

		Surname		Type
Group Number		Name		A
List Number		e-mail		
Student ID		Signature		

**ATTENTION:** Each question has only one correct answer and is worth one point. Be sure to fill in completely the circle that corresponds to your answer on the answer sheet. Use a pencil (not a pen). Only the answers on your answer sheet will be taken into account.

### Questions 1-5

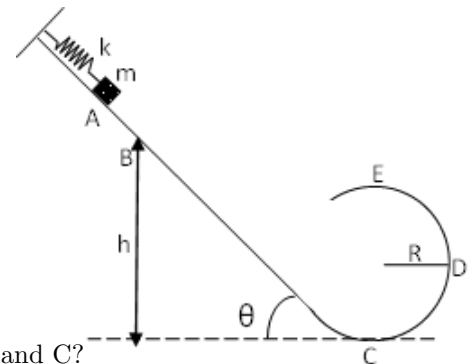
A small piece of packing material with  $m = 3 \text{ kg}$  is dropped from a height of  $2 \text{ m}$  above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by  $a = g - bV$ . After falling  $0.5 \text{ m}$  the material reaches its terminal speed, and then takes  $3 \text{ s}$  more to reach the ground. ( $g = 10 \text{ m/s}^2$ )

- What is the terminal speed of the material?  
(a)  $0.3 \text{ m/s}$  (b)  $0.2 \text{ m/s}$  (c)  $0.5 \text{ m/s}$  (d)  $1 \text{ m/s}$  (e)  $0.4 \text{ m/s}$
- What is the value of the constant  $b$ ?  
(a)  $40 \text{ s}^{-1}$  (b)  $4 \text{ s}^{-1}$  (c)  $5 \text{ s}^{-1}$  (d)  $20 \text{ s}^{-1}$  (e)  $10 \text{ s}^{-1}$
- What is the acceleration at  $t=0$ ?  
(a)  $10 \text{ m/s}^2$  (b)  $2 \text{ m/s}^2$  (c)  $4 \text{ m/s}^2$  (d)  $5 \text{ m/s}^2$  (e)  $6 \text{ m/s}^2$
- What is the acceleration when the speed is  $0.15 \text{ m/s}$ ?  
(a)  $7 \text{ m/s}^2$  (b)  $6 \text{ m/s}^2$  (c)  $10 \text{ m/s}^2$  (d)  $4 \text{ m/s}^2$  (e)  $5 \text{ m/s}^2$
- What is the net force acted on the material when the speed is  $0.15 \text{ m/s}$ ?  
(a)  $21 \text{ N}$  (b)  $15 \text{ N}$  (c)  $18 \text{ N}$  (d)  $30 \text{ N}$  (e)  $12 \text{ N}$

### Questions 6-10

A block of mass  $m$  is placed in front of a spring which is compressed as  $x$  (between points A and B) and the system is set on an inclined surface as in the figure. The rail between A and B, and the circular part (of radius  $R$ ) between C and E are frictionless (no friction). The region between B and C is considered as a completely flat surface of kinetic friction constant,  $\mu_k$ . When the spring is released the block leaves the spring and moves along the rail between the points A and E. It passes the point E without falling down. (Take the gravitational acceleration as  $g$ )

- What is the speed of the block at point B?  
(a)  $v = \sqrt{2gx \sin \theta}$   
(b)  $v = \sqrt{\frac{1}{2}kx}$   
(c)  $v = \sqrt{\frac{2}{m}(mgx \sin \theta - \frac{1}{2}kx^2)}$   
(d)  $v = \sqrt{\frac{2}{m}(mgx \sin \theta + \frac{1}{2}kx^2)}$   
(e)  $v = \sqrt{\frac{2}{m}(\frac{1}{2}kx^2 - mgx \sin \theta)}$
- What is the energy lost in the mechanical energy of the block between the points B and C?  
(a) zero (b)  $\mu_k mgh \cot \theta$  (c)  $\mu_k mgh \tan \theta$  (d)  $\mu_k \frac{1}{2}kx^2$  (e)  $\mu_k mgh \sin \theta$
- What is the kinetic energy of the block at point C?  
(a)  $\sqrt{mgh(1 - \mu_k) + \frac{1}{2}kx^2}$  (b)  $mgh$  (c)  $mgh(1 - \mu_k) - \frac{1}{2}kx^2$  (d)  $mgh(1 + \mu_k) + \frac{1}{2}kx^2$  (e)  $mg(h + x \sin \theta - \mu_k h \cot \theta) + \frac{1}{2}kx^2$
- What is the kinetic energy of the block at point E?  
(a)  $\frac{1}{2}(mgh(1 - \mu_k) + \frac{1}{2}kx^2 + mg2R)$  (b)  $\frac{1}{2}(mgh(1 + \mu_k) + \frac{1}{2}kx^2 + mg2R)$  (c)  $\frac{1}{2}(mgh(1 - \mu_k) - \frac{1}{2}kx^2 - mg2R)$   
(d)  $mg(h + x \sin \theta - \mu_k h \cot \theta - 2R) + \frac{1}{2}kx^2$  (e)  $\frac{1}{2}(mgh(1 + \mu_k) + \frac{1}{2}kx^2 - mg2R)$
- What is the normal force on the block applied by the rail at the point E?  
(a)  $\frac{m}{2R}(mgh(1 - \mu_k) - \frac{1}{2}kx^2 - mg2R) - mg$  (b)  $\frac{2}{R}(mg(h + x \sin \theta - \mu_k h \cot \theta - 2R) + \frac{1}{2}kx^2) - mg$  (c)  $\frac{m}{2R}(mgh(1 - \mu_k) + \frac{1}{2}kx^2 - mg2R) + mg$  (d)  $\frac{m}{2R}(mgh(1 - \mu_k) + \frac{1}{2}kx^2 + mg2R) - mg$  (e)  $\frac{m}{2R}(mgh(1 + \mu_k) + \frac{1}{2}kx^2 - mg2R) - mg$



