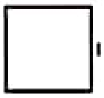
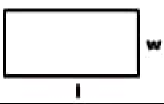
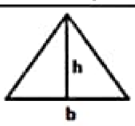
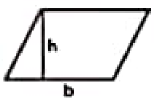
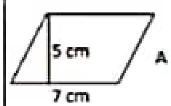
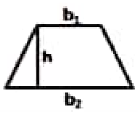
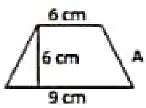
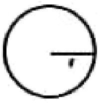
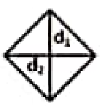
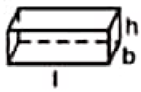
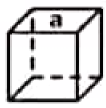
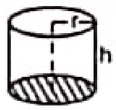
Multiplication factor	SI prefix	SI symbol
$1{,}000{,}000{,}000 = 10^9$	Giga	G
$1{,}000{,}000 = 10^6$	Mega	M
$1000 = 10^3$	Kilo	k
$100 = 10^2$	Hector	h
$10 = 10$	Deka	da
$.1 = 10^{-1}$	Deci	d
$.01 = 10^{-2}$	Centi	c
$.001 = 10^{-3}$	Milli	m
$.000{,}001 = 10^{-6}$	Micro	μ
$.000{,}000{,}001 = 10^{-9}$	Nano	n
$.000{,}000{,}000{,}001 = 10^{-12}$	Pico	p

Length	1 centimeter (cm) = 0.01 meter (m) = 0.3937 inch (in.) 1 in. = 2.54 cm = 0.0254 m 1 foot (ft) = 30.48 cm—0.3048 m 1 m = 3.28 ft = 39.37 in. 1 yard (yd) = 0.9144 m = 3 ft 1 mile = 1609 m = 1.609 kilometer (km) = 5280 ft 1 km = 0.6214 mile
Time	1 minute (min) = 60 seconds (s) 1 hour (h) = 60 min = 3600 s 1 day = 24 h = 1440 min = 86,400 s
Area	1 cm^2 = 0.155 $in.^2$ 1 $in.^2$ = 6.452 cm^2 1 m^2 = 10.763 ft^2 1 ft^2 = 0.0929 m^2

(continued)

Mass	1 pound mass (lbm) = 0.4536 kilogram (kg) 1 kg = 2.2 lbm = 0.0685 slug 1 slug = 14.59 kg = 32.098 lbm
Force	1 kilogram force (kgf) = 9.807 Newton (N) 1 pound force (lbf) = 4.448 N 1 N = 0.2248 lbf 1 dyne (dyn) = 10^{-5} N 1 N = 10^5 dyn
Pressure and stress	1 N/m^2 = 1 Pascal (Pa) = 0.000145 $lbf/in.^2$ (psi) 1 psi = 6895 Pa 1 lbf/ft^2 (psf) = 592,966 Pa 1 dyn/cm^2 = 0.1 Pa
Moment (Torque)	1 Nm = 10^7 dyn cm = 0.7376 lbf ft 1 dyn/cm = 10^{-7} Nm 1 lbf ft = 1.356 Nm
Work and energy	1 Nm = 1 Joule (J) = 10^7 erg 1 J = 0.7376 lbf ft 1 lbf ft = 1.356 J
Power	1 kg m^2/s^3 = 1 J/S = 1 Watt (W) 1 horsepower (hp) = 550 lbf ft/s = 746 W 1 lbf ft/s = 1.356 W 1 W = 0.737 lbf ft/s
Plane angle	1 degree (°) = π/180 radian (rad) 1 revolution (rev) = 360° 1 rev = 2π rad = 6.283 rad 1 rad = 57.297° 1° = 0.0175 rad
Temperature	°C = 273.2 K °C = 5/9 (−32 °F) °F = 9/5 (+32 °C)

Shape		Formula	Example
Square		$A = l \times l = l^2$	What is the area of a square of length 4 cm? $A = 4 \times 4 = 16\ cm^2$
Rectangle		$A = l \times w$	What is the area of a rectangle of length 7 inches and width 5 inches? $A = 7 \times 5 = 35\ in^2$
Triangle		$A = 1/2 \times b \times h$	What is the area of a triangle with height 6 inches and base 5 inches? $A = 1/2 \times 6 \times 5 = 15\ in^2$
Parallelogram		$A = h \times b$	 $A = 5 \times 7 = 35\ cm^2$
Trapezoid		$A = 1/2 \times h \times (b_1 + b_2)$	 $A = 1/2 \times 6 \times (9 + 6) = 45\ cm^2$
Circle		$A = \pi \times r^2$ (π = 3.14 or 22/7)	What is the area of a circle with radius 9 feet? $A = \pi \times 9^2 = 81\pi = 254.34\ ft^2$
Rhombus		$A = 1/2 \times d_1 \times d_2$	What is the area of a rhombus with diagonals 8 inches and 7 inches? $A = 1/2 \times 8 \times 7 = 28\ in^2$

Name	Figure	Curved Surface area	Total surface area	Volume
Cuboid		$2h(l + b)$	$2(lb + bh + lh)$	lbh
Cube		$4a^2$	$6a^2$	a^3
Right circular cylinder		$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
Sphere		$4\pi r^2$	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$
Hemi-sphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$

Scanned with CamScanner

Example 3.1 Figure 3.13 illustrates a person preparing to dive into a pool. The horizontal diving board has a uniform thickness, mounted to the ground at point O, has a mass of 120 kg, and is $l = 4$ m in length. The person has a mass of 90 kg and stands at point B which is the free end of the board. Point A indicates the location of the center of gravity of the board. Point A is equidistant from points O and B.

Determine the moments generated about point O by the weights of the person and the board. Calculate the net moment about point O.

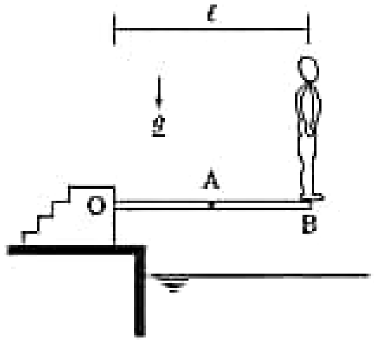


Fig. 3.13 *A person is preparing to dive*

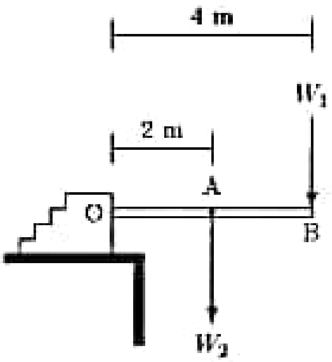


Fig. 3.14 *Forces acting on the diving board*

$$W_1 = m_1 g = (90\ \text{kg})(9.8\,\text{m/s}^2) = 882\ \text{N}$$
$$W_2 = m_2 g = (120\,\text{kg})(9.8\,\text{m/s}^2) = 1176\ \text{N}$$

$$M_1 = l\,W_1 = (4\,\text{m})(882\ \text{N}) = 3528\,\text{Nm} \quad (\text{cw})$$
$$M_2 = \frac{l}{2}W_2 = (2\,\text{m})(1176\ \text{N}) = 2352\,\text{Nm} \quad (\text{cw})$$
$$M_{net} = M_1 + M_2 = 5880\,\text{Nm} \quad (\text{cw})$$

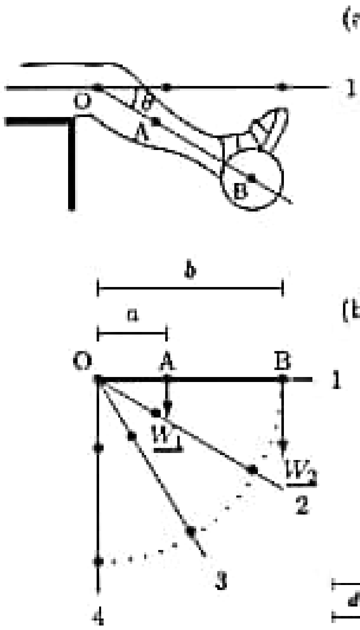


Fig. 3.15 *Example 3.2*

Example 3.2 As illustrated in Fig. 3.15a, consider an athlete wearing a weight boot, and from a sitting position, doing lower leg flexion/extension exercises to strengthen quadriceps muscles. The weight of the athlete's lower leg is $W_1 = 50$N and the weight of the boot is $W_2 = 100$N. As measured from the knee joint at point O, the center of gravity (point A) of the lower leg is located at a distance $a = 20$cm and the center of gravity (point B) of the weight boot is located at a distance $b = 50$cm.

Determine the net moment generated about the knee joint when the lower leg is extended horizontally (position 1), and when the lower leg makes an angle of 30° (position 2), 60° (position 3), and 90° (position 4) with the horizontal (Fig. 3.15b).

Solution:

$$M_O = aW_1 + bW_2$$
$$= (0.20)(50) + (0.50)(100)$$
$$= 60\,\text{Nm} \quad (\text{cw})$$

$$d_1 = a\cos\theta$$
$$d_2 = b\cos\theta$$

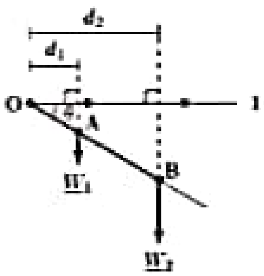


Fig. 3.16 *Forces and moment arms when the lower leg makes an angle θ with the horizontal*

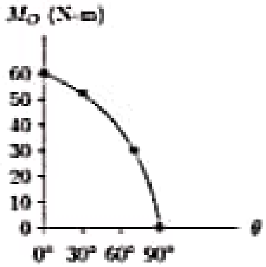
Therefore, the net moment about point O is:

$$M_O = d_1W_1 + d_2W_2$$
$$= a\cos\theta\, W_1 + b\cos\theta\, W_2$$
$$= (aW_1 + bW_2)\cos\theta$$

The term in the parentheses has already been calculated as 60 Nm. Therefore, we can write:

$$M_O = 60\cos\theta$$

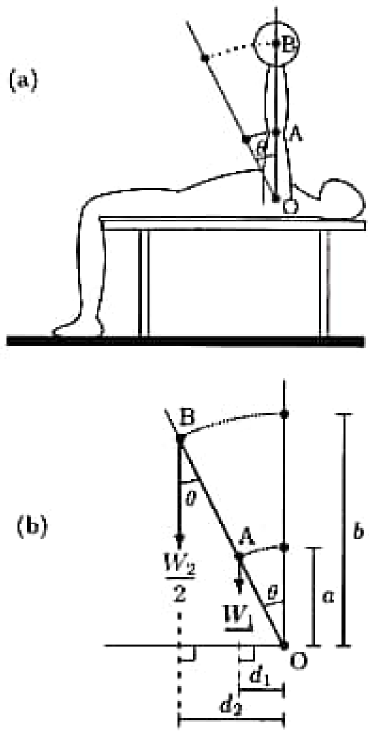
For position 1:	$\theta = 0°$	$M_O = 60\,\text{Nm}$	(cw)
For position 2:	$\theta = 30°$	$M_O = 52\,\text{Nm}$	(cw)
For position 3:	$\theta = 60°$	$M_O = 30\,\text{Nm}$	(cw)
For position 4:	$\theta = 90°$	$M_O = 0$	(cw)



In Fig. 3.17, the moment generated about the knee joint is plotted as a function of angle θ.

Scanned with CamScanner

Example 3.3 Figure 3.18a illustrates an athlete doing shoulder muscle strengthening exercises by lowering and raising a barbell with straight arms. The position of the arms when they make an angle θ with the vertical is simplified in Fig. 3.18b. Point O represents the shoulder joint, A is the center of gravity of one arm, and B is a point of intersection of the centerline of the barbell and the extension of line OA. The distance between points O and A is $a = 24$ cm and the distance between points O and B is $b = 60$ cm. Each arm weighs $W_1 = 50$ N and the total weight of the barbell is $W_2 = 300$ N.



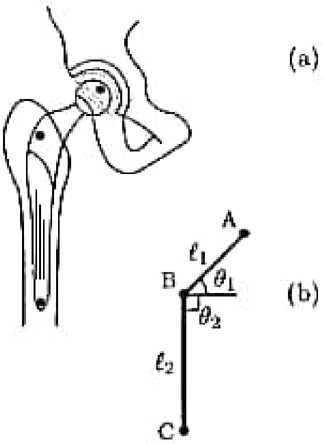
Determine the net moment due to W_1 and W_2 about the shoulder joint as a function of θ, which is the angle the arm makes with the vertical. Calculate the moments for $\theta = 0°, 15°, 30°, 45°$, and $60°$.

Solution

$$d_1 = a\sin\theta$$
$$d_2 = b\sin\theta$$

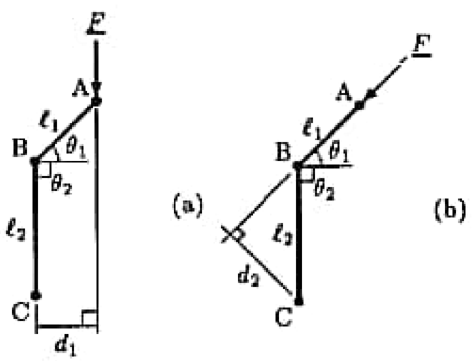
$$M_1 = d_1 W_1 = aW_1 \sin\theta = (0.24)(50)\sin\theta = 12\sin\theta$$
$$M_2 = d_2\frac{W_2}{2} = b\frac{W_2}{2}\sin\theta = (0.60)\left(\frac{300}{2}\right)\sin\theta = 90\sin\theta$$
$$M_O = M_1 + M_2 = 12\sin\theta + 90\sin\theta = 102\sin\theta \text{ Nm} \quad \text{(ccw)}$$

θ	SIN θ	M_O (NM)
0°	0.000	0.0
15°	0.259	26.4
30°	0.500	51.0
45°	0.707	72.1
60°	0.866	88.3



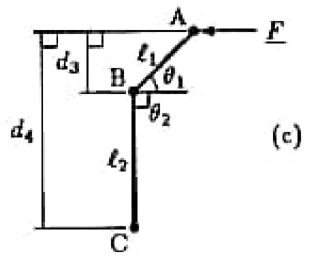
Example 3.4 Consider the total hip joint prosthesis shown in Fig. 3.19. The geometric parameters of the prosthesis are such that $l_1 = 50$ mm, $l_2 = 50$ mm, $\theta_1 = 45°, \theta_2 = 90°$. Assume that, when standing symmetrically on both feet, a joint reaction force of $F = 400$ N is acting at the femoral head due to the body weight of the patient. For the sake of illustration, consider three different lines of action for the applied force, which are shown in Fig. 3.20.

Determine the moments generated about points B and C on the prosthesis for all cases shown.



Solution

$$d_1 = l_1\cos\theta_1 = (50)(\cos 45°) = 35 \text{ mm}$$
$$M_B = M_C = d_1 F = (0.035)(400) = 14 \text{ Nm} \quad \text{(cw)}$$
$$M_B = 0$$
$$d_2 = l_2\cos\theta_1 = (100)(\cos 45°) = 71 \text{ mm}$$
$$M_C = d_1 F = (0.071)(400) = 28 \text{ Nm} \quad \text{(ccw)}$$
$$d_3 = l_1\sin\theta_1 = (50)(\sin 45°) = 35 \text{ mm}$$
$$d_4 = d_3 + l_2 = (35) + (100) = 135 \text{ mm}$$
$$M_B = d_3 F = (0.035)(400) = 14 \text{Nm} \quad \text{(ccw)}$$
$$M_C = d_4 F = (0.135)(400) = 54 \text{Nm} \quad \text{(ccw)}$$



Scanned with CamScanner

Example 3.5 Figure 3.27a illustrates a person using an exercise machine. The "L" shaped beam shown in Fig. 3.27b represents the left arm of the person. Points A and B correspond to the shoulder and elbow joints, respectively. Relative to the person, the upper arm (AB) is extended toward the left (x direction) and the lower arm (BC) is extended forward (z direction). At this instant the person is holding a handle that is connected by a cable to a suspending weight. The weight applies an upward (in the y direction) force with magnitude F on the arm at point C. The lengths of the upper arm and lower arm are $a = 25$ cm and $b = 30$ cm, respectively, and the magnitude of the applied force is $F = 200$ N.

Explain how force F can be translated to the shoulder joint at point A, and determine the magnitudes and directions of moments developed at the lower and upper arms by $\underline{F}$.

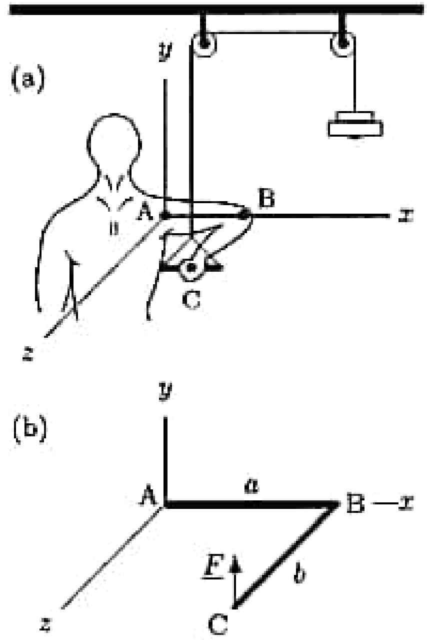
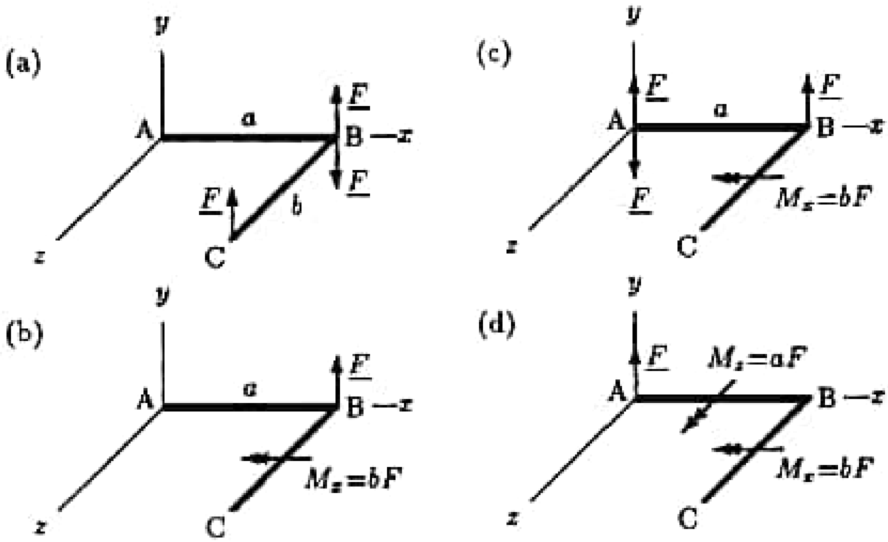


Fig. 3.27 *Example 3.5*

Solution 1: Scalar Method



$M_x = bF$ ($-x$ direction) $M_z = aF$ ($+z$direction) $M_x = (0.30)(200) = 60$ Nm
$= 50$ Nm

Solution 2: Vector Product Method The definition of moment as the vector product of the position and force vectors is more straightforward to apply. The position vector of point C (where the force is applied) with respect to point A (where the shoulder joint is located) and the force vector shown in Fig. 3.29 can be expressed as follows:

$$\underline{r} = a\underline{i} + b\underline{k}$$
$$\underline{F} = F\underline{j}$$
$$\begin{aligned}\underline{M} &= \underline{r} \times \underline{F} \\ &= \left(a\underline{i} + b\underline{k}\right) \times \left(F\underline{j}\right) \\ &= aF\left(\underline{i} \times \underline{j}\right) + bF\left(\underline{k} \times \underline{j}\right) \\ &= aF\underline{k} - bF\underline{i} \\ &= (0.25)(200)\underline{k} - (0.30)(200)\underline{i} \\ &= 50\underline{k} - 60\underline{i}\end{aligned}$$

$M_x = 60$ Nm ($-x$ direction)

$M_y = 0$

$M_z = 50$ Nm ($+z$ direction)

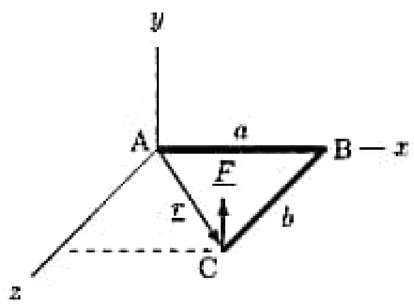


Fig. 3.29 *Vector product method (Example 3.5)*

Scanned with CamScanner

Problem 3.1 As illustrated in Fig. 3.6, assume a worker is applying a force $\underline{F}$ to tighten a right-treaded bolt by using a wrench. As the result of the applied force the bolt is advancing into the metal plate. The recommended torque for the bolt is M_{rec} 80 Nm. Furthermore, the length of the wrench handle is $d = 25$ cm.

(a) Calculate the magnitude of the force requited to complete the task.
(b) How can the task be modified to decrease its force requirements by 50%?

Answers: (a) $F = 320$ N; (b) by increasing the length of the wrench handle ($d = 50$ cm)

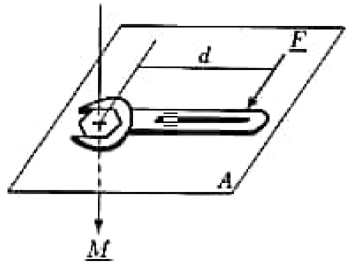


Fig. 3.6 *Wrench and bolt*

Problem 3.2 As illustrated in Fig. 3.13, consider a diver of 88 kg mass standing on the free end of a horizontal diving board at point B, preparing for a jump. The diving board has a uniform thickness and a mass of 56 kg, and it is mounted to the ground at point O. Point A indicates the center of gravity of the diving board, and it is equidistant from points O and B.

Determine the length of the diving board (l) if the net moment generated about point O by the weight of the diver and the diving board is $M_O = 3979$ Nm.

