BME4120
Biomedical Image Processing

Lecture 7

Shape Representation

An efficient object representation or shape representation can be achieved by using boundary-based shape descriptors.

Shape signatures are a set of one-dimensional functions that form these descriptors.

The degree of similarity between the objects or images helps to identify objects.

Shape descriptors capture the perceptual features of the shape such as complex coordinates, centroid distance function, tangent angle or turning angles, curvature function, area function, triangle representation and chord length function, etc.

Shape Descriptors

- Shape descriptors describe specific characteristics regarding the geometry of a particular feature
- In general, ***shape descriptors*** or ***shape features*** are some set of numbers that are produced to describe a given shape

Shape Representation

Some of the share descriptors (or signatures)

- ❑Complex coordinates
- ❑Centroid Distance Function
- ❑Geometry of Curves
- ❑Area Function
- ❑Triangle Area
- ❑Chord Length
- ❑Polygon Area

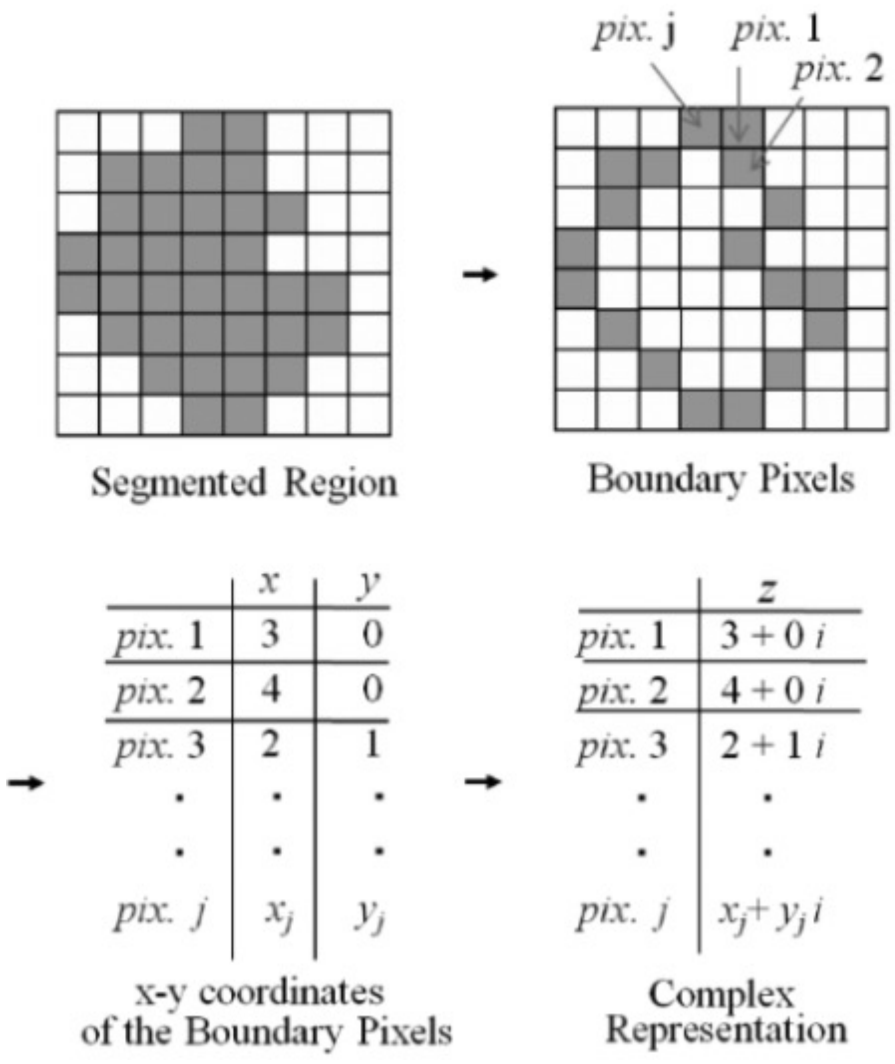
Complex Coordinates

Complex coordinates are measured using complex function that is generated from the coordinate of boundary points p_n $x(n)$, $y(n)$, $n \in [1, N]$.

$$z(n) = [x(n) - g_x] + i[y(n) - g_y]$$

g_x and g_y: centroid's coordinates

Main idea is to transform shape in **R**^{2} to one in **C**.
Complex coordinate representation is invariant to translation.



