BME4120
Biomedical Image Processing

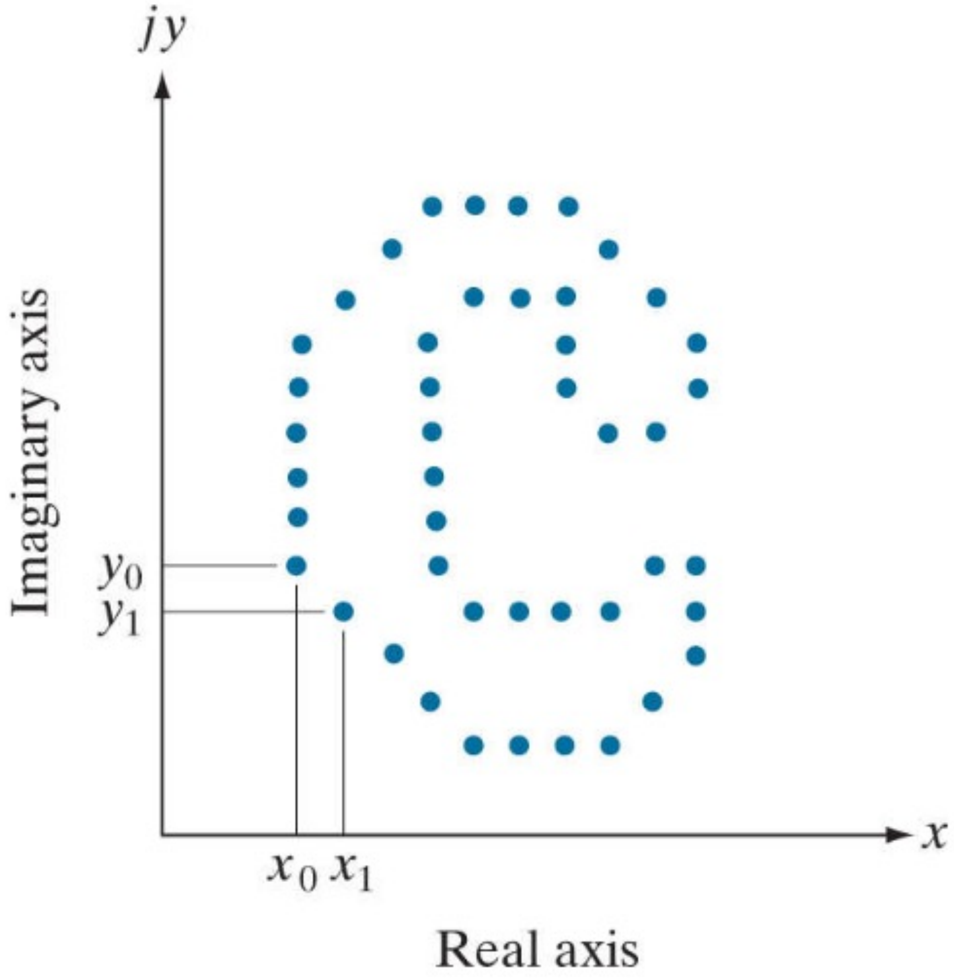
Lecture 8

Fourier Descriptors

Fourier descriptors are a way of encoding the shape of a two-dimensional object by taking the Fourier transform of the boundary, where every point on the boundary is mapped to a complex number. →Based on the idea that boundary can be viewed as 1D periodic signal

View a coordinate (x,y) as a complex number, then apply the Fourier transform to a sequence of boundary points:

$$s(k) = x(k) + jy(k)$$



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Fourier Descriptors

Discrete Fourier transform (DFT) of s(k)

$$a(u) = \sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}$$

Complex coefficients $a(u)$: Fourier descriptors of the boundary

Inverse Fourier transform of $a(u)$ restores s(k)

$$s(k) = \frac{1}{K} \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}$$

We can terminate the value of K at a value (P) to obtain ***adequate*** boundary

The first 10 - 15 descriptors are usually found to be sufficient for character description

Fourier Descriptors

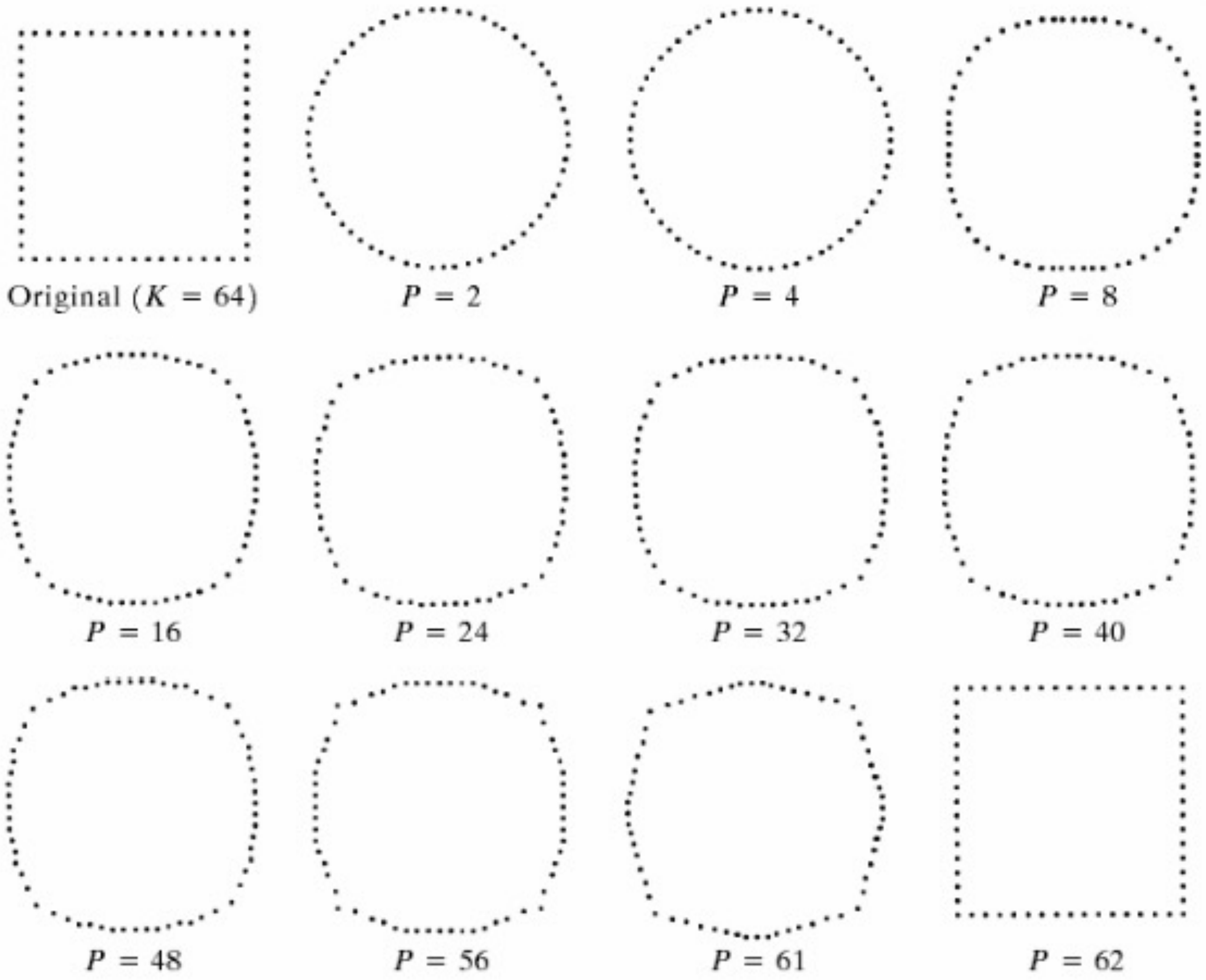

FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.

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Some Properties of Fourier Descriptors

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy} \ \Delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

