1. PHYSICS and MEASUREMENTS

"What are the main areas of interest of physics?"

Like all other sciences, physics is based on experimental observations and quantative measurements.

Physicists observe the phenomena of nature and try to find patterns and principles that relate these phenomena.

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What are these patterns?

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These patterns are called physical theories.

If these patterns are very well established and of broad use, they are called physical laws and principles!

The development of a physical theory requires creativity at every stage. The physicists have to learn to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results.

Of course, no theory is ever regarded as the final or ultimate truth. The possibity always exists for revising or discarding the theory.

" Every physical theory gives good results under certain conditions. "

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For example, Newton's law of motion developed by Newton in the 17^{th} century achieve to predict the motion of particles moving with normal velocities. Newton's law of motion cannot be applied to the particles having with the velocity which is close to the speed of light. ($v_{light} = 3 \times 10^8$ m/s.). As is known, to solve this problem, Albert Einstein proposed special relativity theory for those particles.

STANDARDS and UNITS

The laws of physics are expressed as mathematicall relationships among physical quantities.

Most of these quantities are derived quantities. They can expressed as combinations of a small number of basic quantities.

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In mechanics, we have three basic quantities.

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! Length, mass, and time !

All other quantities in mechanics can be expressed in terms of these three.

* Some physical quantities can be measured directly. For example, the distance between two points is measured by using a ruler directly. Or measuring a time interval between two effects is realized by using a stop-watch (or chronometer)

** In other cases, we define a physical quantity by using directly measured physical quantities..

For example, we can calculate the average velocity of a moving object by using the distance travelled measured with a ruler and the time of travel measured with a stop-watch.

$$\bar{v} = \frac{\Delta x \rightarrow \text{measured with a ruler}}{\Delta t \rightarrow \text{measured with a stop-watch}}$$

To make accurate and reliable measurements, we need units of measurements that do not change and that can be duplicated by observers in various locations all around the world.

The system of units used by scientists and engineers around the world is called as "the metric system". Since 1960, it has been known officially as International System or SI (the abbreviation of its French name Systéme International).

The fundamental units of SI are length, mass, and time. The units are the meter, kilogram, and seconds, respectively.

Length → m
Mass → kg
Time → s

The other fundamental units of SI are temperature (Kelvin), electric current (Amper), liminous intesity (Candela), and the amount of substance (mole).

Length

We can identfy length as the distance between two points of space. But the exact definition of 1m has been established in October 1983.

1m is the distance that light travels in vacuum in 1/299 792 458 second.

Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

Macroscopic length scales in nature

Microscopic length scales in nature.

Mass

The standard of mass ie. 1kg is defined to be the mass of a particular cylinder made of platinum -iridium alloy kept at the International Bureu of Weights and Measures at Sévres, France. The mass standard was established in 1887 and has not been changed since that time because platinum-iridium alloy is the best stable material.

A duplicate of the Sévres cylinder is kept at the National Institute of Standards and Technology in Maryland, USA.

Masses of Various Objects (Approximate Values)

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

Time

The standard of time has been defined twice. Before 1960, the standard of time was defined in terms of mean solor day for the year 1900.

$$\Downarrow$$

$$\frac{1}{60} \cdot \frac{1}{60} \cdot \frac{1}{24}$$

A solor day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.

$$\Downarrow$$

24 hours

But the rotation of the Earth is now known to very slightly with time. Due to this reason, the motion of the

Earth is not suitable to define a time standard.

In 1967, atomic clock, which uses the characteristic frequency of the cesium-133 atom, has been acepted as the reference clock for time standard.

The second (1s) is defined as 9 192 631 770 times the period of vibration of radiation from the Cs atom.

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time interval for one revolution of the Earth about its axis)	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

Unit Prefixes

Once we have defined the fundamental units, it is easy to introduce larger or smaller units for the same physical quantities by using prefixes. In the metric system, these other units are related to the fundamental units by multiples of positive or negative powers of ten.

For example milimeters, nanoseconds, where the prefixes mili- and nano- denote multipliers of negative powers of ten.

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

The most used prefixes and powers of ten in our lectures!

lower case letters abbreviations

upper case letters abbreviations

Example of Unit Conversion:

Let's express 57 km/h in m/s.

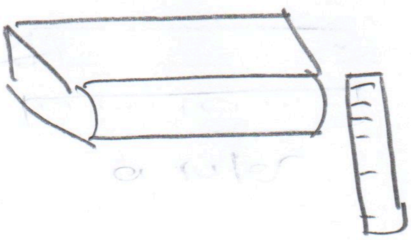
$$57\ \frac{km}{h} = 57 \cdot \frac{1.10^3\ (m)}{60\times 60\ (s)} = 15.833\ (m/s)$$

UNCERTAINTY and SIGNIFICANT FIGURES

Meaningfull Digits

$\boxed{\text{Measurements always have uncertainities.}}$

For example, If you measure the thickness of your book by using an ordinary ruler, your measurement accuracy is milimeter.



3 mm

a ruler

You can't tell whether the actual thickness of your book is 3,00 mm or 3,15 mm or 3,77 mm.

But if you use a micrometer caliper, you can measure the more precise length as 3,12mm. Of course, the measurement made by micrometer caliper will have a smaller uncertaihity relative to a ruler.

The uncertainity is also called as "error" because it indicates the maximum difference there is likely to be between the measured value and the true value.

Error or absolute error = |Measured value − True Value|

The accuracy of a measured value is that how close it is likely to be the true value.

Measured value ∓ $\underbrace{\text{second number}}_{\text{Accuracy}}$

$\underbrace{56,47 \mp 0,02 \text{ mm}}$ ⇒ Measurement result for the diameter of a circle.

