Image Concepts

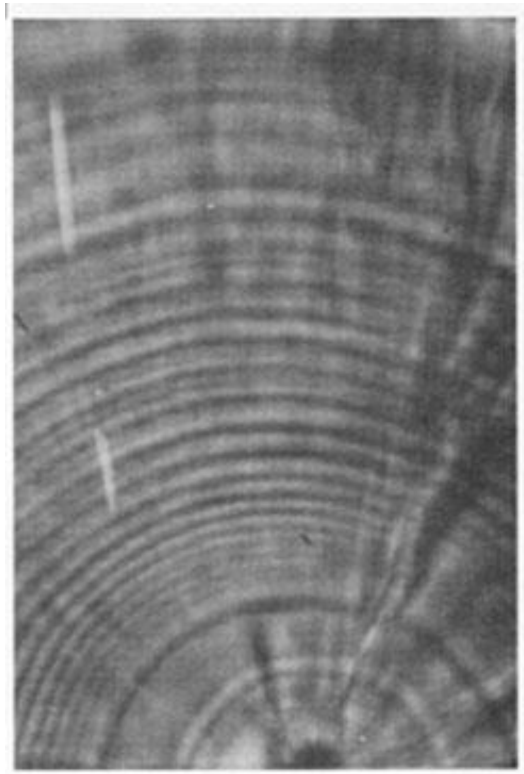
- An image is a function of intensity values over a 2D plane $I(r,s)$
- Sample function at discrete intervals to represent an image in digital form
  - matrix of intensity values for each color plane
  - intensity typically represented with 8 bits
- Sample points are called pixels

Digital Images

- Samples = pixels
- Quantization = number of bits per pixel
- Example: if we would sample and quantize standard TV picture (525 lines) by using VGA (Video Graphics Array), video controller creates matrix 640x480pixels, and each pixel is represented by 8 bit integer (256 discrete gray levels)

Image Representations

- Black and white image
  - single color plane with 2 bits
- Grey scale image
  - single color plane with 8 bits
- Color image
  - three color planes each with 8 bits
  - RGB, CMY, YIQ, etc.
- Indexed color image
  - single plane that indexes a color table
- Compressed images
  - TIFF, JPEG, BMP, etc.



4 gray levels

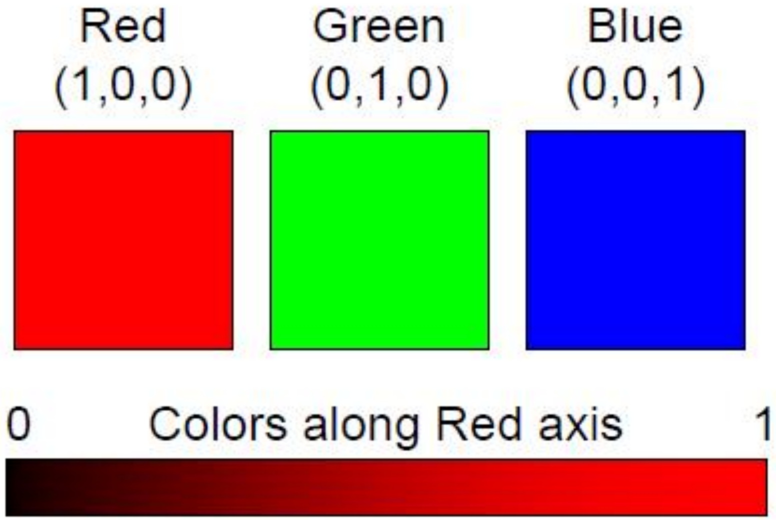
2 gray levels

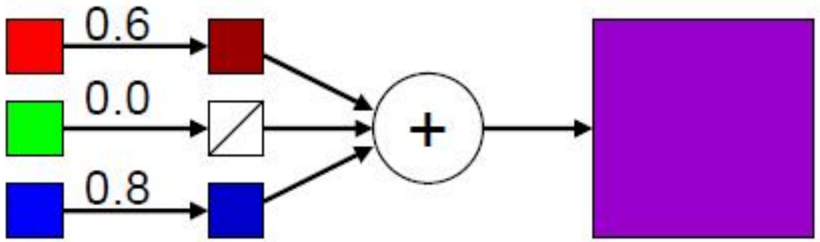
Digital Image Representation

(3 Bit Quantization)

111	111	011	011	011	011	111	111
111	011	111	111	111	111	011	111
000	111	001	111	111	001	111	000
010	111	111	111	111	111	111	010
000	111	100	111	111	100	111	000
000	111	111	100	100	111	111	000
111	000	111	111	111	111	000	111
111	111	000	000	000	000	111	111

Color


Red
(1,0,0)
Green
(0,1,0)
Blue
(0,0,1)
0
Colors along Red axis
1


0.6
0.0
0.8
+


RGB
channels

Image Representation Example

24 bit RGB Representation (uncompressed)

128	135	166	138	190	132
129	255	105	189	167	190
229	213	134	111	138	187

128	138
129	189
229	111

135	190
255	167
213	138

166	132
105	190
134	187

Color Planes

From Processing to Analysis

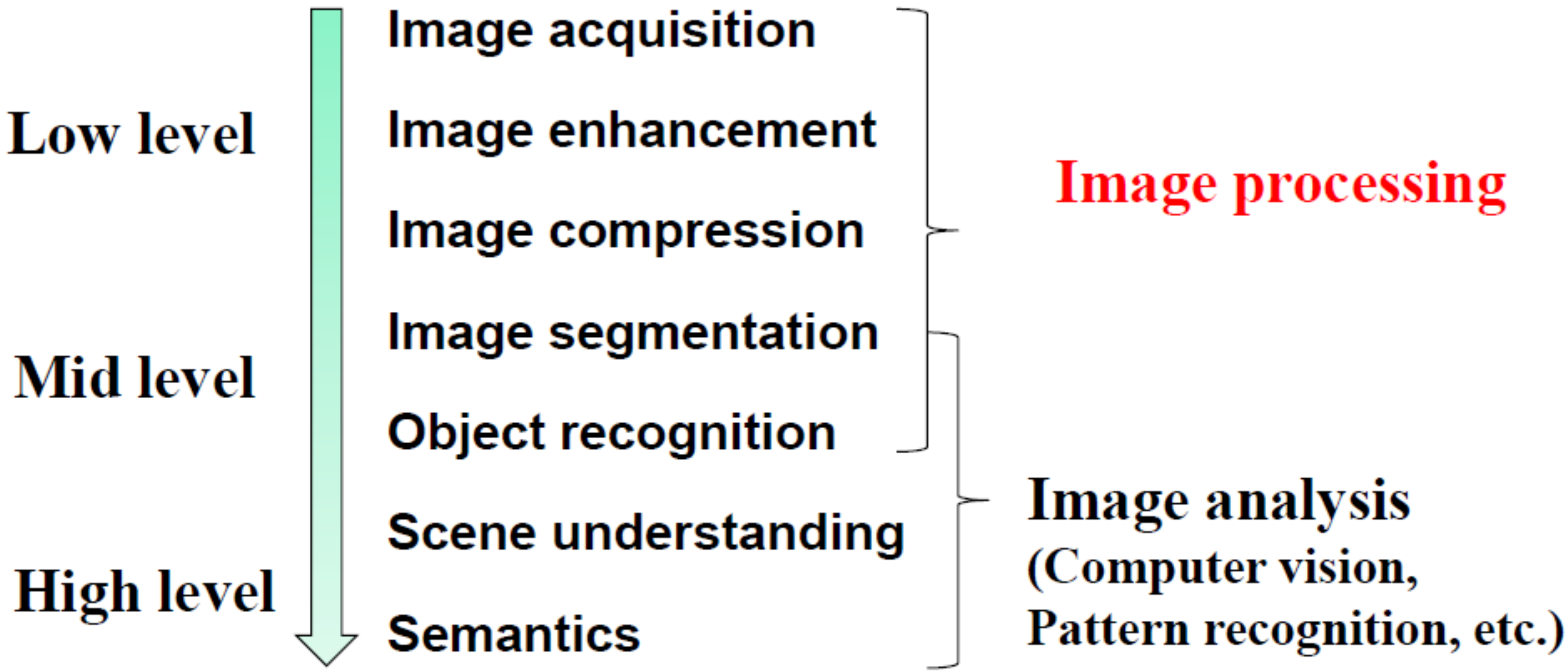


Image Processing Flow

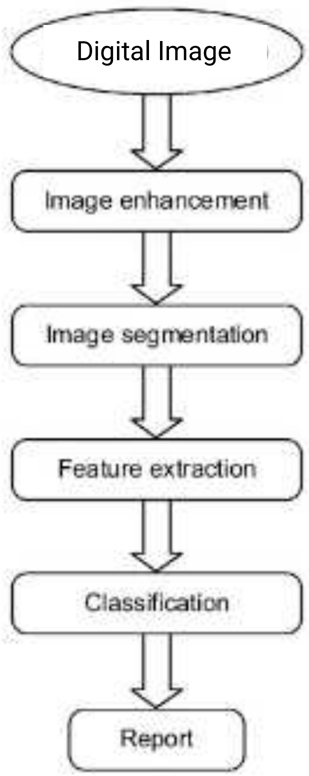


Image enhancement

Are techniques that are used to improve the quality of an image as perceived by a human as well as by a machine.

Broadly classified as spatial domain image enhancement and frequency domain image enhancement methods.

Spatial domain methods: contrast stretch, histogram modification, density slicing, edge enhancement and spatial filtering, …

Frequency domain methods: filters (smoothing, sharpening, homomorphic)

Some reasons for bad quality images: Poor illumination and lighting, improper pose conditions and other noise signals

Image enhancement is important in the analysis of digital images.

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Image Processing Flow

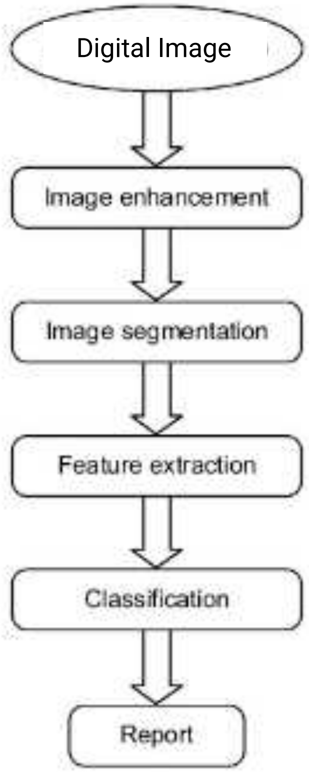

Image segmentation

Very much needed in medical image processing in context to CAD systems.

Visual inspection requires being clear in diagnosis process where the correct region which is affected, needs to be separated. → achieved by image segmentation methods.

A good segmented image can be justified based on statistical analysis.

- ❑ essential to distinguish between the region of interest (ROI) and the remaining portion (a.k.a. background)

Methods used to find ROI are known as segmentation techniques →segmenting the **foreground** from the **background**.

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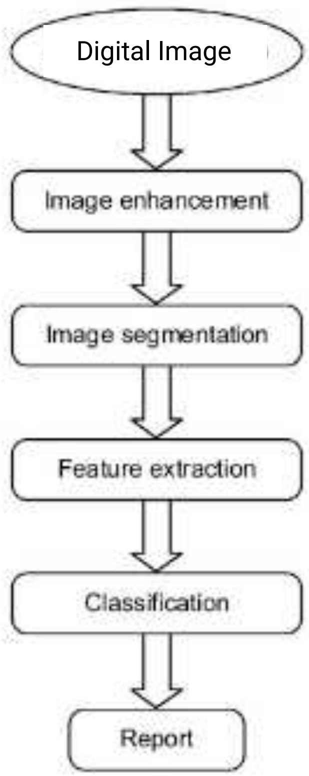
Image Processing Flow



Feature extraction

A very important process in extracting suitable features of any image processing technique.

- ❑ It could be image enhancement or image segmentation.

Few statistical parameters or features are required to compare the input image with the result obtained and therefore, feature extraction plays a crucial role in the analysis of images.

Important features could be used to discriminate between benign and malignant in tumors.

- ❑ Geometric features are based on area (a set of pixels inside ROI), perimeter (number of pixels around ROI boundary, compactness), shape, etc.

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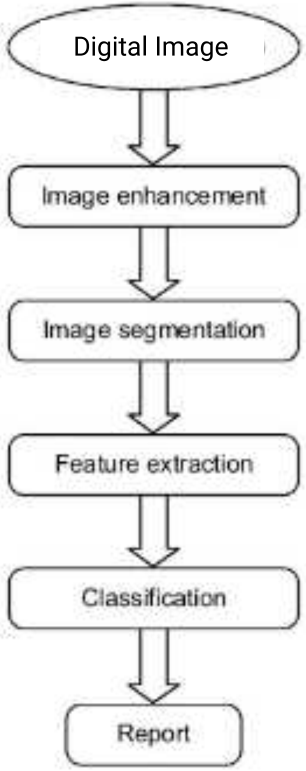
Image Processing Flow



Classification

Generally, the last step in a CAD system → plays a vital role in the implementation of computer-aided diagnosis (CAD) of imaging

Once images are enhanced, segmented and the features are extracted, classification process is applied.

After the features are extracted and selected, then subjected to the classifier to classify the detected suspicious areas into distinct categories, e.g., benign masses or malignant masses.

Image enhancement play also important part as abrupt change in gray level values occurs in the transition between some tissues, and the applied filters can enhance the structures in images representing clusters of malignant masses.

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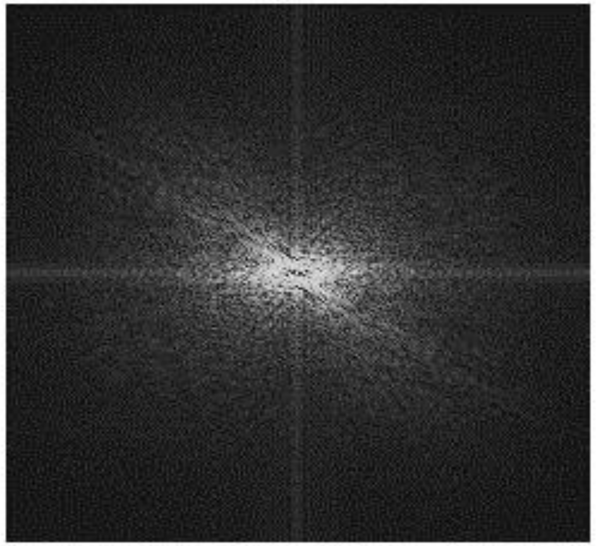
Spatial and Frequency Domains

- Spatial domain
  - refers to planar region of intensity values at time t

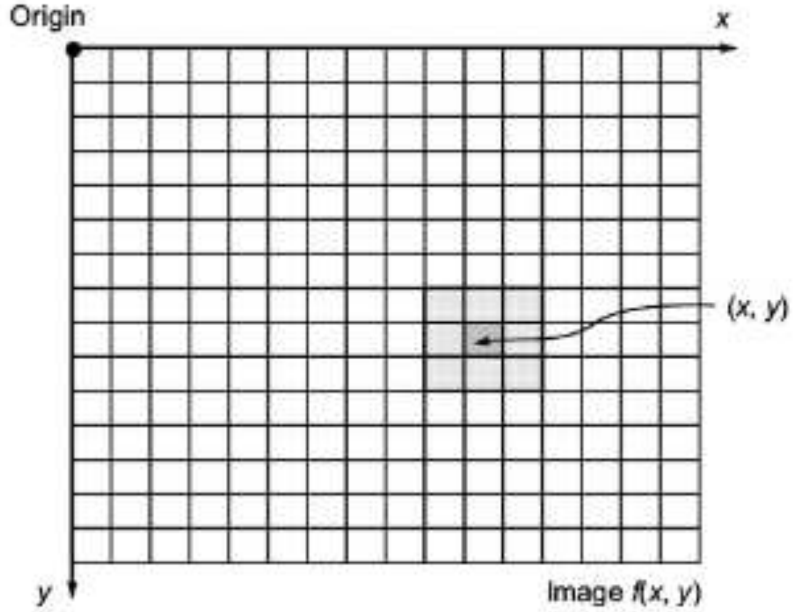
- Frequency domain
  - think of each color plane as a sinusoidal function of changing intensity values
  - refers to organizing pixels according to their changing intensity (frequency)



Spatial Domain Methods

- ❑ Directly operate over pixels.
- ❑ Modification is made over the intensity values of pixels in an image.
- ❑ The process is expressed by a transformation function given as:

$$e(x, y) = T[f(x, y)]$$



f(x, y): input image
e(x, y): processed image or enhanced image
T: an operator that is applied over the input original image in some neighborhood of (x, y)

The neighborhood about a point (x, y) is defined using a square or rectangular sub-image area of suitable size centered at (x, y)

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Spatial Domain Methods

The center of the sub-image is moved from pixel to pixel starting from the top left corner. A transformation operator is applied to all the pixels scanning down to the bottom right pixel of an image. If r is the input grayscale value, then the output grayscale value s will be computed as

$$s = T[r]$$

r: gray level of f(x, y) at any point (x, y)
s: gray level of e(x, y) at any point (x, y)