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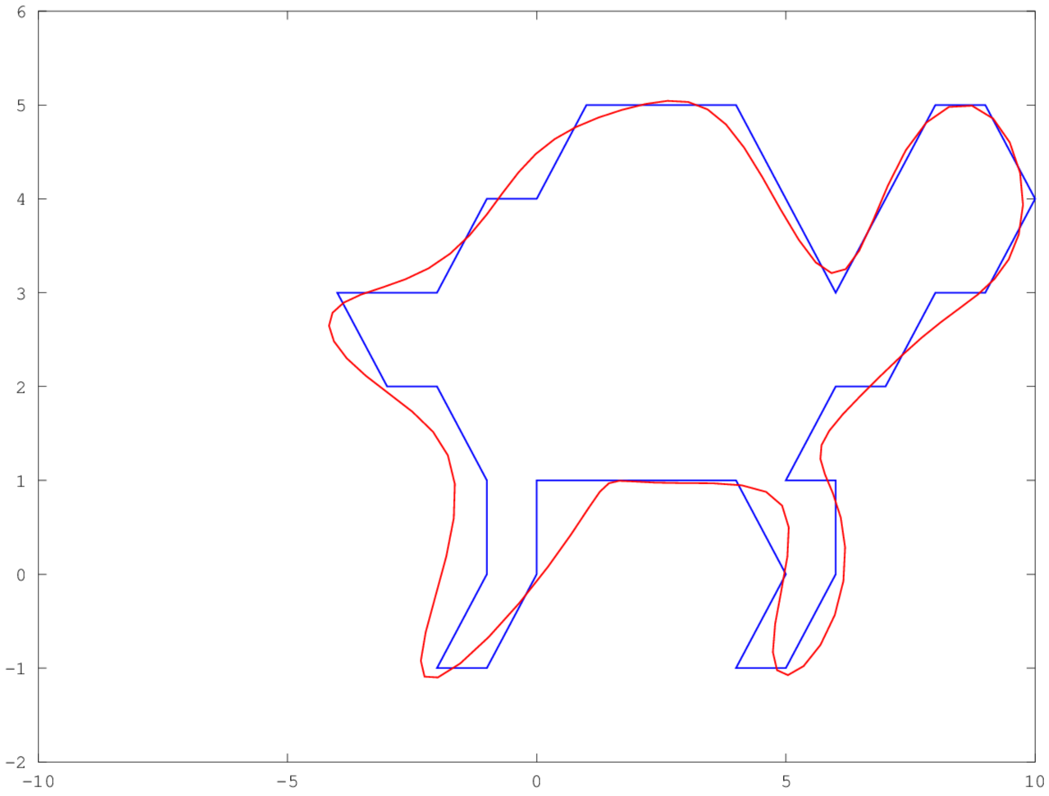
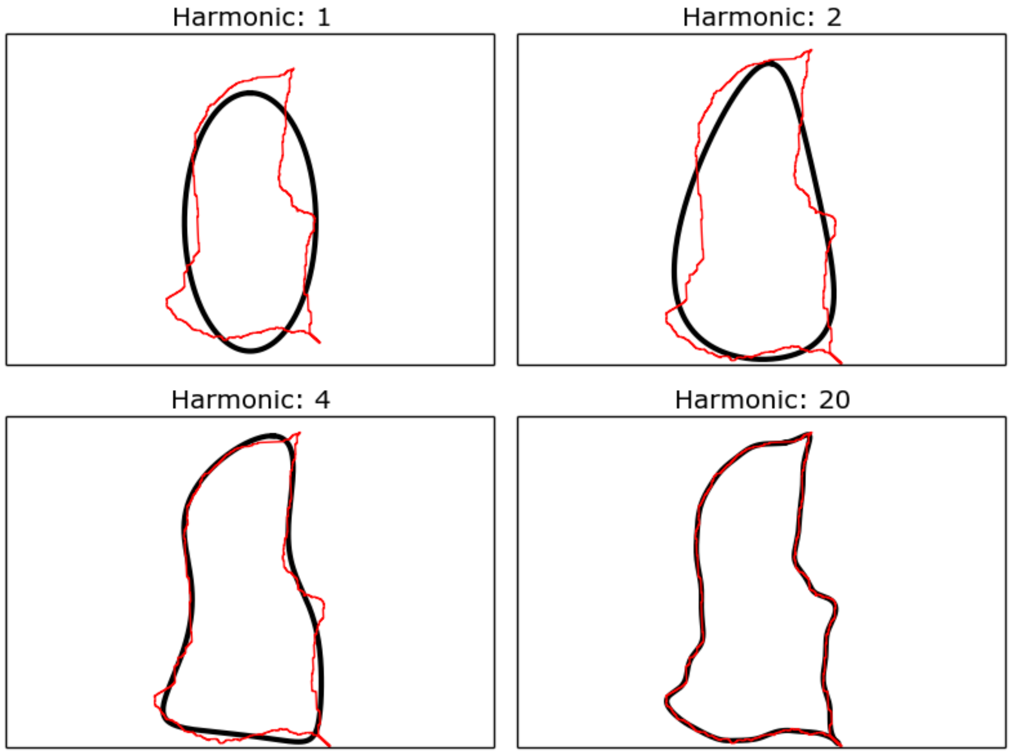
Matlab & Python: Some Examples





https://spatial-efd.readthedocs.io/en/latest/raster_link.html

https://www.mathworks.com/matlabcentral/fileexchange/32800-elliptic-fourier-for-shape-analysis

Python: OpenCV-Fourier Descriptors

Fourier descriptors

Extended Image Processing

Classes

class **cv::ximgproc::ContourFitting**
Class for **ContourFitting** algorithms. **ContourFitting** match two contours z_a and z_b minimizing distance

$$d(z_a, z_b) = \sum (a_n - s b_n e^{j(n\alpha+\phi)})^2$$

where a_n and b_n are Fourier descriptors of z_a and z_b and s is a scaling factor and ϕ is angle rotation and α is starting point factor adjustement. More...

Functions

void	**cv::ximgproc::contourSampling** (**InputArray** src, **OutputArray** out, int nbElt) Contour sampling . More...
Ptr< **ContourFitting** >	**cv::ximgproc::createContourFitting** (int ctr=1024, int fd=16) create **ContourFitting** algorithm object More...
void	**cv::ximgproc::fourierDescriptor** (**InputArray** src, **OutputArray** dst, int nbElt=-1, int nbFD=-1) Fourier descriptors for planed closed curves. More...
void	**cv::ximgproc::transformFD** (**InputArray** src, **InputArray** t, **OutputArray** dst, bool fdContour=true) transform a contour More...

https://docs.opencv.org/4.x/dd/ddc/group__ximgproc__fourier.html

Snakes Boundary Detection

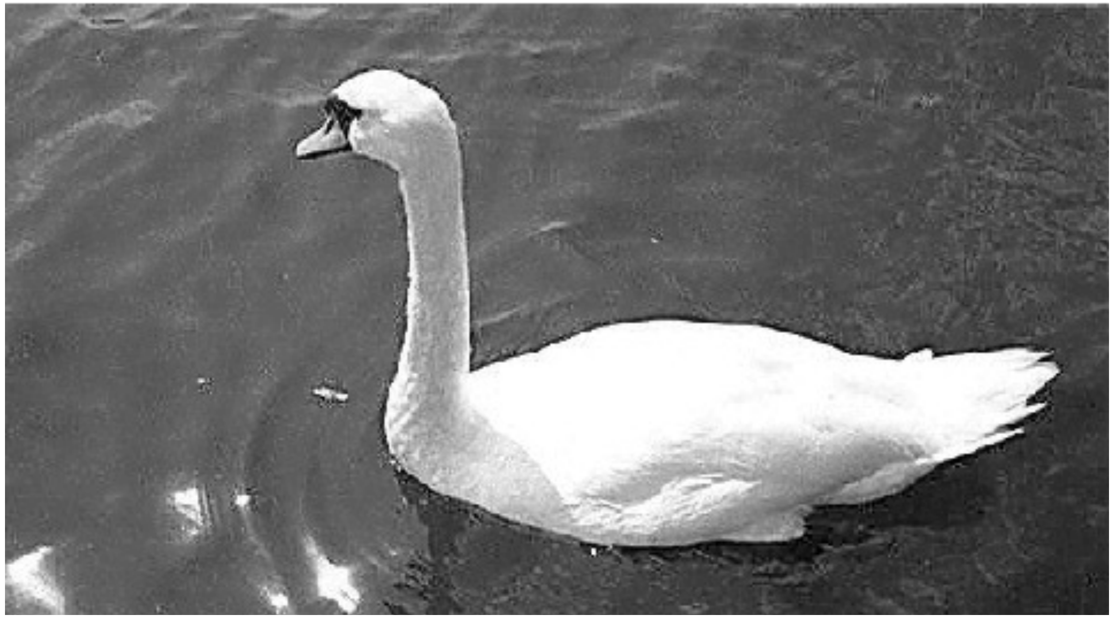
A boundary detection method, first introduced by Kass in 1988.

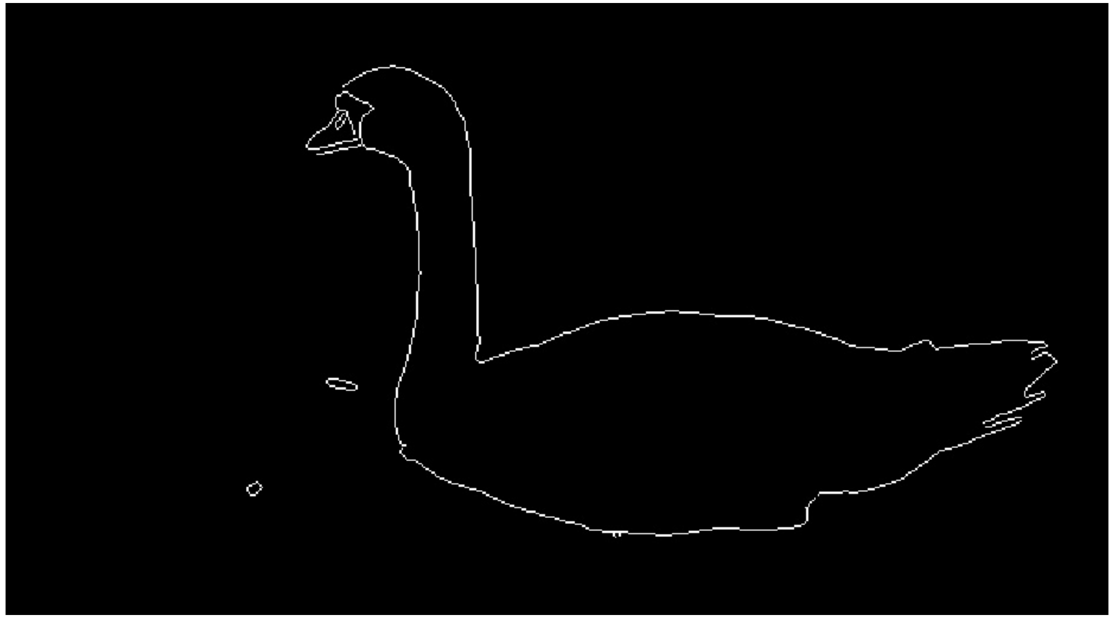
Snake is basically a method of modelling a closed contour to the boundary of an object in an image.