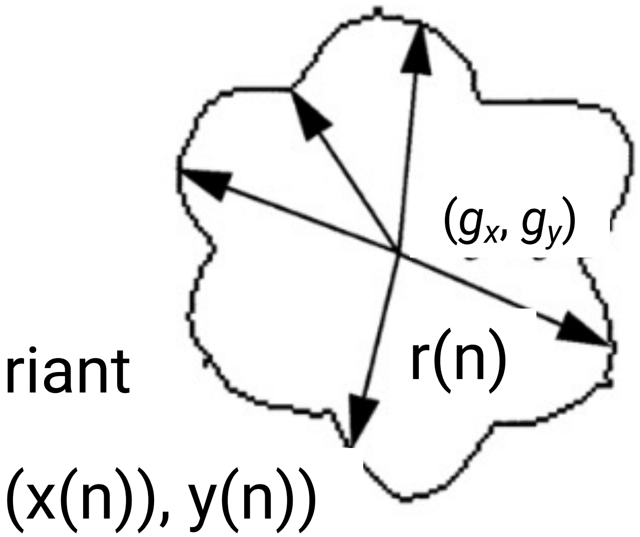
DOI:10.1109/ISDA.2009.205

Centroid Distance Function

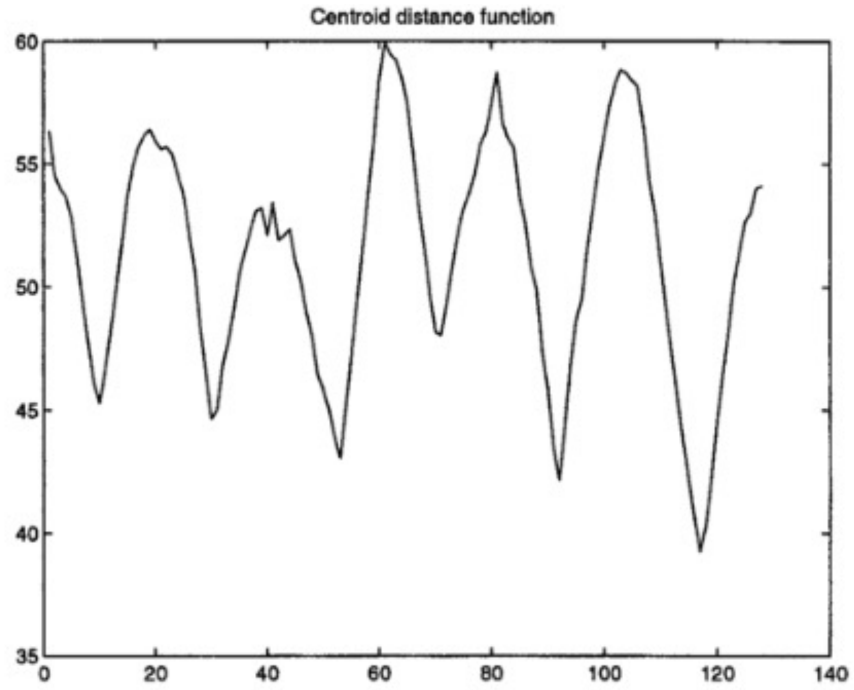
Centroid distance function is measured as the distance of the boundary points from the centroid (g_x, g_y) of a shape.

$$r(n) = \left\{[x(n) - g_x]^2 + \left[y(n) - g_y\right]^2\right\}^{1/2}$$

Centroid distance representation is invariant to translation.

