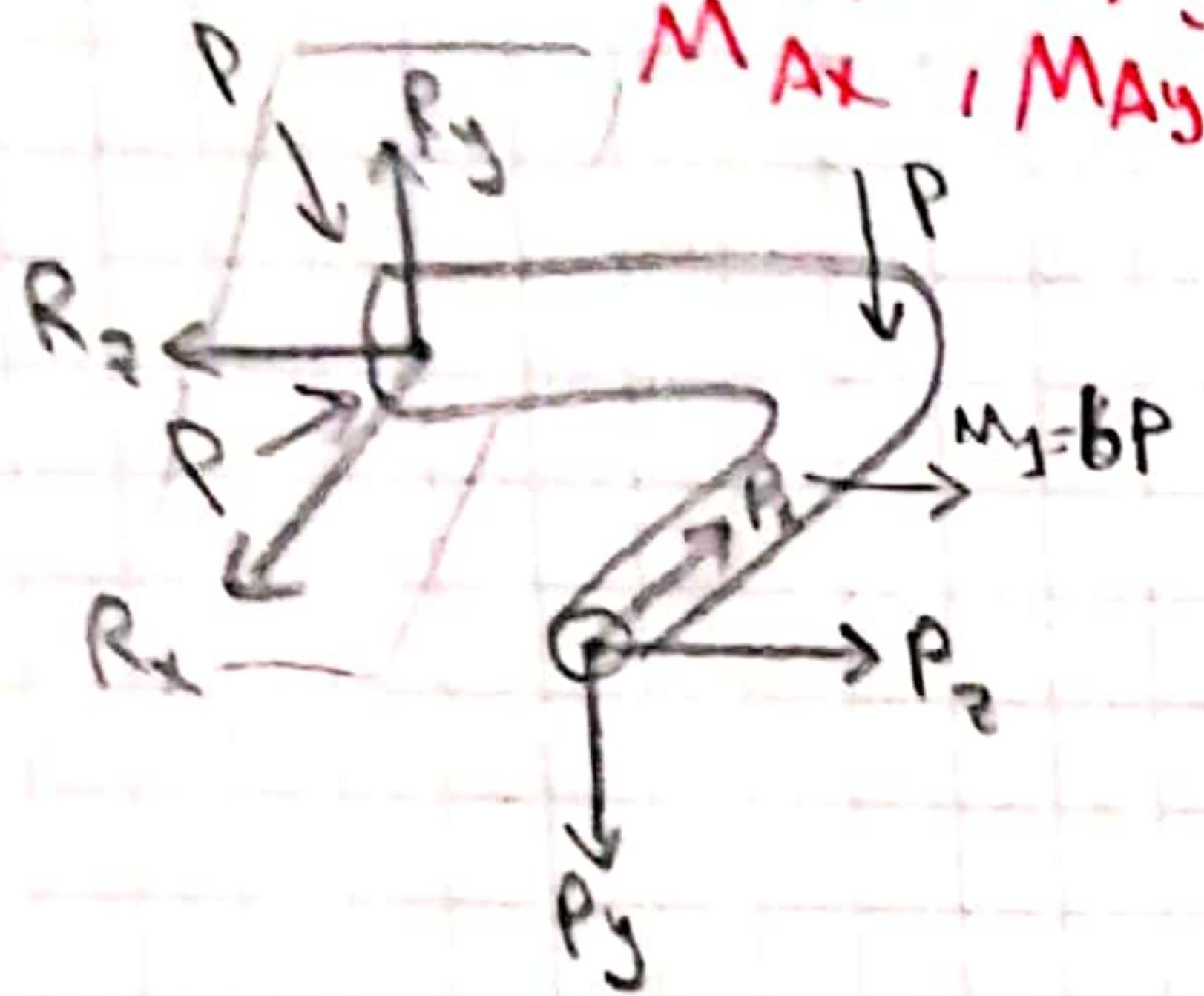
$$|AB| = a = 20 \text{ cm}$$

$$|BC| = b = 30 \text{ cm}, P = 120 \text{ N}$$

Weight of the beam is ~~negl~~ negligible

Determine the reaction at the Fixed (A) of the beam

- Determine R_{Ax} , R_{Ay} , R_{Az} and R_A
- M_{Ax} , M_{Ay} , M_{Az} and M_A



$$M_1 = bP = (0, 30)(120) = 36 \text{ Nm (+z)}$$

$$M_2 = aP = (0, 20)(120) = 24 \text{ Nm (+x)}$$

$$\sum F_x = 0 \quad R_{Ax} = 0$$

$$\sum F_y = 0 \quad R_{Ay} = P = 120 \text{ N (+y)}$$

$$\sum F_z = 0 \quad R_{Az} = 0$$

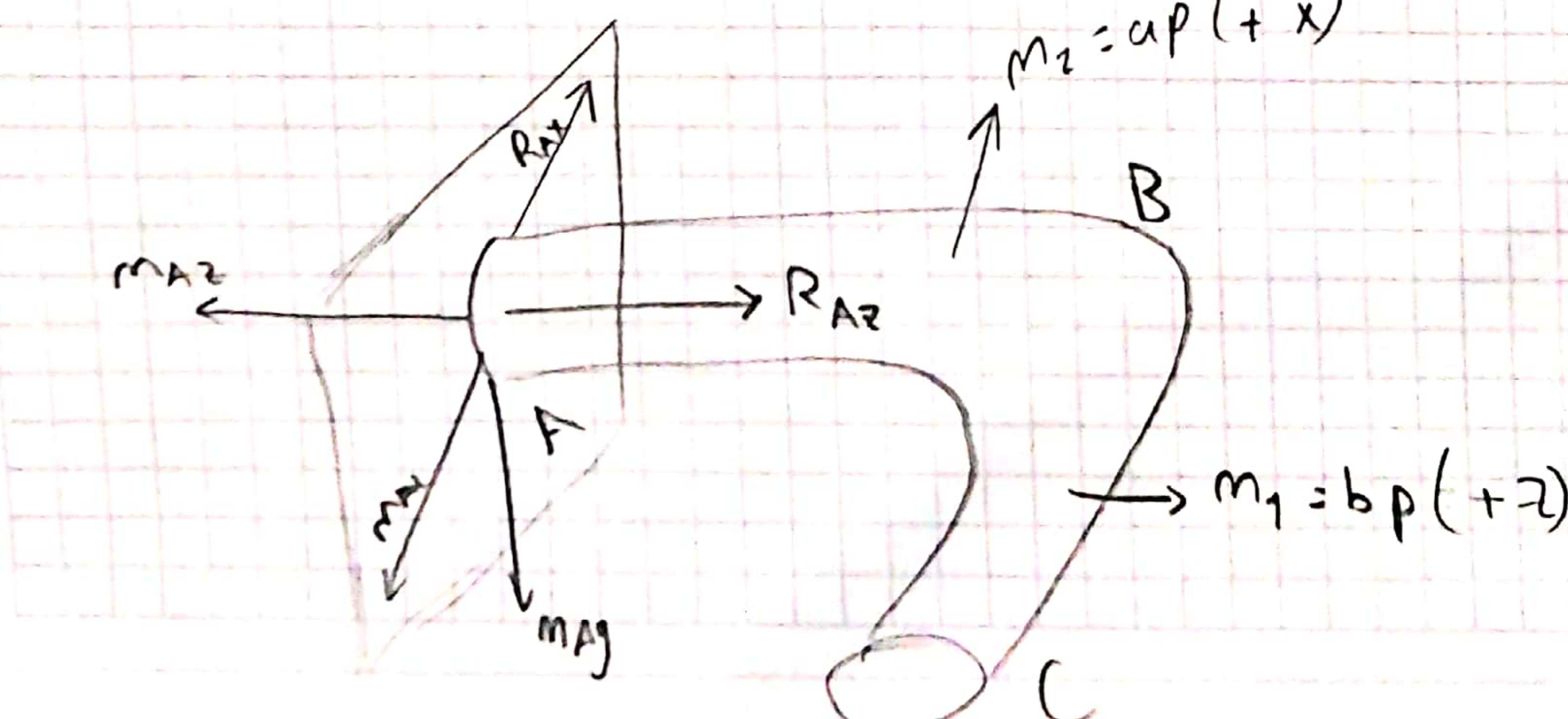
$$\sum m_x = 0 \quad M_{Ax} = M_2 = 24 \text{ Nm (-x)}$$

$$\sum m_y = 0 \quad M_{Ay} = 0$$

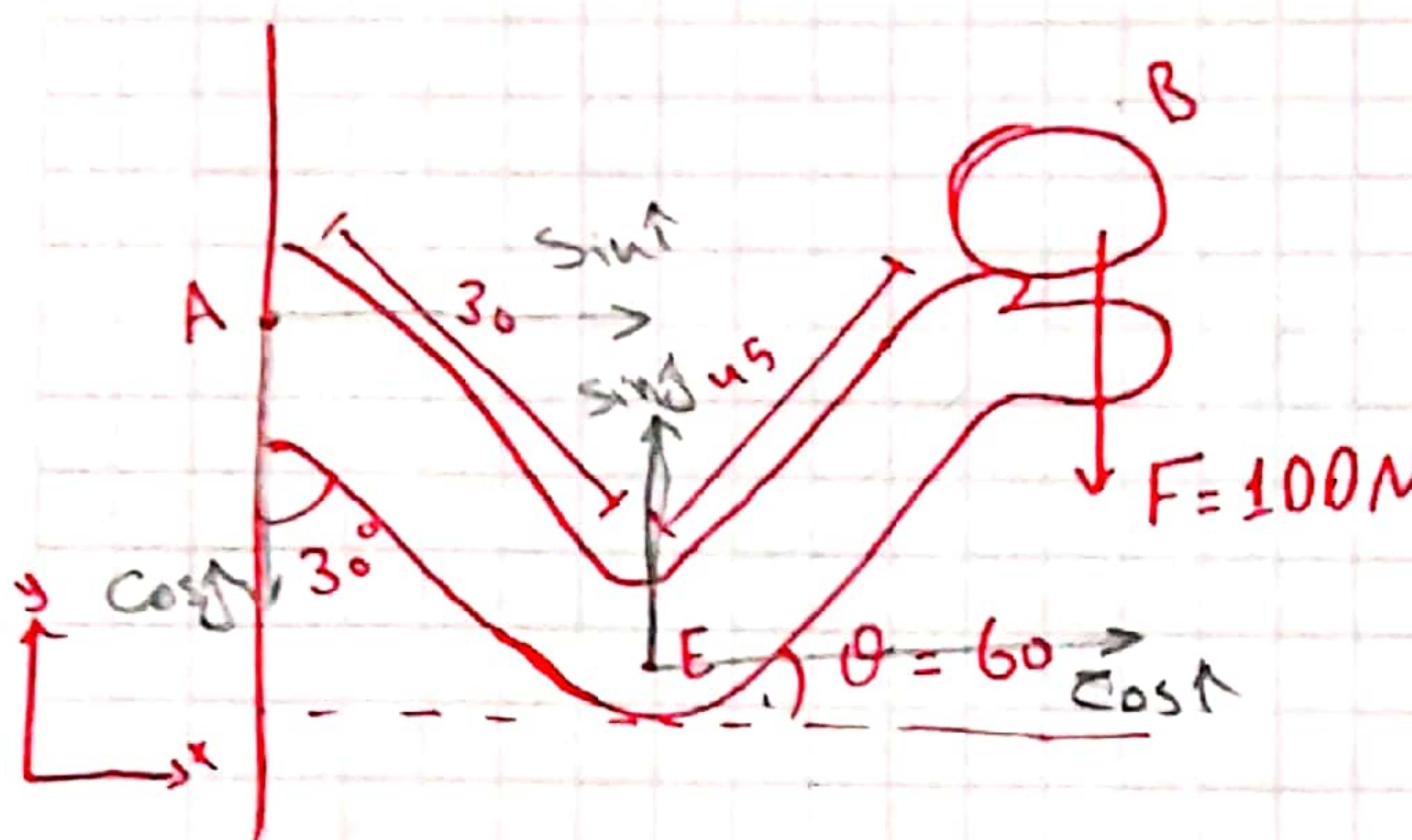
$$\sum m_z = 0 \quad M_{Az} = M_1 = 36 \text{ Nm (-z)}$$

$$R_A = \sqrt{(R_{Ax})^2 + (R_{Ay})^2 + (R_{Az})^2} = R_{Ay} = 120 \text{ N}$$

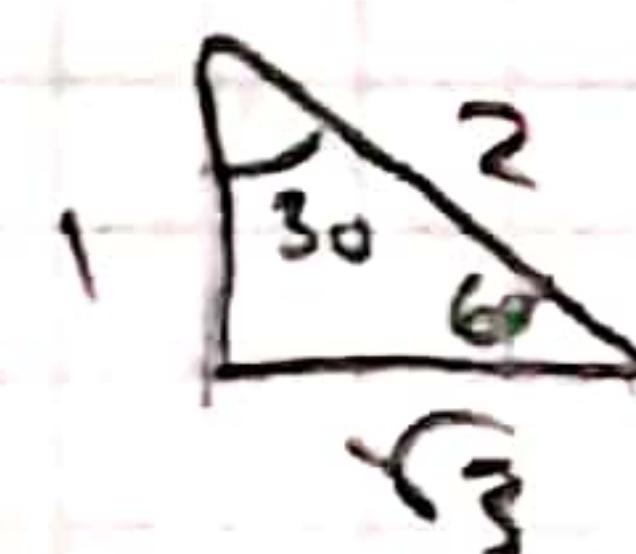
$$M_A = \sqrt{(M_{Ax})^2 + (M_{Ay})^2 + (M_{Az})^2} = 43 \text{ Nm}$$



Question



Calculate M_A and M_E
When $\theta = 60^\circ$ and 90°



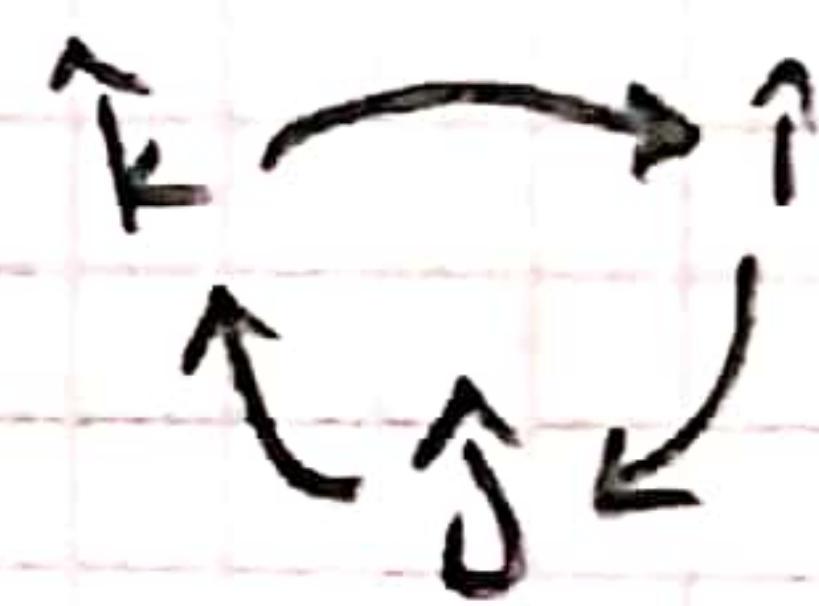
$$|AE| = 30 \sin 30\hat{i} - 30 \cos 30\hat{j}$$

$$= 15\hat{i} - 25,98\hat{j}$$

For $\theta = 60^\circ$

$$|EB| = 45 \cos 60\hat{i} + 45 \sin 60\hat{j}$$

$$= 22,5\hat{i} + 38,97\hat{j}$$



$$ME = (22,5\hat{i} + 38,97\hat{j}) \times (-100\hat{j})$$

$$= -2250\hat{k} + 0 = -2250\hat{k}$$

$$MA = (15\hat{i} - 25,98\hat{j}) \times (-100\hat{j})$$

$$= -1500\hat{k}$$

For $\theta = 90^\circ$

$$|EB| = 45 \cos 90\hat{i} + 45 \sin 90\hat{j}$$

$$= 0 + 45\hat{j}$$

$$ME = (45\hat{j}) \times (-100\hat{j}) = 0$$

$$MA = (15\hat{i} - 25,98\hat{j}) \times (-100\hat{j})$$

$$= -1500\hat{k}$$

Q.

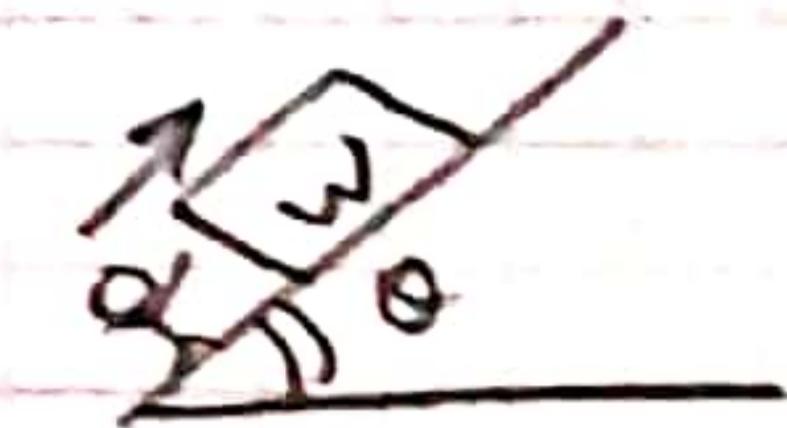
Figure

to push a block up on inclined surface by applying a force parallel to the incline

the weight of the block W

the coefficient of friction μ

the angle of incline θ



Determine the magnitude P of the minimum force the person must apply in order to overcome the friction and gravitational effect to start moving the block in terms of W, μ, θ

$$P = \mu \cdot N$$

$$f = \mu \cdot W g$$



Solution:

$$W_x = W \sin \theta$$

$$W_y = W \cos \theta$$

$$P = \mu \cdot N$$

$$\sum F_y = 0$$

$$N - W_y = 0$$

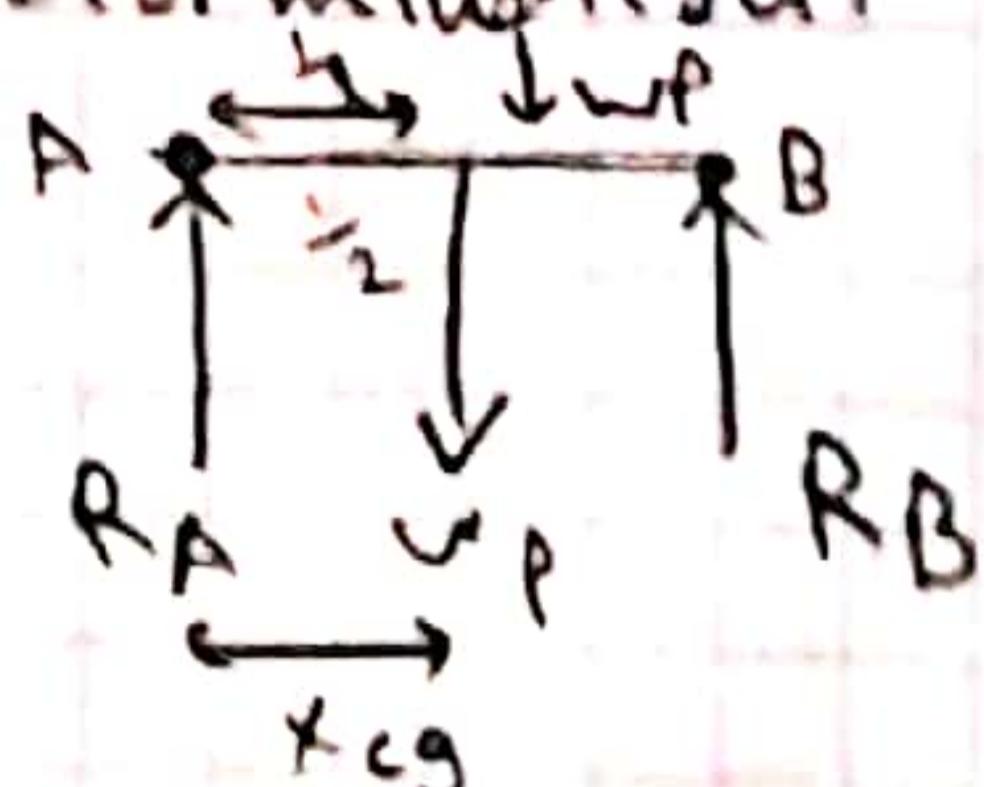
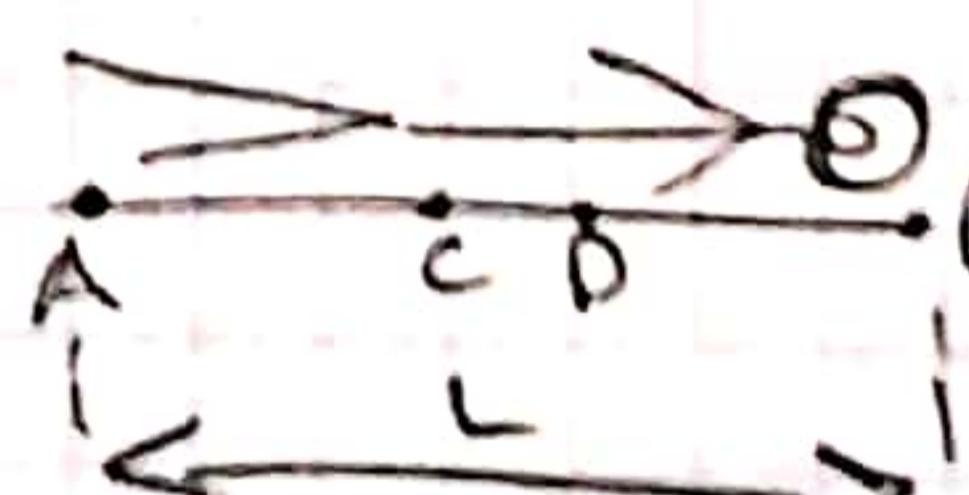
$$N - W_y = W \cos \theta$$

$$W \cdot F_x = 0, P - W \cdot F_x = 0 \Rightarrow P = W_x + f$$

$$P > \mu W \cos \theta + \mu \sin \theta$$

$$\text{moving moment minimum force} \\ \Rightarrow P = \mu W \cos \theta + \mu \sin \theta$$

Center of gravity determination



$$\sum M_A = 0 \Rightarrow \frac{1}{2} w_p + x_{cg} w_p - L R_B = 0$$

$$x_{cg} = \frac{1}{w_p} \left(R_B - \frac{w_B}{2} \right) \Rightarrow M_p = x_{cg} (w_1 + w_2 + w_3)$$

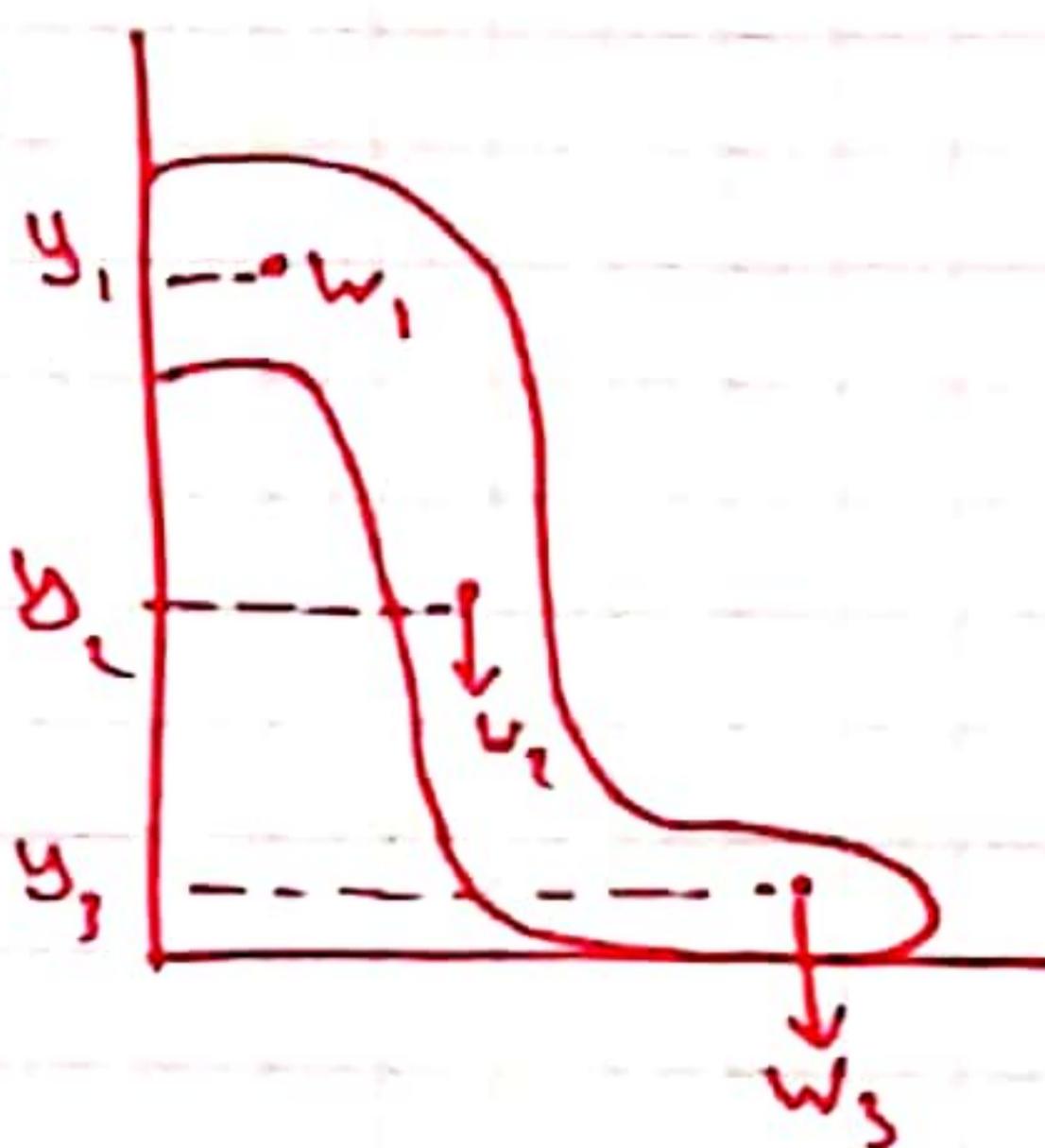
$$M_p = y_1 w_1 + y_2 w_2 + y_3 w_3$$

$$x_{cg} = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3}{w_1 + w_2 + w_3} = \frac{\sum_{i=1}^n x_i w_i}{\sum_{i=1}^n w_i}$$

$$y_{cg} = \frac{y_1 w_1 + y_2 w_2 + y_3 w_3}{w_1 + w_2 + w_3} = \frac{\sum_{i=1}^n y_i w_i}{\sum_{i=1}^n w_i}$$

Q via Q

Ex:



P	x _{cm}	y _{cm}	percentage % w
1	17,3	51,3	10,6
2	42,5	32,2	4,6
3	45	3,3	1,7

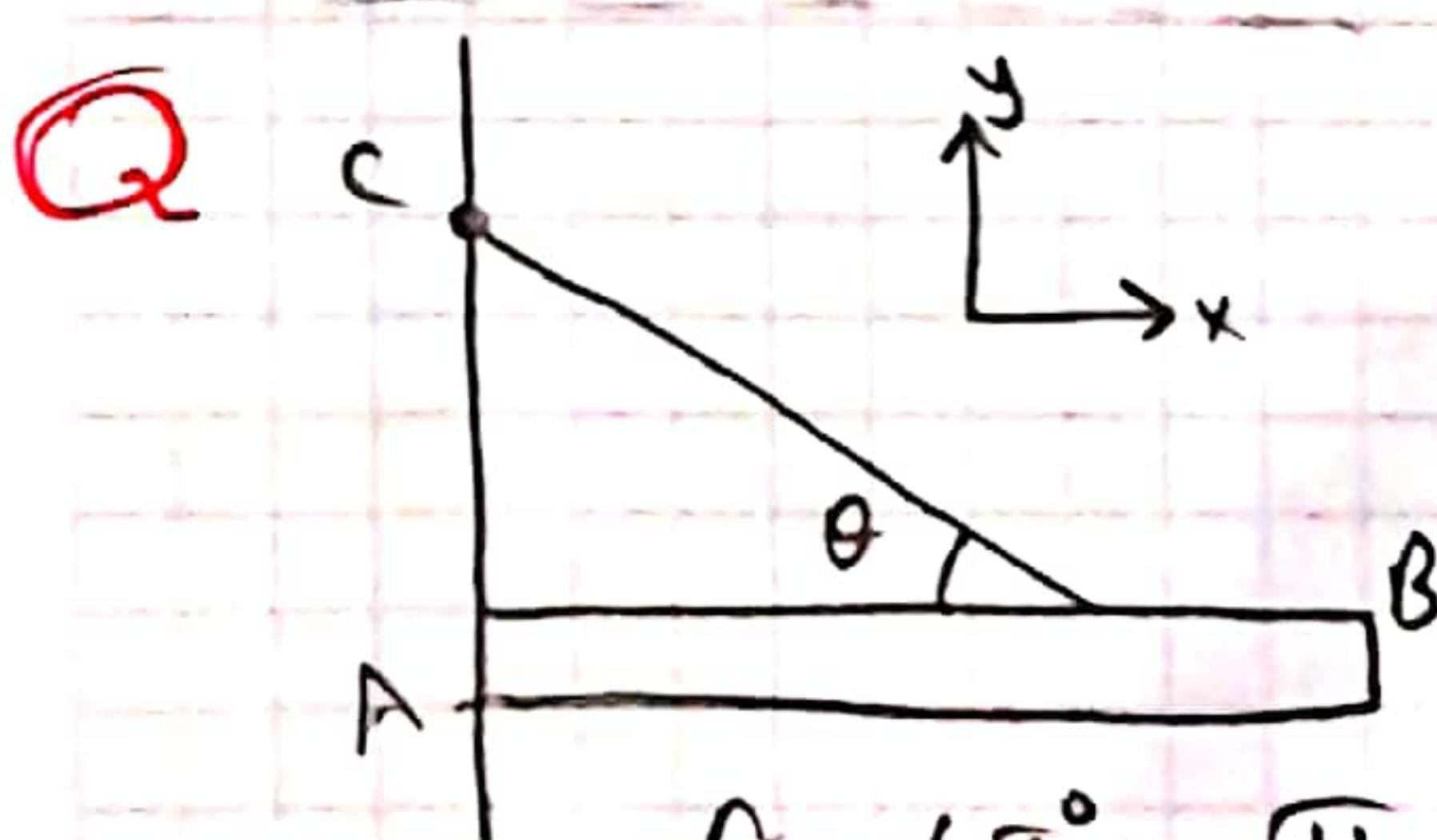
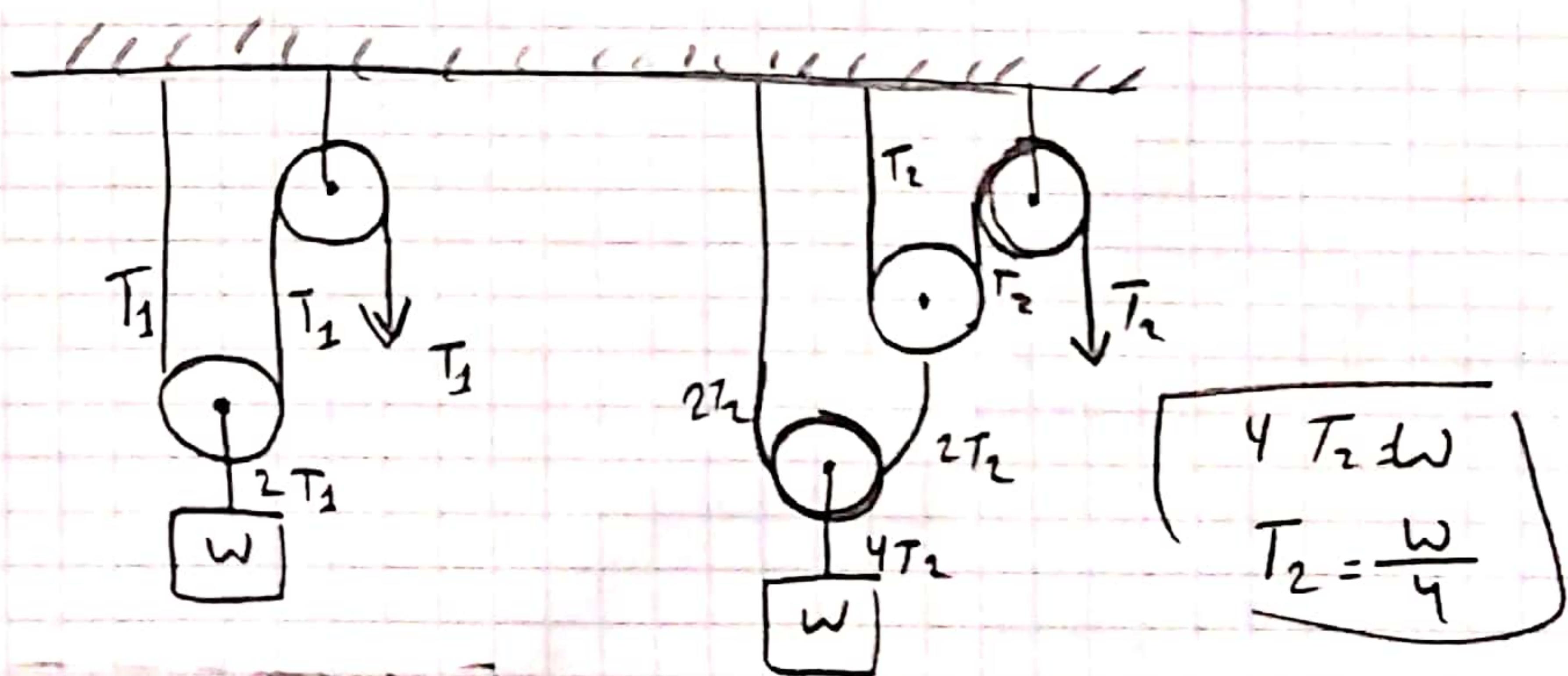
consider the leg shown in fig.

the weight of all leg as percentage of the total weight of the person provide by the table

Determine the location of the center of gravity of the center leg x_{cg} and y_{cg}

$$\text{ans. } x_{cg} = \frac{x_1 w_1 + x_2 w_2 + x_3 w_3}{w_1 + w_2 + w_3} = \frac{(17,3)(0,106) + (42,5)(0,046) + (45)(0,017)}{(0,106) + (0,046) + 0,017} = 26,9 \text{ cm}$$

$$y_{cg} = \frac{y_1 w_1 + y_2 w_2 + y_3 w_3}{w_1 + w_2 + w_3} = \frac{(51,3)(0,106) + (32,2)(0,046) + (3,3)(0,017)}{(0,106) + (0,046) + (0,017)} = 41,4 \text{ cm}$$



The uniform horizontal beam in the fig. is supported to wall at point A and by a cable at C.

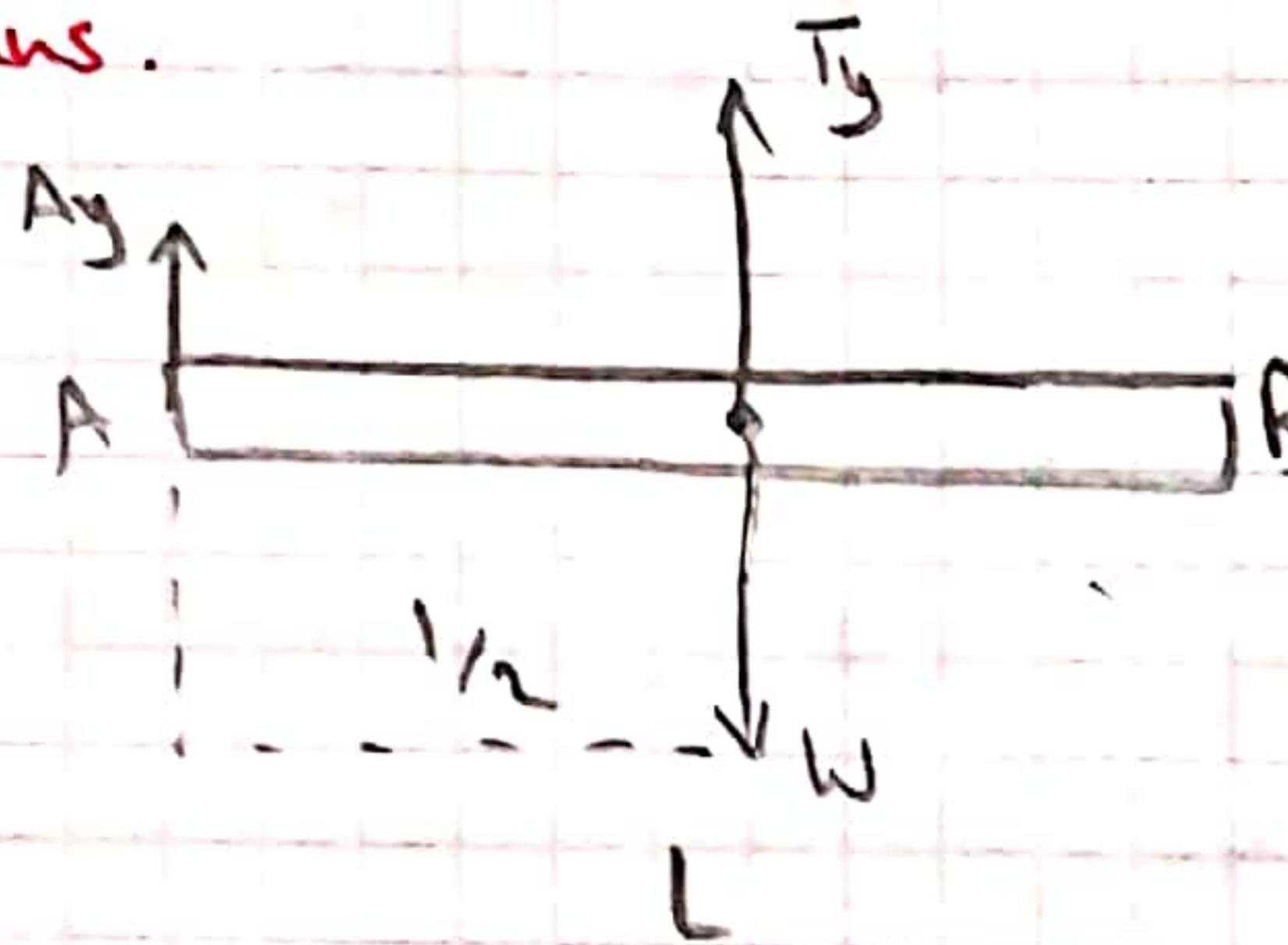
C also represents the center of gravity

$\theta = 68^\circ$, The weight is 400N $L = 4\text{m}$

Calculate the beam T in the cable and components of reaction force on the beam at A (R_{Ax}, R_{Ay})

ans \rightarrow

ans.



$$Ty = T \sin \theta$$

$$Tx = T \cos \theta$$

$$T = 431,41 \text{ N}$$

$$Tx = T \cos \theta = 61,60 \text{ N}$$

$$RAy + 400 = 400 \Rightarrow RAy = 0$$

$$RAx - T \cos 68 = 0, \sum F_x = 0$$

$$RAx = 161,60 \text{ N}$$

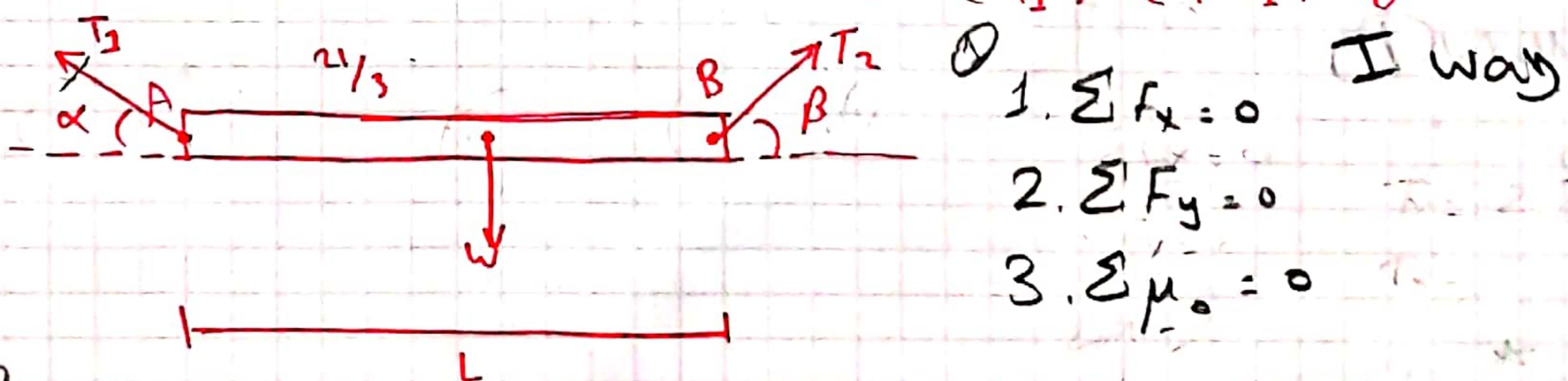
mid-term Q

Quiz: Consider the split russel traction ^{derived} ~~derived~~ the Fig.

The leg is held in the position shown by two weights that are centered to the leg two cables.

- The combined weight of the leg $W = 300 \text{ N}$
- The distance between A and B
- The Angle β is 45°

Determine the tension T_1 and T_2 in the cables, weights W, gW , and angle α of the cables? (T_1, T_2, W_1, W_2)



I way
1. $\sum F_x = 0$

2. $\sum F_y = 0$

3. $\sum \mu_0 = 0$

$$1. -T_1 x + T_2 x = 0 \Rightarrow -T_1 (\cos \alpha) + T_2 \cos 45^\circ = 0 \Rightarrow T_2 = \frac{T_1 \cos \alpha}{\cos 45}$$

$$2. T_2 y + T_1 y - W = 0 \Rightarrow T_2 \sin 45^\circ + T_1 \sin \alpha - 300 \text{ N} = 0$$

$$\Rightarrow T_2 \sin 45^\circ + T_1 \sin \alpha = 300 \text{ N}$$

$$3. T_2 \sin 45^\circ \left(\frac{1}{3} L \right) - T_1 \sin \alpha \left(\frac{2}{3} L \right) = 0 \Rightarrow T_2 = \frac{2 T_1 \sin \alpha}{\sin 45^\circ} = \frac{T_1 \cos \alpha}{\cos 45}$$

$$\frac{2 T_1 \sin \alpha}{\sin 45^\circ} \sin 45^\circ + T_1 \sin \alpha = 300 \text{ N}$$

$$\Rightarrow T_1 = \frac{300}{3 \sin \alpha} = W_1$$

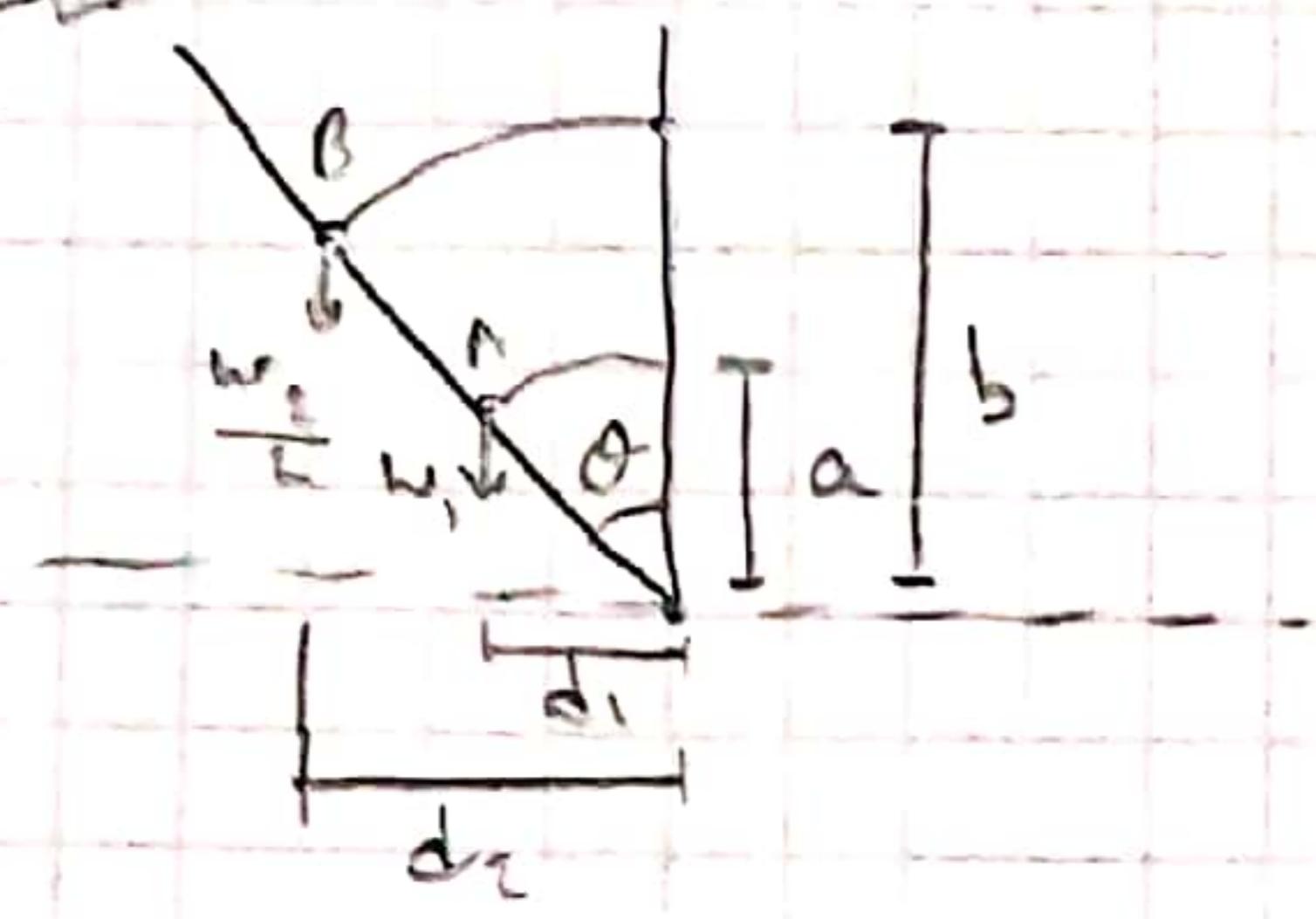
$$2 \frac{\sin \alpha}{\cos \alpha} = \frac{\sin 45^\circ}{\cos 45^\circ} \Rightarrow 2 \tan \alpha = \tan 45^\circ$$

$$\alpha = \arctan \left(\frac{1}{2} + \tan 45^\circ \right)$$

$$\alpha = 26,57^\circ$$

Exam Questions

Q1



$$a = 25 \text{ cm}$$

$$b = 75 \text{ cm}$$

$$w_1 = 100 \text{ N}$$

$$w_2 = 500 \text{ N}$$

calculate the reaction force
for $\theta = 30^\circ, 45^\circ \text{ and } 60^\circ$

①

$$\text{ans. } m_1 = d_1 w_1 = a w_1 \sin \theta = (0,25)(100) \sin \theta$$

$$= 25 \sin \theta$$

$\sin \theta \rightarrow$

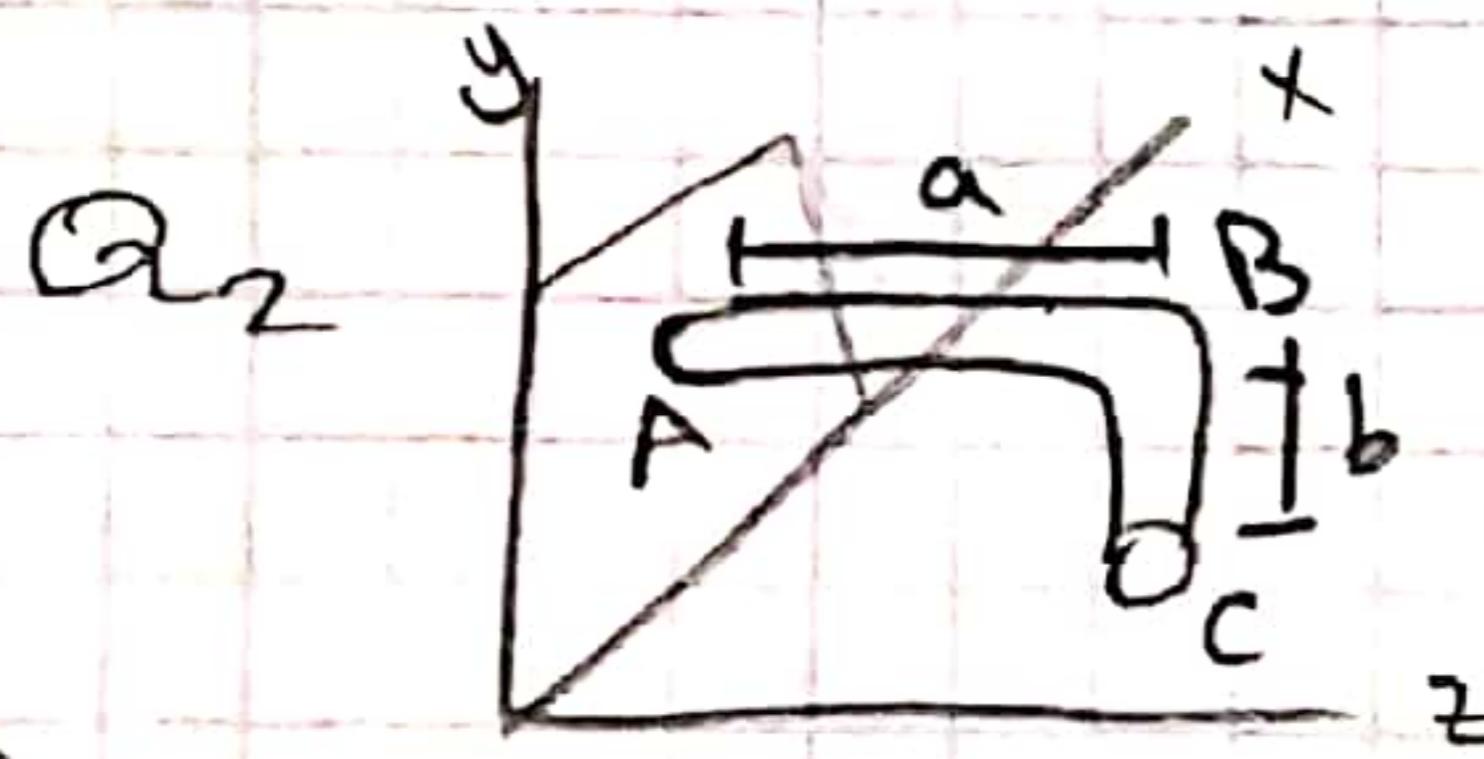
$$30^\circ \quad 0,5 \quad 106,25 \quad m_2 = d_2 (w_2/2) = b (w_2/2) \sin \theta = (0,75)(250)$$

$$45^\circ \quad 0,707 \quad 150,23$$

$$= 187,5 \sin \theta$$

$$60^\circ \quad 0,86 \quad 184,03 \quad m_0 = m_1 + m_2 = 25 \sin \theta + 187,5 \sin \theta = 212,5 \sin \theta$$

Q2



$$a = 0,3 \text{ m} = 30 \text{ cm}$$

$$b = 0,1 \text{ m} = 10 \text{ cm}$$

$$P = 150 \text{ N}$$

Determine the reactions at the final end of the beam

$$m_C = r \times P = (0,4\hat{i} + 0,3\hat{k}) \times (150\hat{j})$$

$$= (0,4)(-150)(\hat{i} \times \hat{j}) + (0,3)(-150)(\hat{k} \times \hat{j})$$

$$m_C = 60\hat{k} + 45\hat{j}, \quad \sum m = 0, \quad m_A + m_C = 0$$

$$(m_{Ax}\hat{i} + m_{Ay}\hat{j} + m_{Az}\hat{k}) + (60\hat{k} + 45\hat{j}) = 0$$

$$(m_{Ax} + 45)\hat{j} + m_{Ay}\hat{j} + (m_{Az} + 60)\hat{k} = 0$$

$$m_{Ax} = 45 \text{ Nm}(-\hat{j}), \quad m_{Ay} = 0, \quad m_{Az} = 60 \text{ Nm}(-\hat{k})$$

$$P = -P\hat{j} = -150\hat{j}$$

$$r = -b\hat{i} + a\hat{k} = -0,4\hat{i} + 0,3\hat{k}$$

$$R_A = R_{Ax}\hat{i} + R_{Ay}\hat{j} + R_{Az}\hat{k}$$

$$m_A = m_{Ax}\hat{i} + m_{Ay}\hat{j} + m_{Az}\hat{k}$$

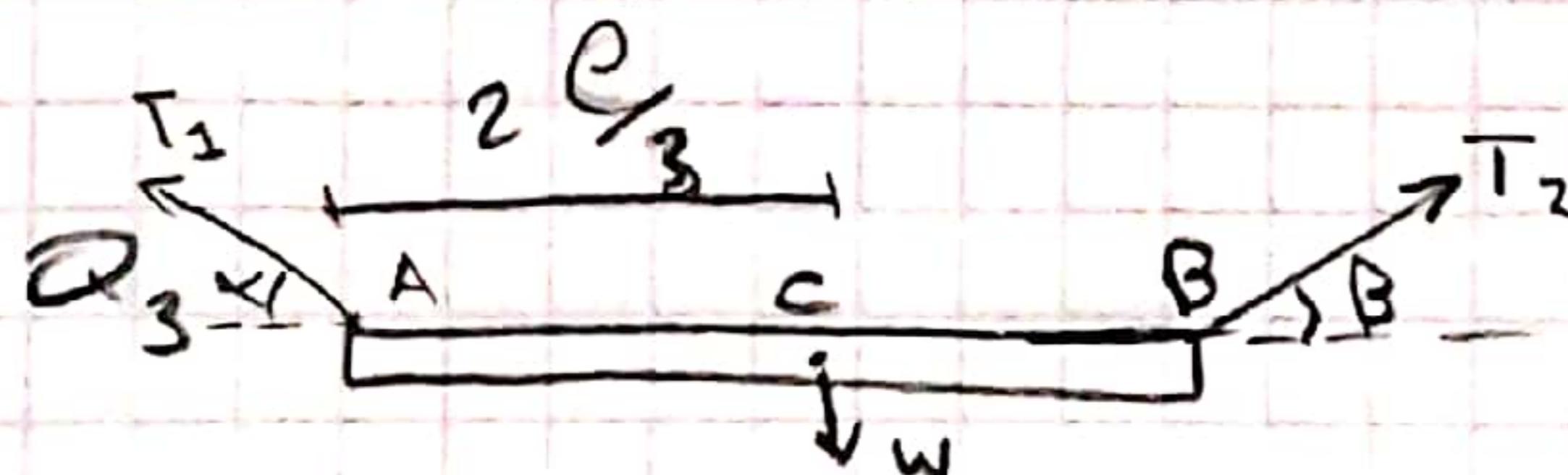
$$\sum F = 0, \quad R_A + P = 0$$

$$(R_{Ax}\hat{i} + R_{Ay}\hat{j} + R_{Az}\hat{k}) + (-150\hat{j}) = 0$$

$$R_{Ax}\hat{i} + (R_{Ay} - 150)\hat{j} + R_{Az}\hat{k} = 0$$

$$R_{Ax} = 0, \quad R_{Ay} = 150 \text{ N} (+\hat{j})$$

$$R_{Az} = 0$$



$$w = 500 \text{ N}$$

$$\beta = 30^\circ$$

Determine the tension T_1 and T_2 , weight w_1 and w_2 , and angle α

②

$$\text{ans. } \sum m_B = 0$$

$$T_1 \sin \alpha \cdot l - w \cdot \frac{l}{3} = 0$$

$$\Rightarrow T_1 \sin \alpha = 166,66 \text{ N}$$

$$\sum F_x = 0$$

$$T_1 \cos \alpha = T_2 \cos 30^\circ$$

$$T_1 \cos \alpha = 577,34$$

①

$$\sum m_A = 0$$

$$T_2 \sin \beta \cdot l - w \cdot \frac{2l}{3} = 0$$

$$T_2 \sin \beta = 333,33 \Rightarrow T_2 = 666,66 \text{ N}$$

$$T_1 = \frac{166,66 \text{ N}}{\sin \alpha} = \frac{577,34}{\cos \alpha}$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{166,66}{577,34} = 0,28$$

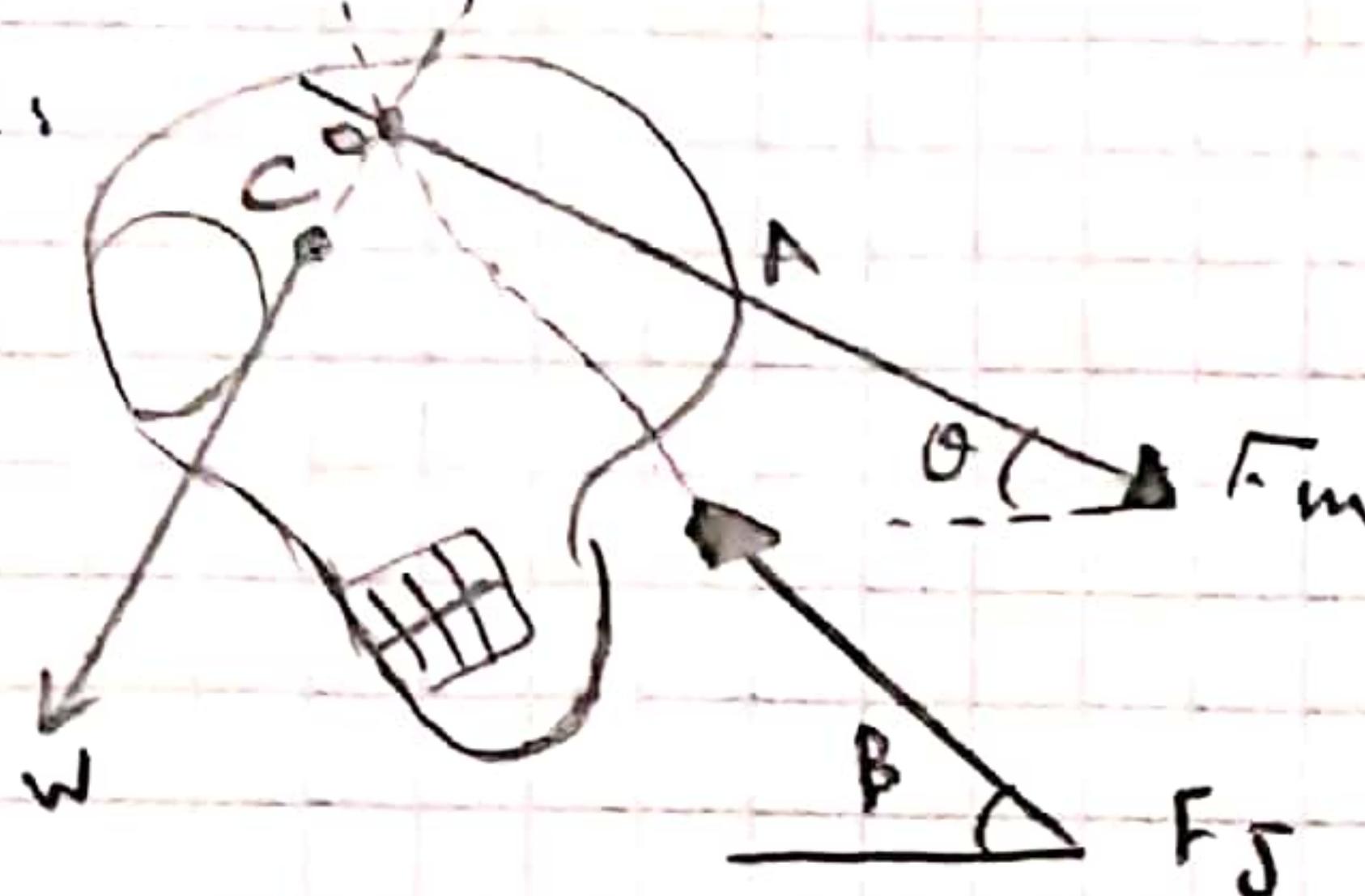
$$\alpha = 66,10^\circ$$

③

Mechanics of spinal column

The principal function of the spinal column are to protect the spinal cord to support the head, neck and upper extremities to transfer loads from the head and trunk to the pelvis and to permit a variety of movement

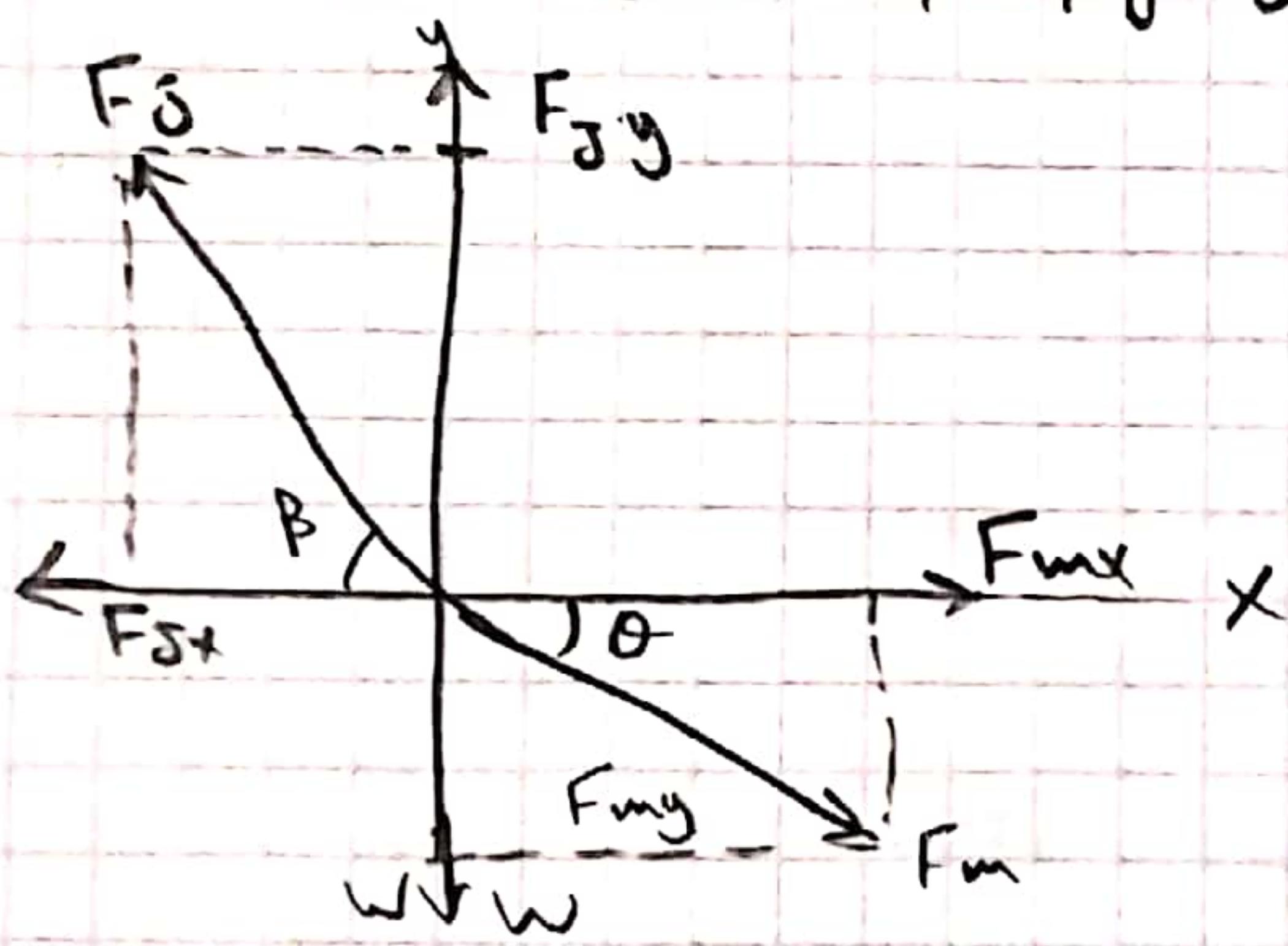
Ex:



Consider the position of the head and the neck, shown in the Figure.