⇓

This result means that the true value is unlikely to be less than 56.45mm or greater than 56.49mm.

The value of uncertainty / error can depend on various factors, such as the quality of the apparatus used, the skill of the experimenter, and the number of measurements performed.

The number of significant figures (or meaningfull digits) in a measurement can be used to express something about the uncertainity.

Examples:

* 23,21 cm Number of significant figures is 4, because we have ensured four digits, but we have not ensured that the true value is 23,215 cm, 23,219 cm or something else

** 0,062 cm Number of significant figures is 2, because this measurement can also be written as $6,2 \times 10^{-2}$ cm. or 62.10^{-3} cm. We have only ensured six and two numbers.

*** 80 km Number of significant figures is 1, because this measurement can also be written as 8.10^{1} km, we have only ensured eight number.

**** 80,0 km Number of significant figure is 3, because in this case the distance has been measured with $\mp 0,1$ km accuracy.

When we solve problems, we usually combine quantities mathematically through multiplication, division, addition, and substraction.

For example if we want to obtain the distance travelled from one point to another by a moving object, we use multiplication of the speed and time. In these type of calculations, how can we determine the number of significant figures for the result?

* When multiplying several quantities, the number of the significant digits in the final answer is the same as the number of significant digits in the quantity having the smallest number of meaningfull digits. The same rule is applied to division operation.

** $0,745 \times 2,2 / 3,88 = ?$

Express the final result with significant digits!

$0,745 \longrightarrow 745 \times 10^{-3} \longrightarrow 3$

$2,2 \longrightarrow 22 \times 10^{-1} \longrightarrow \boxed{2}$

$3,88 \longrightarrow 388 \times 10^{-3} \longrightarrow 3$

The result must be expressed by two significant digits!

$0,745 \times 2,2 / 3,88 = 0,43$

** When numbers are added or substracted, the number of decimal places in the final answer should be the smallest decimal places of any term in the sum or difference operation.

** As an example of this rule, let's consider this operation:

$$\underbrace{27,153}_{\text{3 decimal places}} + \underbrace{138,2}_{\text{1 decimal place}} = 165,353$$

If you perform this calculation by a calculator, you'll find this result

The result must have only 1 decimal place

$\Downarrow$

$\boxed{165,3}$

DIMENSIONAL ANALYSIS

In many situations, you may have to check a spesific formula to see if it is correct or not. To do this, we use dimensional analysis.

What is dimension in physics?

$\downarrow$

Dimension denotes the physical nature of a quantity.

We'll use square brackets [ ] to represent the dimension of a physical quantity.

The dimension of length → [L]

The dimension of mass → [M]

The dimension of time → [T]

Any relationship or equation can be correct only if the dimensions on both sides of the equation are the same.

Example | Show that the expression $v = at$ where v represents speed, a is acceleration, and t is an instant time is dimensionally correct.

$$[v] = \frac{[L]}{[T]} \qquad [a] = \frac{[L]}{[T]^2} \qquad [t] = [T]$$

$[v]$ ↓ unit → m/s

$[a]$ ↓ m/s^2

$$v \stackrel{?}{=} at$$

$$\frac{[L]}{[T]} \stackrel{?}{=} \frac{[L]}{[T]^2}\,\cancel{[T]} \Rightarrow \frac{[L]}{[T]} = \frac{[L]}{[T]}$$

The same dimension on both sides!

The equation is dimensionally correct!

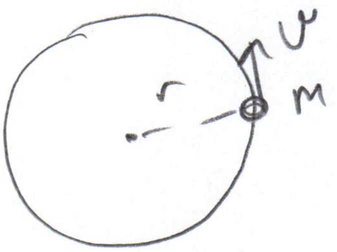
When making dimensional analysis, you can only sum or substract of any terms in the same dimension.

Let's continue with an exam question.

Example Suppose we told that the acceleration a of a particle moving with a uniform speed v in a circle of radius r is proportional to some power of r, say r^n and some power of v, say v^m. Determine the values of n and m, and write the simplest form of the equation for the acceleration.

$$\left.\begin{array}{l} a \propto r^n \\ a \propto v^m \end{array}\right\} a \propto r^n v^m$$

To remove proportionally and write an equation, we need a proportionally constant



$$a = k r^n v^m$$

k: dimensionless proportionality constant.

$$a = k r^n v^m$$

$$[a] = \frac{[L]}{[T]^2} \qquad [r] \Rightarrow [L] \qquad [v] = \frac{[L]}{[T]}$$

$[a] \rightarrow m/s^2$, $[r] \rightarrow m$, $[v] \rightarrow m/s$

$$\frac{[L]}{[T]^2} = [L]^n \frac{[L]^m}{[T]^m}$$

To get the same dimension on both sides of the equation $n+m=1$ $\quad m=2 \Rightarrow n=-1$

$$a = k v^2 r^{-1}$$

$$\boxed{a = k\frac{v^2}{r}}$$

$k=1$

The simplest form of the equation is obtained when $k=1$.

$$a = \frac{v^2}{r} \Rightarrow$$ This equation represents the acceleration of a uniform circular motion.

VECTORS

Some of the physical quantities such as time, temperature, mass, density, etc. can be described completely by a single number with a unit. But many other physical quantities in physics have a direction associated with them. Due to this reason, they cannot be described by a single number.

Scalar quantities: Time, temperature, mass, density, volume etc.