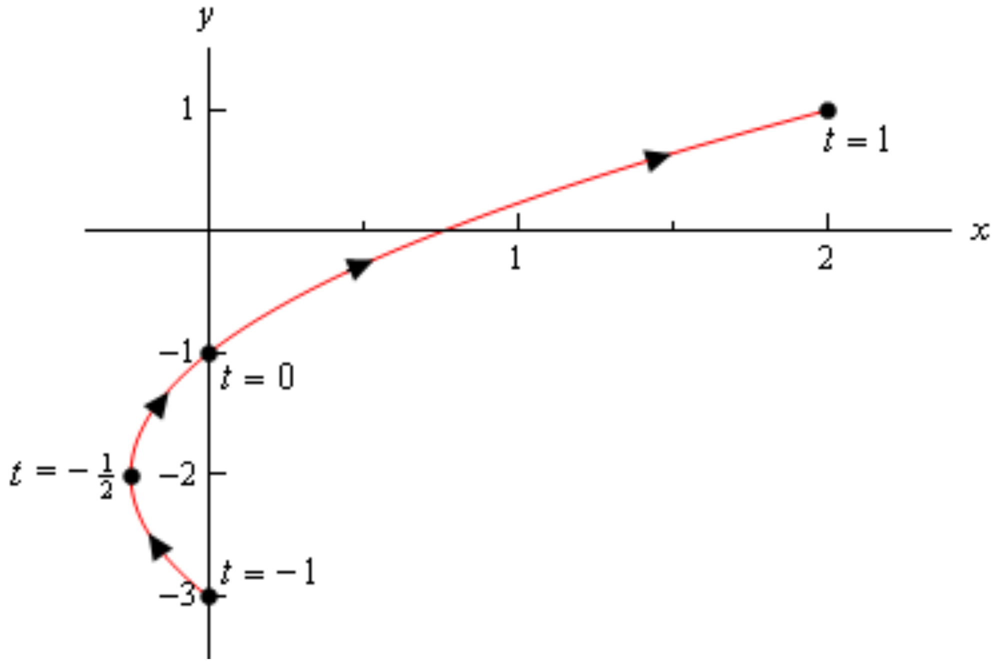


DOI:10.1007/s005300050121



Geometry of Curves

The parametric form of a curve requires two functions—$x(t)$ and $y(t)$ of a parameter t to specify the point along the curve from the starting point $P_1 = [x(t_1), y(t_1)]$ to the end point $P_2 = [x(t_2], y(t_2)]$.



https://tutorial.math.lamar.edu/classes/calcii/parametriceqn.aspx

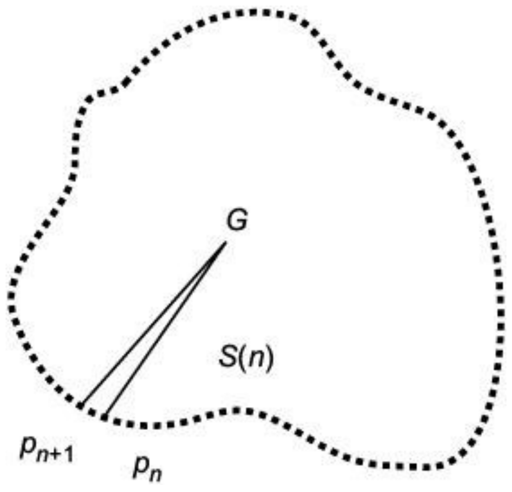
Area Function

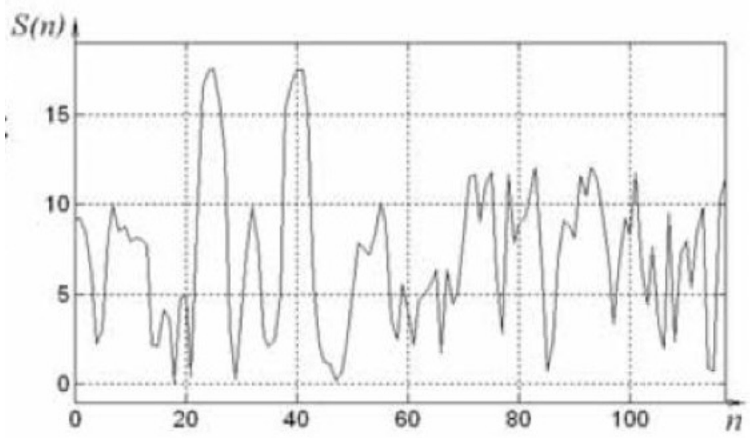
Medical Image Processing, Sinha&Patel, 2014

With the change in boundary points along the shape of a boundary, its centre of gravity also changes.

The area of triangle formed by the two successive boundary points also changes which results in an area function used for the shape representation.

$S(n)$ is the area between the two successive boundary points (p_n, p_{n+1}) and the centre of gravity G.

