**Working Questions 1 for MAT1072**

**1.** Show that if the sequence $\{a_n\}$ converges to a, then the sequence $\{b_n\}$ such that $b_n = \frac{1}{2}(a_n + a_{n+1})$ converges to a.

**2.** Let $\{a_n\}$ is defined by $a_1 = \frac{1}{2}$ and $a_{n+1} = \frac{1+a_n^2}{2}$

a. Show that $\{a_n\}$ is increasing.

b. Show that for $\forall n$ , $a_n < 1$.

c. Find the limit of $\{a_n\}$ .

3. Determine the character of $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$ using the integral test.

4. Determine the character of $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n+1}}$

5. Determine the character of the series

a. $\sum_{n=1}^{\infty} \frac{1}{2n^2 Inn}$ b. $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!+1}$ c. $\sum_{n=1}^{\infty} tan\left(\frac{\pi}{\sqrt{n}}\right)$ d. $\sum_{n=2}^{\infty} \frac{Inn}{(n+1)2^n}$ e. $\sum_{n=1}^{\infty} \frac{n\sqrt{n}}{(n-1)\sqrt[3]{n^4+1}}$

6. Find the sum of $\sum_{k=0}^{\infty} \int_k^{k+1} \frac{dx}{1+e^x}$

7. Determine the character of $\sum_{n=1}^{\infty} \frac{1}{n^3} sin\left(\frac{\pi}{n^2}\right)$

8. Determine the character of $\sum_{n=1}^{\infty} \frac{2n+1}{n^2 2^n}$

9. Find the sum of $\sum_{k=0}^{\infty} \int_{2^k}^{2^{k+1}} \frac{dx}{1+x^2}$

10. Let $f(x)$ be defined on [0,1); differentiable at x=0 and $f(0) = 0$ . Then show that the limit of the sequence defined by $a_n = n.f\left(\frac{1}{n}\right)$ is equal to $f'(0)$.

11. Find the sum of $\sum_{n=1}^{\infty} \frac{-8}{(4n-3)(4n+1)}$

12. Determine the character of $\sum_{n=1}^{\infty} \sqrt{\frac{2n^2+1}{n^3+1}}$

13. Determine the character of $\sum_{n=1}^{\infty} \frac{1+n^{4/3}}{2+n^{5/3}}$

14. Find the limit of the sequence $\{a_n\}$ defined by $a_n = \left(\frac{3n-1}{3n+2}\right)^n$

15. Find the sum of $\sum_{n=2}^{\infty} In\left(1 - \frac{1}{n^2}\right)$

16. Find the sum of $\sum_{k=0}^{\infty} \frac{1}{k^2+3k+2}$

17. Find the Radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n3^n}$

18. Find the interval of the convergence of $\sum_{n=2}^{\infty} \frac{(x+1)^n}{2^n Inn}$. Investigate the series at the end points.

19. Investigate whether the series $\sum_{n=0}^{\infty} e^{-n} cosn\pi$ converges or not. If yes, find the sum of the series.

20. Determine the character of $\sum_{n=2}^{\infty} \frac{cosk\pi}{\sqrt{k}}$

21. Find the Radius of convergence of $\sum_{n=0}^{\infty} \frac{(n+1)(2x+1)^n}{(2n+1)2^n}$

Find the values of x which the series converges absolutely and conditionally.

22. Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{(n+1)^2\, 4^n}$ . Investigate the series at the end points.

23. Find the interval of convergence of $\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right)(x-3)^n$

24. Find the interval of convergence of $\sum_{n=2}^{\infty} \frac{(x-1)^n}{nInn}$

25. a) Recursively, $a_1 = \frac{1}{2}$ , $n \geq 1$ and $a_{n+1} = \sqrt{3 + a_n - 1}$. If $\lim_{n\to\infty} a_n = 1$, then find the limit of the sequence $\left\{\frac{a_{n+1}-1}{a_n-1}\right\}$.

b) Determine that the following series are convergent or divergent.

i) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ ii) $\sum_{n=1}^{\infty} sin\left(\frac{1}{n}\right)$

26. Find the sum of the series $\sum_{n=0}^{\infty} \frac{\pi^{-n}}{\cos(n\pi)}$

27) For $|x| < 1$, using the power series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, find the sum of the series

$\sum_{n=1}^{\infty} (-1)^{n+1} n(n+1) x^{n+1}$ and find the convergence value of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n(n+1)}{2^{n+1}}$