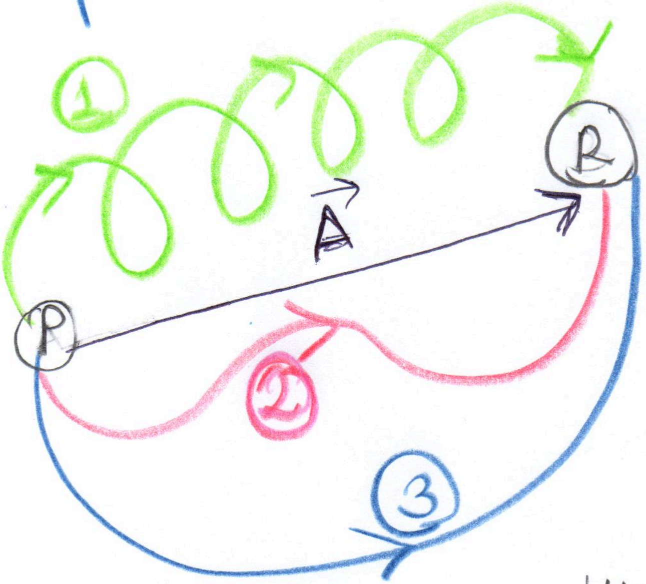
Vector quantities: Displacement, velocity, force, acceleration, etc.

magnitude direction

When we combine scalar quantities, we use ordinary arithmetic operations. However, combining vectors requires a different set of operations!

To understand more about vectors and how they combine let's start focusing on the simplest vector quantity:

Displacement Vector ⇒ A change in position



Suppose that we have a particle moving from point P to R along different paths as shown in the figure

We represent the displacement vector of the particle by drawing an arrow from point P to R. (colored with black) The direction of the arrow head shows the direction of the displacement and its length corresponds to the magnitude of the displacement vector.

Displacements depend only on the initial and final positions. So the displacement vector is independent of the shape of the path followed by the particle!

To represent a vector, we use a letter with an arrow over the letter such as $\vec{A}$

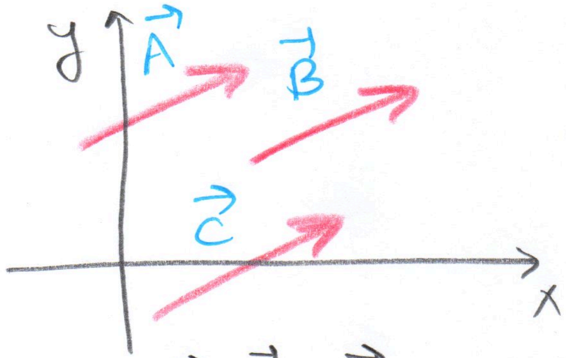
The magnitude of a vector is written either $|\vec{A}|$ or A.

The magnitude of a vector has physical units and the magnitude of a vector is always a positive number

Fundamental Properties of Vectors

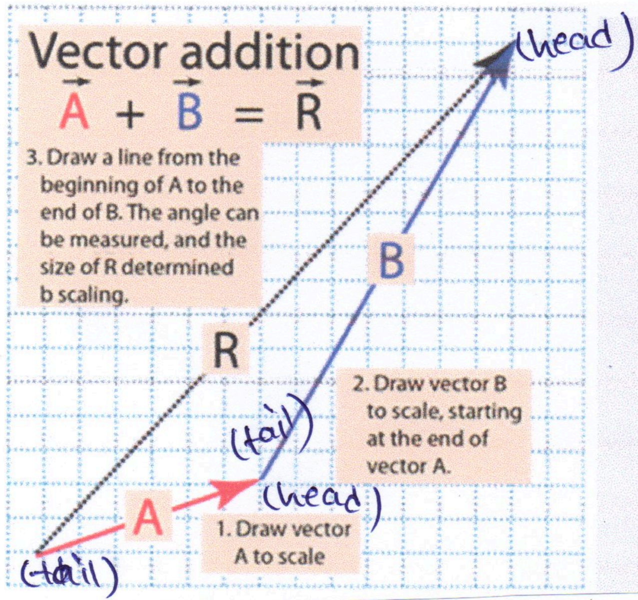
① Equality of Two Vectors: If two vectors such as $\vec{A}$ and $\vec{B}$ have the same magnitude and they point the same direction,

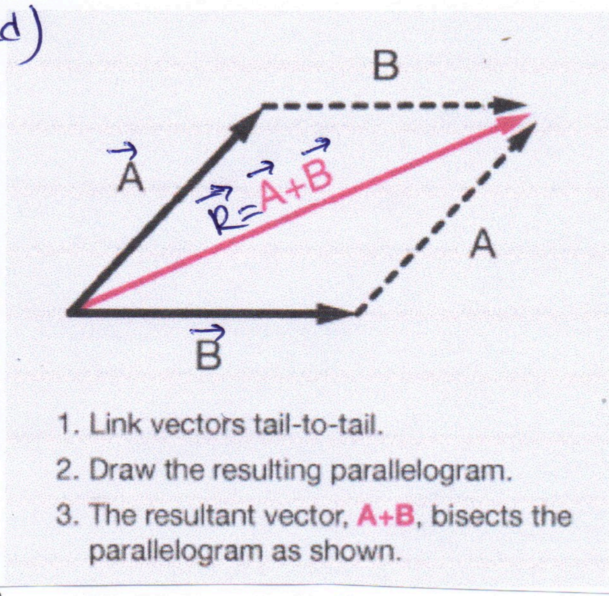
$$\vec{A} = \vec{B}$$



$$\vec{A} = \vec{B} = \vec{C}$$

② Adding Vectors
- Graphical Method (Head to Tail Method)*
- Parallelogram Method





When two vectors are added, the sum is independent of the order of the addition.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

* The first vector's head must coincide with the second's vector tail.

You can add two vectors by constructing a parallelogram. In this case, the vectors $\vec{A}$ and $\vec{B}$ are both drawn with their tails.