Answer: $l = 3.5$ m

Problem 3.3 Figure 3.13 illustrates a diver standing at the free end of uniform diving board (point B) and preparing to dive into a pool. The diving board is mounted to the ground at point O, has a mass of 110 kg and length of $l = 3.0$ m. Point A indicates the center of gravity of the board and it is equidistant from points O and B. If the net moment about point O is $M_{net} = 3760$ Nm, determine the weight of the diver.

Answer: $W = 714.3$ N

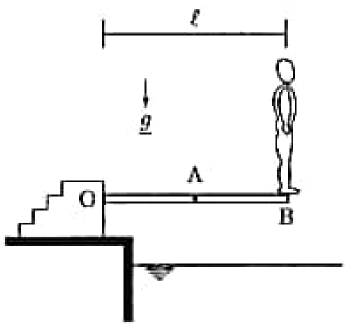
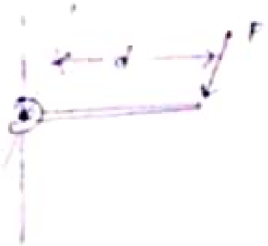


Fig. 3.13 *A person is preparing to dive*

Scanned with CamScanner



The recommended torque is

$M_{rec} = 80$ N-m.

a) We know, that

$$Torque = r \times F$$

where r is the distance from the end to the point from where force is applied.

$$\therefore Torque_{recomm} = r \times F$$

$$\text{or, } F = \frac{Torque}{r}$$

$$= \frac{80}{0.25} N$$

$$= 320 N$$

b) The Force needs to decreased by 50%.

$$\therefore Force_{final} = 160 N$$

As, the torque recommended cannot change, so.

$$F_{initial} \times r_{initial} = F_{final} \times r_{final} = Torque_{recommended}$$

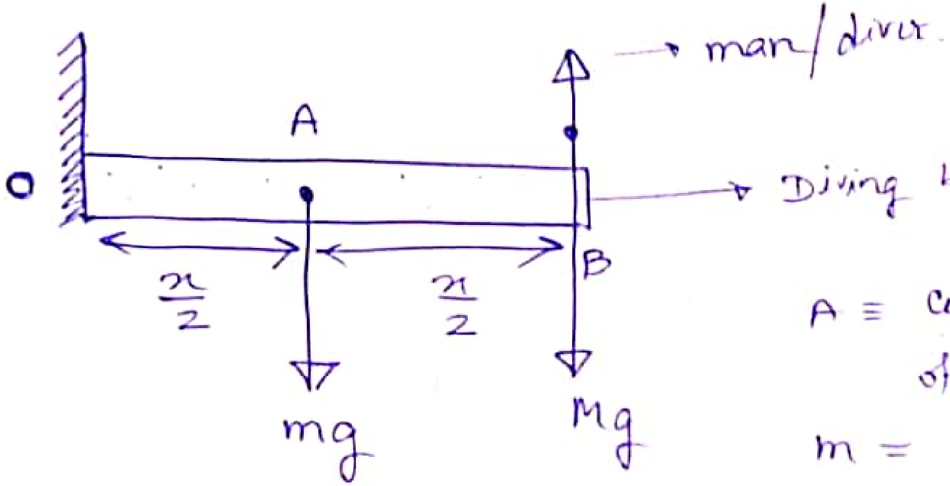
$$\text{or, } 320 \times 0.25 = 160 \times r_{final}$$

$$r_{final} = 0.50 m$$

$$= 50 cm$$

$\therefore$ Therefore increasing wrench handle by 25 cm, the force can be decreased by 50%.

Scanned with CamScanner



A ≡ center of Gravity of Diving board.

m = Mass of board = 56 kg.

M = Mass of diver = 88 kg.

Let the total length of the diving board be x.

By the above figure,

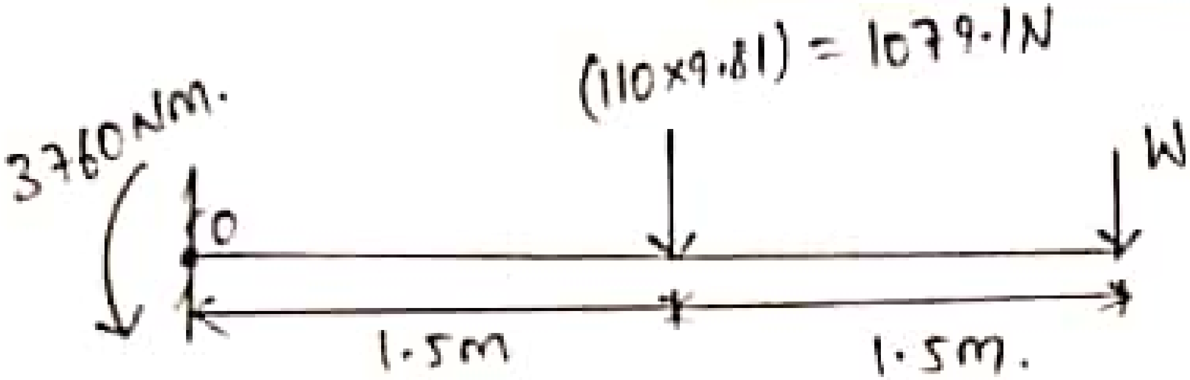
Total moment about O (M_o) $= (mg.)\frac{x}{2} + (Mg.)x$

$\Rightarrow 3979 \text{ Nm} = \left(\frac{56}{2}\text{kg} + 88\text{ kg}\right) \times x \times 9.8 \text{ m/s}^2$

$\Rightarrow x = \dfrac{3979}{9.8 \times \left(\frac{56}{2} + 88\right)} \text{ m}$

$\Rightarrow \underline{\underline{x = 3.5 \text{ m}}}$ ← length of Diving board (l).

Scanned with CamScanner



Take moment about 'O'.

$$W(3) + 1079.1(1.5) = 3760\,Nm$$

$$W(3) = 2141.35\ N.m$$

$$W = \frac{2141.35}{3}\ N$$

$$\boxed{W = 713.78\,N}.$$

Scanned with CamScanner

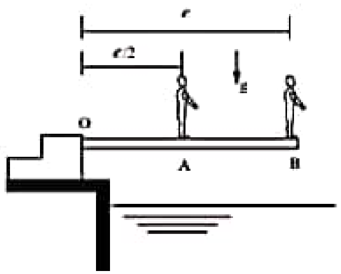


Fig. 3.30 *Divers preparing for sequential jump into the pool*

Problem 3.4 As illustrated in Fig. 3.30, consider two divers preparing to dive into a pool in a sequence. The horizontal diving board of uniform thickness is mounted to the ground at point O, has a mass of 130 kg, and is $l = 4$ m in length. The first diver has a mass of 86 kg and he stands at point B, which is the free end of the diving board. The second diver has a mass of 82 kg and he stands at point A, which is the center of gravity of the board. Furthermore, point A is equidistant from points O and B.

Determine the moment generated about point O by the weights of the divers and the board. Calculate the net moment about point O.

Answers: $M_1 = 3371.2$ Nm; $M_2 = 1607.2$ Nm; $M_b = 2548$ Nm; $M_{net} = 7526.4$ Nm

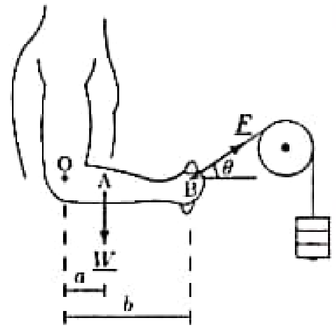


Fig. 3.31 *Problems 3.5 and 3.7*

Problem 3.5 Consider a person using an exercise apparatus who is holding a handle that is attached to a cable (Fig. 3.31). The cable is wrapped around a pulley and attached to a weight pan. The weight in the weight pan stretches the cable and produces a tensile force $\underline{F}$ in the cable. This force is transmitted to the person's hand through the handle. The force makes an angle θ with the horizontal and applied to the hand at point B. Point A represents the center of gravity of the person's lower arm and O is a point along the center of rotation of the elbow joint. Assume that points O, A, and B and force $\underline{F}$ all lie on a plane surface.

If the horizontal distance between point O and A is $a = 15$ cm, distance between point O and B is $b = 35$ cm, total weight of the lower arm is $W = 20$ N, magnitude of the applied force is $F = 50$ N, and angle $\theta = 30°$, determine the net moment generated about O by $\underline{F}$ and $\underline{W}$.

Answer: $M_o = 5.75$ Nm (ccw)

Scanned with CamScanner

Moment = force x perpendicular distance.

Moment due to diver at B about O,

$M_B = W_1 \times l$ $\qquad$ W_1 = weight of diver 1 = $\underline{\underline{mg}}$

$= 86 \times 9.81 \times 4$

$= \underline{\underline{3374.64 \text{ Nm}}}$

Moment due to diver at A about O,

$M_A = W_2 \times l/2 = 82 \times 9.81 \times 2$

$= \underline{\underline{1608.84 \text{ Nm}}}$

Moment due to the board weight.

$M_W = W_b \times l/2 = 130 \times 9.81 \times 2$

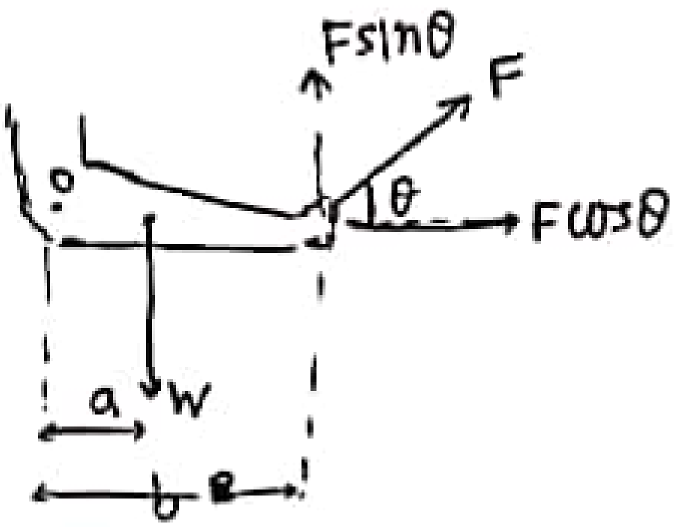
$= \underline{\underline{2550.6 \text{ Nm}}}$

∴ Total moment about O,

$M = 3374.64 + 1608.84 + 2550.6$

$= \underline{\underline{7534.08 \text{ Nm}}}$

Scanned with CamScanner



$a = 15\,cm$

$b = 35\,cm$

$W = 20\,N$

$F = \cancel{15N}\ 50\,N$

$\theta = 30^\circ$

$\tau =$ net moment about O = moment due of F + moment of W.

$= Wa$ clockwise + $F\sin\theta\, b$ Anticlockwise

~~to~~ Taking clockwise moment as +ve,

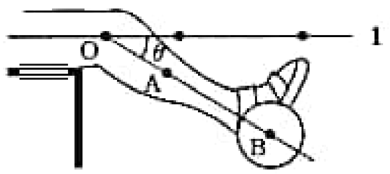
$$\bar{\tau}_0 = 20N \times 15\times10^{-2}m - 50N \times \sin30^\circ \times 35\times10^{-2}m$$

$$\bar{\tau}_0 = -5.75\ N\text{-}m$$

$$\boxed{\tau_0 = 5.75\ N\text{-}m\ (\text{Anticlockwise})}$$

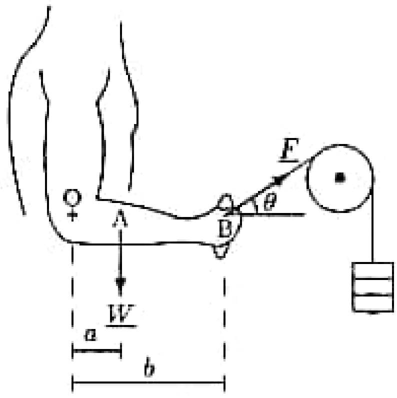
Scanned with CamScanner

Problem 3.6 As illustrated in Fig. 3.15a, consider a person doing lower leg exercises from a sitting position wearing a weight boot. The weight of the boot is $W_2 = 65$ N. Furthermore, as measured from the knee joint at point O, the center of gravity of the lower leg (point A) is located at a distance $a = 23$ cm and the center of gravity of the weight boot (point B) is located at a distance $b = 55$ cm from the point O. If the net moment about the knee joint is $M_{net} = 34$ Nm, determine the weight of the person's lower leg (W_1), when the leg makes an angle $\theta = 45°$ with the horizontal.



Answer: $W_1 = 53.8$ N

Problem 3.7 Consider a person doing arm exercises by using a pulley-based apparatus shown in Fig. 3.31. The person is holding a handle that is attached to a cable. The cable is wrapped around a pulley with the weight pan attached to the free end of the cable. Point A represents the center of gravity of the lower arm and O represents the point where the handle is attached to the cable. The weight of the lower arm is $W = 23$ N. The horizontal distances between points O and A, and O and B is ($a = 16$ cm) and ($b = 39$ cm), respectively. Furthermore, assume that all points as well as forces applied to the lower arm lie on the same plane surface. If the net moment about point O is $M_{net} = 6.3$ Nm and the cable attached to the handle makes an angle $\theta = 17°$ with the horizontal, determine the mass of the weight pan.



Answer: $m = 8.9$ kg

Scanned with CamScanner

①

Given Data

The weight of the boot is $w_2 = 65N$.

At A (point A) is located at a distance

$a = 23cm$.

At (A point B) is located at a distance

$b = 55cm$.

The net moment about the knee joint

is $m_{net} = 34 Nm$.

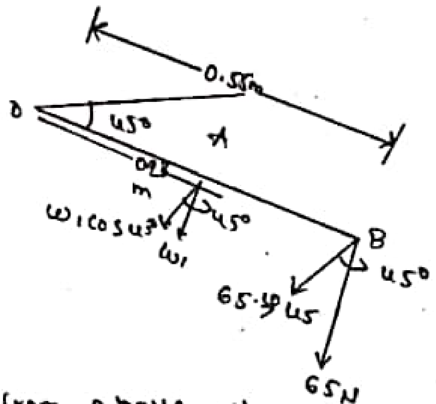
The weight of the person's lower leg

$= w_1$

The leg makes an angle $\theta = 45°$.

Given the diagram below.

line diagram.



⇒ we taken from above the line diagram.

②

$\boxed{moment = force \times distance}$

for combined effect, just add these two moment

Seperately generated by leg and boot

⇒ Now we combined the moment.

$$(w_1 \cos 45°)0.23 + 0.55(65 \cos 45°) = 34$$

$$\Rightarrow w_1 (0.7)(0.23) + 0.55(65(0.707)) = 34$$

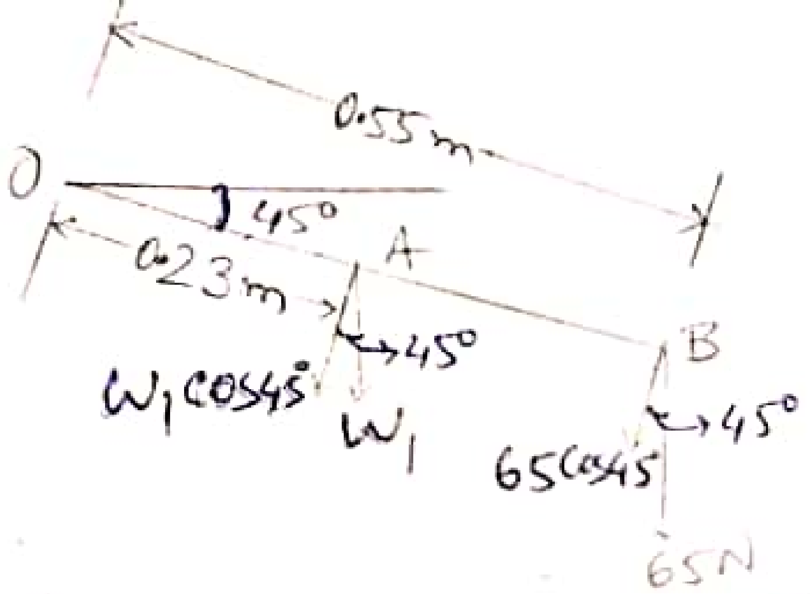
$$\Rightarrow w_1 0.161 + 25.025 = 34$$

$$\Rightarrow w_1 0.16 + 34 - 25.025$$

$$\Rightarrow w_1 = \frac{8.975}{0.16}$$

$$\Rightarrow \boxed{w_1 = 56.09375N}$$

Scanned with CamScanner



There fore the combined moment,

$$(W_1 \cos 45°)\,0.23 + 0.55\,(65 \cos 45°) = 34$$

or, $0.163 W_1 + 25.279 = 34$

or, $W_1 = \dfrac{8.72}{0.163}$

$\therefore \quad W_1 = \underline{53.5 \text{ N}}$

Scanned with CamScanner

Problem 3.8 Figure 3.32 illustrates a simplified version of a hamstring strength training system for rehabilitation and athlete training protocols. From a seated position, a patient or athlete flexes the lower leg against a set resistance provided through a cylindrical pad that is attached to a load. For the position illustrated, the lower leg makes an angle θ with the horizontal. Point O represents the knee joint, point A is the center of gravity of the lower leg, W is the total weight of the lower leg, F is the magnitude of the force applied by the pad on the lower leg in a direction perpendicular to the long axis of the lower leg, a is the distance between points O and A, and b is the distance between point O and the line of action of $\underline{F}$ measured along the long axis of the lower leg.

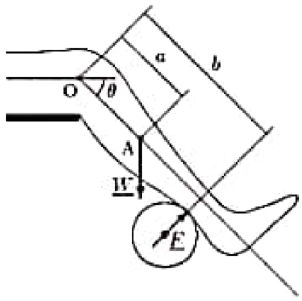


Fig. 3.32 *Problems 3.8 and 3.9*

(a) Determine an expression for the net moment about O due to forces $\underline{W}$ and $\underline{F}$.

(b) If $a = 20$ cm, $b = 40$ cm, $\theta = 30°$, $W = 60$ N, and $F = 200$ N, calculate the net moment about O.

Answers: (a) $M_O = bF - aW\cos\theta$ (b) $M_O = 69.6$ Nm (ccw)

Problem 3.9 As illustrated in Fig. 3.32, consider an athlete performing lower leg exercises from a seated position to strengthen hamstring muscles by using a special training system. The training system provides a set resistance to the leg through a cylindrical pad attached to the load while leg flexing. Point O represents the knee joint, point A is the center of gravity of the lower leg, W is the weight of the lower leg, θ defines an angle that the long axis of the lower leg makes with the horizontal, and M_O is the net moment about point O by forces applied to the leg.

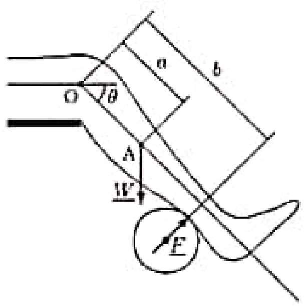


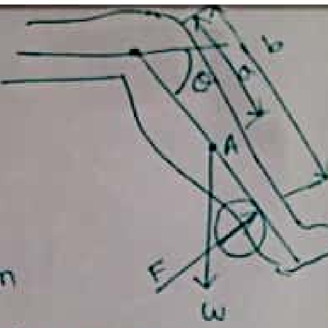
Fig. 3.32 *Problems 3.8 and 3.9*

(a) If $a = 0.23$ m, $b = 0.45$ m, $\theta = 45°$, $W = 65$ N, and $M_O = 86.14$ Nm, determine the magnitude of the set resistance (F).

(b) Considering the same position of the lower leg, calculate the net moment about point O when the set resistance is increased by 10 N.

Answer: (a) $F = 215$ N; (b) $M_O = 90.64$ Nm

(a)



a = 0.23 m
b = 0.45 m
θ = 45°
W = 65 N

m_o = 86.14 Nm

but $m_o = F \times b - W a \cos\theta$

$F = \dfrac{m_o + W a \cos\theta}{b} = \dfrac{86.14 + (65 \times 0.23)\cos 45}{0.45}$

$\boxed{F = 214.9135 \text{ N}}$

(b)

$F' = 214.9135 + 10$

$F' = 224.913$ N

$m_o = F' \times b - W a \cos\theta$

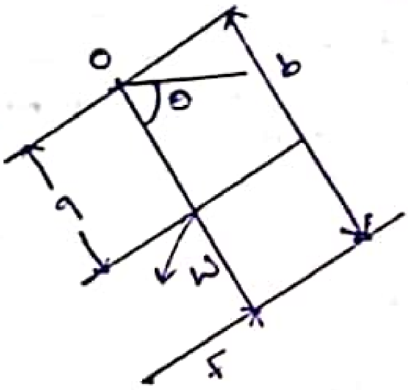
$= (224.913 \times 0.45) - (65 \times 0.23 \times \cos 45°)$

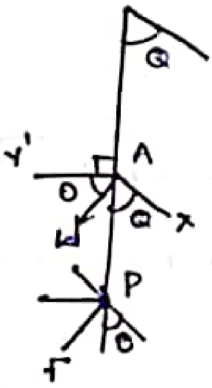
$\boxed{m_o = 90.64 \text{ N-m}}$

Scanned with CamScanner

Answer

①





At point A

$\angle WAX = 90$

$\therefore \angle PAW = 90 - \theta$.

$\angle Y'OP = 90$

$\angle Y'AW = \theta$

Now we can get component of w along and $\perp^r$ to the leg:

$\therefore$ Moment about O (Clockwise positive).

$M_o \equiv F(b) - (W\cos\theta)(a)$.

$\therefore M_o = Fb - W\cos\theta\, a$ = Moment about O:

if $a = 20cm = 0.2\,m$, $b = 40cm = 0.4m$

$\theta = 30°$ $W = 60N$, $F = 200N$.