The head weights 50 N . The center of gravity is located at C . F_m is the magnitude of the resultant force exerted by the neck extensor muscles which is applied on the skull at A .

The atlantooccipital joint center is at B . The line of action of the neck muscle force makes an angle $\theta = 30^\circ$ and the line of action of the joint reaction force makes an angle $\beta = 60^\circ$ with horizontal. F_J and $F_m = ?$



- ans. (1) $F_{mx} = F_m \cos\theta$
 (2) $F_{my} = F_m \sin\theta$
 (3) $F_{Jx} = F_J \cos\beta$
 (4) $F_{Jy} = F_J \sin\beta$
 (5) $\sum F_x = 0, F_{Jx} = F_{mx}$
 (6) $\sum F_y = 0, F_{Jy} = F_{my}$

(I) and (II)

$$F_{mx} = (50)(\cos 30) = 43\text{ N} (+x)$$

$$F_{my} = (50)(\sin 30) = 25\text{ N} (-y)$$

(V) and (Vi)

$$F_{Jx} = 43\text{ N} (-x)$$

$$F_{Jy} = (50 + 25)\text{ N} = 75\text{ N} (+y)$$

$$(III) F_J = \frac{F_{Jx}}{\cos\beta} = \frac{43}{\cos 60} = 86\text{ N}$$

$$(7) F_J \cos\beta = F_m \cos\theta \Rightarrow 1 \text{ and } 3 = 15$$

$$(8) F_J \sin\beta = F_m \sin\theta \Rightarrow 2 \text{ and } 4 = 6$$

$$(9) \tan\beta = \frac{W + F_m \sin\theta}{F_m \cos\theta} \quad \left(\frac{8}{7} \right)$$

$$F_m \cos\theta + F_m \sin\theta = W + F_m \sin\theta$$

$$F_m (\cos\theta + \tan\beta - \sin\theta) = W$$

Q1 Consider the weight lifter illustrated in the figure

who is bent forward and lifting a weight W_0 .

The forces acting on the lower portion of the athlete's body are shown in figure below by considering a section passing through the Fifth Lumbar* Vertebral*

w is the total weight

w_1 is the weight of the legs including

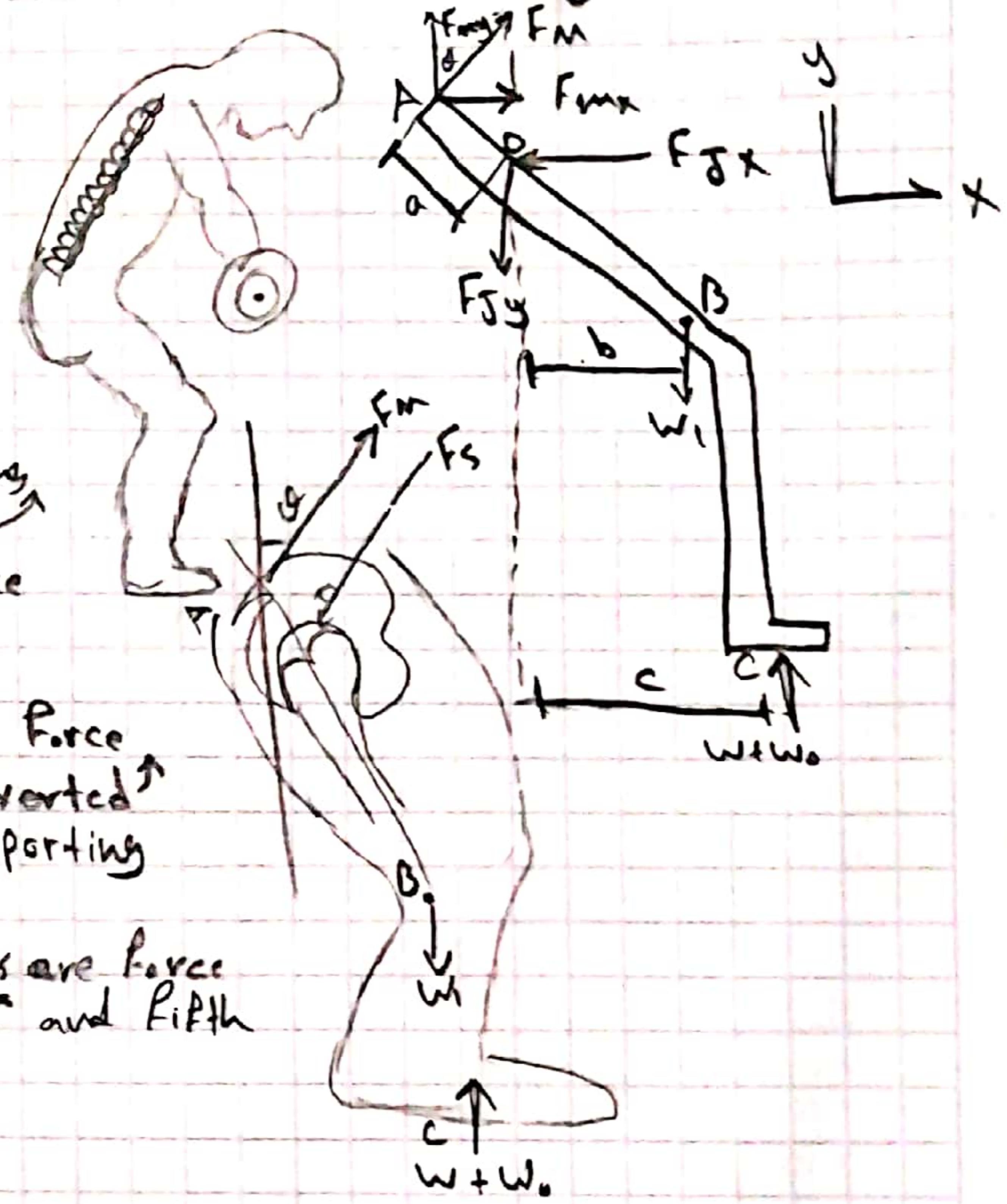
pelvis

$w + w_0$ is the total ground reaction force applied through the feet at C .

F_m is the magnitude of the resultant force exerted

by the erector* spinal muscles supporting the trunk

F_j is the magnitude of the compressive force generated at the area(a) of the Iaem* and Fifth Lumbar Vertebral



ans.

$$\sum M_o = 0; aF_m + bw_1 - c(w + w_0) = 0$$

$$F_m = \frac{c(w + w_0) - bw_1}{a}, \quad F_{mx} = F_m \sin \theta, \quad F_{my} = F_m \cos \theta$$

$$\sum F_x = 0, \quad F_{jx} = F_{mx}$$

$$\sum F_y = 0, \quad F_{jy} = F_{my} + w + w_0 - w_1$$

$$F_m = \frac{(0,12h)(w + w_0) - (0,08h)(0,4w)}{0,02h}$$

$$\text{when } \theta = 45^\circ, a = 0,02h, b = 0,08h, c = 0,12h$$

$$w_0 = w \text{ and } w_1 = 0,4w$$

$$F_{mx} = (10,4w) \sin 45^\circ = 7,4w$$

$$F_{my} = (10,4w) \cos 45^\circ = 7,4w$$

$$F_{jx} = 7,4 = \sqrt{F_{mx}^2 + F_{jy}^2} = 11,7w$$

$$F_{jy} = 7,4w + w_1w - 0,4w = 9,0w$$

$$F_j = \sqrt{(F_{jx})^2 + (F_{jy})^2} = 11,7w$$

Mechanics of the hip:

The construction of the hip joint is such that, it is very stable and has a great deal of mobility, thereby allowing a wide range of action required for action such as walking, sitting and squatting. The movement of the femur of the hip joint include flexion and extension, abducting and adducting and inward and outward rotation.

Example:

During walking and running we momentarily put all of our body weight on one leg.

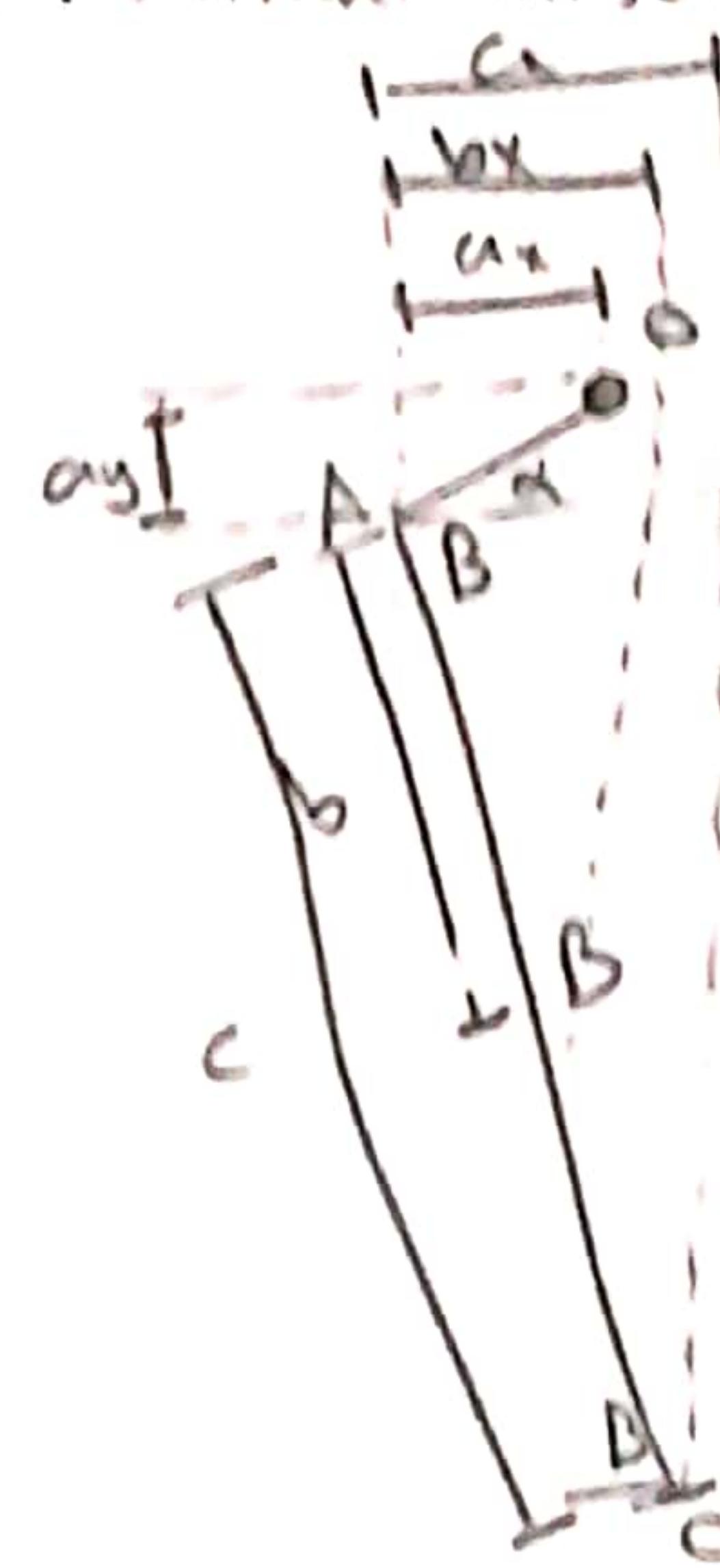
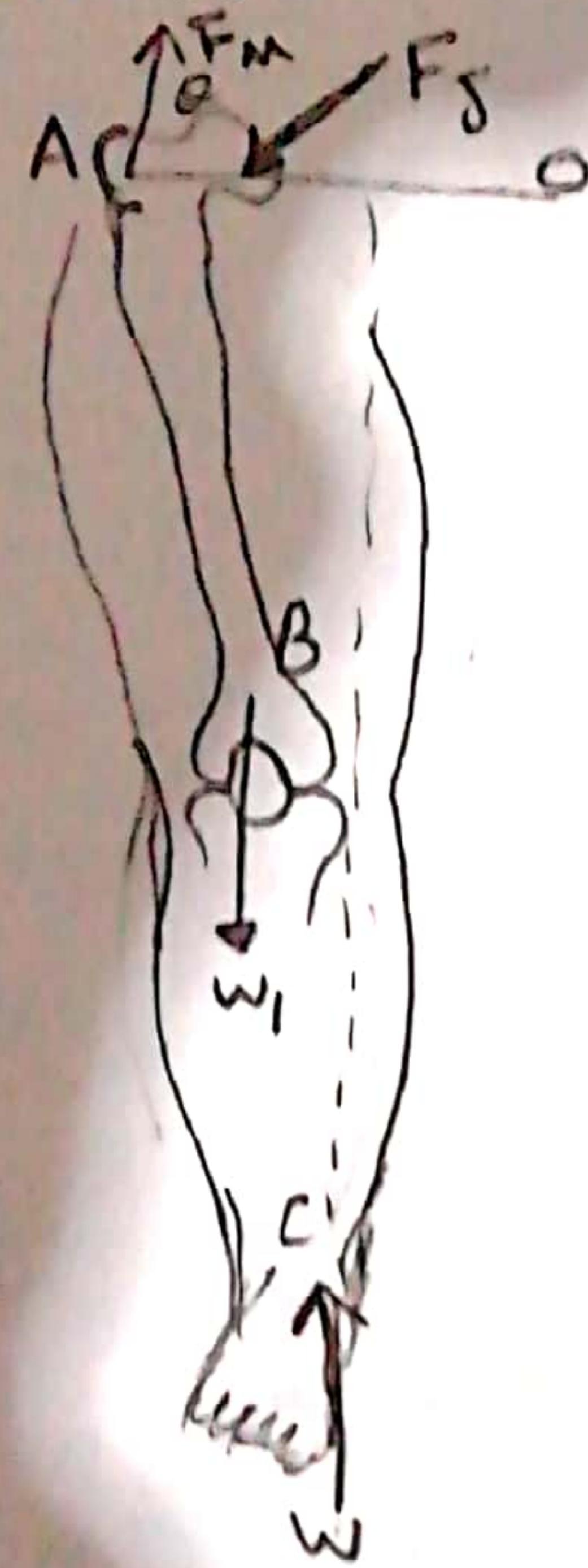
F_m is the magnitude of the resulted force exerted by the hip abductor.
 F_J is the magnitude of the joint reaction force applied by the muscle.

Pelvis is on the femur

w_1 is the weight of the leg

w is the total weight of the body applied as a normal force by the ground on the leg.

The angle between the line of action of the resultant muscle force and the horizontal is designated by α .



$$\sum m_o = 0 ; a_x F_{my} - a_y F_{mx} - (c_x - a_x)w + (b_x - a_x)w_1 = 0$$

$$\begin{aligned} F_{mx} &= F_m \cos \alpha \\ F_{my} &= F_m \sin \alpha \\ a_x &= a \cos \alpha \\ a_y &= a \sin \alpha \\ b_x &= b \cos \beta \\ b_y &= b \sin \beta \end{aligned} \quad \begin{aligned} (a \cos \alpha)(F_m \sin \alpha) - (a \sin \alpha)(F_m \cos \alpha) \\ - (c \cos \beta - a \cos \alpha)w + (b \cos \beta - a \cos \alpha)w_1 = 0 \\ \Rightarrow F_m = \frac{(c w - b w_1)}{a (\cos \alpha \sin \beta - \sin \alpha \cos \beta)} \end{aligned}$$

$$\Rightarrow \sum F_x = 0 ; F_{Jx} = F_{mx} = F_m \cos \alpha$$

$$\sum F_y = 0 ; F_{Jy} = F_{my} + w - w_1$$

$$F_{Jy} = F_m \sin \alpha + w - w_1$$

$$\Rightarrow F_J = \sqrt{(F_{Jx})^2 + (F_{Jy})^2}$$

The knee is the largest joint in the body. It is a modified hinge joint. In addition to flexion and extension action of the leg in the sagittal plane, the knee joint is designed to sustain large loads. It is an essential component of the linkage system responsible for human locomotion.

Example

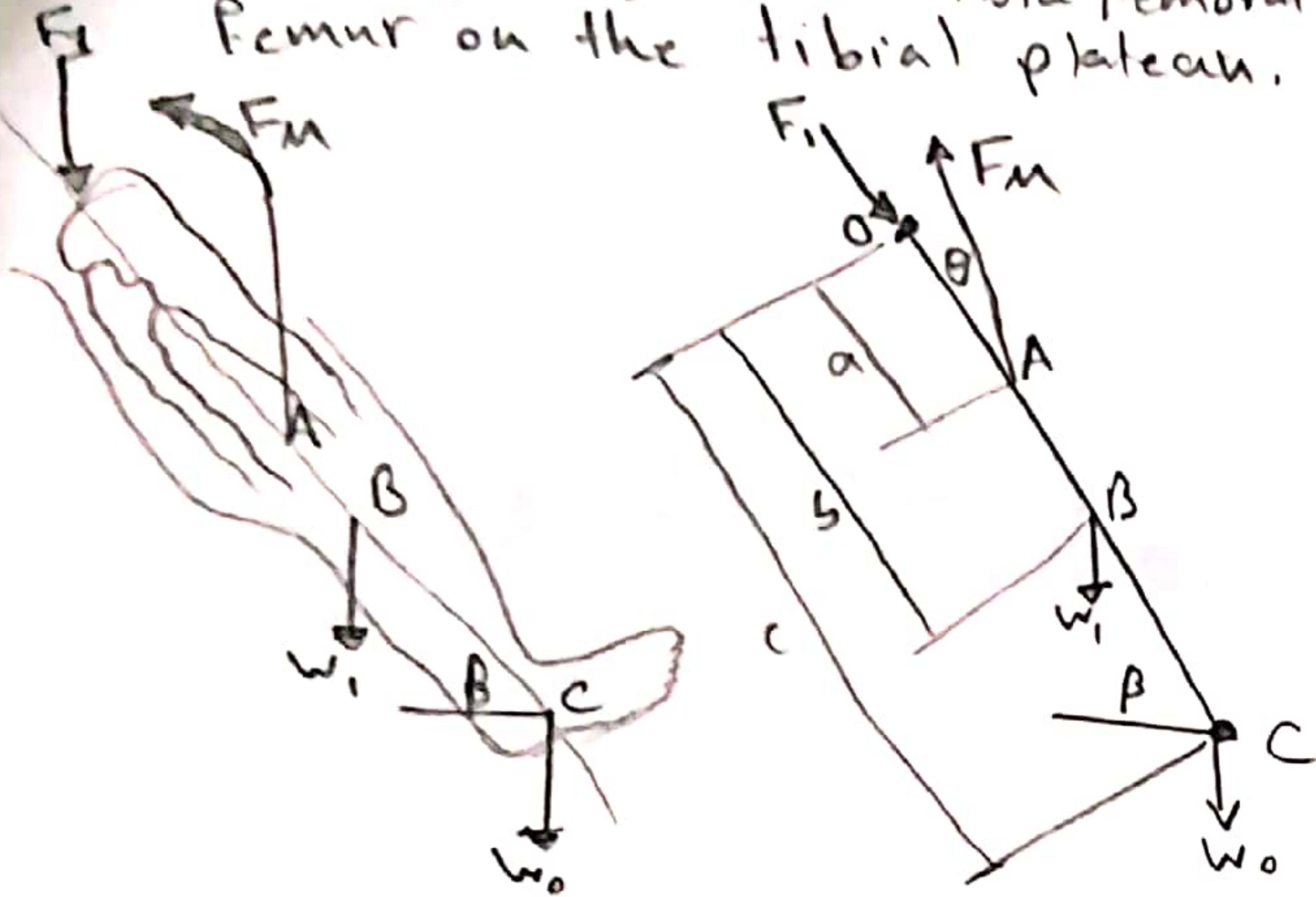
Consider a person wearing a weight boot, and form a sitting position, doing lower leg flexion/extension exercises to strengthen the quadriceps muscle.

w_1 is the weight of the lower leg

w_0 is the weight of the boot

F_m is the magnitude of the tensile force exerted by the quadriceps muscle on the tibia through the patellar tendon.

F_t is the magnitude of the tibia-femoral joint reaction force applied by the femur on the tibial plateau.



$$F_{mx} = F_m \cos(\theta + \beta)$$

$$F_{my} = F_m \sin(\theta + \beta)$$

$$\sum M_o = 0; (a \cos \beta) F_{my} - (a \sin \beta) F_{mx} - (b \cos \beta) w_1 - (c \cos \beta) w_0 = 0$$

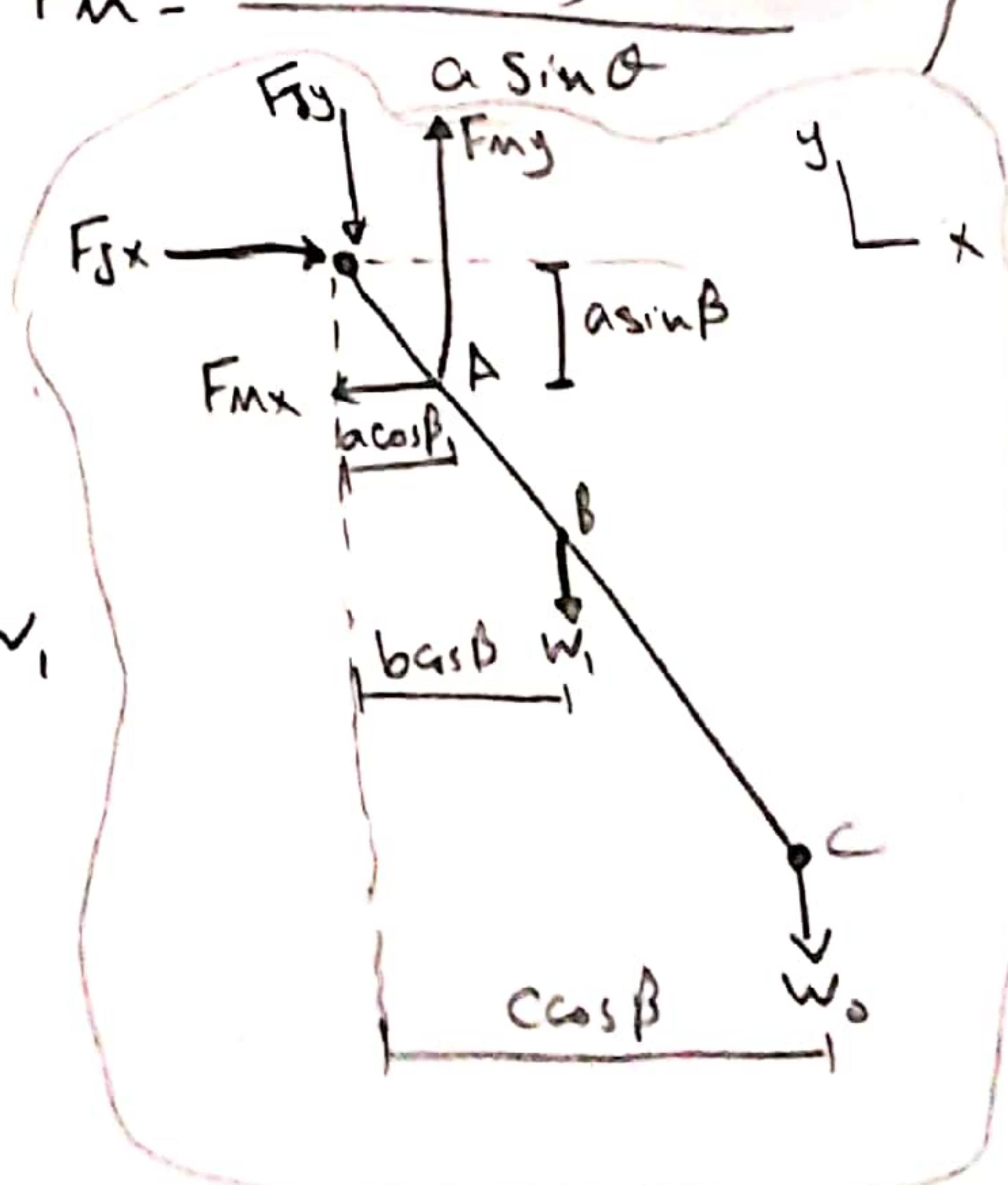
$$F_m = \frac{(b w_1 + c w_0) \cos \beta}{a [\cos \beta \sin(\theta + \beta) - \sin \beta \cos(\theta + \beta)]} \Rightarrow F_m = \frac{(b w_1 + c w_0) \cos \beta}{a \sin \theta}$$

$$\sum F_x = 0; F_{\bar{x}x} = F_{mx} = F_m \cos(\theta + \beta)$$

$$\sum F_y = 0; F_{\bar{y}y} = F_{my} - w_0 - w_1$$

$$F_{\bar{y}y} = F_m \sin(\theta + \beta) - w_0 - w_1$$

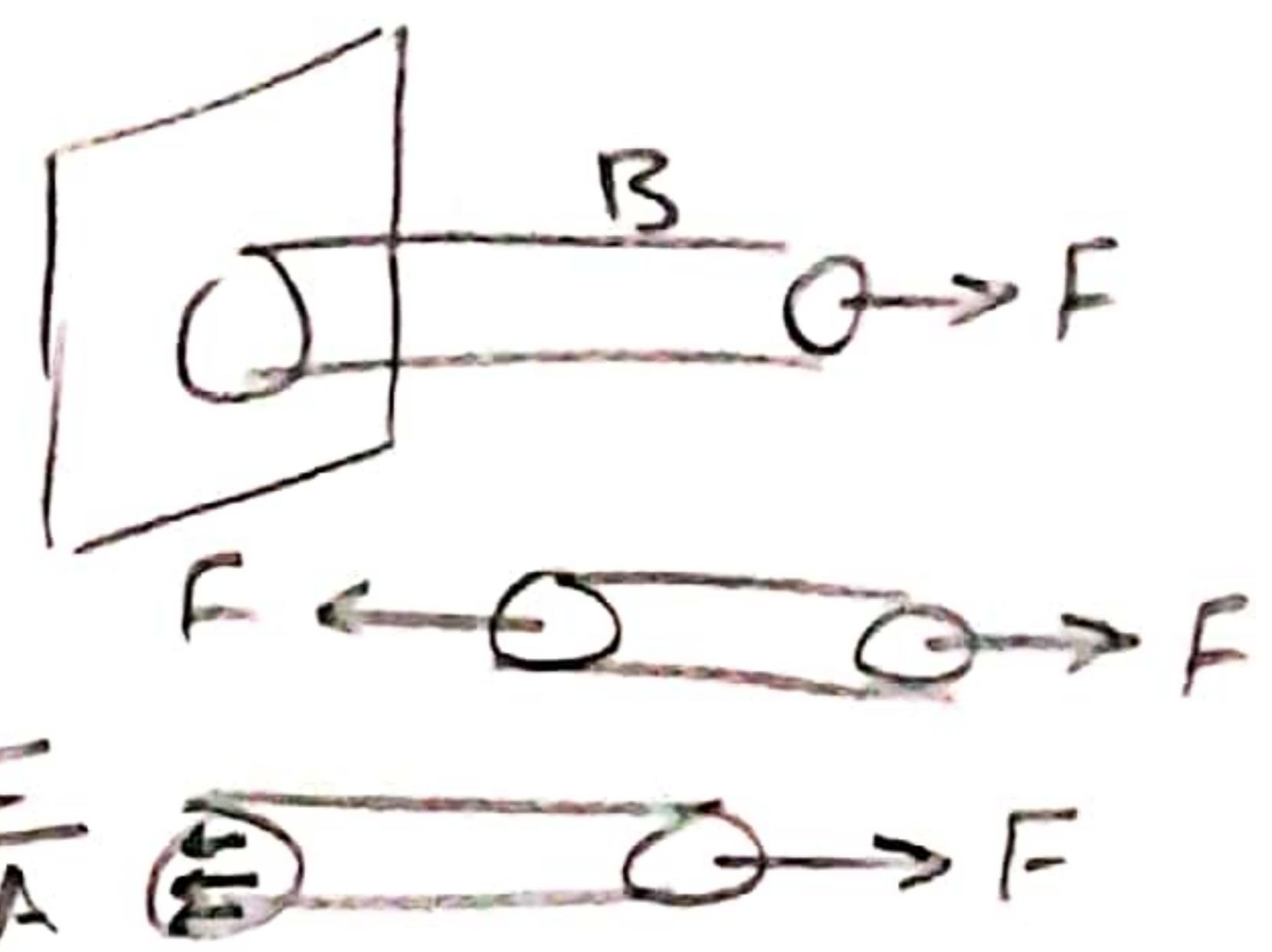
$$F_{\bar{y}} = \sqrt{(F_{\bar{x}x})^2 + (F_{\bar{y}y})^2}$$



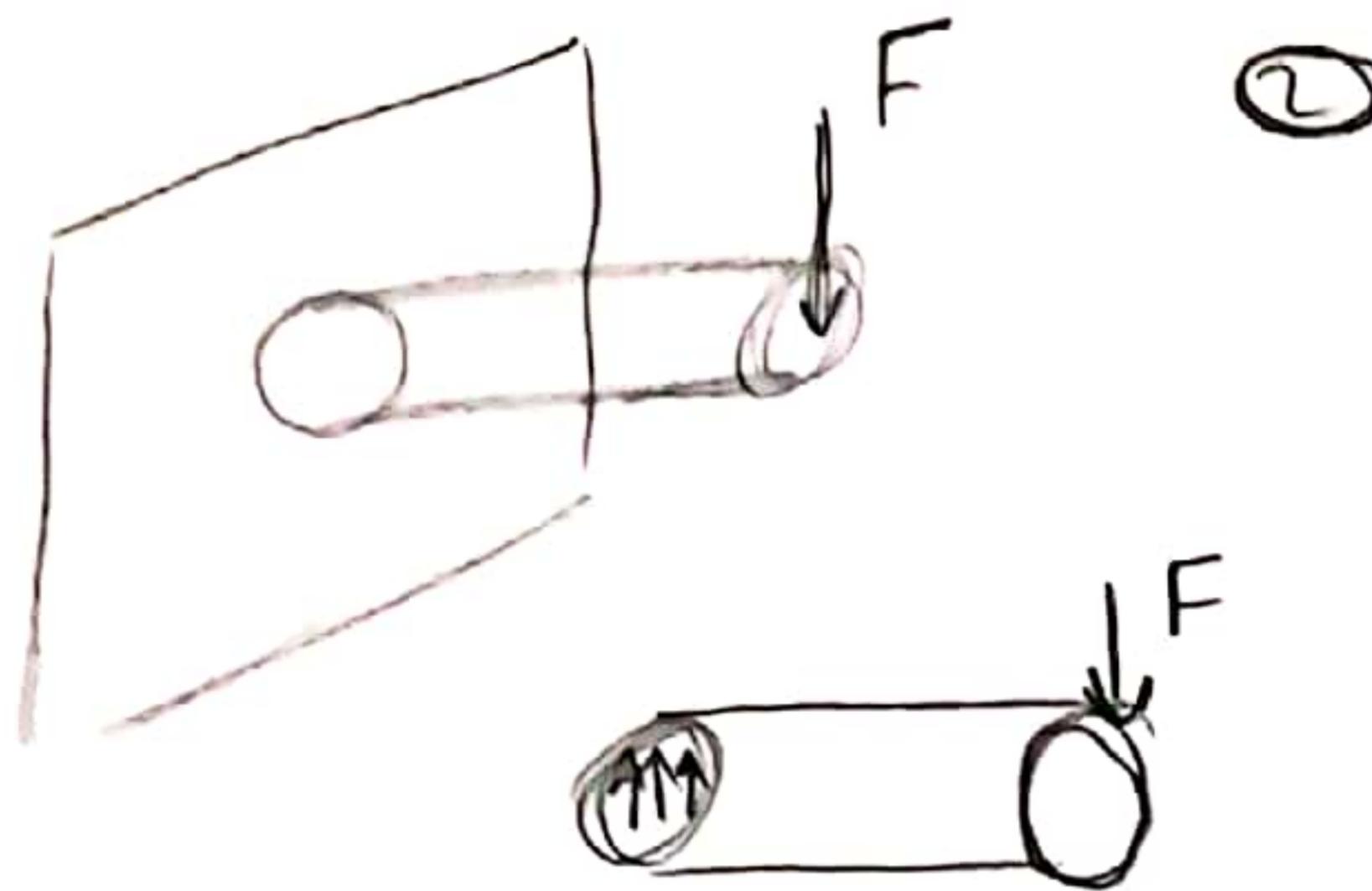
3 Stress - Strain

Assume that the intensity of the distribution force at the cut section is uniform over the cross-sectional area therefore a normal stress can be calculated using

$$\sigma = \frac{F}{A}$$

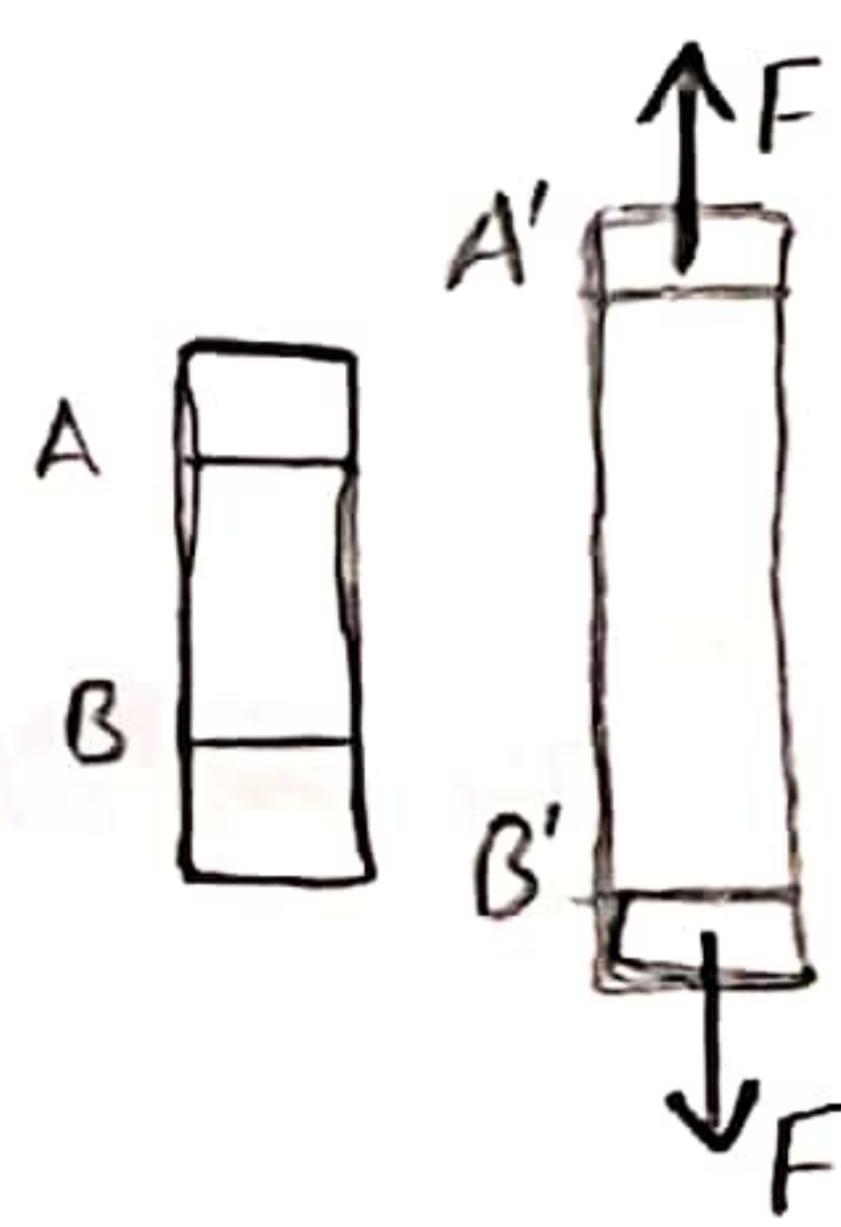


$$\sigma = \frac{F}{A}$$



$$\tau = \frac{F}{A}$$

Simple strain ③

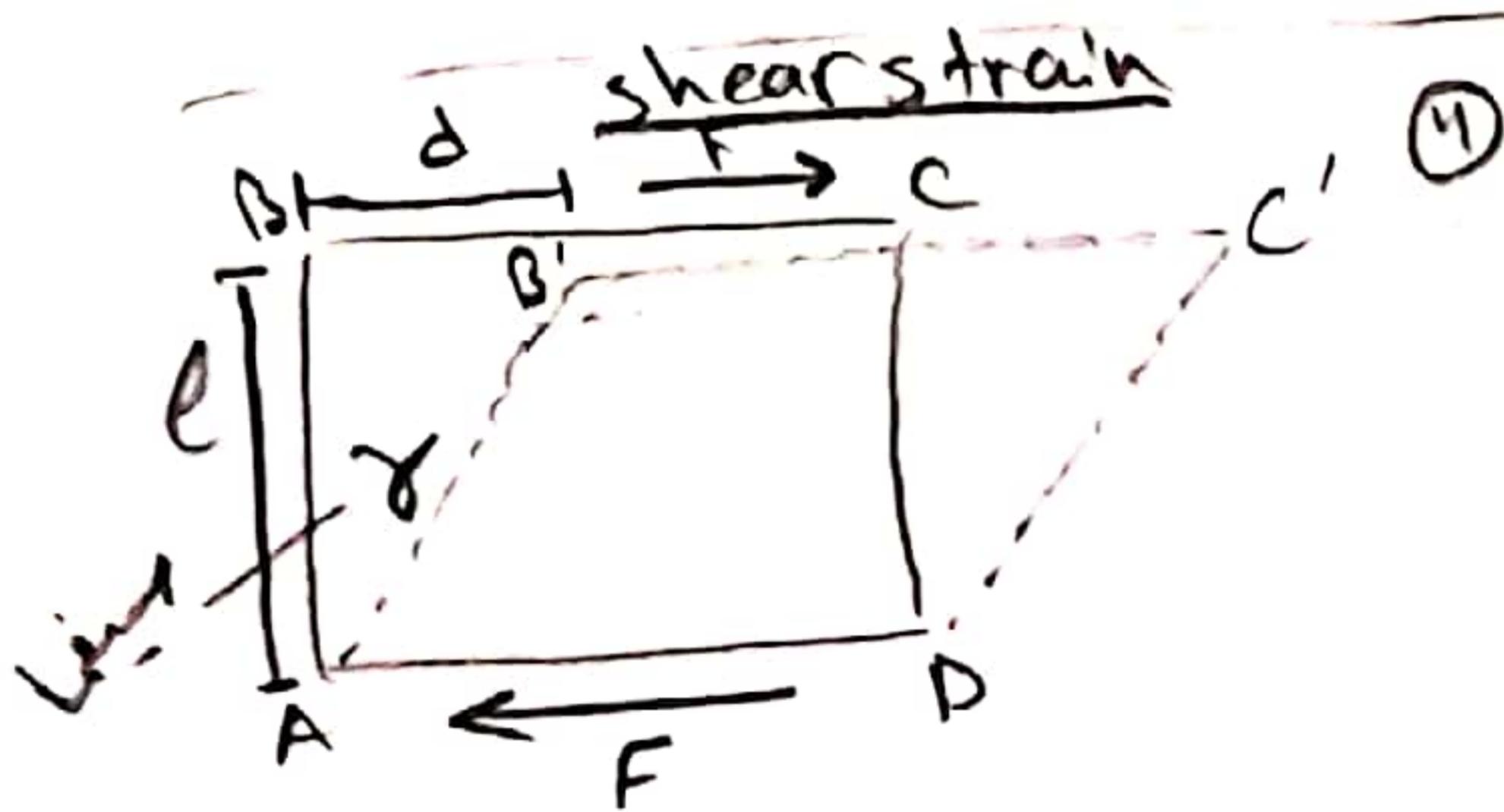


Strain, which is also known as a ~~marks~~ ^{unit} deformation, is a measure of the degree or intensity

Δl is the ratio of the degree of (increase or decrease) of length

l is the ~~original~~ ^{original} length

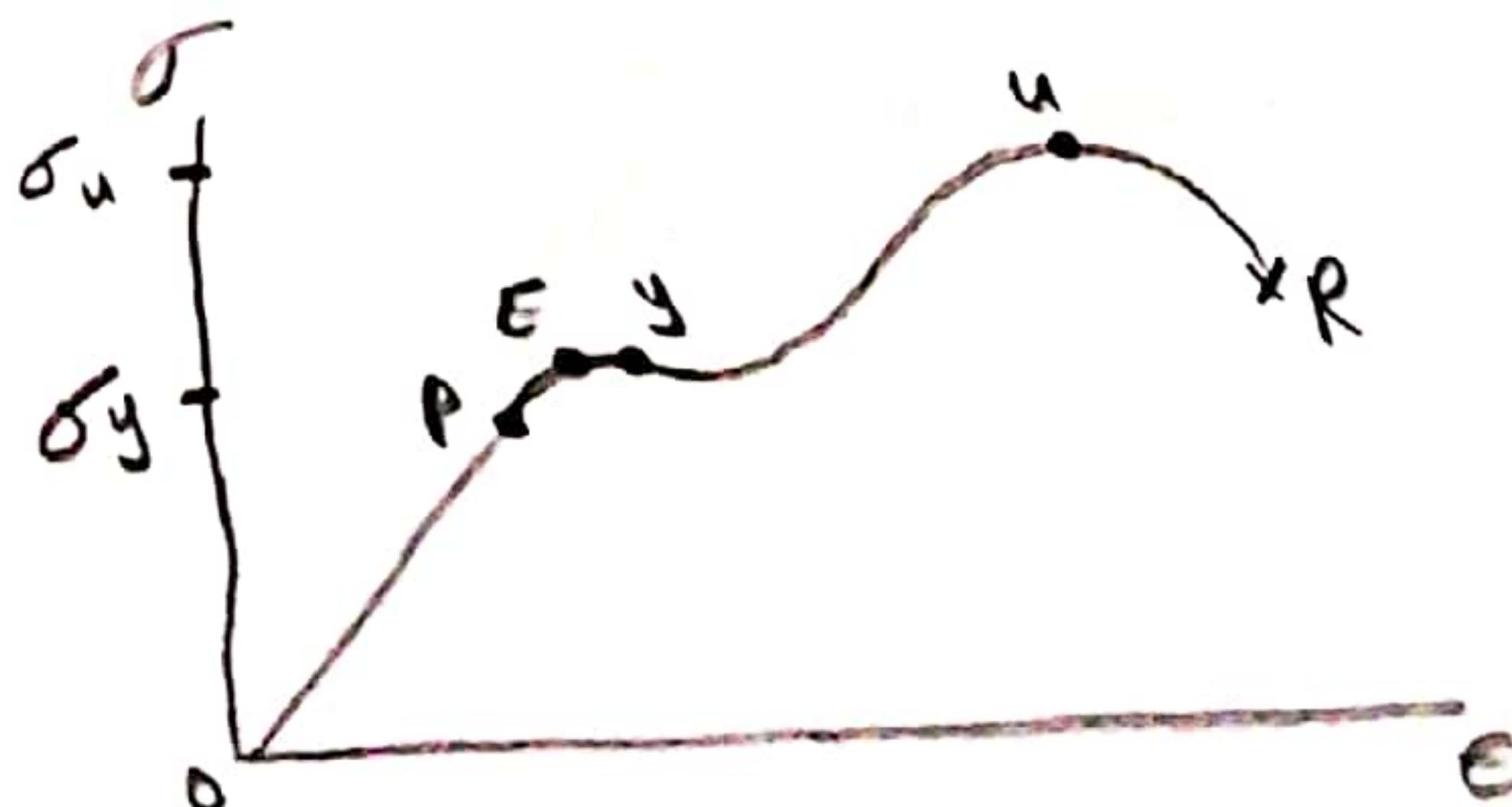
$$\epsilon = \frac{\Delta l}{l}$$



The average shear strain is equal to angle γ (gamma) which can be calculated using

$$\gamma = \frac{d}{l} \quad \gamma \text{ in is radius.}$$

stress - strain diagrams ⑤



point O is the origin of the σ - ϵ diagram, which corresponds to the initial no load

Point P represents the proportionality limit between O and P, stress and strain are linearly proportional.

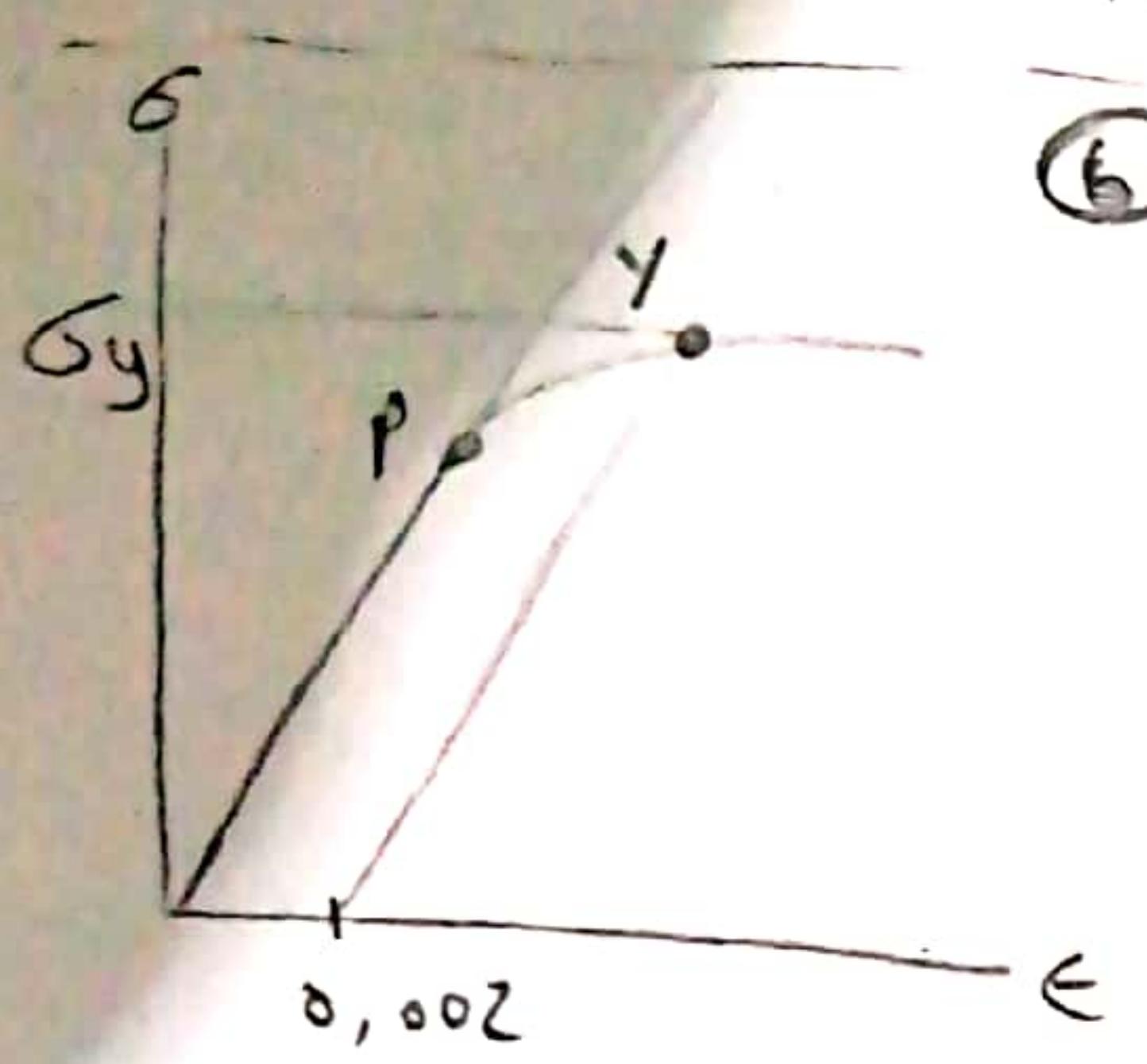
Point E represents the elastic limit, the stress corresponding to the elastic limit is the greatest stress that can be applied to the material without causing any permanent deformation within the material \rightarrow

5.1

Point y is the yield point, and the stress σ_y corresponding to the yield point is called the yield strength of the material.

