# WORKING QUESTIONS ABOUT SERIES

1. Determine that the following series converge or diverge?

A. $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$ B. $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}\,4^n}$ C. $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$ D. $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$ E. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}$ F. $\sum_{n=2}^{\infty} \frac{n+2}{n^2-n}$

G. $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$ H. $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$ I. $\sum_{n=1}^{\infty} \frac{1-n}{n2^n}$ J. $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ K. $\sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!\,3^{2n}}$

L. $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$ $\sum_{n=1}^{\infty} \frac{2^n-n}{n2^n}$ M. $\sum_{n=1}^{\infty} \frac{n^4}{4^n}$ N. $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$ O. $\sum_{n=1}^{\infty} e^{-n}(n^3)$

P. $a_1 = 2, \quad a_{n+1} = \frac{2}{n} a_n$ R. $a_1 = 5, \quad a_{n+1} = \frac{\sqrt[n]{n}}{2} a_n$

2. Determine if the following alternating series converges or diverges.

A. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n^2}$ B. $\sum_{n=1}^{\infty} (-1)^n \ln\left(1+\frac{1}{n}\right)$

3. Which of the following series converge absolutely, which converge, and which diverge?

A. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$ B. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$ C. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$ D. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$

4. (a) Find the series' radius and interval of coovergence. For what values of x does the series converge (b) absolutely, (c) conditionally?

A. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$ B. $\sum_{n=0}^{\infty} \frac{nx^n}{n+2}$ C. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$ D. $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$ E. $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

5. Find the sum of the following series.

A. $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} + \frac{1}{3^n}\right)$ B. $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$ C. $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right)$ D. $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$ E. $\sum_{n=1}^{\infty} \left(\ln\sqrt{n+1} - \ln\sqrt{n}\right)$

F. Express the number $0.\overline{234} = 0.234\,234\,234\ldots$ as the ratio of two integers.

6. Find the Taylor series generated by f at x = a.

A. $f(x) = 1/(1-x)^3, \quad a = 0$ B. $f(x) = 2^x, \quad a = 1$

**TABLE 10.1** Frequently used Taylor series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \qquad |x| < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \qquad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \qquad |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \qquad |x| < \infty$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \qquad -1 < x \le 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \qquad |x| \le 1$$

7. Use the power series operations to find the Taylor series at x = 0 for the following functions.

A. $x \cos \pi x$ B. $x \ln(1 + 2x)$ C. $e^x + \frac{1}{1+x}$ D. $\cos x - \sin x$

8. Use the series to evaluate the limits in the followings.

A. $\lim_{x \to 0} \frac{e^x - (1+x)}{x^2}$ B. $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$ C. $\lim_{x \to 0} \frac{\ln(1+x^2)}{1 - \cos x}$ D. $\lim_{\theta \to 0} \frac{\sin \theta - \theta + (\theta^3/6)}{\theta^5}$

9. Use the series to evaluate the integrals in the followings.

A. $\int_0^x t^2 e^{-t^2}\, dt$, B. $\int_0^x \frac{\ln(1+t)}{t}\, dt$,

10. Find the areas of the regions.

A. Inside the circle $r = 3a \cos \theta$ and outside the cardioid $r = a(1 + \cos \theta)$, $a > 0$

B. Inside the circle $r = 4 \cos \theta$ and to the right of the vertical line $r = \sec \theta$

C. Shared by the cardioids $r = 2(1 + \cos \theta)$ and $r = 2(1 - \cos \theta)$