The resultant vector, $\vec{R}$ is a diagonal of a parallelogram constructed by $\vec{A}$ and $\vec{B}$ vectors.

③ Negative of a Vector: The negative vector $(-\vec{A})$ is defined as the vector that when it is added to $\vec{A}$ gives zero. for the vector sum.

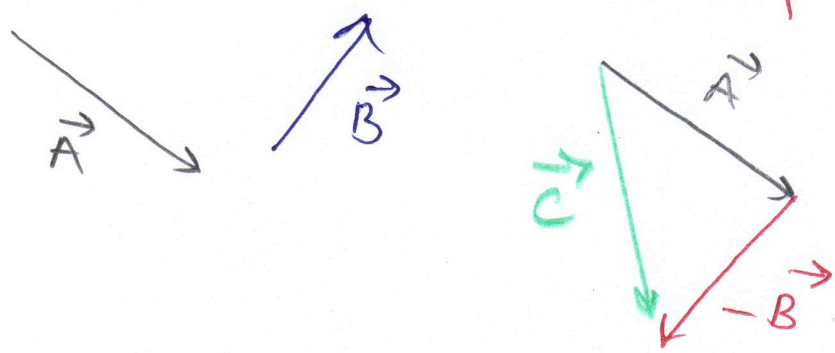
$$\vec{A} + (-\vec{A}) = 0$$

The vectors $\vec{A}$ and $(-\vec{A})$ have the same magnitude but point in opposite directions.

④ Substracting of a Vector:

when we substract one vector from another, we'll use the definition of the negative vector.

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \rightarrow$ Adding $-\vec{B}$ to $\vec{A}$ vector is equivalent to substracting $\vec{B}$ from $\vec{A}$.



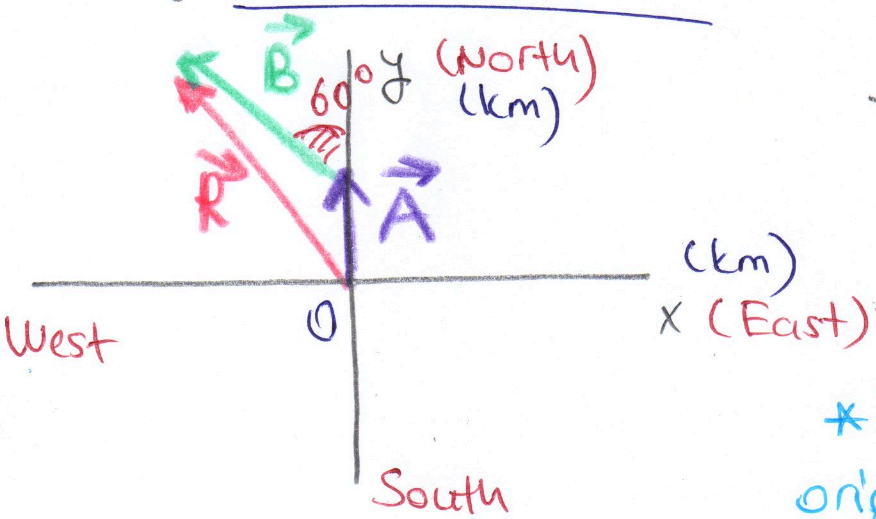
$\vec{C} = \vec{A} - \vec{B}$

⑤ Multiplying a Vector by a Scalar Quantity

If vector $\vec{A}$ is multiplied by a positive scalar quantity, m, the product $m\vec{A}$ is a vector that has the same direction with $\vec{A}$ and the magnitude of mA.

If vector $\vec{A}$ is multiplied by a negative scalar quantity, -m, the product $-m\vec{A}$ is a vector that has the magnitude of mA and it is directed opposite $\vec{A}$.

Example) A car travels 20 km to north and then 35 km in a direction 60° west of north. Find the magnitude and direction of the car's resultant displacement by using graphical method.

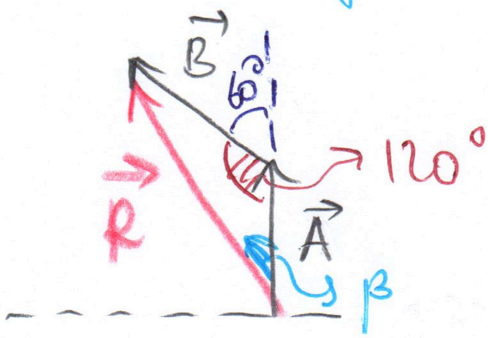


$\vec{A}$ and $\vec{B}$ vectors represent the first and second displacements of the car.

* You should choose the origin as the starting point of the car.

$|\vec{A}| = 20$ km

$|\vec{B}| = 35$ km



We can obtain the magnitude of R by using the law of cosines.

$$R^2 = A^2 + B^2 - 2AB\cos\alpha$$

$$R^2 = 20^2 + 35^2 - 2.20.35.\cos 120^\circ$$

$$R = 48,2 \text{ km}$$

What about the direction of $\vec{R}$? We can obtain the angle β by using the law of sinuse.

$$\frac{\sin 120^\circ}{R} = \frac{\sin\beta}{|\vec{B}|} \Rightarrow \sin\beta = \frac{35.\sin 120^\circ}{48,2} = 0,629$$

$$\beta \cong 38,9^\circ$$

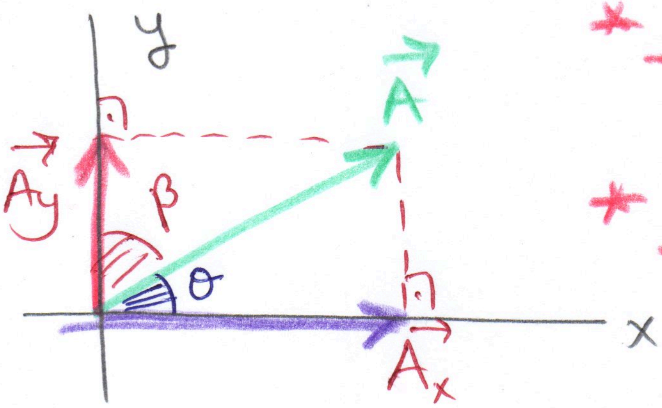
Hence, the resultant displacement of the car is 48,2 km in a direction 38,9° west of north.