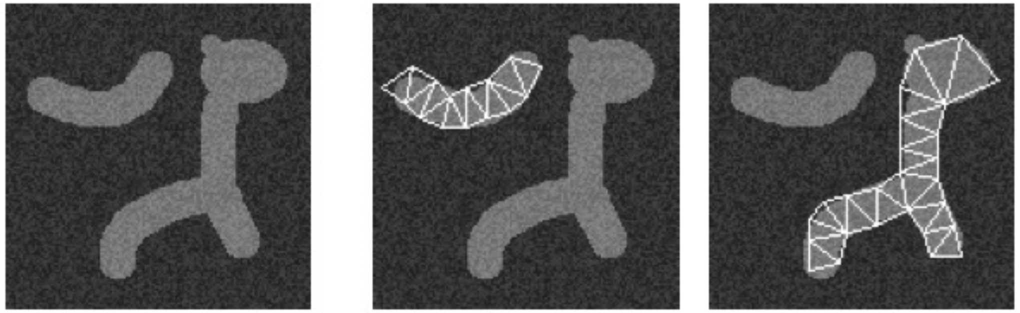




Triangle Area

Triangle area is calculated as the area of the triangles formed by the points on the shape boundary.

The curvature of the contour point (x_n, y_n) is measured using triangle area.



input | sample | sample

Let $n \in [1, N]$ and $t_s \in [1, N/2 - 1]$ (N:even)

For each 3 points:
$P_{n-t_s}(x_{n-t_s}, y_{n-t_s})$, $P_n(x_n, y_n)$, $P_{n+t_s}(x_{n+t_s}, y_{n+t_s})$

The (signed) Area of Triangle Formed by them is:

$$TAR(n, t_s) = \frac{1}{2} \begin{vmatrix} x_{n-t_s} & y_{n-t_s} & 1 \\ x_n & y_n & 1 \\ x_{n+t_s} & y_{n+t_s} & 1 \end{vmatrix}$$

Representation and Detection of Shapes in Images, Pedro F. Felzenszwalb
https://www.math.uci.edu/icamp/summer/research_11/park/shape_descriptors_survey_part3.pdf

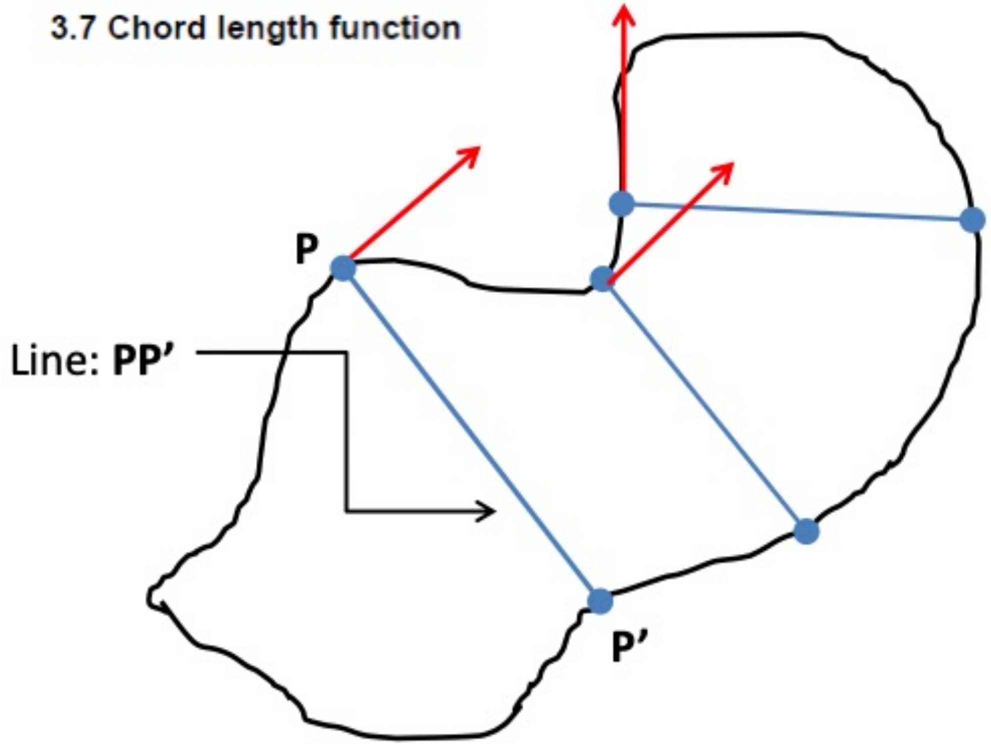
Chord Length

- Chord length (CL) is determined by the shape boundary without using any reference point.