A shape is fixed and made flexible in terms of the parameters defining the shape.

The active contours are called *snakes* that are taken into consideration for detection and feature extraction.

These contours are the set of points that aim to enclose a target feature, the feature to be extracted.





Sometimes edge detectors find the boundary pretty well.

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Sometimes it’s not enough.

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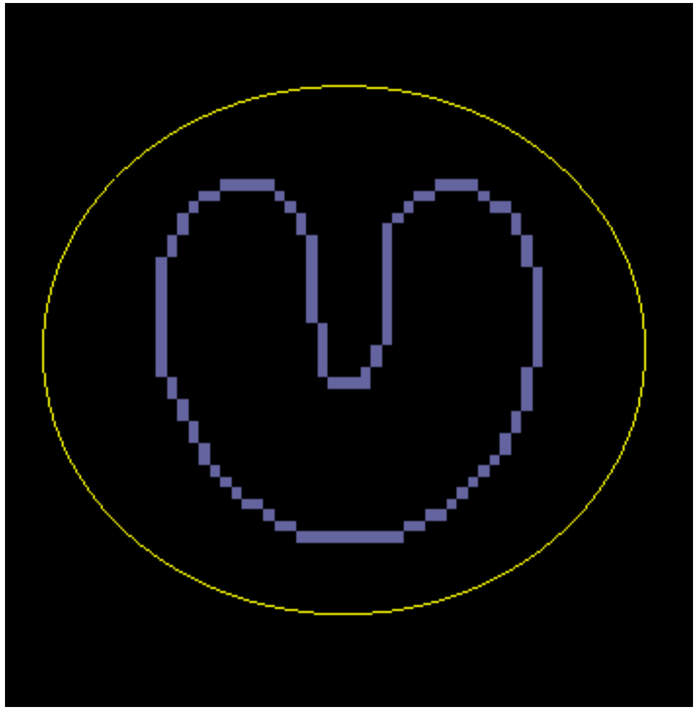
Active contours (Snakes)

- User (or higher-level process) initializes contour
- Snake deforms and shrink-wraps around object boundary

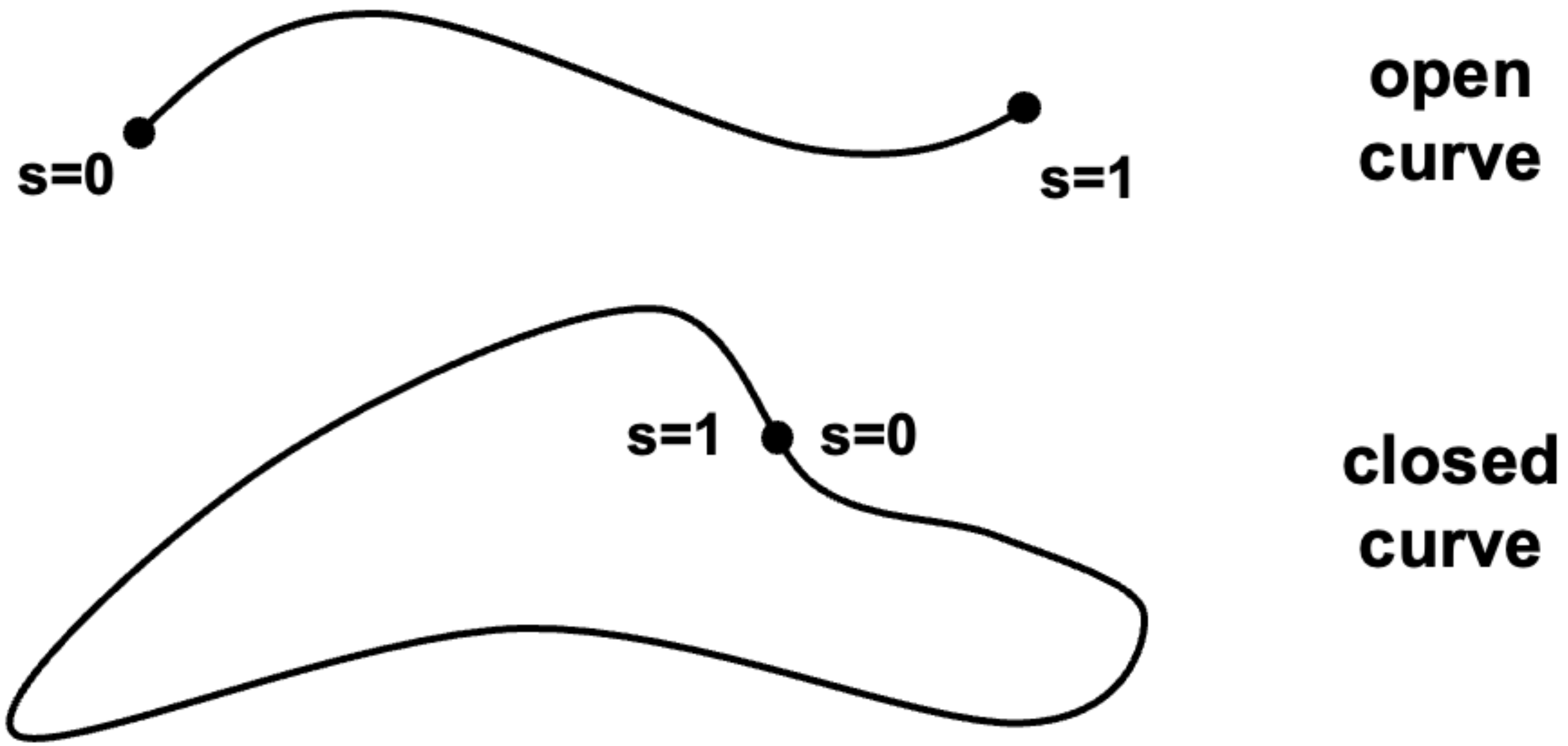


(Diagram courtesy "Snakes, shapes, gradient vector flow", Xu, Prince)

Active contours (Snakes)

- **Snake is a parameterized curve:**

$$\mathbf{v}(s) = [x(s), y(s)] \qquad s \in [0, 1]$$



- **Snake is a type of deformable contour**

Snake Energy

$v(s) = [x(s), y(s)]$ represents a parametric curve in continuous domain where s is the arc length of the curve.

The snake energy

$$E_s = \int_{s=0}^{1} E_{\mathrm{IEF}}[v(s)]ds + \int_{s=0}^{1} E_{IF}[v(s)]ds + \int_{s=0}^{1} E_{TF}[v(s)]ds$$

Internal Energy of Contour

Image Energy

Constrain Energy

Snake Algorithm

$$E_S = \alpha \left| \frac{d\mathbf{v}}{ds} \right|^2 + \beta \left| \frac{d^2\mathbf{v}}{ds^2} \right|^2$$

<u>elasticity</u>

- 1st-order term
- membrane
- α controls tension along spline
- stretching balloon or elastic band

<u>stiffness</u>

- 2nd-order term
- thin plate
- β controls rigidity of spline
- bending thin plate or bending wire

α and β may vary along curve but are usually constant

Discrete Snake Energy

In discrete domain:

$$E_s = \sum_{n=1}^{N} E_{\mathrm{IEF}}[v_n] + \sum_{n=1}^{N} E_{IF}[v_n] + \sum_{n=1}^{N} E_{TF}[v_n]$$

Internal Energy of Contour

Image Energy

Constrain Energy

$v_n = (x_n, y_n)$ for n = 0,1,2,3…. represents the discrete contour

Snake Method

- Goal: Match curve (boundary) to image data
- Approach: minimize energy functional
- Like many vision problems, this is underconstrained
  → regularization (impose smoothness prior)

$$E_s = \int_{s=0}^{1} E_{\mathrm{IEF}}[v(s)]ds + \int_{s=0}^{1} E_{IF}[v(s)]ds + \int_{s=0}^{1} E_{TF}[v(s)]ds$$

Snakes are a top-down approach to segmentation

Minimization of energy

- Two methods:
  - Finite element and calculus of variations
    [Kass, Witkin, and Terzopoulos, IJCV 1988]
  - Dynamic programming
    [Amini, Weymouth, and Jain, PAMI 1990]

Steps of Snakes Algorithm

It begins with a preliminary curve and characterize some energy for that curve based on its geometric properties and the associated image data.

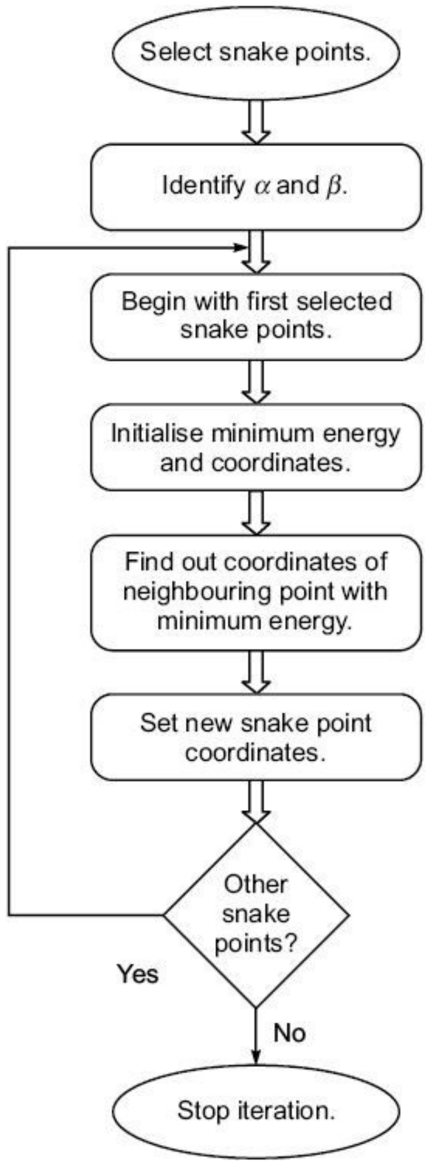
The energy based on the geometry keeps the curve smooth and the energies based on the image data are intended to attract the contour to the object boundaries.