$\Rightarrow M_o = (200)(0.4) - 60\cos 30 \cdot (0.2)$

CS Scanned with CamScanner

②

$= 80 - 60 \frac{\sqrt{3}}{2}(0.2)$.

$= 80 - 6\sqrt{3}$.

$= 69.60$ N-m

Scanned with CamScanner

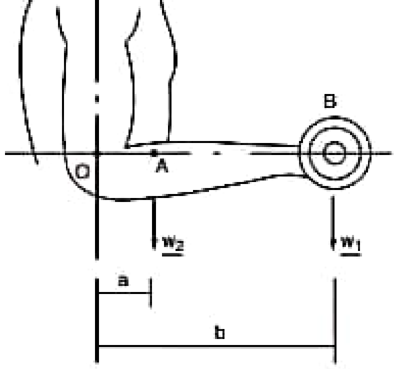


Fig. 3.33 *An athlete performing lower arm exercises*

Problem 3.10 As illustrated in Fig. 3.33, consider an athlete performing flexion/extension exercises of the lower arm to strengthen the biceps muscles. The athlete is holding the weight of $W_1 = 150$ N in his hand, and the weight of his lower arm is $W_2 = 20$ N. As measured from the elbow joint at point O, the center of gravity of the lower arm (point A) is located at a distance $a = 7.5$ cm and the center of gravity of the weight held in the hand is located at a distance $b = 32$ cm.
Determine the net moment generated about the elbow joint, when the lower arm is extended horizontally and when the long axis of the lower arm makes an angle $f = 30°$ and $f = 60°$, respectively, with the horizontal.

Answer: $M_{net}(f = 0°) = 49.5\text{Nm}$; $M_{net}(f = 30°) = 42.9\text{Nm}$; $M_{net}(f = 60°) = 44.8\text{Nm}$

Problem 3.11 Figure 3.34 illustrates a bench experiment designed to test the strength of materials. In the case illustrated, an intertrochanteric nail that is commonly used to stabilize fractured femoral heads is firmly clamped to the bench such that the distal arm (BC) of the nail is aligned vertically. The proximal arm (AB) of the nail has a length a and makes an angle θ with the horizontal.

As illustrated in Fig. 3.34, the intertrochanteric nail is subjected to three experiments by applying forces $\underline{F}_1$ (horizontal, toward the right), $\underline{F}_2$ (aligned with AB, toward A), and $\underline{F}_3$ (vertically downward). Determine expressions for the moment generated at point B by the three forces in terms of force magnitudes and geometric parameters a and θ.

Answers: $M_1 = aF_1 \sin\theta$ (cw) $M_2 = 0$ $M_3 = aF_3 \cos\theta$ (ccw)

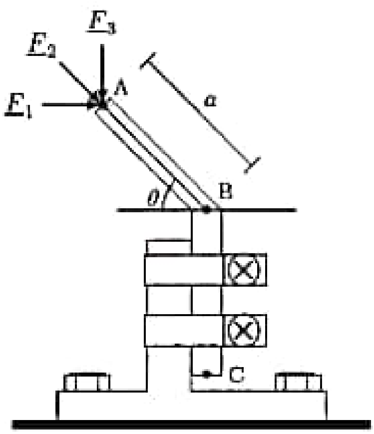
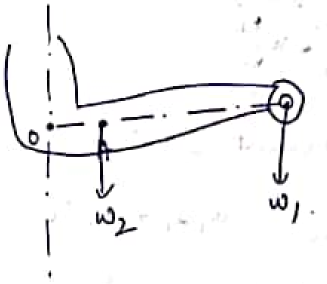


Fig. 3.34 *A bench test*

Scanned with CamScanner

Given weight of weight hold by arm = $\underline{150\ N} = w_1$

(i) when lower arm is horizontal



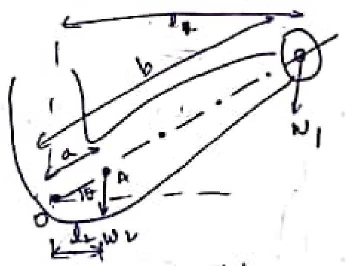
Net moment about 'o'

$$= w_1(b) + w_2(a)$$

$$= (150)\left(\frac{32}{100}\right) + (20)\left(\frac{7.5}{100}\right)$$

$$= \underline{49.5\ N\text{-}m}.$$

(ii) when axis of lower arm makes $\theta = 30^\circ$ with horizontal.



Net moment about 'o'

$$M_o = w_1 l_1 + w_2 l_2$$

$$\therefore \cos\theta = \frac{l_2}{a} = \frac{l_1}{b}.$$

$$M_o = (150)\left(\frac{32}{100}\right)\cos 30^\circ + (20)\left(\frac{7.5}{100}\right)\cos 30^\circ$$

$$= \underline{(49.5)\cos 30^\circ}$$

$$= 42.868\ N\text{-}m.$$

(iii) similiarly at $\theta = 60^\circ$.

$$M_o = w_1 l_1 + w_2 l_2$$

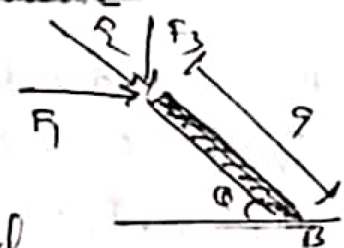
$$= (150)\left(\frac{32}{100}\right)\cos 60 + \frac{(20)(7.5)}{100}\cos 60^\circ$$

$$= (49.5)\cos 60^\circ$$

$$= \underline{24.75}\ N\text{-}m$$

Scanned with CamScanner

at point 'A' the forces



$F_1 \rightarrow$ horizontal

$F_2 \rightarrow$ along AB

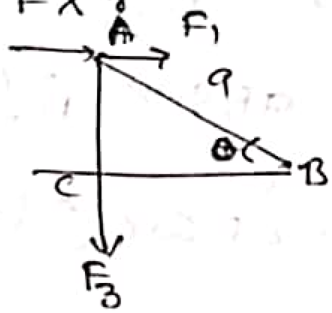
$F_3 \rightarrow$ vertically downward.

the nail of length is "a" is at an angle 'θ' to the horizontal.

moment or Torque = Force $\times \perp^{r}$ distance

$\bar{\tau}$ or $\bar{m} = \bar{F} \times \bar{r}$

$\triangle ABC$.



$\sin\theta = \frac{AC}{a}$

$AC = a\sin\theta$

$\cos\theta = \frac{BC}{a}$

$BC = a\cos\theta$.

$\Rightarrow$ the moment due to force F_1 about 'B'

$\bar{M}_1 = F_1 \times AC = F_1 \times a\sin\theta$

$\bar{m}_1 = aF_1\sin\theta$ clock wise

$\Rightarrow$ the moment due to force F_3 about B

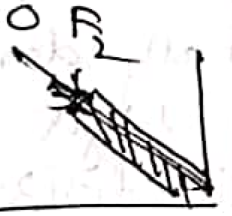
$\bar{M}_3 = F_3 \times BC = F_3 \times a\cos\theta$

$\bar{m}_3 = aF_3\cos\theta$ anti clock wise

the moment due to Force F_2 is M_2

$\bar{m}_2 =$ Force $\times \perp^{r}$ distance

$\bar{m}_2 = F_2 \times 0 = 0$



along the line of axis torque or moment is zero. that $\perp^{r}$ distance $= 0$

$r = 0$, $M_2 = 0$

$\bar{M}_1 = aF_1\sin\theta$ (C.W)

$\bar{M}_2 = 0$

$\bar{M}_3 = aF_3\cos\theta$ (CCW)

Scanned with CamScanner

Problem 3.12 The simple structure shown in Fig. 3.35 is called a *cantilever beam* and is one of the fundamental mechanical elements in engineering. A cantilever beam is fixed at one end and free at the other. In Fig. 3.35, the fixed and free ends of the beam are identified as points A and C, respectively. Point B corresponds to the center of gravity of the beam.

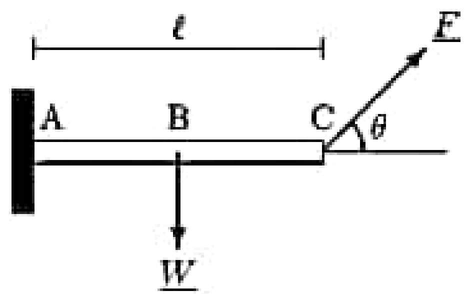


Fig. 3.35 *A cantilever beam*

Assume that the beam shown has a weight $W = 100$ N and a length $l = 1$ m. A force with magnitude $F = 150$ N is applied at the free end of the beam in a direction that makes an angle $\theta = 45°$ with the horizontal.

Determine the magnitude and direction of the net moment developed at the fixed end of the beam.

Answer: $M_A = 56$ Nm (ccw)

Problem 3.13 As illustrated in Fig. 3.36, consider a cantilever beam of 9 kg mass. The beam is fixed to the wall at point A and point B represents the free end of the beam. The length of the beam is $l = 4$ m. Point C is the center of gravity of the beam. The weight of $W_1 = 50$ N is attached at the free end of the beam such that the distance between points B and D is $l = 35$ cm. A force with magnitude $F = 180$ N is applied at the free end of the beam to keep the beam in place. The line of action of the force makes an angle $\beta = 35°$ with the horizontal.

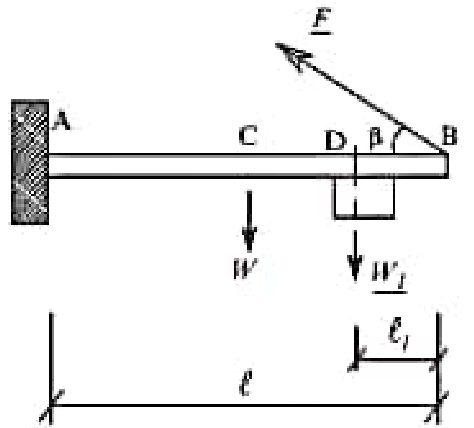


Fig. 3.36 *A cantilever beam with weight W_1 attached at free end*

Determine the magnitude and direction of the net moment generated about point A.

Answer: $M_{net} = 54.1$ Nm (ccw)

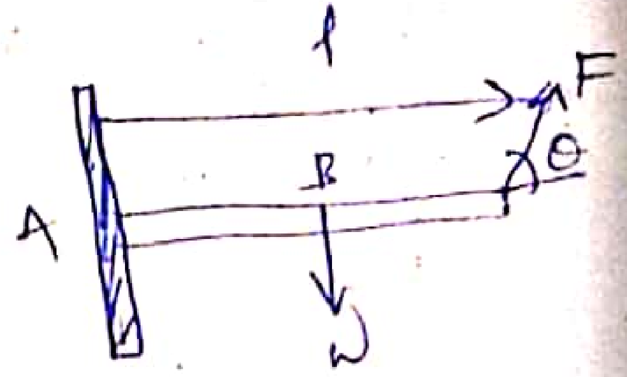
Scanned with CamScanner

$W = 100N$

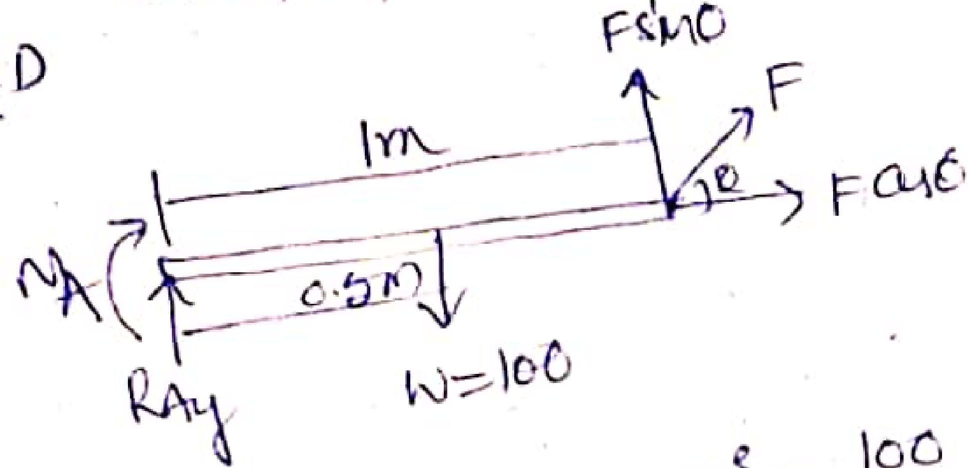
$l = 1m$

$F = 150N$

$\theta = 45°$



<u>FBD</u>



$\Sigma F_y = 0$

$R_{Ay} + 150 \sin 45° = 100$

$R_{Ay} = 100 - 150 \sin 45°$

$R_{Ay} = 100 - 106.06 N$

$R_{Ay} = 6.06 N \downarrow$

$R_{Ay} = 6.06 N \downarrow$

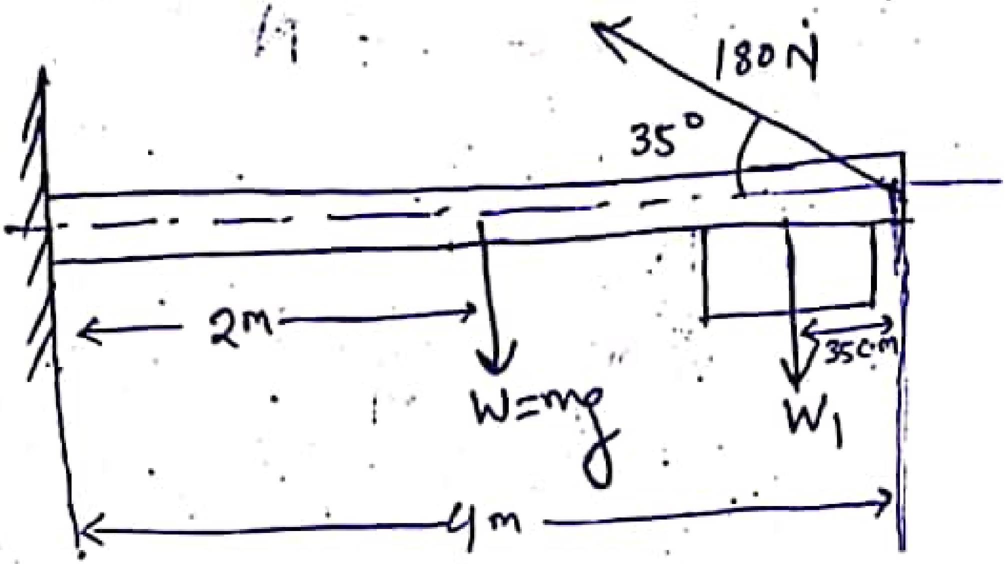
$M_A = (100 \times 0.5) - (150 \times \sin 45° \times 1)$

$M_A = 50 - 106.06$

$M_A = -56.06$ N.m

$M_A = 56.06$ N.m

Scanned with CamScanner



Given $m = 9\,kg$

So $W = mg = 9 \times 9.81 = 88.2\,N$.

$W_1 = 50\,N$ (Given)

Taking counter clock wise moments as (-ve) and clock wise moments as (+ve);

The net moment at 'A'

$$M_A = -180 \sin(35^\circ) \times 4 + W \times 2 + W_1 \times (4 - 0.35)$$

$$= -180 \sin(35^\circ) \times 4 + 88.2 \times 2 + 50 \times 3.65$$

$$= -54.075\ N.m$$

As it is coming (-ve) So net moment at 'A' is counter clock wise.

Scanned with CamScanner

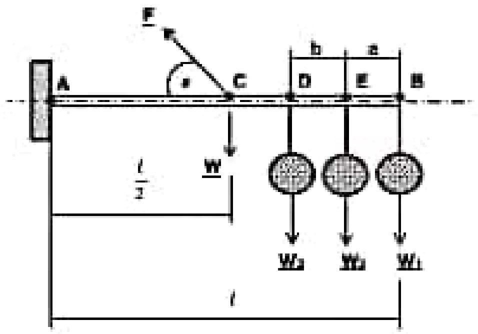


Fig. 3.37 *Problem 3.14*

Problem 3.14 As illustrated in Fig. 3.37, consider a structure consisting of a cantilever beam with three identical spherical electrical fixtures attached by cables at the free end of the beam and at equal distances from each other ($a = b$). The beam is fixed to the wall at point A. Point B identifies the free end of the beam. It is also a point where the first electrical fixture is attached to the beam. D and E are points where the second and third electrical fixtures are attached to the beam. Point C identifies the center of gravity of the beam and it is equidistant from points A and B. The weight of the beam is $W = 150$ N and the weight of the electrical fixtures is $W_1 = W_2 = W_3 = 49$ N. Furthermore, a force $F = 230$ N is applied to the beam at point D to keep the beam in place with the line of action of the force making an angle θ with the horizontal.

(a) If $a = b = 0.5$ m, $\theta = 45°$, and the net moment about point A is $M_{net} = 267.2$ Nm, determine the length (l) of the beam.
(b) If the length of the beam is increased by 50 cm, calculate the net moment about point A.
(c) If the magnitude of force $\underline{F}$ is decreased by 30 N, calculate the net moment about point A when the length of the beam is $l = 3.0$ m.
(d) If the force $\underline{F}$ is applied in the direction perpendicular to the long axis of the beam, calculate the net moment about point A when the length of the beam is $l = 3.0$ m.

Answers: (a) $l = 3.0$ m; (b) $M_{net} = 296.9$ Nm; (c) $M_{net} = 309.7$ Nm; (d) $M_{net} = 192.5$ Nm

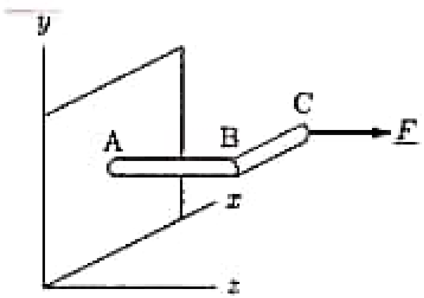


Fig. 3.38 *Problem 3.15*

Problem 3.15 Consider the L-shaped beam illustrated in Fig. 3.38. The beam is mounted to the wall at point A, the arm AB extends in the z direction, and the arm BC extends in the x direction. A force $\underline{F}$ is applied in the z direction at the free end of the beam.

If the lengths of arms AB and BC are a and b, respectively, and the magnitude of the applied force is F, observe that the position vector of point C relative to point A can be written as $\underline{r} = b\underline{i} + a\underline{k}$ and the force vector can be expressed as $\underline{F} = F\underline{k}$, where $\underline{i}$, $\underline{j}$, and $\underline{k}$ are unit vectors indicating positive x, y, and z directions, respectively.

(a) Using the cross product of position and force vectors, determine an expression for the moment generated by $\underline{F}$ about point A in terms of a, b, and F.
(b) If $a = b = 30$ cm and $F = 20$ N, calculate the magnitude of the moment about point A due to $\underline{F}$.

Answers: (a) $\underline{M}_A = -b\,F\underline{j}$; (b) $\underline{M}_A = 6$ Nm

Scanned with CamScanner

a)

Since, in the statement of question it is given that force is applied at D, but in figure it is applied at C. therefore I will go as per statement of question and consider the force at D.

Consider moment about A.

$$-F\sin\theta\times\frac{l}{2}+W\times\frac{l}{2}+W_1 l+W_2(l-a)+W_3\times(l-a-b)=M_{net}$$

Substitute the values.

$$-230\sin 45\times(l-0.5-0.5)+150\times\frac{l}{2}+49\times l+49(l-0.5)+49\times(l-0.5-0.5)=267.2$$

$$+89.1345+59.365l=267.2$$

$$59.365l=178.07$$

$$l=3\text{ m}$$

---

b)

Consider moment about A when length increases by 50 cm ($new\ l=3+0.5=3.5\text{ m}$).

$$-F\sin\theta\times\frac{l}{2}+W\times\frac{l}{2}+W_1 l+W_2(l-a)+W_3\times(l-a-b)=M_{net}$$

Substitute the values.

$$-230\sin 45\times(l-0.5-0.5)+150\times\frac{l}{2}+49\times l+49(l-0.5)+49\times(l-0.5-0.5)=M_{net}$$

$$+89.1345+59.365l=M_{net}$$

$$M_{net}=89.1345+59.365\times3.5$$

$$=296.9\text{ N.m}$$

---

c)

Consider moment about A when F is increase by 30N ($new\ F=230+30=260\text{ N}$).

$$-F\sin\theta\times\frac{l}{2}+W\times\frac{l}{2}+W_1 l+W_2(l-a)+W_3\times(l-a-b)=M_{net}$$

Substitute the values.

$$-200\sin 45\times(l-0.5-0.5)+150\times\frac{l}{2}+49\times l+49(l-0.5)+49\times(l-0.5-0.5)=M_{net}$$

$$+67.92+80.576l=M_{net}$$

$$M_{net}=67.92+80.576\times3$$

$$M_{net}=309.65\text{ N-m}$$

---

d)

Consider moment about A when F is perpendicular to beam ($new\ \theta=90°, F=200\text{ N}$).

$$-F\sin\theta\times\frac{l}{2}+W\times\frac{l}{2}+W_1 l+W_2(l-a)+W_3\times(l-a-b)=M_{net}$$

Substitute the values.

$$-200\sin 90\times(l-0.5-0.5)+150\times\frac{l}{2}+49\times l+49(l-0.5)+49\times(l-0.5-0.5)=M_{net}$$

$$+126.5+22l=M_{net}$$

$$M_{net}=126.5+22\times3$$

$$M_{net}=192.5\text{ N-m}$$

Scanned with CamScanner

3.15 $\vec{F} = F\hat{k}$ . . . (1)

$\vec{r} = b\hat{i} + a\hat{k}$ . . . (2)

(a) now, moment of the force $\vec{F}$ about A

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= (b\hat{i} + a\hat{k}) \times F\hat{k}$$

or, $\vec{\tau} = bF\,\hat{i}\times\hat{k} + aF\,\hat{k}\times\hat{k}$

$$= bF(-\hat{j}) + \vec{0}$$

now, $\hat{i}\times\hat{k} = -\hat{j}$

and, $\hat{k}\times\hat{k} = \vec{0}$

or, $\vec{\tau} = bF(-\hat{j})$ [Answer]

(b) Given $a = b = 30$ cm. $= 0.3$ m.

and $F = 20$ N.

now magnitude of $\vec{\tau} = |\vec{\tau}| = bF$

or, $|\vec{\tau}| = 0.3 \times 20$ N.m

or, $\boxed{|\vec{\tau}| = 6.0\ \text{Nm}}$ (Answer)

Scanned with CamScanner

Problem 3.16 As shown in Fig. 3.39, consider a worker using a special wrench to tighten bolts. The couple-moment generated about the long axis of the wrench is $M_c = 100$ Nm and the distance between the handles of the wrench and the long axis is $r = 25$ cm.