- For each boundary point, its chord length is the shortest distance between this point and another boundary point

    - the line joining the two points is perpendicular to the tangent vector at boundary point.

- CL is invariant to translation.



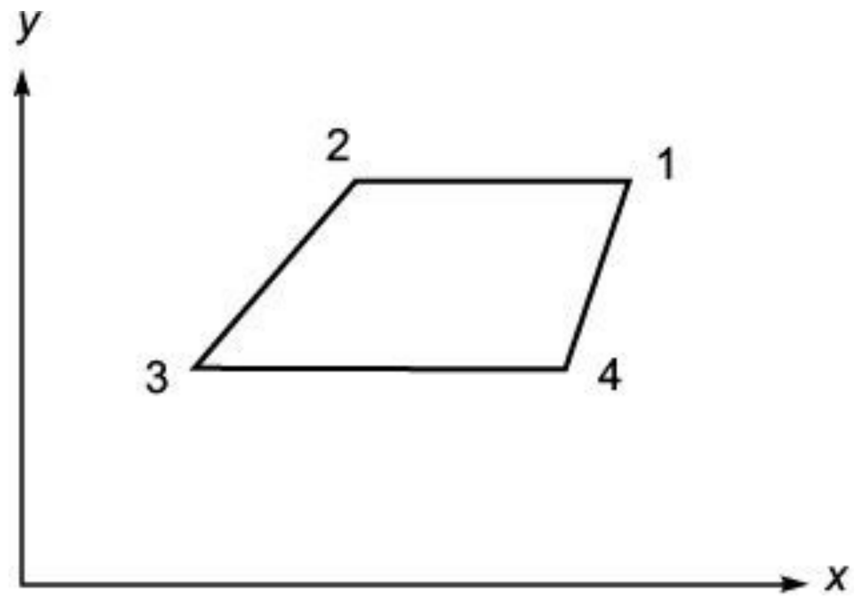
CL very sensitive to noise. Chord length can increase or decrease significantly even for smoothed boundaries

https://www.math.uci.edu/icamp/summer/research_11/park/shape_descriptors_survey_part3.pdf

Polygon Area

Medical Image Processing, Sinha&Patel, 2014

It is s based on the area of polygon for the known ***coordinates*** of its ***vertices***. The vertices are numbered and any suitable point is chosen.



An area of polygon is made by choosing one point as a starting point on the contour, selecting new points and drawing lines from the previous points

Bounding Box

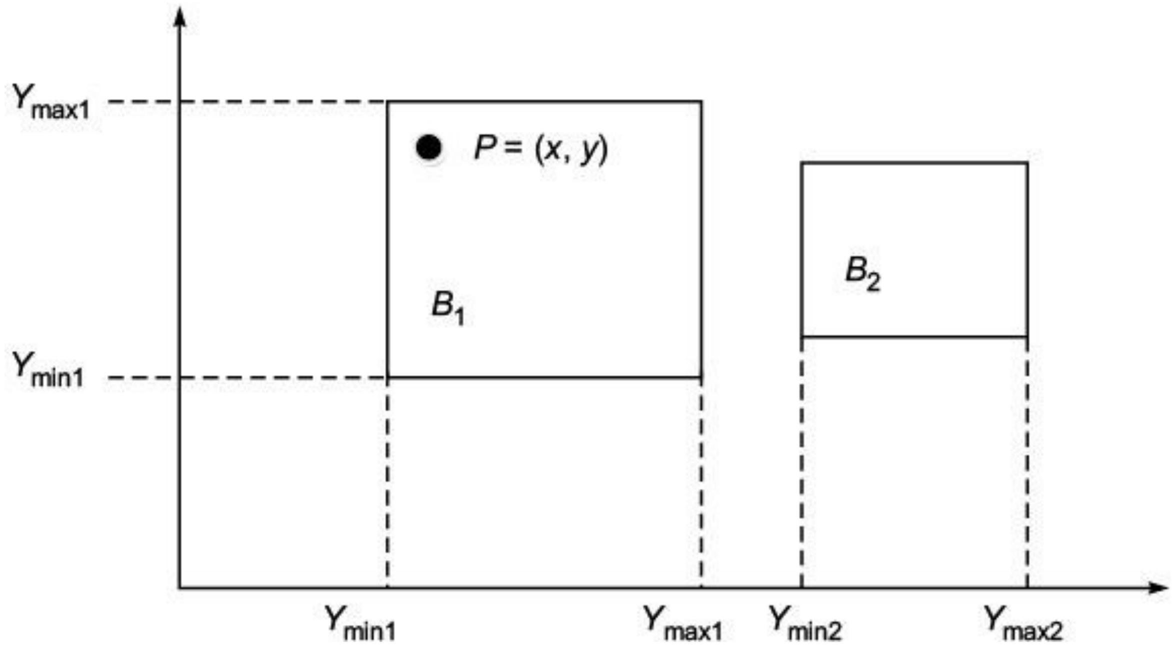
Medical Image Processing, Sinha&Patel, 2014