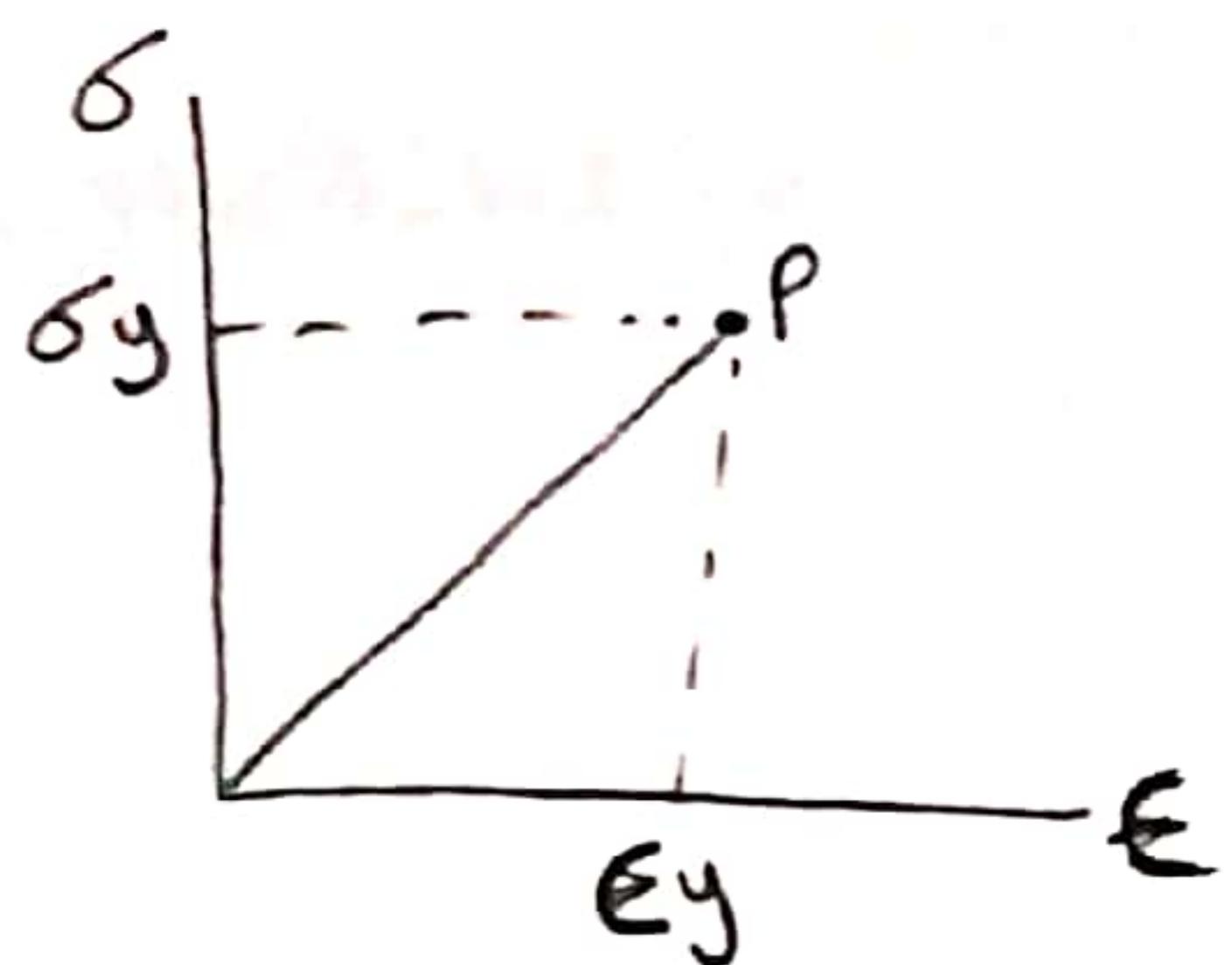
Point U is the highest stress point on the σ - ϵ curve.

The stress σ_u is the ultimate strength of the material, it may cause necking the last point on the σ - ϵ curve is R , which is rupture or failure point.



it is not always easy to distinguish the elastic limit and the yield point, then we use offset method. the offset method is applied by drawing a line parallel to the linear section of the stress-strain diagram and passing through a strain level of about 0,2% (0,002)

Elastic Deformation ⑦



Elastic is the ability of material to resume its original size and shape upon removal of applied loads.

$E = \frac{\sigma}{\epsilon}$ an elastic materials whose σ - ϵ diagram is straight line is called linearly elastic material.

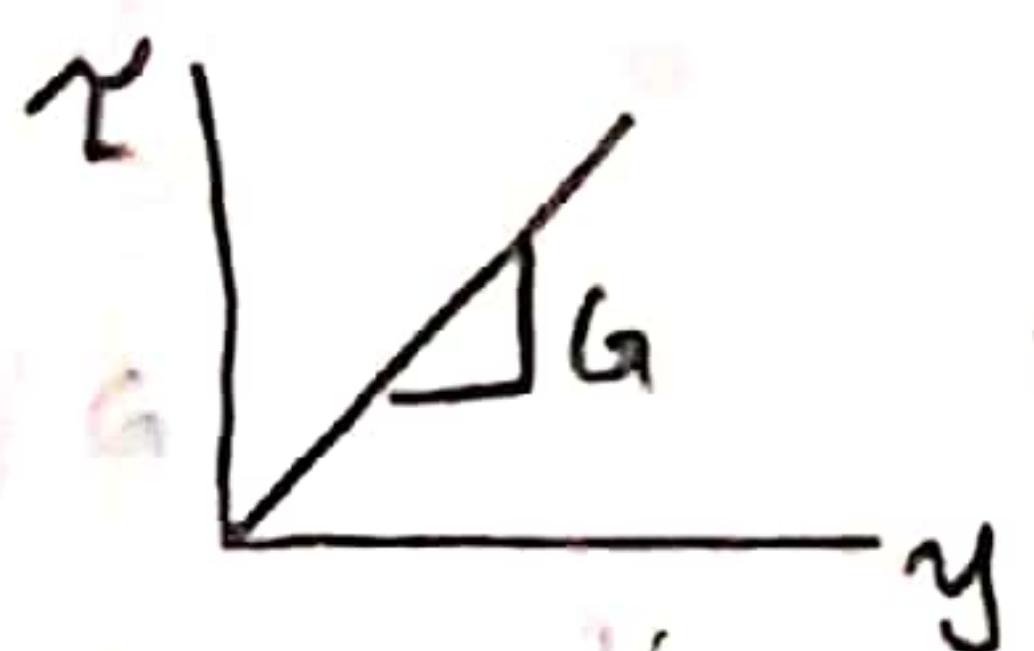
For such a material the stress is linearly proportional to strain the constant of proportionality is called

Elastic or Young's modulus of material $G = E \epsilon$; in which E represents the stiffness of a material, such that the higher elastic modulus, the stiffer the material

$$\sigma = \frac{F}{A} \quad , \quad \epsilon = \frac{\Delta l}{l} \quad , \quad \Delta l = \frac{F l}{E A}$$

Therefore ; we can develop a relations between applied load and corresponding deformation

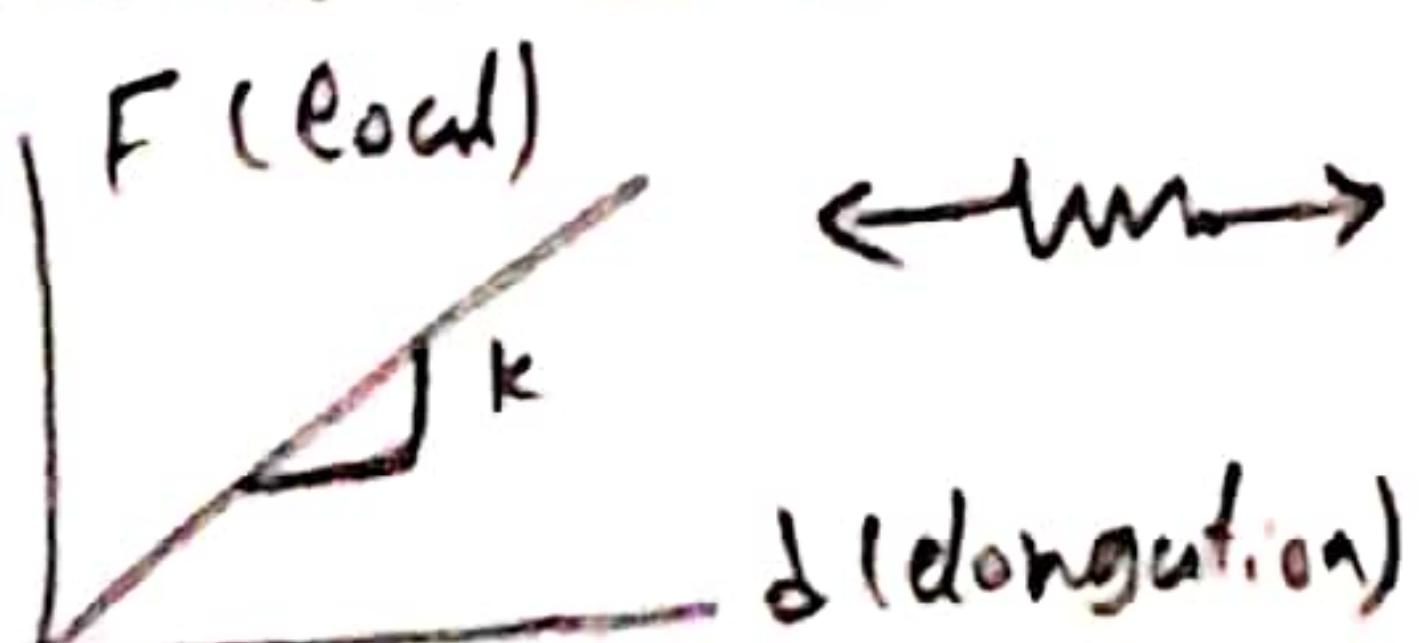
Modulus of Rigidity ⑧



$$G = \frac{\sigma}{\epsilon}$$

higher shear modulus, more rigid material.

Hooke's law

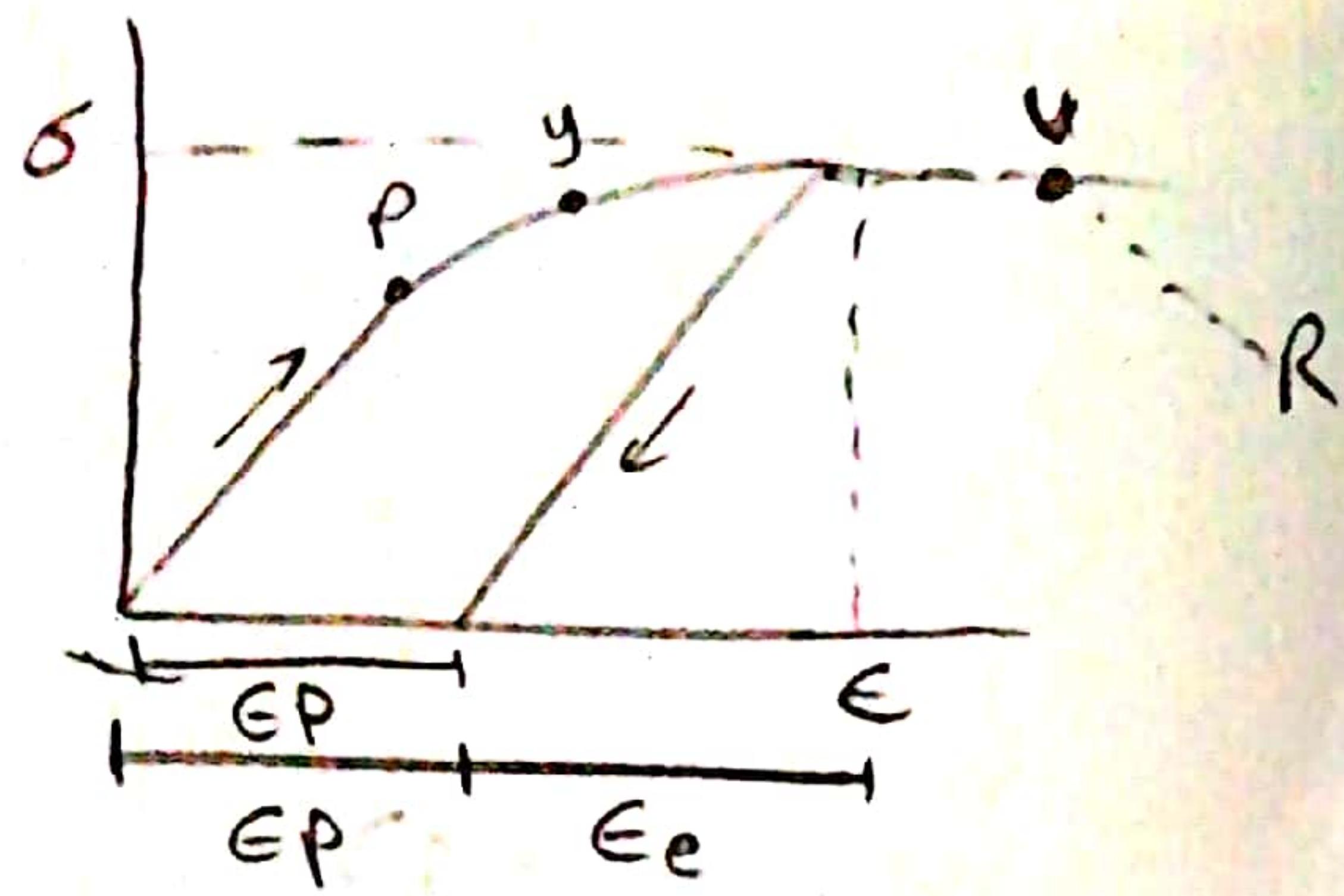


some relations can be formed with spring. Springs have ability to store potential energy when they are subjected to external applied loads.

During Elastic it releases this energy

$$F = k d \rightarrow \begin{matrix} \text{amount of elong} \\ \text{spring constant} \end{matrix}$$

plastic deformation ⑨



plasticity implies permanent deformation

In general materials undergo plastic deformations following elastic deformations.

$\epsilon_p \rightarrow$ the path

Cuts the strain axis is called "plastic strain"

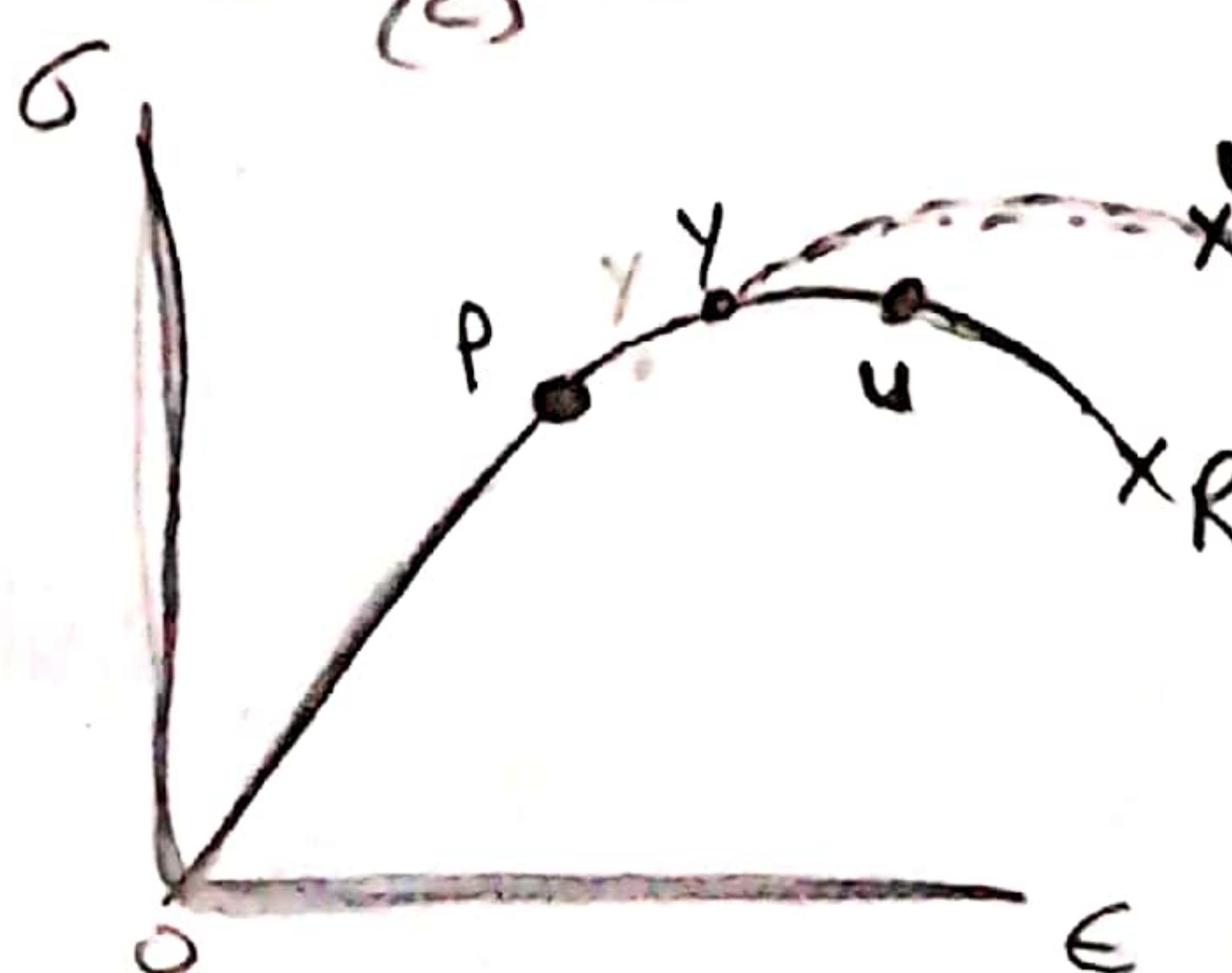
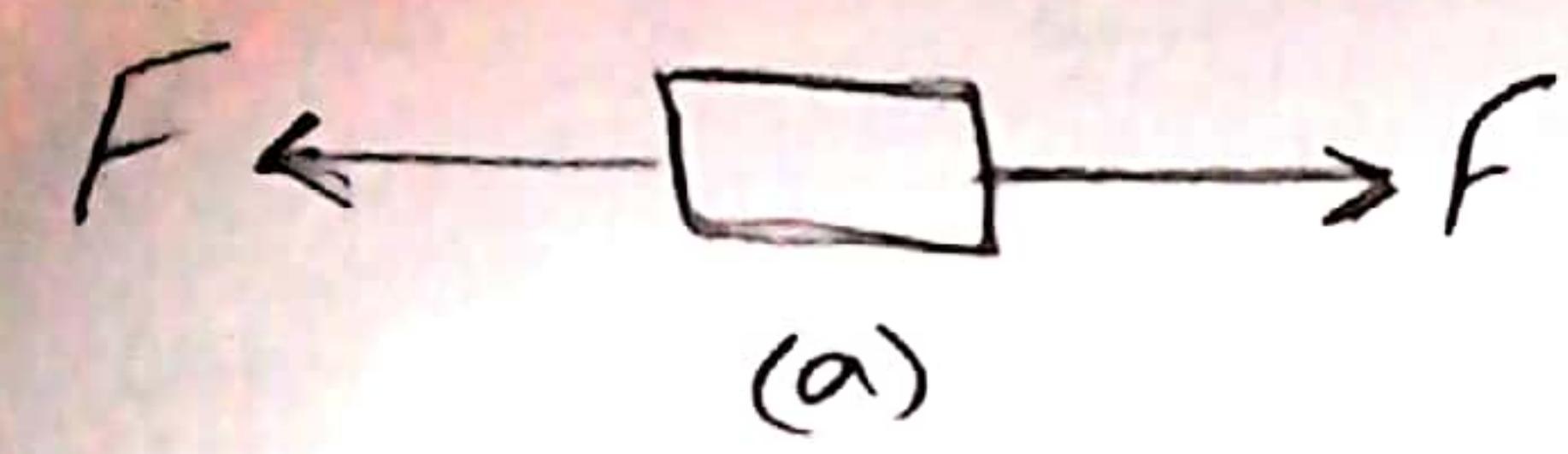
ϵ_p signifies the extent of permanent shape change that has taken place in specimen.

The difference in strains between when specimen is loaded and unloaded ($\epsilon - \epsilon_p$) is equal to the amount of elastic recovered upon unloading.

Therefore, a material loaded to a stress level beyond its elastic limit, the total strain is equal to the sum of elastic and plastic strains.

$$\epsilon = \epsilon_e + \epsilon_p$$

necking ⑩



The largest stress a material is called Ultimate strength.

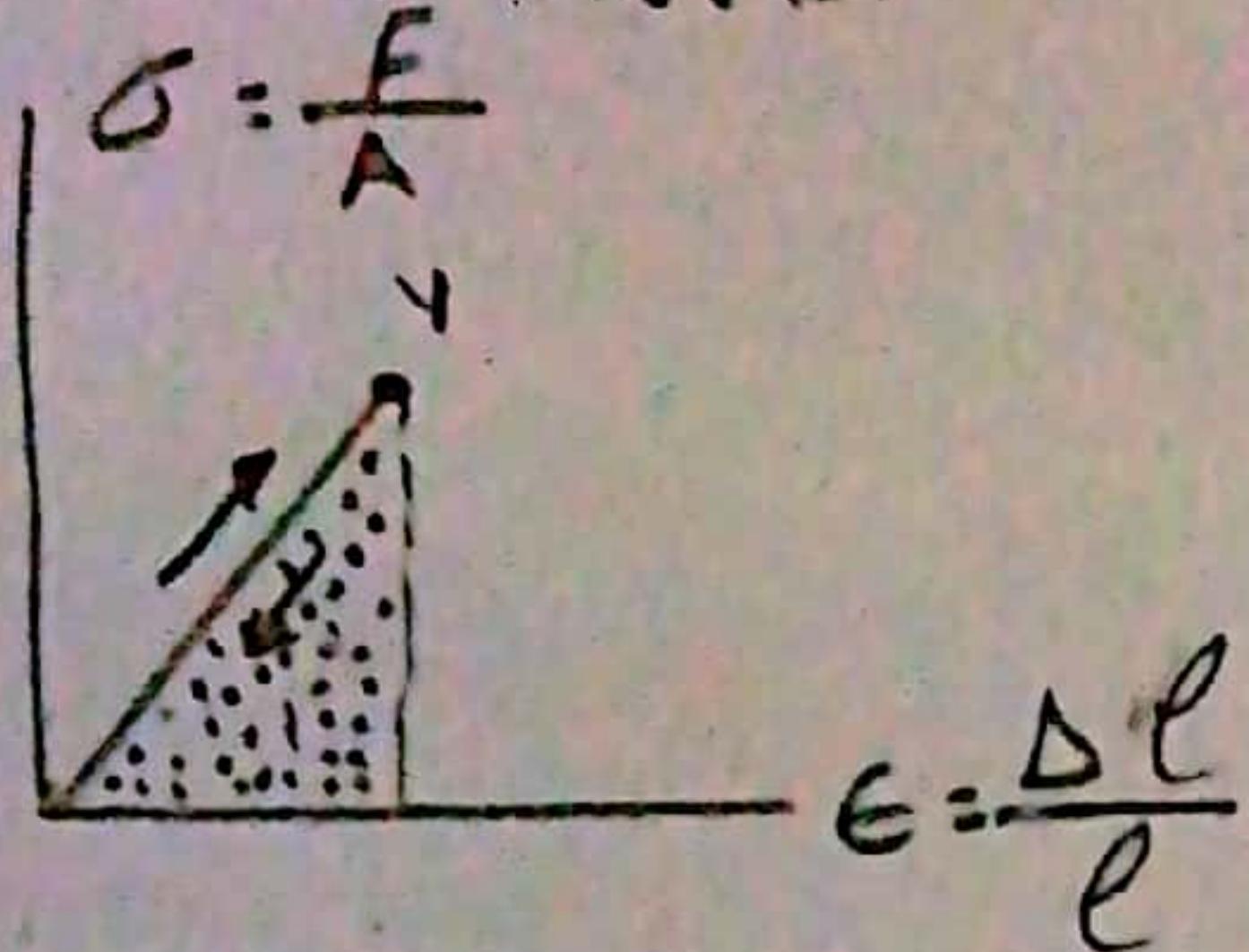
The stress at failure is called rupture strength.

At stress levels close to the yielding points elongation may occur very rapidly and the material may narrow simultaneously.

The cross-section area at the narrowed section decreases, and although the force required to further deform the material may decrease, the force per unit area (stress) may increase.

Work and strain energy ①

Work is defined as a product of force and the distance traveled in the direction of applied force, and energy is the capacity of a system to do work.

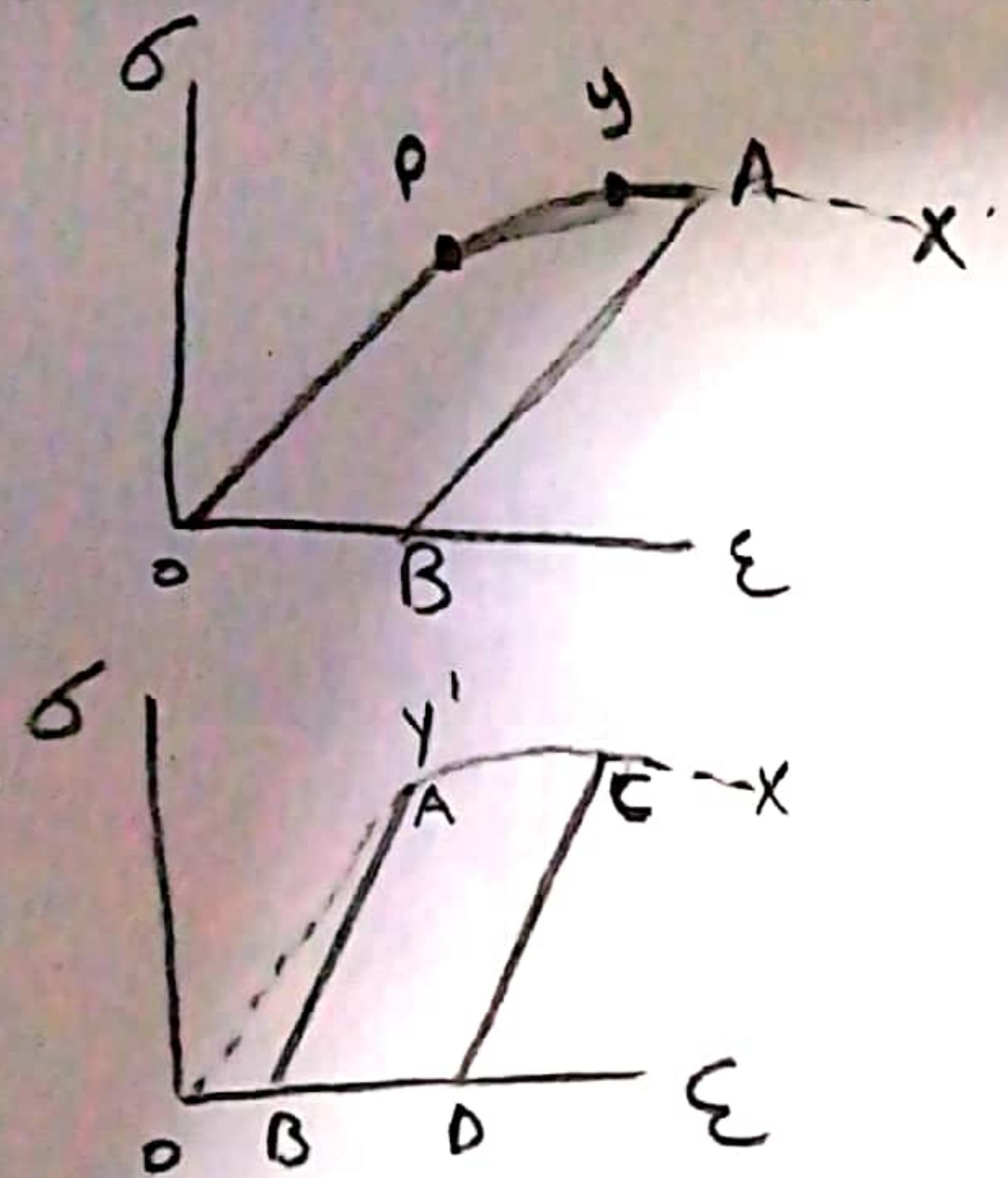


• stress and strain are relation to force and displacement.

stress multiplied by area is equal to force, and strain multiplied by length is displacement.

Therefore, The product of stress and strain is equal to the work done on a body per unit volume of that body.

Strain hardening ②

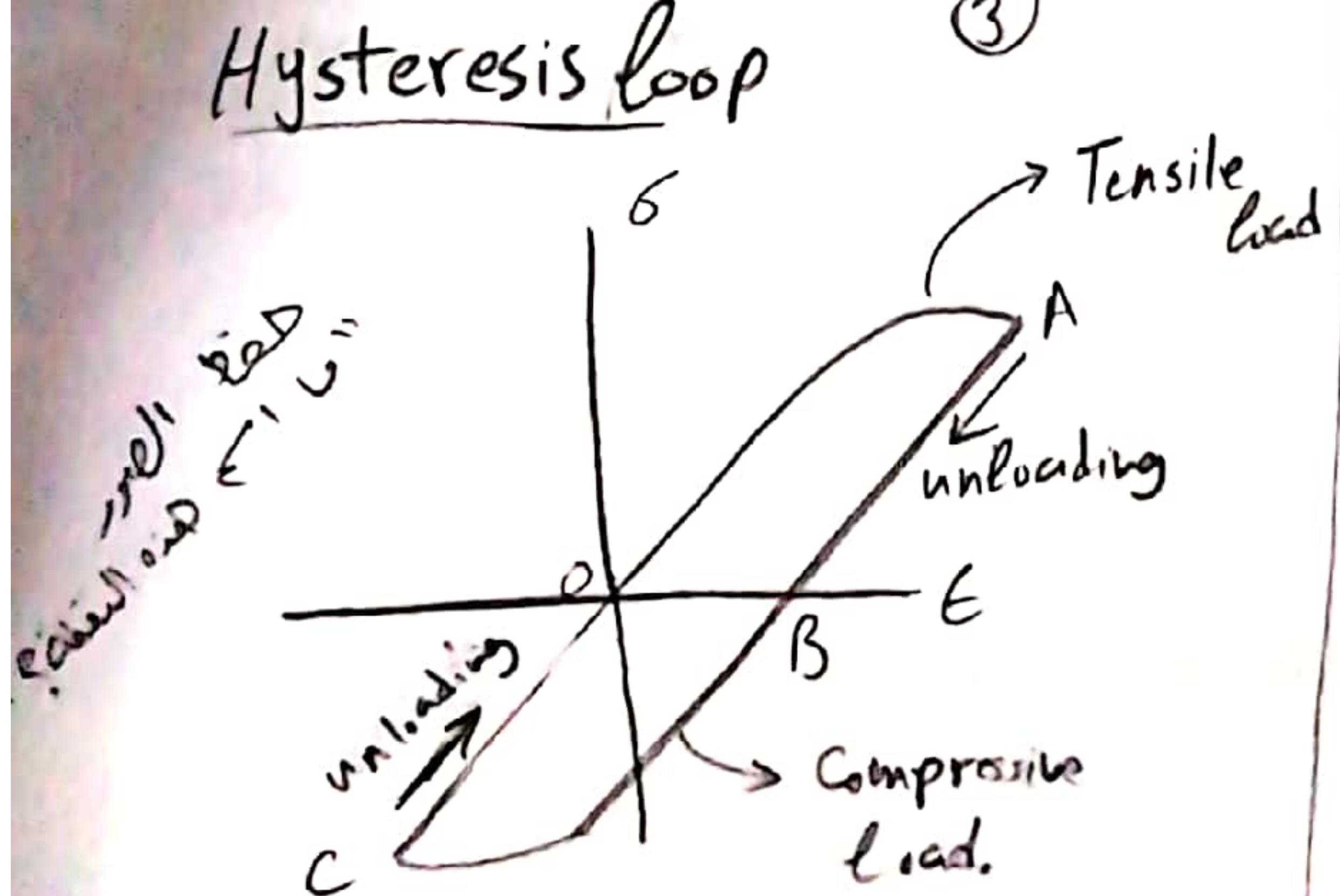


Assume that the material is subjected to a tensile force such that the stress generated is beyond the elastic limit.

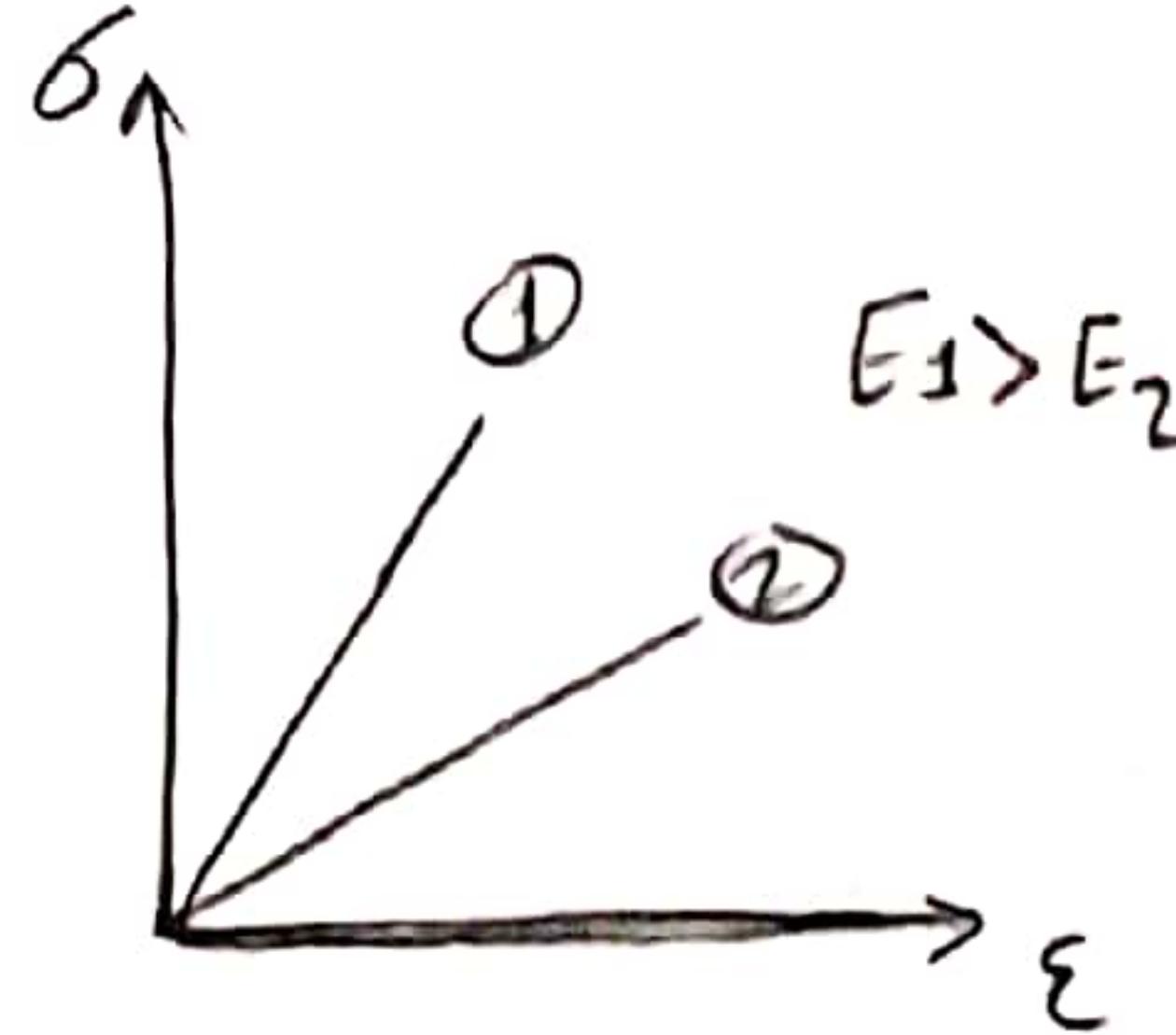
Upon removal to the applied force the material will follow the path CD.

If the material is reloaded it will exhibit elastic behaviour between B and A. The stress at A is being the new yield strength of the material.

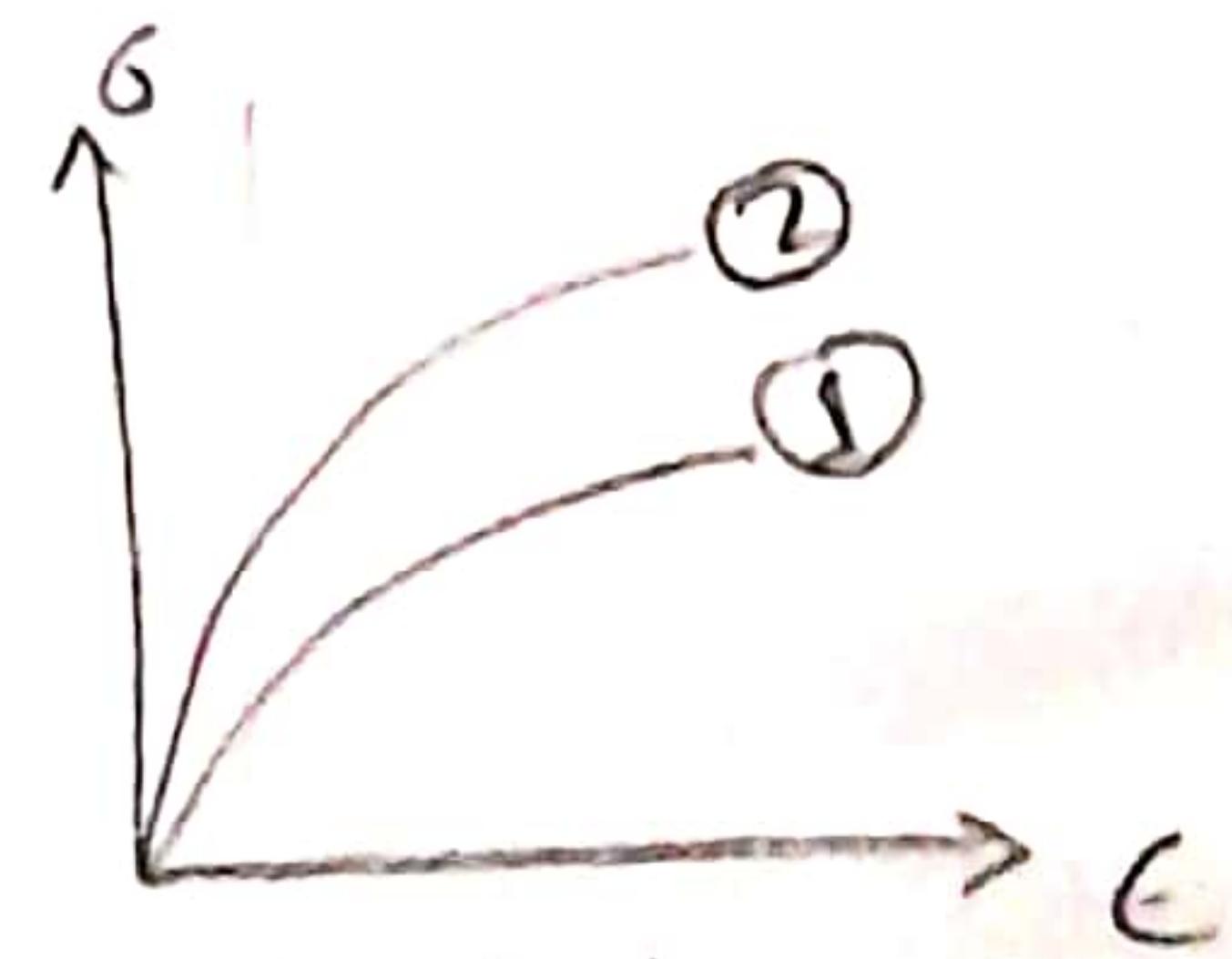
Hysteresis loop ③



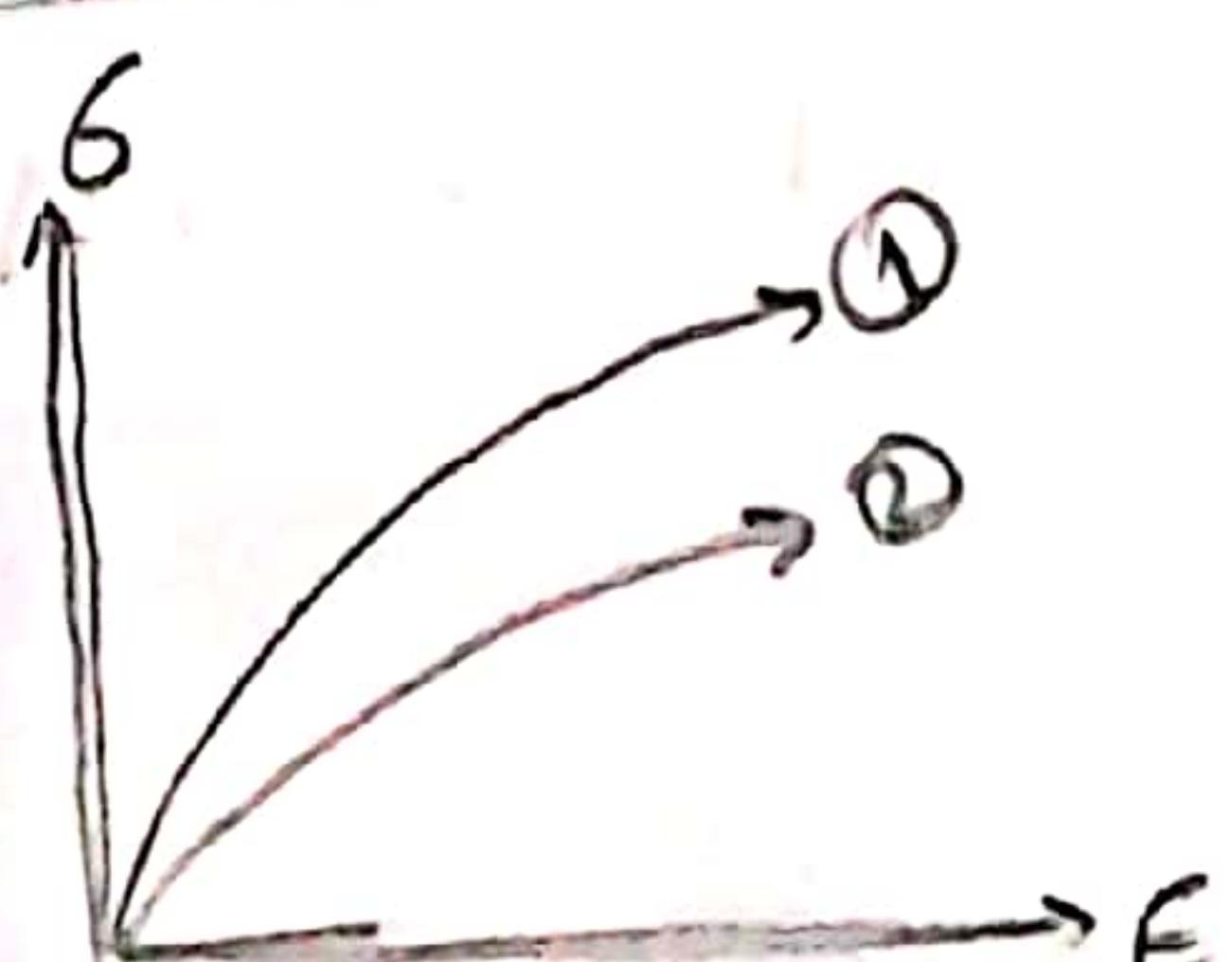
Properties based on stress-strain Diagram ④



Material 1 is stiffer than material 2



Material 1 is more ductile and less brittle than 2



Material ① is tougher than material 2.

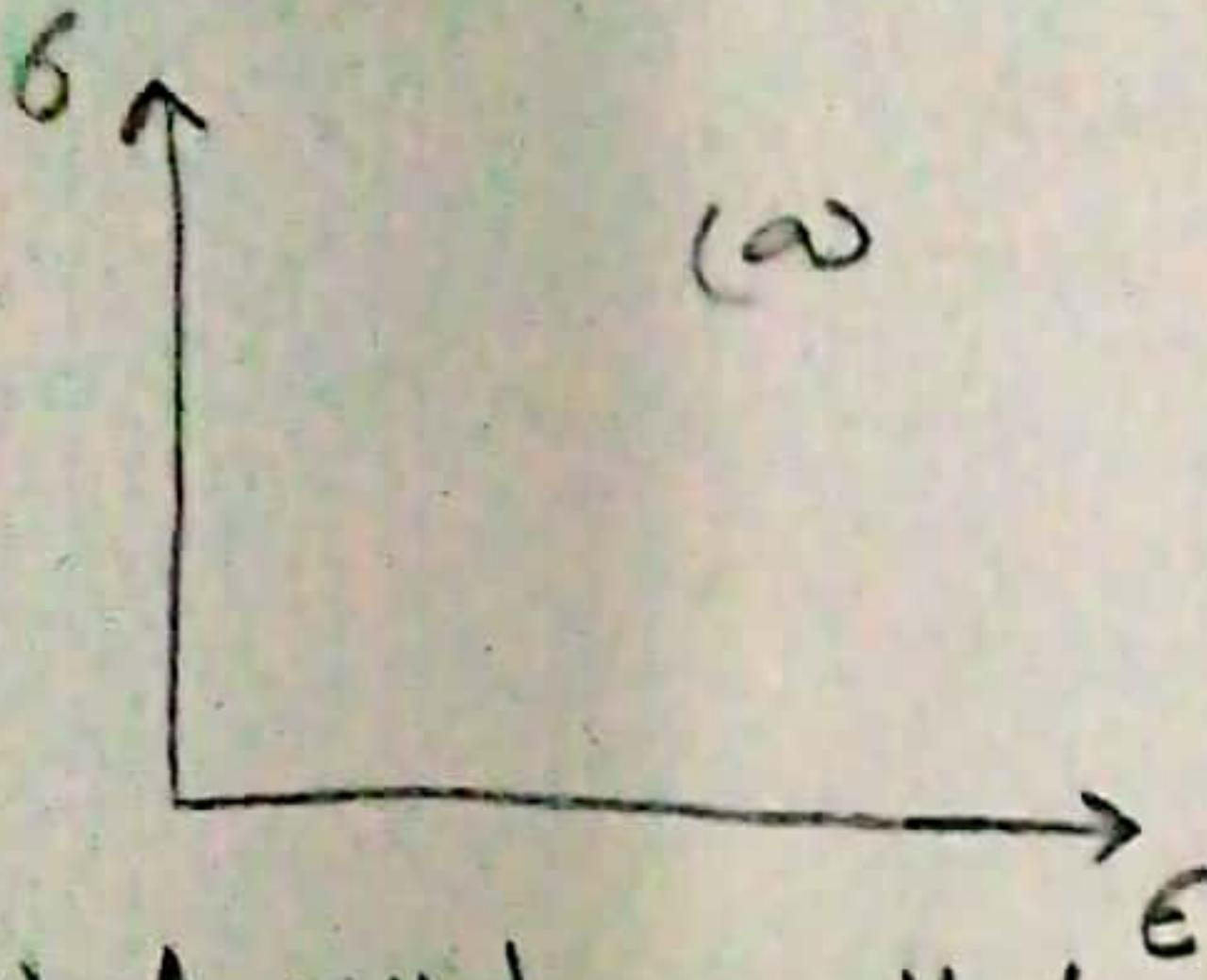
A ductile material is one that exhibits a large plastic deformation prior to failure.

A brittle material shows a sudden failure without undergoing a considerable plastic deformation.

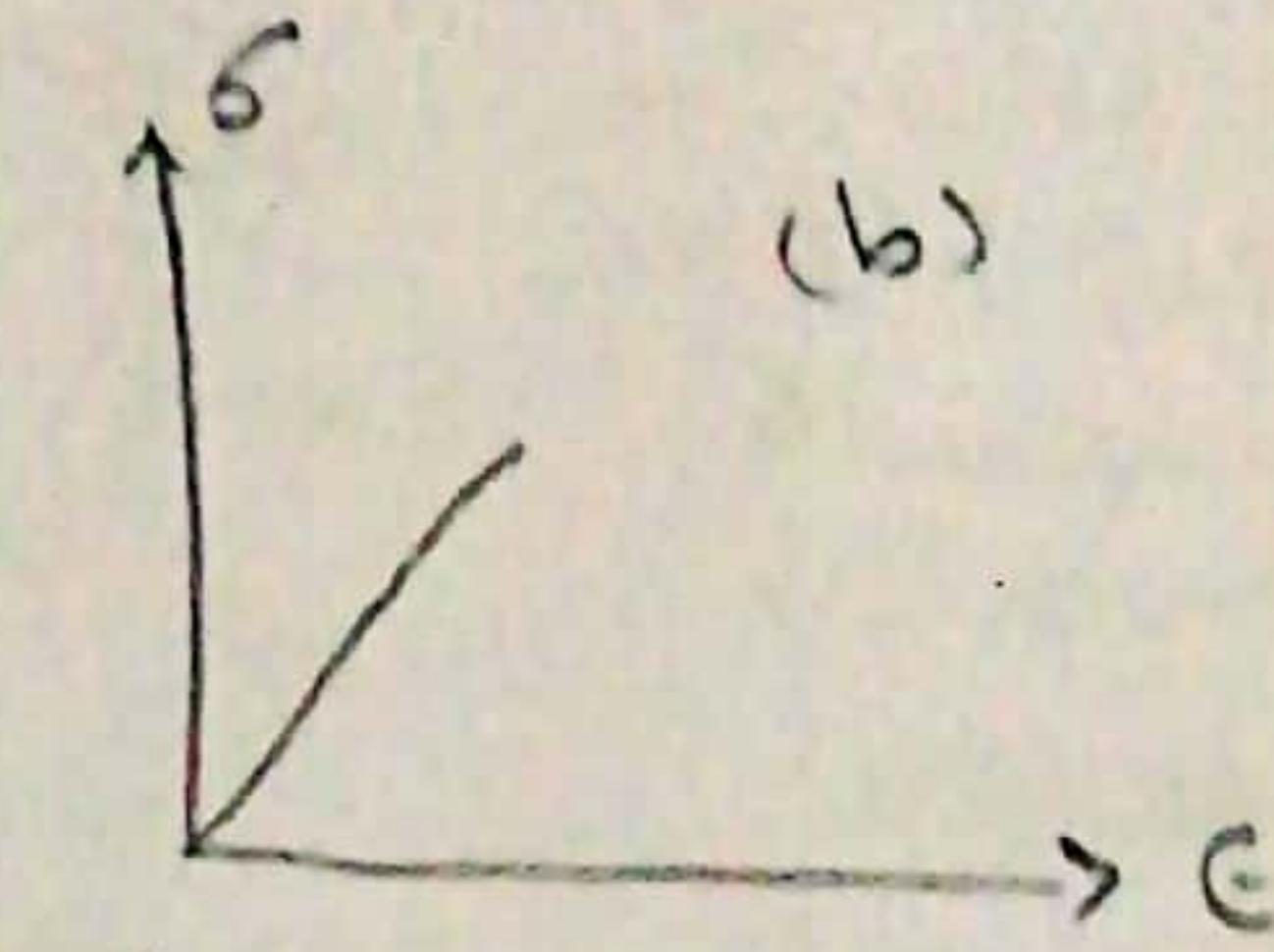
Toughness is a measure of the capacity.

The Toughness is measured by the total area of stress-strain diagram.

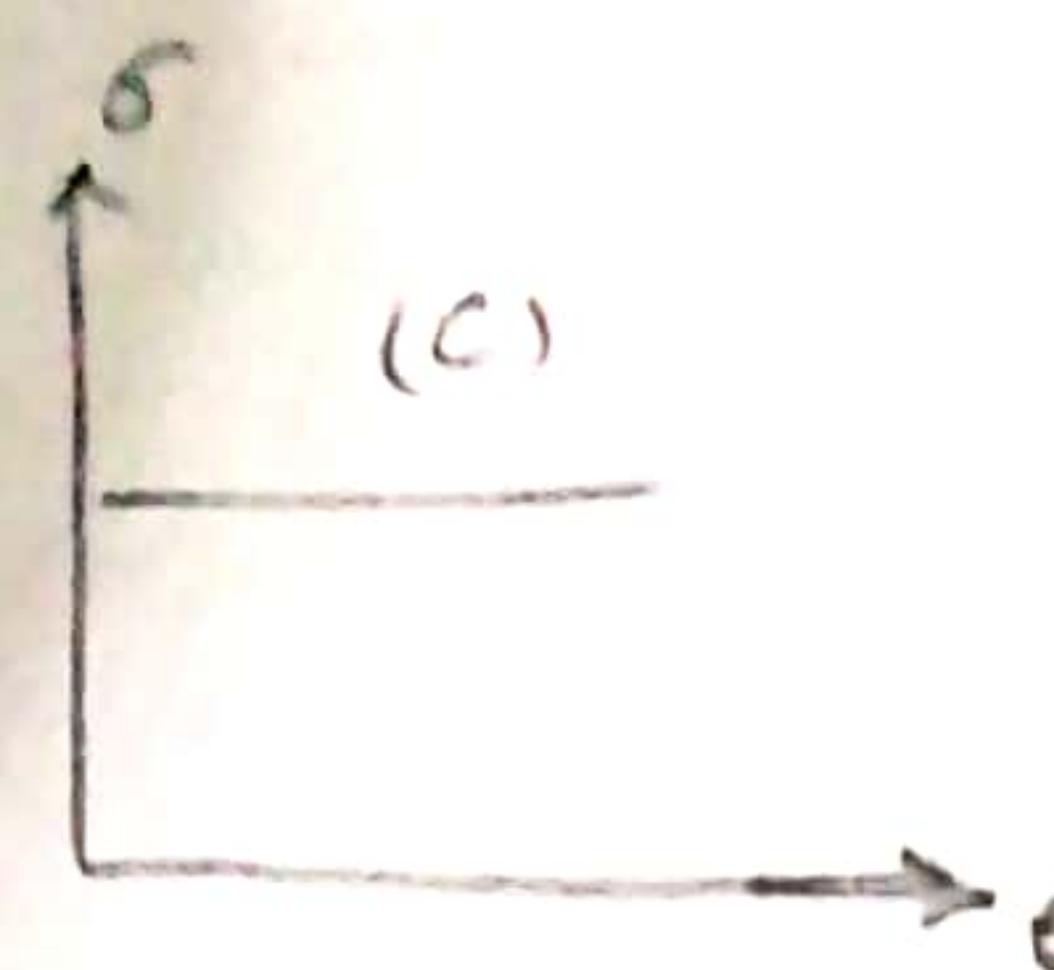
Idealized models of material behaviour



a) A rigid one that can't be deformed

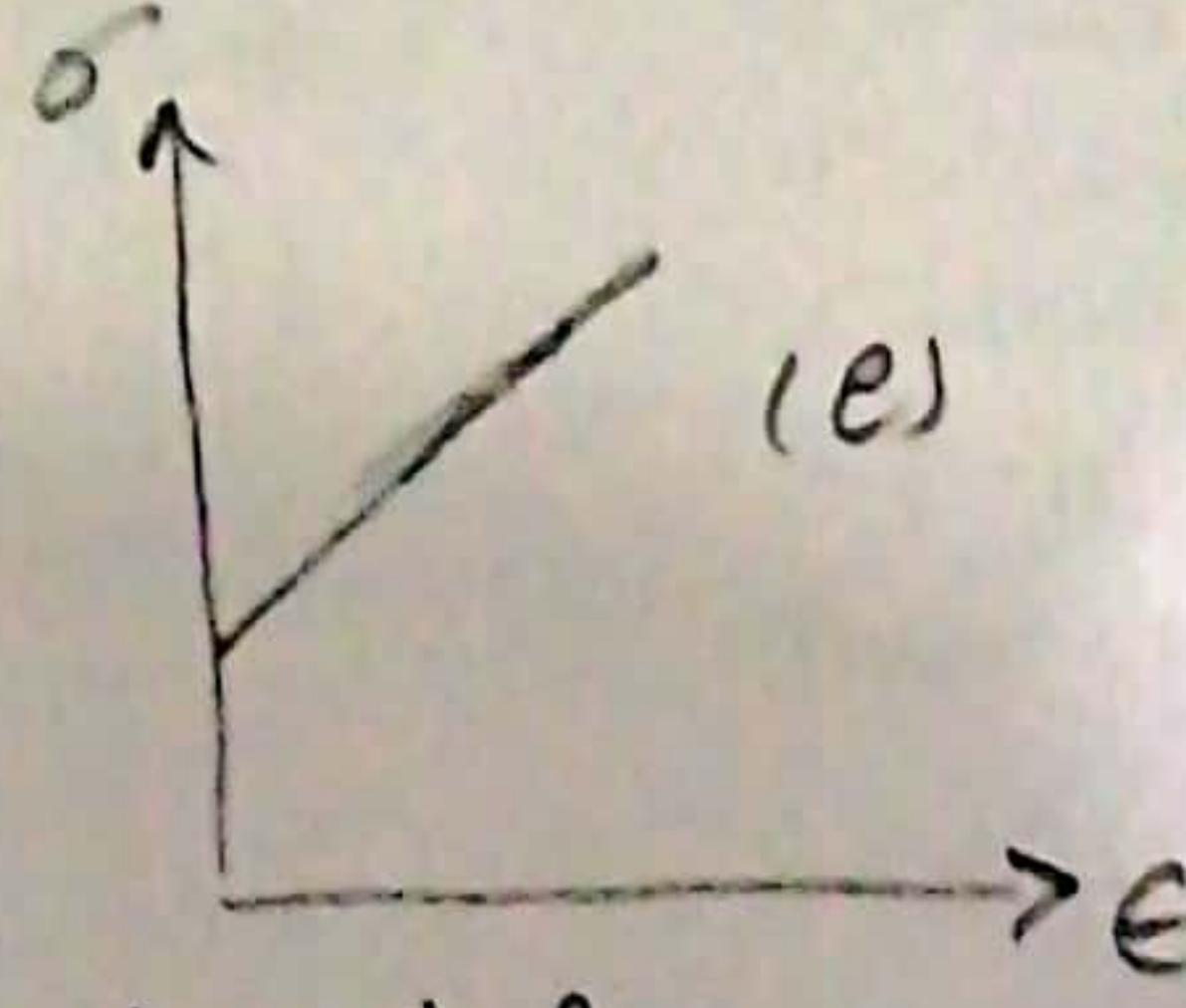


b) A linearly elastic material is one for which stress and strain are linearly proportional

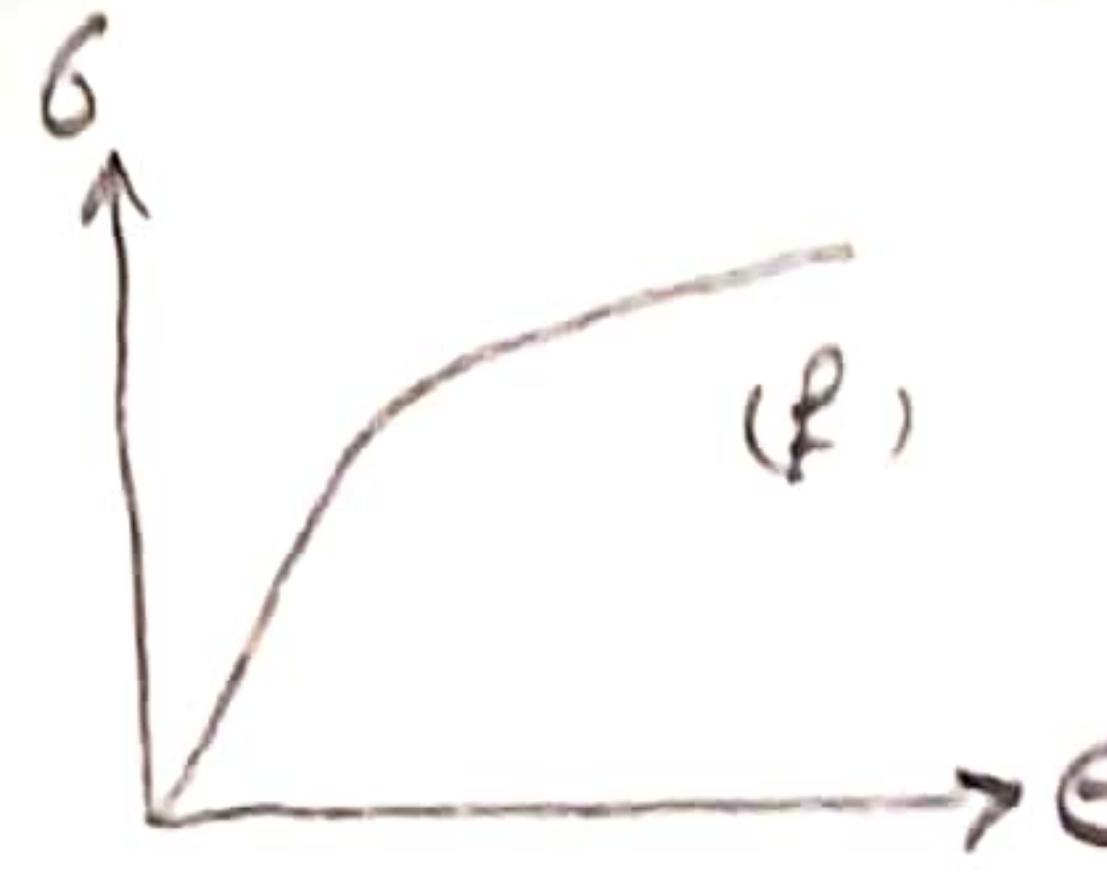


c) A rigid perfectly plastic material does not exhibit an elastic behaviour.

d) After a linearly elastic response linearly elastic - perfect plastic material is one that deforms continuously at a constant stress level.