Some Examples of Spatial Domain Operations

Binary Image Output

Negative of an Image

Log Transformation

Power Law Transformation

Contrast Enhancement

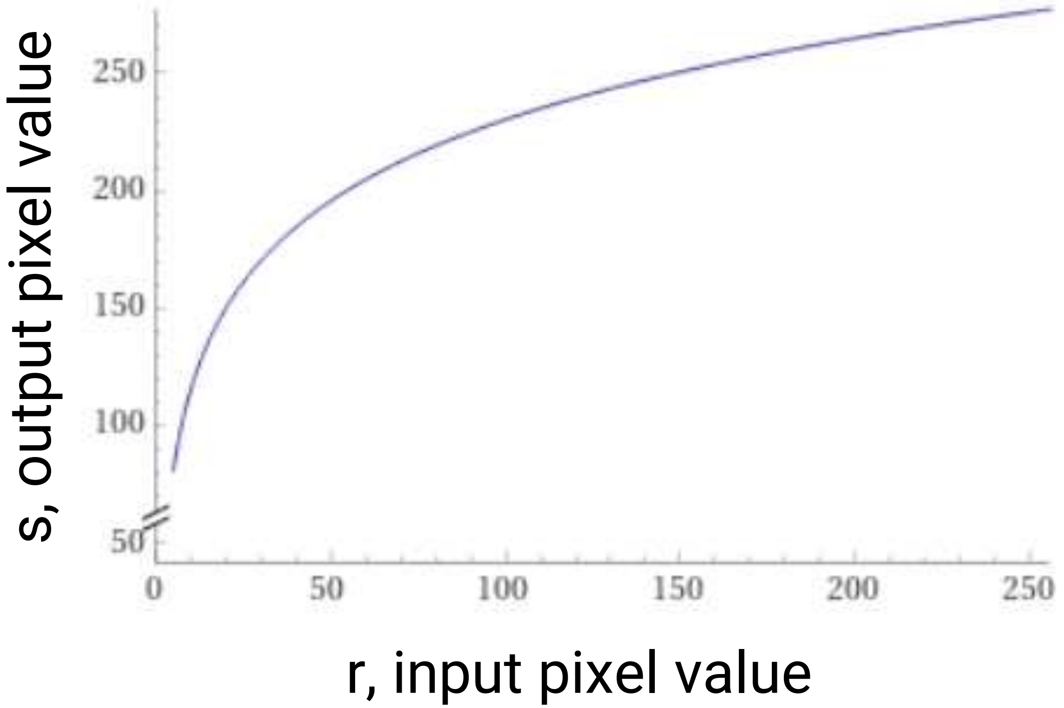
Histogram Processing

Log Transformation

A transformation that maps a narrow range of grayscale values in the input image onto a wider range of output levels (opposite is also true.)

$$s = C\ log(1 + r)$$

s: output grayscale value
r: grayscale value of input image
C: constant.



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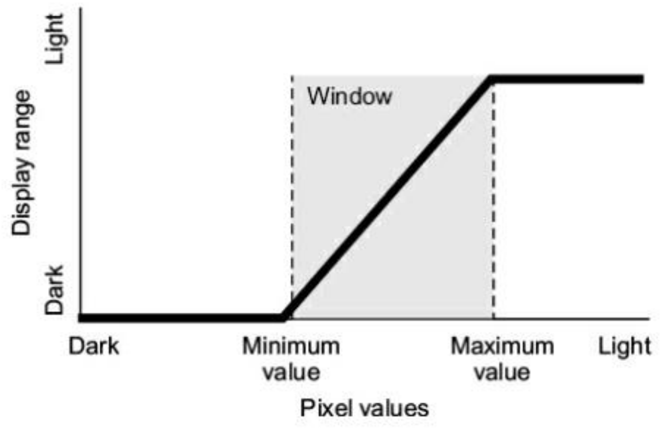
Contrast Enhancement

Is used to improve the contrast of an image by expanding or stretching the range of intensity values.

The features or details obscuring in the original image are now clear in the contrast enhanced image.

$$s = (r - c)\left(\frac{b - a}{d - c}\right) + \alpha$$



Contrast algorithm involves identifying lower and upper bounds from the histogram (minimum and maximum grayscale values).

After applying a transformation in this range, the image is enhanced.
a: lower limit, b: upper limit
limiting values are determined in the image without enhancement (lower = c and upper =d)

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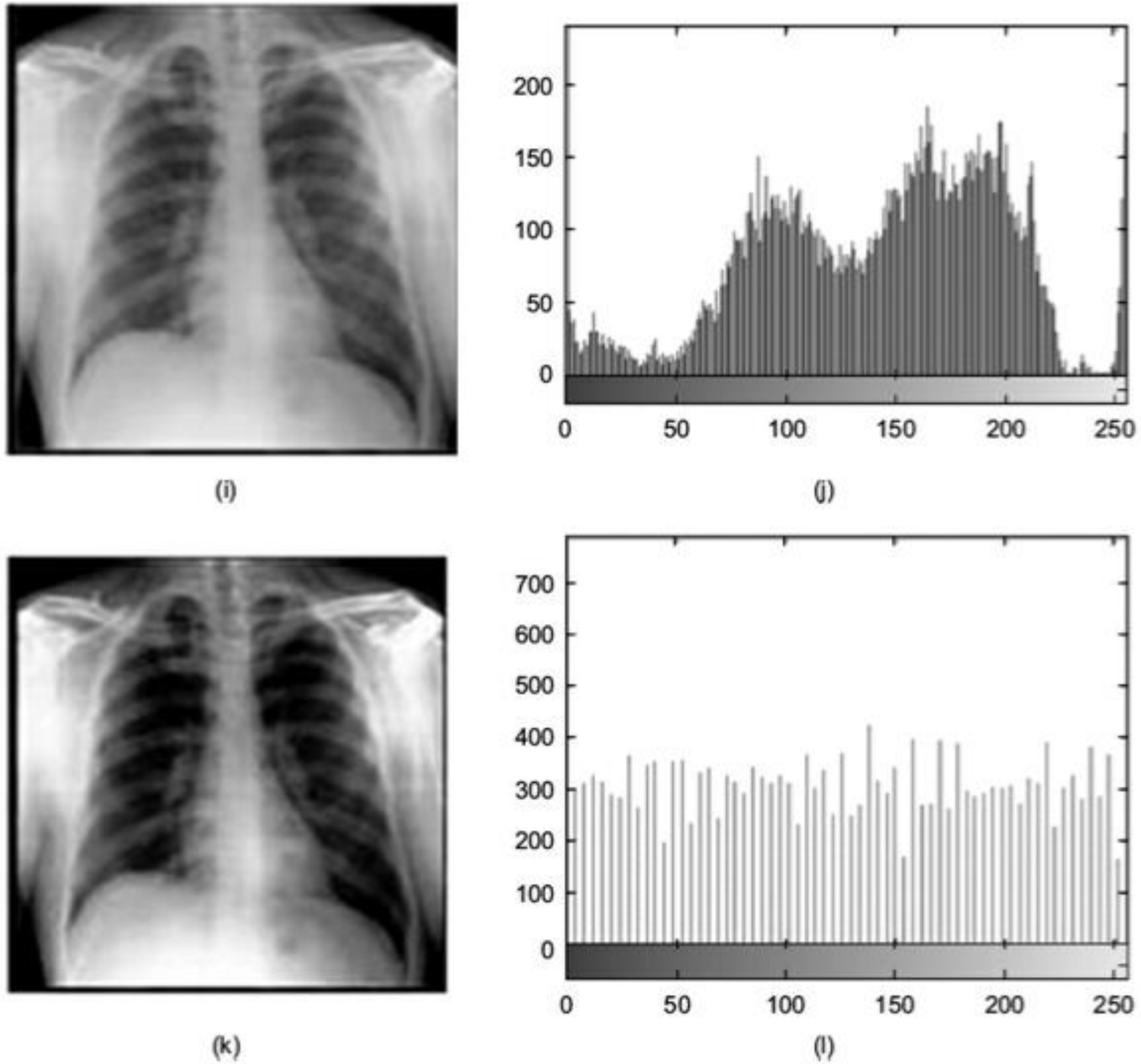
Histogram Processing



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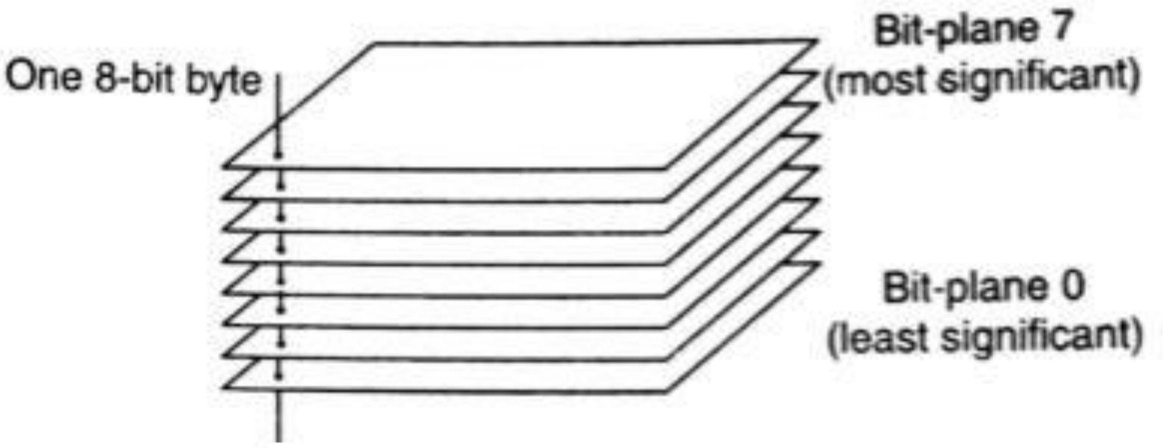
The grayscale values are redistributed with an equal probability of occurrence.

Bit-plane Slicing

Pixels are digital numbers, each one composed of bits. Instead of highlighting gray-level range, we could highlight the contribution made by each bit.

This method is useful and used in image compression.

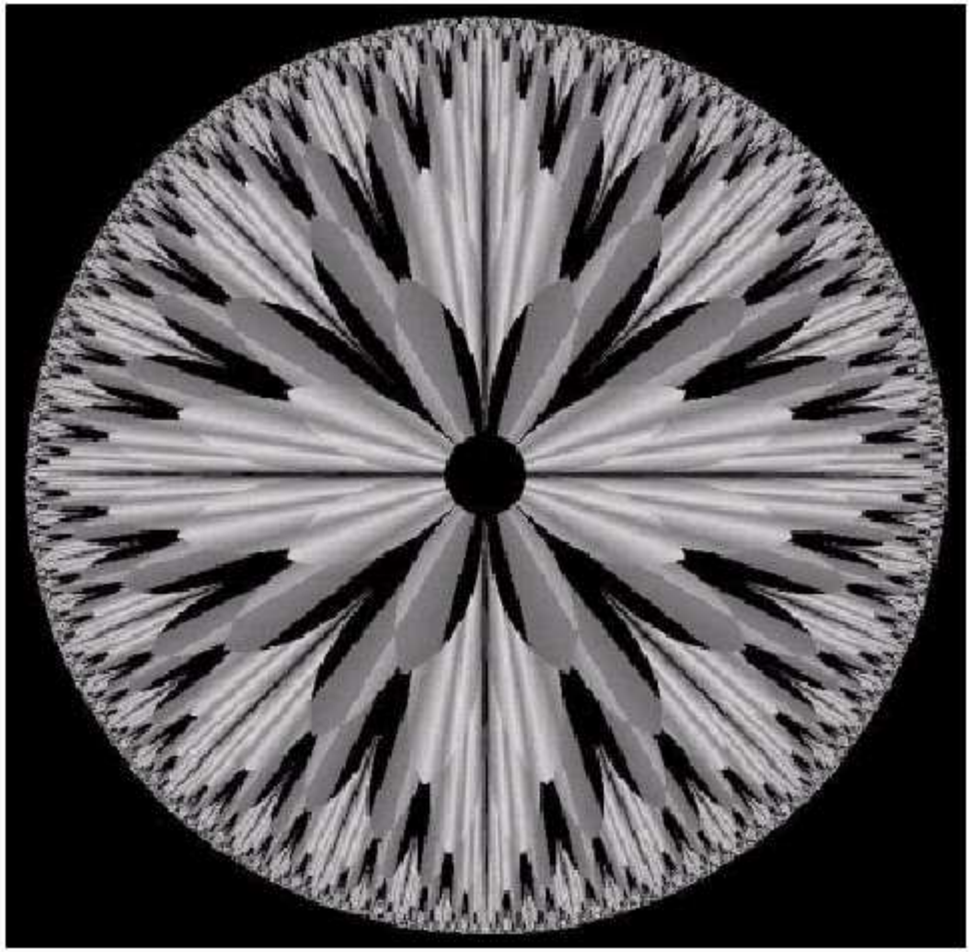


Most significant bits contain the majority of visually significant data.

Bit-plane Slicing

By isolating particular bits of the pixel values in an image we can highlight interesting aspects of that image

- ❑ Higher-order bits usually contain most of the significant visual information
- ❑ Lower-order bits contain subtle details



Bit-plane Slicing

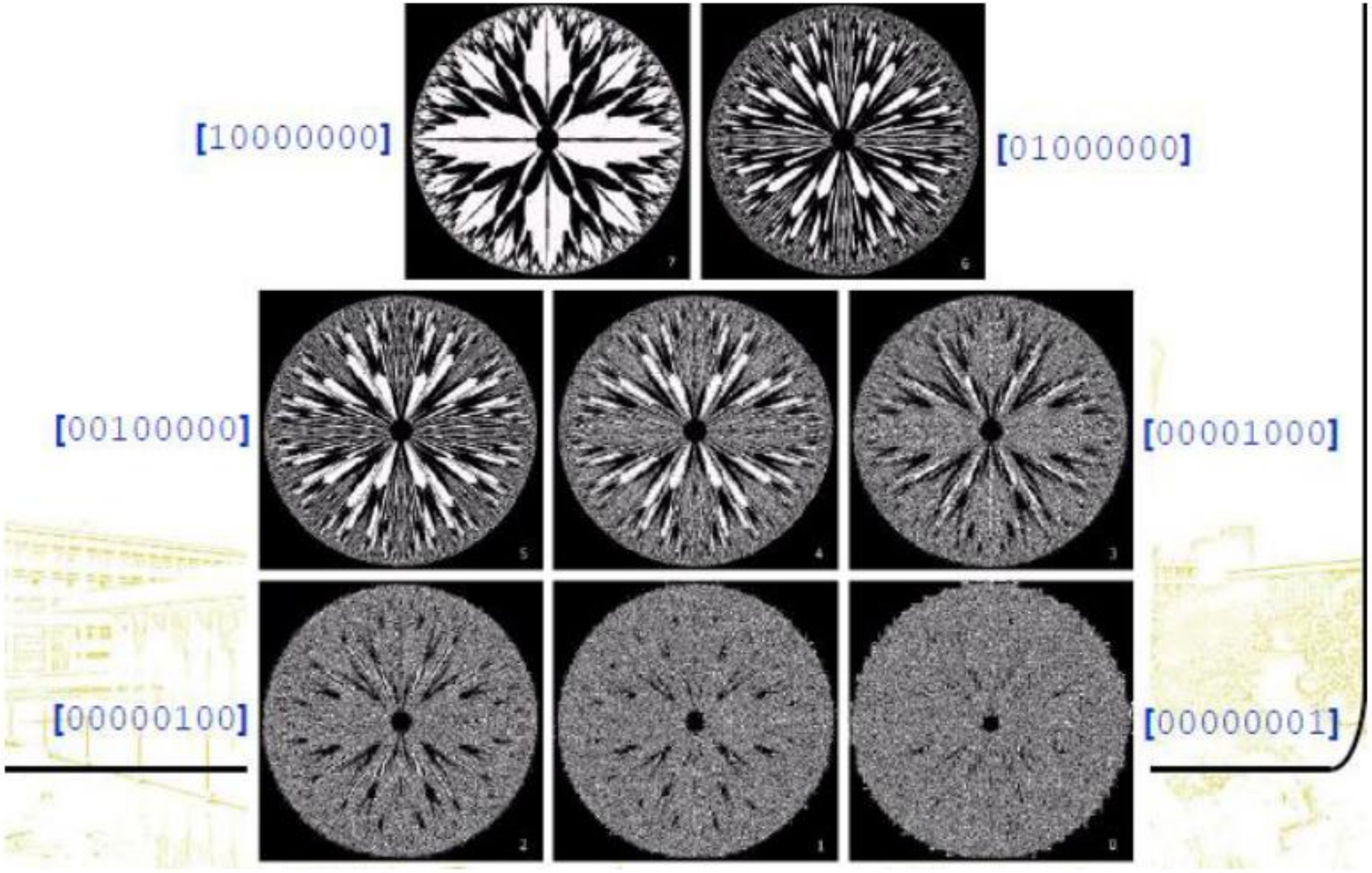
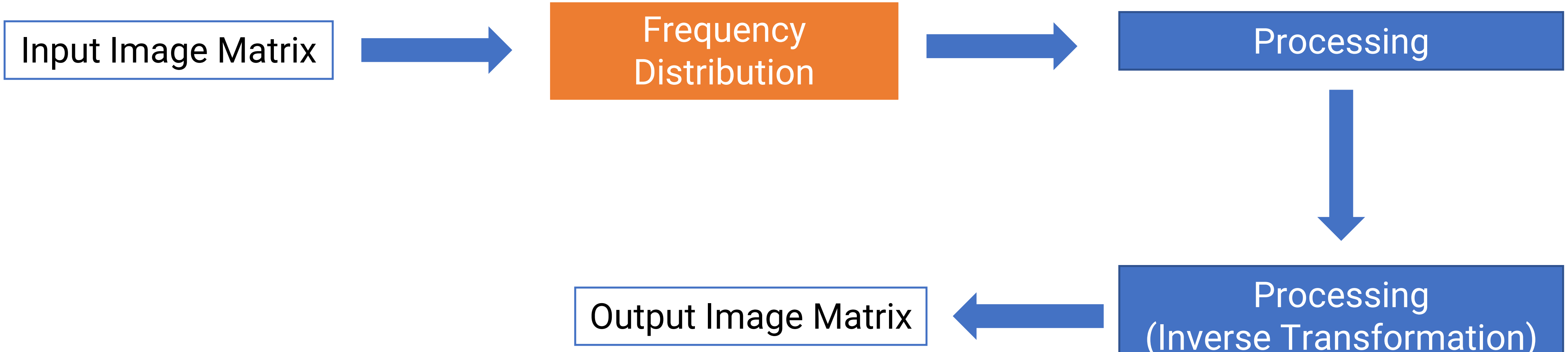


Image Enhancement

Spatial Domain

Frequency Domain



Transformation to Frequency Domain

From spatial to frequency domain

- ❑ Fourier Series
- ❑ Fourier transformation
- ❑ Laplace transform
- ❑ Z transform
- ❑ …

Fourier Analysis

Fourier Analysis in a Nutshell

- Periodic signals can be represented as a sum of ***sines*** and ***cosines*** when multiplied by a certain weight (series),

- This series can be denoted by ***integral transformation***.

