9.5

1. $\cancel{3}y'' + y' = 1,\ \cancel{x} > 0$ diferansiyel denkleminin genel çözümünü bulunuz.

x x

$x^2y'' + xy' = x$ Euler

$x = e^t$

$y' = e^{-t}Dy$

$y'' = e^{-2t}D(D-1)y$

$\underbrace{e^{2t}\cdot e^{-2t}}_{1} D(D-1)y + \underbrace{e^{t}e^{-t}}_{1}Dy = e^t$

$(D^2 - \cancel{D} + \cancel{D})y = e^t$ $y'' = e^t$

$y' = e^t + c_1 \Rightarrow y = e^t + c_1 t + c_2$

$\boxed{y = x + c_1 \ln x + c_2}$

9.5 using variation of paramet.

2. $y'' - 3y' + 2y = e^t$ ~~diferansiyel denkleminin~~ genel çözümünü ~~sabitlerin değişimi yöntemi ile bulunuz.~~

$r^2 - 3r + 2 = 0 \Rightarrow r_1 = 2,\ r_2 = 1$

$-2\ -1$

$\Rightarrow y_h = c_1 e^{2t} + c_2 e^t$

$$\left.\begin{aligned} c_1' e^{2t} + \cancel{c_2' e^t} &= 0 \\ -2c_1' e^{2t} + \cancel{c_2' e^t} &= e^t \end{aligned}\right\}$$

$-c_1' e^{2t} = -e^t \Rightarrow c_1' = e^{-t}$

$\boxed{c_1 = -e^{-t} + k_1}$

$c_2' = -c_1' e^t = -1 \Rightarrow c_2 = -t + k_2$

$y_g = -e^t + k_1 e^{2t} - te^t + k_2 e^t$

solve the IVP.

3. $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$ ~~diferansiyel denklemini~~

$r^2 - r - 6 = 0 \Rightarrow r_1 = 3,\ r_2 = -2$

$-3\ +2$

$y_g = y_h = c_1 e^{3x} + c_2 e^{-2x}$

$y' = 3c_1 e^{3x} - 2c_2 e^{-2x}$

$y(0) = 1 \Rightarrow \boxed{1 = c_1 + c_2}$

$y'(0) = -1 \Rightarrow \boxed{3c_1 - 2c_2 = -1}$

$2)\ c_1 + c_2 = 1$

$3c_1 - 2c_2 = -1$

$+$

$5c_1 = 1 \Rightarrow c_1 = \frac{1}{5} \quad c_2 = \frac{4}{5}$

$$\boxed{y = \frac{e^{3x}}{5} - \frac{4e^{-2x}}{5}}$$

write yp using undet. coef. m.

$y''' + 4y' = e^x + \sin x + x$ ~~diferansiyel denkleminin~~

$r^3 + 4r = 0 \Rightarrow r_1 = 0,\ r_{2,3} = \mp 2i$

$y_h = c_1 + c_2 \cos 2x + c_3 \sin 2x$

$y_p = ke^x + A\cos x + B\sin x + (ax+b).x$

Lagrange

. Find the $\boxed{\text{parametric equations}}$ of the general solution of $y + 2xy' = (y')^3$

$y' = p \Rightarrow y + 2xp = p^3$

$\underbrace{y'}_{p} + 2p + 2xp' = 3p^2 p' \Rightarrow 3p = \frac{dp}{dx}[3p^2 - 2x]$

$\frac{dx}{dp} = \frac{3p^2 - 2x}{3p} = p - \frac{2}{3p}x \Rightarrow \frac{dx}{dp} + \frac{2}{3p}x = p$ L.D.E

$\lambda(p) = e^{\int \frac{2dp}{3p}} = e^{\frac{2}{3}\ln p} = p^{2/3}$

$x = p^{-\frac{2}{3}}\left[\int p^{2/3} \cdot p\,dp + c\right] = \frac{3}{8}p^2 + cp^{-2/3}$

$y = p^3 - \frac{3}{4}p^3 - 2cp^{1/3}$

Param. eq. of gn. sol.

2. Find the general solution and particular solution for $\boxed{f(e) = e\frac{\pi}{2}}$ of the differential equation $\left(x - y\cos\left(\frac{y}{x}\right)\right)dx + x\cos\left(\frac{y}{x}\right)dy = 0$.