This curve is then deformed in order to increase or decrease that energy and hence, the curve is moved towards a local maxima or minima.

Steps of Snakes Algorithm

Snake algorithm for boundary detection includes the following steps:

1.Move the snake points through an iterative process.
2.Calculate the energy function for each point in the local neighborhood.
3.Move to the next point with the lowest energy function.
4.Repeat for every point traversed.
5.Iterate until the termination conditions such as
(i) A specified number of iterations
(ii) Stability of position of the points



Medical Image Processing, Sinha&Patel, 2014

Snake Algorithm

Snake algorithm helps in the detection of active contour region and boundary region.

The values of α and β greatly affect the detection.

The probabilistic approach to active regions used with $\alpha = \beta = 1$ gives a uniform distribution.

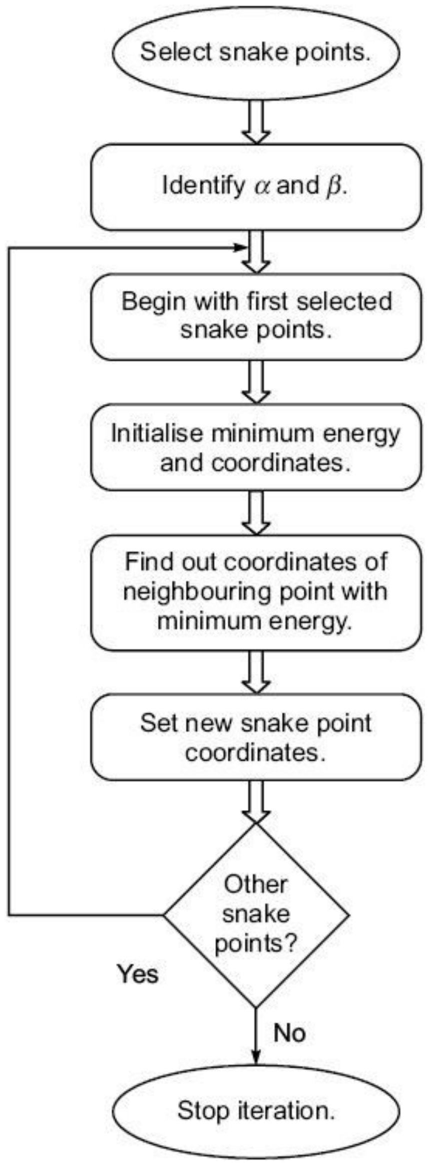
A number of iterations are required for the algorithm to converge.

The active region converges to the mean of the probability map.

Steps of Snakes Algorithm

With the decrease in α, the active region achieves the optimum of its energy functional when it lies at the highest non-overlapping areas of the probability map.

The curve produced by the active region in this case is short and smooth.



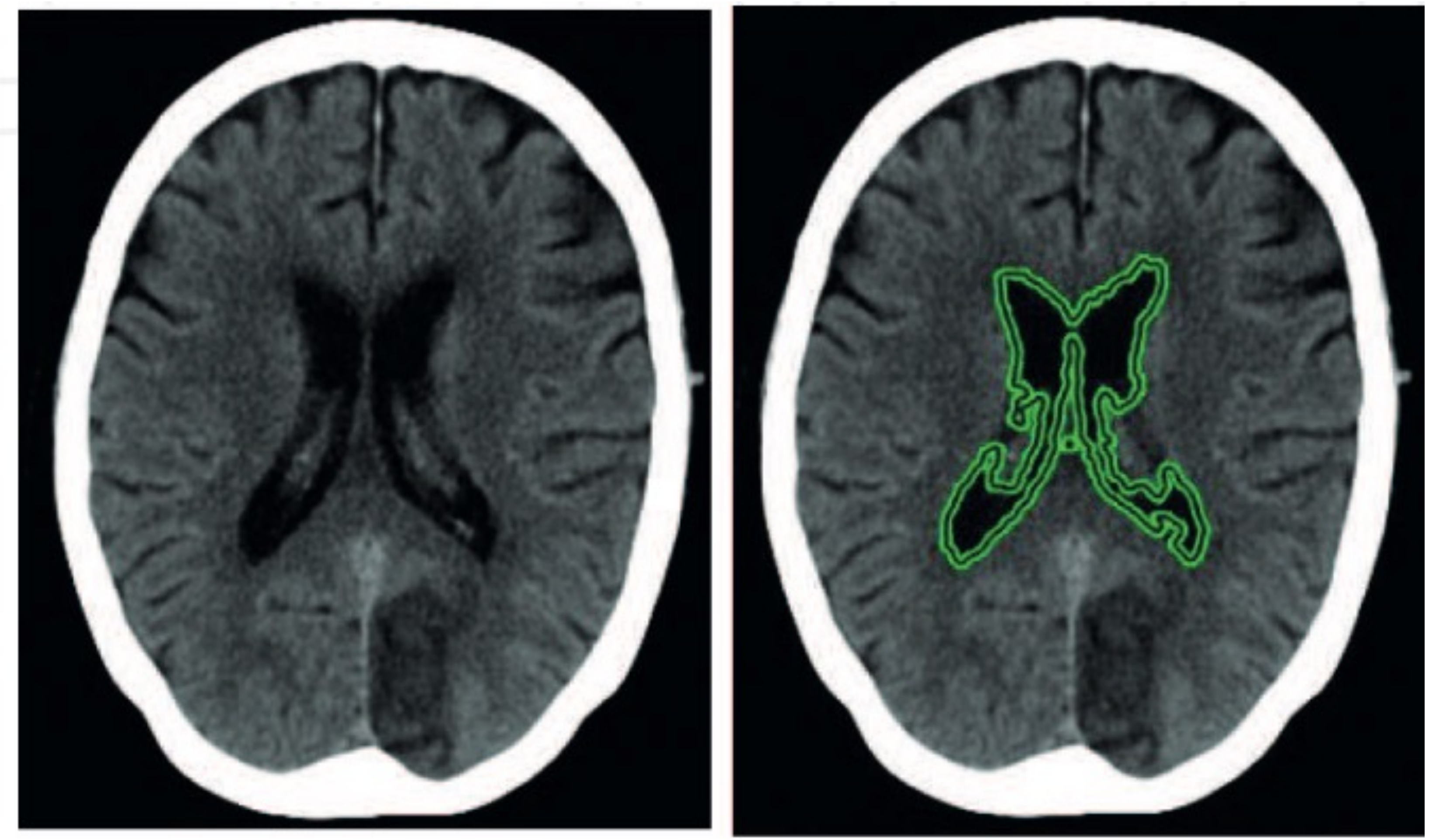
Medical Image Processing, Sinha&Patel, 2014

Python Example: Snake Boundary Model

```
from skimage.segmentation import active_contour
img = data.astronaut()
img = rgb2gray(img)
#create data for circular boundary
s = np.linspace(0, 2*np.pi, 400)
x = 220 + 100*np.cos(s)
y = 100 + 100*np.sin(s)
init = np.array([x, y]).T
#apply gaussian filter & find active contours
cntr = active_contour(gaussian(img, 3),init,
alpha=0.015, beta=10, gamma=0.001)
```