COMPONENTS of a VECTOR and UNIT VECTORS

The graphical method for adding vectors is not recommended when high accuracy is required or three dimensional vectors are considered.

Instead of graphical method, we'll describe a method for adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the components of a vector.

For example, let's consider $\vec{A}$ vector lying in the xy-plane and makes angle θ with the positive x-axis.



* $\vec{A_x}$ is the projection of the vector $\vec{A}$ onto x-axis.
* $\vec{A_y}$ is the projection of the vector $\vec{A}$ onto y-axis.

By using the parallelogram method $\vec{A} = \vec{A_x} + \vec{A_y}$

$$\cos\theta = \frac{|\vec{A_x}|}{|\vec{A}|} \quad \text{or} \quad A_x = A\cos\theta = A\sin\beta$$

$$\sin\theta = \frac{|\vec{A_y}|}{|\vec{A}|} \quad \text{or} \quad A_y = A\sin\theta = A\cos\beta$$

From Pisagore theorem $A^2 = A_x^2 + A_y^2 \Rightarrow A = \sqrt{A_x^2 + A_y^2}$

The direction of the $\vec{A} = tg\theta = \frac{A_y}{A_x} \Rightarrow \theta = Arctg\left(\frac{A_y}{A_x}\right)$

Unit Vectors

In physics, we often describe vector quantities in terms of unit vectors.

A unit vector is a dimensionless vector having a magnitude of exactly "1". Unit vectors are used to specify a given direction and have no other physical significance

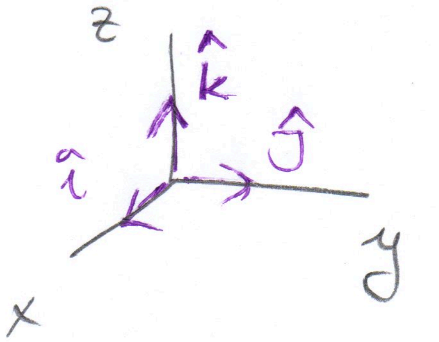
$\hat{\imath}$ → +x-axis

$\hat{\jmath}$ → +y-axis

$\hat{k}$ → +z-axis

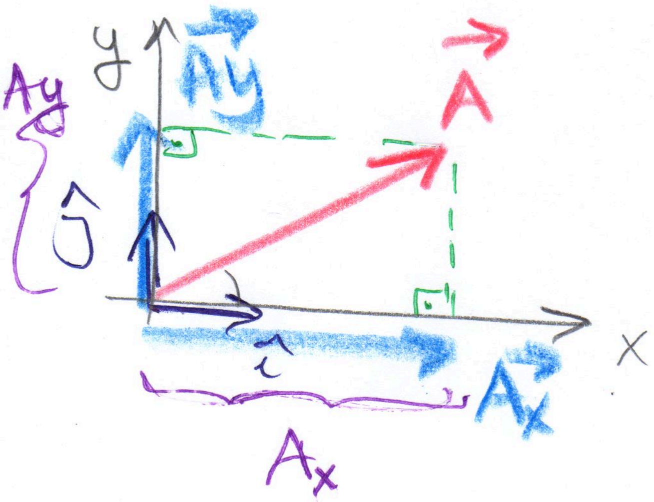
^ → hat symbol

We shall denote unit vectors by using a hat symbol over a physical quantity.



$$|\hat{\imath}| = |\hat{\jmath}| = |\hat{k}| = 1$$

Let's consider a vector $\vec{A}$ lying in the xy-plane.



$$\left.\begin{aligned} \vec{A_x} &= A_x \hat{\imath} \\ \vec{A_y} &= A_y \hat{\jmath} \end{aligned}\right\} \quad \vec{A} = A_x \hat{\imath} + A_y \hat{\jmath}$$

Now let's see how to use components to add vectors when the graphical method is not sufficiently accurate.

$\star$ $\vec{A} = A_x\hat{i} + A_y\hat{j}$ $\qquad$ $\vec{R} = \vec{A} + \vec{B} = ?$