Fourier Series

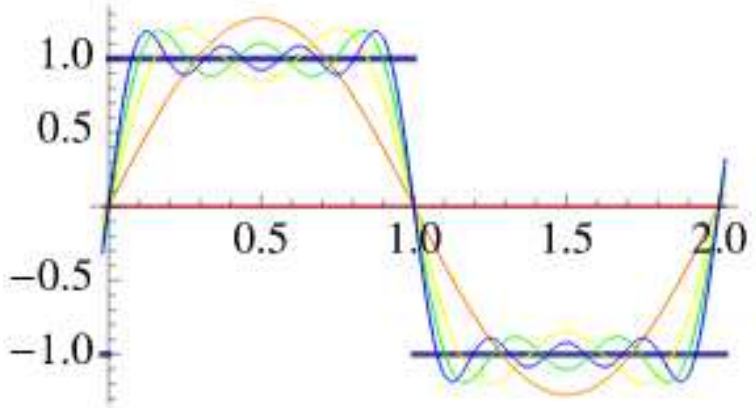
Periodic signals can be represented as a sum of sines and cosines when multiplied by a certain weight:

$$S(x) = \frac{a_0}{2} + \sum_{n=1}^{N}\left(a_n cos\left(\frac{2\pi nx}{P}\right) + b_n sin\left(\frac{2\pi nx}{P}\right)\right)$$

$$S(x) = \sum_{n=-N}^{N} c_n e^{i\frac{2\pi nx}{P}}$$

This also means periodic signals can be broken down into further signals with the following properties:

- The signals are sines and cosines
- The signals are harmonics of each other (i.e., their frequencies are integer multiples of other frequencies)



$\frac{x}{P}$

https://mathworld.wolfram.com/FourierSeries.html

Fourier Transform

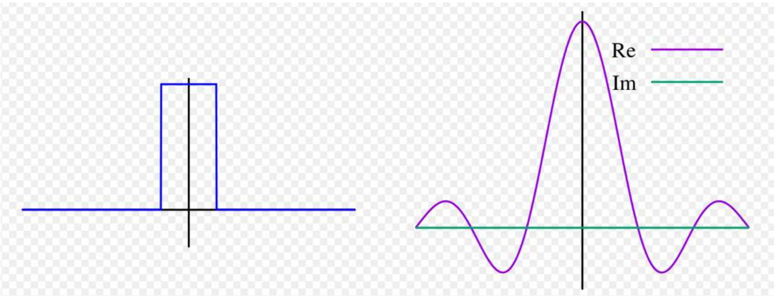
- Aperiodic signals can be decomposed into a continuum of sine waves.

→ Non-periodic signals whose area under the curve is finite can also be represented as integrals of the sines and cosines after being multiplied by a certain weight.

$$f(x) = \int_{-\infty}^{\infty} \mathrm{F}(u) \cdot e^{i2\pi ux} dx$$

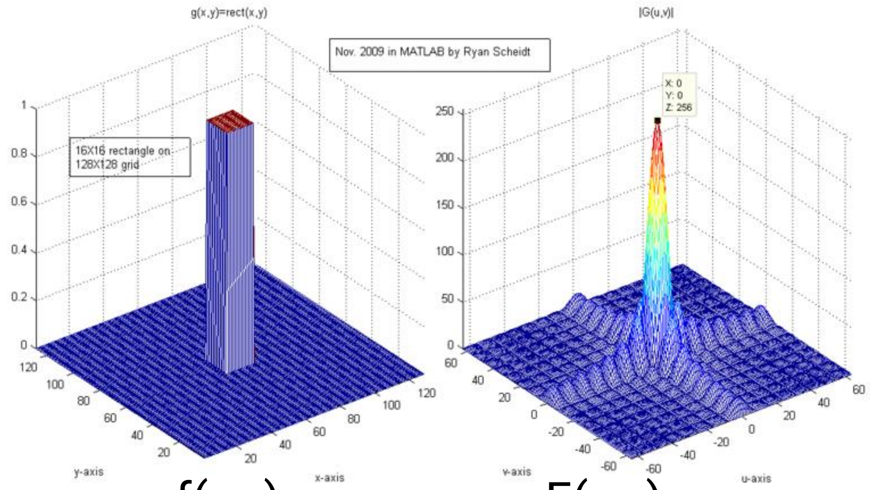
or multidimensional

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{F}(u,v) \cdot e^{i2\pi(ux+vy)} dx\, dy$$



f(x) F(u)

Image: https://en.wikipedia.org/wiki/Fourier_transform



f(x,y) F(u,v)

Image:http://measurebiology.org/wiki/Spring_2011:3D_PSF_lab:Kristin

Discrete Fourier Transform (DFT)

2D DFT

$$\boxed{F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi ux/M}e^{-j2\pi vy/N}}$$

$$u = 0, 1, \ldots, M-1 \quad v = 0, 1, \ldots, N-1$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi ux/M}e^{j2\pi vy/N}$$

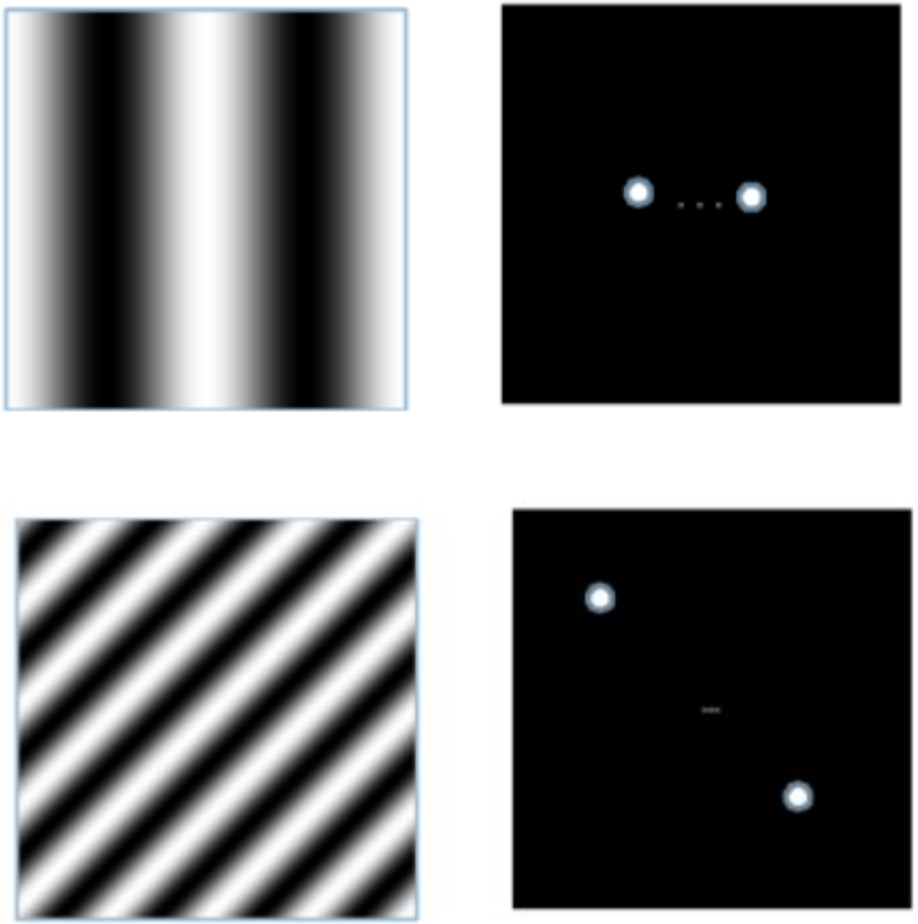
Discrete Fourier Transform (DFT)

2D Inverse DFT

$$\boxed{f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi ux/M} e^{j2\pi vy/N}}$$

$$x = 0, 1, \ldots, M-1 \qquad y = 0, 1, \ldots, N-1$$

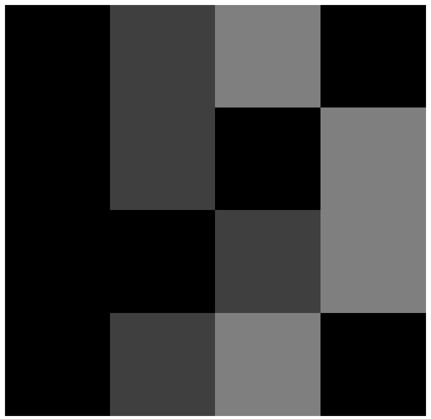
DFT



https://sbme-tutorials.github.io/2018/cv/notes/3_week3.html

DFT Example (4x4 pixel gray image)

For the image shown find Fourier transform at points (0,0), (0,1) and (1,0).

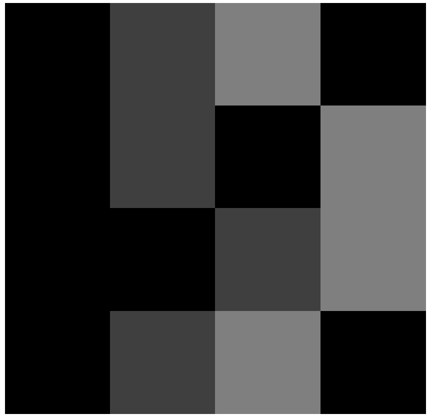


Gray scale: DOI:10.1080/02635143.2012.656084

DFT Example (4x4 pixel gray image)

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi ux/M}e^{-j2\pi vy/N}$$

y \ x	0	1	2	3
0	0	2	1	0
1	0	2	0	1
2	0	0	2	1
3	0	2	1	0

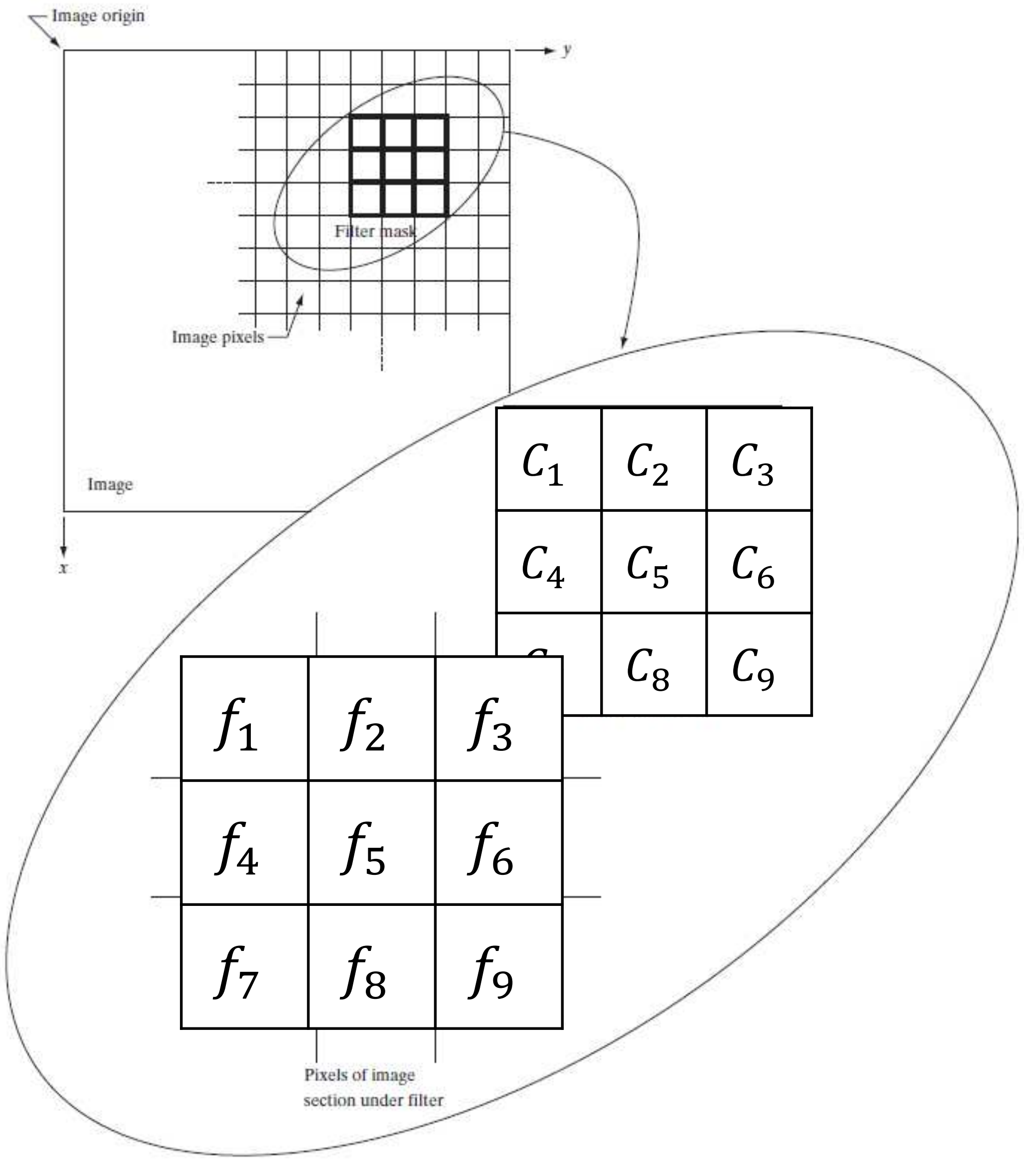


$$F(0,0) = \sum_{x=0}^{3}\sum_{y=0}^{3} f(x,y)ee^{-2j\pi(\frac{x.0}{4}+\frac{y.0}{4})} = 12$$

$$F(0,1) = \sum_{x=0}^{3}\sum_{y=0}^{3} f(x,y)ee^{-2j\pi(\frac{y.1}{4})} = 0$$

$$F(1,0) = \sum_{x=0}^{3}\sum_{y=0}^{3} f(x,y)ee^{-2j\pi(\frac{x.1}{4})} = -4-4j$$

Image Filtering with Masks



Move the mask over the image, calculate the pixel value using correlation or convolution

Adapted from Digital Image Processing 3rd Edition
by Rafael Gonzalez (Author), Richard Woods (Author)

2D Correlation and Convolution

Correlation of two functions

$$g(x,y) = w(x,y) \circ f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution of two functions (like rotate w for 180 degree)

$$g(x,y) = w(x,y) * f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Linear Filtering of an image f of size MxN filter mask of size mxn is given by the expression, a = (m-1)/2, b = (n-1)/2,
x = 0, 1, 2, … , M-1 and y = 0, 1, 2, … , N-1

Spatial Filter (Mask)

Spatial filter: also called mask/kernel/template or window)

- Consist of a neighbourhood with coefficients on pixels
- Example: masks of odd sizes, e.g. 3x3, 5x5,…

Apply a filter to an image: simply move the filter mask from point to point in an image. At each point (x, y), the response of the filter at that point is calculated using a predefined relationship.