$8/3$

3

8

$y = ux$
$dy = u\,dx + x\,du$

$[x - ux\cos u]dx + x\cos u[u\,dx + x\,du] = 0$

$[x \cancel{- ux\cos u} \cancel{+ ux\cos u}]dx + x^2\cos u\,du = 0$

$\int \frac{dx}{x} + \int \cos u\,du = \int 0 \Rightarrow \ln x + \sin u = \ln c$

$\ln x - \ln c = -\sin u \Rightarrow x = ce^{-\sin u}$

y_g $x = ce^{-\sin\frac{y}{x}}$

$x = e, \quad y = e\frac{\pi}{2}$

$e = ce^{-\sin\pi/2} \Rightarrow c = e^2$

$x = e^{2 - \sin\frac{y}{x}}$ y_p

Riccati

3. Solve the differential equation $y' + y^2 = \frac{y}{x} - \frac{1}{x^2}$ knowing that $\boxed{y_1(x) = \frac{1}{x}}$ is a particular solution. Solution: $y = \frac{1}{x}\left[1 + \frac{1}{c+Inx}\right]$

$y = y_1(x) + u \Rightarrow$ Bernoulli

$y = y_1(x) + \frac{1}{u} \Rightarrow$ Linear (seperable)

$$\left.\begin{array}{l} y = \frac{1}{x} + \frac{1}{u} \\ y' = -\frac{1}{x^2} - \frac{u'}{u^2} \end{array}\right\} \; -\cancel{\frac{1}{x^2}} - \frac{u'}{u^2} + \cancel{\frac{1}{x^2}} + \frac{2}{ux} + \frac{1}{u^2} = \cancel{\frac{1}{x^2}} + \frac{1}{ux} - \cancel{\frac{1}{x^2}}$$

$$-\frac{u'}{u^2} + \frac{1}{ux} + \frac{1}{u^2} = 0 \Rightarrow \boxed{u' - \frac{u}{x} = 1} \quad \text{L.D.E}$$

$$\left.\begin{array}{l} y = \frac{1}{x} + u \\ y' = -\frac{1}{x^2} + u' \end{array}\right\} \; -\cancel{\frac{1}{x^2}} + u' + \cancel{\frac{1}{x^2}} + \frac{2u}{x} + u^2 = \cancel{\frac{1}{x^2}} + \frac{u}{x} - \cancel{\frac{1}{x^2}}$$

$$\boxed{u' + \frac{u}{x} = -u^2} \quad \text{B.D.E}$$

4. Find the general solution and, if there exists, $\boxed{\text{singular solution}}$ of $y = x((y')^2 + 2y')$.

$y = x((y')^2 + 2(y'))$ $\quad y' = p \Rightarrow \boxed{y = x(p^2 + 2p)}$

$y' = p^2 + 2p + x(2pp' + 2p') \Rightarrow -p - p^2 = x\,2p'(\cancel{p+1})$

p $\quad -p(\cancel{p+1})$

$-p = 2x\frac{dp}{dx} \Rightarrow \int\frac{dx}{x} = -\int\frac{2dp}{p}$

$\ln x = -2\ln p + \ln c \Rightarrow \boxed{x = \frac{c}{p^2}}$ $\quad p = \mp\sqrt{\frac{c}{x}}$

$\boxed{y = c + \frac{2c}{p}}$

$y = c\left[1 + \frac{2}{\mp\sqrt{\frac{c}{x}}}\right]$

$\boxed{\text{G.S} \Rightarrow y_g = x(c^2 + 2c)}$

5. If the wronskian of f and g is $w = (f, g) = \underline{3e^{4t}}$ and $f(t) = e^{2t}$, then find $\underline{\underline{g(t)}}$. ?

$$w(f,g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = 3e^{4t} \qquad \begin{vmatrix} e^{2t} & g \\ 2e^{2t} & g' \end{vmatrix} = 3e^{4t}$$

$$e^{2t}[g' - 2g] = 3e^{4t} \Rightarrow g' - 2g = 3e^{2t} \quad \text{L.D.E}$$

$$\lambda(t) = e^{\int -2\,dt} = e^{-2t}$$

$$g(t) = e^{2t}\left[\int e^{-2t} \cdot 3e^{+2t}\,dt\right] = \boxed{e^{2t} \cdot 3t}$$

6. Find the general solution of $(1 - \sqrt{3})y' + y\sec x = y^{\sqrt{3}}\sec x$

Let $y_1(t)$ and $y_2(t)$ be two solutions of a second order homogeneous linear differential equation. The Wronskian determinant of the two solutions is $W(y_1(t), y_2(t)) = e^{-t}$. Then, which of the following statement is false?