https://tirthajyoti.github.io/Scikit-image-book/Active_contour_model.html

A Very Rich Topic



Segmentation of brain CT image using active contours

DOI:10.5772/intechopen.74576

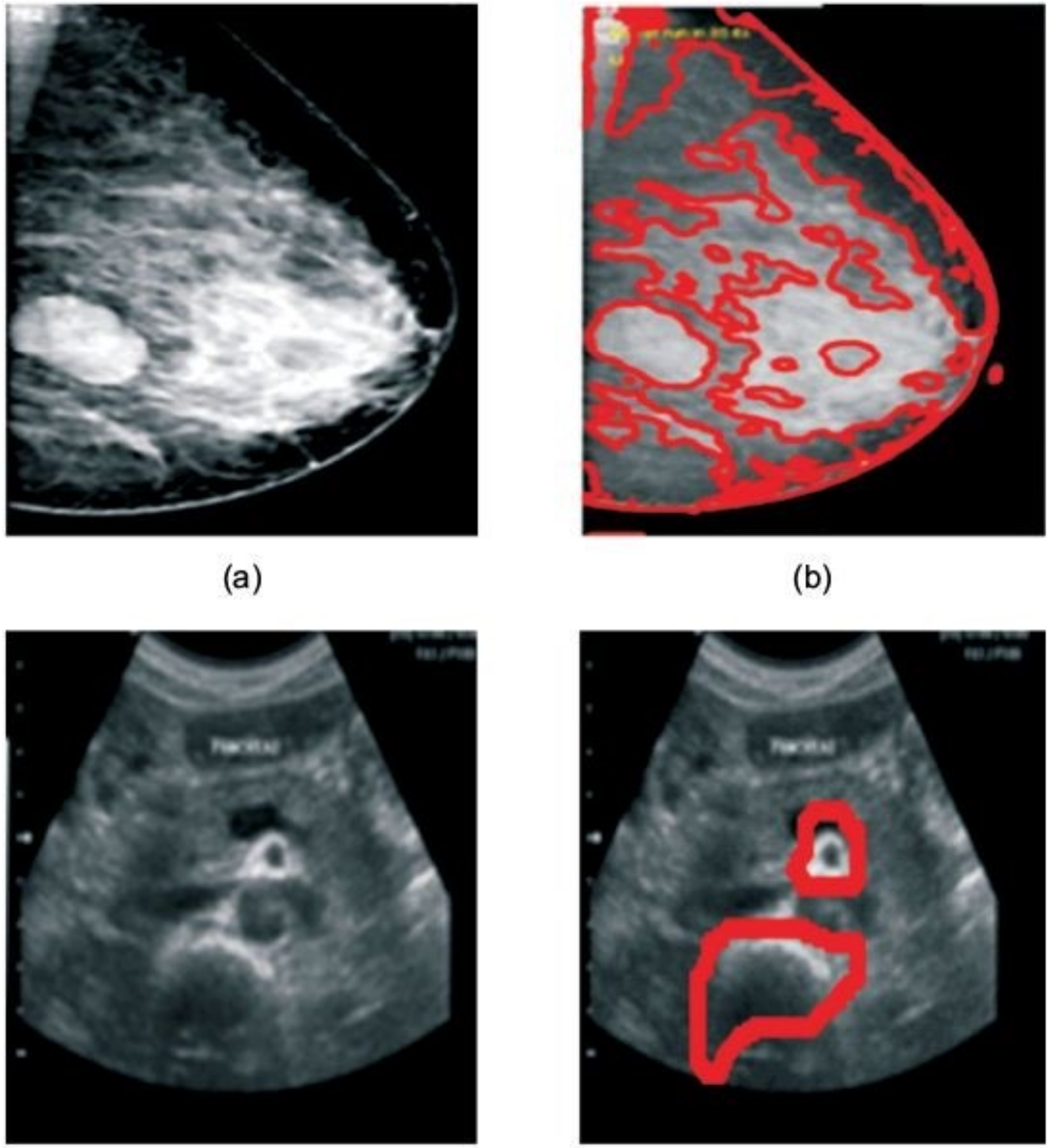
Example Boundary Detections

Boundary detection

(a) and (b) Breast image and its result

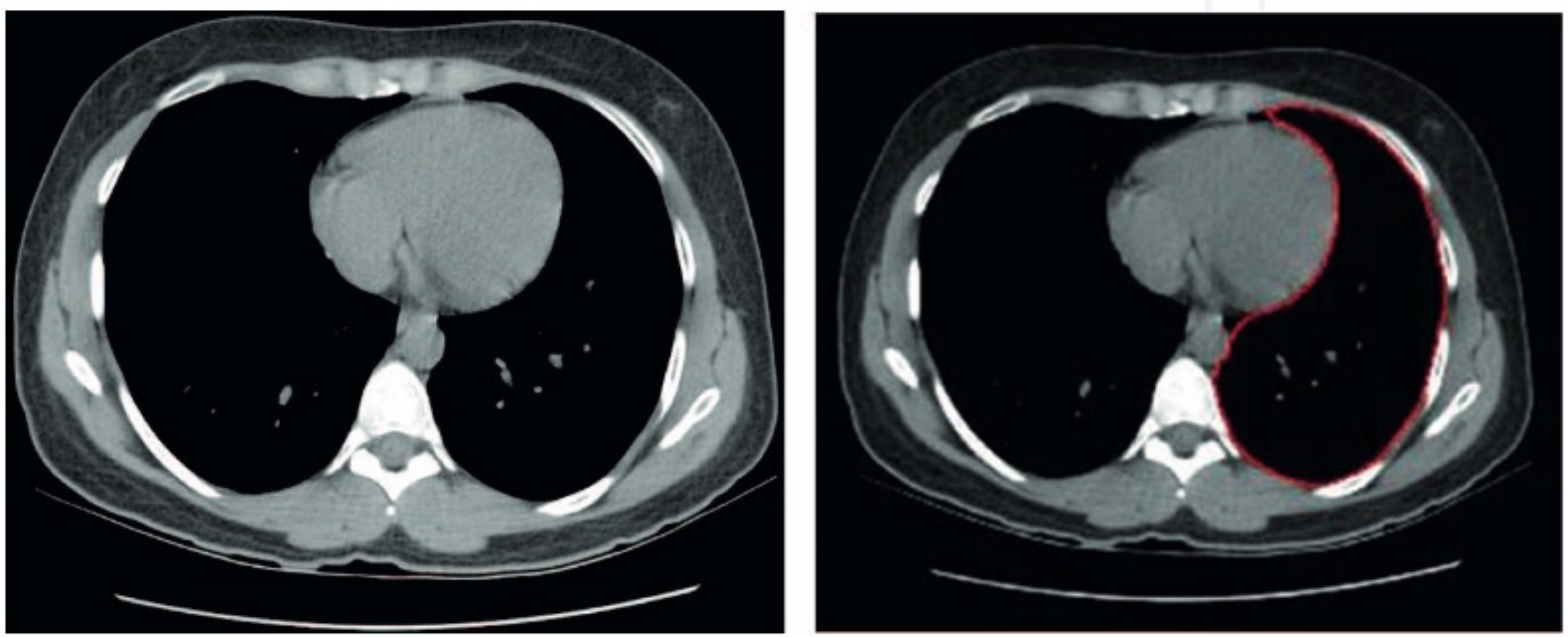
(c) and (d) Liver image and its result, respectively

Medical Image Processing, Sinha&Patel, 2014



(a) (b) (c) (d)

A Very Rich Topic



Segmentation of chest image using snake model

DOI:10.5772/intechopen.74576

Some Alternative Implementations

Signal Processing 89 (2009) 2435–2447

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

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SIGNAL PROCESSING

Active contours driven by local Gaussian distribution fitting energy

Li Wang [a], Lei He [b], Arabinda Mishra [c,*], Chunming Li [c]

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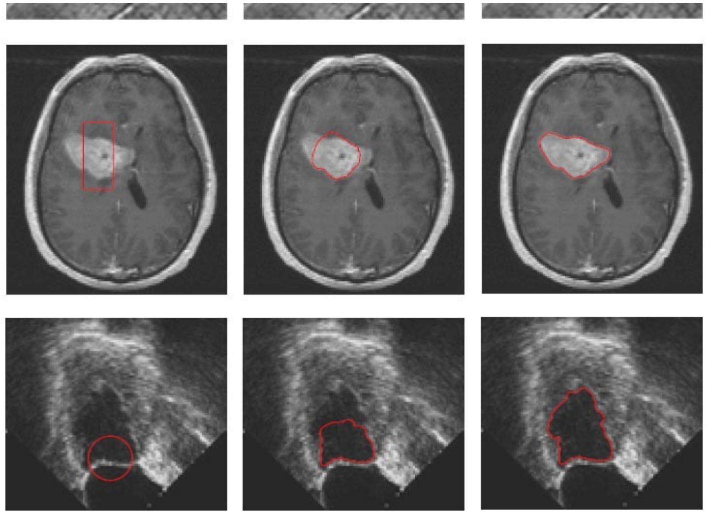

https://www.mathworks.com/matlabcentral/fileexchange/38637-active-contours-driven-by-local-gaussian-distribution

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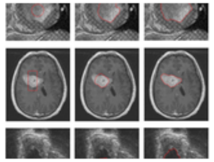

Active contours driven by local Gaussian distribution

version 1.5.0.12 (982 KB) by li wang

This Matlab code implements a segmentation method using local Gaussian distribution fitting energ

Overview | Functions | Reviews (8) | Discussions (1)

This Matlab code implements a segmentation method using local Gaussian distribution fitting energy, proposed by Li Wang et al's
the paper "Active Contours Driven by Local Gaussian Distribution Fitting Energy. Signal Processing, 89(12), 2009, p. 2435-2447"
http://www.sciencedirect.com/science/article/pii/S0165168409000942
The local image intensities are described by Gaussian distributions with different means and variances. The energy minimization
achieved by an interleaved level set evolution and estimation of local intensity means and variances in an iterative process. The m
and variances of local intensities are considered as spatially varying functions to handle intensity inhomogeneities and noise of
spatially varying strength.
More source codes on image segmentation, such as infant or neonatal brain MR image segmentation using patch-based sparse
representation and random forest with auto-context model, can be found in other published papers in the following website:
https://liwang.web.unc.edu/
http://www.ibeat.cloud

The code of patch-based sparse representation can be downloaded from here
https://www.mathworks.com/matlabcentral/fileexchange/74558-sparse-representation-for-brain-image-segmentation

Cite As

li wang (2022). Active contours driven by local Gaussian distribution (https://www.mathworks.com/matlabcentral/fileexchange/38637-active-contours-driven-by-local-gaussian-distribution), MATLAB Central File Exchange. Retrieved November 30, 2022.

HW-3

Use a chest image (1-s2.0-S0929664620300449-gr3_lrg-c.jpg)
By using Python and scikit-image package and examples like below

https://towardsdatascience.com/image-segmentation-using-pythons-scikit-image-module-533a61ecc980

https://tirthajyoti.github.io/Scikit-image-book/Active_contour_model.html

Find contour on the right side of the chest image:

Your result should something like this

HW-3 Dataset Information

https://arxiv.org/abs/2003.11597

Cornell University

We gratefully acknowledge support from the Simons Foundation and member institutions.

arXiv > eess > arXiv:2003.11597

Search... All fields Search

Help | Advanced Search

Electrical Engineering and Systems Science > Image and Video Processing

COVID-19 e-print

Important: e-prints posted on arXiv are not peer-reviewed by arXiv; they should not be relied upon without context to guide clinical practice or health-related behavior and should not be reported in news media as established information without consulting multiple experts in the field.