Determine the force exerted by the worker on the handles of the wrench.

Answer: F = 200 N

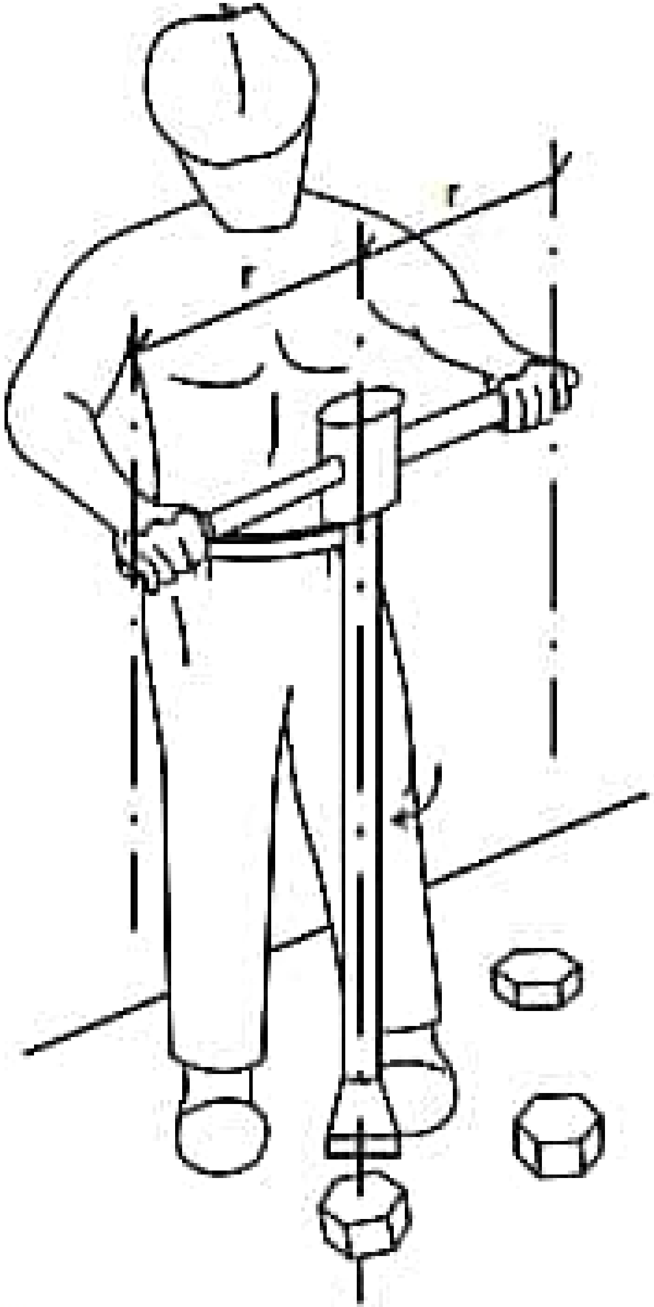


Fig. 3.39 *Worker using a special wrench*

Scanned with CamScanner

We know the torque $Mc = F*r$

here r is the total distance = $2r = 2*25 = 50$ cm = 0.5 m

hence $F = 100 / 0.50 = 200$ N........Ans.

Scanned with CamScanner

Problem 4.1 As illustrated in Fig. 4.12, consider a person standing on a uniform horizontal beam that is resting on frictionless knife-edge and roller supports. A and B are two points that the contact between the beam and the knife-edge and roller support, respectively. Point C is the center of gravity of the beam and it is equidistant from points A and B. D is the point on the beam directly under the center of gravity of the person. Due to the weights of the beam and the person, there are reactions on the beam at points A and B. If the weight of the person is $W = 625$ N and the reactions at points A and B are $R_A = 579.4$ N and $R_B = 735.6$ N,

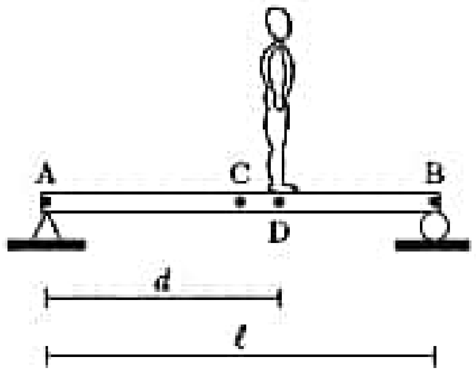


Fig. 4.12 *Example 4.1*

(a) Determine the weight (W) of the beam.
(b) Determine the length (l) of the beam.

Answers: (a) $W = 690$ N; (b) $l = 4$ m

Problem 4.2 As illustrated in Fig. 4.49, consider an 80 kg person preparing to dive into a pool. The diving board is represented by a uniform, horizontal beam that is hinged to the ground at point A and supported by a frictionless roller at point D. B is a point on the board directly under the center of gravity of the person. The distance between points A and B is $l = 6$ m and the distance between points A and D is $d = 2$ m. (Note that one-third of the board is located on the left of the roller support and two-thirds is on the right. Therefore, for the sake of force analyses, one can assume that the board consists of two boards with two different weights connected at point D.)
If the diving board has a total weight of 1500 N, determine the reactions on the beam at points A and D.

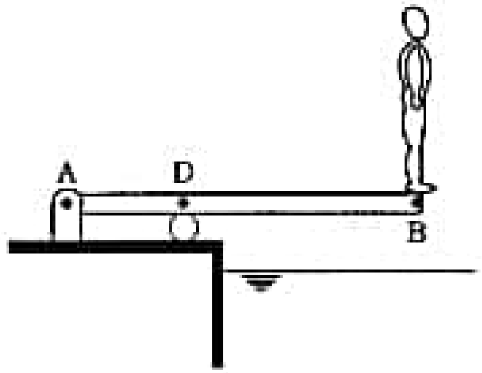
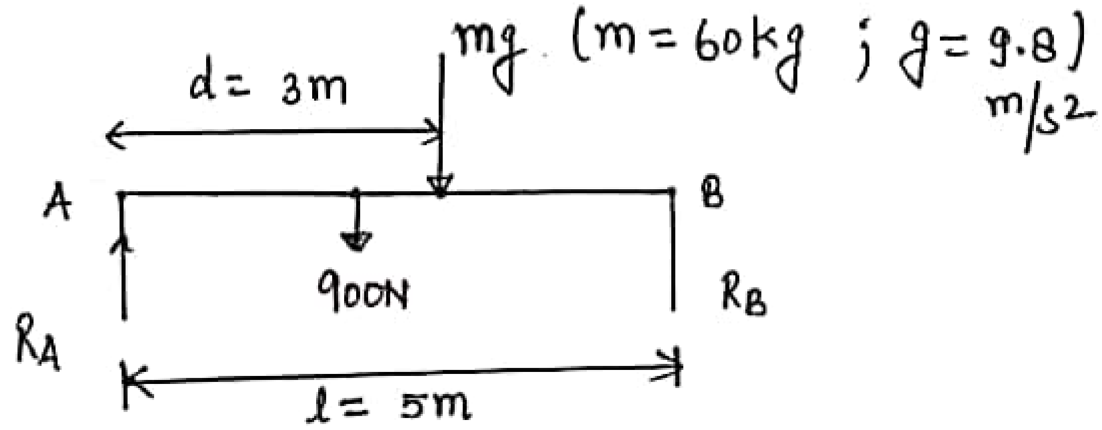


Fig. 4.49 *Problem 4.2*

Answers: $R_A = 2318$ N (↓) $R_D = 4602$ N (↑)

Scanned with CamScanner



force balance Equation :—

$$R_A + R_B = 900 + mg \quad \text{— (I)}$$

$$R_A + R_B = 900 + 60 \times 9.8$$

$$R_A + R_B = 1488 \quad \text{— (I)}$$

Moment about point A is :—

$$R_B \times 5 = 900 \times 5/2 + mg \times d \quad \text{— (II)}$$

$$R_B = \frac{900 \times 5/2 + 60 \times 9.8 \times 3}{5}$$

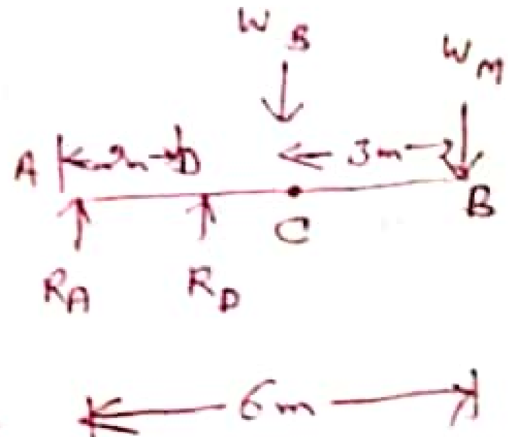
$R_B = 802.8$ N. put in (I) ; then—

$$R_A = 685.2 \text{ N}$$

Scanned with CamScanner

$W_M = 80 \times 9.81 = 784.8\,N$

$W_B = 1500\ N$



$\underline{\Sigma Y = 0}$

$$R_A + R_D = 1500 + 784.8$$

$$R_A + R_D = 2284.8\,N$$

$\underline{\Sigma m_A = 0}$

$$R_D \times 2 - 1500 \times 3 - 784.8 \times 6 = 0$$

$$\boxed{R_D = 4604.4\ N\ \uparrow}$$

and $R_A = 2284.8 - 4604.4$

$$R_A = -2319.6\,N$$

$$\boxed{R_A = 2319.6\,N\ \downarrow}$$

Scanned with CamScanner

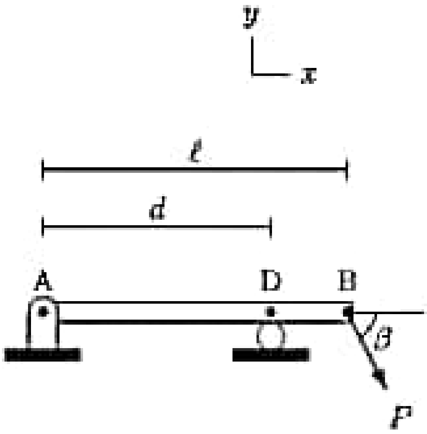


Fig. 4.50 *Problem 4.3*

Problem 4.3 The uniform, horizontal beam shown in Fig. 4.50 is hinged to the ground at point A and supported by a frictionless roller at point D. The distance between points A and B is $l = 4$ m and the distance between points A and D is $d = 3$ m. A force that makes an angle $\beta = 60°$ with the horizontal is applied at point B. The magnitude of the applied force is $P = 1000$ N. The total weight of the beam is $W = 400$ N.

By noting that three-quarters of the beam is on the left of the roller support and one-quarter is on the right, calculate the x and y components of reaction forces on the beam at points A and D.

Answers: $R_D = 1421\,\text{N}\,(\uparrow)$ $R_{Ax} = 500\,\text{N}\,(\leftarrow)$ $R_{Ay} = 155\,\text{N}\,(\downarrow)$

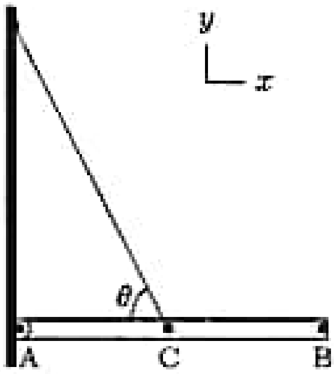


Fig. 4.51 *Problem 4.4*

Problem 4.4 The uniform, horizontal beam shown in Fig. 4.51 is hinged to the wall at point A and supported by a cable attached to the beam at point C. Point C also represents the center of gravity of the beam. At the other end, the cable is attached to the wall so that it makes an angle $\theta = 68°$ with the horizontal. If the length of the beam is $l = 4$ m and the weight of the beam is $W = 400$ N, calculate the tension T in the cable and components of the reaction force on the beam at point A.

Answers: $T = 431$ N $R_{Ax} = 162\,\text{N}\,(\rightarrow)$ $R_{Ay} = 0$

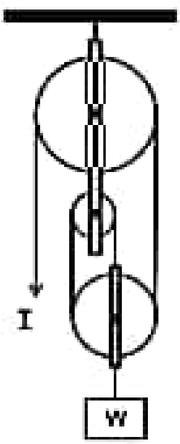


Fig. 4.52 *Problem 4.5*

Problem 4.5 Consider a structure illustrated in Fig. 4.52. The structure includes a horizontal beam hinged to the wall at point A and three identical electrical fixtures attached to the beam at points B, D, and E with point B identifying the free end of the beam. Moreover, the distances between the points of attachment of the electrical fixtures are equal to each other (BD = DE = $a = b = 35$ cm). Point C identifies the center of gravity of the beam and it is equidistant from points A and B. The weight of the beam is $W = 230$ N and each electrical fixture weighs $W_1 = W_2 = W_3 = 45$ N. Furthermore, a cable is attached to the beam at point B making an angle $\alpha = 45°$ with the horizontal. On another end the cable is attached to the wall to keep the beam in place. If the length of the beam is $l = 2.5$ m,

(a) Determine the tension (T) in the cable.
(b) Determine the magnitude of the reaction force (R_A) at point A.
(c) Determine the tension (T_1) in the cable when it makes an angle $\alpha = 65°$ with the horizontal.
(d) Determine change in the magnitude of the reaction force at point A when the cable makes an angle $\alpha = 65°$ with the horizontal.

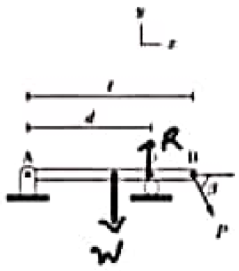
Answers: (a) $T = 817.2$ N; (b) $R_A = 615.7$ N; (c) $T_1 = 634.8$ N; (d) 44.6% decrease

Scanned with CamScanner

Problem 4.2 The uniform, horizontal beam shown in Fig. 4.50 is hinged to the ground at point A and supported by a frictionless roller at point D. The distance between points A and B is $l = 4$ m and the distance between points A and D is $d = 3$ m. A force that makes an angle $\beta = 60°$ with the horizontal is applied at point B. The magnitude of the applied force is $P = 1{,}000$ N. The total weight of the beam is $W = 400$ N.

By noting that three-quarters of the beam is on the left of the roller support and one-quarter is on the right, calculate the x and y components of reaction forces on the beam at points A and D.

Answers: $R_D = 1{,}421$ N(↑), $R_{Ax} = 500$N(←), $R_{Ay} = 155$N(↓)



$l = 4\,m \qquad d = 3\,m \qquad \beta = 60°$

$P = 1000\,N \qquad W = 400\,N$

First we can take sum of torques about point A
reaction force at point d will be in vertical direction

$$R(d) - (P\sin\beta)l - W\left(\frac{l}{2}\right) = 0$$

$$R(3) - (1000\sin 60)(4) - 400\left(\frac{4}{2}\right) = 0$$

$$3R - 3464.1 - 800 = 0$$

$$\boxed{R = 1421\ N} \uparrow$$

(b) We can find out x and y component of reaction at point A using sum of vertical and horizontal forces separately

$$A_x + P\cos\beta = 0$$

$$= -1000\cos 60$$

$$A_x = -500\,N \quad \text{or} \quad \boxed{A_x = 500\,N\ (\leftarrow)}$$

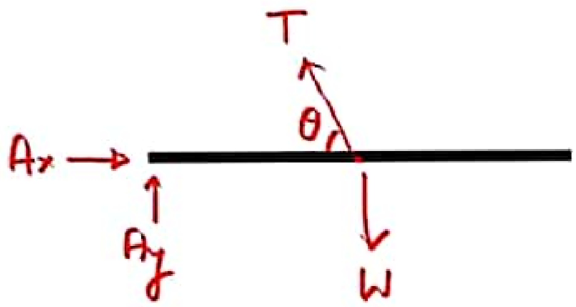
$$A_y - W + R - P\sin\beta = 0$$

$$A_y - 400 + 1421 - 1000\sin 60 = 0$$

$$A_y = -154.97$$

$$\text{or} \quad \boxed{A_y = 155\ (\downarrow)}$$

Scanned with CamScanner



$\theta = 68^\circ$

$W = 400$ N

$l = 4$ m

Since rod is in equilibrium, Moment about A should be zero.

$\boxed{\Sigma M_A = 0}$ $\quad \frac{l}{2} W - \frac{l}{2} T \sin\theta = 0$

$$T = \frac{W}{\sin\theta} = \frac{400}{\sin 68} = 431.4\ N$$

$$\boxed{T = 431.4\ N}$$

$\boxed{\Sigma f_x = 0}$ $\quad A_x - T\cos\theta = 0$

$$A_x = T\cos\theta = 431.4 \cos 68^\circ$$

$$\boxed{A_x = 161.6 \simeq 162\ N}$$

$\boxed{\Sigma f_y = 0}$ $\quad A_y + T\sin\theta - W = 0$

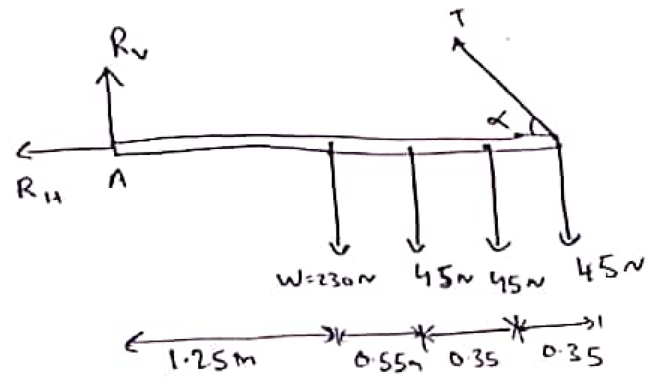
$$\boxed{A_y = 0}$$

Final Answer

$T = 431.4 \simeq 431\ N$, $R_{Ax} = 162\ N$, $R_{Ay} = 0$

Scanned with CamScanner

F.B.D of Rod



Applying equilibrium condition

$\Sigma F_x = 0$

$$R_H + T\cos\alpha = 0$$

$$\boxed{R_H = -T\cos\alpha} \quad \text{— ①}$$

(-ve sign indicates opposite to direction taken i.e towards right)

$\Sigma F_y = 0$

$$R_v + T\sin\alpha - 230 - 45 - 45 - 45 = 0$$

$$\boxed{R_v + T\sin\alpha - 335 = 0} \quad \text{— ②}$$

$\Sigma M_A = 0$

$$T\sin\alpha \times 2.5 - 45\times 2.15 - 45\times 2.15 - 45\times 1.8 - 230\times 1.25 = 0$$

$$T\sin\alpha \times 2.5 = 577.75$$

$$\boxed{T\sin\alpha = 231.1}$$

$$\boxed{T = \frac{231.1}{\sin\alpha}} \quad \text{— ③}$$

Solving eqn's ①, ② & ③

$$\boxed{R_v = 103.9 \text{ N}}$$

$$\boxed{R_H = -231.1\cot\alpha}$$

$$R_A = \sqrt{R_v^2 + R_H^2}$$

$$R_A = \sqrt{(103.9)^2 + (-231.1)^2\cot^2\alpha}$$

Scanned with CamScanner

$R_A = \sqrt{R_V^2 + R_H^2}$

$$\boxed{R_A = \sqrt{(103.9)^2 + (231.1)^2 \cot^2\alpha}}$$

Takin $\alpha = 45°$

(a) $\boxed{T = 326.82\ N}$

(b) $\boxed{R_A = 253.4\ N}$

(c) Taking $\alpha = 65°$

$T = \dfrac{231.1}{\sin 65°}$

$\boxed{T = 254.99\ N}$

(d) $\boxed{R_A = 149.7\ N}$

$\%\ \text{change} = \dfrac{253.4 - 149.7}{253.4} \times 100$

$\boxed{\%\ \text{change} = 40.92\%\ (\text{decrease})}$

Note

I think the answers given below question's are wrong ~~in the and~~ I have tried this question 3 times and I got the same answers so I am sure that answers Provided by me are correct.

Scanned with CamScanner

Problem 4.6 Using two different cable-pulley arrangements shown in Fig. 4.53, a block of weight W is elevated to a certain height. For each system, determine how much force is applied to the person holding the cable.

Answers: $T_1 = W/2 \quad T_2 = W/4$

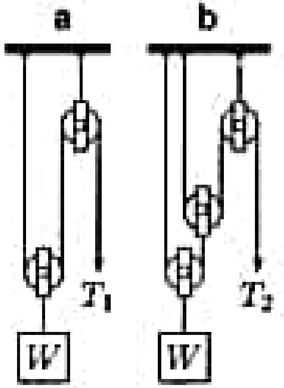


Fig. 4.53 *Problems 4.6 and 4.7*

Problem 4.7 As illustrated in Fig. 4.53b, consider a person who is trying to elevate a load to a certain height by using a cable-pulley arrangement. If the force applied by the person on the cable is $T = 65$ N, determine the mass of the load.

Answer: $m = 26.5$ kg

Problem 4.8 Using a cable-pulley arrangement shown in Fig. 4.54, a block of mass $m = 50$ kg is elevated from the ground to a certain height. Determine the magnitude of force T applied by the worker performing the lifting task on the cable.