$w = 0$ dep

$w \neq 0$ ind.

a) $y_1(t)$ and $y_2(t)$ are linearly dependent functions. ✗

b) The function $2y_1(t) - 3y_2(t)$ is also a solution of this differential equation. ✓

c) $y_1(t)$ and $y_2(t)$ construct a fundamental set of solutions. ✓

d) All the solutions of this differential equation can be represented as $c_1y_1(t) + c_2y_2(t)$, where c_1 and c_2 are constants . ✓

e) $W(2y_1(t), 3y_2(t)) = 6e^{-t}$ ✓

$$w(2y_1, 3y_2) = \begin{vmatrix} 2y_1 & 3y_2 \\ 2y_1' & 3y_2' \end{vmatrix} = 6\underbrace{(y_1 y_2' - y_1' y_2)}_{e^{-t}}$$

Bernoulli

The differential equation $\boxed{y' - \frac{y}{x} = \sqrt{\frac{x}{y}}}$ is transformed into a seperable differential equation using a suitable transformation. Which of the following differential equation is this new seperable differential equation ?

$x^{1/2} y^{-1/2}$

a) $u' = \frac{1}{x\sqrt{u}}$ b) $u' = \frac{1}{\sqrt{xu}}$ c) $u' = x\sqrt{u}$ d) $u' = \frac{1}{xu}$ e) $u' = \sqrt{xu}$

$y^{1-(-\frac{1}{2})} = y^{3/2} = u$

$\frac{3}{2} y^{1/2} y' = u'$

$y' y^{1/2} - \frac{y^{3/2}}{x} = \sqrt{x}$

$\frac{2u'}{3} - \frac{u}{x} = \sqrt{x}$

Not sep.

$\boxed{u' - \frac{3u}{2x} = \frac{3\sqrt{x}}{2}}$ L.D.E

$dy = \left(\frac{y}{x} + \sqrt{\frac{x}{y}}\right)dx$

$y = ux$ $dy = u\,dx + x\,du$

$u\,dx + x\,du = \left(u + \sqrt{\frac{1}{u}}\right)dx$

$\left(\cancel{u} - \cancel{u} - \sqrt{\frac{1}{u}}\right)\frac{dx}{dx} + x\frac{du}{dx} = 0 \Rightarrow -\frac{1}{\sqrt{u}} + xu' = 0$

$\boxed{u' = \frac{1}{\sqrt{u}\,x}}$ sep.

singular sol.

SORU 10. $y = xy' + Cos(y')$ ~~diferansiyel denkleminin tekil çözümü~~ aşağıdakilerden hangisidir?

a) $y = xArcsinx + \sqrt{1 - x^2}$

b) $y = xArcsinx - \sqrt{1 - x^2}$

c) $y = xArcsinx + \sqrt{1 + x^2}$

d) ~~$y = xArccosx + \sqrt{1 - x^2}$~~

e) ~~$y = xArccosx - \sqrt{1 + x^2}$~~

$y = xp + \cos p$

$y' = p + xp' - p'\sin p \Rightarrow p'[x - \sin p] = 0$

$p = c$

1) G.S $\boxed{y = cx + \cos c}$

2) $x = \sin p$

$y = p\sin p + \cos p$

$y = x\arcsin x + \sqrt{1-x^2}$

x, 1, p, $\sqrt{1-x^2}$

solution of IVP

SORU 3. $y'' + Ky' + 4y = x^2e^x$ $\quad y(0) = y'(0) = 0 \quad$ ~~başlangıç değer probleminin~~ çözümü $y = \underbrace{e^x(x^2 + 4x + 6)}_{y_p} + \underbrace{2(x - 3)e^{2x}}_{y_h}$ ise K sayısı kaçtır?