Example: Linear Filter

$$D_a = C_1 Z_1 + C_2 Z_2 + \cdots + C_9 Z_9 = \sum_{i=1}^{m \times n} C_i Z_i$$

C_1	C_2	C_3
C_4	C_5	C_6
C_7	C_8	C_9

Example: Box Filter

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Image filtering

$$g[\cdot\,,\cdot\,] \quad \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

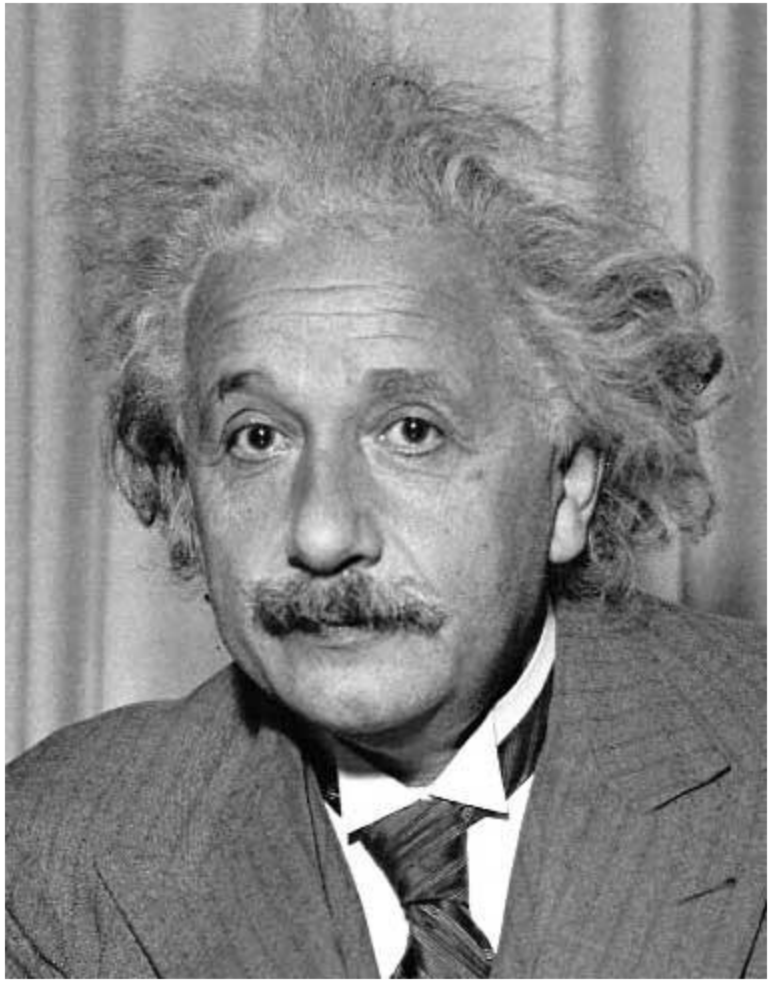
$$h[.,.]$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

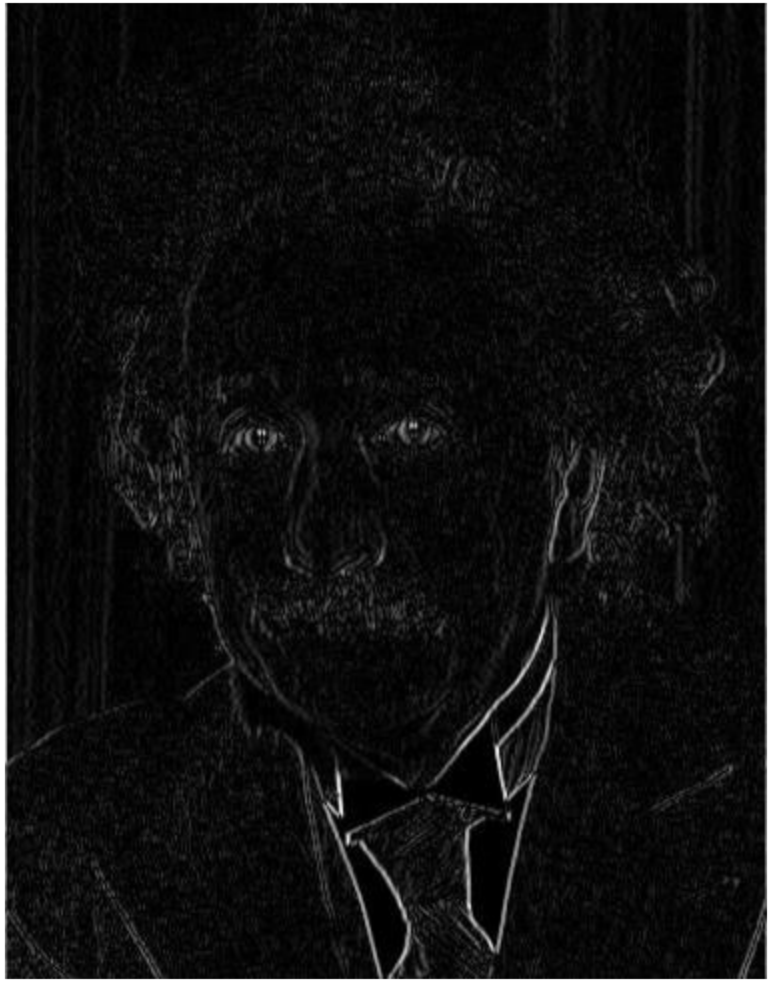
Credit: S. Seitz

Other filters



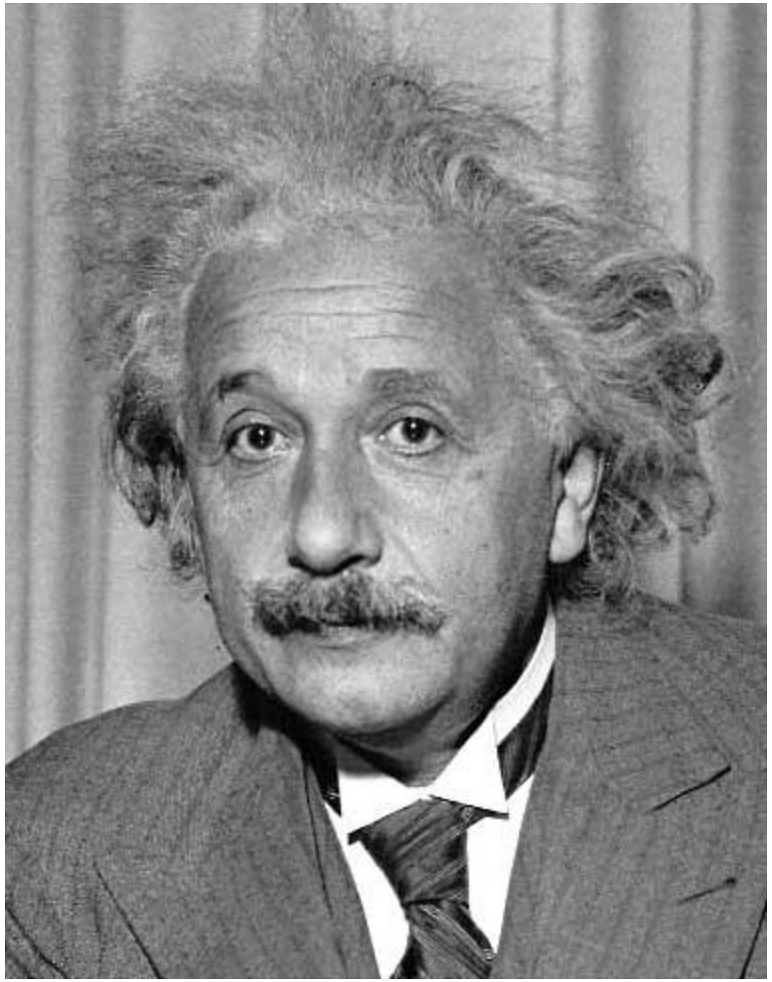
1	0	-1
2	0	-2
1	0	-1

Sobel



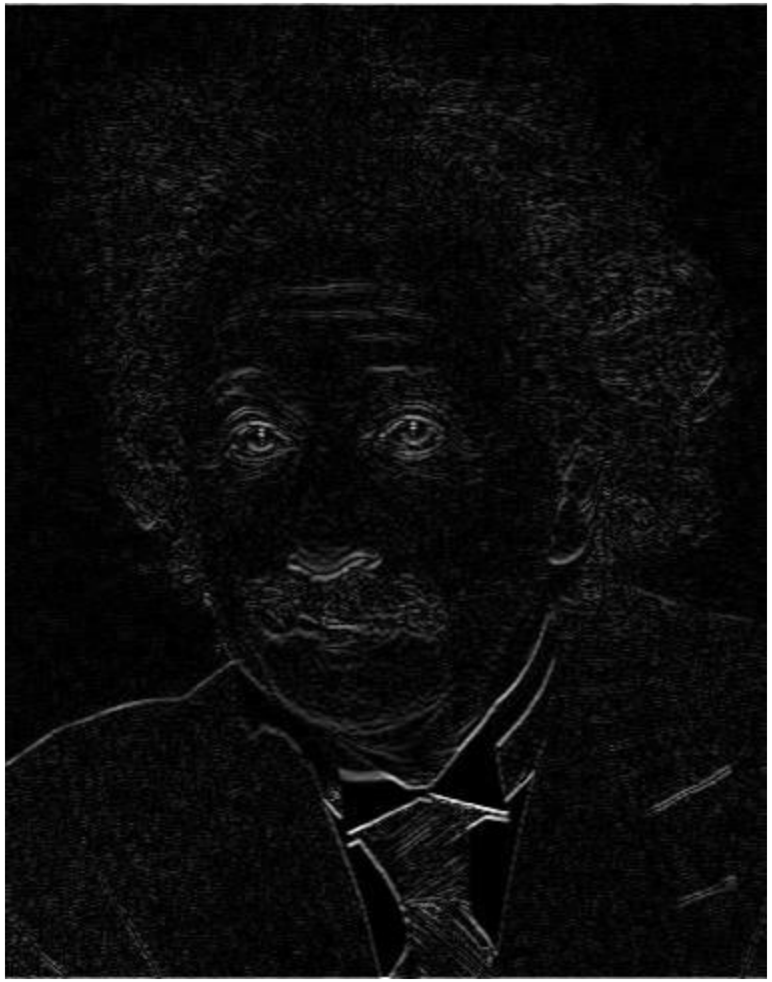
Vertical Edge
(absolute value)

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Basic gradient filters

Horizontal Gradient

0	0	0
-1	0	1
0	0	0

or

-1	0	1

Vertical Gradient

0	-1	0
0	0	0
0	1	0

or

-1
0
1

Examples of 3x3 Smoothing Filter

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Box Filter

Gives a standard average of the pixels under the mask. (a.k.a low pass filter)

$$\frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Weighted Average

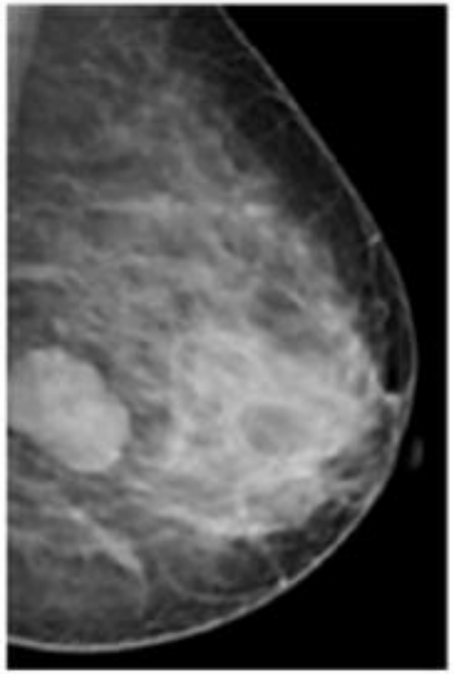
Gives more importance (weight) to some pixels at the expense of others.

High Pass Filter (HPF)

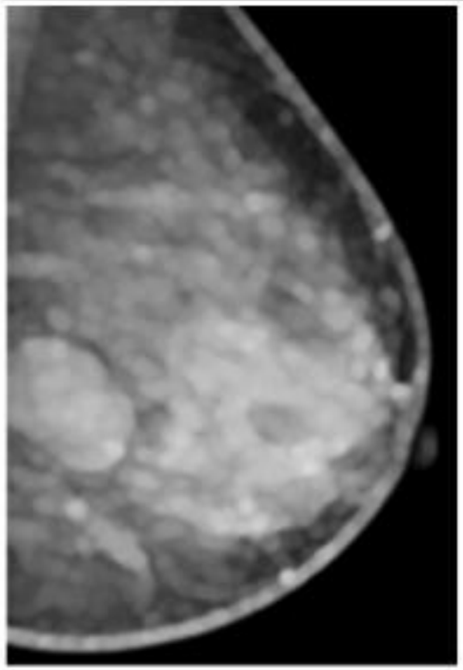
Used to pass high frequency components in spatial domain and reject or attenuate low frequency details of an image. The calculation of filter coefficients is done by selecting the value of a center pixel and subtracting it from it half the average value of the surrounding pixels.

–1	–1	–1
–1	–8	–1
–1	–1	–1

Original



Filtered (HPF)



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High Boost Filter (HBF)

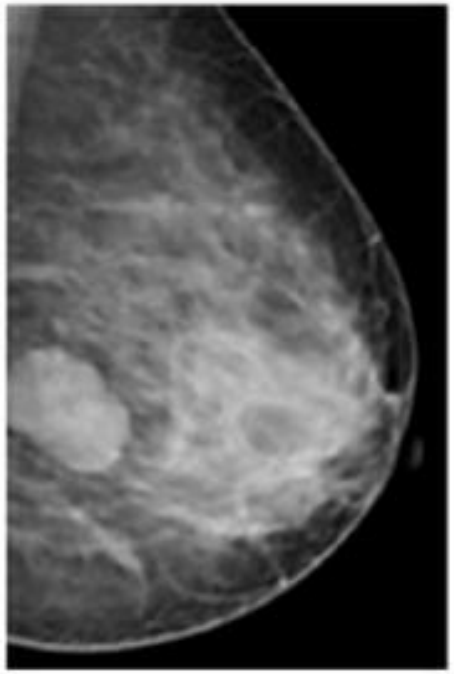
Used to enhance (***boost***) the high frequency content along with the low frequency information of the background.

HBF = (A − 1) Original image + HPF output = A[Original image] − LPF output

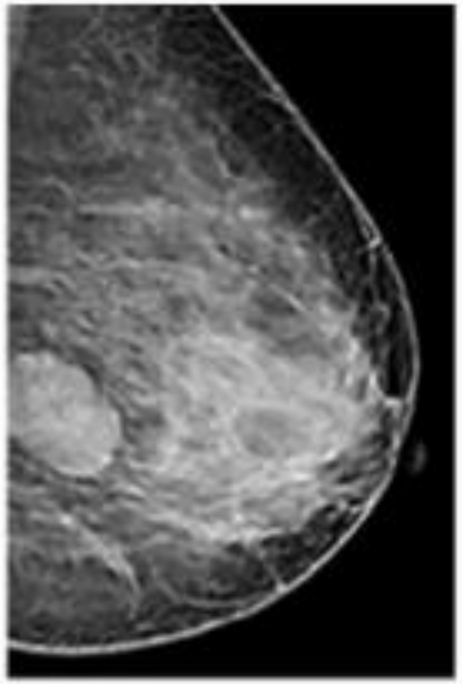
A: High boost coefficient

−1	−1	−1
−1	9 − A	−1
−1	−1	−1

Original



Filtered (HBF)



Patel, G.R.. Medical Image Processing: Concepts and Applications

Frequency Domain Filters

Steps to apply filters in frequency domain:

[Input image : f(x,y)]

- ❑ Compute DFT of the input image (F(u,v)).
- ❑ Multiply F(u,v) with DFT of the transfer function (filter function, G(u,v))
- ❑ Compute inverse DFT of the result.
- ❑ Obtain the real part of the DFT as the ***enhanced*** or ***filtered*** result.