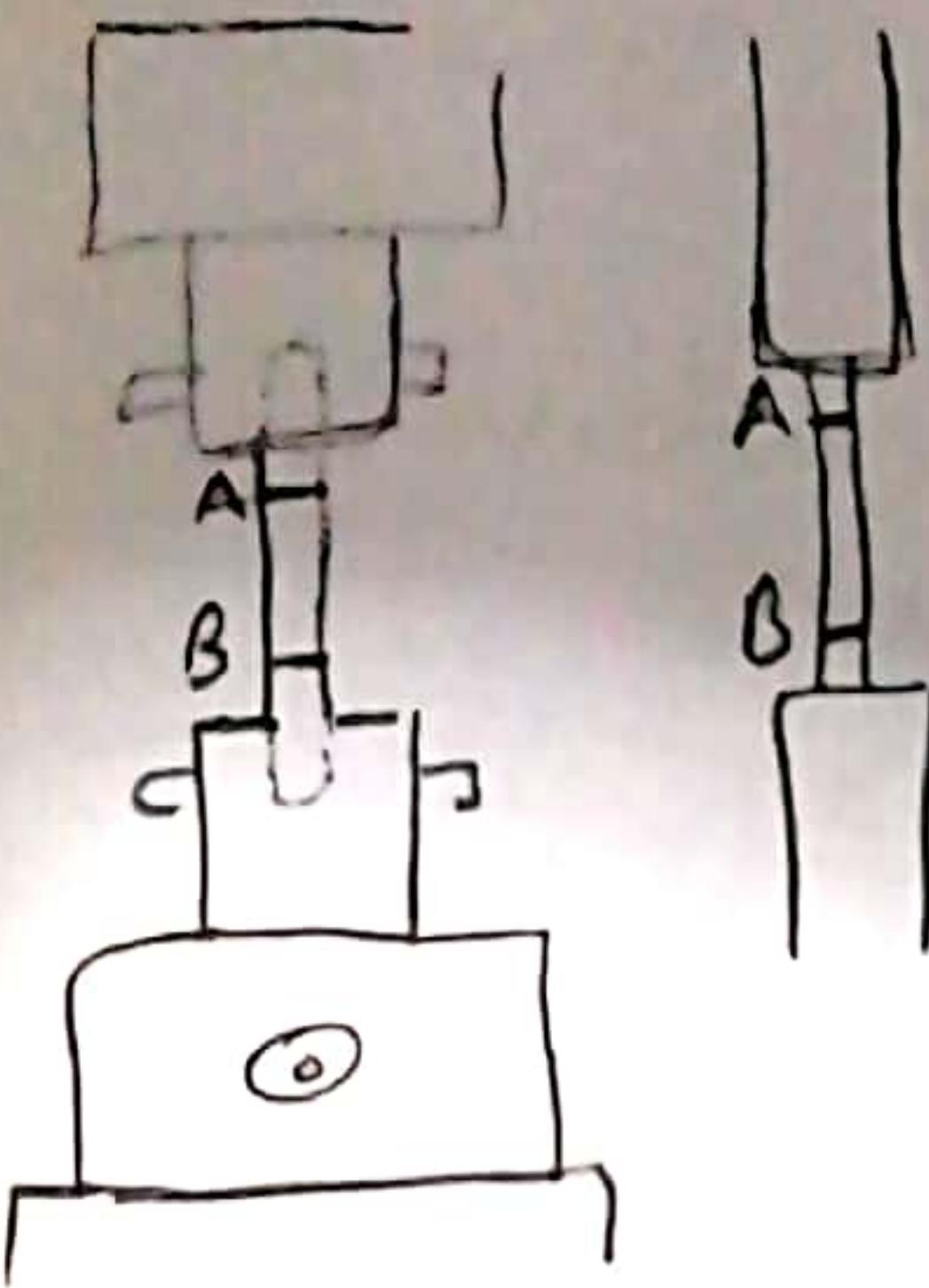


e) rigid linearly plastic behaviour



f) linearly elastic-plastic material has linear stress-strain lines for both elastic and plastic regions

Ex:



A circular cylindrical rod with radius $r = 1,26 \text{ cm}$ is tested in a uniaxial tension test. Before applying a tensile force of $F = 1000 \text{ N}$, two points A and B that are at a distance $I_0 = 30 \text{ cm}$ are marked on the rod. After the force applied, the distance between A and B is measured as $I_1 = 31,5 \text{ cm}$. Determine the tensile strain and average tensile stress generated in the rod.

ans:

$$\Delta I = I_1 - I_0 = 31,5 - 30 = 1,5 \text{ cm}$$

Tensile force is therefore:

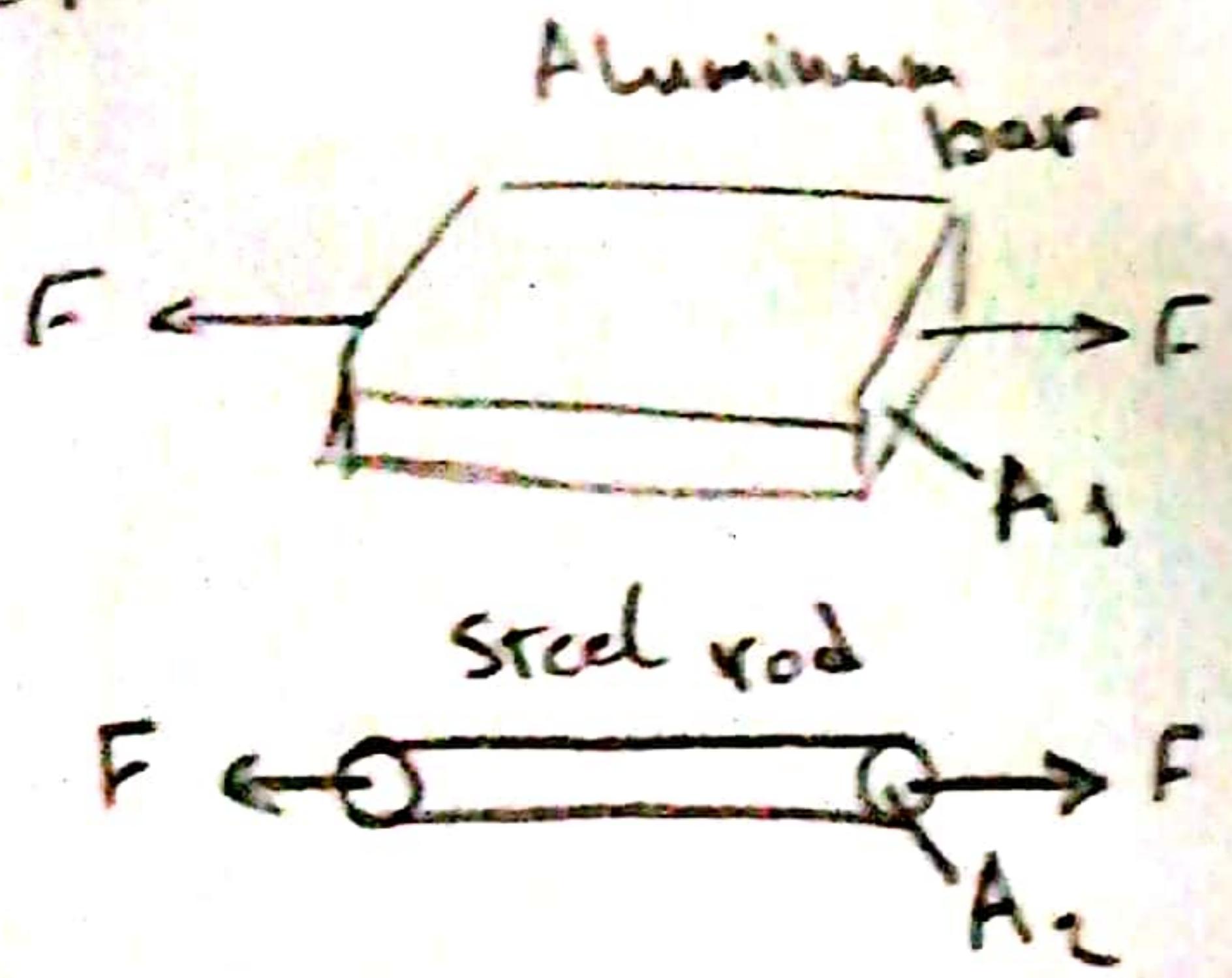
$$\epsilon = \frac{\Delta I}{I_0} = \frac{1,5}{30} = 0,05 \text{ cm/cm}$$

The cross-section area A of the bar is:

$$A = \pi r^2 = (3,1416)(0,0126)^2 = 5 \times 10^{-4} \text{ m}^2 = 0,0005 \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{1000}{0,0005} = 2000000 = 2 \times 10^6 \text{ Pa} = 2 \text{ MPa.}$$

Ex.



→ Tested in a uniaxial tension test

→ $F = 20 \text{ kN} (20 \times 10^3 \text{ N})$

Aluminum bar $E_1 = 70 \text{ GPa}$

rectangular cross-section (1cm x 2cm)

steel rod $E_2 = 200 \text{ GPa}$

circular cross section ($r = 1\text{cm}$)

Calculate tensile stresses?

Calculate tensile strain?

if the original length was 30cm what are their lengths after deformation?

$$\text{ans. } \sigma = \frac{F}{A} ; F = 20 \times 10^3 \text{ N}$$

$$A_1 = (1\text{cm})(2\text{cm}) = 2\text{cm}^2$$

$$A_2 = \pi (1\text{cm})^2 = 3,14 \text{cm}^2$$

$$\sigma_1 = \frac{F}{A_1} = \frac{20 \times 10^3}{2 \times 10^{-4}} = 100 \times 10^6 \text{ Pa} = 100 \text{ MPa}$$

$$\sigma_2 = \frac{F}{A_2} = \frac{20 \times 10^3}{3,14 \times 10^{-4}} = 63,7 \times 10^6 \text{ Pa} = 63,7 \text{ MPa}$$

Therefore, aluminum bar is stressed more than steel rod

To calculate tensile strain

$$\epsilon_1 = \frac{\sigma_1}{E_1} = \frac{100 \times 10^6}{70 \times 10^9} = 1,43 \times 10^{-3}$$

$$\epsilon_2 = \frac{\sigma_2}{E_2} = \frac{63,7 \times 10^6}{200 \times 10^9} = 0,32 \times 10^{-3}$$

Aluminum bar is stretched more than the steel rod

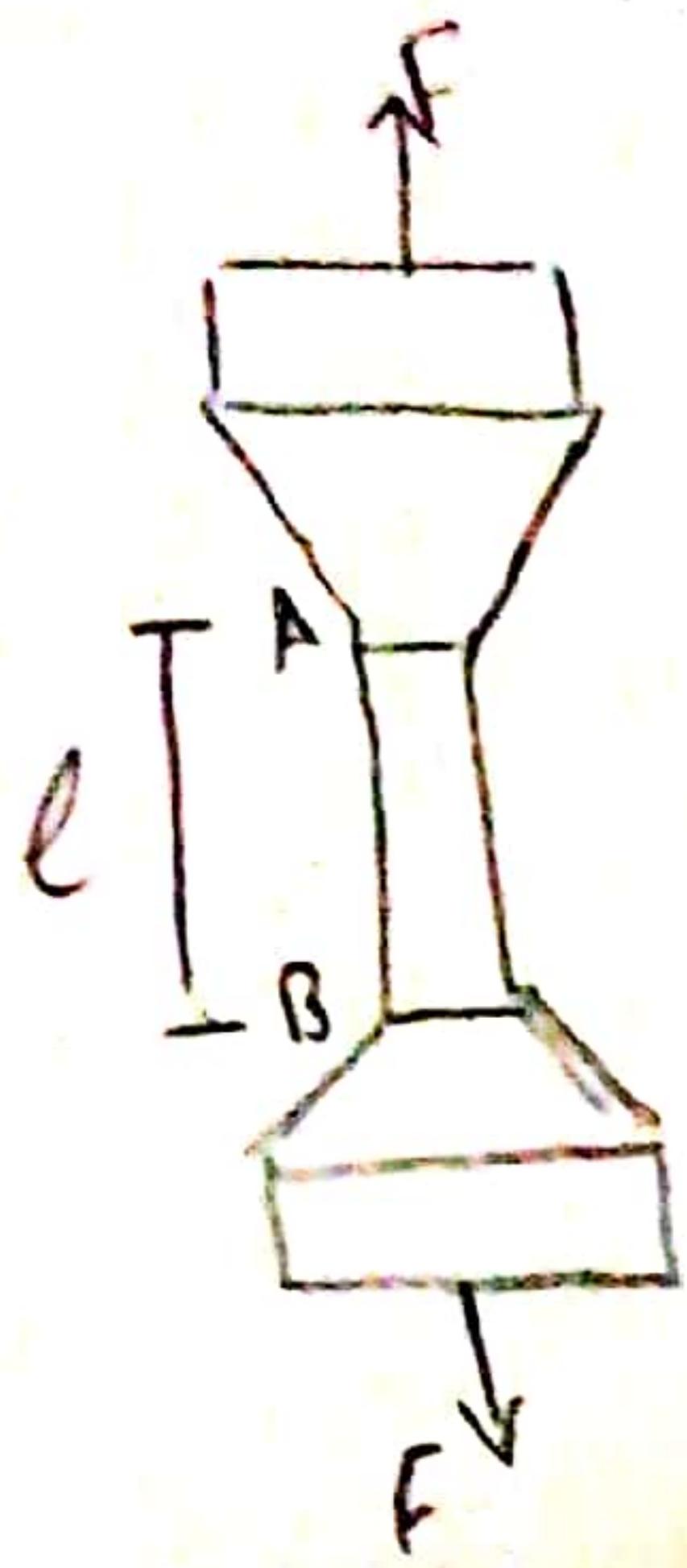
$$\ell_1 = \epsilon_1 l_0 \Rightarrow \epsilon = \frac{\Delta l}{l_0} \Rightarrow \Delta l_1 = \epsilon_1 l_0$$

$$\Delta l_2 = \epsilon_2 l_0$$

Cx.

An experiment was designed to determine the elastic modulus of the human bone.

The square cross-section $\rightarrow (2 \times 2 \text{ mm})$



Applied Force $F(\text{N})$ Measured Gage length $l(\text{mm})$

0	5,000
240	5,017
480	5,033
720	5,050

Determine the tensile stresses and strain and plot a stress-strain diagram

Determine the elastic modulus (E) for the bone

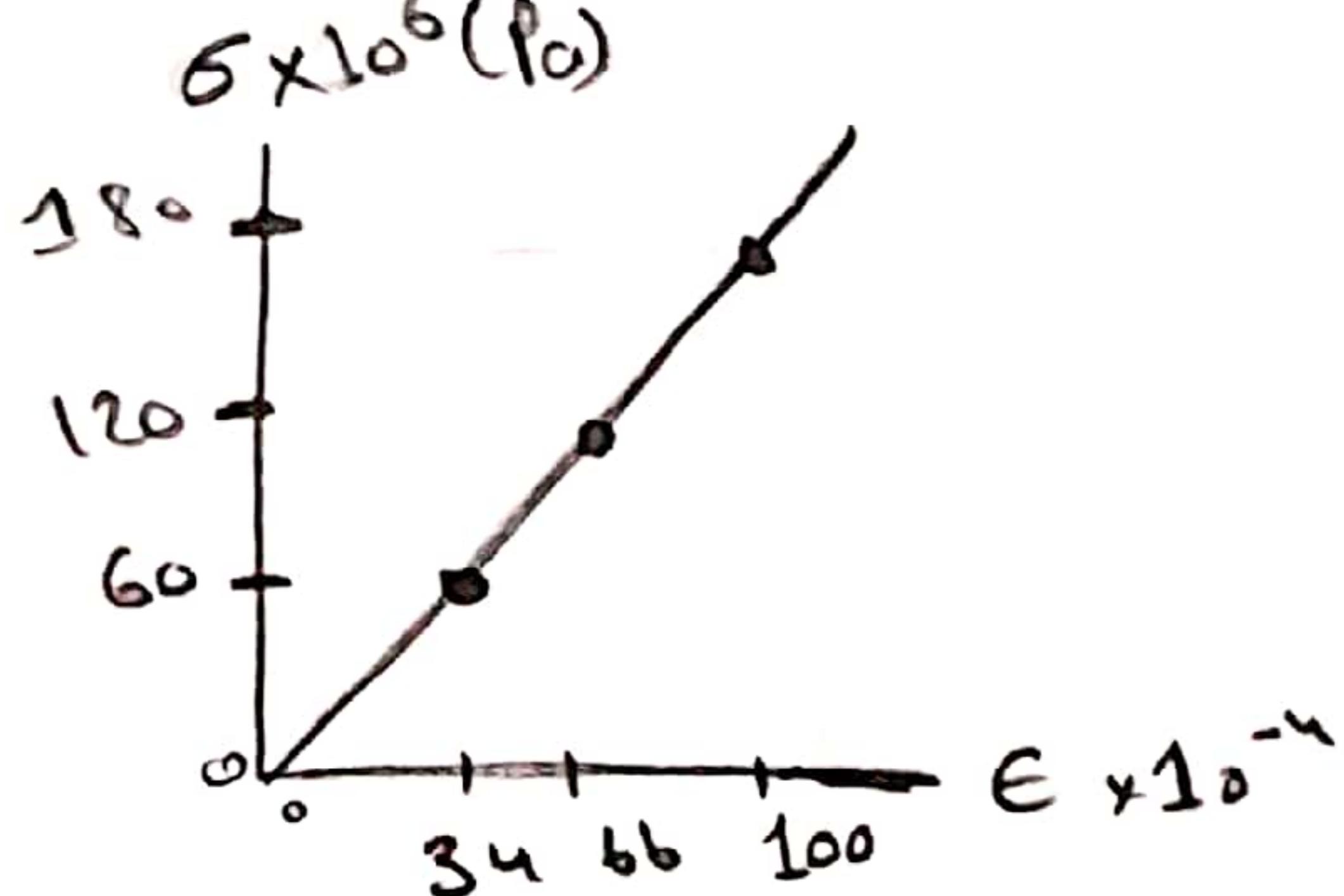
Ans.

$$A = 4 \text{ mm}^2 \text{ or } 4 \times 10^{-6} \text{ m}^2$$

$$l_0 = 5 \text{ m}$$

$$G = \frac{F}{A} \quad E = \frac{l - l_0}{l_0}$$

<u>$F(\text{N})$</u>	<u>$\sigma \times 10^6 (\text{Pa})$</u>	<u>$l(\text{mm})$</u>	<u>$\epsilon (\text{mm/mm})$</u>
0	0	5,000	0,0
240	60	5,017	0,0034
480	120	5,033	0,0066
720	180	5,050	0,0100

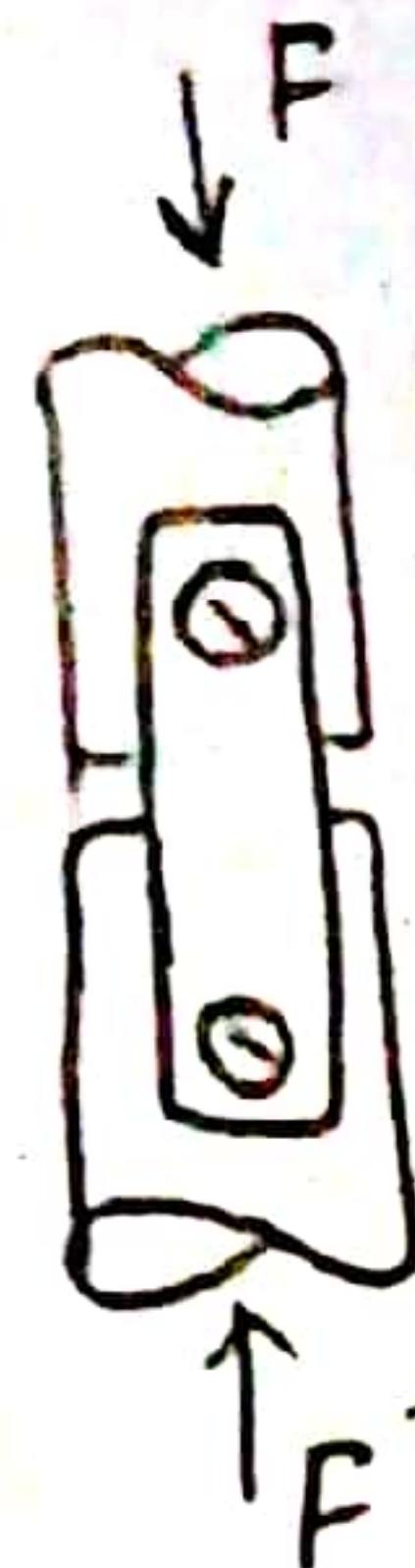


Stress-strain diagram

For the bone

$$E = \frac{\sigma}{\epsilon} = \frac{180 \times 10^6}{0,0100} = 18 \times 10^9 \text{ Pa} = 18 \text{ GPa}$$

not important



Fixation device consisting of a plate and two screws, which can be used to stabilize fractured bones.

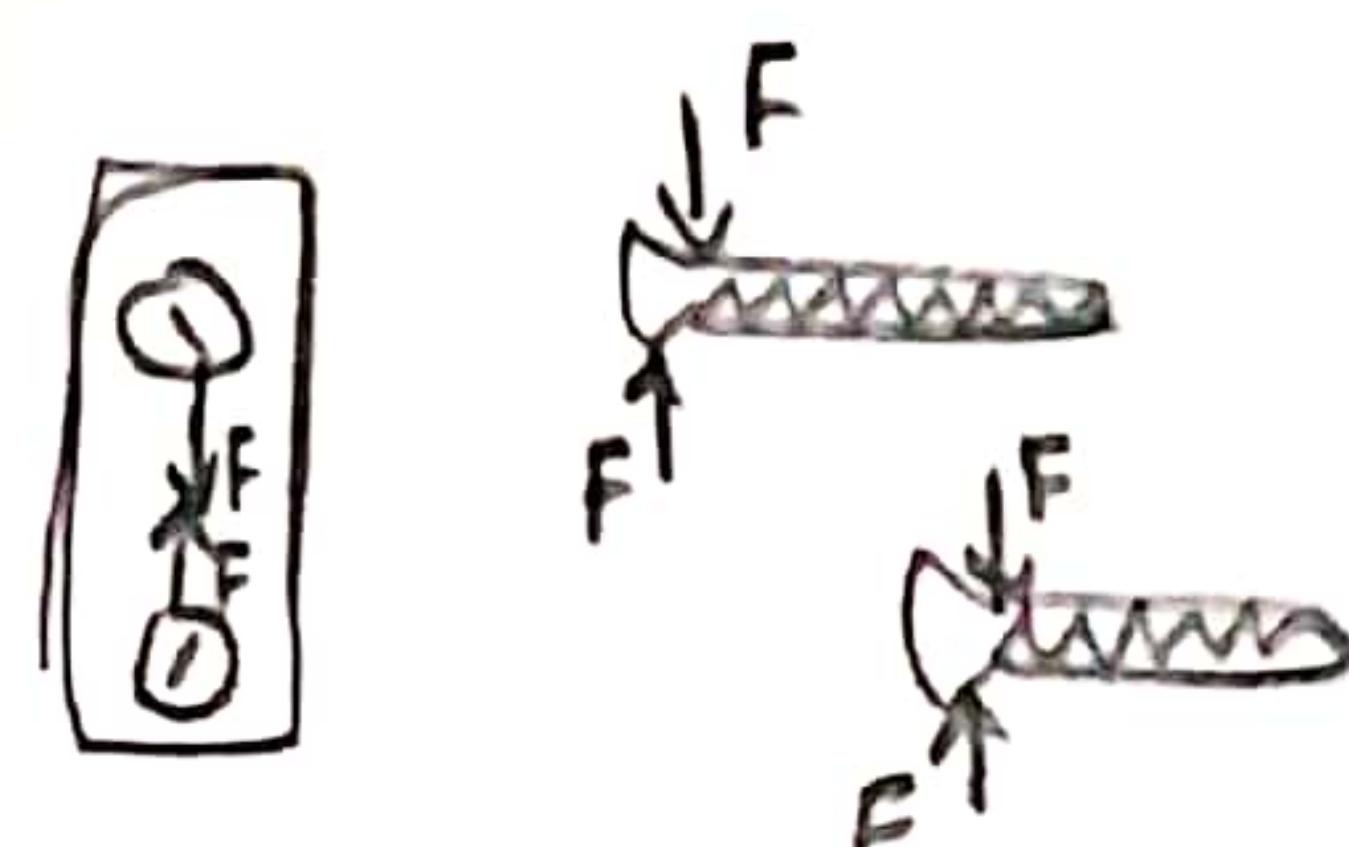
During a single leg stance, applied back on the person through the same foot, which has a compressive effect on the leg, its bones and joints.

diameter of the screw $D = 5 \text{ mm}$
weight of the patient $W = 700 \text{ N}$

Determine the shear stress exerted on the screws.

Ans

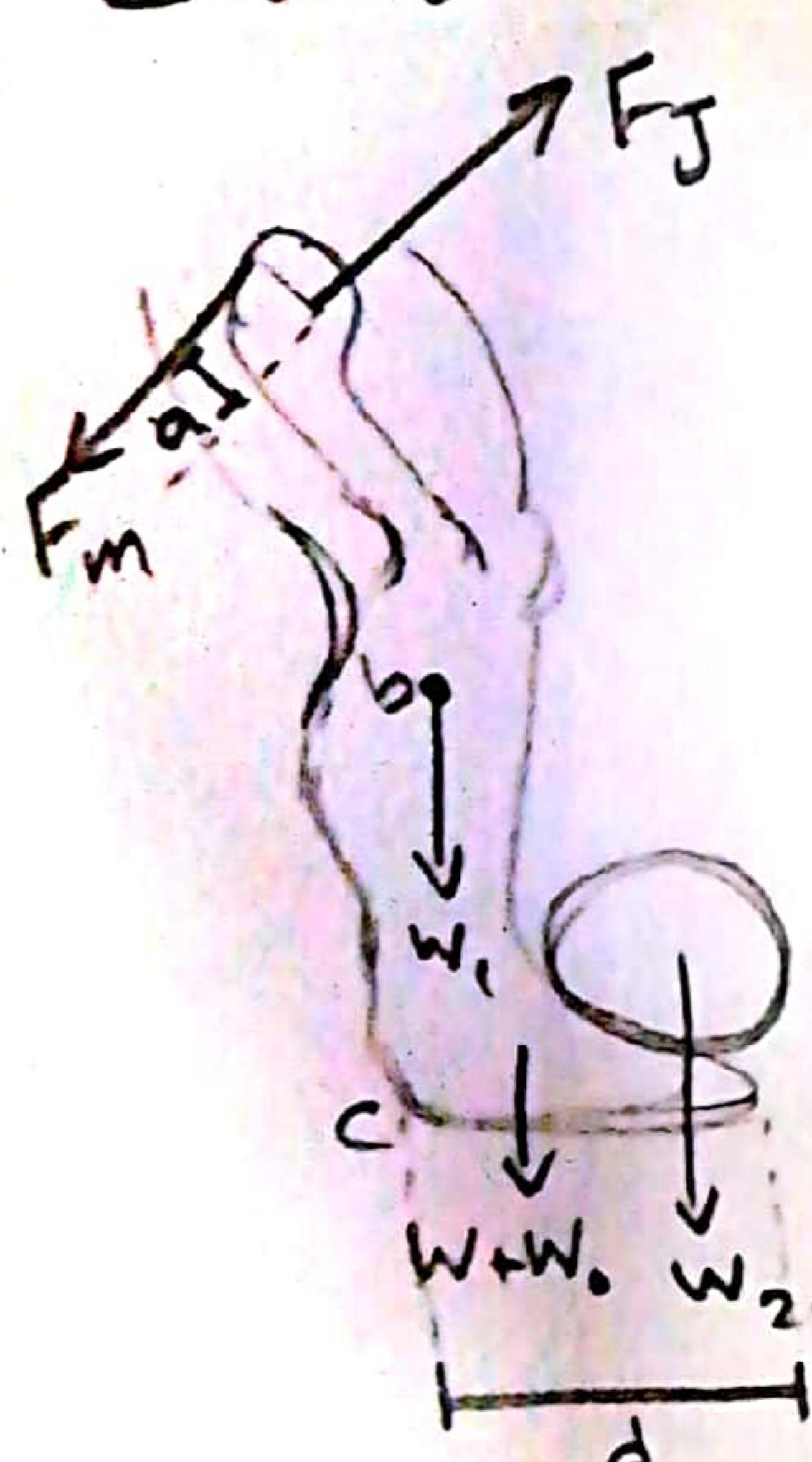
$$\tau = \frac{F}{A}$$



$$A = \pi D^2/4 = 19,6 \text{ mm}^2$$

$$\text{or } A = 19,6 \times 10^{-6} \text{ m}^2$$

Quiz



$$a = 0,02 \text{ m} \quad W_1 = 15 \text{ kg}$$

$$b = 0,05 \text{ m} \quad W_2 = 10 \text{ kg}$$

$$c = 0,07 \text{ m} \quad W = 75 \text{ kg}$$

$$d = 0,10 \text{ m} \quad W_0 = 50 \text{ kg}$$

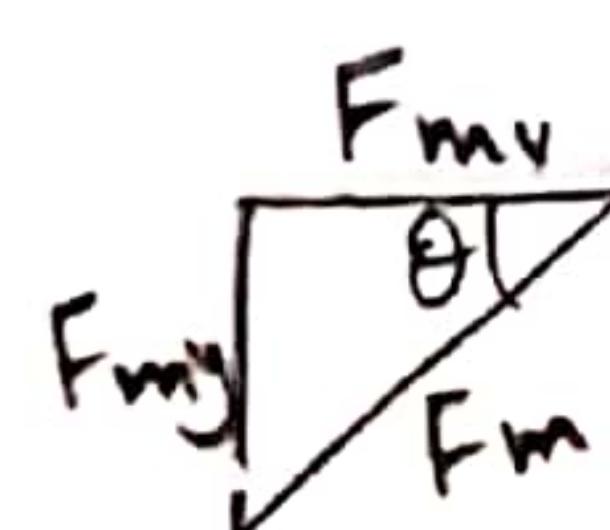
$$\theta = 30^\circ$$

Find F_m and F_m ?

$$\text{ans. } \sum m_0 = 0$$

$$-aF_m + bW_1 + c(W + W_0) + dW_2 = 0$$

$$\Rightarrow F_m = \frac{bW_1 + c(W + W_0) + dW_2}{a}$$



$$F_{mx} = F_m \cos \theta, \quad F_{my} = F_m \sin \theta$$

$$\sum F_x = 0 \Rightarrow F_{mx} = -F_{jx}$$

$$\sum F_y = 0 \Rightarrow F_{my} = F_{jy} - W_1 - (W + W_0) - W_2$$

$$F_m = \frac{(0,05 \text{ m})(15) + (0,07 \text{ m})(125) + (0,1 \text{ m})(10)}{0,02 \text{ m}} = \boxed{F_m = 525}$$

$$\Rightarrow F_{mx} = 525 \cos 30 = 454,66$$

$$F_{my} = 525 \sin 30 = 262,5$$

$$\Rightarrow F_{jx} = -454,66$$

$$\Rightarrow F_{jy} = F_{my} + W_1 + (W + W_0) + W_2 \\ = 262,5 + 15 + 10 + 125$$

$$F_{jy} = 412,5$$

$$F_y = \sqrt{(F_{jx})^2 + (F_{jy})^2} = \sqrt{(-454,66)^2 + (412,5)^2}$$

$$F_y = 613,898$$

$$\Rightarrow F_y \approx 614$$

Quiz



Applied Force (N)

A 0
B 120
C 240
D 360

Measured length (mm)

7,000
7,010
7,020
7,050

$$D = 2 \text{ mm}, \pi = 3,14$$

Plot stress-strain diagram
between A and C
also C and D
Find E_{AC} and E_{CD}

ans.

$$A = \pi \left(\frac{D}{2}\right)^2 = \pi \frac{D^2}{4} = (3,14) \left(\frac{4}{4}\right) = 3,14 \text{ mm}^2 = 3,14 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F}{A}, \epsilon = \frac{l - l_0}{l_0}$$

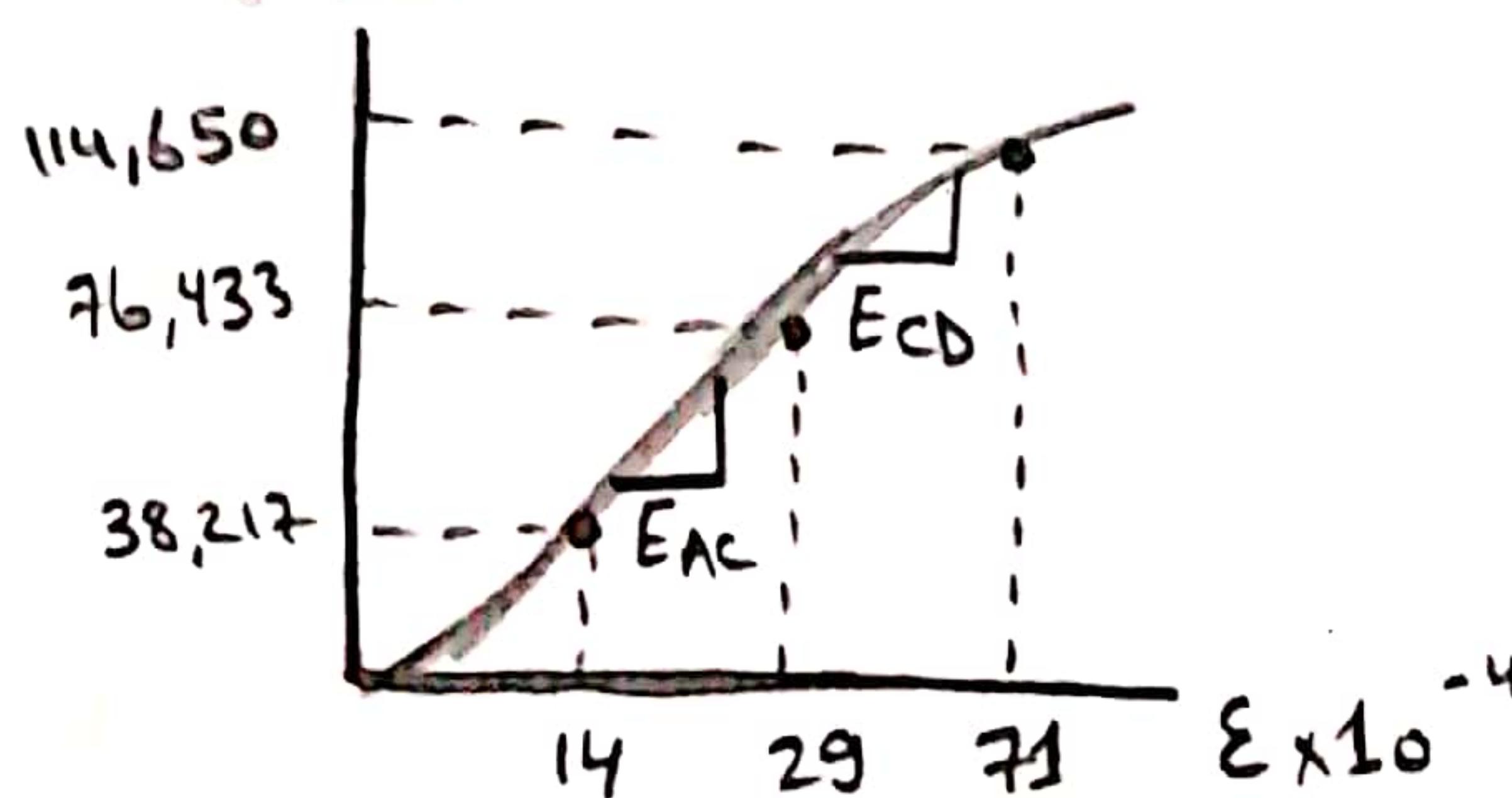
We take this
as l_0 always.

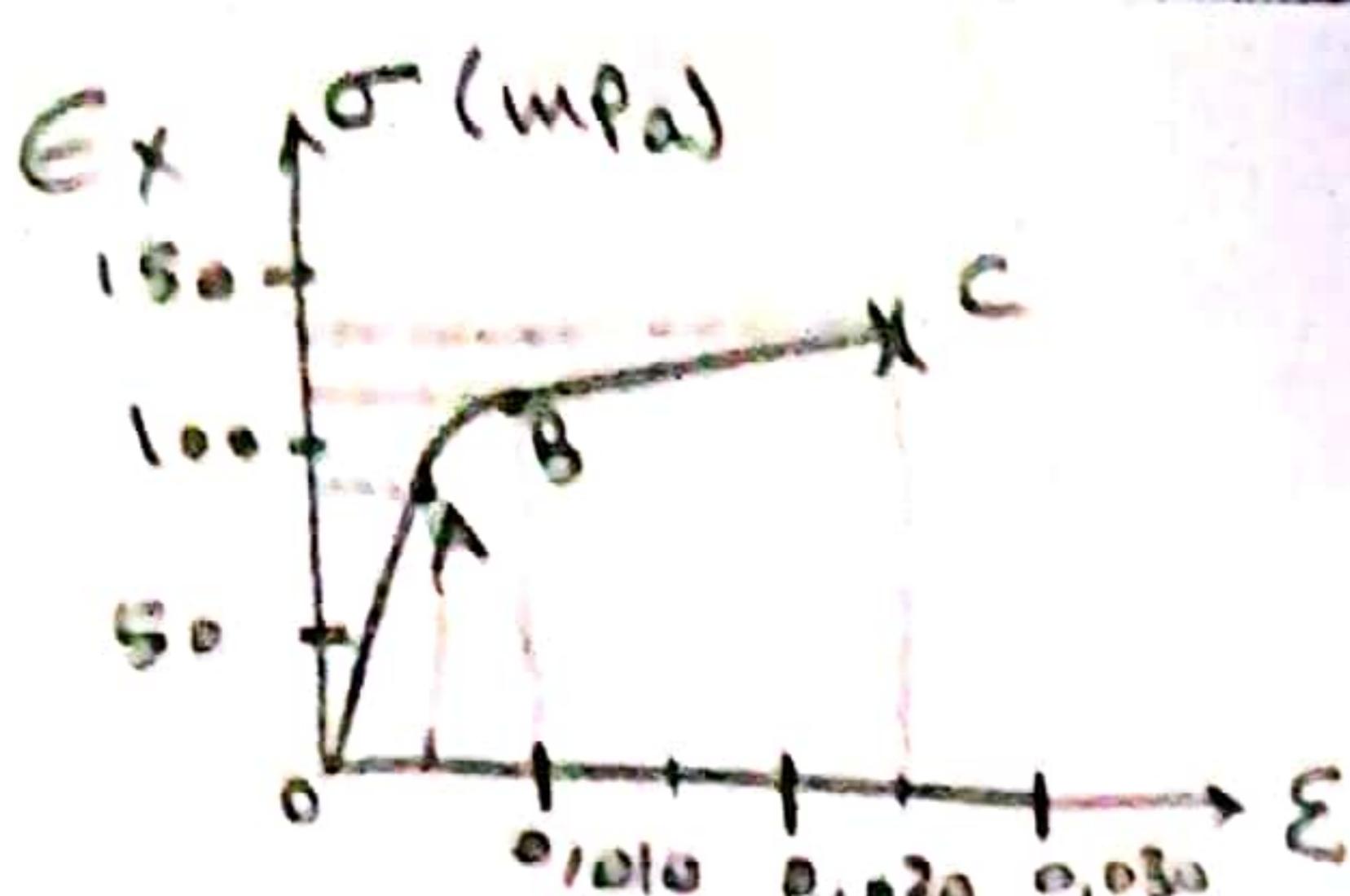
	<u>$F(N)$</u>	<u>$\sigma \times 10^6 (\text{Pa})$</u>	<u>$l(\text{mm})$</u>	<u>$\epsilon (\text{mm/mm})$</u>
A	0	0	7,000	0,0
B	120	38,217	7,010	0,0014
C	240	76,433	7,020	0,0029
D	360	114,650	7,050	0,0071

$$E_{AC} = \frac{\sigma_c - \sigma_A}{\epsilon_c - \epsilon_A} = \frac{76,433 \times 10^6}{0,0029 - 0} = 26,36 \times 10^9 \text{ Pa} = 26,36 \text{ GPa}$$

$$E_{CD} = \frac{\sigma_D - \sigma_C}{\epsilon_D - \epsilon_C} = \frac{(114,650 - 76,433) \times 10^6}{(71 - 29) \times 10^{-4}} = \frac{(38,217) \times 10^6}{42 \times 10^{-4}}$$

$$= 9,099 \times 10^9 \text{ Pa} = 9,099 \text{ GPa}$$





Tensile stress-strain diagram for cortical bone
($1 \text{ MPa} = 10^6 \text{ Pa}$)

- Human cortical bone tissue subjected to a simple tension test until fracture
- Stress-Strain diagram is presented
- Linearly elastic region between O and A
- Non-linear elastoplastic region btwn A and B
- Linearly plastic region btwn B and C

Point	stress σ (MPa)	strain ϵ
O	0	0,0
A	85	0,005
B	114	0,010
C	128	0,026

Determine elastic and strain hardening moduli of bone tissue. Also find mathematical expression relating stress to strain in the linearly elastic and linearly plastic region.

ans.

$$E = \frac{\sigma_A - \sigma_0}{\epsilon_A - \epsilon_0} = \frac{85 \times 10^6 - 0}{0,005 - 0,0} = 17 \times 10^9 \text{ Pa} = 17 \text{ GPa}$$

$$E' = \frac{\sigma_C - \sigma_B}{\epsilon_C - \epsilon_B} = \frac{128 \times 10^6 - 114 \times 10^6}{0,026 - 0,010} = 0,875 \times 10^9 \text{ Pa}$$

$$\sigma_A = E_A \epsilon_A \text{ or } \epsilon_A = \frac{\sigma_A}{E_A}$$

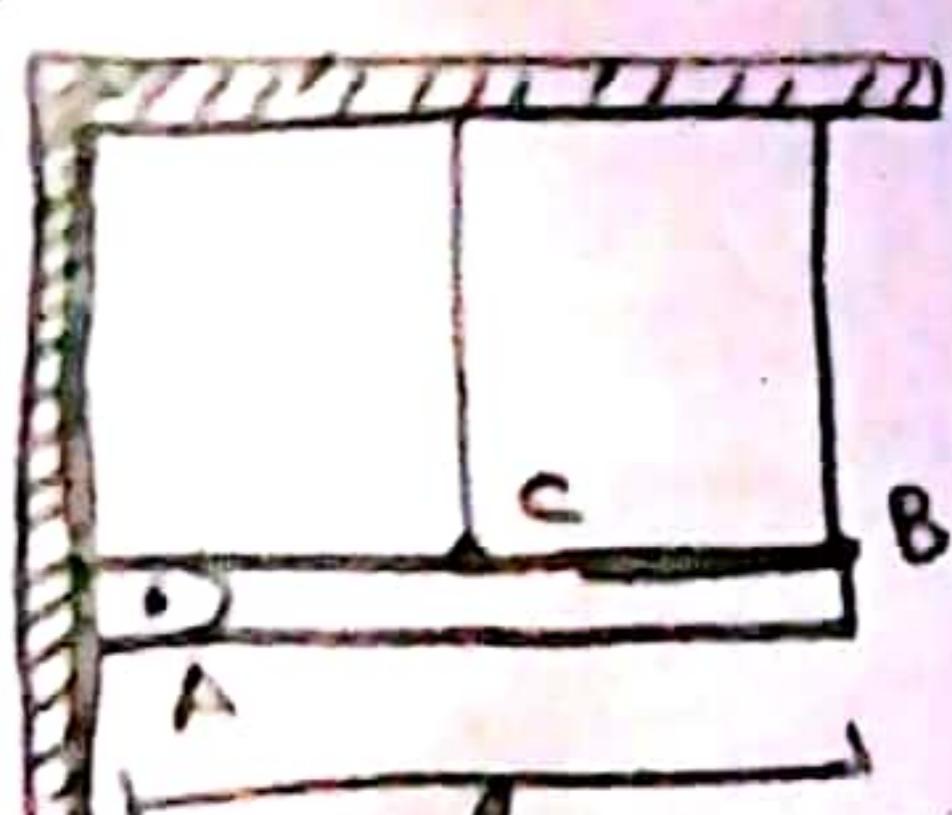
$$\sigma_C = \sigma_B + E'(\epsilon_C - \epsilon_B) \text{ or } \epsilon_C = \epsilon_B + \frac{1}{E'} (\sigma_C - \sigma_B)$$

when a tensile stress is applied

$$\text{as } \sigma_C = 120 \text{ MPa:}$$

$$\epsilon = 0,010 + \frac{120 \times 10^6 - 114 \times 10^6}{0,875 \times 10^9} = 0,017$$

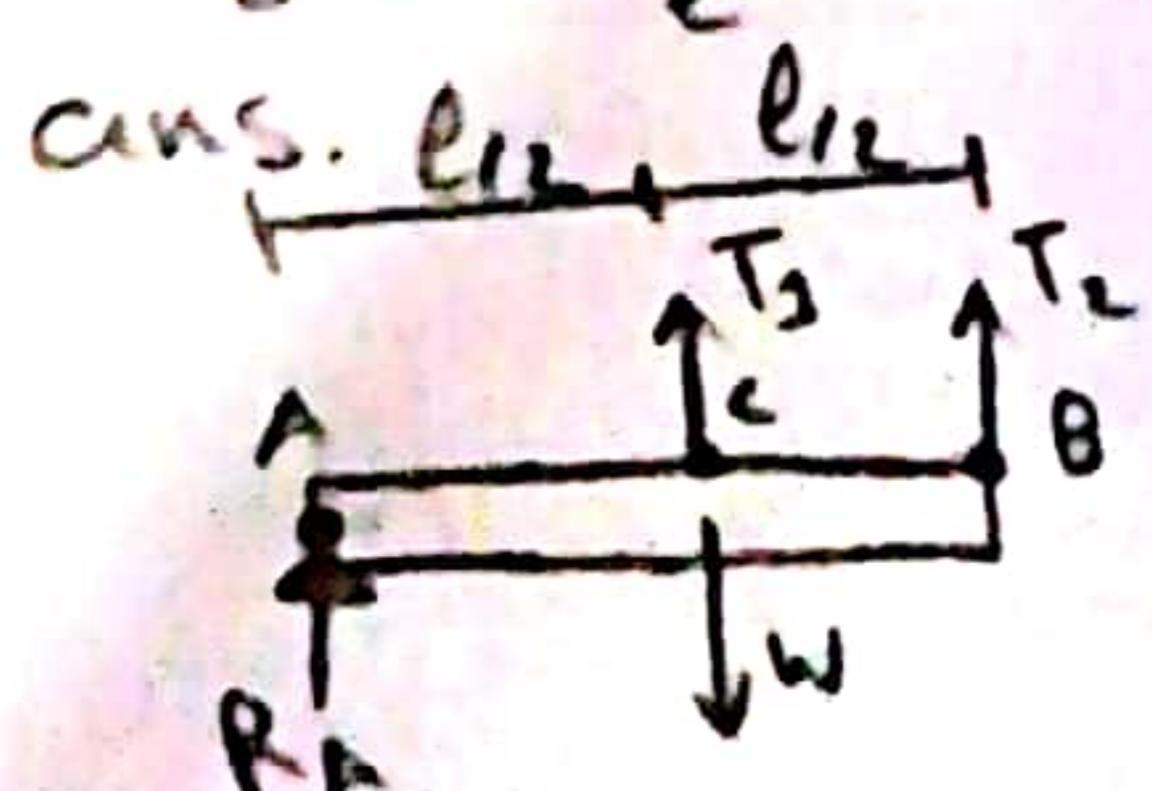
Ex:



$l = 4 \text{ m}$
 $W = 500 \text{ N}$

- hinged at wall at A
- The beam is supported by 2 identical steel bars of length $h = 3 \text{ m}$
- Cross-Section area $A = 2 \text{ cm}^2$
- Elastic modulus $E = 200 \text{ GPa}$

Determine the Forces applied by the beam on the steel bars, The reaction forces at A, The amount of elongation at each steel bar, and the stress generated in each bar.