Bounding box is a very important element for describing a geometric object.

It is a box whose edges are parallel to the coordinate axis and represented by its maximum and minimum extents for all axes.

The box given by all (x, y) coordinates

- Must satisfy $x_{min} \leq x \leq x_{max}$ and $y_{min} \leq y \leq y_{max}$
- Must be specified by the extreme points (x_{min}, y_{min}) and (x_{max}, y_{max})



A point $P(x, y)$ is said in the bounding box if all the discriminations are true. If any of the discrimination is false, then the selected point is not inside the bounding box.

Shape Matrix

The shape matrix is used to present a region shape whose size is (M x N).

A matrix is defined by a square model which is constructed by shape S and create a square centred on the centre of gravity G of that shape.

Shape Matrix

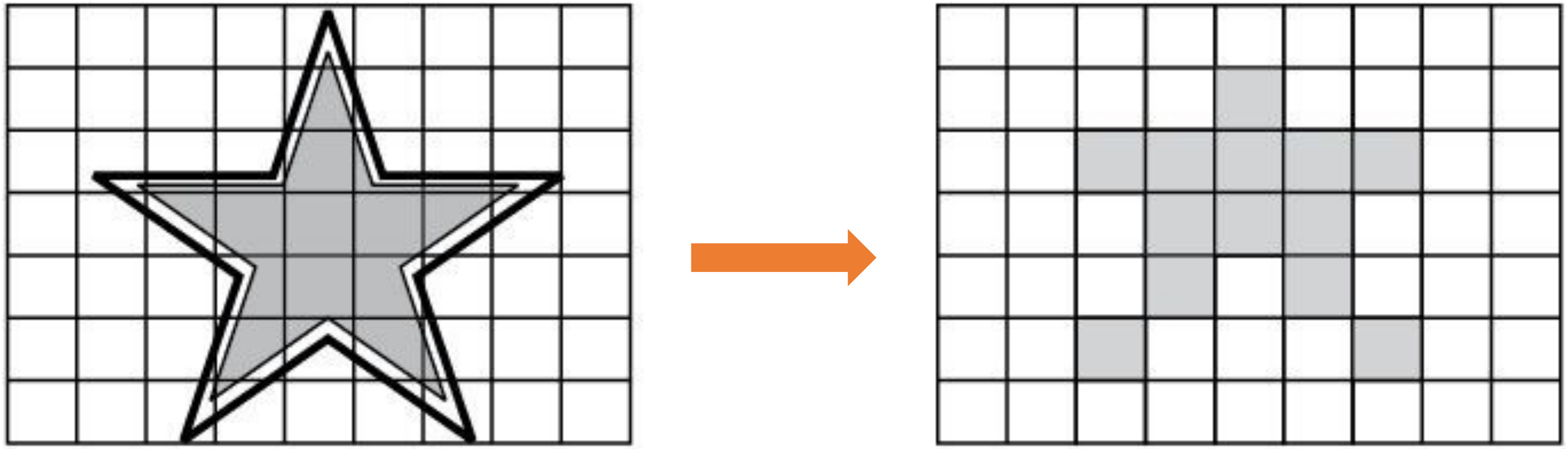
- Given a shape S
- Construct square centered at centroid G of S
- Size of each side is 2L, L= Max Euclidean. Distance between centroid G and point M on boundary of S
- Point M lies in center of one side
- Line GM is perpendicular to side M lies on

Divide the square into $N \times N$ subsquares and denote S_{kj} , $k, j = 1, \ldots, N$, the subsquares of the constructed grid. Define the shape matrix $SM = [B_{kj}]$,

$$B_{kj} = \begin{cases} 1 \Leftrightarrow & \mu(S_{kj} \cap S) \geq \mu(S_{kj})/2 \\ 0 & otherwise \end{cases}$$

where $\mu(F)$ is the area of the planar region F.