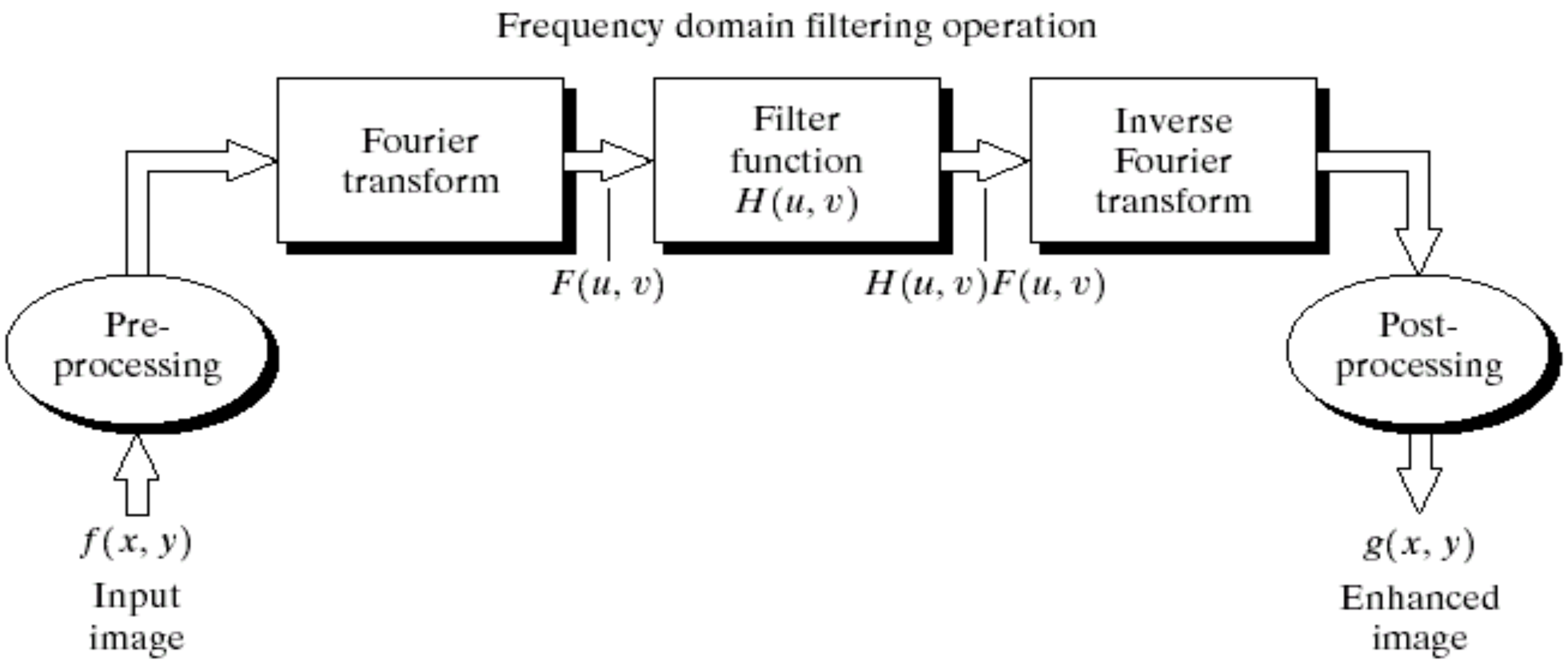


Image: Gonzalez & Woods, Digital Image Processing (2002)

Frequency domain filters

Frequency domain filters are ***mainly*** divided into four different types

- ❑Smoothing domain filters or low pass filters
- ❑Sharpening domain filters or high pass filters
- ❑Homomorphic filters
- ❑Colour image enhancement methods or techniques

Smoothing Domain Filters

A.k.a. low pass filters (LPF)

→ the low pass components of digital images are retained and the high frequency components are eliminated

Sharp transitions (abrupt change in neighbour pixel intensities) in an image are the high frequency details
→ they are removed by smoothing domain filters (low pass filters)

Ideally, an LPF would retain all the low frequency components and eliminate all the high frequency components
→ practical LPFs can't do this. Blurring and ringing are common problems in the final images.

3 types of smoothing domain filters:
Ideal LPF (**ILPF**), Butterworth LPF (**BLPF**), Gaussian LPF (**GLPF**)

Feature Extraction and Statistical Measurement

- ❑Selection of Features
- ❑Shape Related Features
- ❑Fourier Descriptors
- ❑Texture Features
- ❑Example: Breast Tissue Detection
- ❑Analysis of Tissue Structure

Feature Extraction and Statistical Methods

Enhanced images can be evaluated visually.
→ Visual inspection, however, is not enough for comparing the images with numerous changes.

Some features or parameters must be extracted from the images to compare them statistically.
→ E.g., it is very difficult to understand and interpret the mammography images due to small differences in the image intensity and densities of different tissues present in the images.

Feature extraction of statistical parameters is a very important process for the overall system performance in the pattern recognition and other image processing applications.

Selection of Features

Qualitative and quantitative features:
→They allow evaluating the image.

Some of the significant qualitative and quantitative parameters as well as some image features that can be used in the evaluation of performance of results of CAD systems.

- ❑ Signal-to-Noise Ratio (SNR)
- ❑ Peak Signal-to-Noise Ratio (PSNR)
- ❑ Mean Square Error (MSE)
- ❑ Root Mean Square Error (RMSE)
- ❑ Mean Absolute Error (MAE)
- ❑ Entropy

Signal-to-Noise Ratio (SNR)

Defined as the ratio of the signal power to the noise power.

The effect of noise can be quantified → quantitative assessment

Also gives quality assessment → since it is known that how much noise or degradation is mixed in the original image.

It can be expressed in decibel (dB).

A ratio higher than 1:1 indicates more signal than the noise.

The greater the S/N ratio, better is the signal strength or smaller is the noise level.

$$\mathrm{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}}$$

$$(\mathrm{SNR})_{\text{db}} = 10\log_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right)$$

Peak Signal-to-Noise Ratio (PSNR)

Defined as the ratio of the reference signal to the distortion signal in an image, given in decibels.

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{R^2}{\mathrm{MSE}}\right)$$

Used mostly for the comparison of image compression quality between the original image and the compressed image.

R is the maximum input image data type. For an eight-bit unsigned data type (grayscale image), R is 255; for double-precision floating-point data type, R is 1.

The greater the value of PSNR, better is the quality of the compressed or reconstructed image.

Mean Square Error (MSE)

It is calculated as a cumulative squared difference between the two images, that are generally an original image and a distorted image.

The computation of MSE is performed on pixel-by-pixel basis by summing up the squared differences of all the pixels and dividing by the total pixel count.

$$\mathrm{MSE}(P, Q) = \frac{1}{M}\sum_{i=1}^{M}(p_i - q_i)^2$$

(P and Q are the two images between which MSE is to be calculated. M is the number of pixels.)

MSE measures the average of the square of the error which is the amount by which the value differs.

It is an error metric that is used to compare the image compression quality.

Root Mean Square Error (RMSE)

A.k.a. root-mean-square deviation (RMSD).

It is used to measure of the difference between the predicted values and the actual values which are observed. This parameter is a good measure of accuracy.

$$\mathrm{RMSE} = \sqrt{\mathrm{MSE}}$$

Mean Absolute Error (MAE)

Defined as the average value of absolute errors which is calculated over the sample.

MAE is a linear score, measured as a mean value of all the individual differences which are weighted equally in the average

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n} | f_i - y_i |$$

f_i is the predicted value,
y_i is the true value
n is number of samples

Entropy

Entropy measures the amount of disorder or unpredictability of uncertain events.

This is defined as a measure of the uncertainty associated with a random variable and calculated as an expected outcome value.

Entropy is used to quantify the minimum descriptive complexity of a random variable

$$E(I) = -\sum_{k=0}^{L-1} p(k)\, log_2\,(p(k))$$

(L= 256 for grey scale)

(p(k) normalized histogram counts)

Entropy serves as a measure of 'disorder'. As the entropy increases the events become less predictable.

A list of shape-based parameters

- Distance
- Perimeter
- Convex Perimeter
- Major and Minor Axes
- Center of Gravity
- Axis of Inertia
- Aspect Ratio
- Eccentricity
- Circularity Ratio
- Rectangularity
- Convexity
- Solidity

Shape Representation

An efficient object representation or shape representation can be achieved by using boundary-based shape descriptors.

Shape signatures are a set of one-dimensional functions that form these descriptors.

The degree of similarity between the objects or images helps to identify objects.

Shape descriptors capture the perceptual features of the shape such as complex coordinates, centroid distance function, tangent angle or turning angles, curvature function, area function, triangle representation and chord length function, etc.

Shape Descriptors

- Shape descriptors describe specific characteristics regarding the geometry of a particular feature
- In general, ***shape descriptors*** or ***shape features*** are some set of numbers that are produced to describe a given shape

Shape Representation

Some of the share descriptors (or signatures)

- ❑Complex coordinates
- ❑Centroid Distance Function
- ❑Geometry of Curves
- ❑Area Function
- ❑Triangle Area
- ❑Chord Length
- ❑Polygon Area

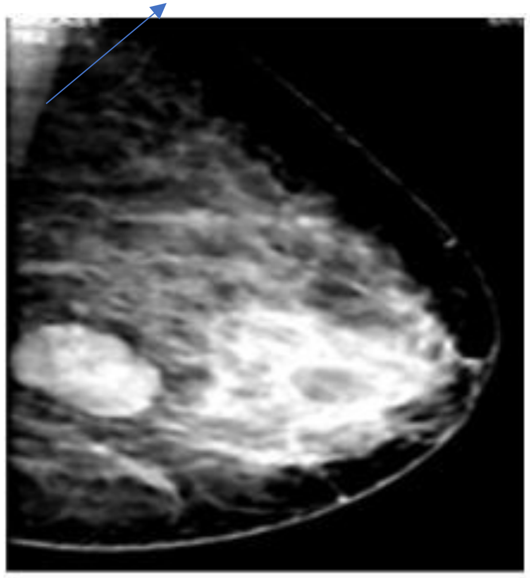
Summary of Shape Features

We must select a number of features as per the requirement and application type.

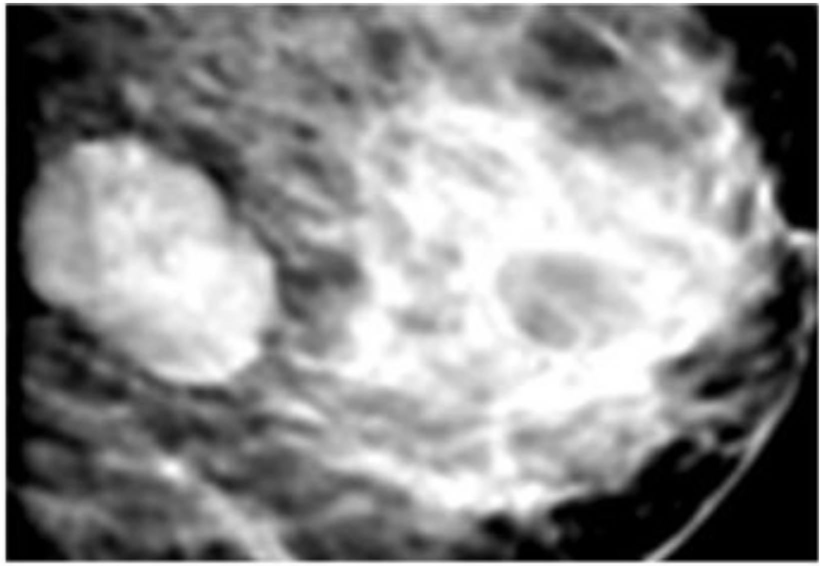
The selection of features is made to classify the cells.

May require magnifying the cells or tissues so that a proper interpretation is given.

A breast image and its result in which the cells are magnified.

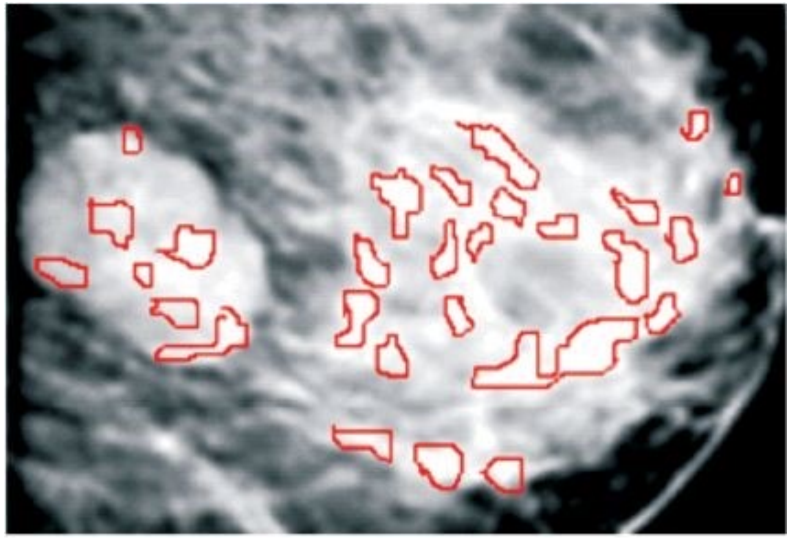


(a)



(b)

Segmented cells of different shapes such as visibly round shapes and irregular elongated shape. We can separate the cells or tissue into two different classes and count their population.



Snakes Boundary Detection

A boundary detection method, first introduced by Kass in 1988.

Snake is basically a method of modelling a closed contour to the boundary of an object in an image.

A shape is fixed and made flexible in terms of the parameters defining the shape.

The active contours are called *snakes* that are taken into consideration for detection and feature extraction.

These contours are the set of points that aim to enclose a target feature, the feature to be extracted.

Python Example: Snake Boundary Model

```
from skimage.segmentation import active_contour
img = data.astronaut()
img = rgb2gray(img)
#create data for circular boundary
s = np.linspace(0, 2*np.pi, 400)
x = 220 + 100*np.cos(s)
y = 100 + 100*np.sin(s)
init = np.array([x, y]).T
#apply gaussian filter & find active contours
cntr = active_contour(gaussian(img, 3),init,
alpha=0.015, beta=10, gamma=0.001)
```

https://tirthajyoti.github.io/Scikit-image-book/Active_contour_model.html

Texture Analysis

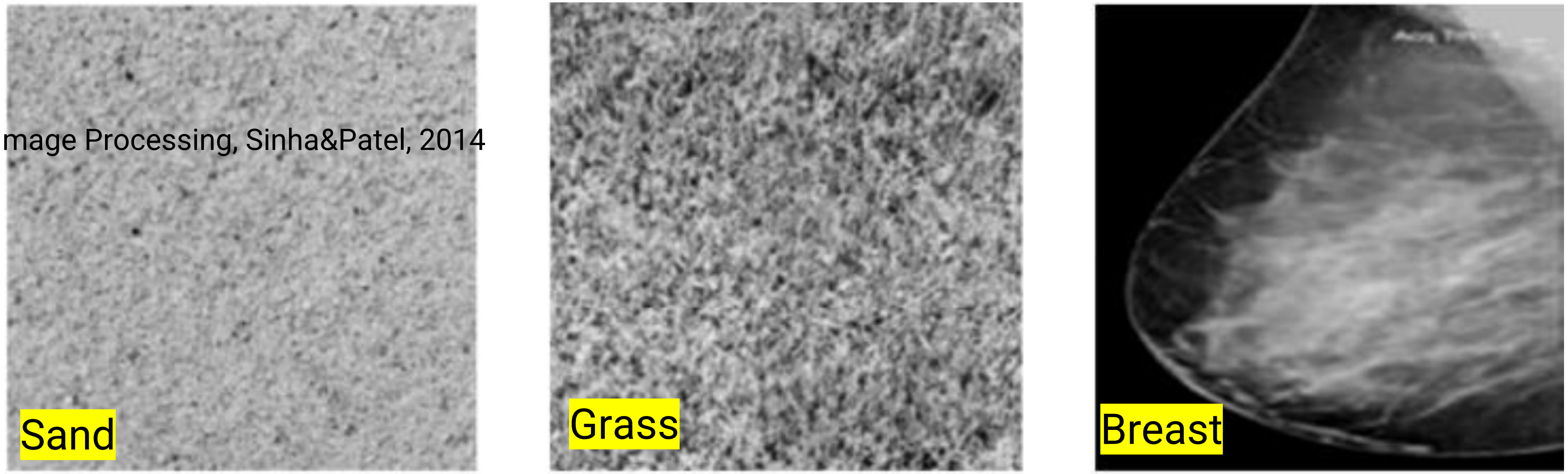
Texture: Regular repetition of particular patterns or structures obtained in an image.

Image textures may be complex patterns also of different brightness, colour, size and shape.

Texture property can help in the image classification and segmentation.

Measuring image texture depends on the size or shape of an object.

Medical Image Processing, Sinha&Patel, 2014



(a) (b) (c)

Texture Analysis

Measuring image texture depends on the size or shape of an object.