Answer: $T = 163.3$ N

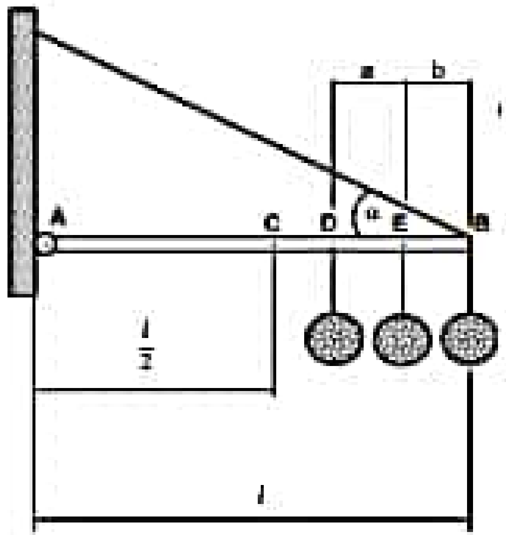


Fig. 4.54 *Problem 4.8*

Scanned with CamScanner

4.6)

for the pulley attached to weight Was the net force is Zero

$2 * T1 - W = 0$ $T1 = W/2$ the tension T1 is **W/2**

fig B)

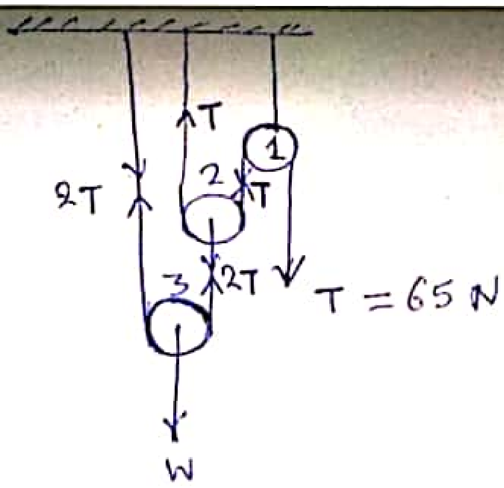
for the lowest pulley tension in the attached string is T3

$2 * T3 - W = 0$ $T3 = W/2$

Now, for the middle pulley $2 * T2 - T3 = 0$

$T2 = W/4$ the tension T2 is **W/4**

Scanned with CamScanner



Since, all the pulley are frictionless so the tension in each side of pulley in same string will be same.

g = Acceleration due to gravity
= 9.81 ~~ktn~~ m/s2

For pulley 2.

$T_2 = T + T = 2T$

For pulley 3.

$W = 2T + 2T$

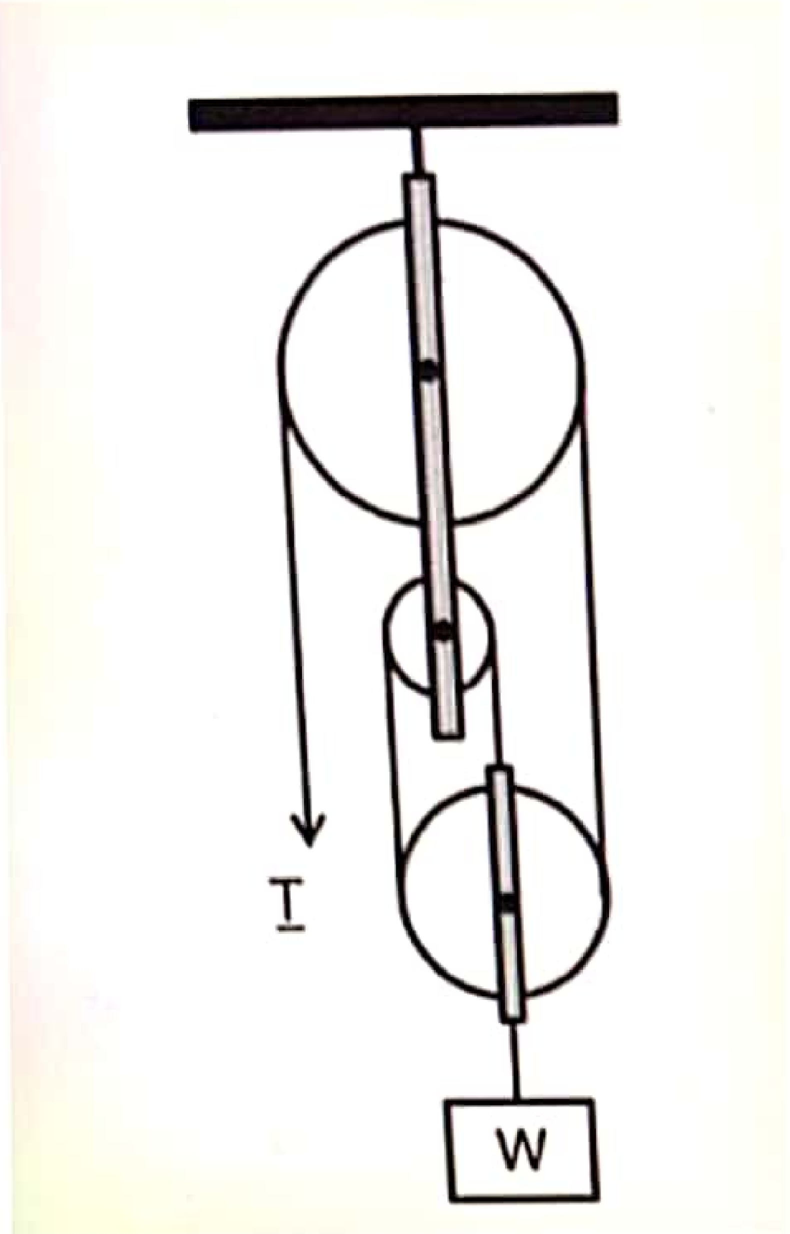
$W = 4T$

$mg = 4T$

$m = \frac{4T}{g} = \frac{4 \times 65}{9.81}$

$\boxed{m = 26.5 \text{ kg}}$

Scanned with CamScanner



Mass of the block $m = 50\,kg$

Forces acting on the block ,

i) weight of block $W = mg$, vertically downwards

ii) $3T$ upwards (T is tension in the string , as there are 3 string attached to lower pulley to pull it upwards)

$W = 3T$ $mg = 3T$

$$T = \frac{mg}{3} = \frac{50 * 9.8}{3} = 163.3\,N$$

Tension in the string is $\boxed{T = 163.3\,N}$

Scanned with CamScanner

Problem 4.9 Consider the split Russel traction device and a mechanical model of the leg shown in Fig. 4.55. The leg is held in the position shown by two weights that are connected to the leg via two cables. The combined weight of the leg and the cast is $W = 300$ N. l is the horizontal distance between points A and B where the cables are attached to the leg. Point C is the center of gravity of the leg including the cast which is located at a distance two-thirds of l as measured from point A. The angle cable 2 makes with the horizontal is measured as $\beta = 45°$.

Determine the tensions T_1 and T_2 in the cables, weights W_1 and W_2, and angle α that cable 1 makes with the horizontal, so that the leg remains in equilibrium at the position shown.

Answers: $T_1 = W_1 = 223.6\text{N}$ $T_2 = W_2 = 282.8\text{N}$ $\alpha = 26.6°$

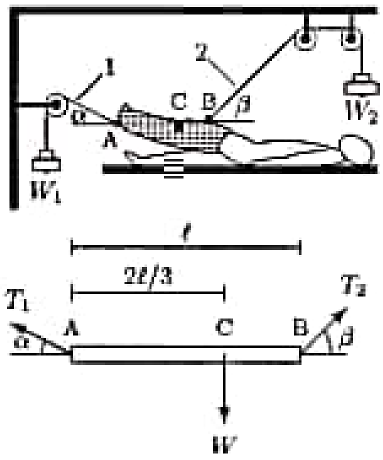


Fig. 4.55 *Problem 4.9*

Problem 4.10 Consider the uniform, horizontal cantilever beam shown in Fig. 4.56. The beam is fixed at point A and a force that makes an angle $\beta = 63°$ with the horizontal is applied at point B. The magnitude of the applied force is $P = 80$ N. Point C is the center of gravity of the beam and the beam weighs $W = 40$ N and has a length $l = 2$ m.

Determine the reactions generated at the fixed end of the beam.

Answers: $R_{Ax} = 36.3\text{N}\ (+x)$ $R_{Ay} = 111.3\text{N}\ (+y)$ $M_A = 182.6\ \text{Mm}\ (\text{ccw})$

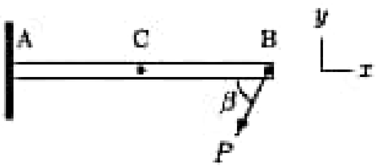


Fig. 4.56 *Problem 4.10*

Problem 4.11 Consider the L-shaped beam illustrated in Fig. 4.57. The beam is welded to the wall at point A, the arm AB extends in the positive z direction, and the arm BC extends in the negative y direction. A force $\underline{P}$ is applied in the positive x direction at the free end (point C) of the beam. The lengths of arms AB and BC are a and b, respectively, and the magnitude of the applied force is P.

Assuming that the weight of the beam is negligibly small, determine the reactions generated at the fixed end of the beam in terms of a, b, and P.

Answers: The non-zero force and moment components are:

$$R_{Ax} = P\ (-x) \quad M_{Ay} = aP\ (-y) \quad M_{Az} = bP\ (-z)$$

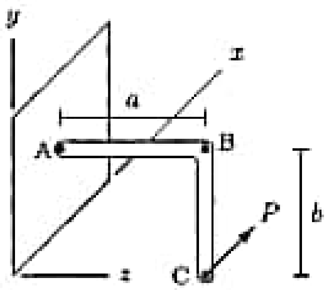


Fig. 4.57 *Problems 4.11 and 4.12*

Problem 4.12 Reconsider the L-shaped beam illustrated in Fig. 4.57. This time, assume that the applied force $\underline{P}$ has components in the positive x and positive z directions such that $\underline{P} = P_x\underline{i} + P_y\underline{j}$. Determine the reactions generated at the fixed end of the beam in terms of a, b, P_x, and P_y.

Answers:

$$R_{Ax} = P_x\ (-x) \qquad R_{Ay} = 0 \qquad R_{Az} = P_y\ (-z)$$
$$M_{Ax} = b\,P_z\ (+y) \qquad M_{Ay} = a\,P_x\ (-y) \qquad M_{Az} = b\,P_x\ (-z)$$

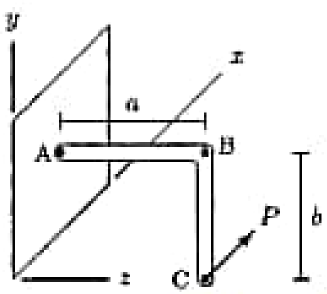


Fig. 4.57 *Problems 4.11 and 4.12*

Scanned with CamScanner

Given data; Load, W =300 N

Angle, $\beta = 45^\circ$

Now body to be in equilibrium, summition of all the forces and moment to be zero.

i.e. Horizontal force, $\sum F_x = 0$

$$T_1 \cos\alpha = T_2 \sin\beta$$

$$\Rightarrow T_1 \cos\alpha = T_2 \sin 45^\circ$$

....................................(1)

Vertical force, $\sum F_y = 0$

$$T_1 \sin\alpha + T_2 \sin\beta = 300$$

$$\Rightarrow T_1 \sin\alpha + T_2 \sin 45^\circ = 300$$

..........................(2)

Moment about any point, $\sum M_A = 0$

................(Taken about 'A')

$$-300 * \frac{2l}{3} + T_2 * l = 0$$

$$\Rightarrow T_2 \sin 45^\circ = 200$$

$$\Rightarrow T_2 = 200\sqrt{2}$$

$$\because \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\therefore \mathbf{T_2} = \mathbf{282.8427}N$$

Substitutng the value of T_2 into the equation 1 & 2, we get;

$T_1 \cos\alpha = 200$ &

$T_1 \sin\alpha = 100$

$$\Rightarrow \tan\alpha\alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = \tan^{-1}\frac{1}{2}$$

$$\therefore \alpha = \mathbf{26.5650^\circ}$$

Thus, $T_1 \cos 26.5650^\circ = 200$

$$\Rightarrow T_1 = \frac{200}{\cos 26.5650^\circ}$$

$$\therefore \mathbf{T_1 = 223.6067N}$$

Therefore, answers are:

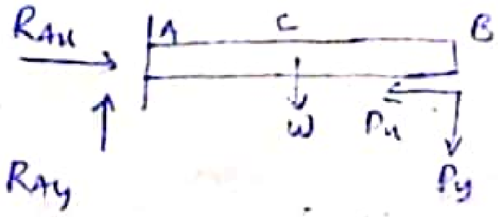
$$\mathbf{T_1 = 223.6067N}$$

$$\mathbf{T_2} = \mathbf{282.8427}N$$

$$\alpha = \mathbf{26.5650^\circ}$$

Scanned with CamScanner

free body diagram



Reducing force in x- direction

$P_x = P\cos\theta$

$= (80)\cos 63°$

$P_x = 36.32\ N$

$P_y = P\sin\theta = (80)\sin 63° = 71.28\ N.$

$\therefore\ \Sigma F_x = 0$

$R_{Ax} = P_x = 36.32\ N.\ (+x)$

Reducing force in y- direction

$R_{Ay} - W - P\sin\theta = 0$

$R_{Ay} = W + P\sin\theta$

$= 40 + 80\sin 63°$

$= 117.3\ (+y)$

Taking moment about point A

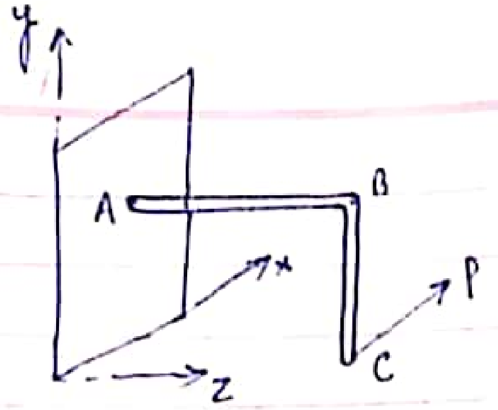
$\Sigma M_A = 0$

$M_A - 2 \cdot P\sin\theta - W \times 1 = 0$

$M_A = 2 \times 80\sin 63° + 40$

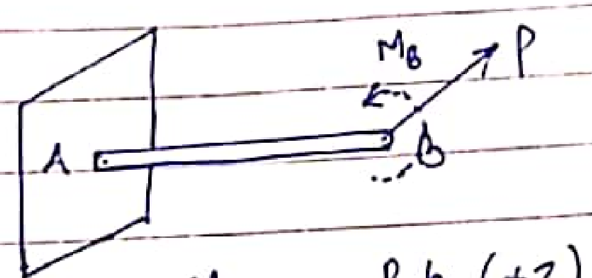
$= 182.6\ N\text{-}m.$ (Counter clockwise)

Scanned with CamScanner



We can remove the link BC assuming a ~~moment~~ couple at point B and a load of P at the same point as illustrated below:

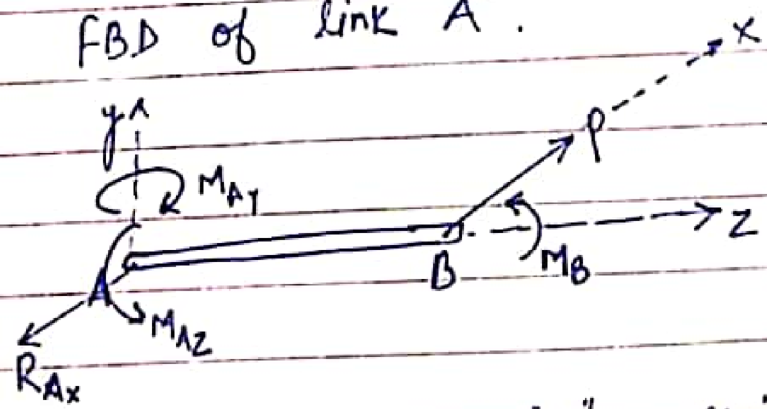
Load & moment at point B ⇒



where, $M_B = P.b \ (+z)$

Now, FBD of link A.

FBD of link AB ⇒



R_{Ax} → Due to the load "P" at Pt. "B"

M_{Az} → Opposite moment due to M_B

M_{Ay} → opposite moment due to Load "P" acting at a distance of "a" i.e. at 'B'

∴ $R_{Ax} = P$ (in $-x$ direction)

$M_{Az} = -M_B = P.b$ (in $-z$ direction) Answer

$M_{Ay} = P.a$ (in $-y$ direction)

Problem 5.1 Consider a person holding an object in his hand with his elbow flexed at the right angle with respect to the upper arm (Fig. 5.4). The forces acting on the forearm and the mechanics model of the system are shown in Fig. 5.5a, b. As for this system assume that the biceps is the major flexor and the line of action of the muscle makes the right angle with the long axis of the forearm. Point O designates the axis of rotation at the elbow joint, A is the point of attachment of the biceps muscle to the radius, point B is the center of gravity of the forearm, and point C is the center of gravity of the object held in the hand. Furthermore, the distances between the axis of rotation of the elbow joint (point O) and points A, B, and C are $a = 4.5$ cm, $b = 16.5$ cm, and $c = 37$ cm. If the total weight of the forearm is $W = 83$ N, and the magnitude of the muscle force is $F_M = 780$ N:

(a) Determine the weight (W_0) of the object held in the hand.
(b) Determine the magnitude of the reaction force (F_J) at the elbow joint.
(c) Determine the magnitude of the muscle (F_{M1}) and joint reaction (F_{J1}) forces when the weight of the object held in the hand is increased by 5 N.

Answers: (a) $W_0 = 57.8$ N; (b) $F_J = 639.2$ N; (c) $F_{M1} = 820$ N, $F_{J1} = 674.2$ N

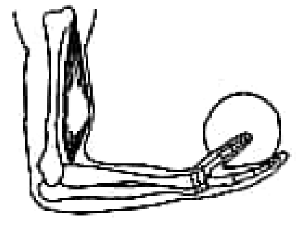


Fig. 5.4 *Example 5.1*

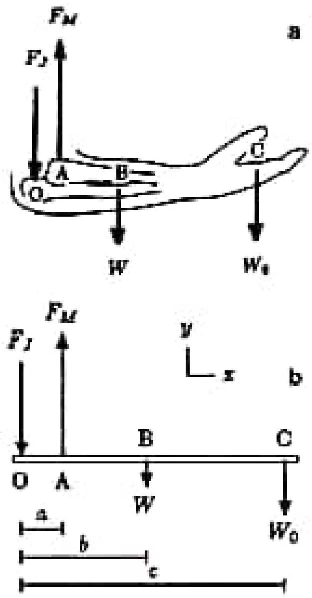


Fig. 5.5 *Forces acting on the lower arm*

Problem 5.2 Consider a person performing shoulder exercises by using a dumbbell (Fig. 5.11). The forces acting on the arm and the mechanical model of the system are shown in Fig. 5.12. For this system assume that the arm of the person is fully extended to the horizontal. Point O designates the axis of rotation of the shoulder joint, A is the point of attachment of the deltoid muscle to the humerus, point B is the center of gravity of the entire arm, and point C is the center of gravity of the dumbbell. The distances between the axis of rotation of the shoulder joint (point O) and points A, B, and C are $a = 17$ cm, $b = 33$ cm, and $c = 63$ cm. The dumbbell weighs $W_0 = 64$ N and for this position of the arm it is estimated that the magnitude of the muscle force is $F_M = 1051$ N. If the lines of action of the muscle (F_M) and the joint reaction forces (F_J) make an angle $\theta = 18°$ and $\beta = 12°$ with the horizontal, respectively:

(a) Determine the magnitude of reaction force (F_J) at the shoulder joint.
(b) Determine the total weight (W) of the arm.
(c) Determine the magnitude of the muscle (F_{M1}) and joint reaction (F_{J1}) forces when the weight of the dumbbell is increased by 5 N.

Answers: (a) $F_J = 1021.9$ N; (b) $W = 47.3$ N; (c) $F_{M1} = 1136.5$ N, $F_{J1} = 1105$ N

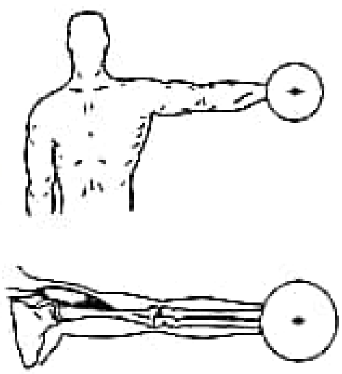


Fig. 5.11 *The arm is abducted to horizontal*

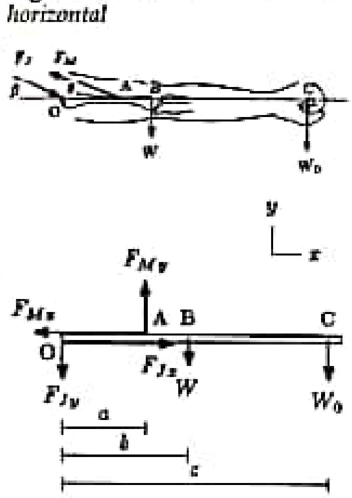
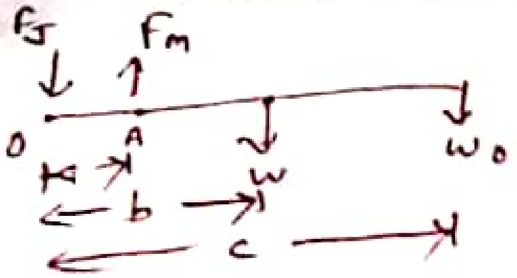


Fig. 5.12 *Forces acting on the arm and a mechanical model representing the arm*

Scanned with CamScanner

Q..