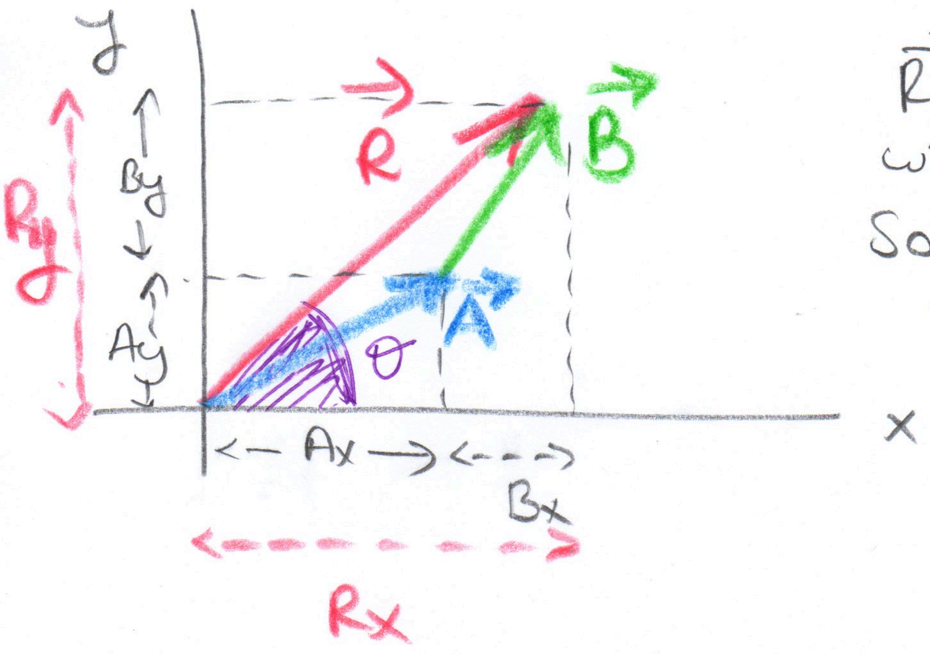
$\vec{B} = B_x\hat{i} + B_y\hat{j}$

When adding or substracting vectors, all we do is add or substract the x, y and z-components separately.

$$\vec{R} = (\underbrace{A_x + B_x}_{R_x})\hat{i} + (\underbrace{A_y + B_y}_{R_y})\hat{j} = R_x\hat{i} + R_y\hat{j}$$

$$|\vec{R}| = R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Let's see the $\vec{R}$ vector in the xy-plane.



$\vec{R}$ vector makes angle θ with the +x-axis.

So the direction of $\vec{R}$ vector $= tg\theta = \dfrac{R_y}{R_x}$

$$tg\theta = \frac{A_y + B_y}{A_x + B_x}$$

If we add vectors in three dimensions;

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

Exercise Suppose that we have two vectors $\vec{A}$ and $\vec{B}$ are

$\vec{A} = 2\hat{i} + 2\hat{j}$ (m)

$\vec{B} = 2\hat{i} - 4\hat{j}$ (m)

a) find the resultant vector.

$$\vec{R} = \vec{A} + \vec{B} = (2+2)\hat{i} + (2-4)\hat{j} = 4\hat{i} - 2\hat{j}\ (m)$$

b) Evaluate the components of vector $\vec{R}$.

$R_x = 4$ (m) $\quad R_y = -2$ (m)

c) find the magnitude of vector $\vec{R}$.

$$|\vec{R}| = R = \sqrt{R_x^2 + R_y^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}\ (m)$$

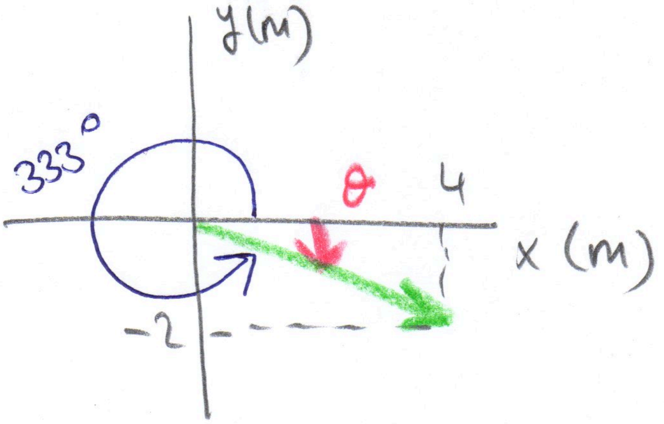
d) What is the direction of the resultant vector?

$$tg\theta = \frac{R_y}{R_x} = -\frac{2}{4} = -0.5 \quad \theta = Arctg(-0.5)$$

$\theta = -27°$

$\downarrow$

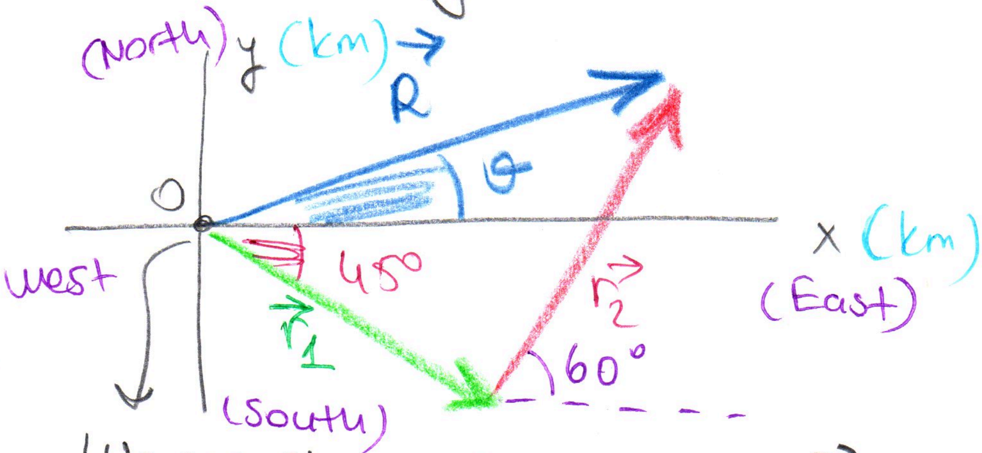
θ is the angle with the +x-axis



Exercise A walker begins a trip by first walking 25 km southeast from her car. On the second day, she walks 40 km in a direction 60° north of east.

a) Determine the displacement vectors of the walker for each day.

To calculate each displacement of her, let's draw an xy-coordinate axes.