Texture analysis is made in following ways:

1. Structural analysis
2. Statistical analysis
3. Model-based texture analysis
4. Transform methods for analysis of texture

Application Example: Mammography

Based on the experimental results, mammograms can be grouped under three categories

- ❑ Fatty
- ❑ Glandular
- ❑ Dense

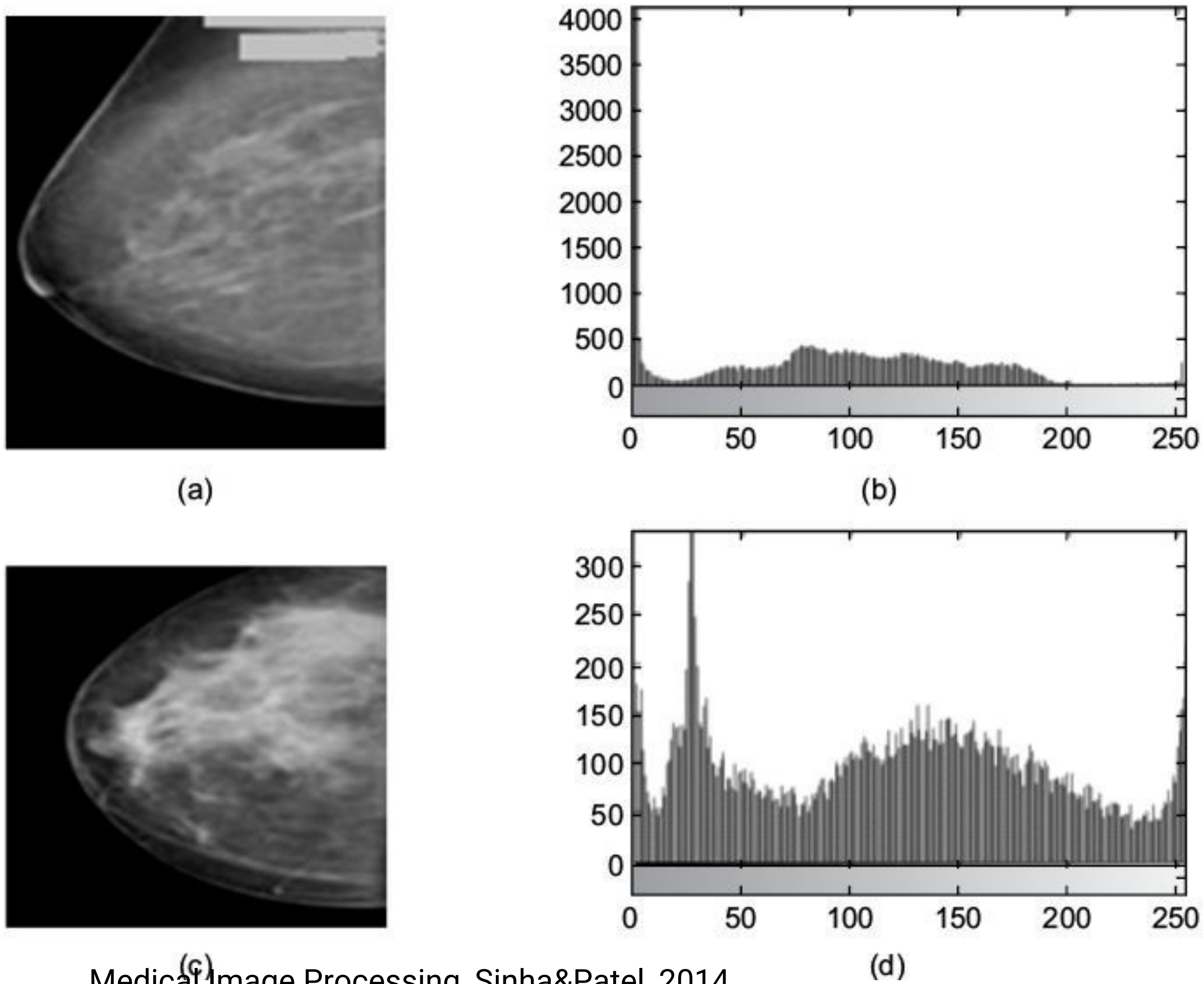
These are further classified based on some statistical features into four classes

- ❑ uncompressed fatty
- ❑ fatty, non-uniform
- ❑ high density

This classification greatly help a radiologist in determining the affected breast anatomy and detection of changes in the breast tissues.

Mammograms and their histograms

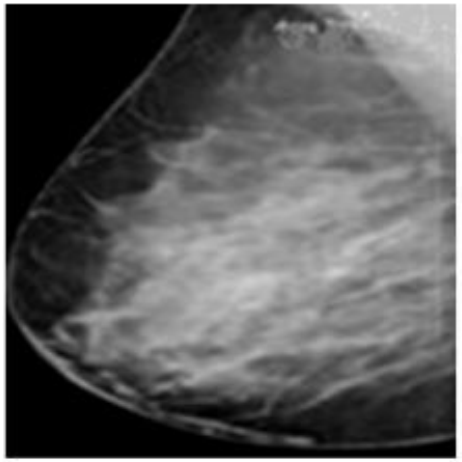
(a) Uncompressed fatty breast
(b) its histogram
(c) Fatty breast
(d) its histogram



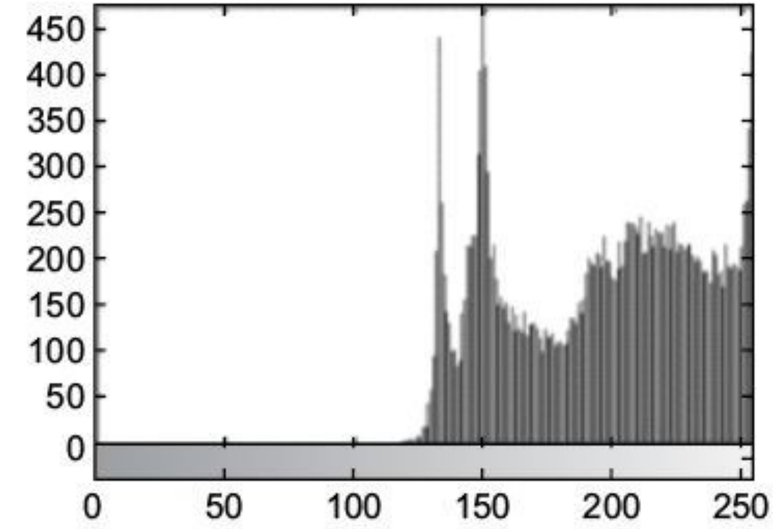
Medical Image Processing, Sinha&Patel, 2014

Mammograms and their histograms

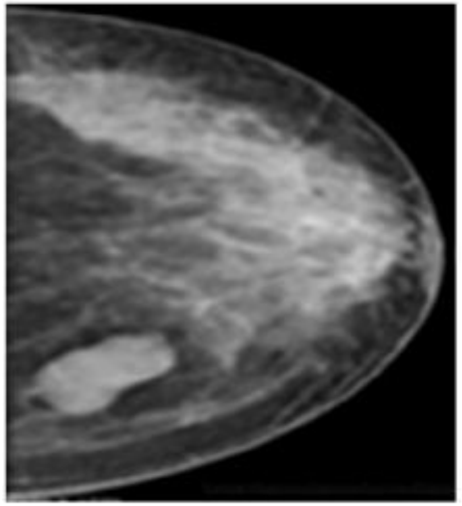
(e) Non-uniform fatty breast
(f) its histogram
(g) High density breast
(h) its histogram,



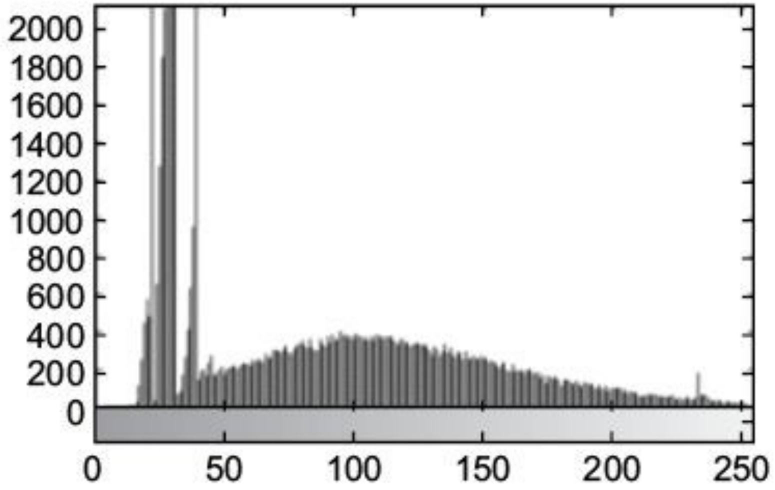
(e)



(f)



(g)



(h)

Medical Image Processing, Sinha&Patel, 2014

Image Restoration: Noise Removal

Image Processing Components

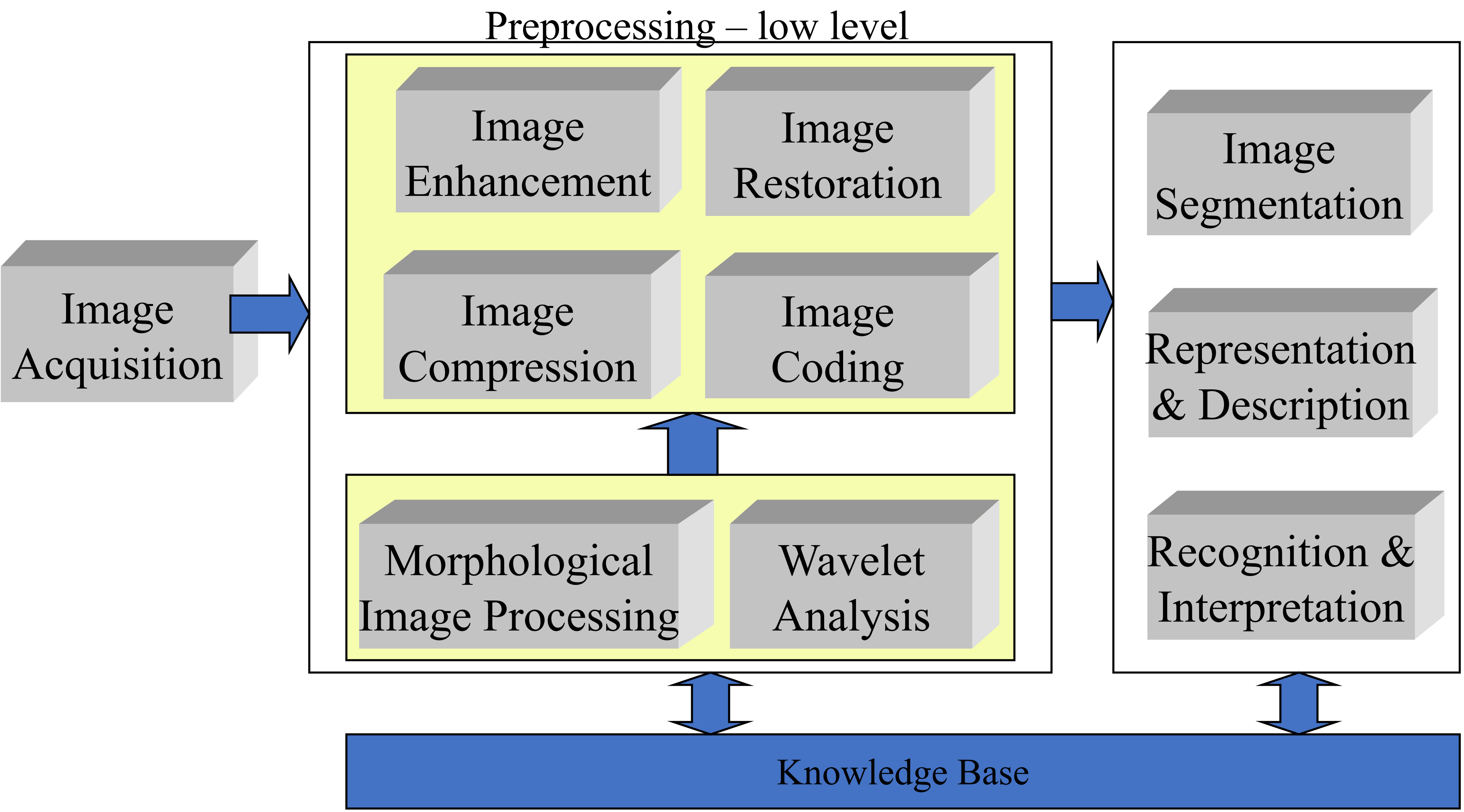


Image Restoration

- Image restoration vs. image enhancement
  - Enhancement:
    - largely a subjective process
    - Priori knowledge about the degradation is not a must (sometimes no degradation is involved)
    - Procedures are heuristic and take advantage of the psychophysical aspects of human visual system
  - Restoration:
    - more an objective process
    - Images are degraded
    - Tries to recover the images by using the knowledge about the degradation

What is Image Restoration?

- Removing noise called **Image Restoration**

- Image restoration can be done in:

  a. Spatial domain, or

  b. Frequency domain

A Model of the Image Degradation/Restoration Process

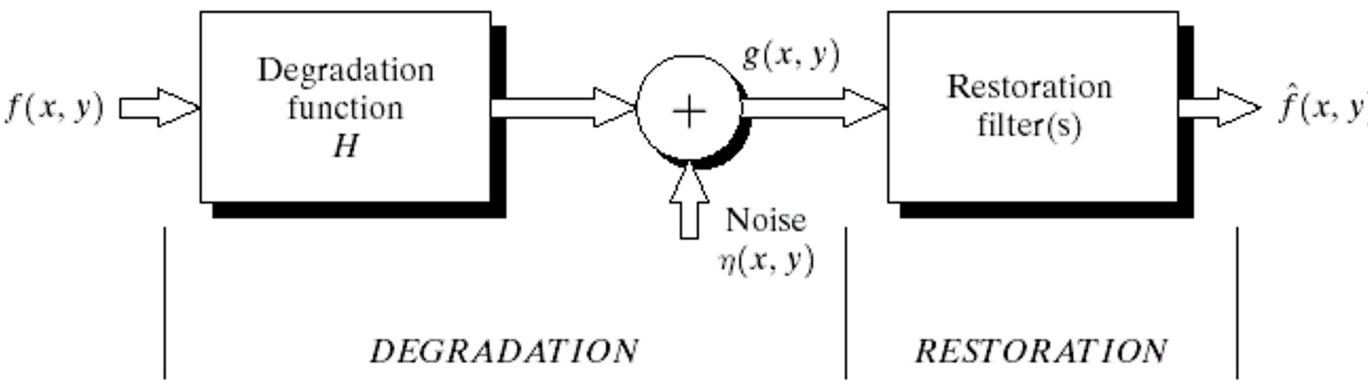


FIGURE 5.1 A model of the image degradation/restoration process.

Gonzalez & Woods, Digital Image Processing (2002)

Noise sources

✶Image acquisition

✶Image transmission

Solving the problem

- ✶ Model the degradation
- ✶ Apply the inverse process to recover the original image

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

$$f(x, y) = H^{-1}[g(x, y) - \eta(x, y)]$$

Noise models

✶Spatially independent noise models

- Gaussian noise
- Rayleigh noise
- Erlang (Gamma) noise
- Exponential noise
- Impulse (salt-and-pepper) noise

✶Spatially dependent noise model

- Periodic noise

The test pattern

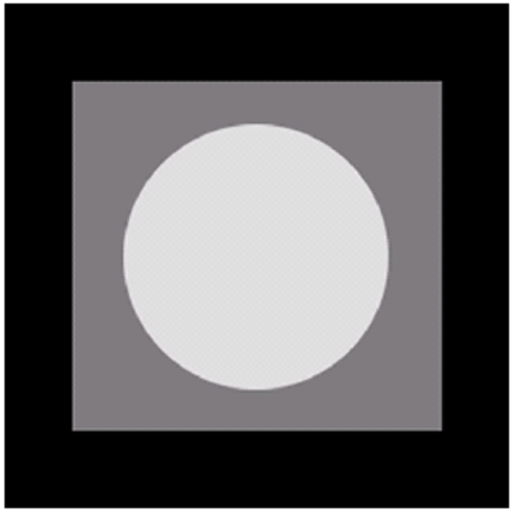


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

How to estimate noise parameters?