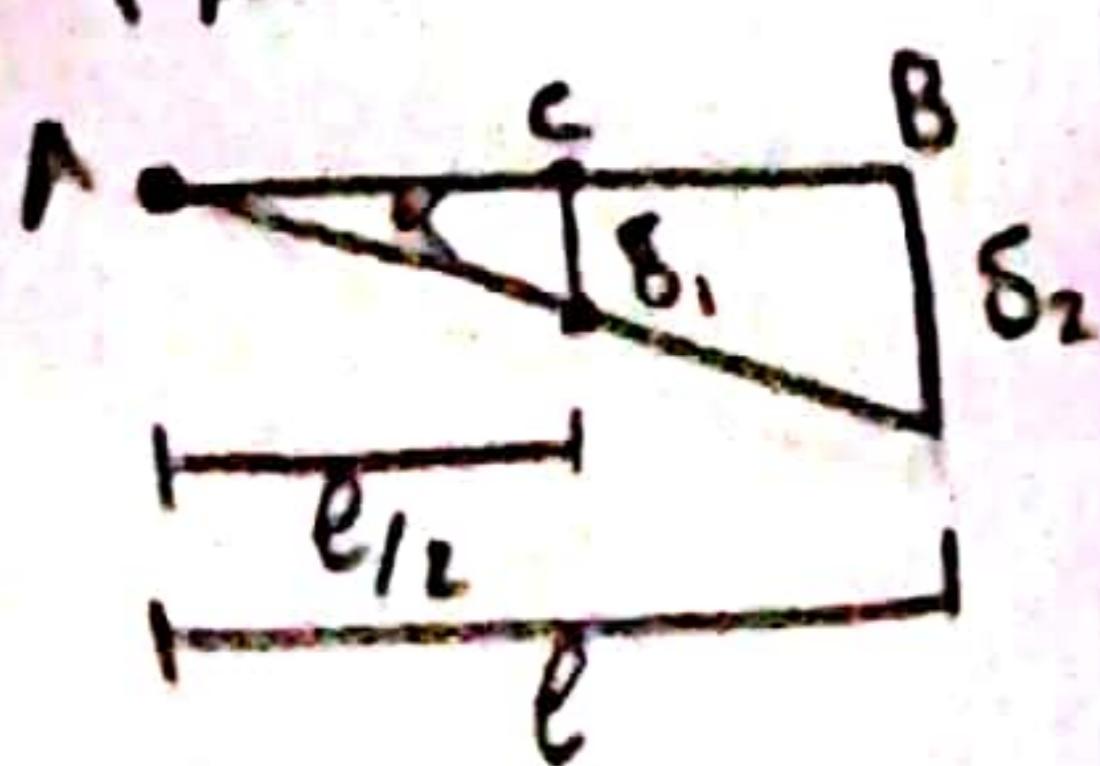


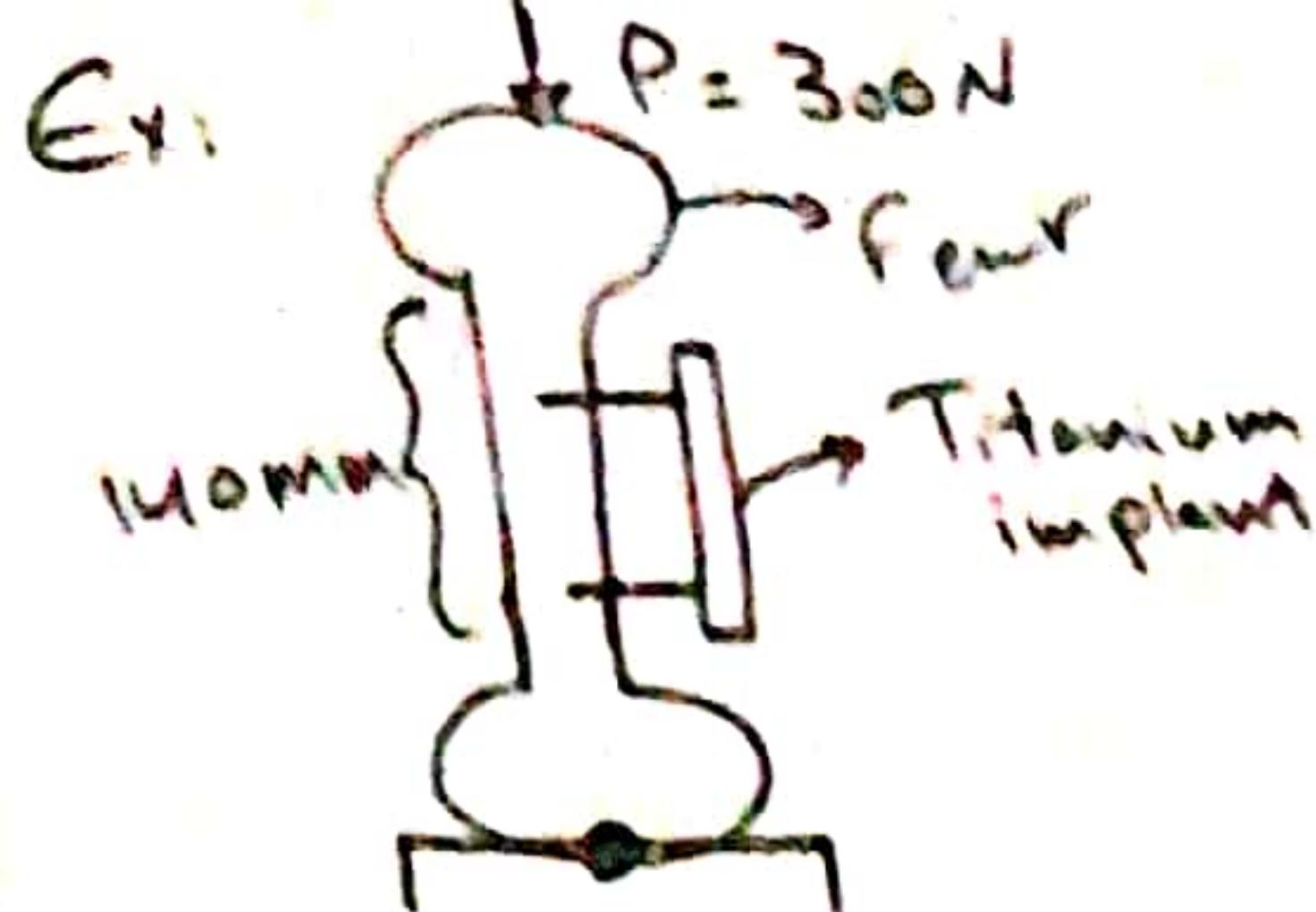
$$\sum F_y = 0: T_1 + T_2 + R_A = W$$

$$\sum M_A = 0: \frac{1}{2} T_1 + T_2 = \frac{1}{2} W$$

$$\tan \alpha \approx \frac{\delta_1}{l/2} \approx \frac{\delta_2}{l} \quad \epsilon_1 = \frac{\delta_1}{h}, \quad \epsilon_2 = \frac{\delta_2}{h} \quad \sigma_1 = E \epsilon_1, \quad \sigma_2 = E \epsilon_2$$

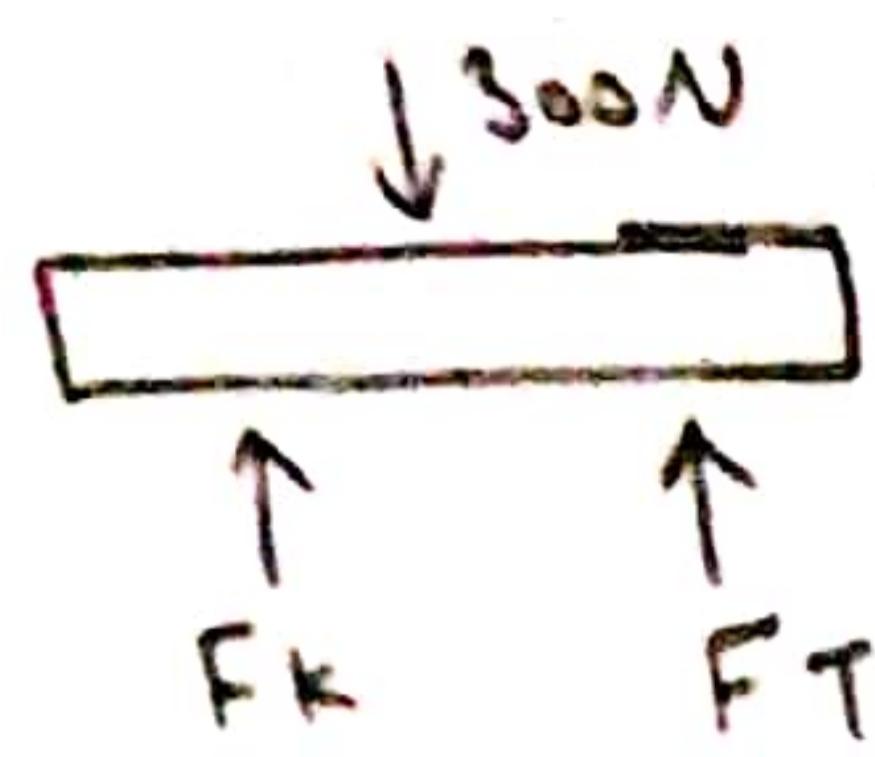
$$\left. \begin{aligned} S_1 &= \frac{T_1 h}{EA} \\ S_2 &= \frac{T_2 h}{EA} \end{aligned} \right\} \left. \begin{aligned} \delta_1 &= 2 S_1 \\ T_1 &= 2 T_2 \end{aligned} \right\} \left. \begin{aligned} T_1 &= \frac{1}{5} W = 100 \text{ N} \\ T_2 &= \frac{2}{5} W = 200 \text{ N} \\ R_A &= \frac{2}{5} W = 200 \text{ N} \end{aligned} \right\} \left. \begin{aligned} \sigma_1 &= \frac{T_1}{A} = 0,5 \times 10^6 \text{ Pa} \\ \sigma_2 &= \frac{T_2}{A} = 1,0 \times 10^6 \text{ Pa} \\ \epsilon_1 &= \frac{\sigma_1}{E} = 2,5 \times 10^{-6} \\ \epsilon_2 &= \frac{\sigma_2}{E} = 5,0 \times 10^{-6} \end{aligned} \right\}$$





Titanium implant
 $E_T = 120 \text{ GPa}$ $E_{\text{Kort}} = 20 \text{ GPa}$
 $A_T = 15 \text{ mm}^2$ $A_K = 50 \text{ mm}^2$
 what is ΔL after compression
 what is σ_K ?

ans.



$$\sum F_y = 0 \quad \delta = \delta_K = \delta_T$$

$$F_K + F_T = 300$$

$$\delta_K = \frac{F_K \cdot L_K}{A_K \cdot E_K} = \frac{F_T \cdot L_T}{A_T \cdot E_T}$$

$$\frac{F_K \cdot 140 \text{ mm}}{50 \text{ mm}^2 \cdot 20 \cdot 10^3 \text{ MPa}} = \frac{F_T \cdot 140 \text{ mm}}{15 \text{ mm}^2 \cdot 120 \cdot 10^3 \text{ MPa}} \Rightarrow 1800 F_K = 1000 F_T$$

$$F_T = 1,8 F_K \Rightarrow F_K + 1,8 F_K = 300 \Rightarrow F_K = 107,2 \text{ N}$$

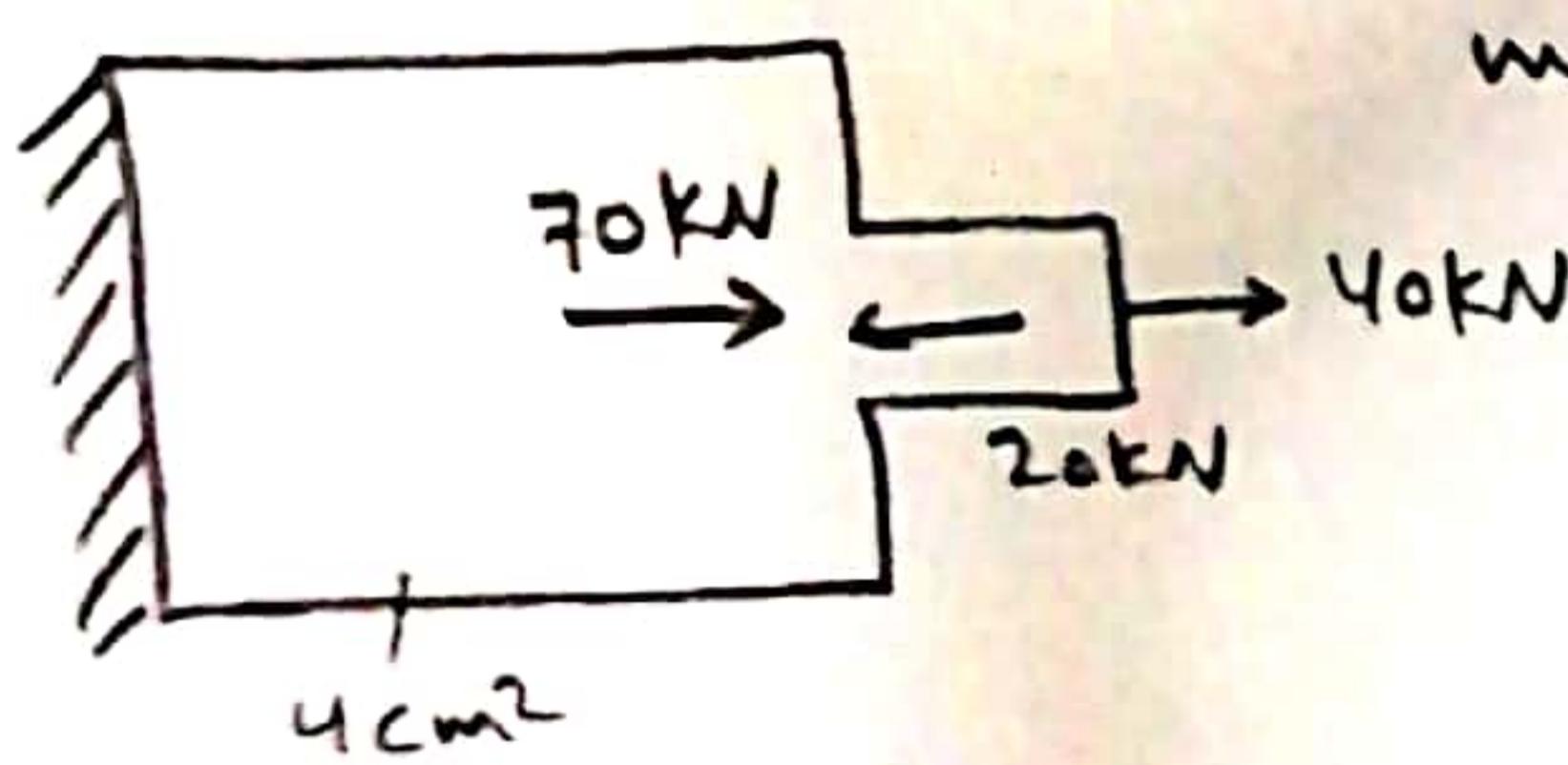
$$F_T = 192,96 \text{ N}$$

$$\delta_T = \frac{192,96 \text{ N} \cdot 140 \text{ mm}}{15 \text{ mm}^2 \cdot 120 \cdot 10^3 \text{ MPa}} = 150 \cdot 10^{-3} \text{ mm}$$

$$= 0,015 \text{ mm}$$

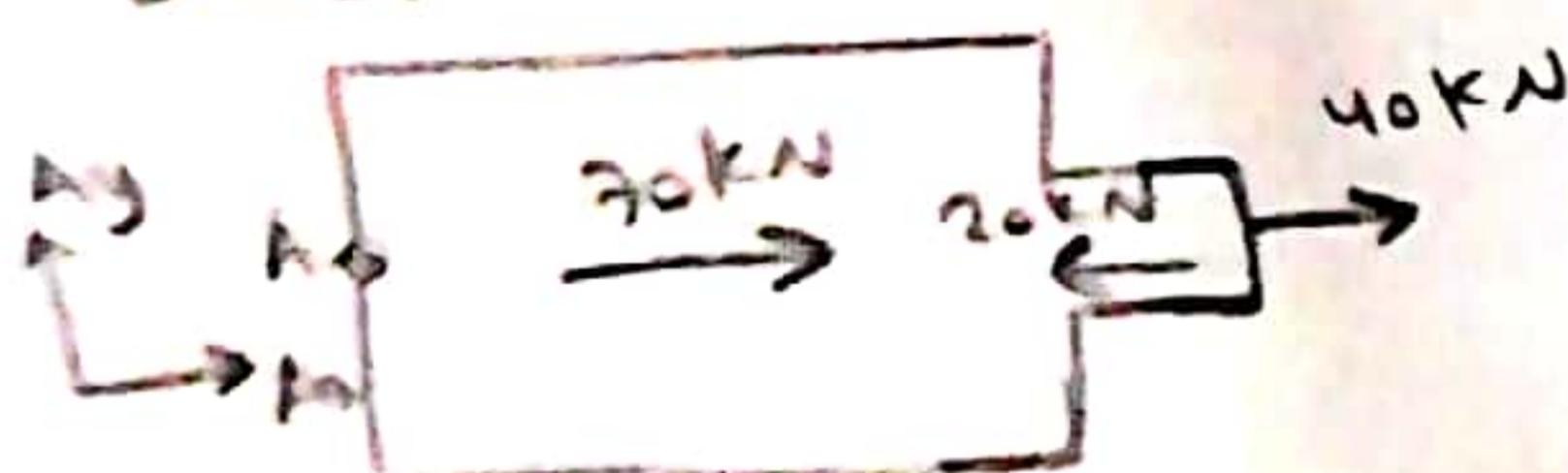
$$\delta_K = \frac{107,2 \text{ N}}{50 \text{ mm}^2} = 2 \text{ MPa}$$

Ex:



max stress?

ans.



$$\sum F_y = 0, A_y = 0$$

$$\sum F_x = 0 \quad -20 + 70 + 40 + A_x = 0 \quad \Rightarrow A_x = -90 \text{ kN}$$

$$\sum M_A = 0$$



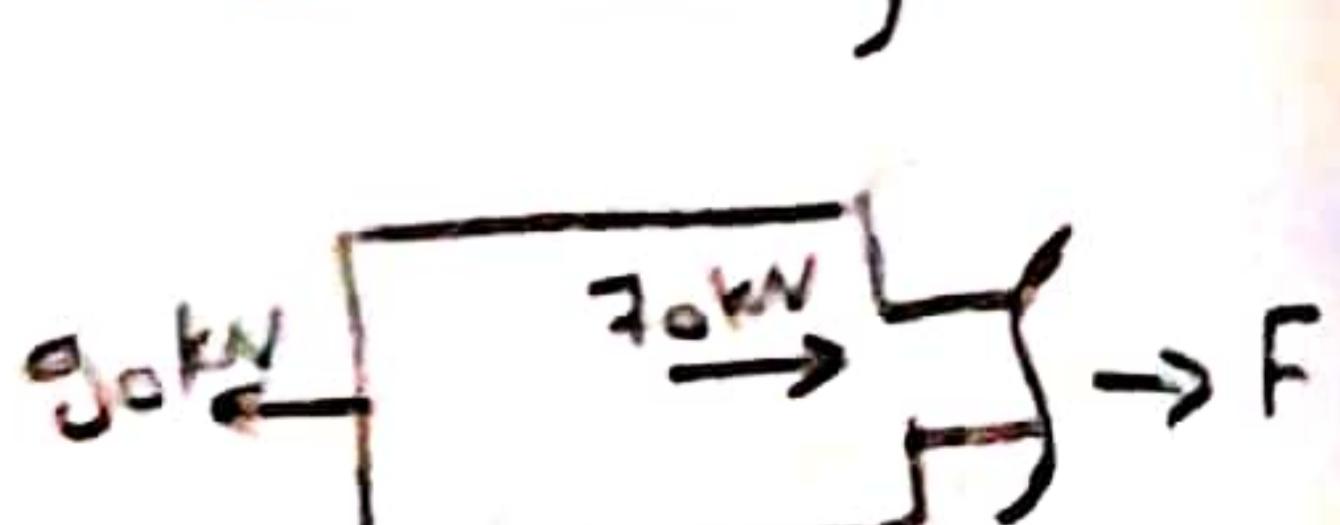
$$\sum F_x = 0$$

$$-90 + F = 0$$

$$F = 90 \text{ kN}$$

$$\sigma = \frac{90 \text{ kN}}{4 \text{ cm}^2} = \frac{90 \cdot 10^3}{4 \cdot 10^{-4}} = 22,5 \cdot 10^7$$

$$= 225 \text{ MPa}$$

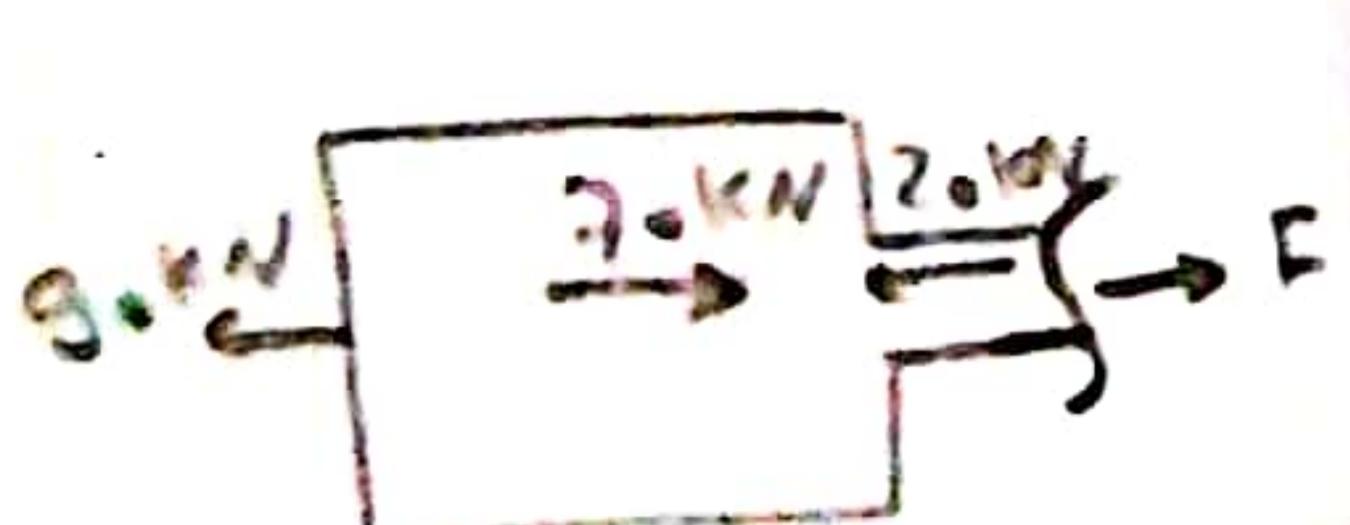


$$\sum F_x = 0$$

$$-90 + 70 + F = 0$$

$$F = 20 \text{ kN}$$

$$\sigma = \frac{20 \text{ kN}}{2 \text{ cm}^2} = \frac{20 \cdot 10^3}{2 \cdot 10^{-4}} = 100 \text{ MPa}$$



$$\sum F_x = 0$$

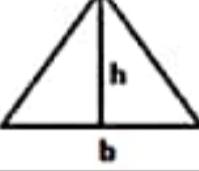
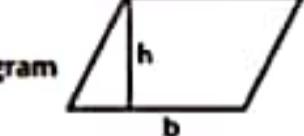
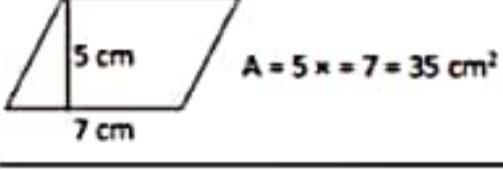
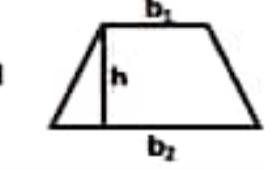
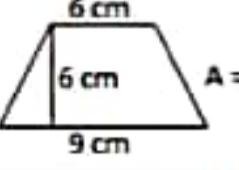
$$-90 + 70 - 20 + F = 0$$

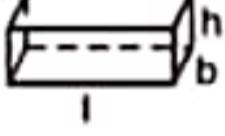
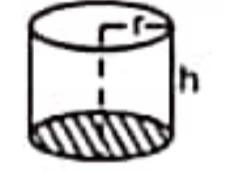
$$F = 40 \text{ kN}$$

$$\sigma = \frac{40 \text{ kN}}{2 \text{ cm}^2} = \frac{40 \cdot 10^3}{2 \cdot 10^{-4}} = 200 \text{ MPa}$$

MULTIPLICATION FACTOR	SI PREFIX	SI SYMBOL
$1,000,000,000 = 10^9$	Giga	G
$1,000,000 = 10^6$	Mega	M
$1000 = 10^3$	Kilo	k
$100 = 10^2$	Hector	h
$10 = 10^1$	Deka	da
$.1 = 10^{-1}$	Deci	d
$.01 = 10^{-2}$	Centi	c
$.001 = 10^{-3}$	Milli	m
$.000,001 = 10^{-6}$	Micro	μ
$.000,000,001 = 10^{-9}$	Nano	n
$.000,000,000,001 = 10^{-12}$	Pico	p

Length	1 centimeter (cm) = 0.01 meter (m) = 0.3937 inch (in.)		Mass	1 pound mass (lbm) = 0.4536 kilogram (kg) 1 kg = 2.2 lbm = 0.0685 slug 1 slug = 14.59 kg = 32.098 lbm
	1 in. = 2.54 cm = 0.0254 m		Force	1 kilogram force (kgf) = 9.807 Newton (N) 1 pound force (lbf) = 4.448 N 1 N = 0.2248 lbf 1 dyne (dyn) = 10^{-5} N 1 N = 10^5 dyn
	1 foot (ft) = 30.48 cm—0.3048 m		Pressure and stress	$1 \text{ N/m}^2 = 1 \text{ Pascal (Pa)} = 0.000145 \text{ lbf/in.}^2$ (psi) 1 psi = 6895 Pa $1 \text{ lbf/in.}^2 \text{ (psf)} = 592,966 \text{ Pa}$ $1 \text{ dyn/cm}^2 = 0.1 \text{ Pa}$
	1 m = 3.28 ft = 39.37 in.		Moment (Torque)	$1 \text{ Nm} = 10^7 \text{ dyn cm} = 0.7376 \text{ lbf ft}$ $1 \text{ dyn/cm} = 10^{-7} \text{ Nm}$ 1 lbf ft = 1.356 Nm
	1 yard (yd) = 0.9144 m = 3 ft		Work and energy	$1 \text{ Nm} = 1 \text{ Joule (J)} = 10^7 \text{ erg}$ 1 J = 0.7376 lbf ft 1 lbf ft = 1.356 J
	1 mile = 1609 m = 1.609 kilometer (km) = 5280 ft		Power	$1 \text{ kg m}^2/\text{s}^3 = 1 \text{ J/s} = 1 \text{ Watt (W)}$ 1 horsepower (hp) = 550 lbf ft/s = 746 W 1 lbf ft/s = 1.356 W 1 W = 0.737 lbf ft/s
	1 km = 0.6214 mile		Plane angle	1 degree ($^\circ$) = $\pi/180$ radian (rad) 1 revolution (rev) = 360° 1 rev = 2π rad = 6.283 rad 1 rad = 57.297° $1^\circ = 0.0175$ rad
Time	1 minute (min) = 60 seconds (s) 1 hour (h) = 60 min = 3600 s 1 day = 24 h = 1440 min = 86,400 s		Temperature	${}^\circ\text{C} = 273.2 \text{ K}$ ${}^\circ\text{C} = 5/9 (-32 {}^\circ\text{F})$ ${}^\circ\text{F} = 9/5 (+32 {}^\circ\text{C})$
Area	1 cm² = 0.155 in.² 1 in.² = 6.452 cm² 1 m² = 10.763 ft² 1 ft² = 0.0929 m²		<i>(continued)</i>	

Shape	Formula	Example
Square	 $A = l \times l = l^2$	What is the area of a square of length 4 cm? $A = 4 \times 4 = 16 \text{ cm}^2$
Rectangle	 $A = l \times w$	What is the area of a rectangle of length 7 inches and width 5 inches? $A = 7 \times 5 = 35 \text{ in}^2$
Triangle	 $A = 1/2 \times b \times h$	What is the area of a triangle with height 6 inches and base 5 inches? $A = 1/2 \times 6 \times 5 = 15 \text{ in}^2$
Parallelogram	 $A = h \times b$	 $A = 5 \times 7 = 35 \text{ cm}^2$
Trapezoid	 $A = 1/2 \times h \times (b_1 + b_2)$	 $A = 1/2 \times 6 \times (9 + 6) = 45 \text{ cm}^2$
Circle	 $A = \pi \times r^2$ ($\pi = 3.14$ or $22/7$)	What is the area of a circle with radius 9 feet? $A = \pi \times 9^2 = 81\pi = 254.34 \text{ ft}^2$
Rhombus	 $A = 1/2 \times d_1 \times d_2$	What is the area of a rhombus with diagonals 8 inches and 7 inches? $A = 1/2 \times 8 \times 7 = 28 \text{ in}^2$

Name	Figure	Curved Surface area	Total surface area	Volume
Cuboid		$2h(l+b)$	$2(lb + bh + lh)$	lbh
Cube		$6a^2$	$6a^2$	a^3
Right circular cylinder		$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right circular cone		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
Sphere		$4\pi r^2$	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$

Example 3.1 Figure 3.13 illustrates a person preparing to dive into a pool. The horizontal diving board has a uniform thickness, mounted to the ground at point O, has a mass of 120 kg, and is $l = 4$ m in length. The person has a mass of 90 kg and stands at point B which is the free end of the board. Point A indicates the location of the center of gravity of the board. Point A is equidistant from points O and B.

Determine the moments generated about point O by the weights of the person and the board. Calculate the net moment about point O.

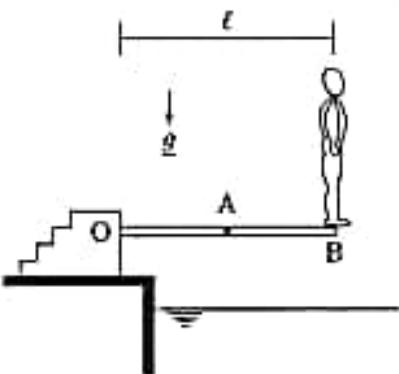


Fig. 3.13 A person is preparing to dive

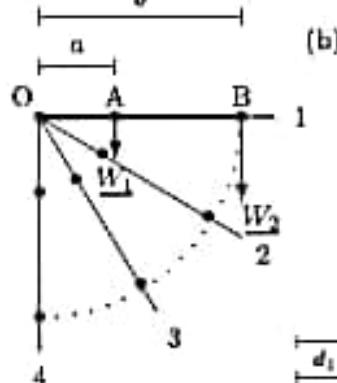
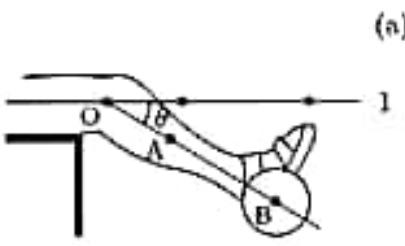
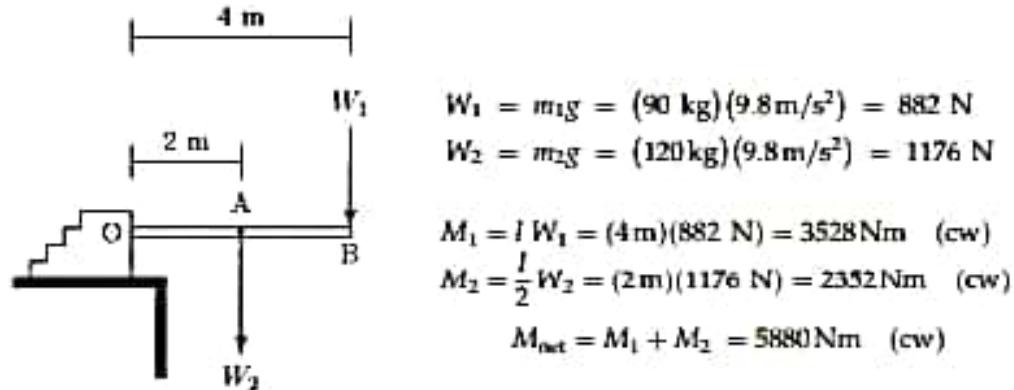


Fig. 3.15 Example 3.2

Example 3.2 As illustrated in Fig. 3.15a, consider an athlete wearing a weight boot, and from a sitting position, doing lower leg flexion/extension exercises to strengthen quadriceps muscles. The weight of the athlete's lower leg is $W_1 = 50 \text{ N}$ and the weight of the boot is $W_2 = 100 \text{ N}$. As measured from the knee joint at point O, the center of gravity (point A) of the lower leg is located at a distance $a = 20 \text{ cm}$ and the center of gravity (point B) of the weight boot is located at a distance $b = 50 \text{ cm}$.

Determine the net moment generated about the knee joint when the lower leg is extended horizontally (position 1), and when the lower leg makes an angle of 30° (position 2), 60° (position 3), and 90° (position 4) with the horizontal (Fig. 3.15b).

Solution:

$$M_O = aW_1 + bW_2 \\ = (0.20)(50) + (0.50)(100) \\ = 60 \text{ Nm (cw)}$$

$$d_1 = a \cos \theta \\ d_2 = b \cos \theta$$

Therefore, the net moment about point O is:

$$M_O = d_1 W_1 + d_2 W_2 \\ = a \cos \theta W_1 + b \cos \theta W_2 \\ = (aW_1 + bW_2) \cos \theta$$

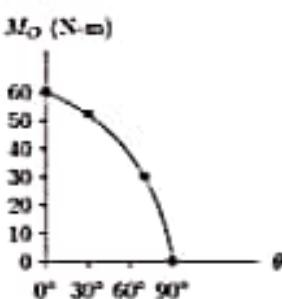
Fig. 3.16 Forces and moment arms when the lower leg makes an angle θ with the horizontal

The term in the parentheses has already been calculated as 60 Nm. Therefore, we can write:

$$M_O = 60 \cos \theta$$

For position 1: $\theta = 0^\circ$ $M_O = 60 \text{ Nm (cw)}$
 For position 2: $\theta = 30^\circ$ $M_O = 52 \text{ Nm (cw)}$
 For position 3: $\theta = 60^\circ$ $M_O = 30 \text{ Nm (cw)}$
 For position 4: $\theta = 90^\circ$ $M_O = 0 \text{ (cw)}$

In Fig. 3.17, the moment generated about the knee joint is plotted as a function of angle θ .



Example 3.3 Figure 3.18a illustrates an athlete doing shoulder muscle strengthening exercises by lowering and raising a barbell with straight arms. The position of the arms when they make an angle θ with the vertical is simplified in Fig. 3.18b. Point O represents the shoulder joint, A is the center of gravity of one arm, and B is a point of intersection of the centerline of the barbell and the extension of line OA. The distance between points O and A is $a = 24$ cm and the distance between points O and B is $b = 60$ cm. Each arm weighs $W_1 = 50$ N and the total weight of the barbell is $W_2 = 300$ N.

Determine the net moment due to W_1 and W_2 about the shoulder joint as a function of θ , which is the angle the arm makes with the vertical. Calculate the moments for $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ$, and 60° .

Solution

$$d_1 = a \sin \theta$$

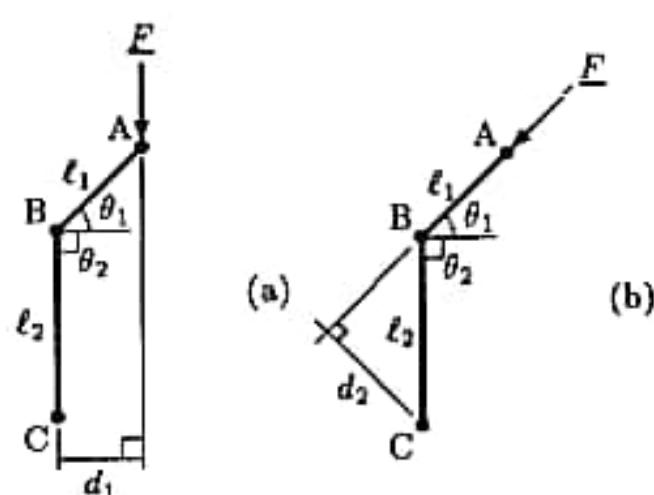
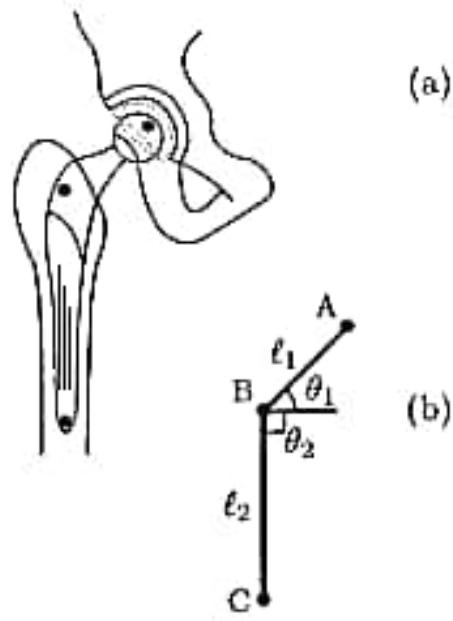
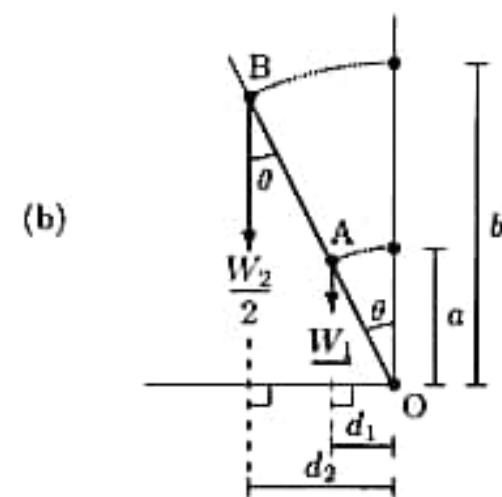
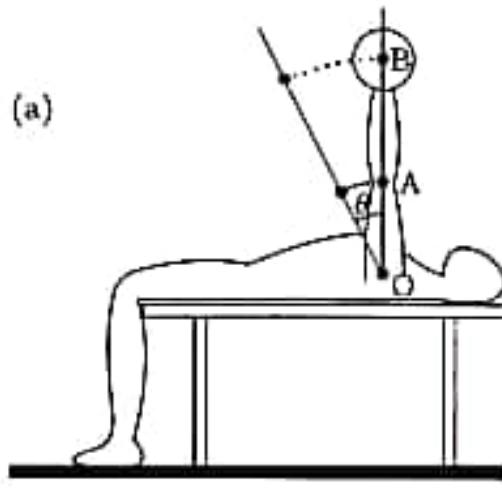
$$d_2 = b \sin \theta$$

$$M_1 = d_1 W_1 = a W_1 \sin \theta = (0.24)(50) \sin \theta = 12 \sin \theta$$

$$M_2 = d_2 \frac{W_2}{2} = b \frac{W_2}{2} \sin \theta = (0.60) \left(\frac{300}{2} \right) \sin \theta = 90 \sin \theta$$

$$M_O = M_1 + M_2 = 12 \sin \theta + 90 \sin \theta = 102 \sin \theta \text{ Nm (ccw)}$$

θ	$\sin \theta$	M_O (Nm)
0°	0.000	0.0
15°	0.259	26.4
30°	0.500	51.0
45°	0.707	72.1
60°	0.866	88.3



Example 3.4 Consider the total hip joint prosthesis shown in Fig. 3.19. The geometric parameters of the prosthesis are such that $l_1 = 50$ mm, $l_2 = 50$ mm, $\theta_1 = 45^\circ$, $\theta_2 = 90^\circ$. Assume that, when standing symmetrically on both feet, a joint reaction force of $F = 400$ N is acting at the femoral head due to the body weight of the patient. For the sake of illustration, consider three different lines of action for the applied force, which are shown in Fig. 3.20.

Determine the moments generated about points B and C on the prosthesis for all cases shown.

Solution $d_1 = l_1 \cos \theta_1 = (50)(\cos 45^\circ) = 35 \text{ mm}$

$$M_B = M_C = d_1 F = (0.035)(400) = 14 \text{ Nm (cw)}$$

$$M_B = 0$$

$$d_2 = l_2 \cos \theta_1 = (100)(\cos 45^\circ) = 71 \text{ mm}$$

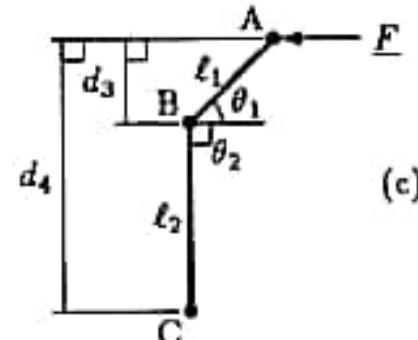
$$M_C = d_1 F = (0.071)(400) = 28 \text{ Nm (ccw)}$$

$$d_3 = l_1 \sin \theta_1 = (50)(\sin 45^\circ) = 35 \text{ mm}$$

$$d_4 = d_3 + l_2 = (35) + (100) = 135 \text{ mm}$$

$$M_B = d_3 F = (0.035)(400) = 14 \text{ Nm (ccw)}$$

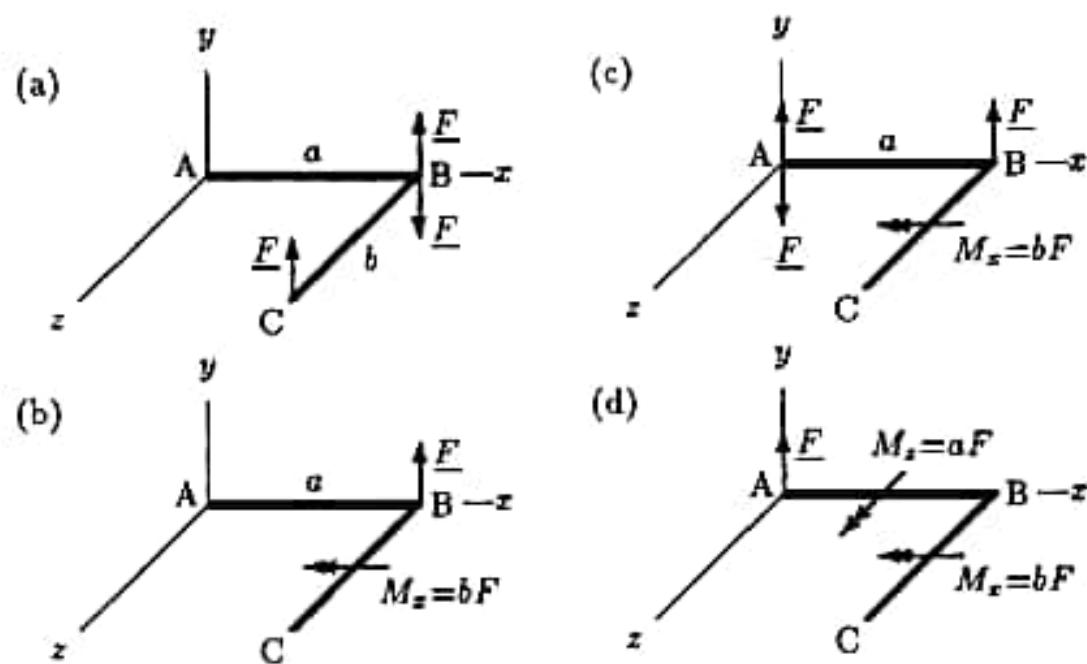
$$M_C = d_4 F = (0.135)(400) = 54 \text{ Nm (ccw)}$$



Example 3.5 Figure 3.27a illustrates a person using an exercise machine. The "L" shaped beam shown in Fig. 3.27b represents the left arm of the person. Points A and B correspond to the shoulder and elbow joints, respectively. Relative to the person, the upper arm (AB) is extended toward the left (x direction) and the lower arm (BC) is extended forward (z direction). At this instant the person is holding a handle that is connected by a cable to a suspending weight. The weight applies an upward (in the y direction) force with magnitude F on the arm at point C. The lengths of the upper arm and lower arm are $a = 25$ cm and $b = 30$ cm, respectively, and the magnitude of the applied force is $F = 200$ N.

Explain how force F can be translated to the shoulder joint at point A, and determine the magnitudes and directions of moments developed at the lower and upper arms by F .

Solution 1: Scalar Method



$$M_x = bF \quad (-x \text{ direction}) \quad M_z = aF \quad (+z \text{ direction}) \quad M_x = (0.30)(200) = 60 \text{ Nm}$$

Solution 2: Vector Product Method The definition of moment as the vector product of the position and force vectors is more straightforward to apply. The position vector of point C (where the force is applied) with respect to point A (where the shoulder joint is located) and the force vector shown in Fig. 3.29 can be expressed as follows:

$$\begin{aligned} \underline{r} &= a\underline{i} + b\underline{k} \\ \underline{F} &= F\underline{j} \\ \underline{M} &= \underline{r} \times \underline{F} \\ &= (a\underline{i} + b\underline{k}) \times (F\underline{j}) \\ &= aF(\underline{i} \times \underline{j}) + bF(\underline{k} \times \underline{j}) \\ &= aF\underline{k} - bF\underline{i} \\ &= (0.25)(200)\underline{k} - (0.30)(200)\underline{i} \\ &= 50\underline{k} - 60\underline{i} \\ M_x &= 60 \text{ Nm} \quad (-x \text{ direction}) \\ M_y &= 0 \\ M_z &= 50 \text{ Nm} \quad (+z \text{ direction}) \end{aligned}$$

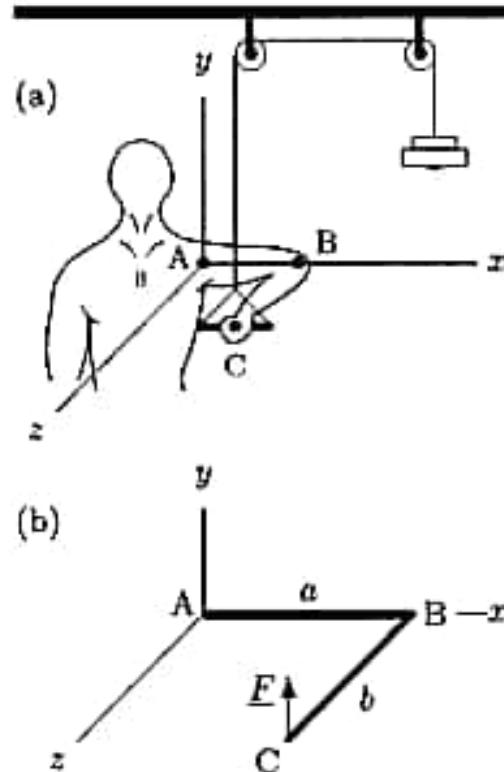


Fig. 3.27 Example 3.5

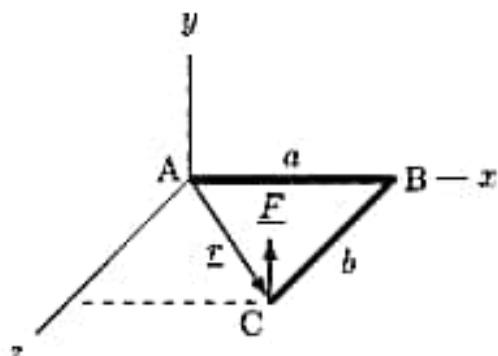


Fig. 3.29 Vector product method (Example 3.5)

Problem 3.1 As illustrated in Fig. 3.6, assume a worker is applying a force F to tighten a right-treaded bolt by using a wrench. As the result of the applied force the bolt is advancing into the metal plate. The recommended torque for the bolt is $M_{rec} = 80 \text{ Nm}$. Furthermore, the length of the wrench handle is $d = 25 \text{ cm}$.

- Calculate the magnitude of the force required to complete the task.
- How can the task be modified to decrease its force requirements by 50%?

Answers: (a) $F = 320 \text{ N}$; (b) by increasing the length of the wrench handle ($d = 50 \text{ cm}$)

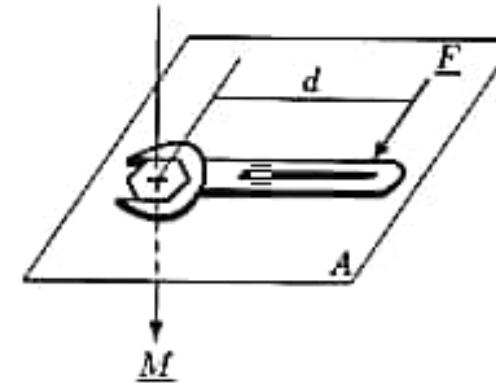


Fig. 3.6 Wrench and bolt

Problem 3.2 As illustrated in Fig. 3.13, consider a diver of 88 kg mass standing on the free end of a horizontal diving board at point B, preparing for a jump. The diving board has a uniform thickness and a mass of 56 kg, and it is mounted to the ground at point O. Point A indicates the center of gravity of the diving board, and it is equidistant from points O and B.

Determine the length of the diving board (l) if the net moment generated about point O by the weight of the diver and the diving board is $M_O = 3979 \text{ Nm}$.

Answer: $l = 3.5 \text{ m}$

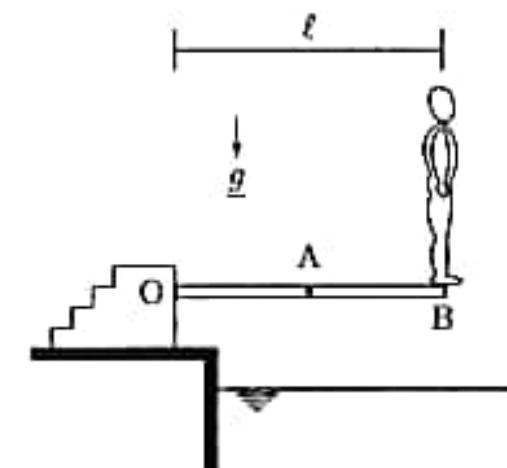
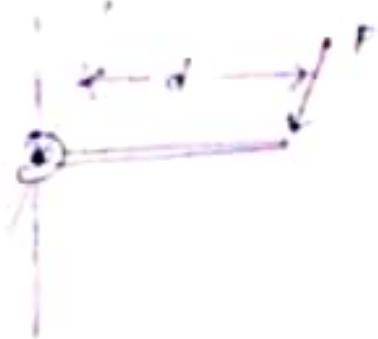


Fig. 3.13 A person is preparing to dive

Problem 3.3 Figure 3.13 illustrates a diver standing at the free end of uniform diving board (point B) and preparing to dive into a pool. The diving board is mounted to the ground at point O, has a mass of 110 kg and length of $l = 3.0 \text{ m}$. Point A indicates the center of gravity of the board and it is equidistant from points O and B. If the net moment about point O is $M_{net} = 3760 \text{ Nm}$, determine the weight of the diver.

Answer: $W = 714.3 \text{ N}$



The recommended torque is

$$M_{\text{rec}} = 80 \text{ N}\cdot\text{m}$$

a) We know, that

$$\text{Torque} = r \times F$$

where r is the distance from the end to the point from where force is applied.

$$\therefore \text{Torque}_{\text{rec}} = r \times F$$

$$\text{or, } F = \frac{\text{Torque}}{r}$$

$$= \frac{80}{0.25} \text{ N}$$

$$= 320 \text{ N}$$

b) The Force needs to decreased by 50%.

$$\therefore \text{Force}_{\text{final}} = 160 \text{ N}$$

As, the torque recommended cannot change, so.

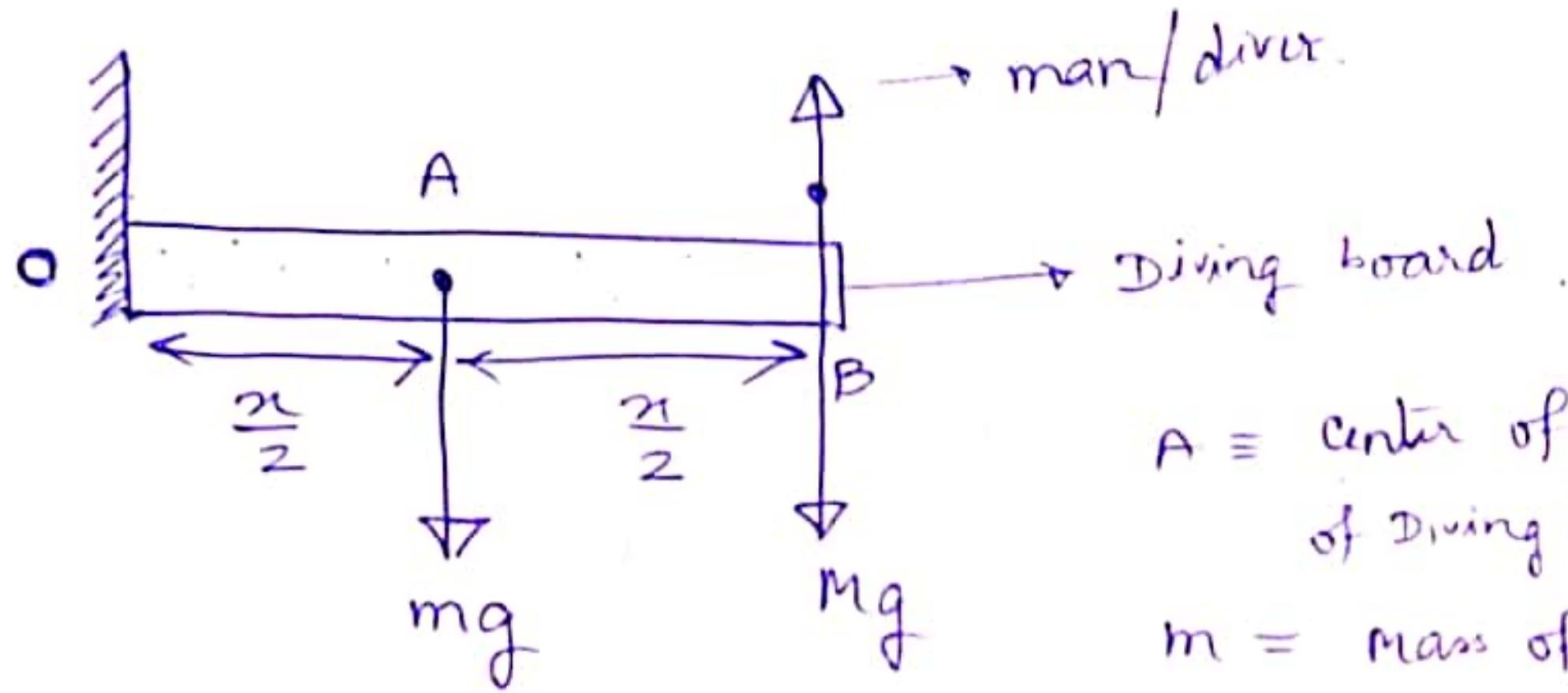
$$F_{\text{initial}} \times r_{\text{initial}} = F_{\text{final}} \times r_{\text{final}} = \text{Torque}_{\text{recommended}}$$

$$\text{or, } 320 \times 0.25 = 160 \times r_{\text{final}}$$

$$r_{\text{final}} = 0.50 \text{ m}$$

$$= 50 \text{ cm}$$

\therefore Therefore increasing wrench handle by 25 cm, the force can be decreased by 50%.



$A =$ center of Gravity
of Diving board.

$m =$ mass of board = 56 kg.

$M =$ Mass of diver = 82 kg.

Let the total length of the diving board be x .

By the above figure,

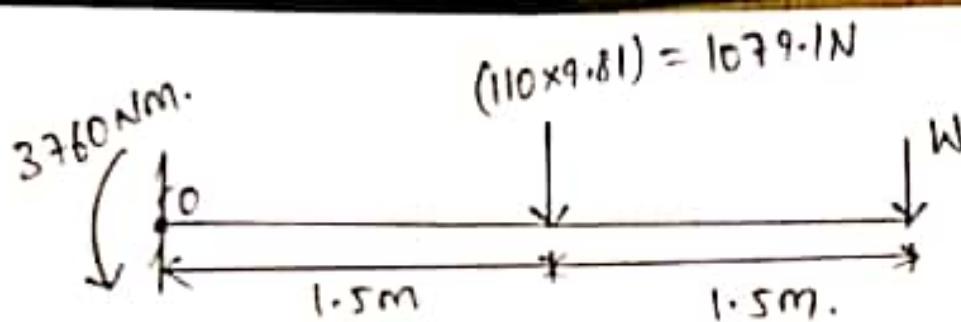
$$\text{Total moment about } O = (mg,) \frac{x}{2} + (Mg,) x$$

$$(M_0)$$

$$\Rightarrow 3979 \text{ Nm} = \left(\frac{56}{2} \text{ kg} + \frac{88}{2} \text{ kg} \right) \times x \times 9.8 \text{ m/s}^2$$

$$\Rightarrow x = \frac{3979}{9.8 \times \left(\frac{56}{2} + 88 \right)} \text{ m}$$

$$\Rightarrow x = \underline{\underline{3.5 \text{ m}}} \quad \leftarrow \text{length of Diving board} \\ (1)$$



Take moment about 'O'.

$$W(3) + 1079.1(1.5) = 3760 \text{ Nm}$$

$$W(3) = 2141.35 \text{ N.m}$$

$$W = \frac{2141.35}{3} \text{ N}$$

$$W = 713.78 \text{ N}$$

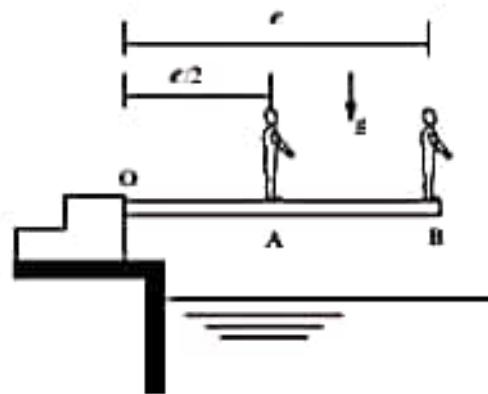


Fig. 3.30 Divers preparing for sequential jump into the pool

Problem 3.4 As illustrated in Fig. 3.30, consider two divers preparing to dive into a pool in a sequence. The horizontal diving board of uniform thickness is mounted to the ground at point O, has a mass of 130 kg, and is $l = 4$ m in length. The first diver has a mass of 86 kg and he stands at point B, which is the free end of the diving board. The second diver has a mass of 82 kg and he stands at point A, which is the center of gravity of the board. Furthermore, point A is equidistant from points O and B.

Determine the moment generated about point O by the weights of the divers and the board. Calculate the net moment about point O.

Answers: $M_1 = 3371.2$ Nm; $M_2 = 1607.2$ Nm; $M_b = 2548$ Nm; $M_{\text{net}} = 7526.4$ Nm

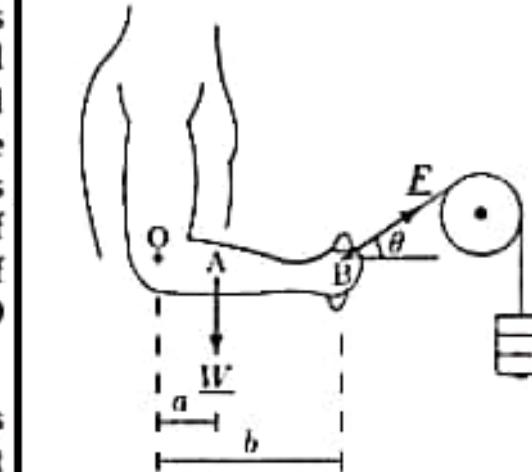


Fig. 3.31 Problems 3.5 and 3.7

Problem 3.5 Consider a person using an exercise apparatus who is holding a handle that is attached to a cable (Fig. 3.31). The cable is wrapped around a pulley and attached to a weight pan. The weight in the weight pan stretches the cable and produces a tensile force F in the cable. This force is transmitted to the person's hand through the handle. The force makes an angle θ with the horizontal and applied to the hand at point B. Point A represents the center of gravity of the person's lower arm and O is a point along the center of rotation of the elbow joint. Assume that points O, A, and B and force F all lie on a plane surface.

If the horizontal distance between point O and A is $a = 15$ cm, distance between point O and B is $b = 35$ cm, total weight of the lower arm is $W = 20$ N, magnitude of the applied force is $F = 50$ N, and angle $\theta = 30^\circ$, determine the net moment generated about O by F and W .