$a = 4.5\text{cm}$
$b = 16.5\text{ cm}$
$c = 37\text{ Cm}$
$W = 83\text{ N}$
$F_m = 780\text{ N}$

<u>$\Sigma Y = 0$</u> $\quad F_m = W + W_0 + F_J$

$780 = 83 + W_0 + F_J \quad — ①$

<u>$\Sigma M_0 = 0$</u> $\quad F_m \times a - W \times b - W_0 \times c = 0$

$780 \times 4.5 - 83 \times 16.5 - W_0 \times 37 = 0$

$\boxed{W_0 = 57.85\text{ N}}$

and $\quad 780 = 83 + 57.85 + F_J$

$\boxed{F_J = 639.1486\text{ N}}$

(C) $\quad W_0 = 57.85 + 5 = 62.85\text{ N}$

$F_{m_1} = 83 + 62.85 + F_{J_1} \quad — ②$

and $\quad F_{m_1} \times 4.5 - 83 \times 16.5 - 62.85 \times 37 = 0$

$\boxed{F_{m_1} = 821.1\text{ N}}$

$F_{J_1} = 821.1 - 83 - 62.85$

$\boxed{F_{J_1} = 675.25\text{ N}}$

Scanned with CamScanner

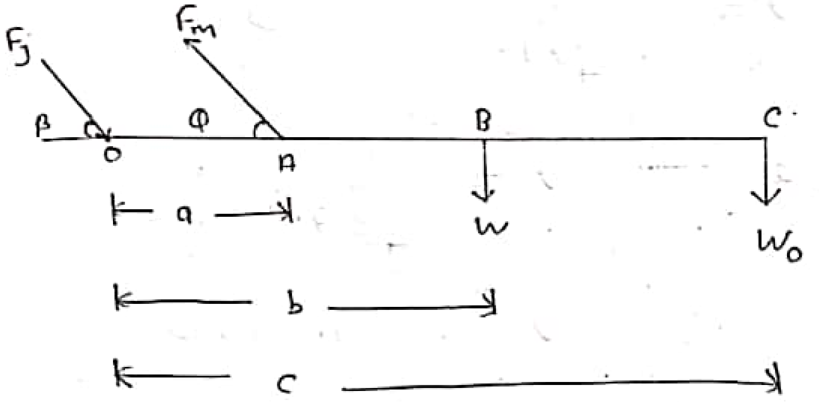
Given :-

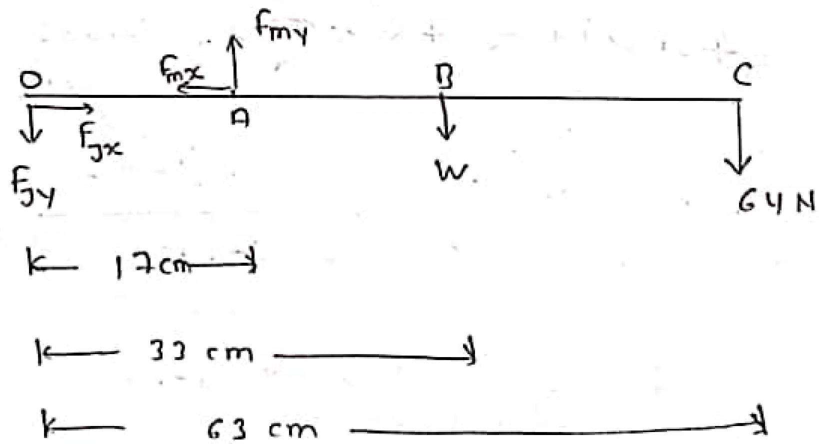
$a = 17\text{ cm}$ dumbbell weight $= 64\text{ N}$ (W_o)

$b = 33\text{ cm}$

$c = 63\text{ cm}$ $F_m = 1051\text{ N}$

$\theta = 18^\circ$ $\beta = 12^\circ$





$$W = \frac{-64\times 63 + 324.777\times 17}{33}$$

(b) $\boxed{W = 47.3\text{ N}}$* Approximate

Now $\Sigma F_y = 0$

$$-F_{Jy} + F_{my} - W - 64 = 0$$

$$-F_{Jy} + 324.777 - 47.3 - 64 = 0$$

Scanned with CamScanner

$$w = \frac{-64\times 63 + 324.777\times 17}{33}$$

(b) $\boxed{W = 47.3\ N}$ * Approximate

Now $\Sigma F_y = 0$

$$-F_{Jy} + F_{my} - W - 64 = 0$$

$$-F_{Jy} + 324.777 - 47.3 - 64 = 0$$

$$F_{Jy} = 213.477\ N$$

but $F_{Jy} = F_J \sin\beta$

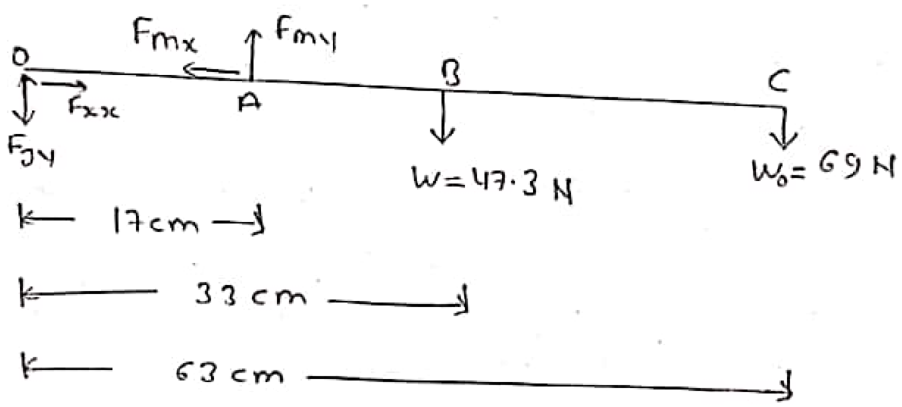
$$213.477 = F_J \times \sin 12$$

(a) Ans. $\boxed{F_J = 1021.9\ N}$ * Approximate

(c) when weight of dumbbell is increase by 5 N

then $W_o = (64+5)\ N$

$W_o = 69\ N$



Taking moment at point O equal to zero

i.e. $\Sigma M_o = 0$

$$-F_{my}\times 17 + 47.3\times 33 + 69\times 63 = 0$$

$F_{my} = 347.5\ldots$

Problem 5.3 Consider the position of the head and neck as well as forces acting on the head shown in Fig. 5.15. For this equilibrium condition assume that the forces involved form a concurrent force system. Point C is the center of gravity of the head, A is the point of application of force (F_M) exerted by the neck extensor muscles on the head, and point B is the center of rotation of the atlantooccipital joint. For this position of the head, it is estimated that the magnitude of the resultant force exerted by the neck extensor muscles is $F_M = 57$ N, and the lines of action of the muscles and the joint reaction forces make an angle $\theta = 36°$ and $\beta = 63°$ with the horizontal, respectively. Determine the magnitude of the gravitational force acting on the head.

Answer: $W = 47$ N

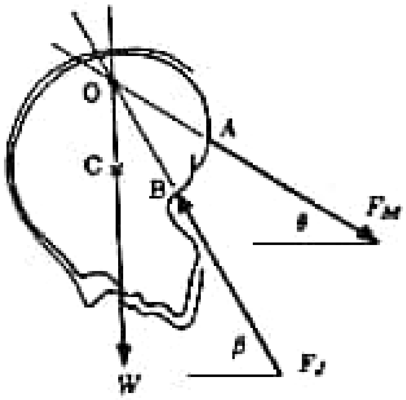


Fig. 5.15 *Forces on the skull form a concurrent system*

Problem 5.4 Consider a weight lifter who is trying to lift a barbell. The forces acting on the lower part of the athlete's body and the mechanical model of the system are shown in Figs. 5.19 and 5.20, respectively. Point O designates the center of rotation at the joint formed by the sacrum and the fifth lumbar vertebra. A is the point of application of force exerted by the back muscles, point B is the center of gravity of the lower body, and C is the point of application of the ground reaction force. With respect to point O, $a = 3.6$ cm, $b = 14.6$ cm, and $c = 22$ cm, are the shortest distances between the lines of action of the back muscles' force, the lower body's gravitational force, and the ground reaction force with the center of rotation of the joint. For a weight lifter in this position, it is estimated that the force exerted by the back muscles is $F_M = 6856$ N and the line of action of this force makes an angle $\theta = 43°$ with the vertical. If the barbell weighs $W_0 = 637$ N and the magnitude of the gravitational force acting on the lower body is $W_1 = 333$ N:

(a) Determine the weight (W) of the athlete.
(b) Determine the magnitude of the reaction force (F_J) acting at the joint.
(c) Determine an angle α that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W = 705.9$ N; (b) $F_J = 7625.8$ N; (c) $\alpha = 52°$

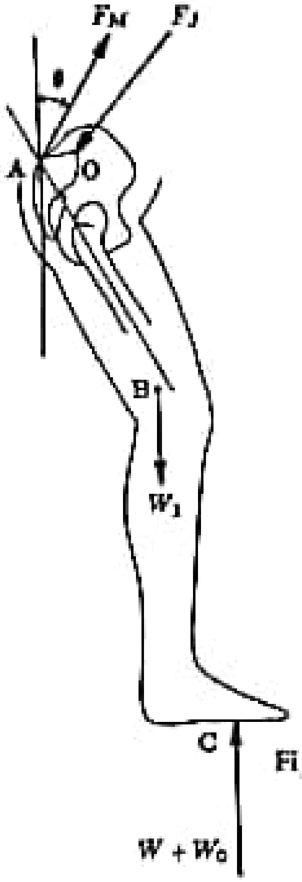


Fig. 5.19 *Forces acting on the lower body of the athlete*

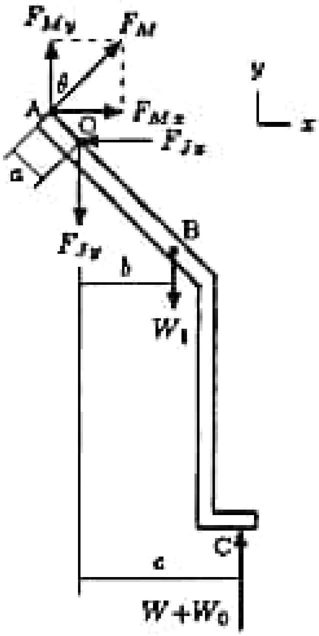
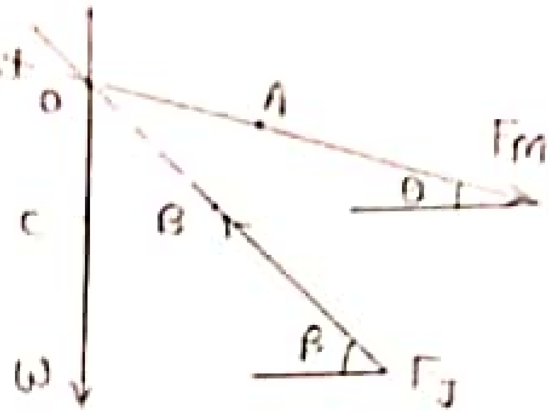


Fig. 5.20 *Free-body diagram*

Scanned with CamScanner

For the equilibrium of the horizontal forces



$$F_m \cos\theta - F_J \cos\beta = 0$$

$$\Rightarrow \quad 57 \cos 36^\circ = F_J \cos 63^\circ$$

$$\Rightarrow \quad F_J = 101.57 \text{ N}$$

For the equilibrium of the vertical forces,

$$W + F_m \sin\theta - F_J \sin\beta = 0$$

$$\Rightarrow \quad W + 57 \times \sin 36^\circ - 101.57 \times \sin 63^\circ = 0$$

$$\Rightarrow \quad \boxed{W = 57 \text{ N}}$$

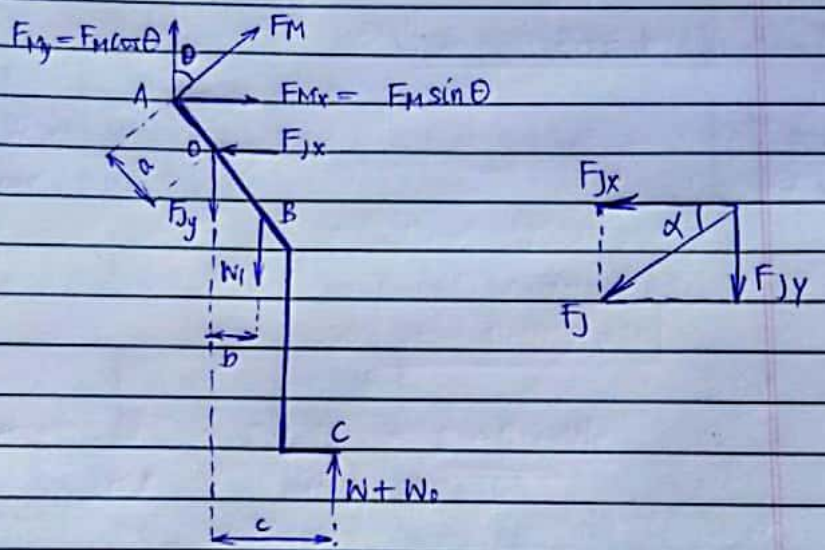
Scanned with CamScanner

Moment is given by the product of force and the perpendicular distance of the force from the point about which the moment is to be calculated. Now forces FM and W1 cause clockwise rotation about point O which is considered negative while force (W+W0) causes anticlockwise rotation about point O which is considered positive.

$a = 3.6\,cm = 0.036\,m$, $b = 14.6\,cm = 0.146\,m$

$c = 22\,cm = 0.22\,m$, $F_M = 6856\,N$, $\theta = 43°$

$W_0 = 637\,N$, $W_1 = 333\,N$



a) $\Sigma M @ O = 0$

$$-F_M \times a - W_1 \times b + (W + W_0) \times c = 0$$

$$-6856 \times 0.036 - 333 \times 0.146 + (W + 637) \times 0.22 = 0$$

$$W = \frac{6856 \times 0.036 + 333 \times 0.146}{0.22} - 637$$

$$\boxed{W = 705.9\,N}$$

b) $\Sigma F_x = 0$ [Summation of forces in horizontal direction]

$$F_M \sin\theta - F_{Jx} = 0$$

$$F_{Jx} = 6856 \sin 43$$

$$F_{Jx} = 4675.78\,N$$

[$\rightarrow$ +ve, $\leftarrow$ −ve]

$\Sigma F_y = 0$ [Summation of forces in vertical direction]

$$F_M \cos\theta - F_{Jy} - W_1 + W + W_0 = 0$$

$$F_{Jy} = 6856 \cos 43 - 333 + 705.9 + 637$$

$$F_{Jy} = 6024.06\,N$$

[$\uparrow$ +ve, $\downarrow$ −ve]

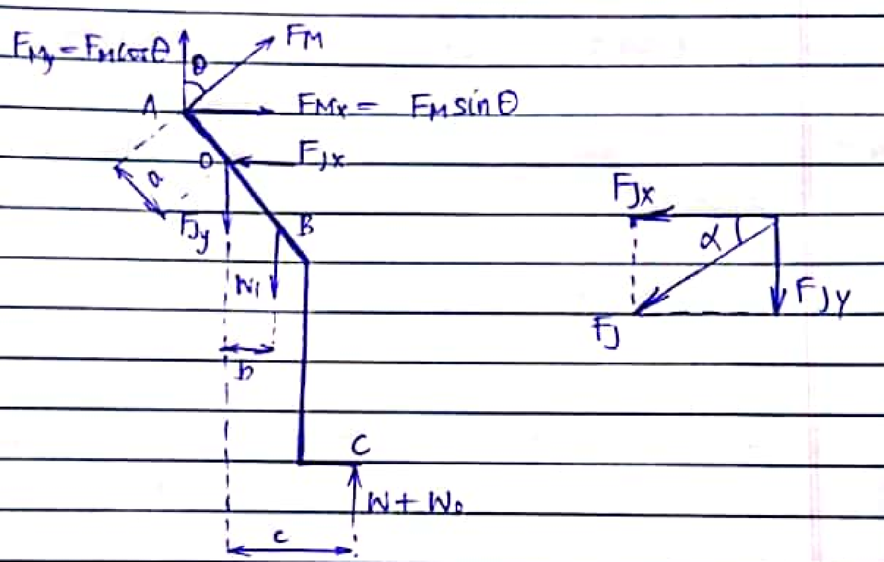
Scanned with CamScanner

considered positive.

$a = 3.6\,cm = 0.036\,m$, $b = 14.6\,cm = 0.146\,m$

$c = 22\,cm = 0.22\,m$, $F_M = 6856\,N$, $\theta = 43°$

$W_0 = 637\,N$, $W_1 = 333\,N$



a) $\Sigma M\ @\ O = 0$

$-F_M \times a - W_1 \times b + (W + W_0) \times c = 0$

$-6856 \times 0.036 - 333 \times 0.146 + (W + 637) \times 0.22 = 0$

$$W = \frac{6856 \times 0.036 + 333 \times 0.146}{0.22} - 637$$

$$\boxed{W = 705.9\,N}$$

b) $\Sigma F_x = 0$ [Summation of forces in horizontal direction]

$F_M \sin\theta - F_{Jx} = 0$

$F_{Jx} = 6856 \sin 43$ [$\rightarrow$ +ve, $\leftarrow$ −ve]

$F_{Jx} = 4675.78\,N$

$\Sigma F_y = 0$ [Summation of forces in vertical direction]

$F_M \cos\theta - F_{Jy} - W_1 + W + W_0 = 0$

$F_{Jy} = 6856 \cos 43 - 333 + 705.9 + 637$ [$\uparrow$ +ve, $\downarrow$ −ve]

$F_{Jy} = 6024.06\,N$

$$F_J = \sqrt{F_{Jx}^2 + F_{Jy}^2} = \sqrt{4675.78^2 + 6024.06^2}$$

$$\boxed{F_J = 7625.8\,N}$$

c) $$\alpha = \tan^{-1}\frac{F_{Jy}}{F_{Jx}} = \tan^{-1}\frac{6024.06}{4675.78}$$

$$\boxed{\alpha = 52°}$$

Scanned with CamScanner

Problem 5.5 Consider a person that momentarily put the entire weight of his body on one leg when walking or running. The forces acting on the leg and the mechanical model of the system are shown in Figs. 5.24 and 5.25, respectively. Point O designates the center of rotation of the hip joint. A is the point of attachment of the hip abductor muscles to the femur, point B is the center of gravity of the leg, and C is the point of application of the ground reaction force. The distances between point A and points O, B, and C are specified as $a = 8.6$ cm, $b = 34.3$ cm, and $c = 89.4$ cm. The angles that the femoral neck and the long axis of the femoral shaft make with the horizontal are specified as $\alpha = 43^\circ$ and $\beta = 79^\circ$, respectively. Furthermore, for this single-leg stance, it is estimated that the magnitude of force exerted by the hip abductor muscles is $F_M = 2062.6$ N and its line of action makes an angle $\theta = 69^\circ$ with the horizontal. If the magnitude of gravitational force acting on the leg is $W_1 = 125$ N:

(a) Determine the total weight (W) of the person.
(b) Determine the magnitude of the reaction force (F_J) acting at the hip joint.
(c) Determine an angle γ that the line of action of the joint reaction force makes with the horizontal.

Answers: (a) $W = 729.7$ N; (b) $F_J = 2636.1$ N; (c) $\gamma = 73.7^\circ$

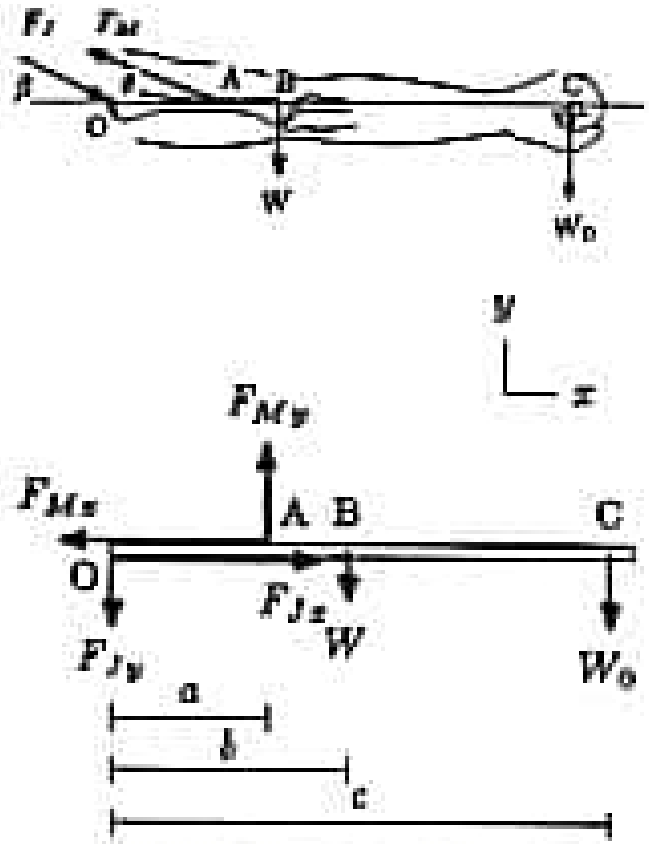


Fig. 5.12 *Forces acting on the arm and a mechanical model representing the arm*

Scanned with CamScanner

Ans

Given that

The distance between point A and point O, B, C are specified as $a = 8.6cm$

$b = 34.3cm$

$c = 84.4cm.$

The specified $\alpha = 43$

$\beta = 79.$

The magnitude of force Exerted by the hip abductor muscles is $FM = 2062.6N$

Makes an angle $\theta = 69,$

Gravitational Weight $W = 1110N$

a) The total weight of the person w is expressed.

The person has a weight and the normal force N of the floor has magnitude equal to w and acts under the center of gravity of the whole density body of the person.