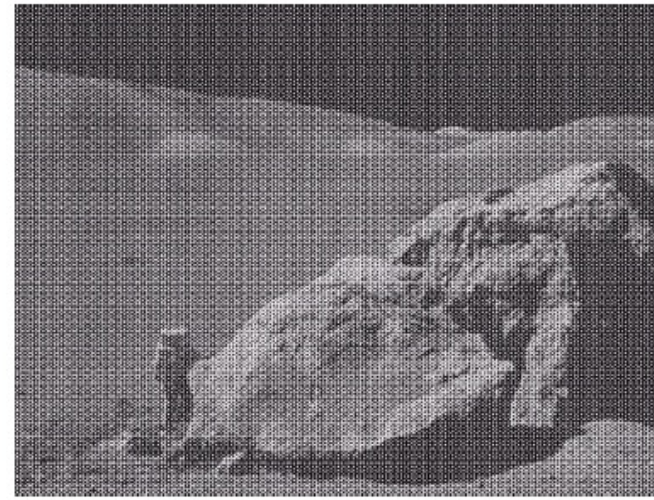
- Periodic noise
  - Analyze the FT
- Noise PDFs
  - Use “flat” images if we can acquire them
  - What if it’s salt-and-pepper noise?
  - Use small strips of uniform intensity if we only have the images but not the acquisition system

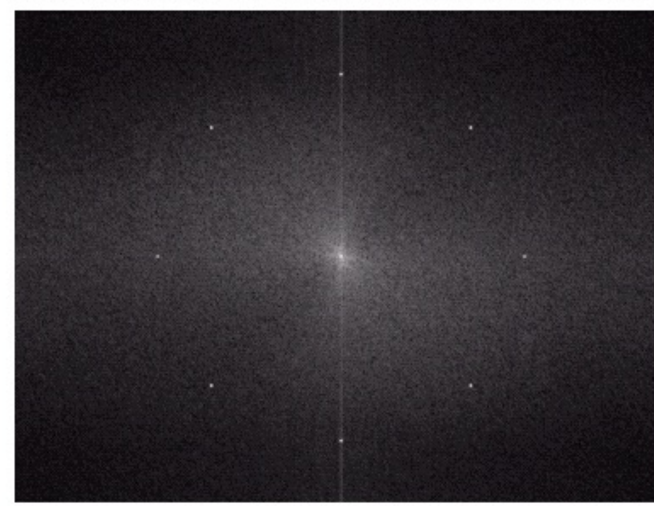
Restoration from noise

- Spatial
  - $g(x,y) = f(x,y) + \eta(x,y)$
- Frequency
  - $G(u,v) = F(u,v) + N(u,v)$

Periodic Noise

- Typically arises due to electrical or electromagnetic interference
- Gives rise to regular noise patterns in an image
- Frequency domain techniques in the Fourier domain are most effective at removing periodic noise



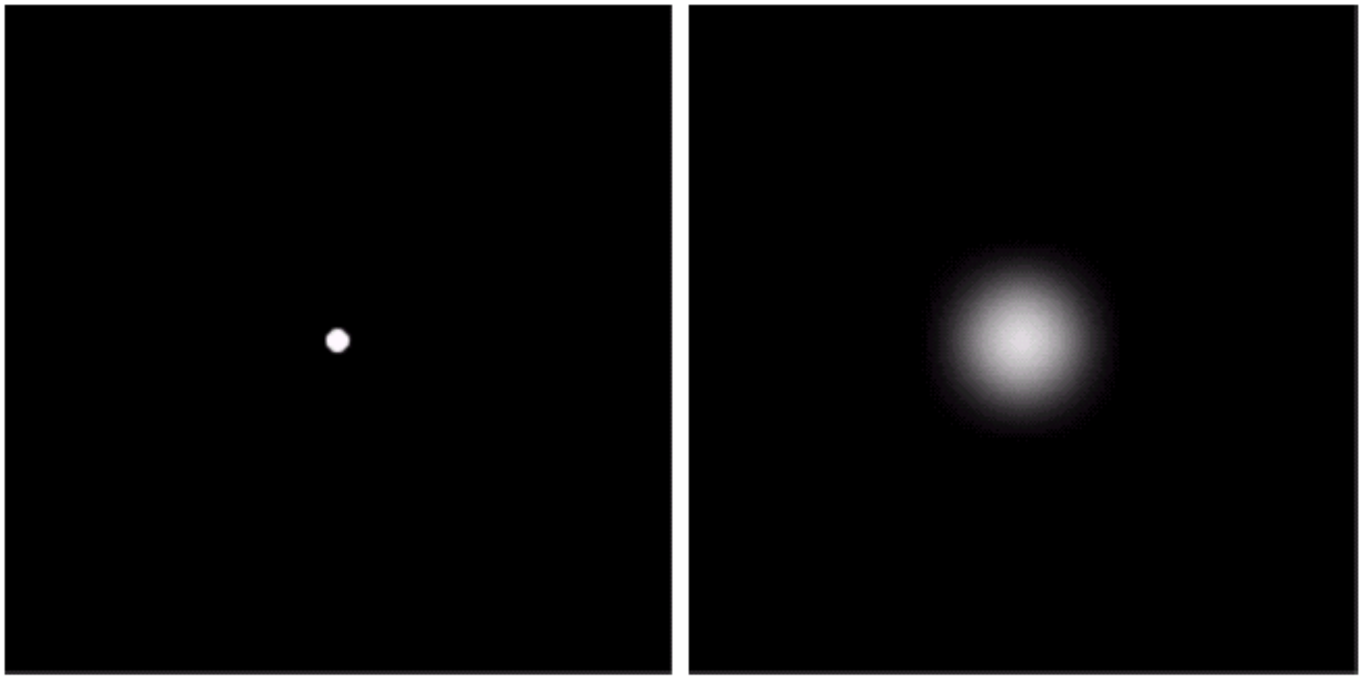


Gonzalez & Woods, Digital Image Processing (2002)

Estimation of Noise Parameters

- Periodic noise
  - Parameters can be estimated by inspection of the spectrum
- Noise PDFs
  - From sensor specifications
  - If imaging sensors are available, capture a set of images of plain environments
  - If only noisy images are available, parameters of the PDF involved can be estimated from small patches of constant regions of the noisy images

Degradation function by experiment



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

by modelling

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

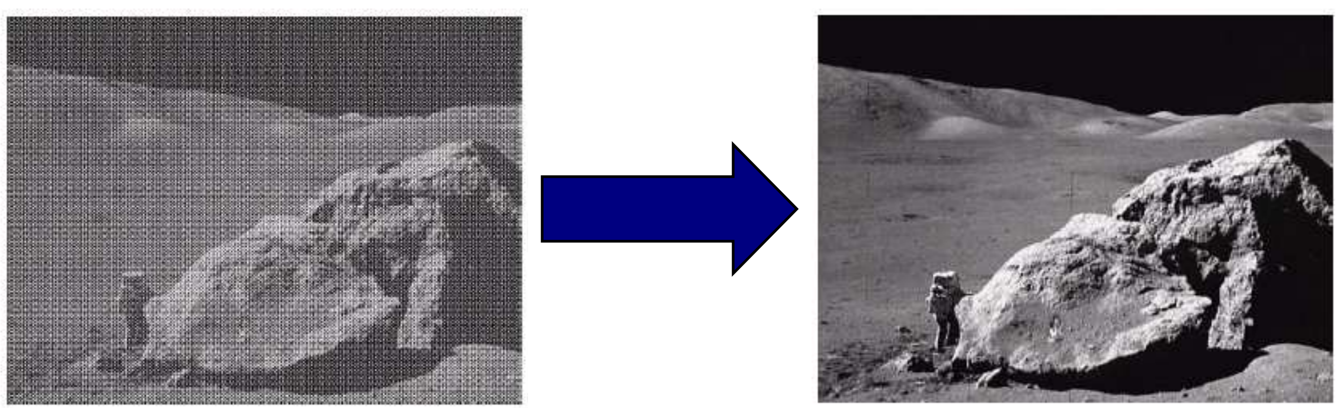
$H(u,v) = \exp(-k(u^2+v^2)^{5/6})$

atmosferic turbulence model

What is Image Restoration?

Image restoration attempts to restore images that have been degraded

- Identify the degradation process and attempt to reverse it
- Similar to image enhancement, but more objective



Gonzalez & Woods, Digital Image Processing (2002)

An Image Degradation Model

- Two types of degradation
  - Additive noise
    - Spatial domain restoration (denoising) techniques are preferred
  - Image blur
    - Frequency domain methods are preferred
- We model the degradation process by a degradation function h(x,y), an additive noise term, $\eta(x,y)$, as $g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$
  - f(x,y) is the (input) image free from any degradation
  - g(x,y) is the degraded image
  - $*$ is the convolution operator
  - The goal is to obtain an estimate of f(x,y) according to the knowledge about the degradation function h and the additive noise η
  - In frequency domain: $G(u,v)=H(u,v)F(u,v)+N(u,v)$
- Three cases are considered
  - $g(x,y)=f(x,y)+\eta(x,y)$
  - $g(x,y)=h(x,y)*f(x,y)$
  - $g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$

Different approaches

- ✶ Noise
  - Noise models and denoising Blur (linear, position-invariant degradations)
  - Estimate the degradation and inverse filters

Noise models

✶Spatially independent noise models

- Gaussian noise
- Rayleigh noise
- Erlang (Gamma) noise
- Exponential noise
- Impulse (salt-and-pepper) noise

✶Spatially dependent noise model

- Periodic noise

The test pattern

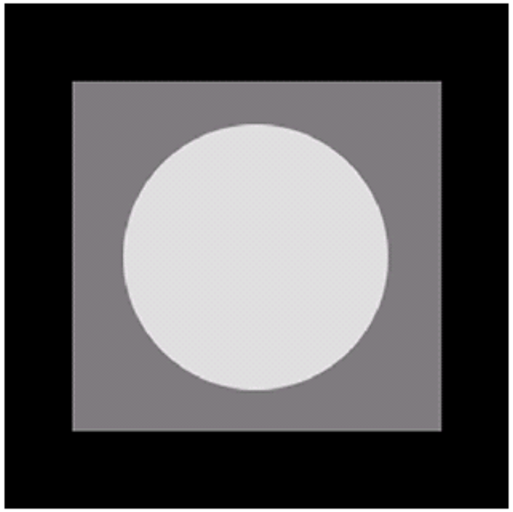


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

How to estimate noise parameters?

- Periodic noise
  - Analyze the FT
- Noise PDFs
  - Use “flat” images if we can acquire them
  - What if it’s salt-and-pepper noise?
  - Use small strips of uniform intensity if we only have the images but not the acquisition system

Evaluating the noise level

- (Root) Mean Square Error (MSE)
  - $E\{||g(x,y) - f(x,y)||^2\}$ or
  - $E\{||(g(x,y)-\underline{g(x,y)}) - (f(x,y)-\underline{f(x,y)})||^2\}$
- Peak Signal to Noise Ratio (PSNR)

  $10\log_{10}[(L-1)/sqrt(MSE)](dB)$

Noise Model

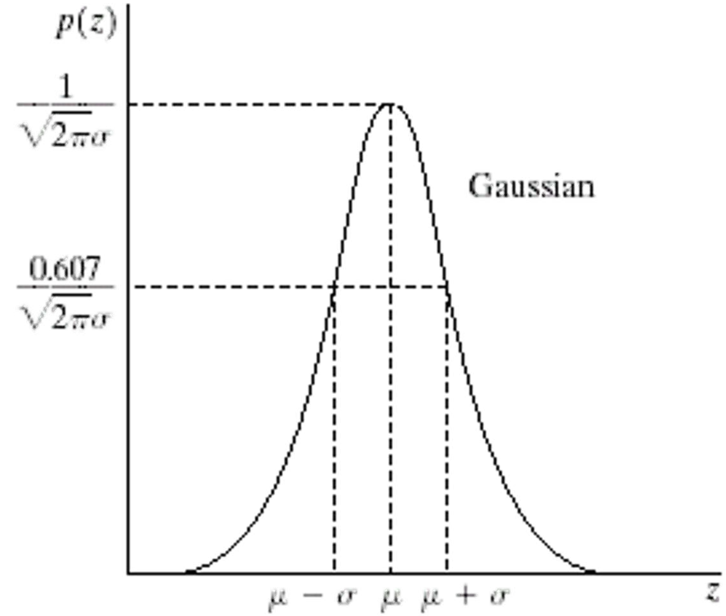
- We first consider the degradation due to noise only
  - h is an impulse for now (H is a constant)
- White noise
  - Autocorrelation function is an impulse function multiplied by a constant
    - $$a(x,y)=\sum_{t=0}^{N-1}\sum_{s=0}^{M-1}\eta(s,t)\cdot\eta(s-x,t-y)=N_0\delta(x,y)$$
    - It means there is no correlation between any two pixels in the noise image
    - There is no way to predict the next noise value
  - The spectrum of the autocorrelation function is a constant

Gaussain Noise

- Noise (image) can be classified according the distribution of the values of pixels (of the noise image) or its (normalized) histogram
- Gaussian noise is characterized by two parameters, μ (mean) and σ^2 (variance), by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

- 70% values of z fall in the range $[(\mu-\sigma),(\mu+\sigma)]$
- 95% values of z fall in the range $[(\mu-2\sigma),(\mu+2\sigma)]$



Gonzalez & Woods, Digital Image Processing (2002)

Noise and Images

The sources of noise in digital images arise during image acquisition (digitization) and transmission

- Imaging sensors can be affected by ambient conditions
- Interference can be added to an image during transmission

Noise Model

We can consider a noisy image to be modelled as follows:

$$g(x, y) = f(x, y) + \eta(x, y)$$

where $f(x, y)$ is the original image pixel, $\eta(x, y)$ is the noise term and $g(x, y)$ is the resulting noisy pixel

If we can estimate the model that the noise in an image is based on, this will help us to figure out how to restore the image

Types of Noise

- **Type of noise determines best types of filters for removing it.**
- **Salt and pepper noise:** Randomly scattered black + white pixels
- Also called **impulse noise, shot noise or binary noise**
- Caused by sudden sharp disturbance

(a) Original image

(b) With added salt & pepper noise

Gonzalez & Woods, Digital Image Processing (2002)

Types of Noise

- **Gaussian Noise:** idealized form of white noise *added to* image, normally distributed

  I + Noise

- **Speckle Noise:** pixel values *multiplied* by random noise   I (1 + Noise)

(a) Gaussian noise

(b) Speckle noise

Types of Noise

- **Periodic Noise:** caused by disturbances of a periodic Nature
- Salt and pepper, Gaussian and speckle noise can be cleaned using spatial filters
- Periodic noise can be cleaned

Using frequency domain filtering

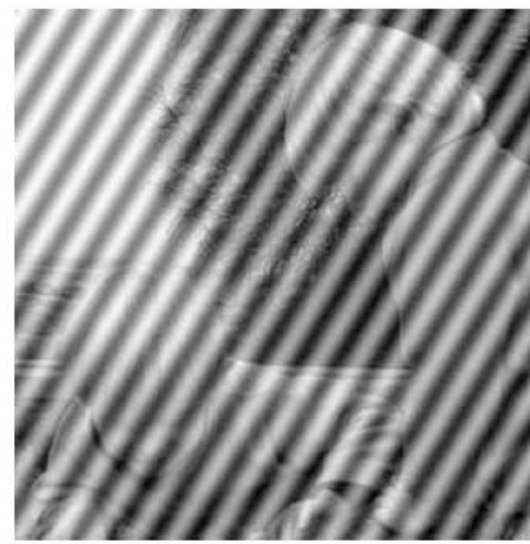


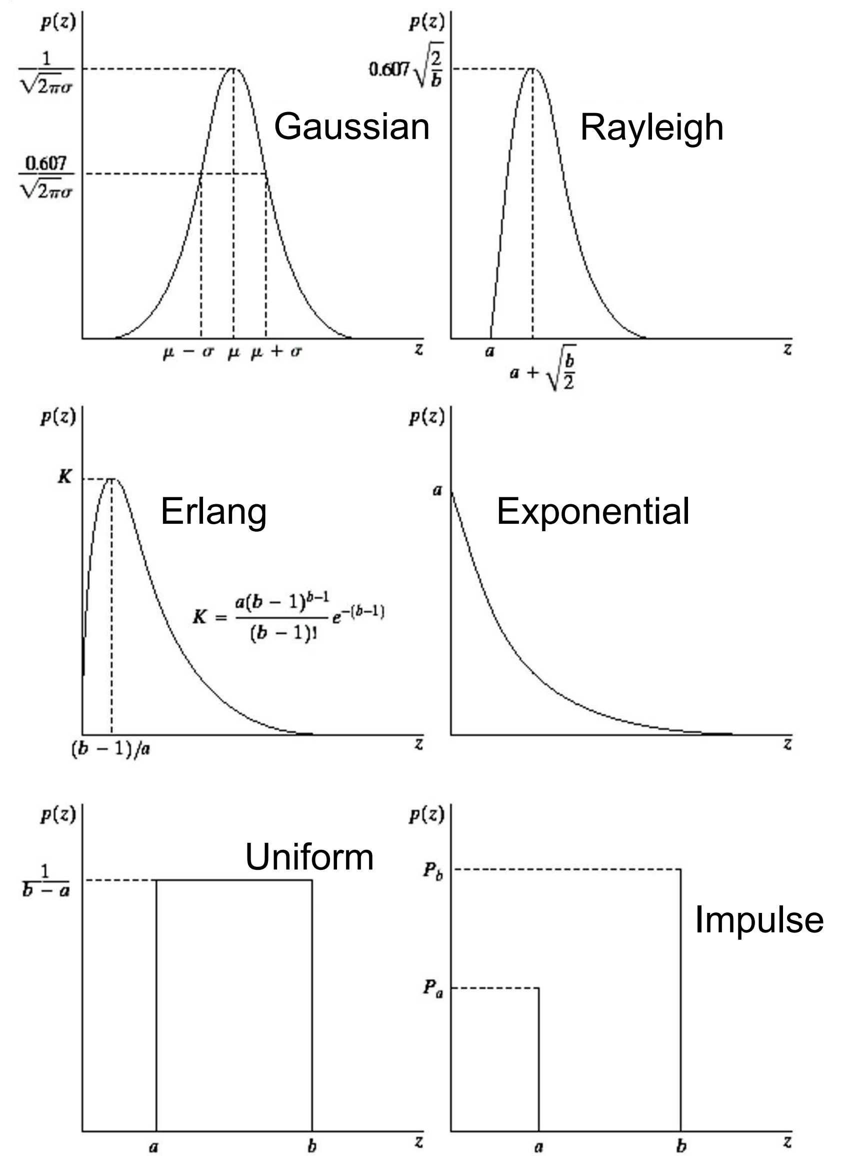
Figure 5.3: The twins image corrupted by periodic noise