Answer: $M_o = 5.75$ Nm (ccw)

Moment = Force \times perpendicular distance,

Moment due to diver at B about D,

$$M_B = w_1 \times l$$

w_1 = weight of diver 1 = $m_1 g$

$$= 86 \times 9.81 \times 4$$

$$= \underline{\underline{3374.64 \text{ Nm}}}$$

Moment due to diver at A about D,

$$M_A = w_2 \times l/2 = 82 \times 9.81 \times 2$$

$$= \underline{\underline{1608.84 \text{ Nm}}}$$

Moment due to the board weight,

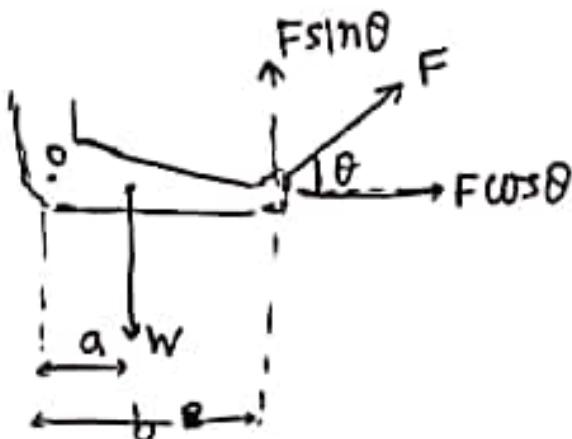
$$M_w = w_b \times l/2 = 130 \times 9.81 \times 2$$

$$= \underline{\underline{2550.6 \text{ Nm}}}$$

\therefore Total moment about D,

$$M = 3374.64 + 1608.84 + 2550.6$$

$$= \underline{\underline{7534.08 \text{ Nm}}}$$



$$a = 15 \text{ cm}$$

$$b = 35 \text{ cm}$$

$$w = 20 \text{ N}$$

~~$$F = 150 \text{ N}$$~~

$$\theta = 30^\circ$$

ζ Net moment about 0 = moment due of F + moment of w .

$$= wa \text{ clockwise} + F \sin \theta b \text{ Anticlockwise}$$

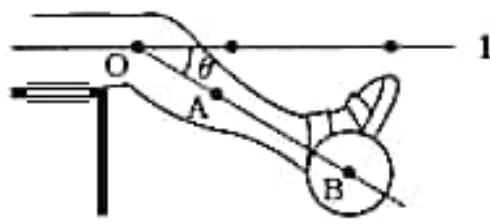
\rightarrow Taking clockwise moment as +ve,

$$\bar{\tau}_0 = \frac{20 \text{ N} \times 15 \times 10^{-2} \text{ m} - 50 \text{ N} \times \sin 30^\circ \times 35 \times 10^{-2} \text{ m}}{1}$$

$$\bar{\tau}_0 = -5.75 \text{ N-m}$$

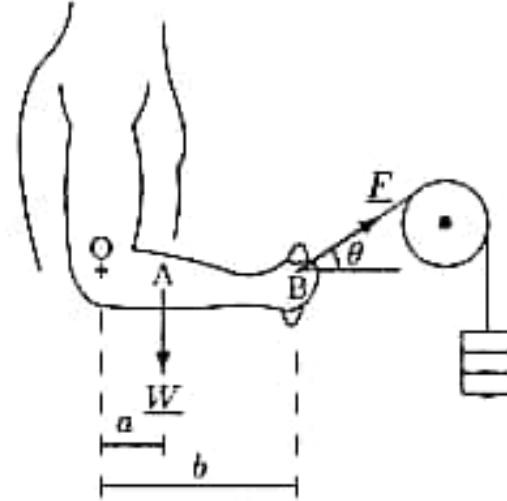
$$\boxed{\tau_0 = 5.75 \text{ N-m (Anticlockwise)}}$$

Problem 3.6 As illustrated in Fig. 3.15a, consider a person doing lower leg exercises from a sitting position wearing a weight boot. The weight of the boot is $W_2 = 65 \text{ N}$. Furthermore, as measured from the knee joint at point O, the center of gravity of the lower leg (point A) is located at a distance $a = 23 \text{ cm}$ and the center of gravity of the weight boot (point B) is located at a distance $b = 55 \text{ cm}$ from the point O. If the net moment about the knee joint is $M_{\text{net}} = 34 \text{ Nm}$, determine the weight of the person's lower leg (W_1), when the leg makes an angle $\theta = 45^\circ$ with the horizontal.



Answer: $W_1 = 53.8 \text{ N}$

Problem 3.7 Consider a person doing arm exercises by using a pulley-based apparatus shown in Fig. 3.31. The person is holding a handle that is attached to a cable. The cable is wrapped around a pulley with the weight pan attached to the free end of the cable. Point A represents the center of gravity of the lower arm and O represents the point where the handle is attached to the cable. The weight of the lower arm is $W = 23 \text{ N}$. The horizontal distances between points O and A, and O and B is ($a = 16 \text{ cm}$) and ($b = 39 \text{ cm}$), respectively. Furthermore, assume that all points as well as forces applied to the lower arm lie on the same plane surface. If the net moment about point O is $M_{\text{net}} = 6.3 \text{ Nm}$ and the cable attached to the handle makes an angle $\theta = 17^\circ$ with the horizontal, determine the mass of the weight pan.



Answer: $m = 8.9 \text{ kg}$

Given Data

The weight of the boot is $w_2 = 65N$.

at a point A is located at a distance

$$a = 23\text{cm.}$$

at (a point B) is located at a distance

$$b = 55\text{cm.}$$

The net moment about the knee joint

$$\text{is moment} = 34\text{Nm.}$$

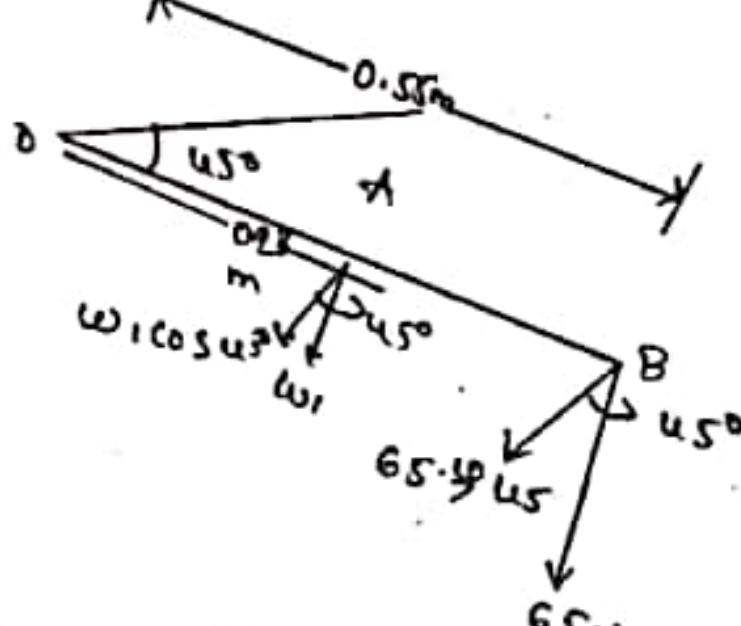
The weight of the person's lower leg

$$= w_1$$

The leg makes an angle $\theta = 45^\circ$.

Given the diagram below.

line diagram.



⇒ we taken from above the line diagram.

moment = force × distance

for combined effect, just add these two moment

separately generated by leg and boot

⇒ now we combined the moment.

$$(w_1 \cos 45^\circ) 0.23 + 0.55 (65 \cos 45^\circ) = 34$$

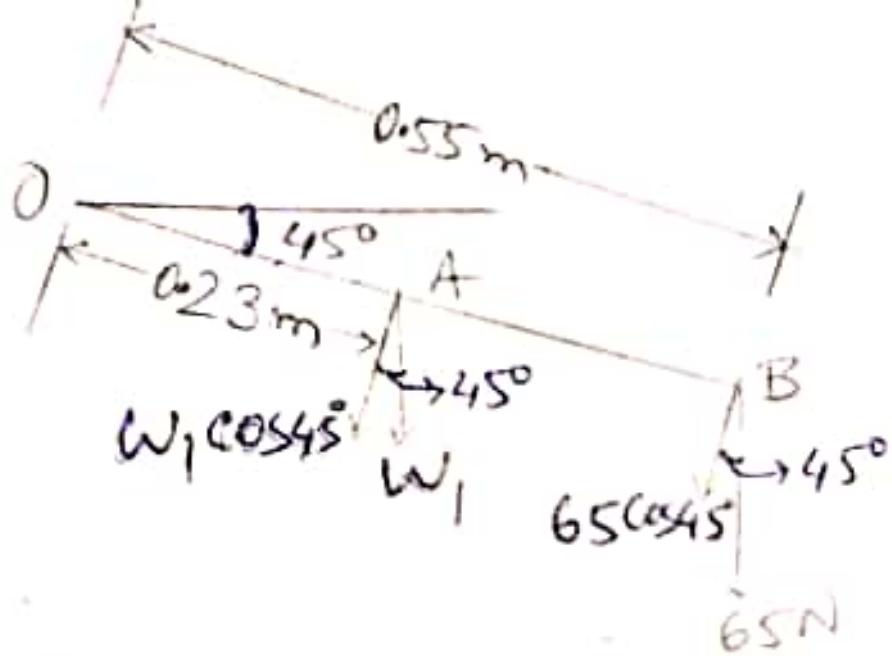
$$\Rightarrow w_1 (0.7) (0.23) + 0.55 (65 (0.707)) = 34$$

$$\Rightarrow w_1 0.161 + 25.025 = 34$$

$$\Rightarrow w_1 0.16 + 34 = 25.025$$

$$\Rightarrow w_1 = \frac{8.975}{0.16}$$

$$\Rightarrow w_1 = 56.09375\text{N}$$



Therefore the combined moment,

$$(w_1 \cos 45^\circ) 0.23 + 0.55 (65 \cos 45^\circ) = 34$$

$$\text{OR, } 0.163w_1 + 25.279 = 34$$

$$\text{OR, } w_1 = \frac{8.72}{0.163}$$

$$\therefore w_1 = \underline{53.5 \text{ N}}$$

Problem 3.8 Figure 3.32 illustrates a simplified version of a hamstring strength training system for rehabilitation and athlete training protocols. From a seated position, a patient or

athlete flexes the lower leg against a set resistance provided through a cylindrical pad that is attached to a load. For the position illustrated, the lower leg makes an angle θ with the horizontal. Point O represents the knee joint, point A is the center of gravity of the lower leg, W is the total weight of the lower leg, F is the magnitude of the force applied by the pad on the lower leg in a direction perpendicular to the long axis of the lower leg, a is the distance between points O and A, and b is the distance between point O and the line of action of F measured along the long axis of the lower leg.

(a) Determine an expression for the net moment about O due to forces W and F .

(b) If $a = 20 \text{ cm}$, $b = 40 \text{ cm}$, $\theta = 30^\circ$, $W = 60 \text{ N}$, and $F = 200 \text{ N}$, calculate the net moment about O.

Answers: (a) $M_O = bF - aW \cos \theta$ (b) $M_O = 69.6 \text{ Nm}$ (ccw)

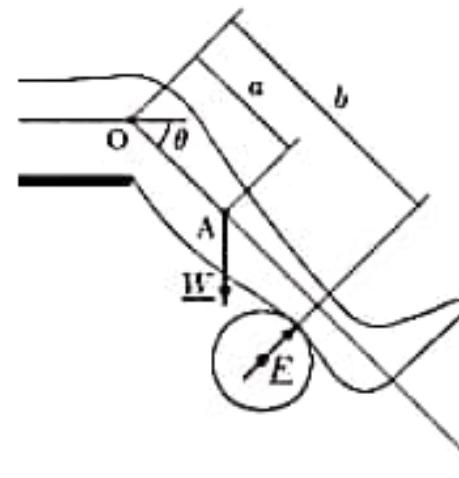


Fig. 3.32 Problems 3.8 and 3.9

Problem 3.9 As illustrated in Fig. 3.32, consider an athlete performing lower leg exercises from a seated position to strengthen hamstring muscles by using a special training system. The training system provides a set resistance to the leg through a cylindrical pad attached to the load while leg flexing. Point O represents the knee joint, point A is the center of gravity of the lower leg, W is the weight of the lower leg, θ defines an angle that the long axis of the lower leg makes with the horizontal, and M_O is the net moment about point O by forces applied to the leg.

(a) If $a = 0.23 \text{ m}$, $b = 0.45 \text{ m}$, $\theta = 45^\circ$, $W = 65 \text{ N}$, and $M_O = 86.14 \text{ Nm}$, determine the magnitude of the set resistance (F).

(b) Considering the same position of the lower leg, calculate the net moment about point O when the set resistance is increased by 10 N.

Answer: (a) $F = 215 \text{ N}$; (b) $M_O = 90.64 \text{ Nm}$

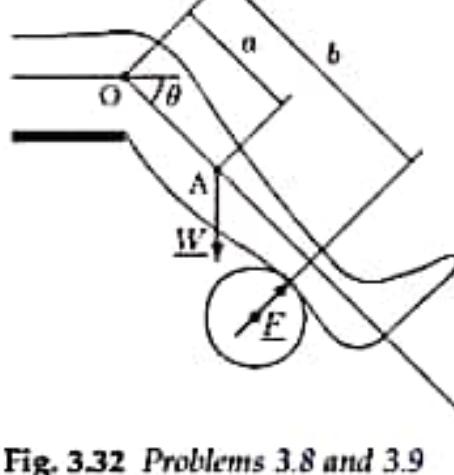
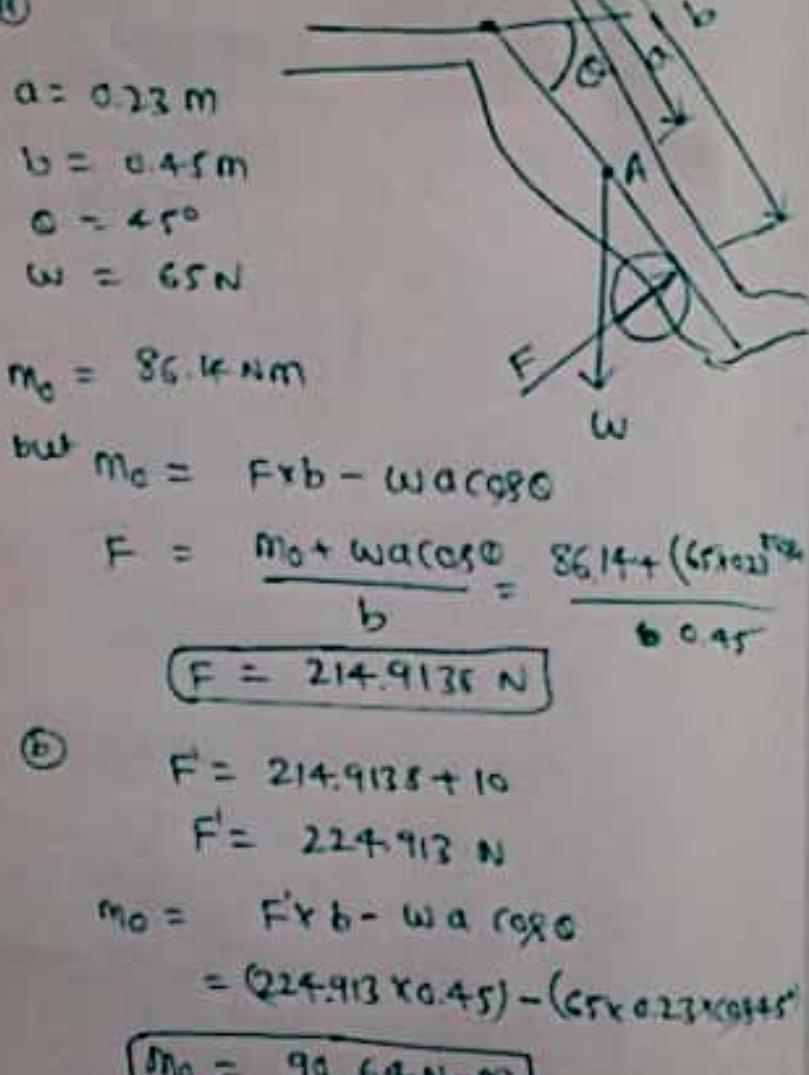
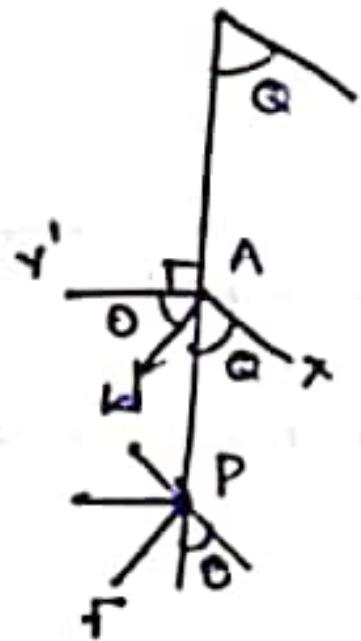
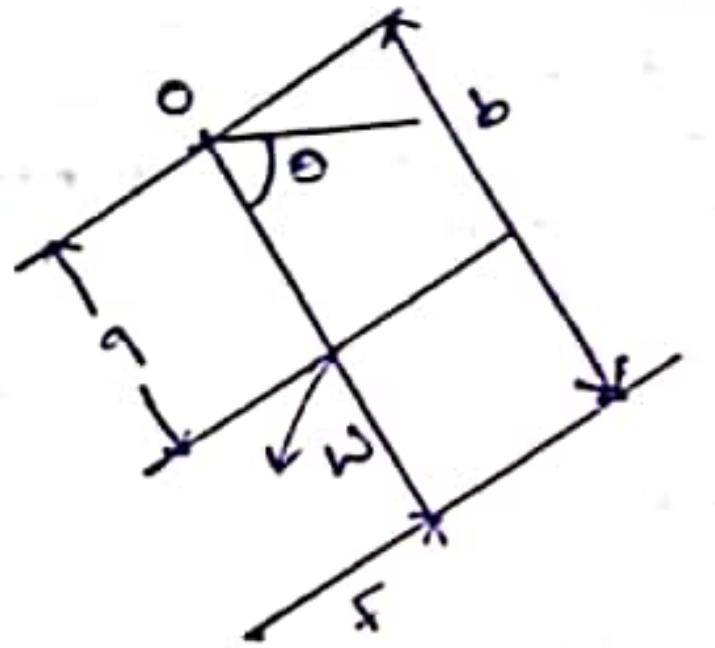


Fig. 3.32 Problems 3.8 and 3.9



Answers



- At point A

$$\angle WAX = 90^\circ$$

$$\therefore \angle PAW = 90^\circ - \theta$$

$$\bullet \angle Y'OP = 90^\circ$$

$$\angle Y'AW = 0^\circ$$

Now we can get component of
w along and \perp to the leg

\therefore Moment about O (Clockwise positive)

$$M_O = f(b) - (w \cos \theta)(a)$$

$\therefore M_O = fb - w \cos \theta a = \text{Moment about } O$

if $a = 20 \text{ cm} = 0.2 \text{ m}$, $b = 40 \text{ cm} = 0.4 \text{ m}$

$$\theta = 30^\circ \quad w = 60 \text{ N}, \quad f = 200 \text{ N}$$

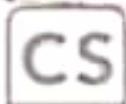
$$\Rightarrow M_O = (200)(0.4) - 60 \cos 30 \cdot (0.2)$$

②

$$= 80 - 60 \frac{\sqrt{3}}{2} (0.2)$$

$$= 80 - 6\sqrt{3}$$

$$= 69 - 60 \text{ N-m}$$



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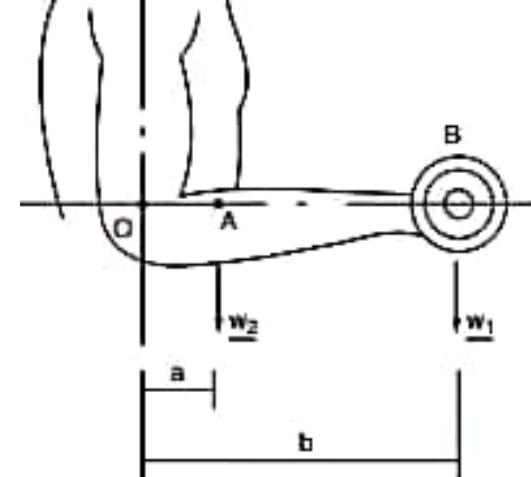


Fig. 3.33 An athlete performing lower arm exercises

Problem 3.10 As illustrated in Fig. 3.33, consider an athlete performing flexion/extension exercises of the lower arm to strengthen the biceps muscles. The athlete is holding the weight of $W_1 = 150 \text{ N}$ in his hand, and the weight of his lower arm is $W_2 = 20 \text{ N}$. As measured from the elbow joint at point O, the center of gravity of the lower arm (point A) is located at a distance $a = 7.5 \text{ cm}$ and the center of gravity of the weight held in the hand is located at a distance $b = 32 \text{ cm}$. Determine the net moment generated about the elbow joint, when the lower arm is extended horizontally and when the long axis of the lower arm makes an angle $f = 30^\circ$ and $f = 60^\circ$, respectively, with the horizontal.

Answer: $M_{\text{net}}(f = 0^\circ) = 49.5 \text{ Nm}$; $M_{\text{net}}(f = 30^\circ) = 42.9 \text{ Nm}$; $M_{\text{net}}(f = 60^\circ) = 44.8 \text{ Nm}$

Problem 3.11 Figure 3.34 illustrates a bench experiment designed to test the strength of materials. In the case illustrated, an intertrochanteric nail that is commonly used to stabilize fractured femoral heads is firmly clamped to the bench such that the distal arm (BC) of the nail is aligned vertically. The proximal arm (AB) of the nail has a length a and makes an angle θ with the horizontal.

As illustrated in Fig. 3.34, the intertrochanteric nail is subjected to three experiments by applying forces F_1 (horizontal, toward the right), F_2 (aligned with AB, toward A), and F_3 (vertically downward). Determine expressions for the moment generated at point B by the three forces in terms of force magnitudes and geometric parameters a and θ .

Answers: $M_1 = aF_1 \sin \theta$ (cw) $M_2 = 0$ $M_3 = aF_3 \cos \theta$ (ccw)

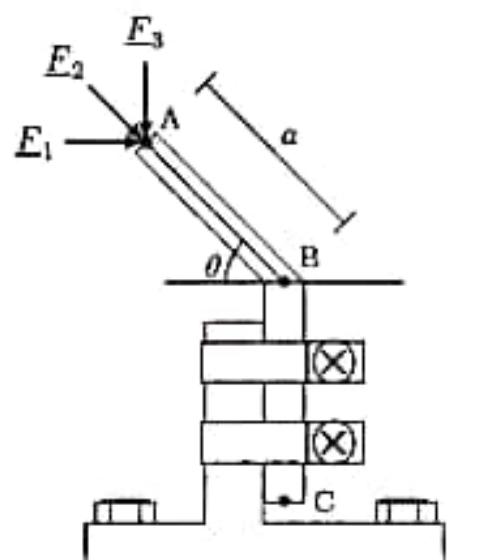
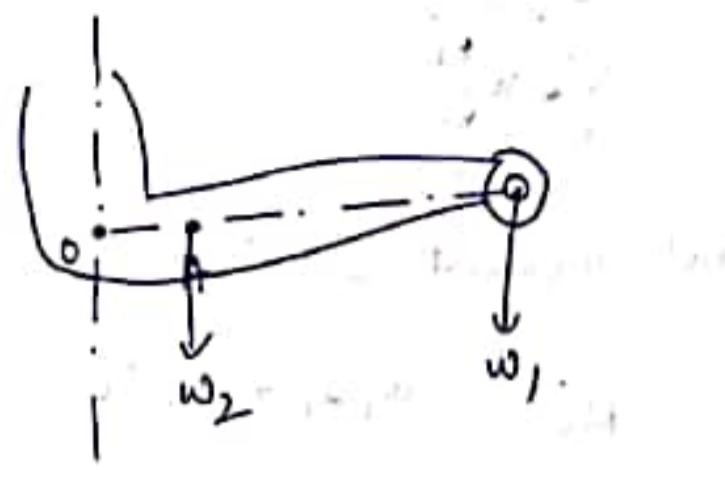


Fig. 3.34 A bench test

Given weight of weight held by

$$arm = \underline{150 \text{ N}} = w_1$$

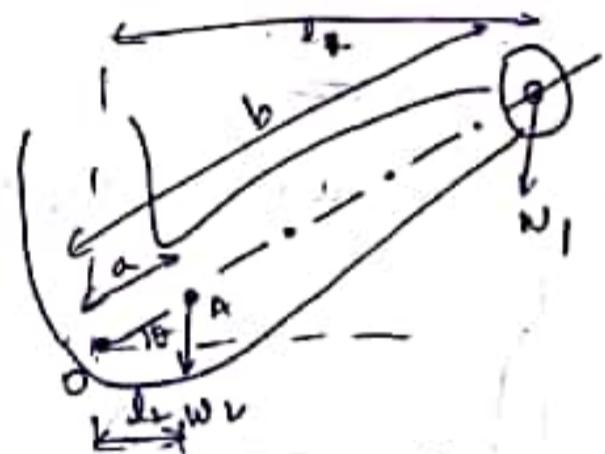
i) when lower arm is horizontal



Net moment about 'O'

$$\begin{aligned} &= w_1(b) + w_2(a) \\ &= (150) \left(\frac{32}{100} \right) + (20) \left(\frac{7.5}{100} \right) \\ &= \underline{49.5 \text{ N-m}}. \end{aligned}$$

ii) when axis of lower arm makes
 $\theta = 30^\circ$ with horizontal



Net moment about 'O'

$$M_O = w_1 l_1 + w_2 l_2$$

$$\therefore \cos \theta = \frac{l_2}{a} = \frac{l_1}{b}$$

$$\begin{aligned} M_O &= (150) \left(\frac{32}{100} \right) \cos 30^\circ + (20) \left(\frac{7.5}{100} \right) \cos 30^\circ \\ &= \underline{(49.5) \cos 30^\circ} \\ &= 42.868 \text{ N-m}. \end{aligned}$$

iii) similarly at $\theta = 60^\circ$

$$\begin{aligned} M_O &= w_1 l_1 + w_2 l_2 \\ &= (150) \left(\frac{32}{100} \right) \cos 60^\circ + (20) \left(\frac{7.5}{100} \right) \cos 60^\circ \\ &= (49.5) \cos 60^\circ \\ &= \underline{24.75 \text{ N-m}} \end{aligned}$$

at point 'A' the forces

F_1 → horizontal

F_2 → along AB

F_3 → vertically downward.

The nail of length 'a' is at an angle 'θ' to the horizontal.

moment of Torque = Force \times perpendicular distance

$$\bar{M} \text{ or } \bar{m} = \bar{F} \times \bar{r}$$

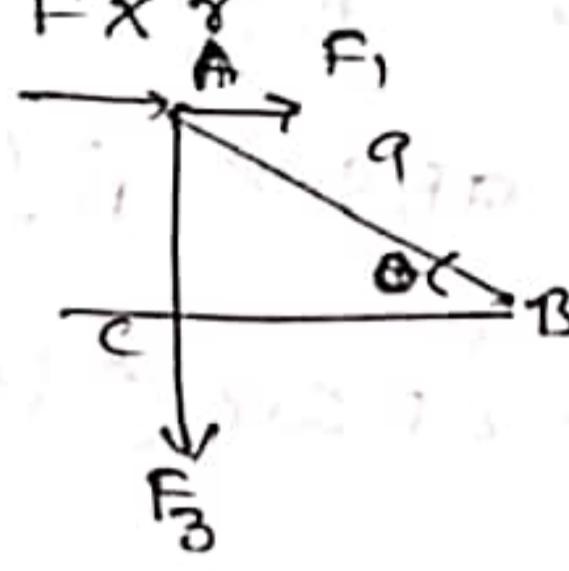
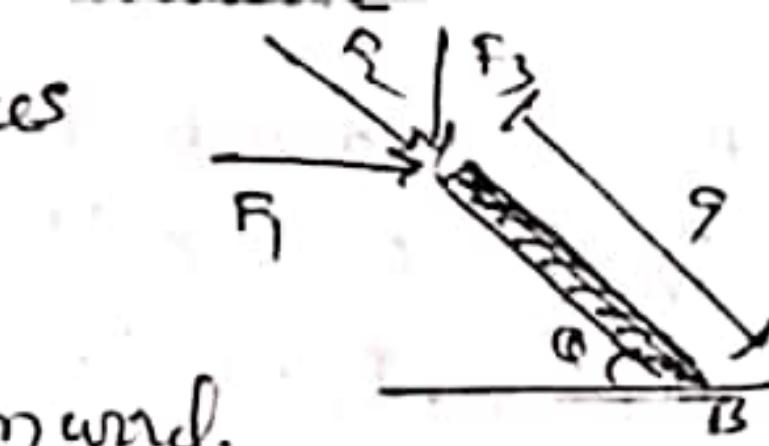
$\triangle ABC$.

$$\sin \theta = \frac{AC}{a}$$

$$AC = a \sin \theta$$

$$\cos \theta = \frac{BC}{a}$$

$$BC = a \cos \theta.$$



\Rightarrow The moment due to force F_1 about 'B'

$$\bar{m}_1 = F_1 \times AC = F_1 \times a \sin \theta$$

$\bar{m}_1 = a F_1 \sin \theta$ clockwise

\Rightarrow The moment due to force F_3 about B

$$\bar{m}_3 = F_3 \times BC = F_3 \times a \cos \theta$$

$\bar{m}_3 = a F_3 \cos \theta$ counter-clockwise

The moment due to force F_2 is M_2

$\bar{m}_2 = \text{Force} \times \perp \text{distance}$

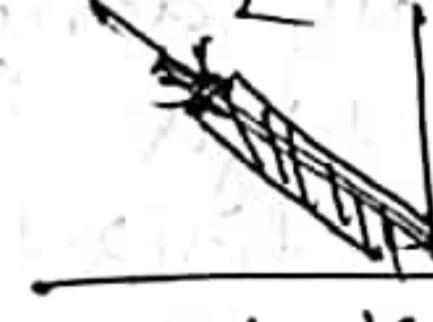
$$\bar{m}_2 = F_2 \times 0 = 0$$

along the line

of axis of torque

of moment is zero. That is distance = 0

$$r = 0, \bar{m}_2 = 0$$



$$\bar{m}_1 = a F_1 \sin \theta \rightarrow \underline{\text{C.W}}$$

$$\bar{m}_2 = 0$$

$$\bar{m}_3 = a F_3 \cos \theta \rightarrow \underline{\text{C.C.W}}$$

Problem 3.12 The simple structure shown in Fig. 3.35 is called a *cantilever beam* and is one of the fundamental mechanical elements in engineering. A cantilever beam is fixed at one end and free at the other. In Fig. 3.35, the fixed and free ends of the beam are identified as points A and C, respectively. Point B corresponds to the center of gravity of the beam.

Assume that the beam shown has a weight $W = 100 \text{ N}$ and a length $l = 1 \text{ m}$. A force with magnitude $F = 150 \text{ N}$ is applied at the free end of the beam in a direction that makes an angle $\theta = 45^\circ$ with the horizontal.

Determine the magnitude and direction of the net moment developed at the fixed end of the beam.

Answer: $M_A = 56 \text{ Nm}$ (ccw)

Problem 3.13 As illustrated in Fig. 3.36, consider a cantilever beam of 9 kg mass. The beam is fixed to the wall at point A and point B represents the free end of the beam. The length of the

beam is $l = 4 \text{ m}$. Point C is the center of gravity of the beam. The weight of $W_1 = 50 \text{ N}$ is attached at the free end of the beam such that the distance between points B and D is $l = 35 \text{ cm}$. A force with magnitude $F = 180 \text{ N}$ is applied at the free end of the beam to keep the beam in place. The line of action of the force makes an angle $\beta = 35^\circ$ with the horizontal.

Determine the magnitude and direction of the net moment generated about point A.

Answer: $M_{\text{net}} = 54.1 \text{ Nm}$ (ccw)

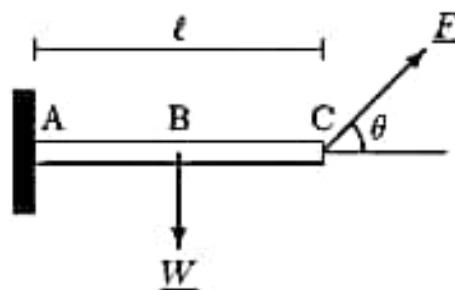


Fig. 3.35 A cantilever beam

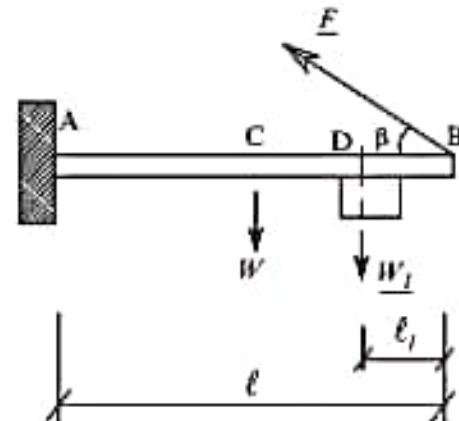


Fig. 3.36 A cantilever beam with weight W_1 attached at free end

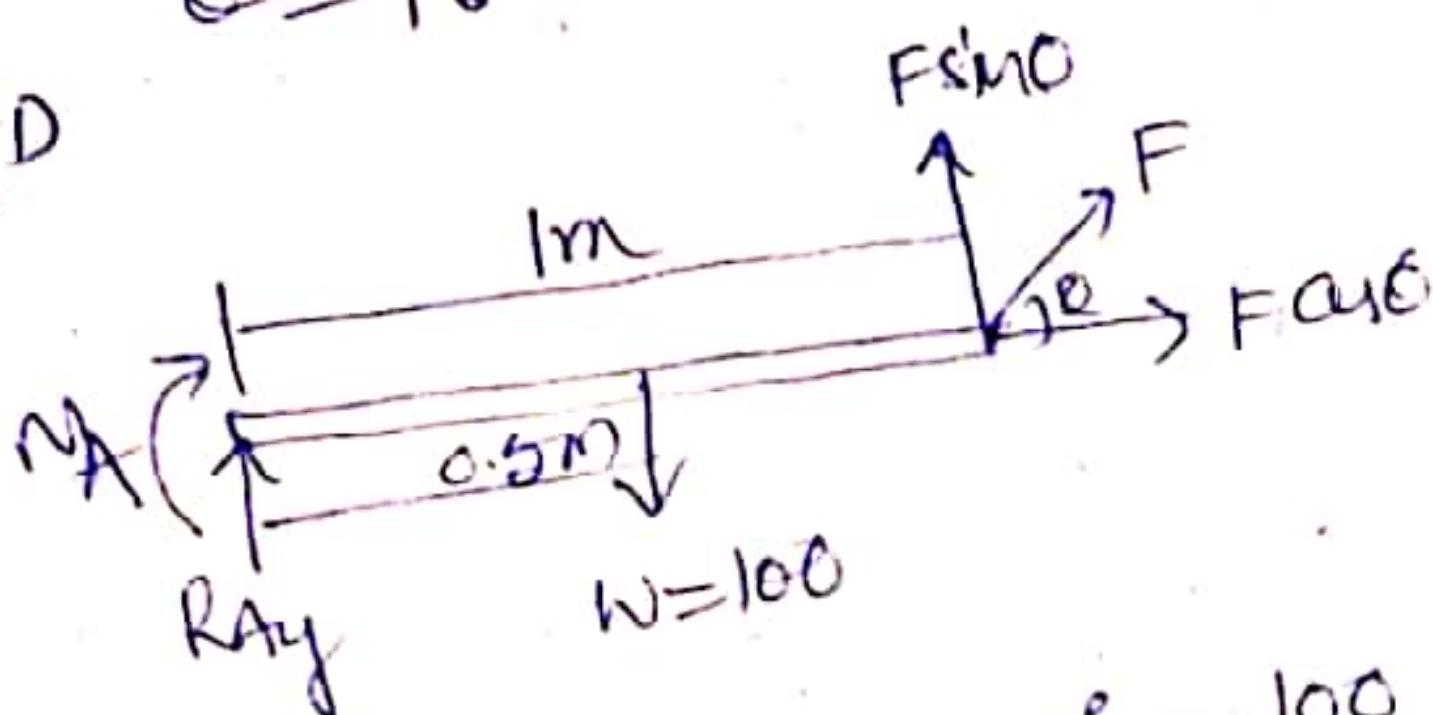
$$W = 100N$$

$$l = 1m$$

$$F = 150N$$

$$\theta = 45^\circ$$

FBD



$$\sum F_y = 0$$

$$R_{Ay} + 150 \sin 45^\circ = 100$$

$$R_{Ay} = 100 - 150 \sin 45^\circ$$

$$R_{Ay} = 100 - 106.06 N$$

$$R_{Ay} = 6.06 N \downarrow$$

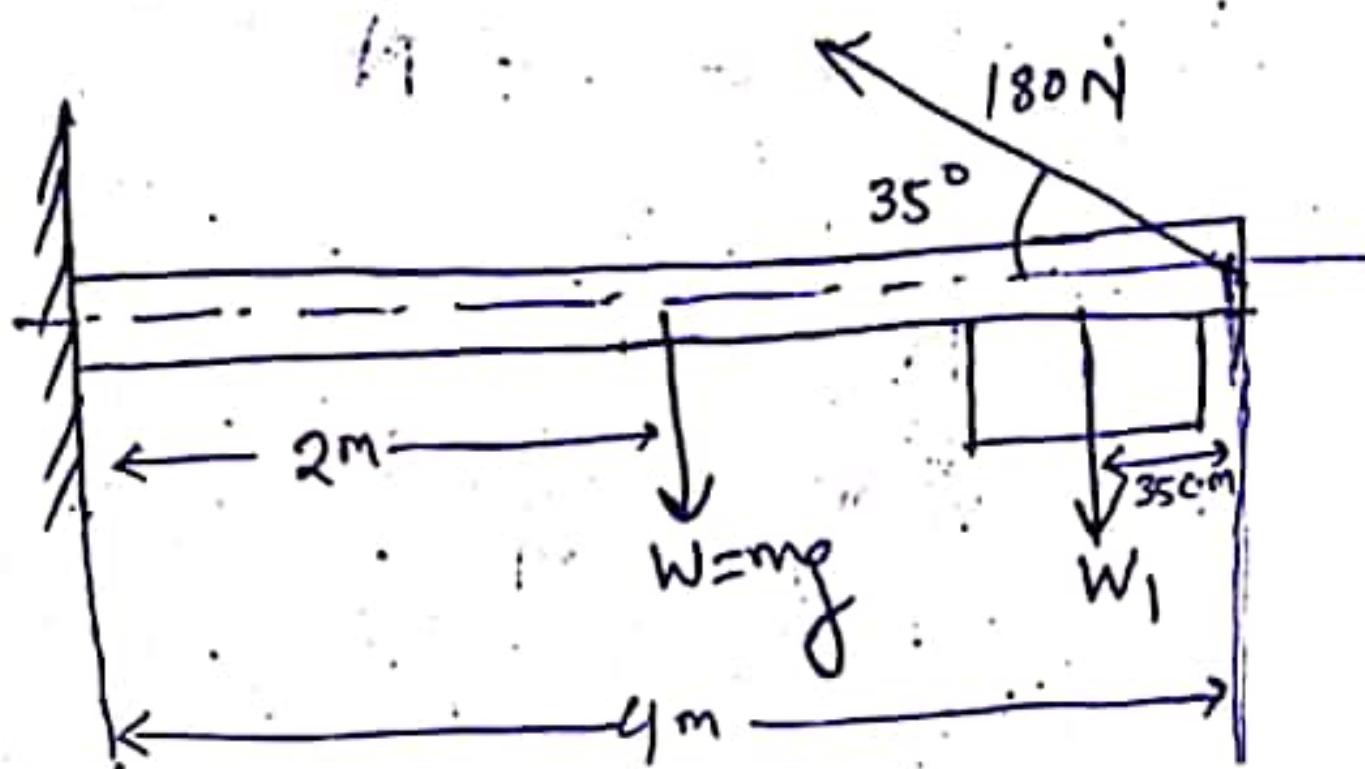
$$R_{Ay} = 6.06 N \downarrow$$

$$M_A = (100 \times 0.5) - (150 \times \sin 45^\circ \times 1)$$

$$M_A = 50 - 106.06$$

$$M_A = -56.06 N.m$$

$$M_A = 56.06 N.m$$



Given $m = 9 \text{ kg}$

$$\text{So } W = mg = 9 \times 9.81 = 88.2 \text{ N}$$

$$W_1 = 50 \text{ N} \text{ (given)}$$

Taking counter clockwise moments as (-ve)
and clockwise moments as (+ve);

The net moment at 'A'

$$M_A = -180 \sin(35^\circ) \times 4$$

$$+ W \times 2 + W_1 \times (4 - 0.35)$$

$$= -180 \sin(35^\circ) \times 4$$

$$+ 88.2 \times 2 + 50 \times 3.65$$

$$= -54.075 \text{ N.m}$$

As it is coming (-ve) So net moment
at 'A' is counter clockwise.

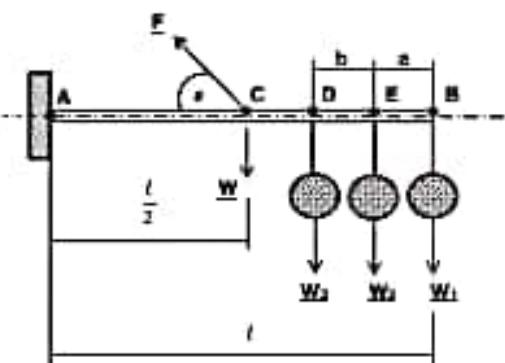


Fig. 3.37 Problem 3.14

Problem 3.14 As illustrated in Fig. 3.37, consider a structure consisting of a cantilever beam with three identical spherical electrical fixtures attached by cables at the free end of the beam and at equal distances from each other ($a = b$). The beam is fixed to the wall at point A. Point B identifies the free end of the beam. It is also a point where the first electrical fixture is attached to the beam. D and E are points where the second and third electrical fixtures are attached to the beam. Point C identifies the center of gravity of the beam and it is equidistant from points A and B. The weight of the beam is $W = 150 \text{ N}$ and the weight of the electrical fixtures is $W_1 = W_2 = W_3 = 49 \text{ N}$. Furthermore, a force $F = 230 \text{ N}$ is applied to the beam at point D to keep the beam in place with the line of action of the force making an angle θ with the horizontal.

- If $a = b = 0.5 \text{ m}$, $\theta = 45^\circ$, and the net moment about point A is $M_{\text{net}} = 267.2 \text{ Nm}$, determine the length (l) of the beam.
- If the length of the beam is increased by 50 cm, calculate the net moment about point A.
- If the magnitude of force F is decreased by 30 N, calculate the net moment about point A when the length of the beam is $l = 3.0 \text{ m}$.
- If the force F is applied in the direction perpendicular to the long axis of the beam, calculate the net moment about point A when the length of the beam is $l = 3.0 \text{ m}$.

Answers: (a) $l = 3.0 \text{ m}$; (b) $M_{\text{net}} = 296.9 \text{ Nm}$; (c) $M_{\text{net}} = 309.7 \text{ Nm}$; (d) $M_{\text{net}} = 192.5 \text{ Nm}$

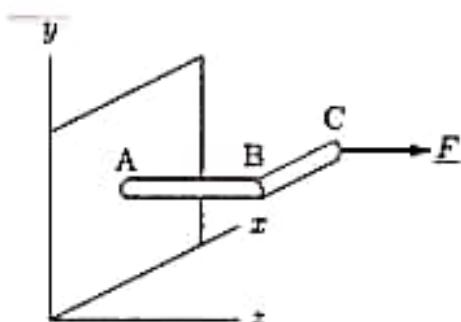


Fig. 3.38 Problem 3.15

Problem 3.15 Consider the L-shaped beam illustrated in Fig. 3.38. The beam is mounted to the wall at point A, the arm AB extends in the z direction, and the arm BC extends in the x direction. A force F is applied in the z direction at the free end of the beam.

If the lengths of arms AB and BC are a and b , respectively, and the magnitude of the applied force is F , observe that the position vector of point C relative to point A can be written as $\underline{r} = b\hat{i} + a\hat{k}$ and the force vector can be expressed as $\underline{F} = F\hat{k}$, where \hat{i} , \hat{j} , and \hat{k} are unit vectors indicating positive x , y , and z directions, respectively.

- Using the cross product of position and force vectors, determine an expression for the moment generated by \underline{F} about point A in terms of a , b , and F .
- If $a = b = 30 \text{ cm}$ and $F = 20 \text{ N}$, calculate the magnitude of the moment about point A due to \underline{F} .

Answers: (a) $M_A = -bF\hat{j}$; (b) $M_A = 6 \text{ Nm}$

a)

Since, in the statement of question it is given that force is applied at D, but in figure it is applied at C. therefore I will go as per statement of question and consider the force at D.

Consider moment about A.

$$-F \sin \theta \times \frac{l}{2} + W \times \frac{l}{2} + W_1 l + W_2 (l-a) + W_3 \times (l-a-b) = M_{\text{net}}$$

Substitute the values.

$$\begin{aligned} -230 \sin 45 \times (l-0.5-0.5) + 150 \times \frac{l}{2} + 49 \times l + 49(l-0.5) + 49 \times (l-0.5-0.5) &= 267.2 \\ +89.1345 + 59.365l &= 267.2 \\ 59.365l &= 178.07 \\ l &= 3 \text{ m} \end{aligned}$$

b)

Consider moment about A when length increases by 50 cm (new $l = 3 + 0.5 = 3.5 \text{ m}$).

$$-F \sin \theta \times \frac{l}{2} + W \times \frac{l}{2} + W_1 l + W_2 (l-a) + W_3 \times (l-a-b) = M_{\text{net}}$$

Substitute the values.

$$\begin{aligned} -230 \sin 45 \times (l-0.5-0.5) + 150 \times \frac{l}{2} + 49 \times l + 49(l-0.5) + 49 \times (l-0.5-0.5) &= M_{\text{net}} \\ +89.1345 + 59.365l &= M_{\text{net}} \\ M_{\text{net}} &= 89.1345 + 59.365 \times 3.5 \\ &= 296.9 \text{ N}\cdot\text{m} \end{aligned}$$

c)

Consider moment about A when F is increase by 30N (new $F = 230 + 30 = 260 \text{ N}$).

$$-F \sin \theta \times \frac{l}{2} + W \times \frac{l}{2} + W_1 l + W_2 (l-a) + W_3 \times (l-a-b) = M_{\text{net}}$$

Substitute the values.

$$\begin{aligned} -200 \sin 45 \times (l-0.5-0.5) + 150 \times \frac{l}{2} + 49 \times l + 49(l-0.5) + 49 \times (l-0.5-0.5) &= M_{\text{net}} \\ +67.92 + 80.576l &= M_{\text{net}} \\ M_{\text{net}} &= 67.92 + 80.576 \times 3 \\ M_{\text{net}} &= 309.65 \text{ N}\cdot\text{m} \end{aligned}$$

d)

Consider moment about A when F is perpendicular to beam (new $\theta = 90^\circ, F = 200 \text{ N}$).

$$-F \sin \theta \times \frac{l}{2} + W \times \frac{l}{2} + W_1 l + W_2 (l-a) + W_3 \times (l-a-b) = M_{\text{net}}$$

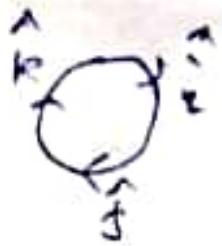
Substitute the values.

$$\begin{aligned} -200 \sin 90 \times (l-0.5-0.5) + 150 \times \frac{l}{2} + 49 \times l + 49(l-0.5) + 49 \times (l-0.5-0.5) &= M_{\text{net}} \\ +126.5 + 22l &= M_{\text{net}} \\ M_{\text{net}} &= 126.5 + 22 \times 3 \\ M_{\text{net}} &= 192.5 \text{ N}\cdot\text{m} \end{aligned}$$

3.15

$$\vec{F} = F\hat{k} \quad \dots \textcircled{1}$$

$$\vec{r} = b\hat{i} + a\hat{k} \quad \dots \textcircled{2}$$



(a) now, moment of the force \vec{F} about A

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= (\hat{i}b + \hat{k}a) \times F\hat{k}$$

$$\text{or, } \vec{\tau} = bF \hat{i} \times \hat{k} + aF \hat{k} \times \hat{k}$$

$$= bF (-\hat{j}) + \vec{0}$$

$$\text{or, } \vec{\tau} = bF (-\hat{j}) \quad [\text{Answer}]$$

$$\text{Given } a = b = 30 \text{ cm.} = 0.3 \text{ m.}$$

(b)

$$\text{and } F = 20 \text{ N.}$$

$$\text{now magnitude of } \vec{\tau} = |\vec{\tau}| = bF$$

$$\text{or, } |\vec{\tau}| = 0.3 \times 20 \text{ N.m}$$

$$\text{or, } |\vec{\tau}| = 6.0 \text{ Nm}$$

(Answer)

Problem 3.16 As shown in Fig. 3.39, consider a worker using a special wrench to tighten bolts. The couple-moment generated about the long axis of the wrench is $M_c = 100 \text{ Nm}$ and the distance between the handles of the wrench and the long axis is $r = 25 \text{ cm}$.

Determine the force exerted by the worker on the handles of the wrench.

Answer: $F = 200 \text{ N}$

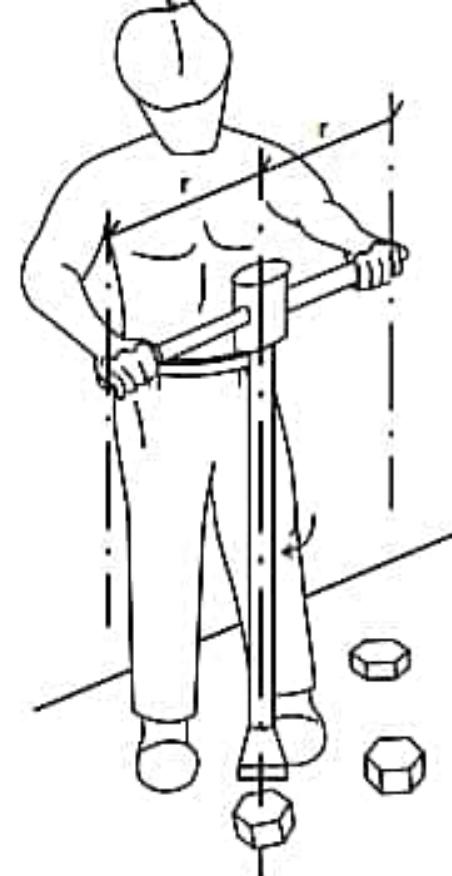


Fig. 3.39 Worker using a special wrench

We know the torque $M_c = F \cdot r$

here r is the total distance $= 2r = 2 \cdot 25 = 50 \text{ cm} = 0.5 \text{ m}$

hence $F = 100 / 0.50 = 200 \text{ N} \dots \dots \text{Ans.}$

Problem 4.1 As illustrated in Fig. 4.12, consider a person standing on a uniform horizontal beam that is resting on frictionless knife-edge and roller supports. A and B are two points that the contact between the beam and the knife-edge and roller support, respectively. Point C is the center of gravity of the beam and it is equidistant from points A and B. D is the point on the beam directly under the center of gravity of the person. Due to the weights of the beam and the person, there are reactions on the beam at points A and B. If the weight of the person is $W = 625 \text{ N}$ and the reactions at points A and B are $R_A = 579.4 \text{ N}$ and $R_B = 735.6 \text{ N}$,

- Determine the weight (W) of the beam.
- Determine the length (l) of the beam.

Answers: (a) $W = 690 \text{ N}$; (b) $l = 4 \text{ m}$

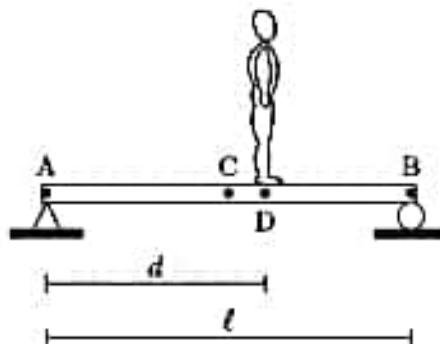


Fig. 4.12 Example 4.1

Problem 4.2 As illustrated in Fig. 4.49, consider an 80 kg person preparing to dive into a pool. The diving board is represented by a uniform, horizontal beam that is hinged to the ground at point A and supported by a frictionless roller at point D. B is a point on the board directly under the center of gravity of the person. The distance between points A and B is $l = 6 \text{ m}$ and the distance between points A and D is $d = 2 \text{ m}$. (Note that one-third of the board is located on the left of the roller support and two-thirds is on the right. Therefore, for the sake of force analyses, one can assume that the board consists of two boards with two different weights connected at point D.)

If the diving board has a total weight of 1500 N, determine the reactions on the beam at points A and D.

Answers: $R_A = 2318 \text{ N}$ (↓) $R_D = 4602 \text{ N}$ (↑)

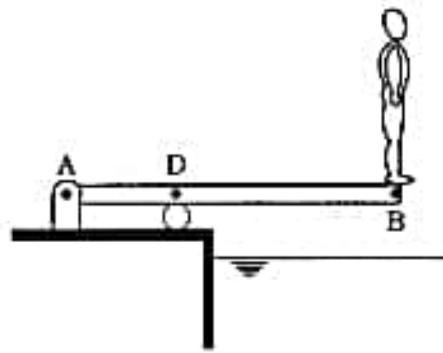
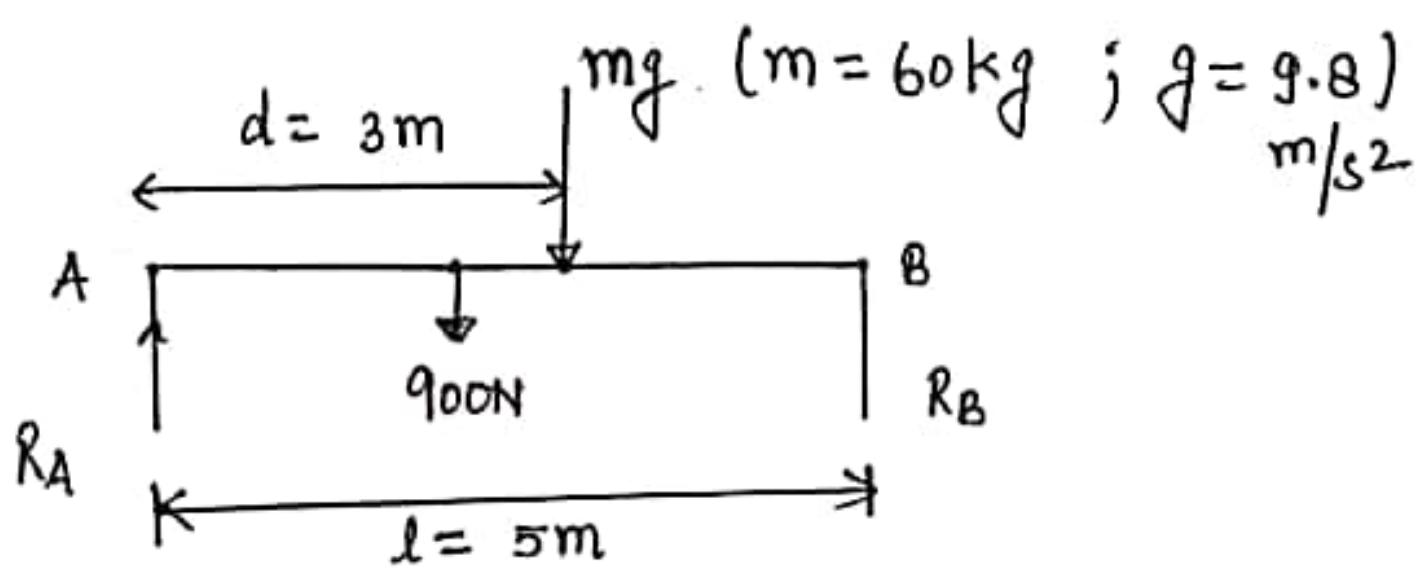


Fig. 4.49 Problem 4.2



force balance equation :-

$$R_A + R_B = 900 + mg \quad \text{--- (1)}$$

$$R_A + R_B = 900 + 60 \times 9.8$$

$$R_A + R_B = 1488 \quad \text{--- (1)}$$

Moment about point A is :-

$$R_B \times 5 = 900 \times \frac{5}{2} + mg \times d \quad \text{--- (1)}$$

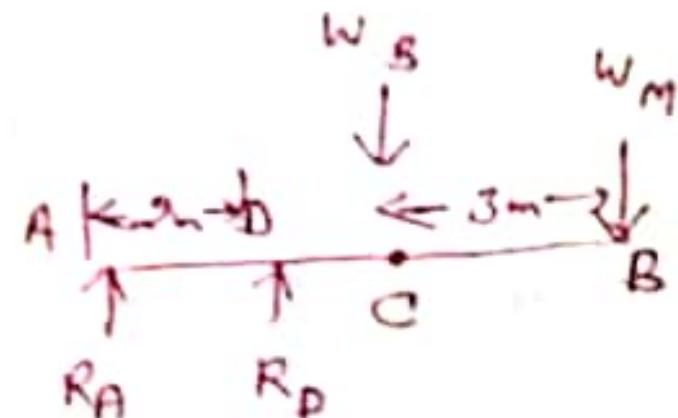
$$R_B = \frac{900 \times \frac{5}{2} + 60 \times 9.8 \times 3}{5}$$

$R_B = 802.8 \text{ N}$. put in (1) ; then —

$$R_A = \underline{\underline{685.2 \text{ N}}}$$

$$w_B = 80 \times 9.81 = 784.8 N$$

$$w_B = 1500 N$$



$$\sum \gamma = 0$$

$$R_A + R_D = 1500 + 784.8$$

$$R_A + R_D = 2284.8 N$$

$$\sum m_A = 0$$

$$R_D \times 2 - 1500 \times 3 - 784.8 \times 6 = 0$$

$$R_D = 4604.4 N \uparrow$$

$$\text{and } R_A = 2284.8 - 4604.4$$

$$R_A = -2319.6 N$$

$$R_A = 2319.6 N \downarrow$$

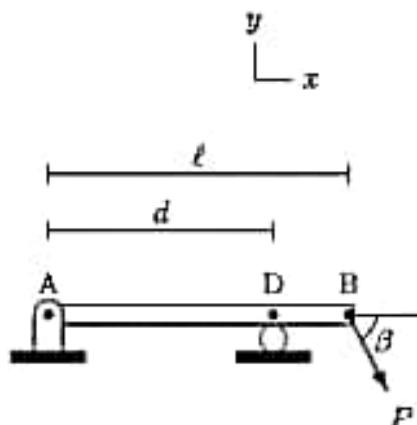


Fig. 4.50 Problem 4.3

Problem 4.3 The uniform, horizontal beam shown in Fig. 4.50 is hinged to the ground at point A and supported by a frictionless roller at point D. The distance between points A and B is $l = 4 \text{ m}$ and the distance between points A and D is $d = 3 \text{ m}$. A force that makes an angle $\beta = 60^\circ$ with the horizontal is applied at point B. The magnitude of the applied force is $P = 1000 \text{ N}$. The total weight of the beam is $W = 400 \text{ N}$.