The first day's displacement $\rightarrow \vec{r_1}$

The second day's displacement $\rightarrow \vec{r_2}$

We can choose the origin as the starting point of the walk.

$$\vec{r_1} = r_{1x}\hat{i} - r_{1y}\hat{j} \quad |\vec{r_1}| = 25\,km$$

$$\vec{r_2} = r_{2x}\hat{i} + r_{2y}\hat{j} \quad |\vec{r_2}| = 40\,km$$

$$\vec{r_1} = |\vec{r_1}|.\cos 45\,\hat{i} - |\vec{r_1}|.\sin 45\,\hat{j}$$

$$\vec{r_1} = 25\left[\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}\right] \Rightarrow \vec{r_1} = 17{,}7(\hat{i} - \hat{j})\ (km)$$

$$\vec{r_2} = |\vec{r_2}|\cos 60\,\hat{i} + |\vec{r_2}|.\sin 60\,\hat{j}$$

$$\vec{r_2} = 40\left[\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right] \Rightarrow \vec{r_2} = 20\hat{i} + 34{,}6\hat{j}\ (km)$$

b) Determine the resultant displacement $\vec{R}$ for trips in terms of unit vectors.

$$\vec{R} = \vec{r_1} + \vec{r_2} = (17{,}7 + 20)\,\hat{i} + (34{,}6 - 17{,}7)\hat{j}$$

$$\vec{R} = 37{,}7\,\hat{i} + 16{,}9\,\hat{j}\ (km)$$

c) Calculate the magnitude of the total displacement and the direction of vector $\vec{R}$.

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{37{,}7^2 + 16{,}9^2} = 41{,}3\,km \qquad \theta = \mathrm{Arctg}\left(\frac{16{,}9}{37{,}7}\right) = 24{,}1°$$

Multiplying of Two Vectors

→ Scalar product
→ Vector product

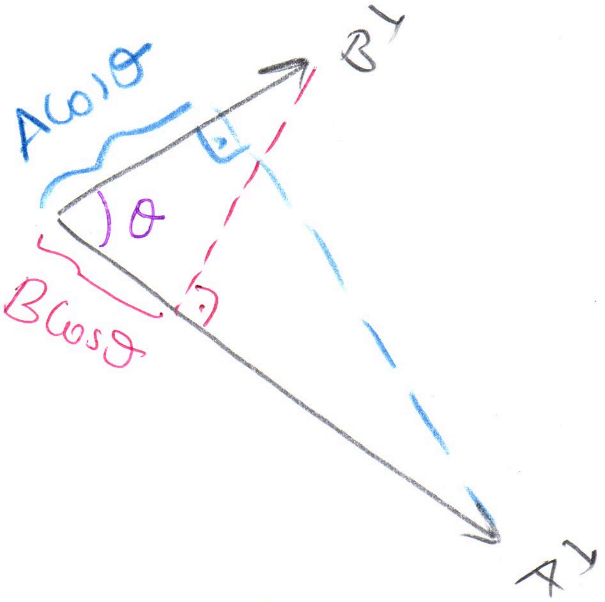
Scalar and vector products of two vectors will give important physical quantities such as work, angular momentum or torque. For example the scalar product of force and displacement vectors gives the work. Due to this reason, we'll focus on multiplying of two vectors in two different ways.

SCALAR PRODUCT of TWO VECTORS

DOT

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta}$$

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as a scalar quantity equals the product of the magnitudes of two vectors and the cosine of the angle between these two vectors.



The result of a scalar product must be a scalar quantity!

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cdot \cos\theta$$

What does this term represent?

Bcosθ is the projection of $\vec{B}$ onto $\vec{A}$ vector.

Scalar product is represented by a big dot symbol.

What about $A\cos\theta$ in the $\vec{A}\cdot\vec{B} = AB\cos\theta$

$A\cos\theta$ is also the projection of $\vec{A}$ vector onto $\vec{B}$.

* In the scalar product, if the order of the vector is changed, the result does not change.

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

** If vector $\vec{A}$ is perpendicular to vector $\vec{B}$,

$\vec{A}\cdot\vec{B} = 0$ (because $\cos 90° = 0$)

*** If vector $\vec{A}$ is parallel to vector $\vec{B}$ ($\vec{A} \parallel \vec{B}$)

$\vec{A}\cdot\vec{B}$ can have two results.

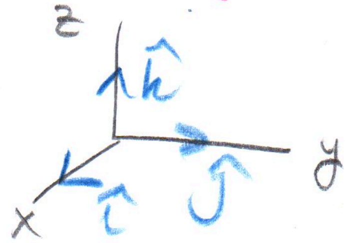
$\vec{A}\cdot\vec{B} = AB$ $(\theta = 0)$ $\qquad$ $\vec{A}\cdot\vec{B} = -AB$ $(\theta = 180°)$

**** The scalar products of unit vectors:

$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|.\cos 0 = 1.1.1 = 1$

$\hat{i}\cdot\hat{j} = |\hat{i}|.|\hat{j}|.\cos 90° = 1.1.0 = 0$

$\hat{j}\cdot\hat{j} = 1$ $\qquad$ $\hat{j}\cdot\hat{k} = 0$



$\hat{i}$, $\hat{j}$, and $\hat{k}$ unit vectors are perpendicular to each other!

We can multiply unit vectors by using scalar product if they are in the same direction.

* Suppose that we have two vectors $\vec{A}$ and $\vec{B}$ expressed in terms of unit vectors in 3-dimensions.