The person momentically stationary the horizontal and vertical component of the forces and the total torque are equal to zero

$\Sigma FH = 0, \Sigma FV = 0, \Sigma \tau = 0$

The horizontal component is $F\cos(\theta) - RH$

The vertical component is $F\sin(\theta) - RV - \frac{W}{7} + W$

weight of the length is $\frac{1}{7}$ of the weight of the person.

Scanned with CamScanner

of the person.

The person momentically stationary the horizontal and vertical component of the forces and the total torque are equal to zero

(i)

$\Sigma F_H = 0, \Sigma F_V = 0, \Sigma \tau = 0$

The horizontal component is $T\cos(\theta) - R_H$

The vertical component is $T\sin(\theta) - R_V - \frac{W}{7} + W$

weight of the length is $\frac{1}{7}$ of the weight of the person.

$R = \sqrt{R^2_H + R^2_V}$

$W = 125 \times 7$

$= 875N$

Magnitude of the Reaction

$F_S = 2.4 \times W$

$= 2.4 \times 875$

$= 2100\,N.$

The angle at the time of force (reaction makes with the horizontal $\theta = 10°$

Problem 5.6 Consider a person performing lower leg flexion–extension exercises from a sitting position while wearing a weight boot. Forces acting on the leg and the mechanical model of the system are shown in Fig. 5.38. Point O designates the center of rotation of the tibiofemoral joint. A is the point of attachment of the patellar tendon to the tibia, point B is the center of gravity of the lower leg, and point C is the center of gravity of the weight boot. For this system assume that the points O, A, B, and C all lie along a straight line. The distances between point O and points A, B, and C are measured as $a = 13$ cm, $b = 23.5$ cm, and $c = 53$ cm, respectively. For this position of the leg, the long axis of the tibia makes an angle $\beta = 47°$ with the horizontal, and the line of action of the quadriceps muscle force makes an angle $\theta = 17°$ with the long axis of the tibia. Furthermore, for this position of the leg, it is estimated that the force exerted by the quadriceps muscle is $F_M = 1.940$ N.

If the weight of the lower leg is $W_1 = 163$ N:

(a) Determine the weight (W_0) of the weight boot.
(b) Determine the magnitude of the reaction force (F_J) of the tibiofemoral joint.
(c) Determine an angle φ that the line of action the joint reaction force makes with the horizontal.

Answers: (a) $W_0 = 98.4$ N; (b) $F_J = 1707.5$ N; (c) $\varphi = 60.2°$

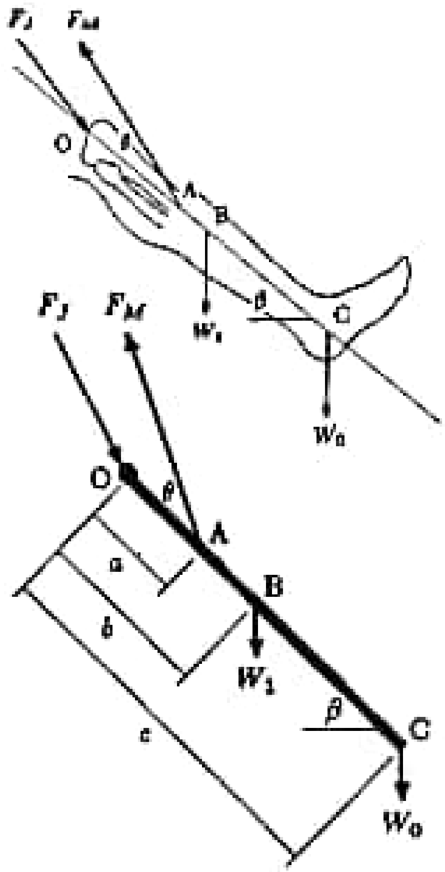


Fig. 5.38 *Forces acting on the lower leg*

Problem 5.7 Consider a person standing on tiptoe on one foot. For this position, the forces acting on the foot are shown in Fig. 5.45. Point A is the point of attachment of the Achilles tendon through which a force is exerted by the gastrocnemius and soleus muscles on the calcaneus. Point B designates the center of the ankle joint and C is the point of application of the ground reaction force. For this system assume that the weight of the foot can be ignored as it is relatively small when compared to the weight of the entire body of the person. For this position of the foot, it is estimated that the lines of action of the tensile force in the Achilles tendon and the reaction force (F_J) of the ankle joint make an angle $\theta = 49°$ and $\beta = 65°$ with the horizontal, respectively. Furthermore, for this position of the foot, it is also estimated that the magnitude of force exerted by the gastrocnemius and soleus muscles on the calcaneus is $F_M = 1275.4$ N.

(a) Determine the entire weight (W) of the person.
(b) Determine the magnitude of the reaction force (F_J) of the ankle joint.

Answers: (a) $W = 831.8$ N; (b) $F_J = 1980.3$ N

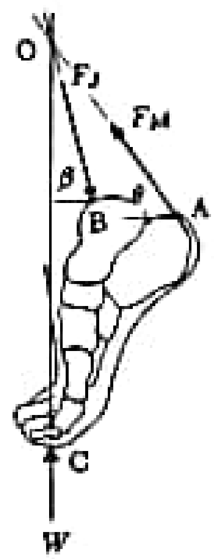


Fig. 5.45 *Forces acting on the foot form a concurrent system of forces*

Scanned with CamScanner

Solution:

given that:

$a = 13cm$

$b = 23.5\ cm$

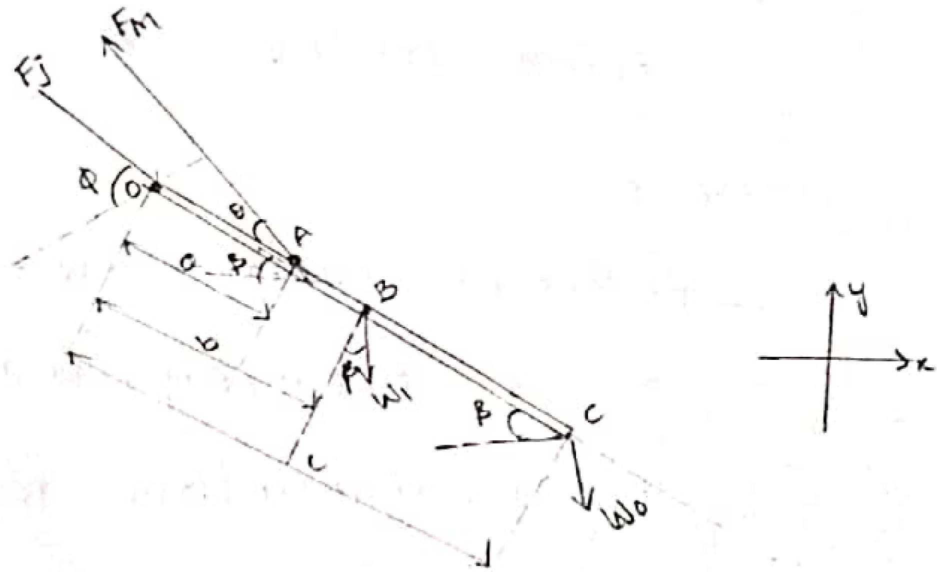
$c = 53\ cm$

$\beta = 47^\circ$

$\theta = 17^\circ$

$F_m = 1940N$

The weight of the lower leg $W_1 = 163N$.



a. Moment balance about o.

$$(\Sigma M)_o = 0$$

$$a F_m \sin\theta = W_1 \cos\beta \times b + W_1 \cos\beta \times c$$

$$W_o = \frac{F_m a \sin\theta - W_1 b \cos\beta}{c \cos\beta}$$

Scanned with CamScanner

$$W_o = \frac{1940 \times 13 \sin(17) - 163 \times 23.5 \cos(47)}{53 \times \cos(47)}$$

$$\boxed{W_o = 131.72\ N}$$

b. $\Sigma F_x = 0$

Scanned with CamScanner

$$W_o = \frac{1940 \times 13 \sin(17) - 163 \times 23.5 \cos(47)}{53 \times \cos(47)}$$

$$\boxed{W_o = 131.72 \text{ N}}$$

b. $\Sigma F_x = 0$

$$F_j \cos\phi - F_m \cos(\theta+\beta) = 0$$

$$F_j \cos\phi = F_m \cos(\theta+\beta)$$

$$F_j \cos\phi = 1940 \cos(17+47)$$

$$F_j \cos\phi = 850.44 \text{ N} \rightarrow 1$$

$\Sigma F_y = 0$

$$-F_j \sin\phi + F_m \sin(\theta+\beta) - W_1 - W_o = 0$$

$$F_j \sin\phi = F_m \sin(\theta+\beta) - W_1 - W_o$$

$$F_j \sin\phi = 1940 \sin(17+47) - 163 - 131.72$$

$$F_j \sin\phi = 1448.94 \text{ N} \rightarrow 2$$

eq $1^2 + 2^2$

$$F_j^2 (\sin^2\phi + \cos^2\phi) = 1448.94^2 + 850.44^2$$

$$F_j^2 = 1448.94^2 + 850.44^2$$

Scanned with CamScanner

$$F_j = \sqrt{1448.94^2 + 850.44^2}$$

$$\boxed{F_j = 1680.08 \text{ N}}$$

c.

Scanned with CamScanner

$F_j \sin\phi = 1 \cdot 1405 \sin(17+47) - 163 - 131.72$

$$F_j \sin\phi = 1448.94\,N \rightarrow (2)$$

eq 2, $1^2 + 2^2$

$$F_j^2 (\sin^2\phi + \cos^2\phi) = 1448.94^2 + 850.44^2$$

$$F_j^2 = 1448.94^2 + 850.44^2$$

CS Scanned with CamScanner

(3)

$$F_j = \sqrt{1448.94^2 + 850.44^2}$$

$$\boxed{F_j = 1680.08\,N}$$

c.

$$\tan\theta = \frac{1448.94}{850.44}$$

$$\theta = \tan^{-1}\left[\frac{1448.94}{850.44}\right]$$

$$\theta = 59.59 \simeq$$

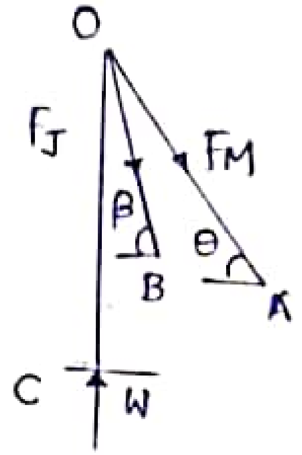
$$\boxed{\theta = 60^\circ}$$

Scanned with CamScanner

Sol Given Data

$F_M = 1275.4\,N$

$\theta = 49^\circ$, $\beta = 65^\circ$



At equilibrium condition we can write

Along Horizontal direction : $F_J \cos\beta = F_M \cos\theta$ — ①

Along Vertical direction :. $F_J \sin\beta - F_M \sin\theta = W$ — ②

(b) From ① and ② we get the magnitude of reaction force of the ankle Joint

$$F_J = F_M \frac{\cos\theta}{\cos\beta} = 1275.4 \times \frac{\cos 49}{\cos 65}$$

$$= 1979.9\,N$$

(a) Put equation ① in to ② equation we get

$$F_M \frac{\cos\theta}{\cos\beta} \cdot \sin\beta - F_M \sin\theta = W$$

$$W = F_M \frac{\sin(\beta-\theta)}{\cos\beta} = \frac{1275.4 \times \sin(65-49)}{\cos 65}$$

$$W = 831.8\,N$$

Scanned with CamScanner

Problem 13.1 Complete the following definitions with appropriate expressions.

(a) Unit deformation of a material as a result of an applied load is called _______.

(b) The internal resistance of a material to deformation due to externally applied forces is called _______.

(c) _______ is a measure of the intensity of internal forces acting parallel or tangent to a plane of cut, while _______ are associated with the intensity of internal forces that are perpendicular to the plane of cut.

(d) On the stress–strain diagram, the stress corresponding to the _______ is the highest stress that can be applied to the material without causing permanent deformation.

(e) On the stress–strain diagram, the highest stress level corresponds to the _______ of the material.

(f) For some materials, it may not be easy to distinguish the yield point. The yield strength of such materials may be determined by the _______.

(g) _______ is defined as the ability of a material to resume its original (stress-free) size and shape upon removal of applied loads.

(h) For linearly elastic materials, stress is linearly proportional to strain and the constant of proportionality is called the _______ of the material.

(i) The distinguishing factor in linearly elastic materials is their _______.

(j) Materials for which the stress–strain curve in the elastic region is not a straight line are known as _______ materials.

(k) _______ is the constant of proportionality between shear stress and shear strain for linearly elastic materials.

(l) A mathematical equation that relates stresses to strains is called a _______.

(m) The analogy between elastic materials and springs is known as _______.

(n) _______ implies permanent (unrecoverable) deformations.

(o) The area under the stress–strain diagram in the elastic region corresponds to the _______ energy stored in the material while deforming the material.

(p) _______ energy is dissipated as heat while deforming the material.

(q) The area enclosed by the _______ signifies the total strain energy dissipated as heat while loading and unloading a material.

(r) The technique of changing the yield point of a material by loading the material beyond its yield point is called _______.

(s) The elastic modulus of a material is a relative measure of the _______ of one material with respect to another.

(t) A _______ material is one that exhibits a large plastic deformation prior to failure.

(u) A _______ material is one that shows a sudden failure (rupture) without undergoing a considerable plastic deformation.

(v) _______ is a measure of the capacity of a material to sustain permanent deformation. The toughness of a material is measured by considering the total area under its stress–strain diagram.

(w) The ability of a material to store or absorb energy without permanent deformation is called the _______ of the material.

(x) If the mechanical properties of a material do not vary from location to location within the material, then the material is called _______.

(y) If a material has constant density, then the material is called _______.

(z) If the mechanical properties of a material are independent of direction or orientation, then the material is called _______.

Answers to Problem 13.1:

(a) strain	(n) Plasticity
(b) stress	(o) elastic strain
(c) Shear stress, normal stress	(p) Plastic strain
(d) elastic limit	(q) hysteresis loop
(e) ultimate strength	(r) strain hardening
(f) offset method	(s) stiffness
(g) Elasticity	(t) ductile
(h) elastic (Young's) modulus	(u) brittle
(i) elastic (Young's) modulus	(v) Toughness
(j) nonlinear elastic	(w) resilience
(k) Shear modulus	(x) homogeneous
(l) material function	(y) incompressible
(m) Hooke's Law	(z) isotropic

Scanned with CamScanner

Problem 13.2 Curves in Fig. 13.40 represent the relationship between tensile stress and tensile strain for five different materials. The "dot" on each curve indicates the yield point and the "cross" represents the rupture point. Fill in the blank spaces below with the correct number referring to a material.

Material _______ has the highest elastic modulus.
Material _______ is the most ductile.
Material _______ is the most brittle.
Material _______ has the lowest yield strength.
Material _______ has the highest strength.
Material _______ is the toughest.
Material _______ is the most resilient.
Material _______ is the most stiff.

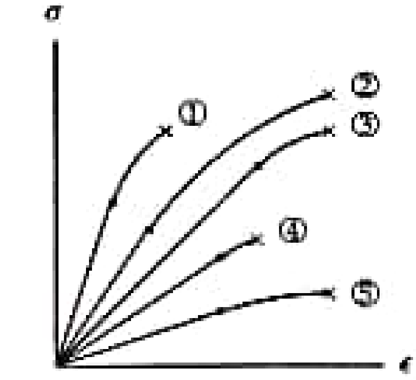


Fig. 13.40 *Problem 13.2*

Answers to Problem 13.2: 1, 2, 4, 5, 2, 2, 3, 1

Problem 13.3 Consider two bars, 1 and 2, made of two different materials. Assume that these bars were tested in a uniaxial tension test. Let F_1 and F_2 be the magnitudes of tensile forces applied on bars 1 and 2, respectively. E_1 and E_2 are the elastic moduli and A_1 and A_2 are the cross-sectional areas perpendicular to the applied forces for bar 1 and 2, respectively. For the conditions indicated below, determine the correct symbol relating tensile stresses σ_1 and σ_2 and tensile strains ϵ_1 and ϵ_2. Note that ">" indicates greater than, "<" indicates less than, "=" indicates equal to, and "?" indicates that the information provided is not sufficient to make a judgement.

(a) If $A_1 > A_2$ and $F_1 = F_2$ then $\sigma_1 >=?< \sigma_2$ and $\epsilon_1 >=?< \epsilon_2$.
(b) If $E_1 > E_2$, $A_1 = A_2$ and $F_1 = F_2$ then $\sigma_1 >=?< \sigma_2$ and $\epsilon_1 >=?< \epsilon_2$

Answers to Problem 13.3:

(a) $\sigma_1 < \sigma_2$, $\epsilon_1 ? \epsilon_2$
(b) $\sigma_1 = \sigma_2$, $\epsilon_1 < \epsilon_2$

Problem 13.4 As illustrated in Fig. 13.29, consider a circular cylindrical rod tested in a uniaxial tension test. Two points A and B located at a distance $l_0 = 32$ cm from each other are marked on the rod and a tensile force of $F = 980$ N is applied on the rod. If the tensile strain and tensile stress generated in the rod were $\epsilon = 0.06$ cm/cm and $\sigma = 2.2$ MPa, determine:

(a) The radius, r, of the rod after the application of the force
(b) The total elongation of the rod, Δl, after the application of the force

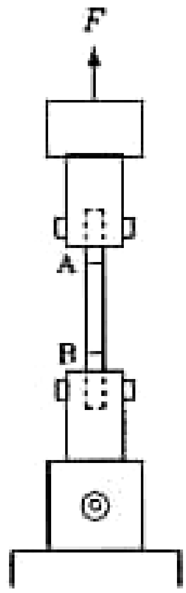


Fig. 13.29 *Example 13.1*

Answers to Problem 13.4: (a) $r = 1.2$ cm, (b) $\Delta l = 1.92$ cm

Problem 13.5 Consider an aluminum rod of radius $r = 1.5$ cm subjected to a uniaxial tension test by force of $F = 23$ kN. If the elastic modulus of the aluminum rod is $E = 70$ GPa, determine:

(a) The tensile stress, σ, developed in the rod
(b) The tensile strain, ϵ, developed in the rod

Answers to Problem 13.5: (a) $\sigma = 32.6$ MPa, (b) $\epsilon = 0.47 \times 10^{-3}$

Scanned with CamScanner

Ans: Given Data:

Two bars 1 And 2

These 2 bars are tested in uniaxial tension test.

F_1, F_2 be the forces applied on Bar 1, Bar 2

E_1, E_2 be the elastic modulus of 1, 2 bars.

And A_1, A_2 are the crosssectionals.

(a) If $A_1 > A_2$ and $F_1 = F_2$ Then

$F_1 = F_2 = F$ (say)

And $A_1 > A_2$

$\sigma_1 = \frac{F_1}{A_1}$; $\sigma_2 = \frac{F_2}{A_2}$

$\sigma_1 = \frac{F}{A_1}$; $\sigma_2 = \frac{F}{A_2}$

As given $A_1 > A_2$

$\downarrow \sigma \propto \frac{1}{A\uparrow}$ (Here)

so; $\boxed{\sigma_1 < \sigma_2}$ Ans:

$\varepsilon_1 = \frac{\sigma_1}{E_1}$, $\varepsilon_2 = \frac{\sigma_2}{E_2}$

let us take. $\sigma_1 = 1$; $\sigma_2 = 2$

Now; $\varepsilon_1 = \frac{1}{E_1}$; $\varepsilon_2 = \frac{2}{E_2}$

if $E_1 = E_2$ them $\varepsilon_2 > \varepsilon_1$

if $E_1 > E_2$ them $\varepsilon_2 > \varepsilon_1$

if $E_1 < E_2$ Then we cant say

So, 'Insufficient data' Ans:

(b) $E_1 > E_2$ & $A_1 = A_2$

$F_1 = F_2$

i.e.; $A_1 = A_2 = A$; $F_1 = F_2 = F$

Now; $\sigma_1 = \frac{F_1}{A_1}$; $\sigma_2 = \frac{F_2}{A_2}$

$\sigma_1 = \frac{F}{A}$; $\sigma_2 = \frac{F}{A}$

$\boxed{\therefore \sigma_1 = \sigma_2} = \sigma$ (say)

As

$\varepsilon_1 = \frac{\sigma_1}{E_1}$; $\varepsilon_2 = \frac{\sigma_2}{E_2}$

$\varepsilon_1 = \frac{\sigma}{E_1}$; $\varepsilon_2 = \frac{\sigma}{E_2}$

$\varepsilon_1 \propto \frac{1}{E_1}$; $\varepsilon_2 \propto \frac{1}{E_2}$

for $E_1 > E_2$

$\boxed{\varepsilon_1 < \varepsilon_2}$ Ans:

Scanned with CamScanner

Given
Total length of test section l_0 = 32 cm
Appied force = 980 N
Strain = 0.06 cm/cm
Stress = 2.2 MPa = 2.2×10^6 Pa

(a)

$$\text{Stress} = \frac{\text{force}}{\text{Area}}$$

$$2.2 \times 10^6 = \frac{980}{\pi \times r^2}$$

$$r^2 = \frac{980}{2.2 \times 10^6 \times \pi}$$

$$r^2 = 1.4 \times 10^{-4}$$

$$r = 0.0119 \text{ m}$$

Radius of rod = 0.0119 m = 1.19 cm

(b) $$\text{Strain} = \frac{\text{change in length}}{\text{original length}}$$

$$0.06 = \frac{\Delta l}{32}$$

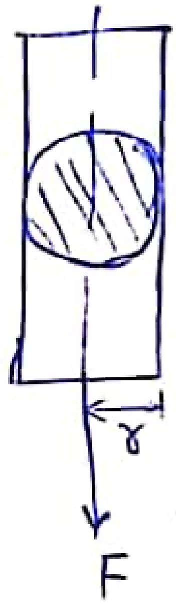
$\Delta l = 32 \times 0.06 = 1.92$ cm

Total elongation of rod is 1.92 cm

Scanned with CamScanner

(a) Tensile stress $\sigma = \dfrac{\text{Force}}{\text{Area}}$

$$\sigma = \frac{23 \times 10^3 \,(N)}{\pi (15)^2 \,(mm^2)} \qquad \left(\begin{array}{l} r = 1.5\,cm \\ \;\; = 15\,mm \end{array} \right)$$



$$\boxed{\sigma = 32.55\,MPa}$$

(b) $\epsilon = \dfrac{\text{Stress}}{E}$ $\qquad$ (Hook's Law $\boxed{\sigma = E\epsilon}$)

$$= \frac{32.55\,(MPa)}{70 \times 10^3\,(MPa)}$$

$$\boxed{\epsilon = 0.465 \times 10^{-3}}$$

This is the value of the strain.