By noting that three-quarters of the beam is on the left of the roller support and one-quarter is on the right, calculate the x and y components of reaction forces on the beam at points A and D.

Answers: $R_D = 1421 \text{ N} (\uparrow)$ $R_{Ax} = 500 \text{ N} (\leftarrow)$ $R_{Ay} = 155 \text{ N} (\downarrow)$

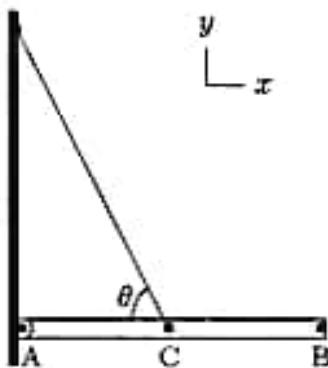


Fig. 4.51 Problem 4.4

Problem 4.4 The uniform, horizontal beam shown in Fig. 4.51 is hinged to the wall at point A and supported by a cable attached to the beam at point C. Point C also represents the center of gravity of the beam. At the other end, the cable is attached to the wall so that it makes an angle $\theta = 68^\circ$ with the horizontal. If the length of the beam is $l = 4 \text{ m}$ and the weight of the beam is $W = 400 \text{ N}$, calculate the tension T in the cable and components of the reaction force on the beam at point A.

Answers: $T = 431 \text{ N}$ $R_{Ax} = 162 \text{ N} (\rightarrow)$ $R_{Ay} = 0$

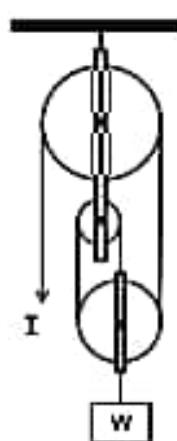


Fig. 4.52 Problem 4.5

Problem 4.5 Consider a structure illustrated in Fig. 4.52. The structure includes a horizontal beam hinged to the wall at point A and three identical electrical fixtures attached to the beam at points B, D, and E with point B identifying the free end of the beam. Moreover, the distances between the points of attachment of the electrical fixtures are equal to each other ($BD = DE = a = b = 35 \text{ cm}$). Point C identifies the center of gravity of the beam and it is equidistant from points A and B. The weight of the beam is $W = 230 \text{ N}$ and each electrical fixture weighs $W_1 = W_2 = W_3 = 45 \text{ N}$. Furthermore, a cable is attached to the beam at point B making an angle $\alpha = 45^\circ$ with the horizontal. On another end the cable is attached to the wall to keep the beam in place. If the length of the beam is $l = 2.5 \text{ m}$,

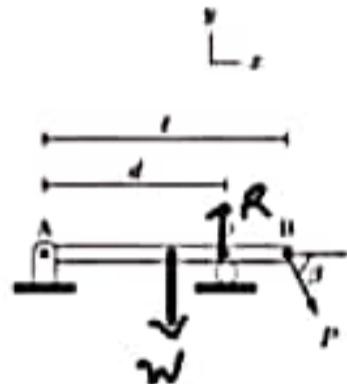
- Determine the tension (T) in the cable.
- Determine the magnitude of the reaction force (R_A) at point A.
- Determine the tension (T_1) in the cable when it makes an angle $\alpha = 65^\circ$ with the horizontal.
- Determine change in the magnitude of the reaction force at point A when the cable makes an angle $\alpha = 65^\circ$ with the horizontal.

Answers: (a) $T = 817.2 \text{ N}$; (b) $R_A = 615.7 \text{ N}$; (c) $T_1 = 634.8 \text{ N}$; (d) 44.6% decrease

Problem 4.2 The uniform, horizontal beam shown in Fig. 4.50 is hinged to the ground at point A and supported by a frictionless roller at point D. The distance between points A and B is $l = 4\text{ m}$ and the distance between points A and D is $d = 3\text{ m}$. A force that makes an angle $\beta = 60^\circ$ with the horizontal is applied at point B. The magnitude of the applied force is $P = 1,000\text{ N}$. The total weight of the beam is $W = 400\text{ N}$.

By noting that three-quarters of the beam is on the left of the roller support and one-quarter is on the right, calculate the x and y components of reaction forces on the beam at points A and D.

Answer: $R_D = 1,421\text{ N}(\uparrow)$, $R_A = 500\text{ N}(-)$, $R_{Ax} = 155\text{ N}(\leftarrow)$



$$l = 4\text{ m} \quad d = 3\text{ m} \quad \beta = 60^\circ$$

$$P = 1000\text{ N} \quad W = 400\text{ N}$$

First we can take sum of torques about point A
reaction force at point D will be in vertical direction

$$R(D) - (P \sin \beta)l - W\left(\frac{l}{2}\right) = 0$$

$$R(3) - (1000 \sin 60)(4) - 400\left(\frac{4}{2}\right) = 0$$

$$3R - 3464.1 - 800 = 0$$

$$R = 1421\text{ N} \quad \uparrow$$

(b)

We can find out x and y component of reaction at point A using sum of vertical and horizontal forces separately

$$A_x + P \cos \beta = 0$$

$$= -(1000 \cos 60)$$

$$A_x = -500\text{ N} \quad \text{or}$$

$$A_x = 500\text{ N} \quad (\leftarrow)$$

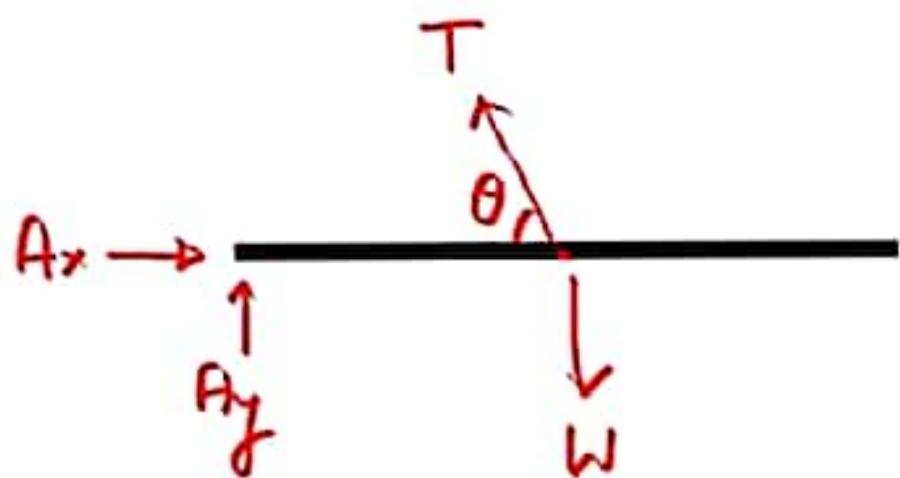
$$A_y - W + R - P \sin \beta = 0$$

$$A_y - 400 + 1421 - 1000 \sin 60 = 0$$

$$A_y = -154.97$$

or

$$A_y = 155 \quad (\downarrow)$$



$$\theta = 68^\circ$$

$$W = 400 \text{ N}$$

$$l = 4 \text{ m}$$

Since rod is in equilibrium, Moment about A should be zero.

$$\Rightarrow \boxed{\sum M_A = 0} \quad \frac{l}{2} W - \frac{l}{2} T \sin \theta = 0$$

$$T = \frac{W}{\sin \theta} = \frac{400}{\sin 68^\circ} = 431.4 \text{ N}$$

$$\boxed{T = 431.4 \text{ N}}$$

$$\boxed{\sum F_x = 0} \quad A_x - T \cos \theta = 0$$

$$A_x = T \cos \theta = 431.4 \cos 68^\circ$$

$$\boxed{A_x = 161.6 \simeq 162 \text{ N}}$$

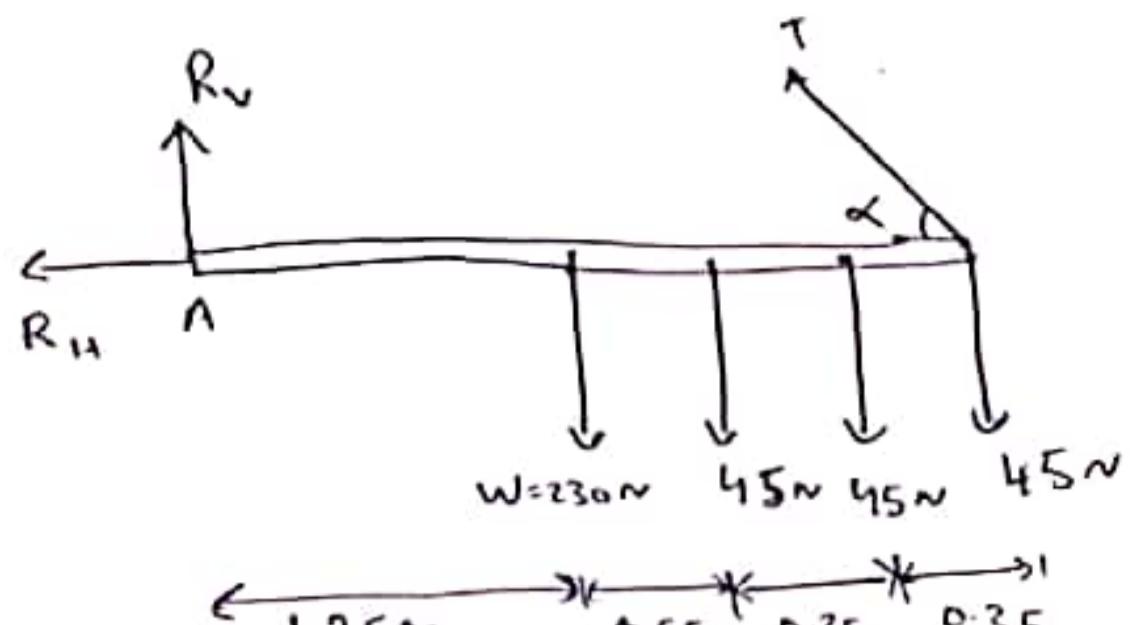
$$\boxed{\sum F_y = 0} \quad A_y + T \sin \theta - W = 0$$

$$\boxed{A_y = 0}$$

Final Answer

$$T = 431.4 \simeq 431 \text{ N}, \quad A_x = 162 \text{ N}, \quad A_y = 0$$

F.B.O of Rock



Applying equilibrium condition

$$\sum F_x = 0$$

$$R_H + T \cos \alpha = 0$$

$$R_H = -T \cos \alpha$$

(-ve sign indicate)

Opposite to direction

— ① tangent
i.e. towards right

$$\sum F_y = 0$$

$$R_v + T \sin \alpha - 230 - 45 - 45 - 45 = 0$$

$$R_v + T \sin \alpha - 335 = 0$$

— ②

$$\sum M_A = 0$$

$$T \sin \alpha \times 2.5 - 45 \times 2.15 - 45 \times 2.15 - 45 \times 1.8$$

$$- 230 \times 1.25 = 0$$

$$T \sin \alpha \times 2.5 = 577.75$$

$$T \sin \alpha = 231.1$$

$$T = \frac{231.1}{\sin \alpha}$$

— ③

Solving eqn's ①, ② & ③

$$R_v = 103.9 \text{ N}$$

$$R_H = -231.1 \cot \alpha$$

$$R_A = \sqrt{R_v^2 + R_H^2}$$

$$R_A = \sqrt{(103.9)^2 + (231.1)^2 \cot^2 \alpha}$$

$$R_A = R_v + R_u$$

$$R_A = \sqrt{(103.9)^2 + (231.1)^2} \cot \angle$$

Taking $\angle = 45^\circ$

(a)

$$T = 326.82 \text{ N}$$

(b)

$$R_A = 253.4 \text{ N}$$

(c)

Taking $\angle = 65^\circ$

$$T = \frac{231.1}{\sin 65^\circ}$$

$$T = 254.99 \text{ N}$$

(d)

$$R_A = 149.7 \text{ N}$$

$$\% \text{ change} = \frac{253.4 - 149.7}{253.4} \times 10^0$$

$$\% \text{ change} = 40.92 \% \text{ (decrease)}$$

NOTE

I think the answers given ^{below question's} are wrong
~~in the book~~. I have tried this question
3 times and I got the same answers
so I am sure that answers provided
by me are correct.

Problem 4.6 Using two different cable-pulley arrangements shown in Fig. 4.53, a block of weight W is elevated to a certain height. For each system, determine how much force is applied to the person holding the cable.

Answers: $T_1 = W/2$ $T_2 = W/4$

Problem 4.7 As illustrated in Fig. 4.53b, consider a person who is trying to elevate a load to a certain height by using a cable-pulley arrangement. If the force applied by the person on the cable is $T = 65$ N, determine the mass of the load.

Answer: $m = 26.5$ kg

Problem 4.8 Using a cable-pulley arrangement shown in Fig. 4.54, a block of mass $m = 50$ kg is elevated from the ground to a certain height. Determine the magnitude of force T applied by the worker performing the lifting task on the cable.

Answer: $T = 163.3$ N

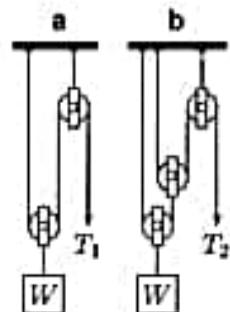


Fig. 4.53 Problems 4.6 and 4.7

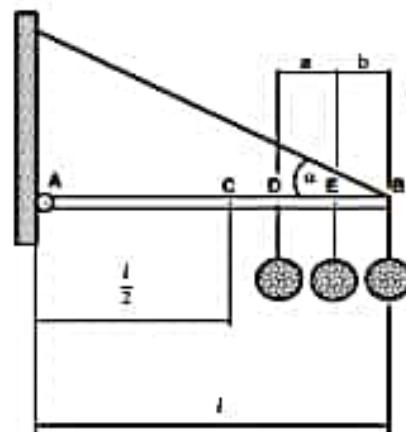


Fig. 4.54 Problem 4.8

4.6)

for the pulley attached to weight W as the net force is zero

$2 * T_1 - W = 0$ $T_1 = W/2$ the tension T_1 is $W/2$

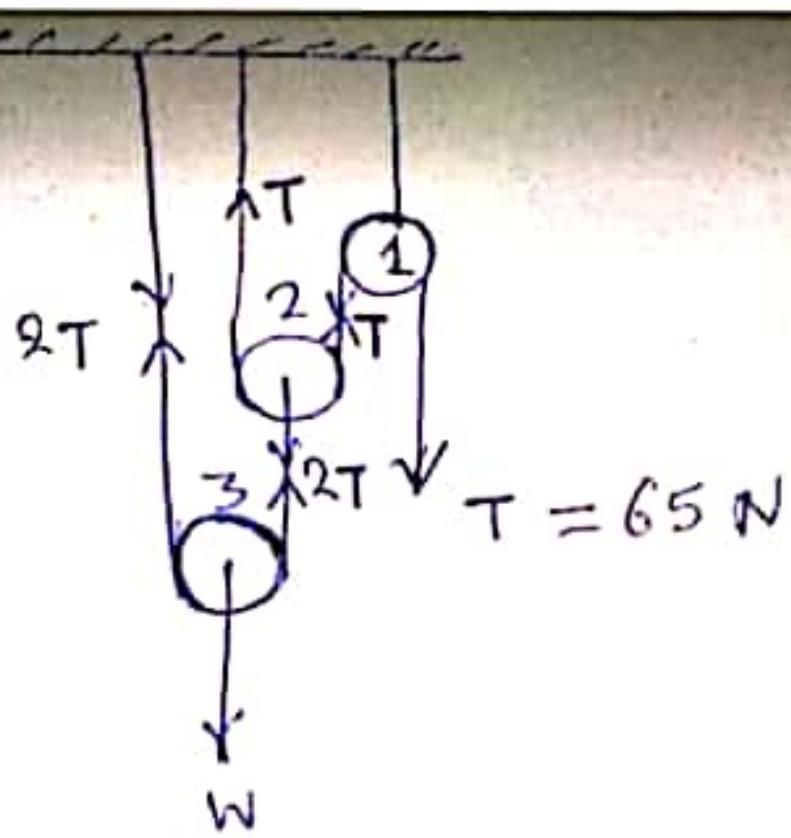
fig B)

for the lowest pulley tension in the attached string is T_3

$2 * T_3 - W = 0$ $T_3 = W/2$

Now, for the middle pulley $2 * T_2 - T_3 = 0$

$T_2 = W/4$ the tension T_2 is $W/4$



Since, all the pulley are frictionless
 so the tension in each side of pulley
 in same string will be same.

g = Acceleration due to gravity
 $= 9.81 \cancel{m/s^2}$

For pulley 2.

$$T_2 = T + T = 2T$$

For pulley 3.

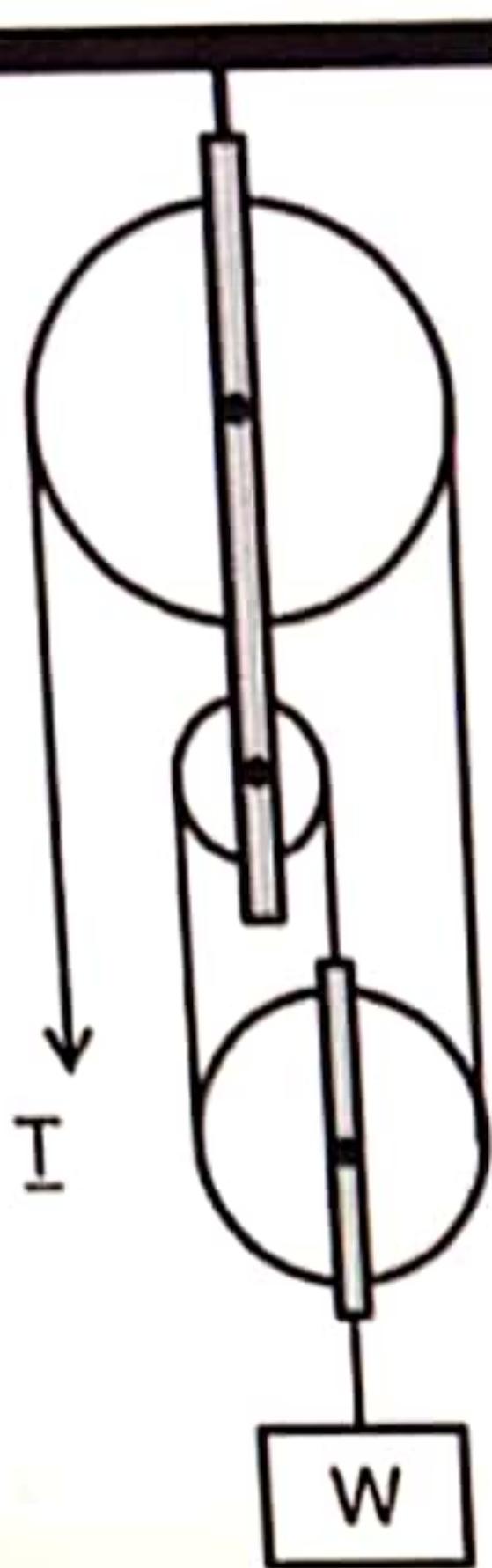
$$W = 2T + 2T$$

$$W = 4T$$

$$mg = 4T$$

$$m = \frac{4T}{g} = \frac{4 \times 65}{9.81}$$

$$m = 26.5 \text{ kg}$$



Mass of the block $m = 50 \text{ kg}$

Forces acting on the block ,

i) weight of block $W = mg$, vertically downwards

ii) $3T$ upwards (T is tension in the string , as there are 3 string attached to lower pulley to pull it upwards)

$$W = 3T \quad mg = 3T$$

$$T = \frac{mg}{3} = \frac{50 * 9.8}{3} = 163.3 \text{ N}$$

Tension in the string is $T = 163.3 \text{ N}$

Problem 4.9 Consider the split Russel traction device and a mechanical model of the leg shown in Fig. 4.55. The leg is held in the position shown by two weights that are connected to the leg via two cables. The combined weight of the leg and the cast is $W = 300 \text{ N}$. l is the horizontal distance between points A and B where the cables are attached to the leg. Point C is the center of gravity of the leg including the cast which is located at a distance two-thirds of l as measured from point A. The angle cable 2 makes with the horizontal is measured as $\beta = 45^\circ$.

Determine the tensions T_1 and T_2 in the cables, weights W_1 and W_2 , and angle α that cable 1 makes with the horizontal, so that the leg remains in equilibrium at the position shown.

Answers: $T_1 = W_1 = 223.6 \text{ N}$ $T_2 = W_2 = 282.8 \text{ N}$ $\alpha = 26.6^\circ$

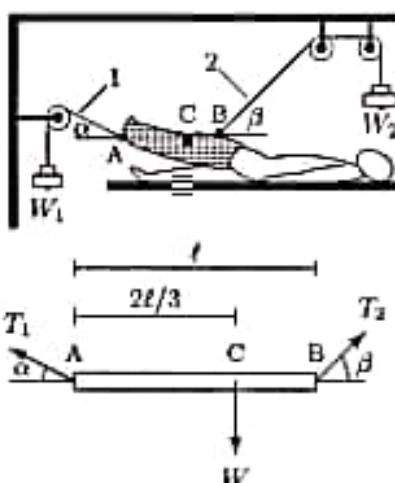


Fig. 4.55 Problem 4.9

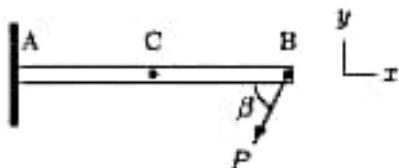


Fig. 4.56 Problem 4.10

Problem 4.10 Consider the uniform, horizontal cantilever beam shown in Fig. 4.56. The beam is fixed at point A and a force that makes an angle $\beta = 63^\circ$ with the horizontal is applied at point B. The magnitude of the applied force is $P = 80 \text{ N}$. Point C is the center of gravity of the beam and the beam weighs $W = 40 \text{ N}$ and has a length $l = 2 \text{ m}$.

Determine the reactions generated at the fixed end of the beam.

Answers: $R_{Ax} = 36.3 \text{ N}$ (+x) $R_{Ay} = 111.3 \text{ N}$ (+y) $M_A = 182.6 \text{ Mm}$ (ccw)

Problem 4.11 Consider the L-shaped beam illustrated in Fig. 4.57. The beam is welded to the wall at point A, the arm AB extends in the positive z direction, and the arm BC extends in the negative y direction. A force P is applied in the positive x direction at the free end (point C) of the beam. The lengths of arms AB and BC are a and b , respectively, and the magnitude of the applied force is P .

Assuming that the weight of the beam is negligibly small, determine the reactions generated at the fixed end of the beam in terms of a , b , and P .

Answers: The non-zero force and moment components are:

$$R_{Ax} = P(-x) \quad M_{Ay} = aP(-y) \quad M_{Az} = bP(-z)$$

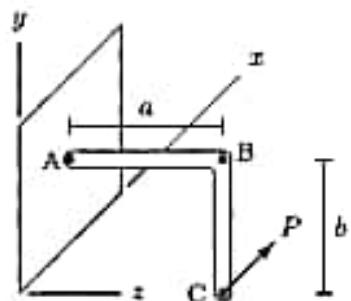


Fig. 4.57 Problems 4.11 and 4.12

Problem 4.12 Reconsider the L-shaped beam illustrated in Fig. 4.57. This time, assume that the applied force P has components in the positive x and positive z directions such that $P = P_x i + P_y j$. Determine the reactions generated at the fixed end of the beam in terms of a , b , P_x , and P_y .

Answers:

$$R_{Ax} = P_x(-x) \quad R_{Ay} = 0 \quad R_{Az} = P_y(-z)$$

$$M_{Ax} = bP_z(+y) \quad M_{Ay} = aP_z(-y) \quad M_{Az} = bP_x(-z)$$

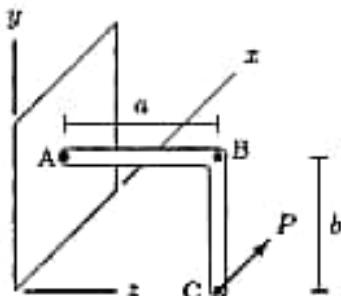


Fig. 4.57 Problems 4.11 and 4.12

Given data; Load, $W = 300 \text{ N}$

Angle, $\beta = 45^\circ$

Now body to be in equilibrium, summition of all the forces and moment to be zero.

i.e. Horizontal force, $\sum F_x = 0$

$$T_1 \cos \alpha = T_2 \sin \beta$$

$$\Rightarrow T_1 \cos \alpha = T_2 \sin 45^\circ$$

.....(1)

Vertical force, $\sum F_y = 0$

$$T_1 \sin \alpha + T_2 \sin \beta = 300$$

$$\Rightarrow T_1 \sin \alpha + T_2 \sin 45^\circ = 300$$

.....(2)

Moment about any point, $\sum M_A = 0$

.....(Taken about 'A')

$$-300 * \frac{2l}{3} + T_2 * l = 0$$

$$\Rightarrow T_2 \sin 45^\circ = 200$$

$$\Rightarrow T_2 = 200\sqrt{2}$$

$$\because \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\therefore T_2 = 282.8427 \text{ N}$$

Substitututng the value of T_2 into the equation 1 & 2, we get;

$$T_1 \cos \alpha = 200 \quad \&$$

$$T_1 \sin \alpha = 100$$

$$\Rightarrow \tan \alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = \tan^{-1} \frac{1}{2}$$

$$\therefore \alpha = 26.5650^\circ$$

Thus,

$$T_1 \cos 26.5650^\circ = 200$$

$$\Rightarrow T_1 = \frac{200}{\cos 26.5650^\circ}$$

$$\therefore T_1 = 223.6067 \text{ N}$$

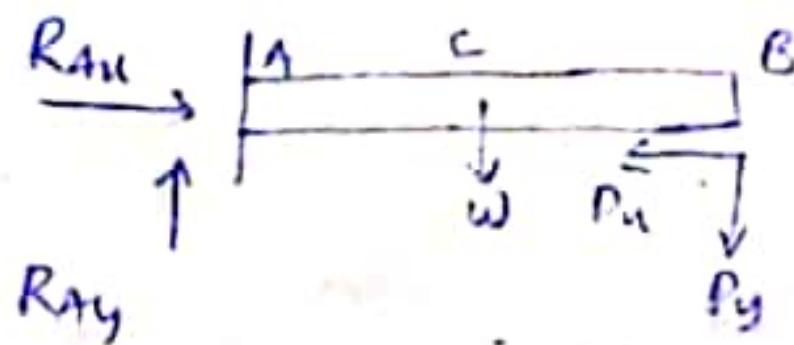
Therefore, answers are:

$$T_1 = 223.6067 \text{ N}$$

$$T_2 = 282.8427 \text{ N}$$

$$\alpha = 26.5650^\circ$$

free body diagram



Reducing force in x-direction

$$P_x = P \cos \theta$$

$$= (80) \cos 63^\circ$$

$$P_x = 36.32 \text{ N}$$

$$P_y = P \sin \theta = (80) \sin 63^\circ = 71.28 \text{ N.}$$

$$\therefore \sum F_x = 0$$

$$R_{Ax} = P_x = 36.32 \text{ N. (+x)}$$

Reducing force in y-direction

$$R_{Ay} - w - P \sin \theta = 0$$

$$R_{Ay} = w + P \sin \theta$$

$$= 40 + 80 \sin 63^\circ$$

$$= 117.3 \text{ (+y)}$$

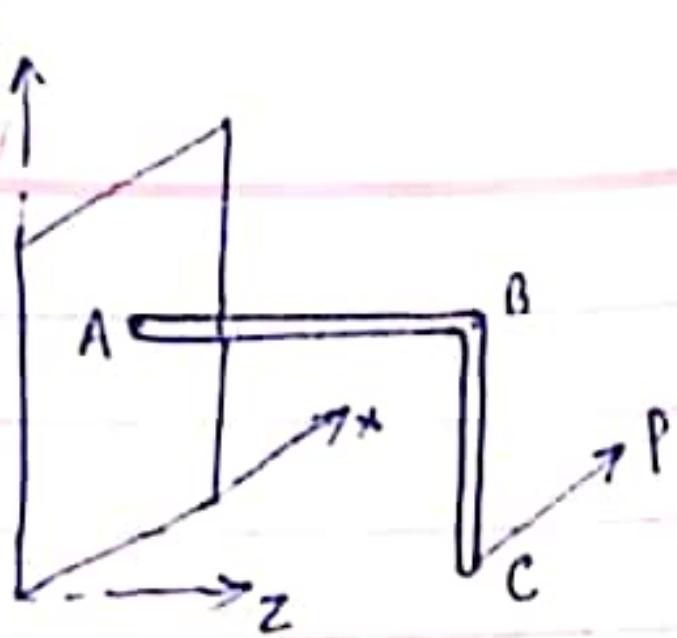
Taking moment about Point A

$$\sum M_A = 0$$

$$m_A - 2 \cdot P \sin \theta \rightarrow w \times 1 = 0$$

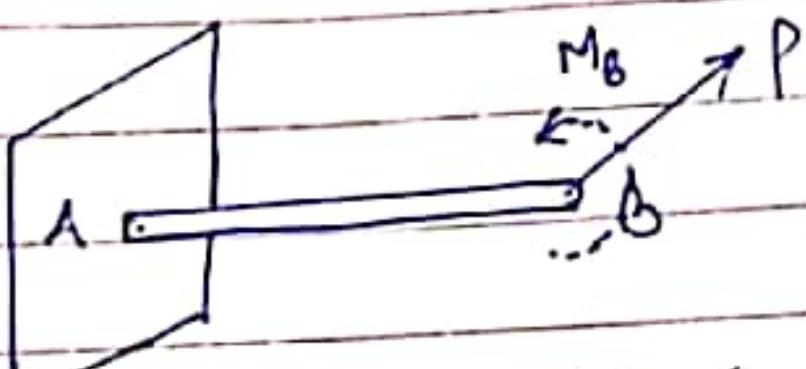
$$m_A = 2 \times 80 \sin 63^\circ + 40$$

$$= 182.6 \text{ N-m. (counter clockwise)}$$



we can remove the link BC assuming a ~~rotational~~ couple at point B and a load of P at the same point as illustrated below:

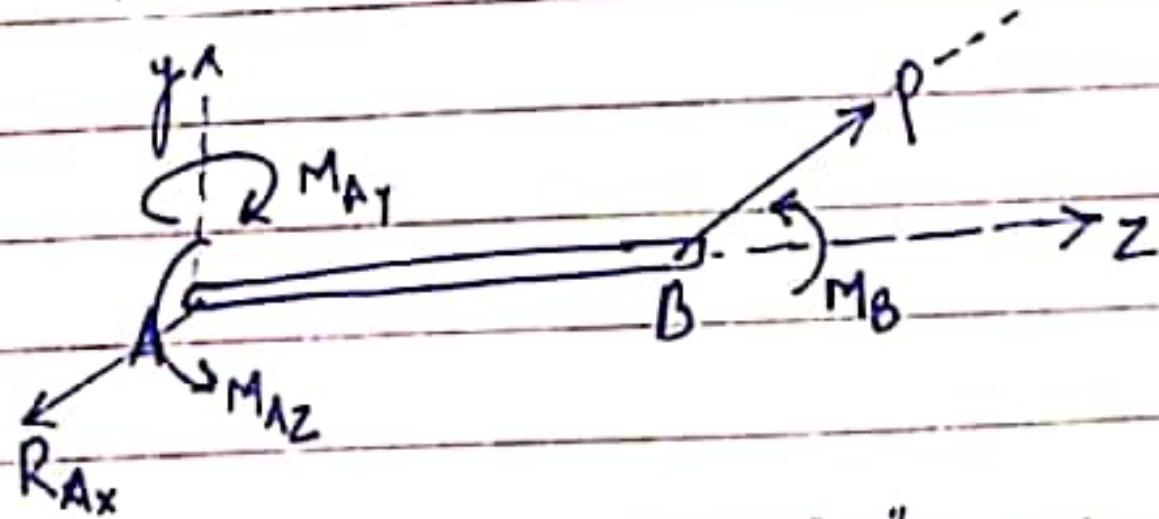
Load & moment at point B
⇒



$$\text{where, } M_B = P \cdot b \text{ (+z)}$$

Now, FBD of link A.

FBD of link AB ⇒



R_{Ax} → Due to the load 'P' at Pt. 'B'

M_{Az} → Opposite moment due to M_B

M_{Ay} → Opposite moment due to load "P" acting at a distance of "a" i.e. at 'B'

$$\therefore R_{Ax} = P \text{ (in -x direction)}$$

$$M_{Az} = -M_B = P \cdot b \text{ (in -z direction)}$$

$$M_{Ay} = P \cdot a \text{ (in -y direction)}$$

Answer

Problem 5.1 Consider a person holding an object in his hand with his elbow flexed at the right angle with respect to the upper arm (Fig. 5.4). The forces acting on the forearm and the mechanics model of the system are shown in Fig. 5.5a, b. As for this system assume that the biceps is the major flexor and the line of action of the muscle makes the right angle with the long axis of the forearm. Point O designates the axis of rotation at the elbow joint, A is the point of attachment of the biceps muscle to the radius, point B is the center of gravity of the forearm, and point C is the center of gravity of the object held in the hand. Furthermore, the distances between the axis of rotation of the elbow joint (point O) and points A, B, and C are $a = 4.5 \text{ cm}$, $b = 16.5 \text{ cm}$, and $c = 37 \text{ cm}$. If the total weight of the forearm is $W = 83 \text{ N}$, and the magnitude of the muscle force is $F_M = 780 \text{ N}$:

- Determine the weight (W_0) of the object held in the hand.
- Determine the magnitude of the reaction force (F_J) at the elbow joint.
- Determine the magnitude of the muscle (F_{M1}) and joint reaction (F_{J1}) forces when the weight of the object held in the hand is increased by 5 N.

Answers: (a) $W_0 = 57.8 \text{ N}$; (b) $F_J = 639.2 \text{ N}$; (c) $F_{M1} = 820 \text{ N}$, $F_{J1} = 674.2 \text{ N}$



Fig. 5.4 Example 5.1

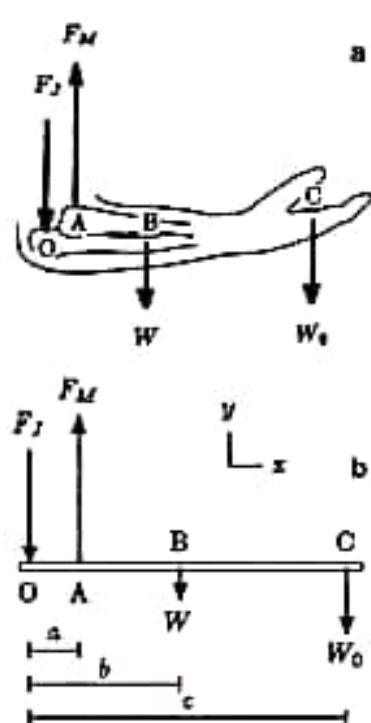


Fig. 5.5 Forces acting on the lower arm

Problem 5.2 Consider a person performing shoulder exercises by using a dumbbell (Fig. 5.11). The forces acting on the arm and the mechanical model of the system are shown in Fig. 5.12. For this system assume that the arm of the person is fully extended to the horizontal. Point O designates the axis of rotation of the shoulder joint, A is the point of attachment of the deltoid muscle to the humerus, point B is the center of gravity of the entire arm, and point C is the center of gravity of the dumbbell. The distances between the axis of rotation of the shoulder joint (point O) and points A, B, and C are $a = 17 \text{ cm}$, $b = 33 \text{ cm}$, and $c = 63 \text{ cm}$. The dumbbell weighs $W_0 = 64 \text{ N}$ and for this position of the arm it is estimated that the magnitude of the muscle force is $F_M = 1051 \text{ N}$. If the lines of action of the muscle (F_M) and the joint reaction forces (F_J) make an angle $\theta = 18^\circ$ and $\beta = 12^\circ$ with the horizontal, respectively:

- Determine the magnitude of reaction force (F_J) at the shoulder joint.
- Determine the total weight (W) of the arm.
- Determine the magnitude of the muscle (F_{M1}) and joint reaction (F_{J1}) forces when the weight of the dumbbell is increased by 5 N.

Answers: (a) $F_J = 1021.9 \text{ N}$; (b) $W = 47.3 \text{ N}$; (c) $F_{M1} = 1136.5 \text{ N}$, $F_{J1} = 1105 \text{ N}$

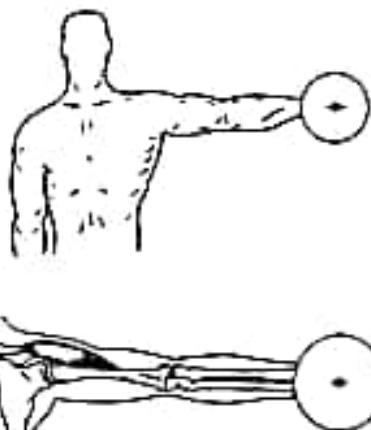


Fig. 5.11 The arm is abducted to horizontal

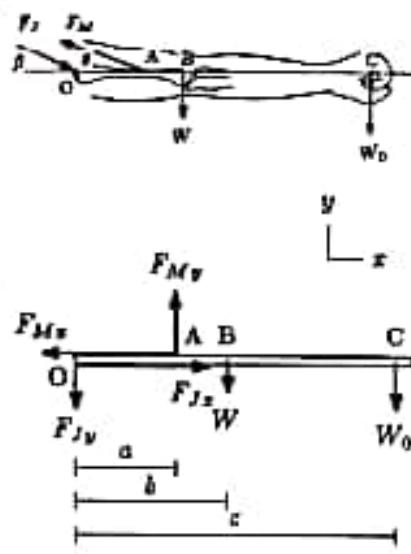
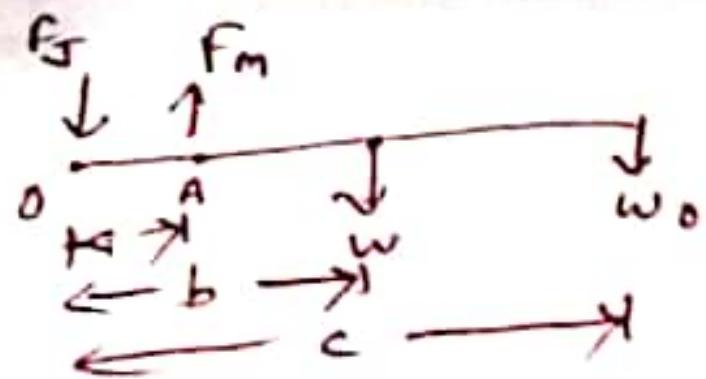


Fig. 5.12 Forces acting on the arm and a mechanical model representing the arm



$$\begin{aligned}
 a &= 4.5 \text{ cm} \\
 b &= 16.5 \text{ cm} \\
 c &= 37 \text{ cm} \\
 w &= 83 \text{ N} \\
 F_m &= 780 \text{ N}
 \end{aligned}$$

$\Sigma Y = 0$

$$F_m = w + w_0 + F_J$$

$$780 = 83 + w_0 + F_J \quad \text{--- (1)}$$

$\Sigma M_0 = 0$

$$F_m \times a - w \times b - w_0 \times c = 0$$

$$780 \times 4.5 - 83 \times 16.5 - w_0 \times 37 = 0$$

$$\boxed{w_0 = 57.85 \text{ N}}$$

and

$$780 = 83 + 57.85 + F_J$$

$$\boxed{F_J = 639.1486 \text{ N}}$$

$$(c) \quad w_0 = 57.85 + 5 = 62.85 \text{ N}$$

$$F_{m_1} = 83 + 62.85 + F_{J_1} \quad \text{--- (2)}$$

$$\text{and} \quad F_{m_1} \times 4.5 - 83 \times 16.5 - 62.85 \times 37 = 0$$

$$\boxed{F_{m_1} = 821.1 \text{ N}}$$

$$F_{J_1} = 821.1 - 83 - 62.85$$

$$\boxed{F_{J_1} = 675.25 \text{ N}}$$

Given :-

$$a = 17 \text{ cm}$$

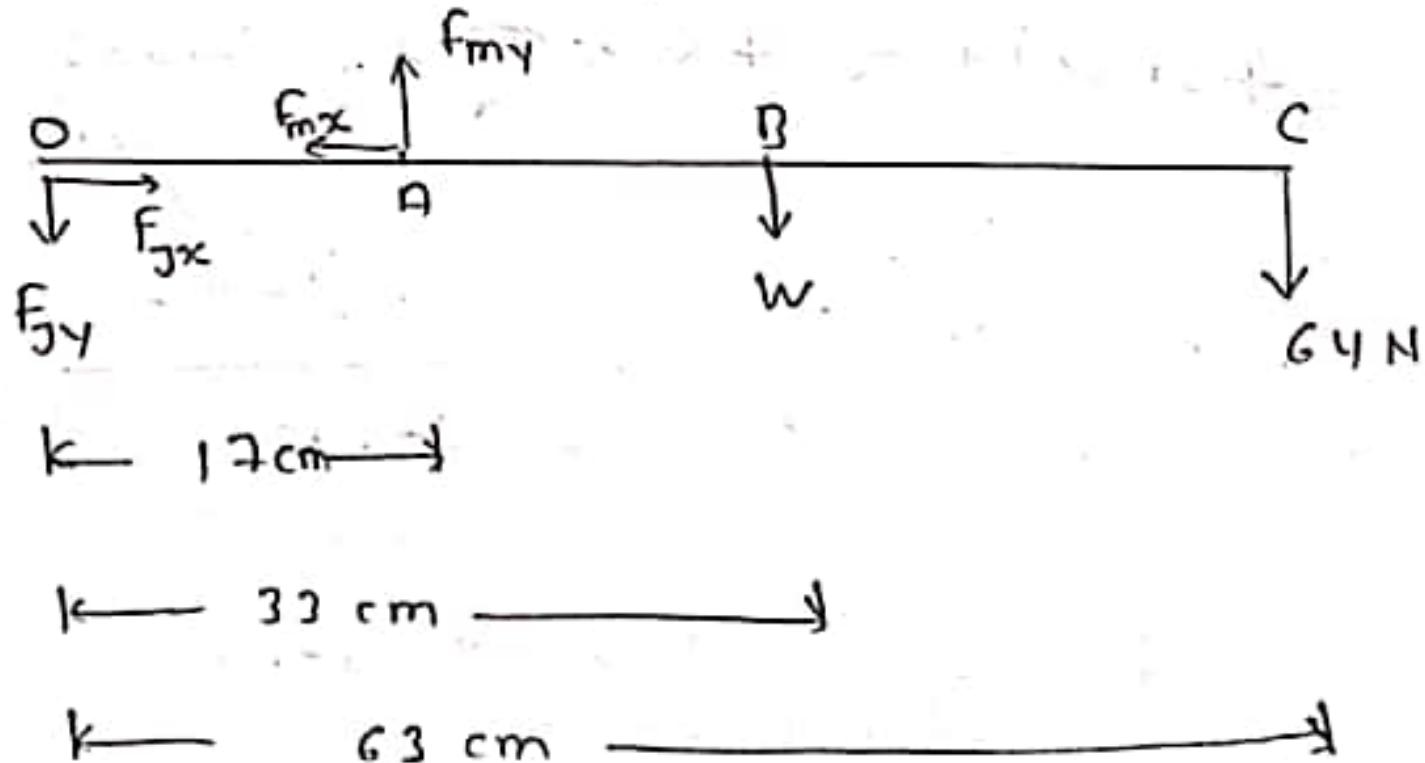
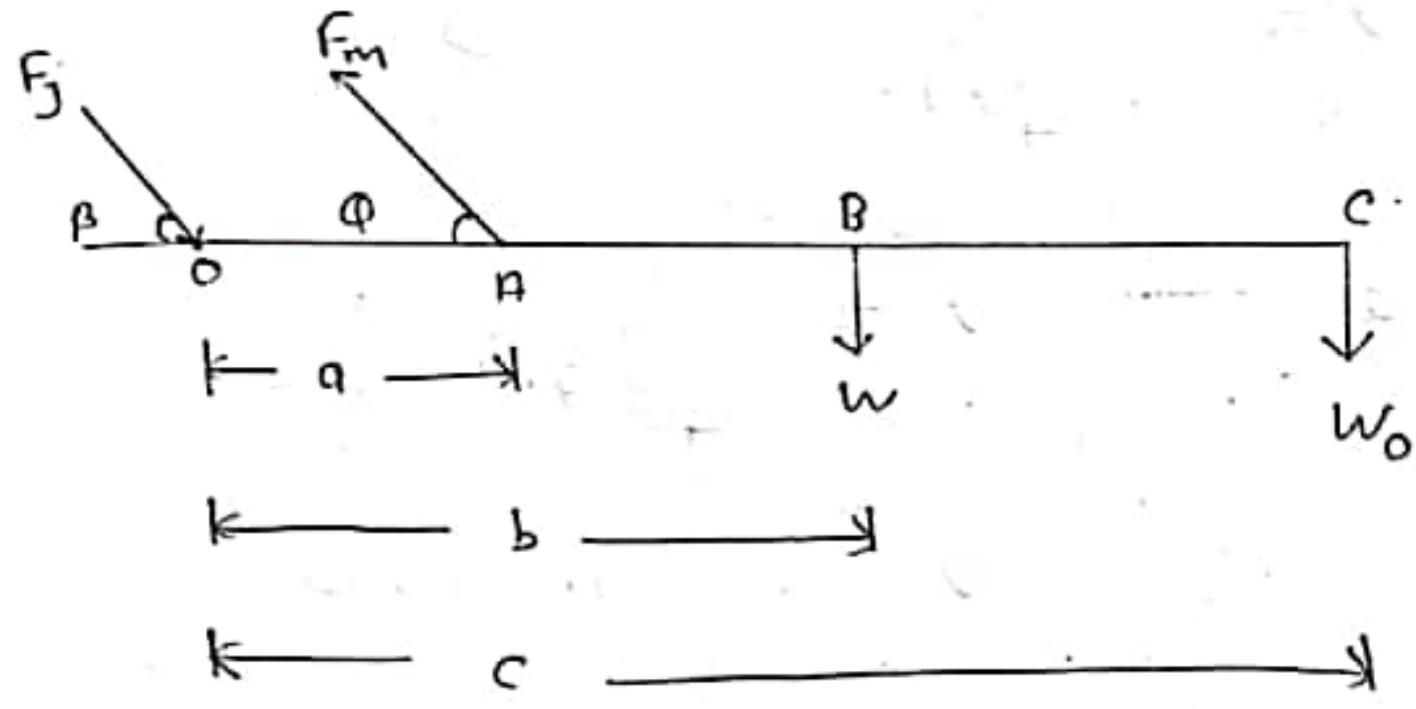
$$b = 33 \text{ cm}$$

$$c = 63 \text{ cm}$$

dumbbell weight = 64 N
(w_0)

$$F_m = 1051 \text{ N}$$

$$\theta = 18^\circ \quad \beta = 12^\circ$$



$$w = \frac{-64 \times 63 + 324.777 \times 17}{33}$$

$$(b) \boxed{w = 47.3 \text{ N}} \quad \text{Approximate}$$

$$\text{Now } \sum F_y = 0$$

$$-F_gy + F_{mxy} - w - 64 = 0$$

$$-F_gy + 324.777 - 47.3 - 64 = 0$$

$$w = \frac{-64 \times 6.3 + 324.777 \times 17}{33}$$

(b) $w = 47.3 \text{ N}$ * Approximate

Now $\sum F_y = 0$

$$-F_{JY} + F_{MY} - w - 64 = 0$$

$$-F_{JY} + 324.777 - 47.3 - 64 = 0$$

$$F_{JY} = 213.477 \text{ N}$$

but $F_{JY} = F_J \sin \beta$

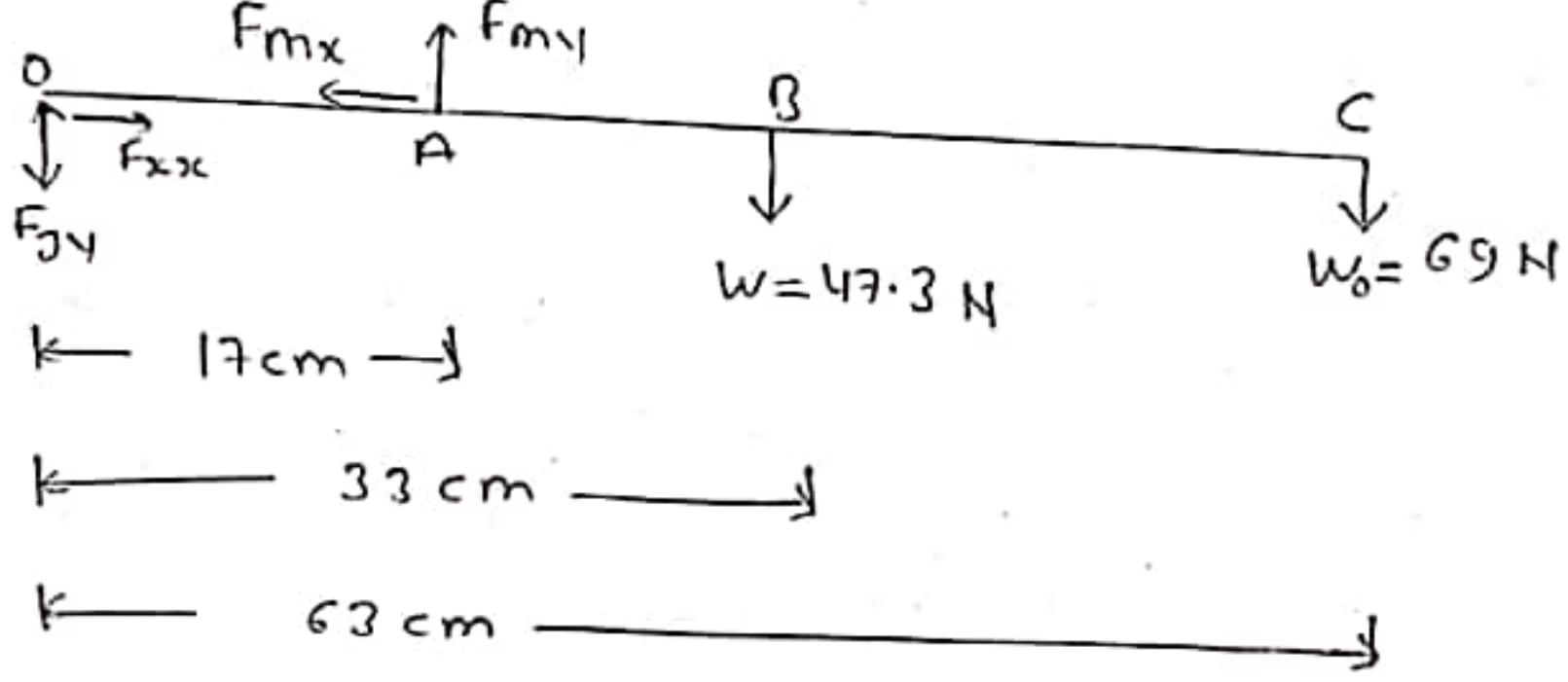
$$213.477 = F_J \times \sin 12$$

(q) Ans. $F_J = 1021.9 \text{ N}$ * Approximate

(c) when weight of dumbbell is increase by 5 N

then $w_0 = (64 + 5) \text{ N}$

$$w_0 = 69 \text{ N}$$



Taking moment at point O equal to zero
i.e. $\sum M_O = 0$

$$-F_{MY} \times 17 + 47.3 \times 33 + 69 \times 63 = 0$$

$$F_{MY} = 242.7 \text{ N}$$

Problem 5.3 Consider the position of the head and neck as well as forces acting on the head shown in Fig. 5.15. For this equilibrium condition assume that the forces involved form a concurrent force system. Point C is the center of gravity of the head, A is the point of application of force (F_M) exerted by the neck extensor muscles on the head, and point B is the center of rotation of the atlantooccipital joint. For this position of the head, it is estimated that the magnitude of the resultant force exerted by the neck extensor muscles is $F_M = 57 \text{ N}$, and the lines of action of the muscles and the joint reaction forces make an angle $\theta = 36^\circ$ and $\beta = 63^\circ$ with the horizontal, respectively. Determine the magnitude of the gravitational force acting on the head.