$r^2 + kr + 4 = 0$

$r_1 \cdot r_2 = 4 \quad r_1 + r_2 = -k \quad \boxed{r_1 = r_2 = 2}$

$y_h =$

$\boxed{k = -4}$

$y = c_1 e^{2x} + c_2 x e^{2x}$

$M \quad N$

7. ~~Show that the differential equation~~ $\underbrace{y^2}_{M}dx + \underbrace{(3xy - e^y)}_{N}dy = 0$ is not exact. ~~Solve the given equation by~~ finding an integrating factor.

$$\frac{\partial M}{\partial y} = 2y \neq \frac{\partial N}{\partial x} = 3y$$

(+) $\lambda = \lambda(x)$

$$\ln \lambda = \int \frac{M_y - N_x}{N}dx = \int \frac{2y-3y}{3xy-e^y}dx \neq \lambda(x)$$

(+) $\lambda = \lambda(y)$

$$\ln \lambda = \int \frac{N_x - M_y}{M}dy = \int \frac{3y-2y}{y^2}dy = \int \frac{dy}{y} = \ln y$$

$\lambda(y) = y$

8. Find the differential equation of the family of curves $c_1(x+2) + c_2(x+2)In(x+2) = y$ $c_1, c_2 \in R^+$.

$c_1(x+2) + c_2(x+2)In(x+2) = y$

$$c_1 + c_2\ln(x+2) + c_2\cancel{(x+2)}\frac{1}{\cancel{x+2}} = y'$$

$$\frac{c_2}{x+2} = y''$$

$$\boxed{y' - \frac{y}{x+2} = (x+2)y''} \quad \text{D.E}$$

$c_1(x+2) + c_2(x+2)In(x+2) = y$

$$c_1 + c_2(1+\ln(x+2)) = y'$$

$$c_2\left[(x+2)\cancel{\ln(x+2)} - (x+2) - \cancel{(x+2)\ln(x+2)}\right] = y-(x+2)y'$$

$$c_2 = \frac{y-(x+2)y'}{-(x+2)} = y' - \frac{y}{x+2}$$

9. Find the general solution of differential equation $y(x+3)dx + (x+2)(ydx - xdy) = 0$

$$[y(x+3) + y(x+2)]dx - x(x+2)dy = 0$$

$$y(2x+5)dx - x(x+2)dy = 0$$

$$\int \frac{2x+5}{x(x+2)}dx - \int \frac{dy}{y} = \int 0$$

$$\frac{A}{x} + \frac{B}{x+2} = \frac{2x+5}{x(x+2)}$$

$$A + B = 2$$

$$2A = 5 \Rightarrow A = \frac{5}{2},\ B = -\frac{1}{2}$$

$$\frac{5}{2}\ln x - \frac{1}{2}\ln(x+2) - \ln y = \ln c$$

$$\frac{x^{5/2}}{y\sqrt{x+2}} = c$$

G.S

10. Consider the differential equation $(y - xy')\left(x - \frac{y}{y'}\right) = -2$. Find the general solution of this equation and the $\boxed{\text{singular solution}}$ of this equation in cartesian form.

clairaut

$$(y - xy')(xy' - y) = -2y'$$

$$(xy' - y)^2 = 2y' \Rightarrow xy' - y = \mp\sqrt{2y'}$$

$$y = xy' \mp \sqrt{2y'} \qquad y' = p$$

$$y = xp \mp \sqrt{2p}$$

$$y' = \cancel{p} + xp' \mp \frac{\cancel{2}p'}{\cancel{2}\sqrt{p}\,\sqrt{2}}$$

$$p'\left[x \mp \frac{1}{\sqrt{2p}}\right] = 0$$

$$x = \pm\frac{1}{\sqrt{2p}}$$

$$y = \pm\frac{p}{\sqrt{2p}} \mp \sqrt{2p}$$

$$y = \frac{\pm p \mp 2p}{\sqrt{2p}} = \frac{\mp p}{\sqrt{2p}}$$

$$\mp\sqrt{2p} = \frac{1}{x}$$

$$x^2 = \frac{1}{2p} \qquad 2x^2 = \frac{1}{2y^2}$$

$$y^2 = \frac{p^2}{2p}$$

$$p = 2y^2$$

$$4x^2y^2 = 1$$

16. Solve the initial value problem $y^3y' + \frac{y^4}{2x} - x = 0$, $\boxed{y(1) = 2}$, $(x > 0)$

$$y' + \frac{y}{2x} = x y^{-3}$$

$$y^{1-(-3)} = y^4 = u$$

$$4y^3y' = u'$$

$$y'y^3 + \frac{y^4}{2x} = x$$

$$\frac{u'}{4} + \frac{u}{2x} = x \implies u' + \frac{2u}{x} = 4x \quad \text{L.D.E}$$