### Questions 11-15

A  $3.0 \text{ kg}$  object has the following two forces acting on it:  $\vec{F}_1 = (16\hat{i} + 12\hat{j}) \text{ N}$  and  $\vec{F}_2 = (-10\hat{i} + 21\hat{j}) \text{ N}$ . The object is initially at rest at a point given by the coordinates ( $x = 3 \text{ m}$ ,  $y = 4 \text{ m}$ ).

- What is the magnitude of acceleration of the object?  
(a)  $6 \text{ m/s}^2$  (b)  $12 \text{ m/s}^2$  (c)  $5\sqrt{5} \text{ m/s}^2$  (d)  $11.75 \text{ m/s}^2$  (e)  $11 \text{ m/s}^2$

12. What is the momentum change in 4 s?

- (a)  $(8\hat{i} + 44\hat{j}) \text{ N}\cdot\text{s}$  (b)  $(3\hat{i} + 4\hat{j}) \text{ N}\cdot\text{s}$  (c)  $(6\hat{i} + 33\hat{j}) \text{ N}\cdot\text{s}$  (d)  $(24\hat{i} + 132\hat{j}) \text{ N}\cdot\text{s}$  (e)  $(2\hat{i} + 44\hat{j}) \text{ N}\cdot\text{s}$

13. What is the velocity of the object at  $t = 2 \text{ s}$ ?

- (a)  $(2\hat{i} + 44\hat{j}) \text{ m/s}$  (b)  $(4\hat{i} + 22\hat{j}) \text{ m/s}$  (c)  $(3\hat{i} + 4\hat{j}) \text{ m/s}$  (d)  $(6\hat{i} + 33\hat{j}) \text{ m/s}$  (e)  $(8\hat{i} + 24\hat{j}) \text{ m/s}$

14. What is the position vector of the object at  $t = 2 \text{ s}$ ?

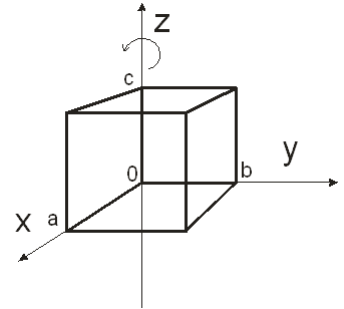
- (a)  $(7\hat{i} + 26\hat{j}) \text{ m}$  (b)  $(10\sqrt{5}\hat{i} + 10\hat{j}) \text{ m}$  (c)  $(4\hat{i} + 72\hat{j}) \text{ m}$  (d)  $(4\hat{i} + 88\hat{j}) \text{ m}$  (e)  $(8\hat{i} + 132\hat{j}) \text{ m}$

15. What is the average velocity of the object between  $t = 2 \text{ s}$  and  $t = 3 \text{ s}$ ?

- (a)  $(12\hat{i} + 66\hat{j}) \text{ m/s}$  (b)  $(5\hat{i} + 27.5\hat{j}) \text{ m/s}$  (c)  $(4\hat{i} + 88\hat{j}) \text{ m/s}$  (d)  $(8\hat{i} + 24\hat{j}) \text{ m/s}$  (e)  $(6\hat{i} + 8\hat{j}) \text{ m/s}$

### Questions 16-20

A rectangular prism with a mass  $M = 3 \text{ kg}$  rotates in a coordinate system as shown in the figure. The lengths of the sides are  $a = 1 \text{ m}$ ,  $b = 2 \text{ m}$ , and  $c = 3 \text{ m}$ . The prism has an angular velocity  $\omega = 2 + 3t^2 - 2t^3$  about  $+z$ -axis in units of  $\text{rad/s}$ .