Answer: $W = 47 \text{ N}$

Problem 5.4 Consider a weight lifter who is trying to lift a barbell. The forces acting on the lower part of the athlete's body and the mechanical model of the system are shown in Figs. 5.19 and 5.20, respectively. Point O designates the center of rotation at the joint formed by the sacrum and the fifth lumbar vertebra. A is the point of application of force exerted by the back muscles, point B is the center of gravity of the lower body, and C is the point of application of the ground reaction force. With respect to point O, $a = 3.6 \text{ cm}$, $b = 14.6 \text{ cm}$, and $c = 22 \text{ cm}$, are the shortest distances between the lines of action of the back muscles' force, the lower body's gravitational force, and the ground reaction force with the center of rotation of the joint. For a weight lifter in this position, it is estimated that the force exerted by the back muscles is $F_M = 6856 \text{ N}$ and the line of action of this force makes an angle $\theta = 43^\circ$ with the vertical. If the barbell weighs $W_0 = 637 \text{ N}$ and the magnitude of the gravitational force acting on the lower body is $W_1 = 333 \text{ N}$:

- Determine the weight (W) of the athlete.
- Determine the magnitude of the reaction force (F_J) acting at the joint.
- Determine an angle α that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W = 705.9 \text{ N}$; (b) $F_J = 7625.8 \text{ N}$; (c) $\alpha = 52^\circ$

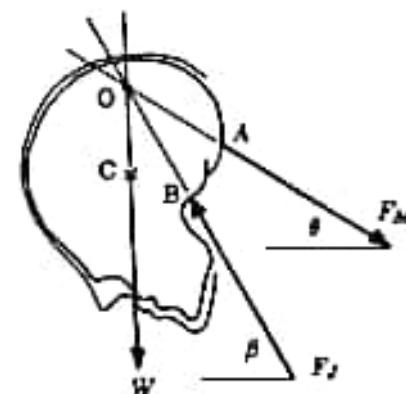


Fig. 5.15 Forces on the skull form a concurrent system

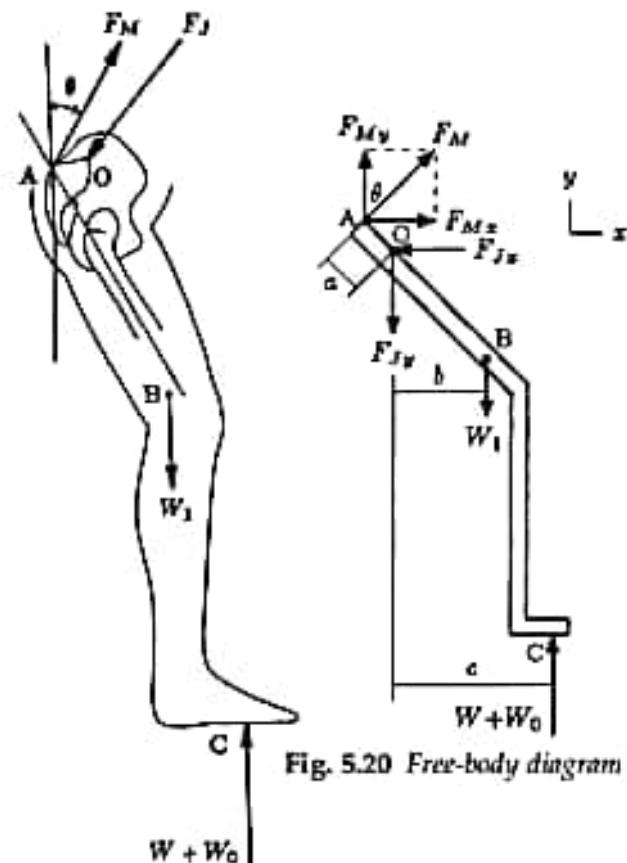
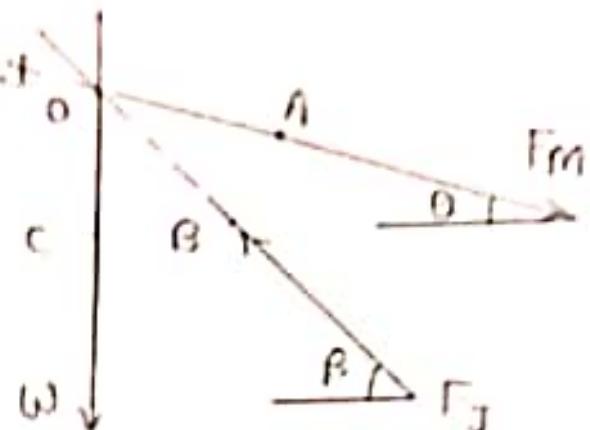


Fig. 5.20 Free-body diagram

Fig. 5.19 Forces acting on the lower body of the athlete

For the equilibrium of the horizontal forces,



$$F_m \cos \theta - F_J \cos \beta = 0$$

$$\Rightarrow 57 \cos 36^\circ = F_J \cos 63^\circ$$

$$\Rightarrow F_J = 101.57 \text{ N}$$

For the equilibrium of the vertical forces,

$$\omega + F_m \sin \theta - F_J \sin \beta = 0$$

$$\Rightarrow \omega + 57 \times \sin 36^\circ - 101.57 \times \sin 63^\circ = 0$$

\Rightarrow

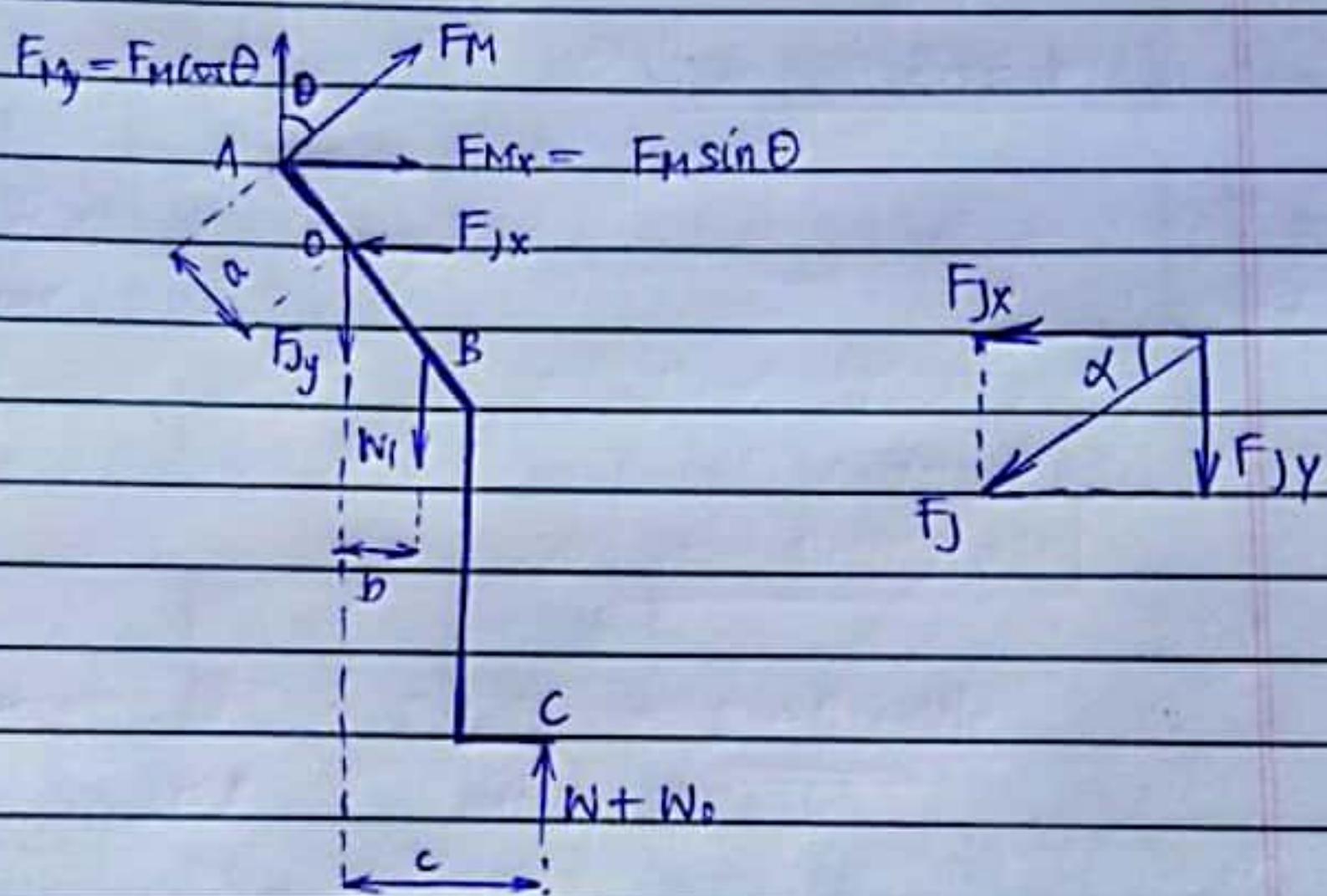
$$\boxed{\omega = 57 \text{ N}}$$

Moment is given by the product of force and the perpendicular distance of the force from the point about which the moment is to be calculated. Now forces F_M and W_1 cause clockwise rotation about point O which is considered negative while force $(W+W_0)$ causes anticlockwise rotation about point O which is considered positive.

$$a = 3.6 \text{ cm} = 0.036 \text{ m}, \quad b = 14.6 \text{ cm} = 0.146 \text{ m}$$

$$c = 22 \text{ cm} = 0.22 \text{ m}, \quad F_M = 6856 \text{ N}, \quad \theta = 43^\circ$$

$$W_0 = 637 \text{ N}, \quad W_1 = 333 \text{ N}$$



a) $\sum M @ O = 0$

$$-F_M \times a - W_1 \times b + (W + W_0) \times c = 0$$

$$-6856 \times 0.036 - 333 \times 0.146 + (W + 637) \times 0.22 = 0$$

$$W = 6856 \times 0.036 + 333 \times 0.146 - 637$$

0.22

$$W = 705.9 \text{ N}$$

b) $\sum F_x = 0$ [Summation of forces in horizontal direction]

$$F_M \sin \theta - F_{Jx} = 0$$

$$F_{Jx} = -6856 \sin 43^\circ$$

$\rightarrow +ve$

$$F_{Jx} = 4675.78 \text{ N}$$

$\leftarrow -ve$

$\sum F_y = 0$ [Summation of forces in vertical direction]

$$F_M \cos \theta - F_{Jy} - W_1 + W_0 = 0$$

$$F_{Jy} = 6856 \cos 43^\circ - 333 + 705.9 + 637$$

$\uparrow +ve$

$$F_{Jy} = 6024.06 \text{ N}$$

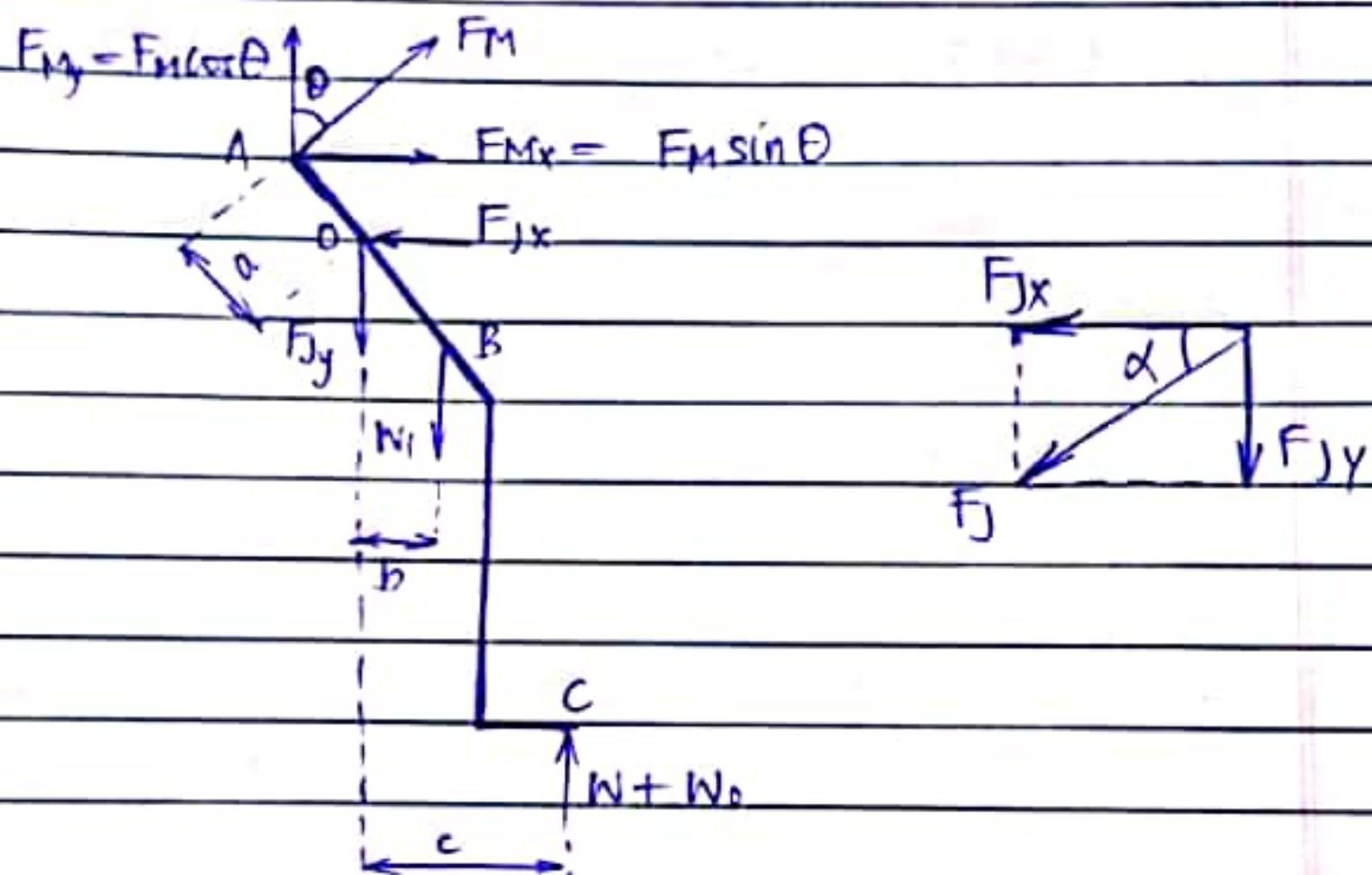
$\downarrow -ve$

considered positive.

$$a = 3.6 \text{ cm} = 0.036 \text{ m}, \quad b = 14.6 \text{ cm} = 0.146 \text{ m}$$

$$c = 22 \text{ cm} = 0.22 \text{ m}, \quad F_M = 6856 \text{ N}, \quad \theta = 43^\circ$$

$$N_0 = 637 \text{ N}, \quad W_1 = 333 \text{ N}$$



a) $\sum M @ O = 0$

$$-F_M \times a - W_1 \times b + (W + W_0) \times c = 0$$

$$-6856 \times 0.036 - 333 \times 0.146 + (W + 637) \times 0.22 = 0$$

$$W = \frac{6856 \times 0.036 + 333 \times 0.146}{0.22} - 637$$

$$W = 705.9 \text{ N}$$

$$W = 705.9 \text{ N}$$

b) $\sum F_x = 0$ [Summation of forces in horizontal direction]

$$F_M \sin \theta - F_{jx} = 0$$

$$F_{jx} = -6856 \sin 43^\circ$$

$\rightarrow +ve$

$$F_{jx} = 4675.78 \text{ N}$$

$\leftarrow -ve$

$\sum F_y = 0$ [Summation of forces in vertical direction]

$$F_M \cos \theta - F_{jy} - W_1 + W + W_0 = 0$$

$$F_{jy} = 6856 \cos 43^\circ - 333 + 705.9 + 637$$

$\uparrow +ve$

$$F_{jy} = 6024.06 \text{ N}$$

$\downarrow -ve$

$$F_j = \sqrt{F_{jx}^2 + F_{jy}^2} = \sqrt{4675.78^2 + 6024.06^2}$$

$$F_j = 7625.8 \text{ N}$$

c) $\alpha = \tan^{-1} \frac{F_{jy}}{F_{jx}} = \tan^{-1} \frac{6024.06}{4675.78}$

$$\alpha = 52^\circ$$

Problem 5.5 Consider a person that momentarily put the entire weight of his body on one leg when walking or running. The forces acting on the leg and the mechanical model of the system are shown in Figs. 5.24 and 5.25, respectively. Point O designates the center of rotation of the hip joint. A is the point of attachment of the hip abductor muscles to the femur, point B is the center of gravity of the leg, and C is the point of application of the ground reaction force. The distances between point A and points O, B, and C are specified as $a = 8.6$ cm, $b = 34.3$ cm, and $c = 89.4$ cm. The angles that the femoral neck and the long axis of the femoral shaft make with the horizontal are specified as $\alpha = 43^\circ$ and $\beta = 79^\circ$, respectively. Furthermore, for this single-leg stance, it is estimated that the magnitude of force exerted by the hip abductor muscles is $F_M = 2062.6$ N and its line of action makes an angle $\theta = 69^\circ$ with the horizontal. If the magnitude of gravitational force acting on the leg is $W_1 = 125$ N:

- Determine the total weight (W) of the person.
- Determine the magnitude of the reaction force (F_J) acting at the hip joint.
- Determine an angle γ that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W = 729.7$ N; (b) $F_J = 2636.1$ N; (c) $\gamma = 73.7^\circ$

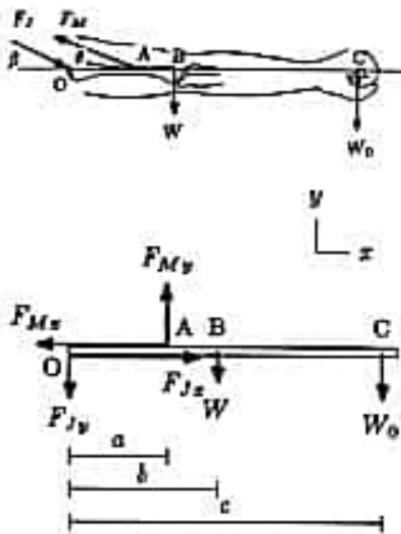


Fig. 5.12 Forces acting on the arm and a mechanical model representing the arm

(a)

Ans

Given that

The distance between point A and point D, B, C are specified as a = 8.1cm

$$b = 34.7\text{cm}$$

$$c = 89.4\text{cm.}$$

The specified $\alpha = 43^\circ$

$$\beta = 79^\circ$$

The magnitude of force exerted by the hip abductor muscles is $TM = 262.6\text{N}$

Makes an angle $\theta = 69^\circ$,
gravitational weight $w = 150\text{N}$

(i) The total weight of the person w is exerted. The person has a weight and the normal force of the floor has magnitude equal to w and acts under the center of gravity of the whole density body of the person.

The person momentarily stationary the horizontal and vertical component of the forces and the total torque are equal to zero

(b)

$$\sum F_H = 0, \sum F_V = 0, \sum M = 0$$

The horizontal component is $f \cos(\theta) - R_H$

The vertical component is $f \sin(\theta) - R_V - \frac{w}{2} + w$

Weight of the length is $\frac{1}{2}$ of the weight of the person.

of the person.

The person momentarily stationary the horizontal and vertical component of the forces and the total torque are equal to zero

③

$$\sum F_H = 0, \sum F_V = 0, \sum \tau = 0$$

The horizontal component is $F \cos(\theta) - R_H$

The vertical component is $F \sin(\theta) - R_V - \frac{W}{7} + W$

Weight of the length is $\frac{1}{7}$ of the weight of the person.

$$R = \sqrt{R^2 y + R^2 v}$$

$$W = 125 \times 7$$

$$= 875 \text{ N}$$

Magnitude of the Reaction

$$F_R = 2.4 \times W$$

$$= 2.4 \times 875$$

$$= 2100 \text{ N}$$

The angle at the knee of force reaction makes with the horizontal $\theta = 10^\circ$

Problem 5.6 Consider a person performing lower leg flexion-extension exercises from a sitting position while wearing a weight boot. Forces acting on the leg and the mechanical model of the system are shown in Fig. 5.38. Point O designates the center of rotation of the tibiofemoral joint. A is the point of attachment of the patellar tendon to the tibia, point B is the center of gravity of the lower leg, and point C is the center of gravity of the weight boot. For this system assume that the points O, A, B, and C all lie along a straight line. The distances between point O and points A, B, and C are measured as $a = 13$ cm, $b = 23.5$ cm, and $c = 53$ cm, respectively. For this position of the leg, the long axis of the tibia makes an angle $\beta = 47^\circ$ with the horizontal, and the line of action of the quadriceps muscle force makes an angle $\theta = 17^\circ$ with the long axis of the tibia. Furthermore, for this position of the leg, it is estimated that the force exerted by the quadriceps muscle is $F_M = 1.940$ N.

If the weight of the lower leg is $W_1 = 163$ N:

- Determine the weight (W_0) of the weight boot.
- Determine the magnitude of the reaction force (F_J) of the tibiofemoral joint.
- Determine an angle φ that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W_0 = 98.4$ N; (b) $F_J = 1707.5$ N; (c) $\varphi = 60.2^\circ$

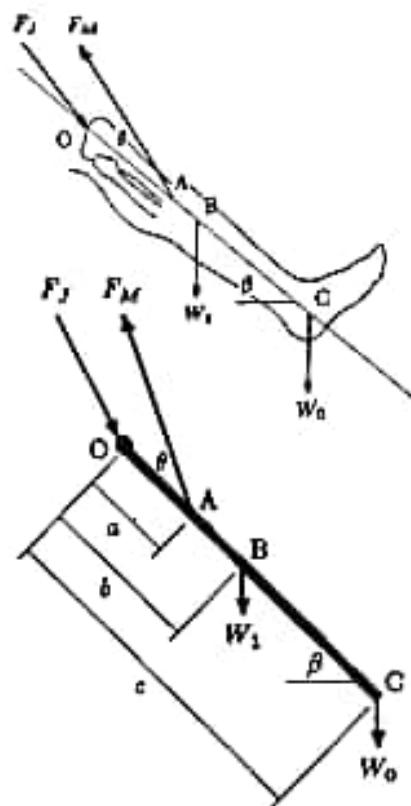


Fig. 5.38 Forces acting on the lower leg

Problem 5.7 Consider a person standing on tiptoe on one foot. For this position, the forces acting on the foot are shown in Fig. 5.45. Point A is the point of attachment of the Achilles tendon through which a force is exerted by the gastrocnemius and soleus muscles on the calcaneus. Point B designates the center of the ankle joint and C is the point of application of the ground reaction force. For this system assume that the weight of the foot can be ignored as it is relatively small when compared to the weight of the entire body of the person. For this position of the foot, it is estimated that the lines of action of the tensile force in the Achilles tendon and the reaction force (F_J) of the ankle joint make an angle $\theta = 49^\circ$ and $\beta = 65^\circ$ with the horizontal, respectively. Furthermore, for this position of the foot, it is also estimated that the magnitude of force exerted by the gastrocnemius and soleus muscles on the calcaneus is $F_M = 1275.4$ N.

- Determine the entire weight (W) of the person.
- Determine the magnitude of the reaction force (F_J) of the ankle joint.

Answers: (a) $W = 831.8$ N; (b) $F_J = 1980.3$ N

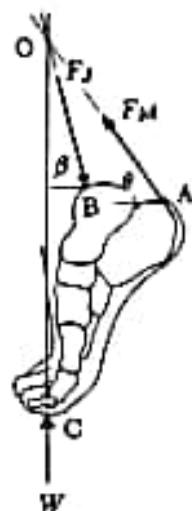


Fig. 5.45 Forces acting on the foot form a concurrent system of forces

Solution:

given that:

$$a = 13 \text{ cm}$$

$$b = 23.5 \text{ cm}$$

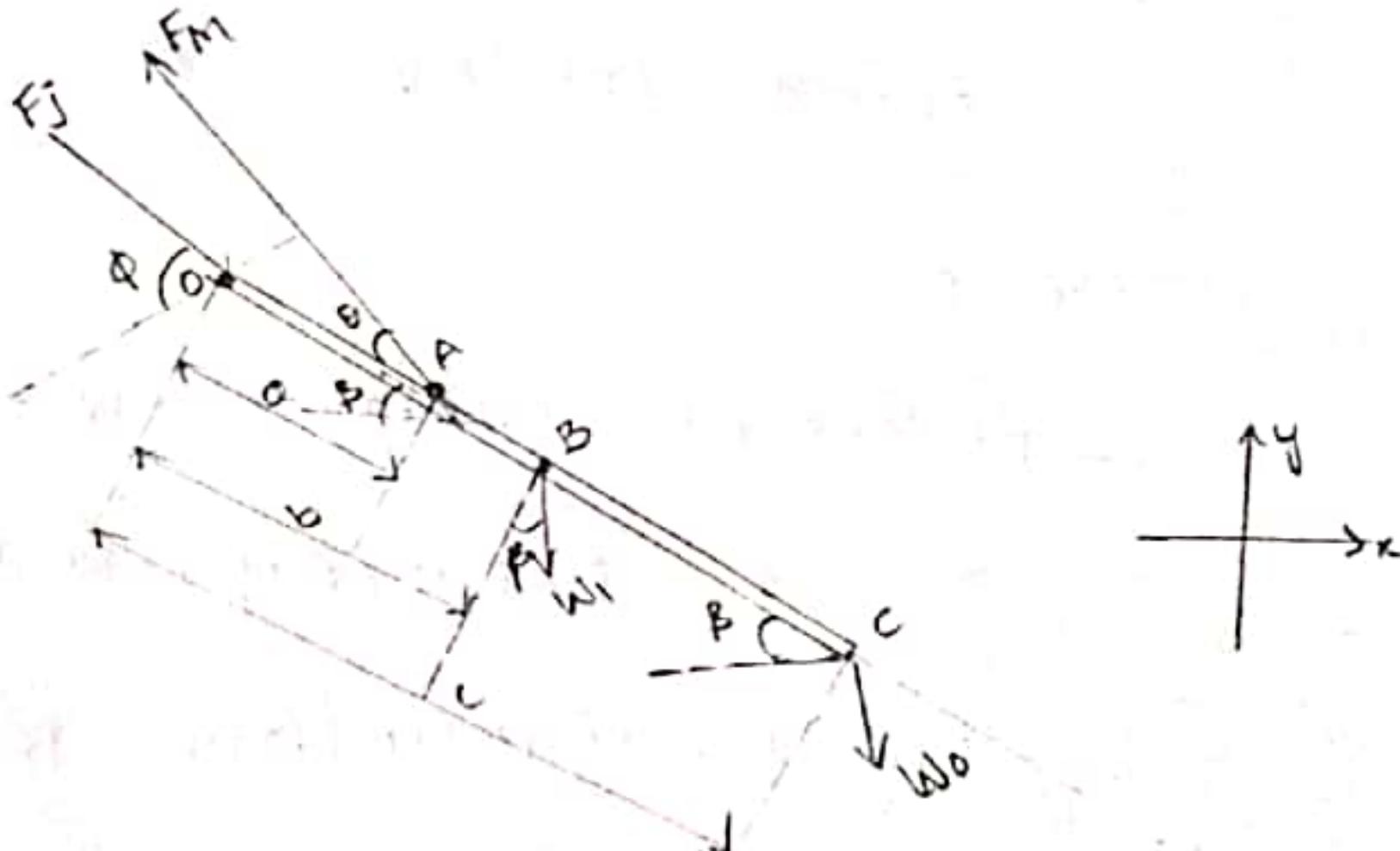
$$c = 53 \text{ cm}$$

$$\beta = 47^\circ$$

$$\theta = 17^\circ$$

$$F_m = 1940 \text{ N}$$

The weight of the lower leg $w_1 = 163 \text{ N}$.



a. moment balance about O.

$$(\Sigma M)_O = 0$$

$$a F_m \sin \theta = w_1 \cos \beta \times b + w_1 \cos \beta \times c$$

$$w_0 = \frac{F_m a \sin \theta - w_1 b \cos \beta}{c \cos \beta}$$

$$w_0 = \frac{1940 \times 13 \sin(17) - 163 \times 23.5 \cos(47)}{53 \times \cos(47)}$$

$$w_0 = 131.72 \text{ N}$$

b. $\sum F_x = 0$

$$w_0 = \frac{1940 \times 13 \sin(17) - 163 \times 23.5 \cos(17)}{53 \times \cos(47)}$$

$$w_0 = 131.72 \text{ N}$$

$$\underline{\underline{b}} \cdot \sum F_x = 0$$

$$F_j \cos \phi - F_m \cos(\theta + \beta) = 0$$

$$F_j \cos \phi = F_m \cos(\theta + \beta)$$

$$F_j \cos \phi = 1940 \cos(17 + 47)$$

$$F_j \cos \phi = 850.44 \text{ N} \rightarrow ①$$

$$\sum F_y = 0$$

$$-F_j \sin \phi + F_m \sin(\theta + \beta) - w_1 - w_0 = 0$$

$$F_j \sin \phi = F_m \sin(\theta + \beta) - w_1 - w_0$$

$$F_j \sin \phi = 1940 \sin(17 + 47) - 163 - 131.72$$

$$F_j \sin \phi = 1448.94 \text{ N} \rightarrow ②$$

$$eq \rightarrow P^2 + \omega^2$$

$$F_j^2 (\sin^2 \phi + \cos^2 \phi) = 1448.94^2 + 850.44^2$$

$$F_j^2 = 1448.94^2 + 850.44^2$$

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$$F_j = \sqrt{1448.94^2 + 850.44^2}$$

$$F_j = 1680.08 \text{ N}$$

③

$$F_j \sin \theta = 1448.94 \text{ N} \rightarrow ②$$

$$F_j \sin \theta = 1448.94 \text{ N} \rightarrow ②$$

$$c^2 = l^2 + \alpha^2$$

$$F_j^2 (\sin^2 \theta + \cos^2 \theta) = 1448.94^2 + 850.44^2$$

$$F_j^2 = 1448.94^2 + 850.44^2$$

Scanned with CamScanner

③

$$F_j = \sqrt{1448.94^2 + 850.44^2}$$

$$F_j = 1680.08 \text{ N}$$

c.
=

$$\tan \theta = \frac{1448.94}{850.44}$$

$$\theta = \tan^{-1} \left[\frac{1448.94}{850.44} \right]$$

$$\theta = 59.59 \approx$$

$$\theta = 60^\circ$$

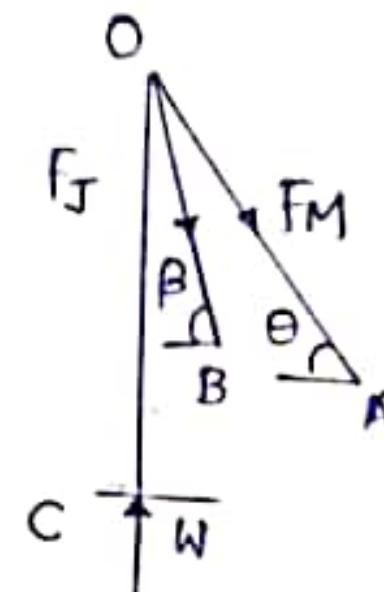
=

Ques Given Data

$$F_M = 1275.4 \text{ N}$$

$$\theta = 49^\circ, \beta = 65^\circ$$

At equilibrium condition
we can write



$$\text{Along Horizontal direction: } F_J \cos \beta = F_M \cos \theta \quad \text{--- (1)}$$

$$\text{Along Vertical direction: } F_J \sin \beta - F_M \sin \theta = W \quad \text{--- (2)}$$

(b) From (1) and (2) we get the magnitude of reaction force of the ankle joint

$$F_J = F_M \frac{\cos \theta}{\cos \beta} = 1275.4 \times \frac{\cos 49}{\cos 65}$$
$$= 1979.9 \text{ N}$$

(a) Put equation (1) into (2) equation we get

$$F_M \frac{\cos \theta}{\cos \beta} \cdot \sin \beta - F_M \sin \theta = W$$

$$W = F_M \frac{\sin(\beta - \theta)}{\cos \beta} = \frac{1275.4 \times \sin(65 - 49)}{\cos 65}$$

$$W = 831.8 \text{ N}$$

Problem 13.1 Complete the following definitions with appropriate expressions.

- (a) Unit deformation of a material as a result of an applied load is called _____.
- (b) The internal resistance of a material to deformation due to externally applied forces is called _____.
- (c) _____ is a measure of the intensity of internal forces acting parallel or tangent to a plane of cut, while _____ are associated with the intensity of internal forces that are perpendicular to the plane of cut.
- (d) On the stress-strain diagram, the stress corresponding to the _____ is the highest stress that can be applied to the material without causing permanent deformation.
- (e) On the stress-strain diagram, the highest stress level corresponds to the _____ of the material.
- (f) For some materials, it may not be easy to distinguish the yield point. The yield strength of such materials may be determined by the _____.
- (g) _____ is defined as the ability of a material to resume its original (stress-free) size and shape upon removal of applied loads.
- (h) For linearly elastic materials, stress is linearly proportional to strain and the constant of proportionality is called the _____ of the material.
- (i) The distinguishing factor in linearly elastic materials is their _____.
- (j) Materials for which the stress-strain curve in the elastic region is not a straight line are known as _____ materials.
- (k) _____ is the constant of proportionality between shear stress and shear strain for linearly elastic materials.
- (l) A mathematical equation that relates stresses to strains is called a _____.
- (m) The analogy between elastic materials and springs is known as _____.
- (n) _____ implies permanent (unrecoverable) deformations.
- (o) The area under the stress-strain diagram in the elastic region corresponds to the _____ energy stored in the material while deforming the material.
- (p) _____ energy is dissipated as heat while deforming the material.
- (q) The area enclosed by the _____ signifies the total strain energy dissipated as heat while loading and unloading a material.
- (r) The technique of changing the yield point of a material by loading the material beyond its yield point is called _____.
- (s) The elastic modulus of a material is a relative measure of the _____ of one material with respect to another.
- (t) A _____ material is one that exhibits a large plastic deformation prior to failure.
- (u) A _____ material is one that shows a sudden failure (rupture) without undergoing a considerable plastic deformation.
- (v) _____ is a measure of the capacity of a material to sustain permanent deformation. The toughness of a material is measured by considering the total area under its stress-strain diagram.
- (w) The ability of a material to store or absorb energy without permanent deformation is called the _____ of the material.
- (x) If the mechanical properties of a material do not vary from location to location within the material, then the material is called _____.
- (y) If a material has constant density, then the material is called _____.
- (z) If the mechanical properties of a material are independent of direction or orientation, then the material is called _____.

Answers to Problem 13.1:

(a) strain	(n) Plasticity
(b) stress	(o) elastic strain
(c) Shear stress, normal stress	(p) Plastic strain
(d) elastic limit	(q) hysteresis loop
(e) ultimate strength	(r) strain hardening
(f) offset method	(s) stiffness
(g) Elasticity	(t) ductile
(h) elastic (Young's) modulus	(u) brittle
(i) elastic (Young's) modulus	(v) Toughness
(j) nonlinear elastic	(w) resilience
(k) Shear modulus	(x) homogeneous
(l) material function	(y) incompressible
(m) Hooke's Law	(z) isotropic

Problem 13.2 Curves in Fig. 13.40 represent the relationship between tensile stress and tensile strain for five different materials. The "dot" on each curve indicates the yield point and the "cross" represents the rupture point. Fill in the blank spaces below with the correct number referring to a material.

- Material _____ has the highest elastic modulus.
 Material _____ is the most ductile.
 Material _____ is the most brittle.
 Material _____ has the lowest yield strength.
 Material _____ has the highest strength.
 Material _____ is the toughest.
 Material _____ is the most resilient.
 Material _____ is the most stiff.

Answers to Problem 13.2: 1, 2, 4, 5, 2, 2, 3, 1

Problem 13.3 Consider two bars, 1 and 2, made of two different materials. Assume that these bars were tested in a uniaxial tension test. Let F_1 and F_2 be the magnitudes of tensile forces applied on bars 1 and 2, respectively. E_1 and E_2 are the elastic moduli and A_1 and A_2 are the cross-sectional areas perpendicular to the applied forces for bar 1 and 2, respectively. For the conditions indicated below, determine the correct symbol relating tensile stresses σ_1 and σ_2 and tensile strains ϵ_1 and ϵ_2 . Note that " $>$ " indicates greater than, " $<$ " indicates less than, " $=$ " indicates equal to, and " $?$ " indicates that the information provided is not sufficient to make a judgement.

- (a) If $A_1 > A_2$ and $F_1 = F_2$ then $\sigma_1 >= ? < \sigma_2$ and $\epsilon_1 >= ? < \epsilon_2$.
 (b) If $E_1 > E_2$, $A_1 = A_2$ and $F_1 = F_2$ then $\sigma_1 >= ? < \sigma_2$ and $\epsilon_1 >= ? < \epsilon_2$

Answers to Problem 13.3:

- (a) $\sigma_1 < \sigma_2$, $\epsilon_1 ? \epsilon_2$
 (b) $\sigma_1 = \sigma_2$, $\epsilon_1 < \epsilon_2$

Problem 13.4 As illustrated in Fig. 13.29, consider a circular cylindrical rod tested in a uniaxial tension test. Two points A and B located at a distance $l_0 = 32$ cm from each other are marked on the rod and a tensile force of $F = 980$ N is applied on the rod. If the tensile strain and tensile stress generated in the rod were $\epsilon = 0.06$ cm/cm and $\sigma = 2.2$ MPa, determine:

- (a) The radius, r , of the rod after the application of the force
 (b) The total elongation of the rod, Δl , after the application of the force

Answers to Problem 13.4: (a) $r = 1.2$ cm, (b) $\Delta l = 1.92$ cm

Problem 13.5 Consider an aluminum rod of radius $r = 1.5$ cm subjected to a uniaxial tension test by force of $F = 23$ kN. If the elastic modulus of the aluminum rod is $E = 70$ GPa, determine:

- (a) The tensile stress, σ , developed in the rod
 (b) The tensile strain, ϵ , developed in the rod

Answers to Problem 13.5: (a) $\sigma = 32.6$ MPa, (b) $\epsilon = 0.47 \times 10^{-3}$

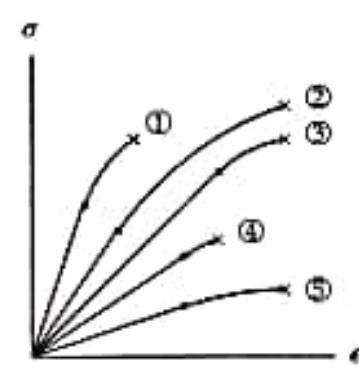


Fig. 13.40 Problem 13.2

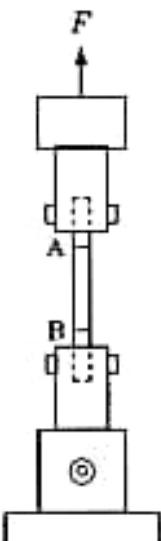


Fig. 13.29 Example 13.1

Ans: Given Data:

TWO bars 1 And 2

These 2 bars are tested in uniaxial tension test.

F_1, F_2 be the forces applied on Bar 1, Bar 2

E_1, E_2 be the elastic modulus of 1, 2 bars.

and A_1, A_2 are the cross sectional areas.

(a) If $A_1 > A_2$ and $F_1 = F_2$ then (b) $E_1 > E_2$ & $A_1 = A_2$

$$\sigma_1 = \frac{F_1}{A_1} ; \sigma_2 = \frac{F_2}{A_2}$$

And $A_1 > A_2$

$$\sigma_1 = \frac{F_1}{A_1} ; \sigma_2 = \frac{F_2}{A_2}$$

$$\sigma_1 = \frac{F}{A_1} ; \sigma_2 = \frac{F}{A_2}$$

As given $A_1 > A_2$

$$\downarrow \sigma \propto \frac{1}{A} \text{ (Here)}$$

$$\text{So: } \boxed{\sigma_1 < \sigma_2} \text{ Ans:}$$

$$\epsilon_1 = \frac{\sigma_1}{E_1} ; \epsilon_2 = \frac{\sigma_2}{E_2}$$

let us take $\sigma_1 = 1 ; \sigma_2 = 2$

$$\text{Now: } \epsilon_1 = \frac{1}{E_1} ; \epsilon_2 = \frac{2}{E_2}$$

if $E_1 = E_2$ then $\epsilon_2 > \epsilon_1$

if $E_1 > E_2$ then $\epsilon_2 > \epsilon_1$

if $E_1 < E_2$ Then we can't say

So, 'Insufficient data' Ans:

$$F_1 = F_2$$

i.e., $A_1 = A_2 = A$; $F_1 = F_2 = F$

$$\text{Now: } \sigma_1 = \frac{F_1}{A_1} ; \sigma_2 = \frac{F_2}{A_2}$$

$$\sigma_1 = \frac{F}{A} ; \sigma_2 = \frac{F}{A}$$

$$\text{As } \boxed{\therefore \sigma_1 = \sigma_2} = \sigma \text{ (say)}$$

$$\epsilon_1 = \frac{\sigma_1}{E_1} ; \epsilon_2 = \frac{\sigma_2}{E_2}$$

$$\epsilon_1 = \frac{\sigma}{E_1} ; \epsilon_2 = \frac{\sigma}{E_2}$$

$$\epsilon_1 \propto \frac{1}{E_1} ; \epsilon_2 \propto \frac{1}{E_2}$$

for $E_1 > E_2$

$$\boxed{\epsilon_1 < \epsilon_2} \text{ Ans:}$$

Given

Total length of test section $l_0 = 32 \text{ cm}$

Applied force = 980 N

Strain = 0.06 cm/cm

Stress = 2.2 MPa = $2.2 \times 10^6 \text{ Pa}$

(a)

$$\text{Stress} = \frac{\text{force}}{\text{Area}}$$

$$2.2 \times 10^6 = \frac{980}{\pi \times r^2}$$

$$r^2 = \frac{980}{2.2 \times 10^6 \times \pi}$$

$$r^2 = 1.4 \times 10^{-4}$$

$$r = 0.0119 \text{ m}$$

Radius of rod = 0.0119 m = 1.19 cm

$$(b) \text{ Strain} = \frac{\text{change in length}}{\text{original length}}$$

$$0.06 = \frac{\Delta l}{32}$$

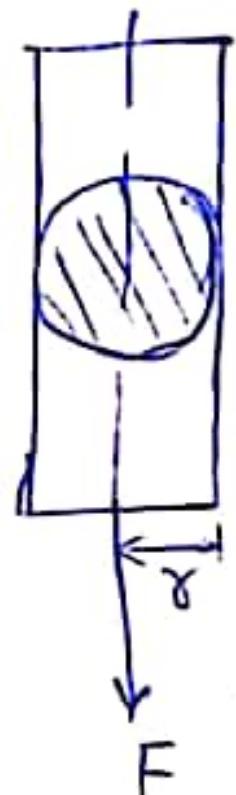
$$\Delta l = 32 \times 0.06 = 1.92 \text{ cm}$$

Total elongation of rod is 1.92 cm

(a) Tensile stress $\sigma = \frac{\text{Force}}{\text{Area}}$

$$\sigma = \frac{23 \times 10^3 \text{ (N)}}{\pi (15)^2 \text{ (mm}^2\text{)}} \quad (\gamma = 1.5 \text{ cm} \\ = 15 \text{ mm})$$

$$\sigma = 32.55 \text{ MPa}$$



(b) $\epsilon = \frac{\text{stress}}{E}$ (Hook's Law $\sigma = E \epsilon$)

$$= \frac{32.55 \text{ (MPa)}}{70 \times 10^3 \text{ (MPa)}}$$

$$\epsilon = 0.465 \times 10^{-3}$$

This is the value of the strain.

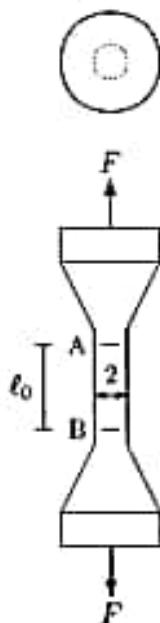


Fig. 13.41 Problem 13.6

Problem 13.6 Figure 13.41 illustrates a bone specimen with a circular cross-section. Two sections, A and B, that are $l_0 = 6\text{ mm}$ distance apart are marked on the specimen. The radius of the specimen in the region between A and B is $r_0 = 1\text{ mm}$.

This specimen was subjected to a series of uniaxial tension tests until fracture by gradually increasing the magnitude of the applied force and measuring corresponding deformations. As a result of these tests, the following data were recorded:

RECORD #	FORCE, $F(\text{N})$	DEFORMATION, $\Delta l (\text{mm})$
1	94	0.009
2	190	0.018
3	284	0.027
4	376	0.050
5	440	0.094

If record 3 corresponds to the end of the linearly elastic region and record 5 corresponds to fracture point, carry out the following:

- Calculate average tensile stresses and strains for each record.
- Draw the tensile stress-strain diagram for the bone specimen.
- Calculate the elastic modulus, E , of the bone specimen.
- What is the ultimate strength of the bone specimen?
- What is the yield strength of the bone specimen? (use the offset method)

Answers to Problem 13.6:

- $E = 20\text{ GPa}$
- $\sigma_u = 140\text{ MPa}$
- $\sigma_y = 118\text{ MPa}$

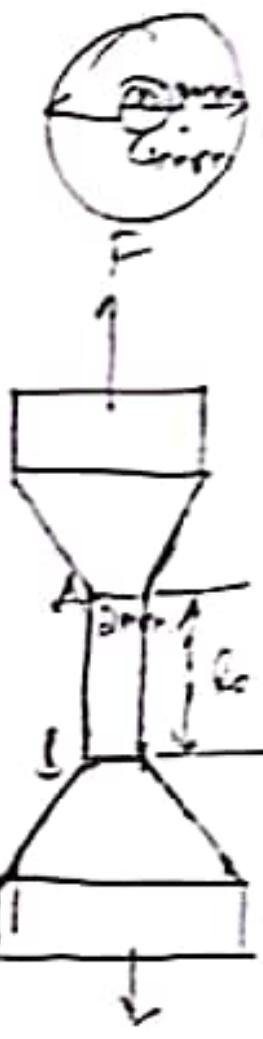
Problem 13.7 Consider a fixation device consisting of a plate and two screws that was used to stabilize a fractured femoral bone of a patient (Fig. 13.33). The weight of the patient was $W = 833\text{ N}$. If the shear stress generated in the screws was $\tau = 35.12 \times 10^6\text{ Pa}$, determine the diameter, d , of the screws.

Answer to Problem 13.7: $d = 5.5\text{ mm}$

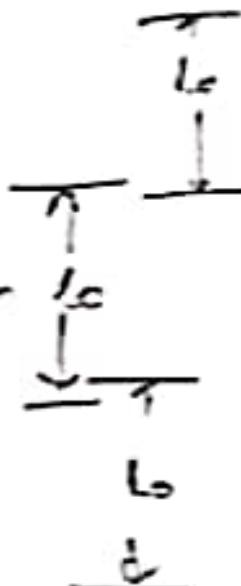


Fig. 13.33 Example 13.4

forces



$$\begin{aligned}L_0 &= 3L_0 \\&= 3(6\text{mm}) \\&= 18\text{mm} \\&= 0.018\text{m}\end{aligned}$$



$$\begin{aligned}E &= \frac{1}{2}mgh \\&= \frac{1}{2} \times m \times 9.8 \times 0.018\text{m} \\&= \frac{1}{2} \times m \times 9.8 \times 0.018\text{m}\end{aligned}$$

$$\boxed{E = m \times 0.0882 \text{ J/g}}$$

$$\begin{aligned}F &= mg \\&= m \times 9.8 \text{ m/s}^2 \times 0.0882 \text{ m}\end{aligned}$$

$$\underline{F = \cancel{m} \times 0.22 \text{ N}}$$

force m length is in (mm) it does not except much force to implement so $\underline{F = 0.22 \text{ N}}$

(d) Ultimate strength = $\frac{\text{Breaking force}}{\text{Reduced Area.}}$

$$= \frac{440 \text{ N}}{[\pi (1)^2] \text{ mm}^2}$$

$$= \underline{\underline{140.06 \text{ MPa}}}$$

(e) Yield strength

elastic region; $F = 284 \text{ N}$. $\epsilon = \frac{\Delta l}{l} = \frac{0.027}{6}$

$$= 0.0045$$

$$\sigma = \frac{F}{A} = \frac{284}{\pi (1)^2} = 90.4 \text{ MPa.}$$

So, Young's Modulus of Elasticity, $E = \frac{\sigma}{\epsilon} = \frac{90.4}{0.0045}$

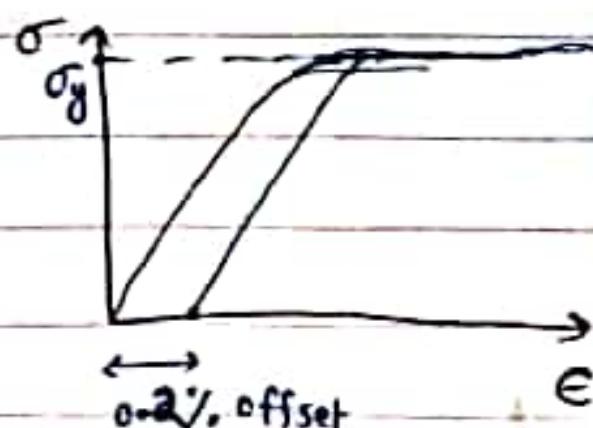
$$\Rightarrow \underline{\underline{E = 20.089 \text{ GPa}}}$$

For yield strength;
consider 0.2% ϵ offset

So, for 0.2% ϵ offset

strain for yielding is,

$$\epsilon_y = 0.002 + 0.0045 = 0.0065$$



Corresponding yield strength (σ_y) is;

$$\sigma_y = E \epsilon_y$$

$$= (20.089 \times 10^9) (0.0065) \cancel{\text{N/mm}^2} \text{ Pa}$$

$$\underline{\underline{\sigma_y = 130.58 \text{ MPa}}}$$

Problem 13.8 Human femur bone was subjected to a uniaxial tension test. As a result of a series of experiments, the stress-strain curve shown in Fig. 13.42 was obtained. Based on the graph in Fig. 13.42, determine the following parameters:

- Elastic modulus, E .
- Apparent yield strength, σ_y (use the offset method).
- Ultimate strength, σ_u .
- Strain, ϵ_1 , corresponding to yield stress.
- Strain, ϵ_2 , corresponding to ultimate stress.
- Strain energy when stress is at the proportionality limit.

Answers to Problem 13.8:

- $E = 14.6 \text{ GPa}$
- $\sigma_y = 235 \text{ MPa}$
- $\sigma_u = 240 \text{ MPa}$
- $\epsilon_1 = 0.018$
- $\epsilon_2 = 0.02$
- 1.235 MPa

Problem 13.9 As illustrated in Fig. 13.43, consider a structure including the horizontal beam AB hinged to the wall at point A and supported by steel bar at point B. The length of the beam is $l = 3.7 \text{ m}$ and its weight is $W = 450 \text{ N}$. The length of the steel bar is $h = 2.5 \text{ m}$ and its radius is $r = 1.2 \text{ cm}$. If the elastic modulus of the steel bar is $E = 200 \text{ GPa}$, determine:

- The magnitude of reaction force, R_A , at point A
- The tensile stress, σ , exerted by the beam on the steel bar
- The tensile strain, ϵ , developed in the bar

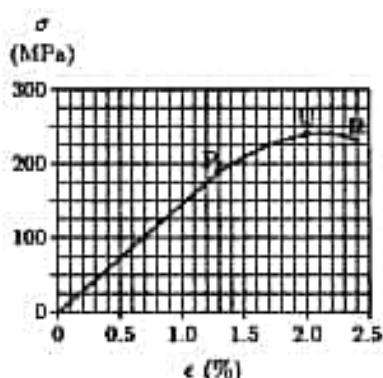


Fig. 13.42 Problem 13.8

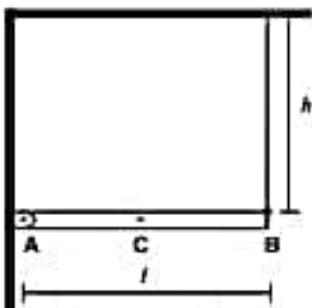


Fig. 13.43 Problem 13.9

Answers to Problem 13.9: (a) $R_A = 225 \text{ N}$, (b) $\sigma = 0.5 \text{ MPa}$, (c) $\epsilon = 2.5 \times 10^{-6}$

④ Elastic Modulus E , from Graph

$$E = \frac{\sigma}{\epsilon} = \frac{190}{1.3/100} = 14615.38 \text{ MPa} = \underline{\underline{14.61 \text{ GPa}}}$$

⑤

$$\sigma_u = E \epsilon_{avg} = 14615.38 \left[\frac{2+1.3}{2(100)} \right]$$

ϵ_{avg} is between ϵ_u & $\epsilon_{Elastic}$

$$\Rightarrow \boxed{\sigma_u = 241.15 \text{ MPa}}$$

yield stress of offset is about 0.02%. taken out

$$\sigma_y = (1-0.02) \sigma_u = 0.98 \sigma_u = 0.98 (241.15)$$

$$\boxed{\sigma_y = 236 \text{ MPa}}$$

⑥ $\boxed{\sigma_u = 241.15 \text{ MPa}}$ as solved in (b)

⑦

$$\epsilon_1 = \frac{\sigma_y}{E} = \frac{236}{14615} = \boxed{\underline{\underline{0.017 = \epsilon_1}}}$$

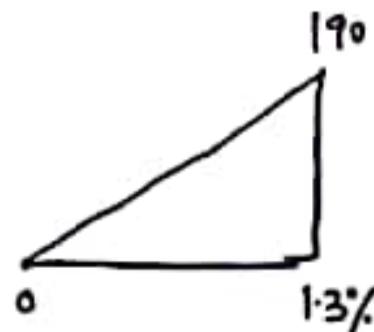
⑧ $\epsilon_2 = \frac{\sigma_u}{E} = \frac{190}{14615} = \boxed{\underline{\underline{0.02 = \epsilon_2}}} \quad \text{(from Figure)}$

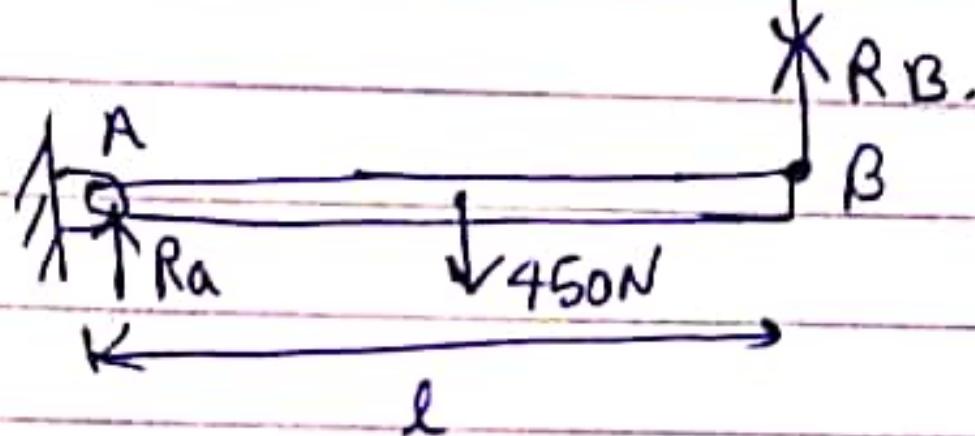
⑨

$$U = \frac{1}{2} \sigma \epsilon$$

$$= \frac{1}{2} (190) \left(\frac{1.3}{100} \right)$$

$$\boxed{U = 1.235 \text{ MPa}}$$





① $\sum M_A = 0$

$$450 \times \frac{l}{2} - R_B \times l$$

$$R_B = \frac{450}{2} = 225N$$

$$R_A = 450 - \frac{225}{2}$$

$$R_A = 225N$$

② $\sigma_t = \frac{R_B}{A_{nod}}$

$$A_{nod} = \pi \times \pi^2$$

$$\pi = 12mm$$

$$A_{nod} = \pi \times (12)^2 \text{ mm}^2$$

$$= 452.389 \text{ mm}^2$$

$$\sigma_t = \frac{225}{452.389} = 0.49735 \text{ MPa}$$

③ $\sigma_t = \epsilon E \quad (\text{Hooke's Law})$

$$\epsilon = \frac{0.49735}{200 \times 10^3} = 2.4867 \times 10^{-6}$$

Problem 13.10 Consider the uniform, horizontal beam shown in Fig. 13.44a. The beam is hinged to a wall at A and a weight $W_2 = 400 \text{ N}$ is attached on the beam at B. Point C represents the center of gravity of the beam, which is equidistant from A and B. The beam has a weight $W_1 = 100 \text{ N}$ and length $l = 4 \text{ m}$. The beam is supported by two vertical rods, 1 and 2, attached to the beam at D and E. Rod 1 is made of steel with elastic modulus $E_1 = 200 \text{ GPa}$ and a cross-sectional area $A_1 = 500 \text{ mm}^2$, and rod 2 is bronze with elastic modulus $E_2 = 80 \text{ GPa}$ and $A_2 = 400 \text{ mm}^2$. The original (undeformed) lengths of both rods is $h = 2 \text{ m}$. The distance between A and D is $d_1 = 1 \text{ m}$ and the distance between A and E is $d_2 = 3 \text{ m}$.

The free-body diagram of the beam and its deflected orientation is shown in Fig. 13.44b, where T_1 and T_2 represent the forces exerted by the rods on the beam. Symbols δ_1 and δ_2 represent the amount of deflection the steel and bronze rods undergo, respectively. Note that the beam material is assumed to be very stiff (almost rigid) as compared to the rods so that it maintains its straight shape.

- Calculate tensions T_1 and T_2 , and the reactive force R_A on the beam at A.
- Calculate the average tensile stresses σ_1 and σ_2 generated in the rods.

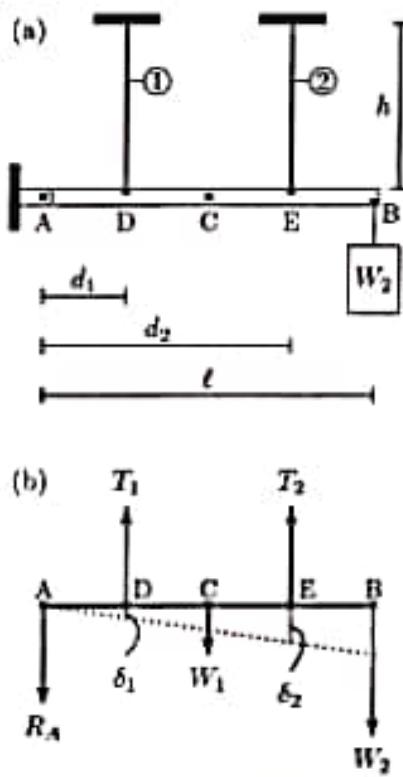


Fig. 13.44 Problem 13.10

Answers to Problem 13.10:

- $T_1 = 464 \text{ N}$, $T_2 = 445 \text{ N}$, $R_A = 409 \text{ N}$
- $\sigma_1 = 0.928 \text{ MPa}$, $\sigma_2 = 1.113 \text{ MPa}$