$$\lambda(x) = e^{\int \frac{2dx}{x}} = e^{2\ln x} = x^2$$

$$u = \frac{1}{x^2}\left[\underbrace{\int x^2 \cdot 4x\,dx + c}_{x^4+c}\right] = x^2 + \frac{c}{x^2}$$

$$y = \sqrt[4]{x^2 + \frac{c}{x^2}}$$

$$2 = \sqrt[4]{1+c} \implies 16 = 1 + c \implies \boxed{c = 15}$$

6. $\underbrace{2yy'(2+x)+2x-x^2+y^2}_{y} = \underset{y}{0}$

$$2(2+x)y' + \frac{2x-x^2}{y} + y = 0$$

$$2(2+x)y' + y = (x^2-2x)y^{-1}$$

$$y^{1-(-1)} = y^2 = u$$

$$2yy' = u'$$

$$2(2+x)y'y + y^2 = x^2 - 2x$$

$$(2+x)u' + u = x^2 - 2x \quad \text{L.D.E}$$

$$u' + \frac{u}{2+x} = \frac{x^2-2x}{2+x}$$

$$\lambda(x) = e^{\int \frac{dx}{x+2}} = (x+2)$$

$$u = \frac{1}{x+2}\left[\underbrace{\int (\cancel{x+2})\frac{x^2-2x}{\cancel{2+x}}dx + c}_{\frac{x^3}{3} - x^2 + c}\right]$$

$$y = \sqrt{u}$$

21. $(3x-2y+2)dy+(2y-3x)dx=0$

$$\left.\begin{array}{l} x=x_1+h \\ y=y_1+k \end{array}\right\}$$

Case I

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 3 & -2 \\ -3 & 2 \end{vmatrix}=0$$

$$\left.\begin{array}{l} 3x-2y=t \\ 3dx-2dy=dt \end{array}\right\}$$

$$(t+2)\left(\frac{3dx-dt}{2}\right)-t\,dx=0$$

$$\left(\frac{3t}{2}+3-t\right)dx-\frac{t+2}{2}dt=0 \Rightarrow \left(\frac{t}{2}+3\right)dx-\frac{t+2}{2}dt=0$$

$$(t+6)dx-(t+2)dt=0$$

$$\int dx-\int\underbrace{\frac{t+2}{t+6}}dt=\int 0 \Rightarrow \boxed{x-(3x-2y)-4\ln(3x-2y+6)=c}$$

$$\int\left(1-\frac{4}{t+6}\right)dt$$

24. $(y-y^2\ln x)\,\cancel{dx}=x\ln x\,\cancel{dy}\;y' \Rightarrow$

$$x\ln x\,y'-y=-y^2\ln x \quad \text{Bernoulli}$$

$$\underbrace{(y-y^2\ln x)}_{m}dx\underbrace{-x\ln x}_{n}dy=0$$

$$\frac{\partial m}{\partial y}=1-2y\ln x \neq \frac{\partial n}{\partial x}=-\ln x-1$$

(+) $\lambda=\lambda(x)$

$$\ln\lambda=\int\frac{1-2y\ln x+\ln x+1}{-x\ln x}dx \neq \lambda(x)$$

Not ex. or int. fac.

Which of the following transformation is the transformation that will convert the differential equation $\cancel{y'}\ \frac{dy}{dx} = \frac{2x-y+5}{x+y+1}$ into a homogeneous differential equation?

$x = x_1 + h$
$y = y_1 + k$

a) $x = x_1 - 2$, $y = y_1 + 1$ b) $x = x_1 - 2$, $y = y_1 - 1$ c) ~~$x = x_1 + 2$, $y = y_1 - 1$~~ d) ~~$x = x_1 + 2$, $y = y_1 + 1$~~ e) ~~$x = x_1 + 1$, $y = y_1 + 2$~~