Scanned with CamScanner

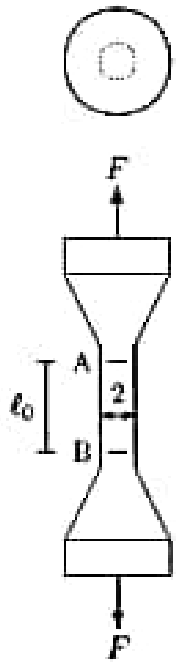


Fig. 13.41 *Problem 13.6*

Problem 13.6 Figure 13.41 illustrates a bone specimen with a circular cross-section. Two sections, A and B, that are $l_0 = 6\,\mathrm{mm}$ distance apart are marked on the specimen. The radius of the specimen in the region between A and B is $r_0 = 1\,\mathrm{mm}$.

This specimen was subjected to a series of uniaxial tension tests until fracture by gradually increasing the magnitude of the applied force and measuring corresponding deformations. As a result of these tests, the following data were recorded:

Record #	Force, F(N)	Deformation, Δl (mm)
1	94	0.009
2	190	0.018
3	284	0.027
4	376	0.050
5	440	0.094

If record 3 corresponds to the end of the linearly elastic region and record 5 corresponds to fracture point, carry out the following:

(a) Calculate average tensile stresses and strains for each record.
(b) Draw the tensile stress–strain diagram for the bone specimen.
(c) Calculate the elastic modulus, E, of the bone specimen.
(d) What is the ultimate strength of the bone specimen?
(e) What is the yield strength of the bone specimen? (use the offset method)

Answers to Problem 13.6:

(c) $E = 20$ GPa
(d) $\sigma_u = 140$ MPa
(e) $\sigma_y = 118$ MPa

Problem 13.7 Consider a fixation device consisting of a plate and two screws that was used to stabilize a fractured femoral bone of a patient (Fig. 13.33). The weight of the patient was $W = 833$ N. If the shear stress generated in the screws was $\tau = 35.12 \times 10^6$ Pa, determine the diameter, d, of the screws.

Answer to Problem 13.7: $d = 5.5$ mm

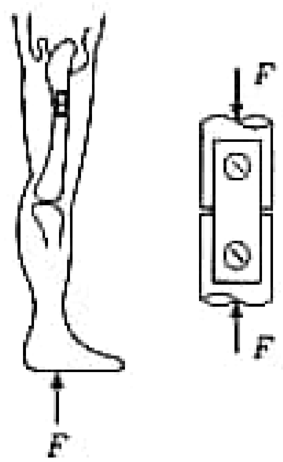
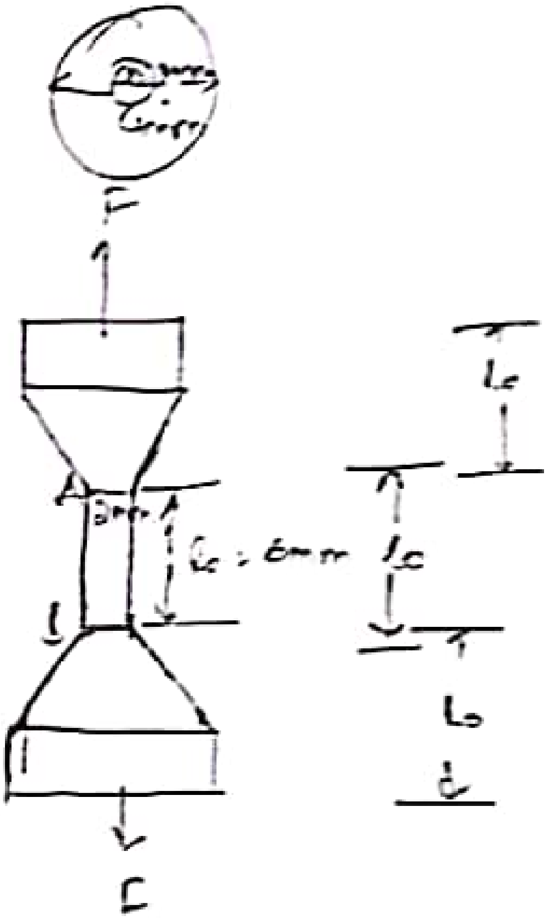
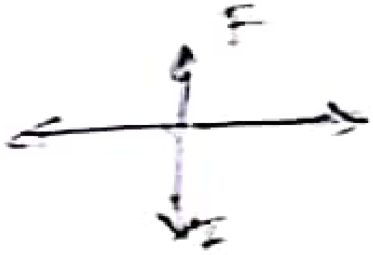


Fig. 13.33 *Example 13.4*

Scanned with CamScanner

Answer





$L_0 = 3 l_0$
$= 3(6mm)$
$= 18mm$
$= 0.018\ m$

$E = \frac{1}{2} mgh$
$= \frac{1}{2} \times m \times 9.8 \times 0.018 m$

$\boxed{E = m\,0.0882\ J/s}$

$F = mg$
$= \frac{m \times 9.8 m/s^2 \times 0}{0.0882 m}$

$F = 0.22 N$

force 100 lenth is in (mm) it does not except much force to implement so $F = 0.22 N$

(d) Ultimate strength $= \dfrac{\text{Breaking force}}{\text{Reduced Area.}}$

$= \dfrac{440\ N}{[\pi(1)^2]\ mm^2}$

$= 140.06\ MPa$

(e) Yield strength

elastic region; $F = 284\ N$. $\epsilon = \dfrac{\Delta l}{l} = \dfrac{0.027}{6}$

$= 0.0045$

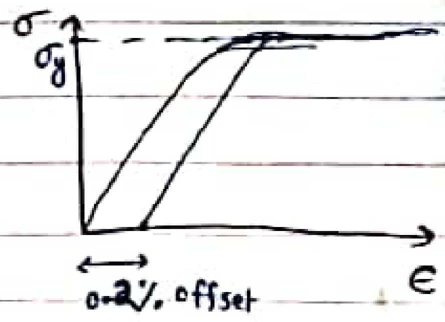
$$\sigma = \frac{F}{A} = \frac{284}{\pi(1)^2} = 90.4\ MPa.$$

So, Youngs Modulus of Elasticity, $E = \dfrac{\sigma}{\epsilon} = \dfrac{90.4}{0.0045}$

$\Rightarrow$ $E = 20.089\ GPa$

For yield strength;

consider $0.2\%\ \epsilon$ offset



So, for $0.2\%\ \epsilon$ offset

strain for yielding is,

$\epsilon_y = 0.002 + 0.0045 = 0.0065$

Corresponding yield strength (σ_y) is;

$\sigma_y = E\ \epsilon_y$

$= (20.089 \times 10^9)(0.0065)$ Pa

$\sigma_y = 130.58\ MPa$

Scanned with CamScanner

Problem 13.8 Human femur bone was subjected to a uniaxial tension test. As a result of a series of experiments, the stress–strain curve shown in Fig. 13.42 was obtained. Based on the graph in Fig. 13.42, determine the following parameters:

(a) Elastic modulus, E.
(b) Apparent yield strength, σ_y (use the offset method).
(c) Ultimate strength, σ_u.
(d) Strain, ϵ_1, corresponding to yield stress.
(e) Strain, ϵ_2, corresponding to ultimate stress.
(f) Strain energy when stress is at the proportionality limit.

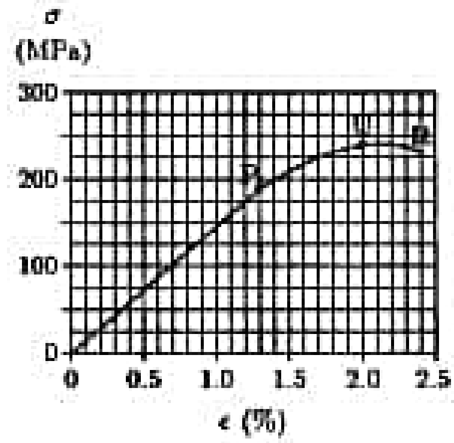


Fig. 13.42 *Problem 13.8*

Answers to Problem 13.8:

(a) $E = 14.6$ GPa
(b) $\sigma_y = 235$ MPa
(c) $\sigma_u = 240$ MPa
(d) $\varepsilon_2 = 0.018$
(e) $\varepsilon_2 = 0.02$
(f) 1.235 MPa

Problem 13.9 As illustrated in Fig. 13.43, consider a structure including the horizontal beam AB hinged to the wall at point A and supported by steel bar at point B. The length of the beam is $l = 3.7$ m and its weight is $W = 450$ N. The length of the steel bar is $h = 2.5$ m and its radius is $r = 1.2$ cm. If the elastic modulus of the steel bar is $E = 200$ GPa, determine:

(a) The magnitude of reaction force, RA, at point A
(b) The tensile stress, σ, exerted by the beam on the steel bar
(c) The tensile strain, ϵ, developed in the bar

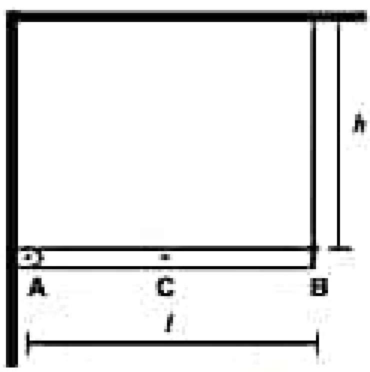


Fig. 13.43 *Problem 13.9*

Answers to Problem 13.9: (a) $R_A = 225$ N, (b) $\sigma = 0.5$ MPa, (c) $\epsilon = 2.5 \times 10^{-6}$

Scanned with CamScanner

ⓐ Elastic Modulus E , from Graph

$$E = \frac{\sigma}{\varepsilon} = \frac{190}{\frac{1.3}{100}} = 14615.38\ \text{Mpa} = \cancel{14.6}\ 14.61\ \text{Gpa}$$

ⓑ

$$\cancel{\sigma_y = E\varepsilon}. \quad \sigma_u = E\,\varepsilon_{avg} = 14615.38\left[\frac{2+1.3}{2(100)}\right]$$

[ε_{avg} is between ε_u & $\varepsilon_{Elastic}$]

$$\Rightarrow \boxed{\sigma_u = 241.15\ \text{Mpa}}$$

yieldstress of offset is about 0.02% taken out

$$\sigma_y = (1-0.02)\sigma_u = 0.98\,\sigma_u = 0.98(241.15)$$

$$\boxed{\sigma_y = 236\ \text{Mpa}}$$

ⓒ $\boxed{\sigma_u = 241.15\ \text{Mpa}}$ as solved in (b)

ⓓ

$$\varepsilon_1 = \frac{\sigma_y}{E} = \frac{236}{14615} = \boxed{0.017 = \varepsilon_1}$$

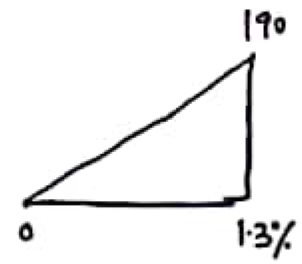
ⓔ . $\cancel{\varepsilon_2 = \frac{\sigma_2}{E} = \frac{240}{14615}}$ $\boxed{E_2 = \frac{2}{100} = 0.02}$ (from Figure)

f)

$$U = \frac{1}{2}\sigma\varepsilon$$

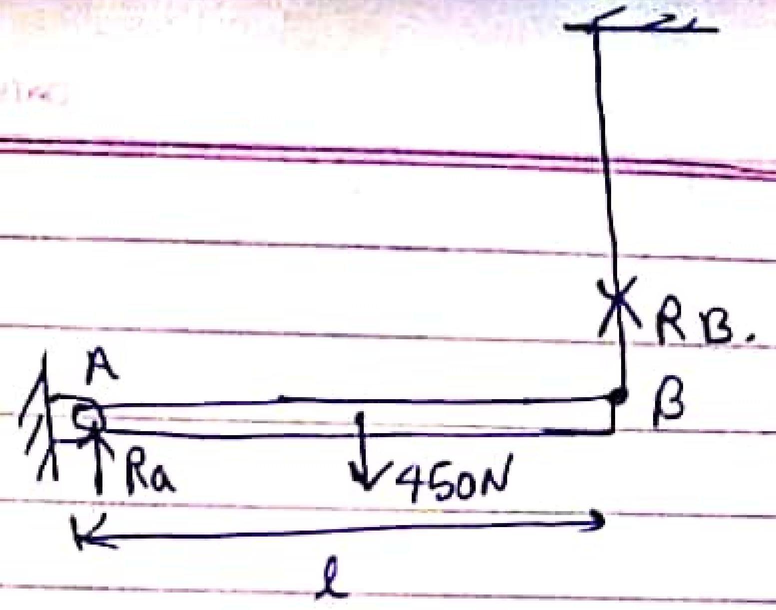
$$= \frac{1}{2}(190)\left(\frac{1.3}{100}\right)$$

$$\boxed{U = 1.235\ \text{Mpa}}$$



Scanned with CamScanner

PAGE
DATE



(a) $\Sigma M_A = 0.$

$$450 \times \frac{L}{2} - R_B \times L.$$

$$R_B = \frac{450}{2} = 225N.$$

$$R_A = 450-225$$

$$\boxed{R_A = 225N}$$

(b) $$\sigma_t = \frac{R_B}{A_{rod}}$$

$$A_{rod} = \pi \times r^2$$

$$r = 12mm$$

$$A_{rod} = \pi \times (12)^2 \ mm^2$$

$$= 452.389 \ mm^2$$

$$\sigma_t = \frac{225}{452.389} = 0.49735 MPa.$$

(c) $\sigma_t = \epsilon E$ (Hoke's Law)

$$\epsilon = \frac{.49735}{206 \times 10^3} = 2.4867 \times 10^{-6}$$

Scanned with CamScanner

Problem 13.10 Consider the uniform, horizontal beam shown in Fig. 13.44a. The beam is hinged to a wall at A and a weight $W_2 = 400\,\text{N}$ is attached on the beam at B. Point C represents the center of gravity of the beam, which is equidistant from A and B. The beam has a weight $W_1 = 100\,\text{N}$ and length $l = 4\,\text{m}$. The beam is supported by two vertical rods, 1 and 2, attached to the beam at D and E. Rod 1 is made of steel with elastic modulus $E_1 = 200\,\text{GPa}$ and a cross-sectional area $A_1 = 500\,\text{mm}^2$, and rod 2 is bronze with elastic modulus $E_2 = 80\,\text{GPa}$ and $A_2 = 400\,\text{mm}^2$. The original (undeformed) lengths of both rods is $h = 2\,\text{m}$. The distance between A and D is $d_1 = 1\,\text{m}$ and the distance between A and E is $d_2 = 3\,\text{m}$.

The free-body diagram of the beam and its deflected orientation is shown in Fig. 13.44b, where T_1 and T_2 represent the forces exerted by the rods on the beam. Symbols δ_1 and δ_2 represent the amount of deflection the steel and bronze rods undergo, respectively. Note that the beam material is assumed to be very stiff (almost rigid) as compared to the rods so that it maintains its straight shape.

(a) Calculate tensions T_1 and T_2, and the reactive force R_A on the beam at A.
(b) Calculate the average tensile stresses σ_1 and σ_2 generated in the rods.

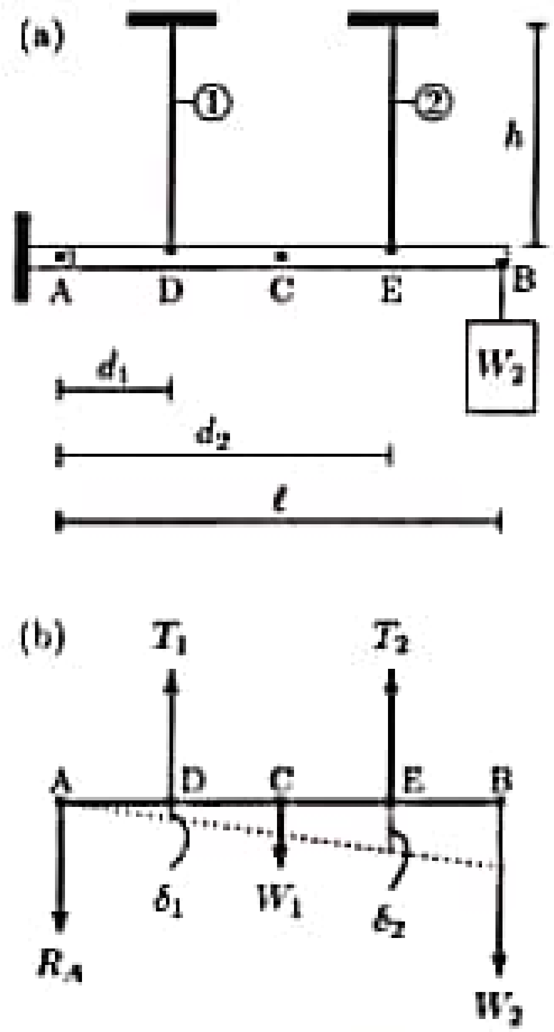


Fig. 13.44 *Problem 13.10*

Answers to Problem 13.10:

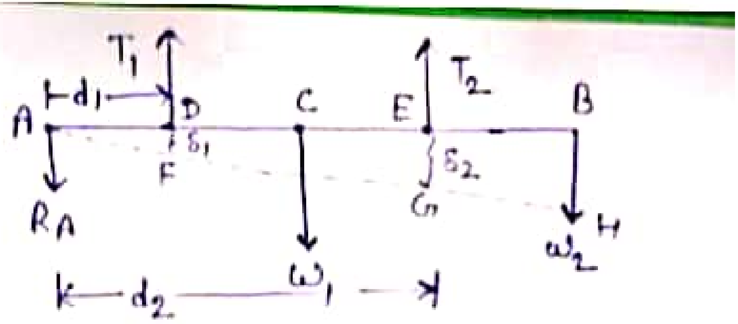
(a) $T_1 = 464\,\text{N}$, $T_2 = 445\,\text{N}$, $R_A = 409\,\text{N}$
(b) $\sigma_1 = 0.928\,\text{MPa}$, $\sigma_2 = 1.113\,\text{MPa}$

Scanned with CamScanner

$w_2 = 400N,\ l = 4m$

$w_1 = 100N,\ d_1 = 1m$

$d_2 = 3m$

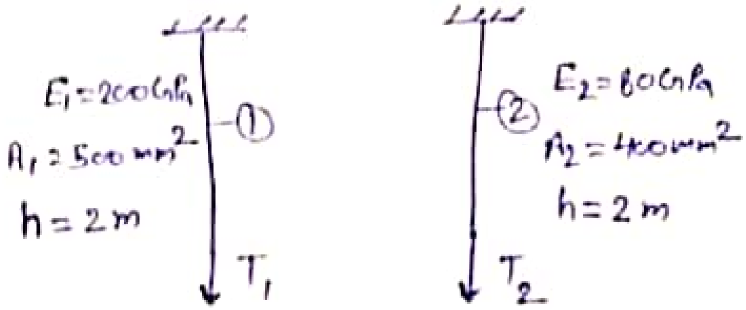


From Geometry

ΔADF and ΔAEG, & ΔABH

$$\frac{\delta_1}{d_1} = \frac{\delta_2}{d_2}$$

$$\frac{\delta_1}{\delta_2} = \frac{d_1}{d_2} = \frac{1}{3}$$



Extension in Rod 1 & Rod 2 is δ_1 & δ_2

$$\delta_1 = \frac{T_1 h}{A_1 E_1}$$

$$\delta_2 = \frac{T_2 h}{A_2 E_2}$$

$$\frac{\delta_1}{\delta_2} = \frac{T_1}{T_2}\,\frac{A_2 E_2}{A_1 E_1}$$

$$\frac{1}{3} = \frac{T_1}{T_2}\,\frac{400\times 80}{500\times 200}$$

$$\frac{T_1}{T_2} = \frac{25}{24} \quad \text{—(1)}$$

Taking moment about A

$$T_1 \cdot d_1 + T_2 \cdot d_2 = w_1\left(\frac{l}{2}\right) + w_2(l)$$

$$T_1 + 3T_2 = 100(2) + 400(4)$$

$$T_1 + 3T_2 = 1800 \quad \text{—(2)}$$

from eqn (1) & (2), we get

$T_2 = 445.361N,\ T_1 = 463.918N$

force equilibrium on the beam

$$R_A + w_1 + w_2 = T_1 + T_2$$

$$R_A + 100 + 400 = 445.361 + 463.918$$

$$\left.\begin{aligned} R_A &= 409.279\,N \\ T_1 &= 463.918\,N \\ T_2 &= 445.361\,N \end{aligned}\right\} \text{Ans}$$

(b) Average Tensile stress σ_1 & σ_2 generated in the rods

$$\sigma_1 = \frac{T_1}{A_1} = \frac{463.918}{500\times 10^{-4}}$$

$$\boxed{\sigma_1 = 927.836\ Pa \times 10}$$

$$\sigma_2 = \frac{T_2}{A_2} = \frac{445.361}{400\times 10^{-4}}$$

$$\left.\begin{aligned} \sigma_2 &= 11.134\ kPa \\ \sigma_1 &= 9.278\ kPa \end{aligned}\right\} \text{Ans}$$

Scanned with CamScanner