[Submitted on 25 Mar 2020]

COVID-19 Image Data Collection

Joseph Paul Cohen, Paul Morrison, Lan Dao

This paper describes the initial COVID-19 open image data collection. It was created by assembling medical images from websites and publications and currently contains 123 frontal view X-rays.

Comments: Dataset available here: this https URL

Subjects: **Image and Video Processing (eess.IV)**; Computer Vision and Pattern Recognition (cs.CV); Machine Learning (cs.LG); Quantitative Methods (q-bio.QM)

Cite as: arXiv:2003.11597 **[eess.IV]**

(or arXiv:2003.11597v1 **[eess.IV]** for this version)

https://doi.org/10.48550/arXiv.2003.11597

Submission history

From: Joseph Paul Cohen [view email]

[v1] Wed, 25 Mar 2020 19:37:25 UTC (84 KB)

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References & Citations

- NASA ADS
- Google Scholar
- Semantic Scholar

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Dataset
https://github.com/ieee8023/covid-chestxray-dataset

Use Image:
1-s2.0-S0929664620300449-gr3_lrg-c.jpg

HW-3 Important Notes

You must use Python and Scikit-Image

Your code should be printed and and returned to me on 7th December on paper.

HW returns must be made just before the class hour 11:00am.
(You may return before that if you want.)
HW returns with emails will not be accepted.

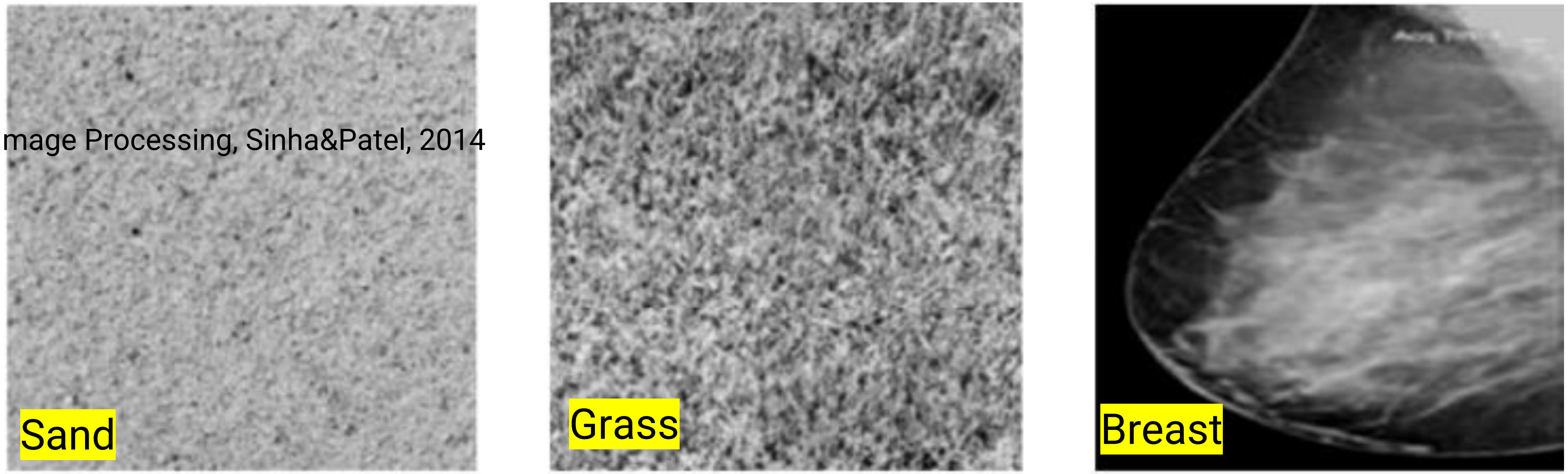
Texture Analysis

Texture: Regular repetition of particular patterns or structures obtained in an image.

Image textures may be complex patterns also of different brightness, colour, size and shape.

Texture property can help in the image classification and segmentation.

Measuring image texture depends on the size or shape of an object.



Medical Image Processing, Sinha&Patel, 2014

(a) (b) (c)

Texture Analysis

Measuring image texture depends on the size or shape of an object.

Texture analysis is made in following ways:

1. Structural analysis
2. Statistical analysis
3. Model-based texture analysis
4. Transform methods for analysis of texture

Texture Features

Texture features play a very important role in the texture classification, image segmentation and image shape identification processes.

Image segmentation and shape identification are performed as the preprocessing steps for pattern or object recognition.

Extracted textural features need to be estimated → post-processing and further operation over the images would not be affected.

Many methods exist for the extraction of texture: Fourier transforms, convolution filters, co-occurrence matrix, fractals, …

First Order Statistics-based Methods

First order statistical features are calculated from the original grayscale values of an object. Texture features based on the first order statistics are measured like variance.

Since pixel neighborhood relationships are not considered, histogram-based approach is used.

Common texture features include moments such as mean, variance, dispersion, mean square value or average energy, entropy, skewness, etc.

First Order Statistics-based Methods

Measure of the texture: Variance in the gray level in a neighborhood region of a pixel.

The ***histogram of intensity levels*** is used to summarize statistical information of an image.

The first order statistical information of images can be found in the histograms.

In an image, approximate probability density of the occurrence of intensity levels can be determined.

Example: A narrowly distributed histogram indicates a low contrast image

Drawback: *Histogram-based texture analysis do not provide information about the relative position of pixels to each other.*

Histogram-based Methods

Remember that a two-dimensional image is given as 2D function $f(x, y)$ ($x = 0,1,\ldots, M - 1$ and $y = 0,1,\ldots, N - 1$)

The size of image is (M x N).

Histogram of an image is the distribution of grayscale values with respect to the number of pixels having a particular intensity in the entire image.

$$h(i) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \delta[f(x, y), i] \quad , \quad \delta(j, i) = \begin{cases} 1 & j = i \\ 0 & j \neq 1 \end{cases}$$

$$p(i) = h(i)/MN,\ i = 0,1,\ldots,I - 1.$$

Probability density of the occurrence of intensity levels

Central Moments Obtained From a Histogram

$$\text{Mean} = \mu = \sum_{i=0}^{G-1} ip(i)$$

Mean value is an average level of intensity of the image features.

$$\text{Variance} = \sigma^2 = \sum_{i=0}^{G-1} (i-\mu)^2 p(i)$$

Variance is defined as the variation of intensity around the mean.

Skewness = $S_{kn} = \sigma^{-3} \sum_{i=0}^{G-1} (i-\mu)^3 p(i)$

The skewness is an indication of symmetry, i.e., its value is zero if the histogram is symmetrical about the mean or otherwise either positive or negative depending upon whether it has been skewed above or below the mean.

Kurtosis = $K_t = \sigma^{-4} \sum_{i=0}^{G-1} (i-\mu)^4 p(i) - 3$

Kurtosis measures the flatness of a histogram.

Energy = $E = \sigma^{-3} \sum_{i=0}^{G-1} p(i)^2$

Energy is a measure of the localized change of the image

Entropy = $H = -\sum_{i=0}^{G-1} p(i) \log_2[p(i)]$

Entropy indicates the histogram uniformity.

For visual images, the mean and variance do not carry the information of image features.

But the mean and variance after normalization can give better feature discrimination accuracy than using the actual mean and the actual variance as feature parameters.

Texture features can be used for the classification of tissues or cells in an image.

Application Example: Mammography

These features are very useful features for the analysis of mammographic images and tissue classification.

The classification further helps in the diagnosis of palpable or microcalcifications or tumors in the breast.

Application Example: Mammography

Based on the experimental results, mammograms can be grouped under three categories

- ❑ Fatty
- ❑ Glandular
- ❑ Dense

These are further classified based on some statistical features into four classes