$\Rightarrow (x+y+1)dy - (2x-y+5)dx = 0$

$\left.\begin{array}{l} h+k+1=0 \\ 2h-k+5=0 \end{array}\right\}$

$h + \cancel{k} = -1$
$2h - \cancel{k} = -5$
$+$
$3h = -6$
$h = -2$

$k = -1-h$
$k = -1+2 = 1$

$$\boxed{\begin{array}{l} x = x_1 - 2 \\ y = y_1 + 1 \end{array}}$$

$(x_1+y_1)dy_1 - (2x_1-y_1)dx_1 = 0 \Rightarrow$ H.D.E charact. equation

SORU 2. ~~Karakteristik denklemi~~ $r^2(r+3)(r^2-2r+5)=0$ ~~olan sabit katsayılı lineer homojen diferansiyel denklem aşağıdakilerden hangisidir?~~ D.E

a) $y^v + 6y^{iv} + 9y'' = 0$

b) $y^v + 3y''' + 9y'' + y' = 0$

c) $y''' + y'' - y' + 15 = 0$

d) $y^v + 4y^{iv} + 3y' = 0$

e) $y^v + y^{iv} - y''' + 15y'' = 0$

$(r^3+3r^2)(r^2-2r+5)=0 \Rightarrow r^5-2r^4+5r^3+3r^4-6r^3+15r^2=0$

$r^5+r^4-r^3+15r^2=0$

$y^{v}+y^{iv}-y'''+15y''=0$

ex If one ind. solution is $\boxed{x^2e^{-x}}$, then what is the $\boxed{\text{third order}}$ homog. d.e?

$(r+1)^3=0 \qquad r^3+3r^2+3r+1=0$

$y'''+3y''+3y'+y=0$

ex If some ind. solutions are $x\cos x$ and x, then what is your six order hom. d.e?

$$(r^2+1)^2 \cdot r^2 = 0$$

$r^6+2r^4+r^2=0$

$y^{vi}+2y^{iv}+y''=0$

n. $r_{1,2} = 1 \pm \sqrt{2}i$, $r_{3,4} = \pm\sqrt{2}i$, $r_{5,6} = -\sqrt{2}$

$$y_h = e^{x}\left[c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x\right] + c_3\cos\sqrt{2}x + c_4\sin\sqrt{2}x$$
$$+\left[c_5 + c_6 x\right]e^{-\sqrt{2}x}$$

c. $y''' - 3y'' + 3y' - y = 8xe^x + \sin x$

$$r^3 - 3r^2 + 3r - 1 = 0$$
$$\Rightarrow -3r(r-1)$$
$$(r-1)(r^2+r+1)$$
$$(r-1)(r^2-2r+1) = 0 \Rightarrow (r-1)^3 = 0 \qquad r_1 = r_2 = r_3 = 1$$
$$y_h = (c_1 + c_2 x + c_3 x^2)e^x$$

$$y_p = e^x(ax+b)x^3 + A\cos x + B\sin x$$

ex $\quad r_1 = r_2 = 0$, $r_{3,4} = \mp 2i$, $r_5 = 1 \qquad Q(x) = x - 4 + e^x\cos x$

$$y_p = (ax+b)x^2 + e^x(A\cos x + B\sin x)$$

SORU 6. Cauchy-Euler ~~diferansiyel denklemi olduğu bilinen~~ $x^k y'' + xy' + y = kx+1$ ~~diferansiyel denklemi, uygun bir değişken dönüşümü ile aşağıdaki hangi sabit katsayılı denkleme dönüşür?~~ what is d.e with constant coef?

$k=2 \qquad x^2y'' + xy' + y = 2x+1$

$x = e^t \qquad [D(D-1)y + Dy + y] = 2e^t + 1$

$$(D^2+1)y = 2e^t + 1$$

$$\frac{d^2y}{dt^2} + y = 2e^t + 1$$

SORU 14. $2y'' - y' - y = Sec2x$ ~~diferansiyel denklemini çözmek için aşağıdaki denklem sistemlerinden hangisi kullanılır~~? equt. system.

$$2r^2 - r - 1 = 0 \qquad r_{1,2} = \frac{1 \mp \sqrt{1 - 4 \cdot 2(-1)}}{4} = \frac{1 \mp 3}{4} \begin{array}{l} \to r_1 = 1 \\ \to r_2 = -\frac{1}{2} \end{array}$$

$$y_h = c_1 e^{x} + c_2 e^{-\frac{x}{2}}$$

$$\left.\begin{array}{l} c_1' e^{x} + c_2' e^{-\frac{x}{2}} = 0 \\ c_1' e^{x} - \frac{1}{2} c_2' e^{-\frac{x}{2}} = \frac{\sec 2x}{2} \end{array}\right\}$$