16. Find the rotational inertia about  $z$ -axis in  $\text{kgm}^2$ ?

- (a) 21 (b) 7 (c) 5 (d) 10 (e) 42

17. Find the rotational inertia about axis through the center of mass and parallel to the  $z$ -axis.

- (a)  $10 \text{ kgm}^2$  (b)  $21 \text{ kgm}^2$  (c)  $5/4 \text{ kgm}^2$  (d)  $12 \text{ kgm}^2$  (e)  $14 \text{ kgm}^2$

18. What is the angular displacement of the point given by the coordinates  $(x = 1 \text{ m}, y = 2 \text{ m}, z = 3 \text{ m})$  between  $t = 0 \text{ s}$  and  $t = 2 \text{ s}$ ?

- (a) 3 rad (b) 0 rad (c) 5 rad (d) 4 rad (e) 2 rad

19. What is the magnitude of the tangential acceleration of the point given by the coordinates  $(x = 1 \text{ m}, y = 2 \text{ m}, z = 3 \text{ m})$  at  $t = 2 \text{ s}$  in  $\text{m/s}^2$ ?

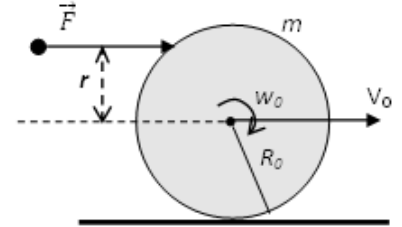
- (a)  $4\sqrt{5}$  (b)  $4\sqrt{2}$  (c)  $12\sqrt{5}$  (d)  $18\sqrt{5}$  (e)  $8\sqrt{5}$

20. What is the kinetic energy of the rectangular prism at  $t = 2 \text{ s}$ ?

- (a) 14 J (b) 21 J (c) 16 J (d) 18 J (e) 10 J

### Questions 21-25

A force  $\vec{F} = F\hat{i}$  is applied only for a short time at a point above the center of a sphere and transfers a net linear momentum  $\vec{p} = p\hat{i}$  to the sphere in the  $x$ -direction. Ignore any frictional force during the application of the force  $\vec{F} = F\hat{i}$ , and consider that the only force is the frictional force for  $t \geq 0$ . The sphere has a mass  $m$  and radius  $R_0$ . The sphere is at rest initially. The point to which the force applied is  $r = \frac{3}{10}R_0$  above the center of mass of the sphere. The magnitude of the net frictional force for the sphere is  $F_k = \mu mg$  where  $\mu$  is the kinetic friction coefficient between the surfaces. The moment of inertia about an axis passing through the center of mass of the sphere is given by  $I = \frac{2}{5}mR_0^2$ . The direction of  $+z$ -axis is out of the page.



21. What is the speed of the center of mass of the sphere just after the application of the force? ( $V_0 = V(t = 0) = ?$ )

- (a)  $\frac{2p}{m}$  (b)  $\frac{p}{m}$  (c)  $\frac{m}{p}$  (d)  $\frac{p^2}{2m}$  (e)  $\frac{p}{2m}$

22. What is the angular speed about the axis passing through the center of mass just after the application of the force? ( $\omega_0 = \omega(t = 0) = ?$ )

- (a)  $\frac{4mR_0}{3p}$  (b)  $\frac{3}{4} \frac{p}{mR_0}$  (c)  $\frac{mR_0}{p}$  (d)  $\frac{4}{3} \frac{p}{mR_0}$  (e)  $\frac{p}{mR_0}$

23. What is the velocity of the center of mass as function of time? ( $\vec{V}(t) = ?$ )

- (a)  $(\frac{p}{2m} - \mu gt)\hat{i}$  (b)  $(\frac{p}{2m} - 2\mu gt)\hat{i}$  (c)  $(\frac{2p}{m} - \mu gt)\hat{i}$  (d)  $(\frac{p}{m} - 2\mu gt)\hat{i}$  (e)  $(\frac{p}{m} - \mu gt)\hat{i}$

24. What is the angular velocity about the axis passing through the center of mass as function of time? ( $\vec{\omega}(t) = ?$ )

- (a)  $-\left(\frac{3}{4} \frac{p}{mR_0} + \frac{5\mu g}{2R_0} t\right)\hat{k}$  (b)  $-\left(\frac{p}{mR_0} + \frac{5\mu g}{4R_0} t\right)\hat{k}$  (c)  $-\left(\frac{4}{3} \frac{p}{mR_0} + \frac{4\mu g}{5R_0} t\right)\hat{k}$  (d)  $-\left(\frac{3}{4} \frac{p}{mR_0} + \frac{\mu g}{R_0} t\right)\hat{k}$  (e)  $-\left(\frac{4mR_0}{3p} + \frac{5\mu g}{4R_0} t\right)\hat{k}$

25. At  $t=0$  the sphere is slipping on the surface. Find the value of  $t$  for the sphere to start rolling without slipping?

- (a)  $\frac{p}{\mu g}$  (b)  $\frac{p}{14\mu g}$  (c)  $\frac{p}{m\mu g}$  (d)  $\frac{p}{m\mu}$  (e)  $\frac{9p}{m\mu g}$