- ❑ uncompressed fatty
- ❑ fatty, non-uniform
- ❑ high density

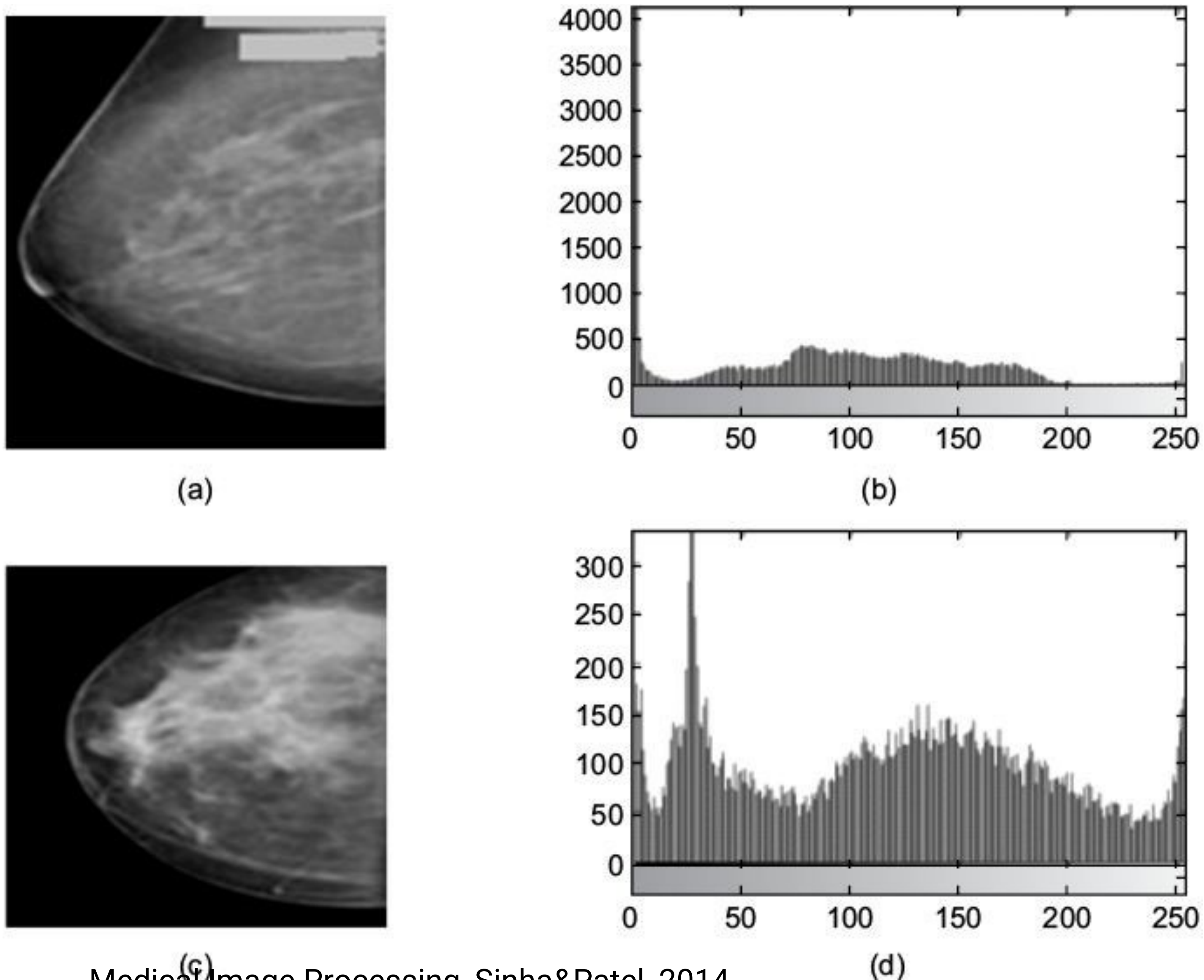
This classification greatly help a radiologist in determining the affected breast anatomy and detection of changes in the breast tissues.

Table 4.1 Classification of Mammographic Images based on Texture Features

Texture Feature	*Tissue Types*					
	Average Intensity or Grayscale Value	*Average Contrast Value*	*Smoothness*	*Second Moment*	*Uniformity*	*Entropy*
Uncompressed and Fatty	43.652	46.314	0.0319	0.451	0.2156	4.876
Fatty	68.512	71.236	0.0672	2.451	0.2332	3.253
Non-uniform	51.065	81.972	0.0976	8.364	0.5225	4.468
High Density	48.173	68.153	0.06472	6.153	0.3273	3.857

Mammograms and their histograms

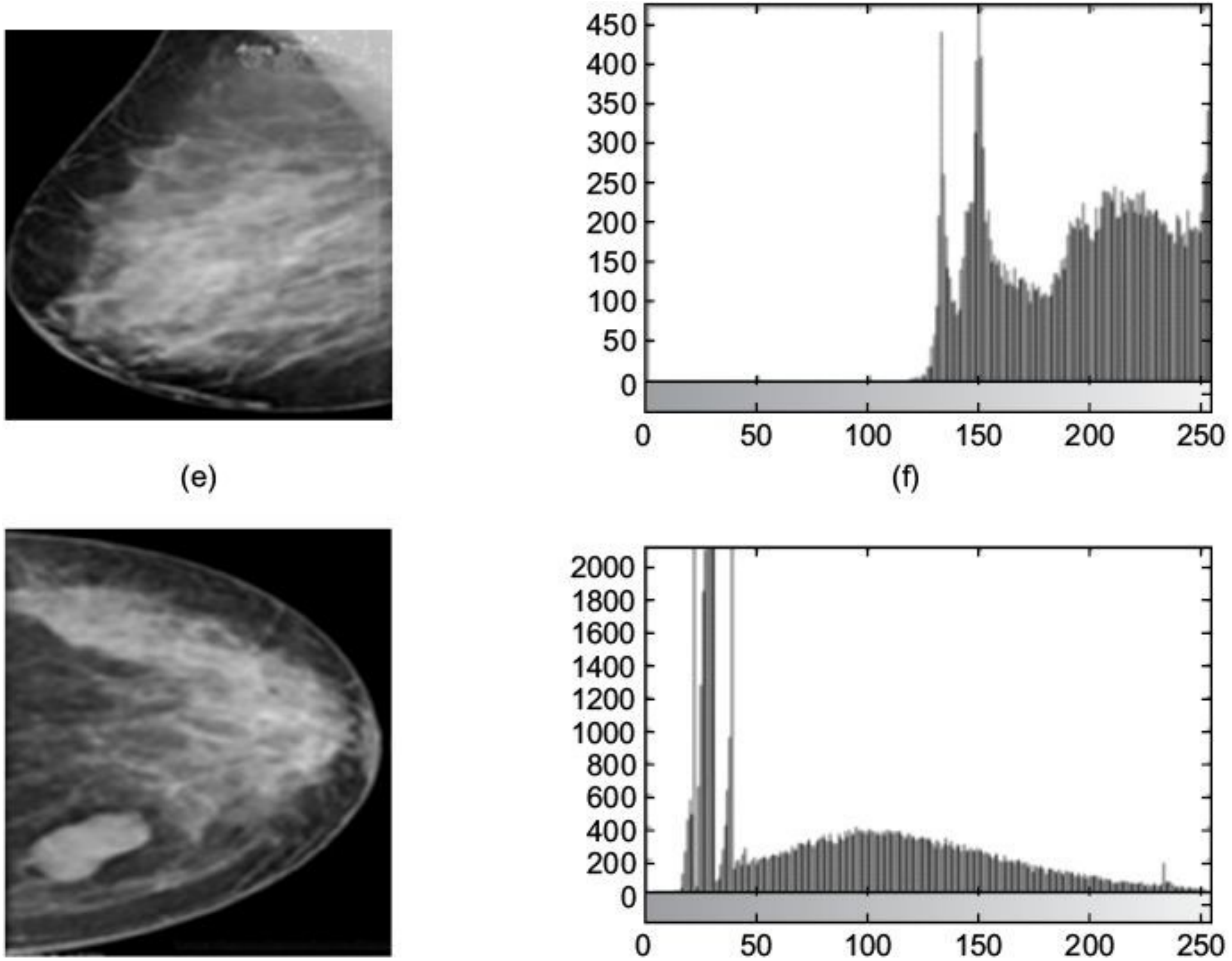
(a) Uncompressed fatty breast
(b) its histogram
(c) Fatty breast
(d) its histogram



Medical Image Processing, Sinha&Patel, 2014

Mammograms and their histograms

(e) Non-uniform fatty breast
(f) its histogram
(g) High density breast
(h) its histogram,



(g) Medical Image Processing, Sinha&Patel, 2014 (h)

Edge Detection Methods

Edge detection methods are widely used in the medical image processing and its applications.

Different methods are available for edge detection.

After using a suitable edge detection method, an edge map array $e(j, k)$ is produced

$$\mathrm{e}(j,k) = \begin{cases} 1, & if\ edge\ is\ detected \\ 0, & otherwise \end{cases}$$