Noise Models

There are many different models for the image

noise term $\eta(x, y)$:

- Gaussian
  - Most common model
- Rayleigh
- Erlang
- Exponential
- Uniform
- Impulse
  - *Salt and pepper* noise

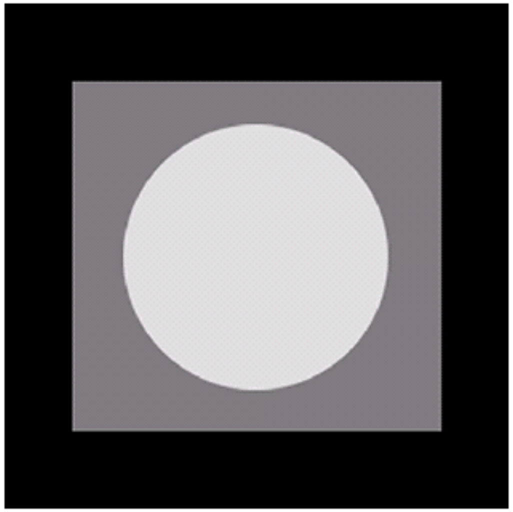


Gonzalez & Woods, Digital Image Processing (2002)

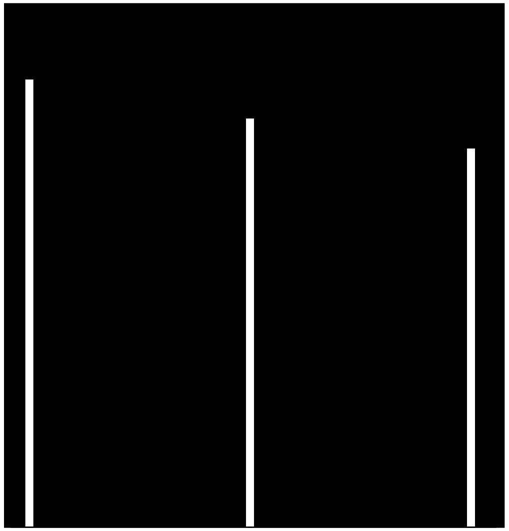
Noise Example

The test pattern to the right is ideal for demonstrating the addition of noise

The following slides will show the result of adding noise based on various models to this image



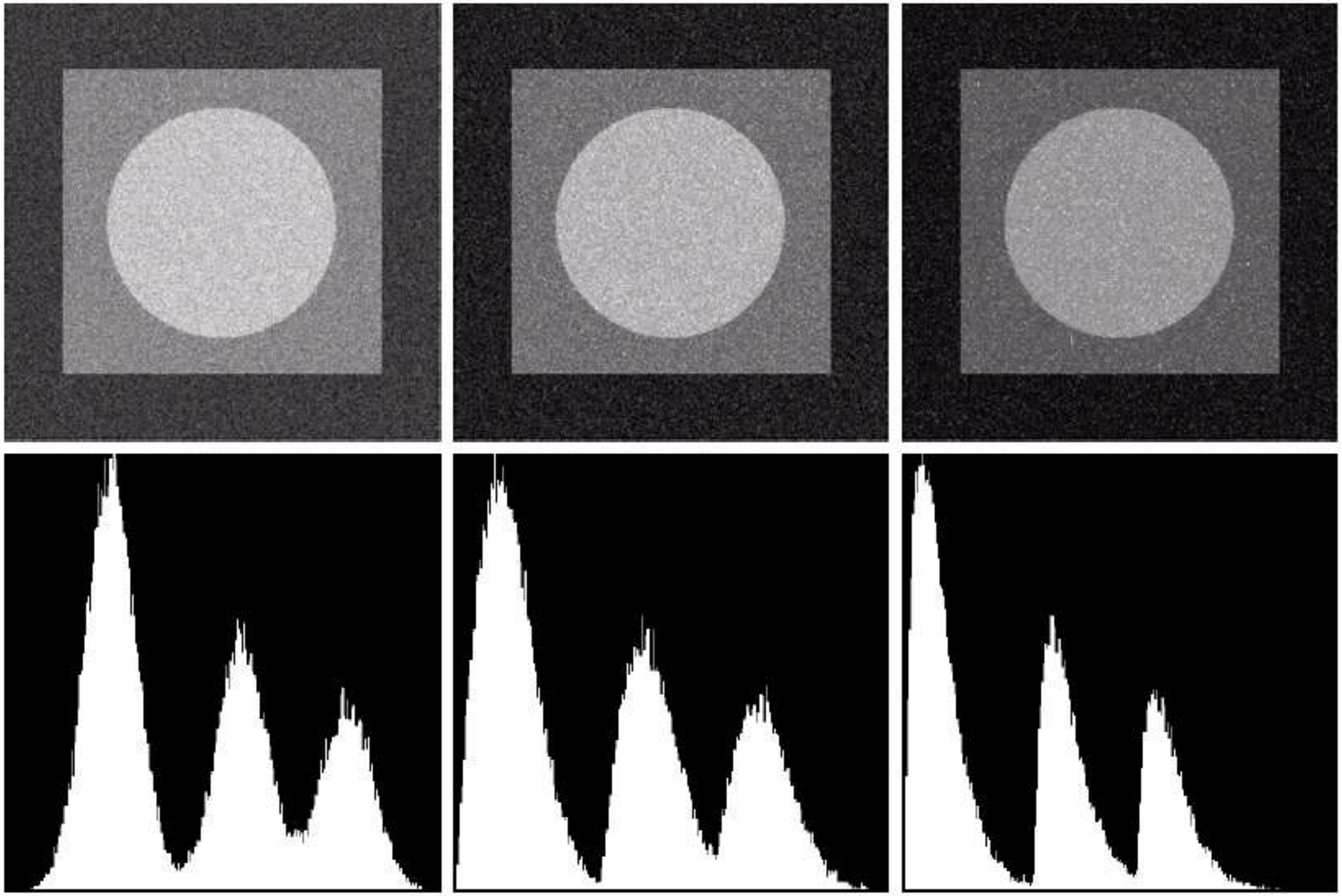
Image



Histogram

Gonzalez & Woods, Digital Image Processing (2002)

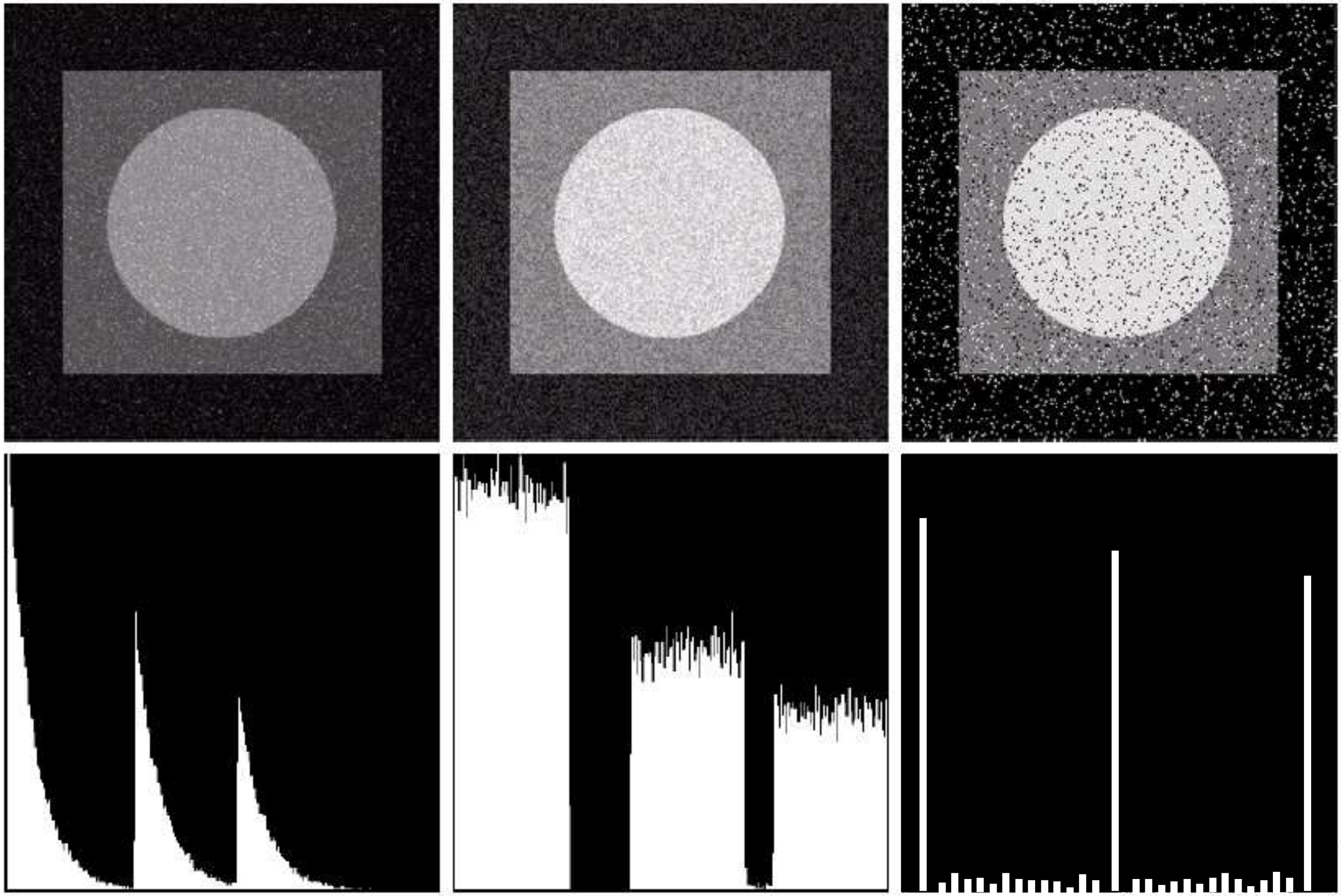
Noise Example (cont…)



Gaussian Rayleigh Erlang

Gonzalez & Woods, Digital Image Processing (2002)

Noise Example (cont…)



Exponential Uniform Impulse

Gonzalez & Woods, Digital Image Processing (2002)

Filtering to Remove Noise

We can use spatial filters of different kinds to remove different kinds of noise

The *arithmetic mean* filter is a very simple one and is calculated as follows:

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

This is implemented as the simple smoothing filter

Blurs the image to remove noise

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

Other Means

There are different kinds of mean filters all of which exhibit slightly different behaviour:

- Geometric Mean
- Harmonic Mean
- Contraharmonic Mean

Other Means (cont…)

There are other variants on the mean which can give different performance

Geometric Mean:

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail

5	16	22
6	3	18
12	3	15

Gives the result of: (5*16*22*6*3*18*12*3*15)^(1/9) = 8.77.

https://en.wikipedia.org/wiki/Geometric_mean_filter

Other Means (cont…)

Harmonic Mean:

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise

Also does well for other kinds of noise such as Gaussian noise

5	16	22
6	3	18
12	3	15

$$\frac{9}{\left(\frac{1}{5}+\frac{1}{16}+\frac{1}{22}+\frac{1}{6}+\frac{1}{3}+\frac{1}{18}+\frac{1}{12}+\frac{1}{3}+\frac{1}{15}\right)} = 7$$

Other Means (cont…)

Contraharmonic Mean:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q is the *order* of the filter and adjusting its value changes the filter's behaviour

Positive values of Q eliminate pepper noise

Negative values of Q eliminate salt noise

5	16	22
6	3	18
12	3	15

Q=1

(5*5+16*16+22*22+6*6+3*3+18*18+12*12+3*3+15*15)/(5+16+22+6+3+18+12+3+15)

Order Statistics Filters

Spatial filters that are based on ordering the pixel values that make up the neighbourhood operated on by the filter

Useful spatial filters include

- Median filter
- Max and min filter
- Midpoint filter
- Alpha trimmed mean filter

Median Filter

Median Filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$$

Excellent at noise removal, without the smoothing effects that can occur with other smoothing filters

Particularly good when salt and pepper noise is present

The pixels are sorted in ascending order and the middle pixel is chosen

5	16	22
6	3	18
12	3	15

(3,3,5,6,12,15,16,18,22) →12

Max and Min Filter

Max Filter:

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Min Filter:

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

Max filter is good for pepper noise and min is good for salt noise

5	16	22
6	3	18
12	3	15

Max Filter → 22

Min Filter → 3

Midpoint Filter

Midpoint Filter:

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

Good for random Gaussian and uniform noise

5	16	22
6	3	18
12	3	15

Max → 22

Min → 3
Midpoint → 12 (rounded)

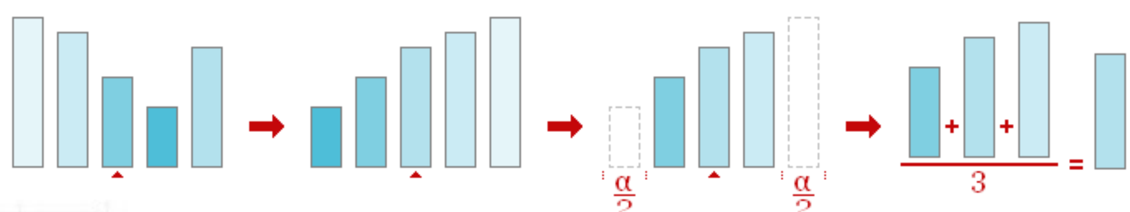
Alpha-Trimmed Mean Filter

Alpha-Trimmed Mean Filter:

$$\hat{f}(x,y) = \frac{1}{mn - \alpha} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

We can delete the $\alpha/2$ lowest and $\alpha/2$ highest grey levels

So $g_r(s, t)$ represents the remaining $mn - \alpha$ pixels



5	16	22
6	3	18
12	3	15

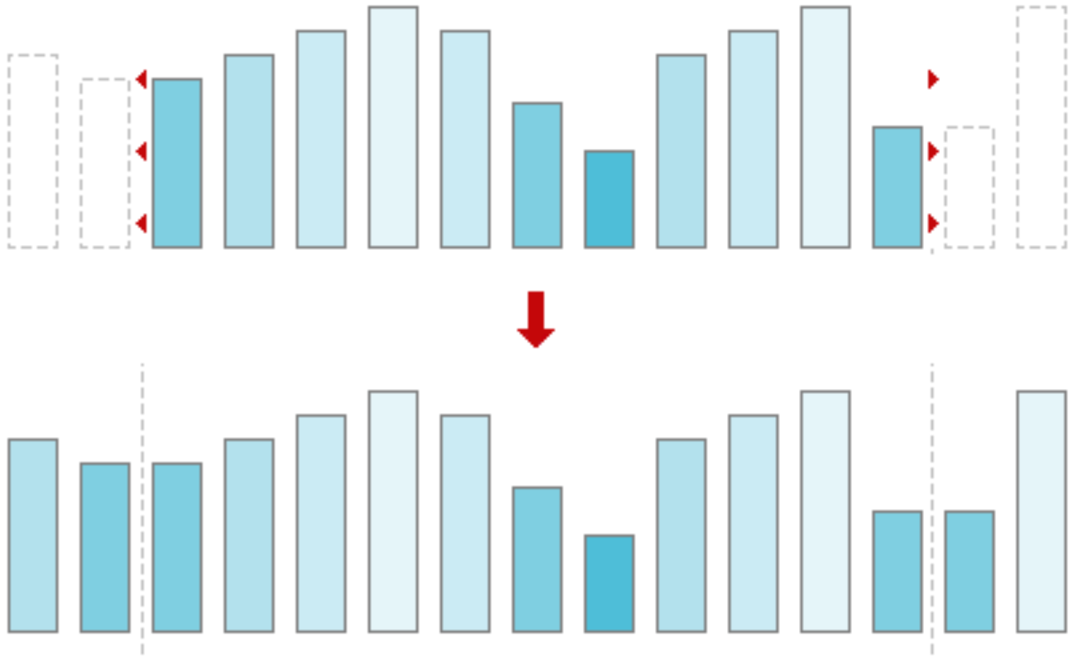
$\alpha = 2$

~~3~~	3	5	6	12	15	16	18	~~22~~

Output = (3+5+6+12+15+16+18)/7= 11

http://www.librow.com/articles/article-7

Alpha-Trimmed Mean Filter



Padding (symmetrical)

http://www.librow.com/articles/article-7

Summary

- ✶Type of noise
  - Spatial invariant
    - SAP
    - Gaussian
  - Periodic noise
- ✶How to identify the type of noise?
  - Test pattern
  - Histogram
- ✶How to evaluate noise level?
  - RMSE
  - PSNR
- ✶Noise removal
  - Spatial domain
    - Mean filters
    - Order-statistics filters
  - Frequency domain
    - Band-pass
    - Band-reject
    - Notch filters
    - Optimal notch filter