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x (\hat{i} \cdot \hat{i})^{1} + A_x B_y (\hat{i} \cdot \hat{j})^{0} + A_x B_z (\hat{i} \cdot \hat{k})^{0} + A_y B_x (\hat{j} \cdot \hat{i})^{0}$$
$$+ A_y B_y (\hat{j} \cdot \hat{j})^{1} + A_y B_z (\hat{j} \cdot \hat{k})^{0} + A_z B_x (\hat{k} \cdot \hat{i})^{0} + A_z B_y (\hat{k} \cdot \hat{j})^{0}$$
$$+ A_z B_z (\hat{k} \cdot \hat{k})^{1}$$

We see that six of nine terms are zero due to the fact that the scalar product of unit vectors that are perpendicular is zero.

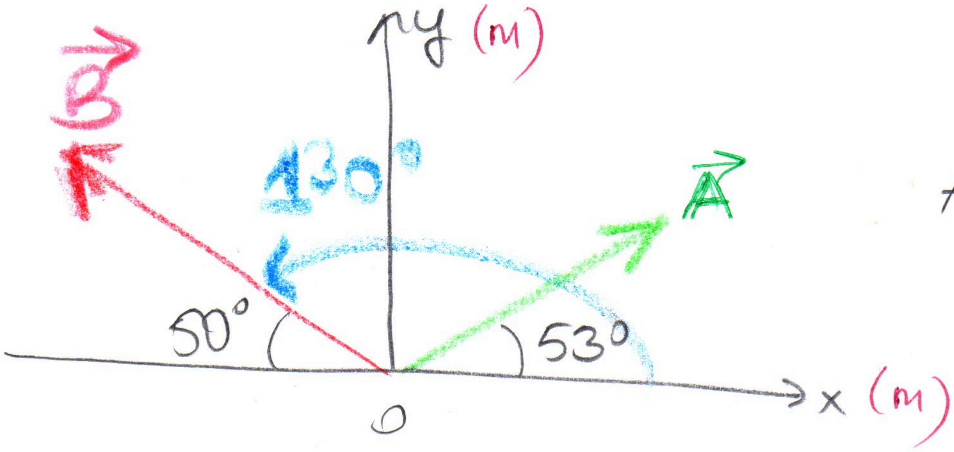
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = \text{SCALAR!}$$

The scalar product of two vectors is the sum of the products of their respective components.

** The scalar product definition gives a straightforward way to find the angle θ between any two vectors $\vec{A}$ and $\vec{B}$.

$$\vec{A} \cdot \vec{B} = A.B \cos\theta \Rightarrow \cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

Exercise Find the scalar product of $\vec{A}$ and $\vec{B}$ vectors given in the figure. The magnitudes of the vectors are $A = 4m$ and $B = 5m$.



$\vec{A} = A\cos 53\,\hat{i} + A\sin 53\,\hat{j}$

$\vec{A} = (4 \times 0{,}6\,\hat{i}) + (4 \times 0{,}8)\,\hat{j}$

$\vec{A} = 2{,}4\,\hat{i} + 3{,}2\,\hat{j}\ (m)$

$\vec{B} = -B\cos 50\,\hat{i} + B\sin 50\,\hat{j}$

$\vec{B} = -5 \cdot \cos 50\,\hat{i} + 5 \cdot \sin 50\,\hat{j}$

$\vec{B} = -3{,}2\,\hat{i} + 3{,}8\,\hat{j}\ (m)$

We used acute angle (50°) for writing $\vec{B}$.

Alternatively we can use wide angle (130°) to write $\vec{B}$.

$\vec{B} = B\cos 130\,\hat{i} + B\sin 130\,\hat{j}$

$\vec{B} = -3{,}2\,\hat{i} + 3{,}8\,\hat{j}\ (m)$

$\vec{A} \cdot \vec{B} = (2{,}4\,\hat{i} + 3{,}2\,\hat{j}) \cdot (-3{,}2\,\hat{i} + 3{,}8\,\hat{j})$

$\vec{A} \cdot \vec{B} = [2{,}4 \times (-3{,}2)] + (3{,}2) \times (3{,}8)$

$\vec{A} \cdot \vec{B} = 4{,}48\ (m^2)$

Exercise Find the angle between $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = -4\hat{i} + 2\hat{j} - \hat{k}$.

$$\vec{A} \cdot \vec{B} = A . B \cos\theta$$

$$A = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$B = \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{21}$$

$$\cos\theta = \frac{(2\hat{i} + 3\hat{j} + \hat{k}) \cdot (-4\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{14} . \sqrt{21}}$$

$$\cos\theta = \frac{-8 + 6 - 1}{\sqrt{14} . \sqrt{21}}$$

$$\theta = 100^\circ$$

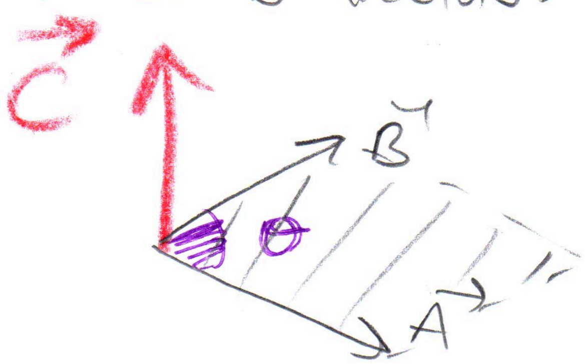
VECTOR / CROSS PRODUCT

Vector product is denoted by $\vec{A} \times \vec{B}$ and it is defined as a third vector $\vec{C}$ which has a magnitude of $AB\sin\theta$.

The result of vector product gives a new vector

$$\vec{A} \times \vec{B} = \vec{C} \qquad |\vec{C}| = AB\sin\theta$$

$\vec{C}$ vector is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ vectors.



Suppose that $\vec{A}$ and $\vec{B}$ vectors are lying in the xy-plane. $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$.