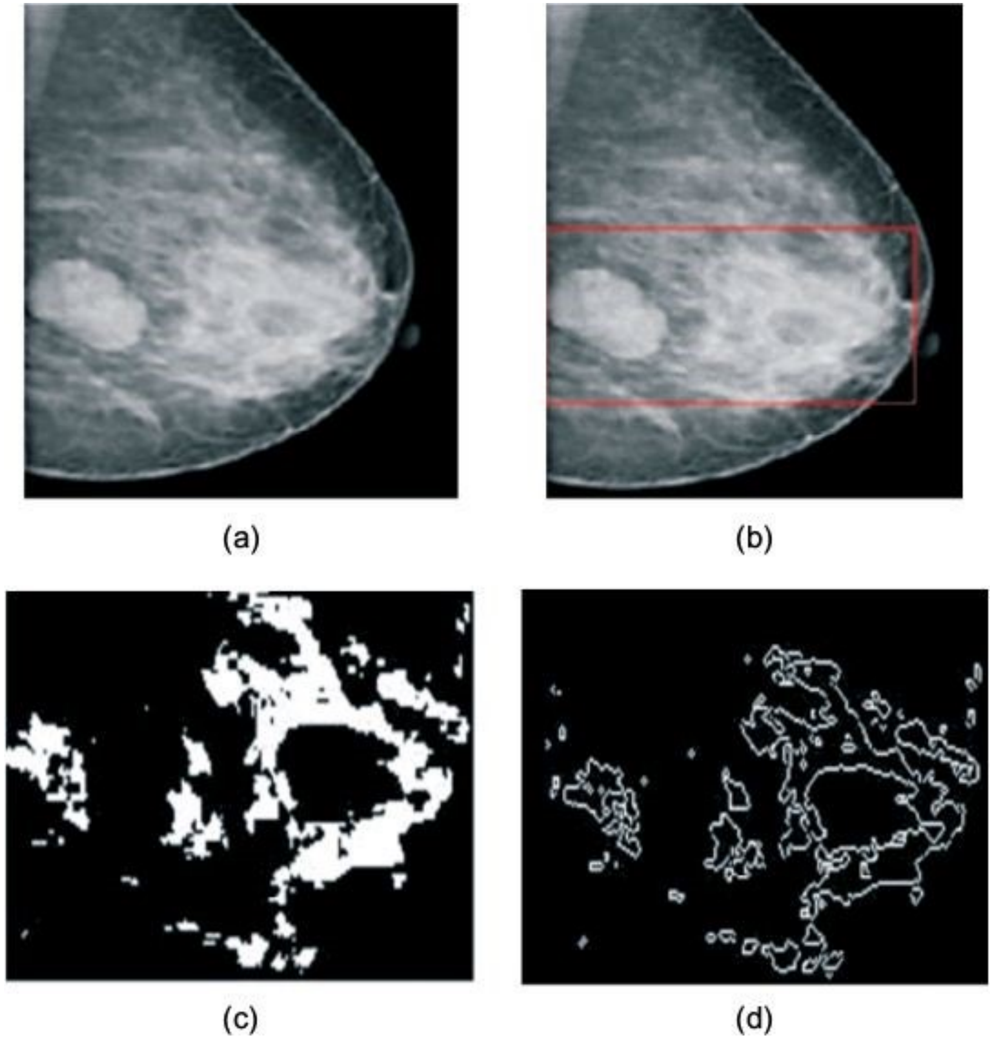
Texture Measure

Medical Image Processing, Sinha&Patel, 2014

Texture measure or a threshold measure using edge array is given as

$$Th(j,k) = \frac{1}{W^2} \sum_{n=-w}^{w} \sum_{n=-w}^{w} e(j+m, k+n)$$

W = (2w+1): dimension of the observation window



Choosing Region of Interest for Texture Analysis

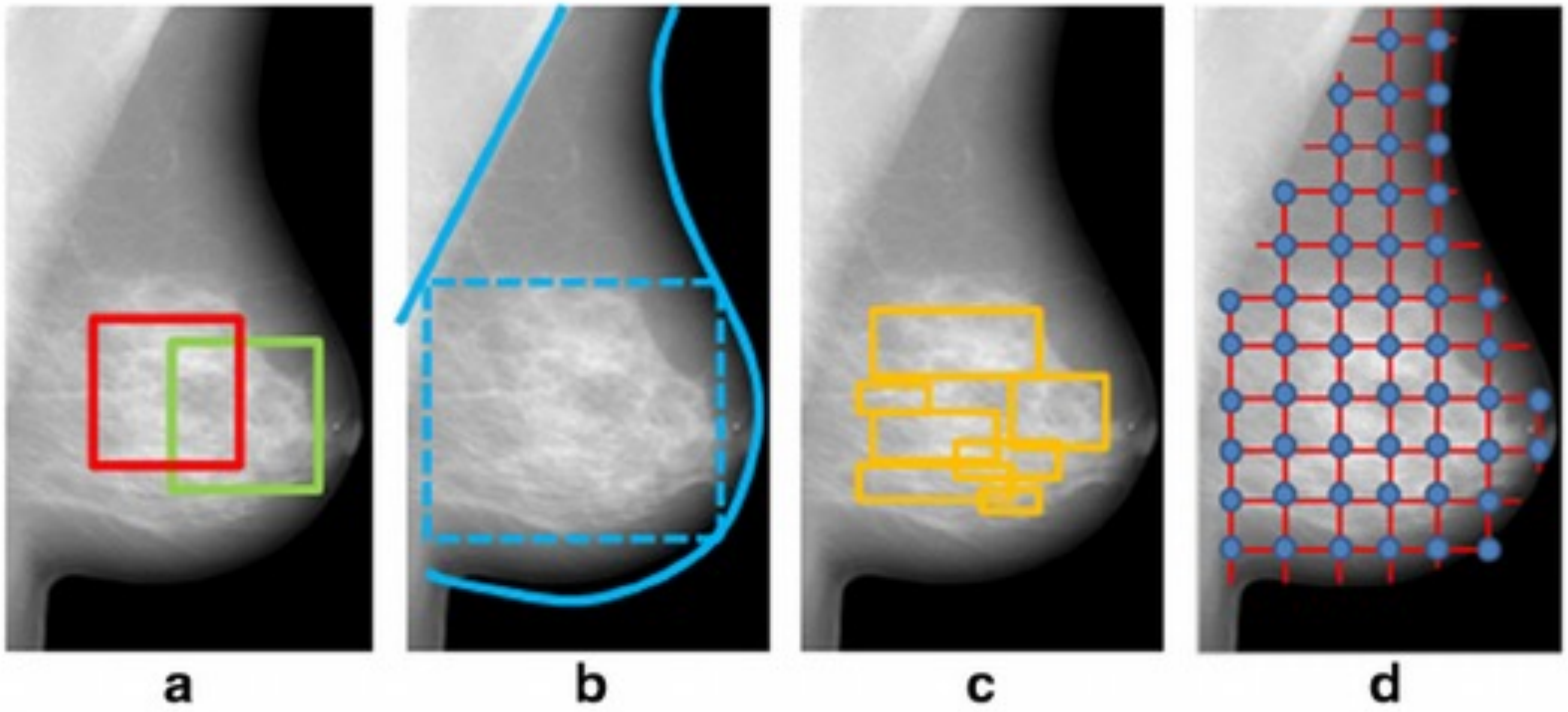


Fig. 1 Regions of interest (ROIs) used in texture analysis. **a** single ROIs selected in the retro-areolar breast area, **b** the entire breast and the largest rectangular box inscribed within the breast, studied as single ROIs, **c** multiple ROIs at multiple scales of density, and **d** multiple ROIs defined by a lattice covering the entire breast

Gastounioti et al. Breast Cancer Research (2016) 18:91

Different Texture Analysis Methods

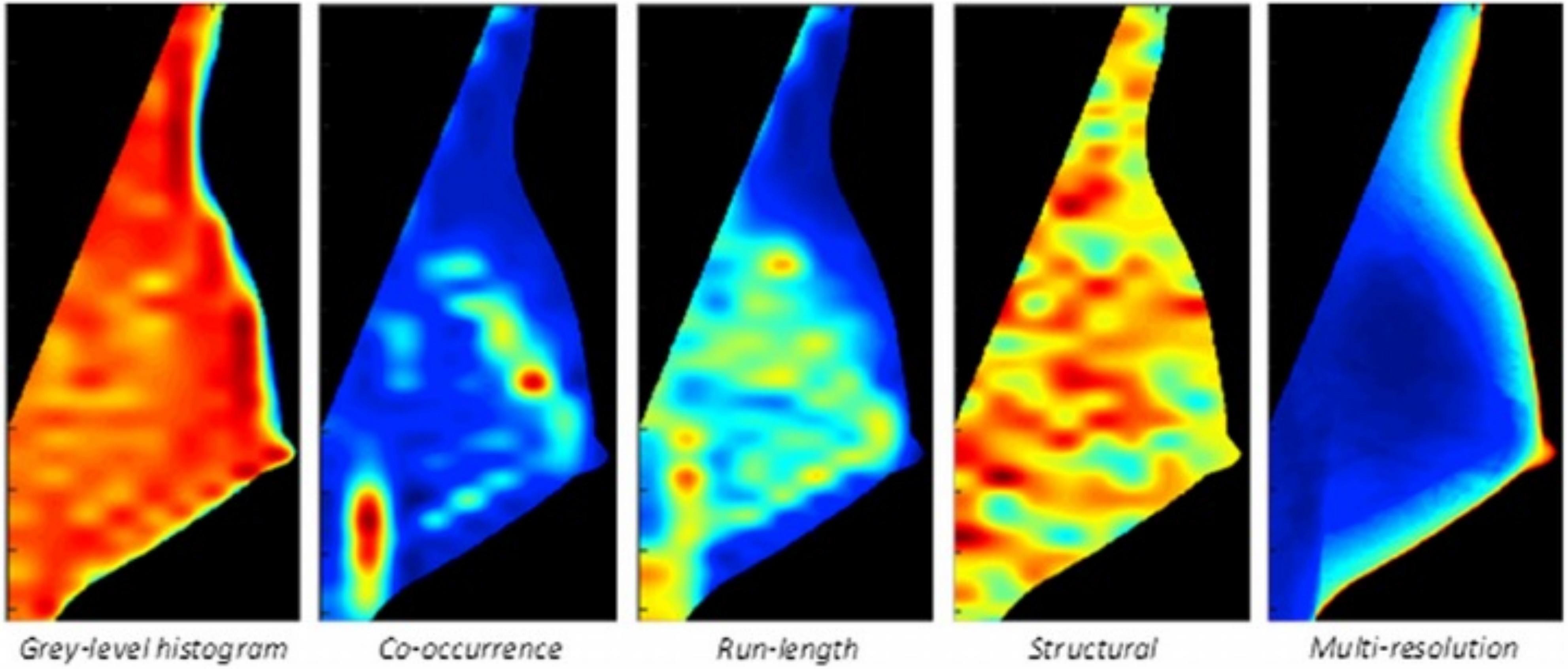


Fig. 2 Characterization of parenchymal patterns using computerized texture analysis. Examples of feature maps showing the distribution of texture values in the breast, generated by the application of the lattice-based strategy of Zheng et al. [51] to an MLO-view full-field digital mammogram. (**a**) Grey-level histogram, (**b**) Co-occurrence, (**c**) Run-length, (**d**) Structural, and (**e**) Multi-resolution

Gastounioti et al. Breast Cancer Research (2016) 18:91