Shape Matrix

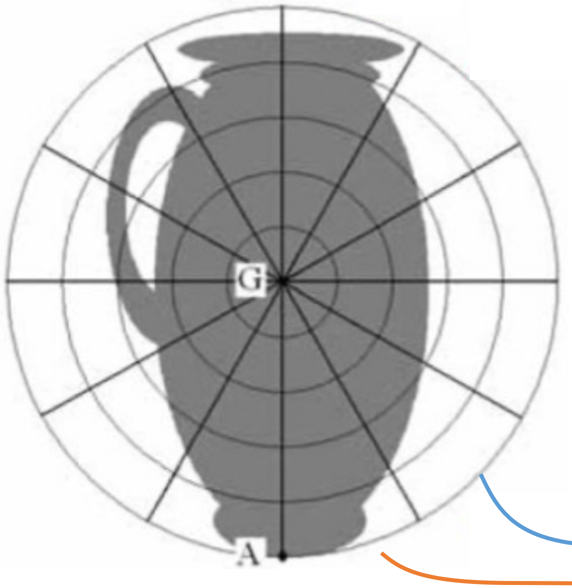


(a) (b)

$$S_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Polar Shape Matrix

- Let S be a shape. G: centroid, GA: maximum radius of shape
- Construct n circles centered at centroid G with equally spaced radii
- Starting with GA, counterclockwise draw radii that divide circles into m equal arcs
- Values of matrix are same as in square model. i.e. value 1 if shape occupies more than 1/2 the area of the polar rectangle
- Example for n=5 and m=12



$$PSM = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Moments of Region and Shape

Image or shape moment is a particular weighted average (i.e., moment) of the image pixels' intensities

- ❑ It could be a function of moments as well
- ❑ It is a measure of the spatial distribution of a shape in relation to an axis.

Moments of different shapes, contour and regions are very important in the analysis of objects and images.

- ❑ *Boundary Moments* → reduce the dimension of boundary representation in the analysis of a contour.
- ❑ *Region Moments* → used as region-based descriptors which help in the analysis of regions in objects.

Moment Analysis

The evaluation of moments represents a systematic method of shape analysis.

- ❑ The most commonly used region attributes are calculated from the three low-order moments.
- ❑ Knowledge of the low-order moments allows the calculation of the central moments, normalised central moments, and moment invariants

Boundary Moments

reduce the dimension of boundary representation in the analysis of a contour. The boundaries of shapes can be represented using it moments such as normal moment and central moment. Generally, m_r is denoted as rth moment and cm_r is used to express the central moment.

$$m_r = \frac{1}{N}\sum_{i=1}^{N}[s(i)]^r$$

$$cm_r = \frac{1}{N}\sum_{i=1}^{N}[s(i) - m_1]^r$$

N is the number of points present on the boundary

Region Moments

Region moments are used as region-based descriptors which help in the analysis of regions in objects. The generalised form of region moment Rm_{ab} of order (a + b) for a region is expressed as

$$Rm_{ab} = \sum_{x} \sum_{y} \underbrace{\Psi_{ab}(x, y)}_{\text{Moment weighting set}} \underbrace{f(x, y)}_{\text{Shape region}} \qquad a, b = 0, 1, 2 \ldots$$

Co-occurance Matrix

A **co-occurrence matrix** or **co-occurrence distribution** (a.k.a. *gray-level co-occurrence matrices,* GLCMs) is a matrix that is defined over an image to be the distribution of co-occurring pixel values (grayscale values, or colors) at a given offset.

It is used as an approach to texture analysis with various applications especially in medical image analysis.

intensity

$$C_{\Delta x,\Delta y}(i,j)=\sum_{x=1}^{n}\sum_{y=1}^{m}\begin{cases}1, & \text{if } I(x,y)=i \text{ and } I(x+\Delta x,y+\Delta y)=j\\ 0, & \text{otherwise}\end{cases}$$

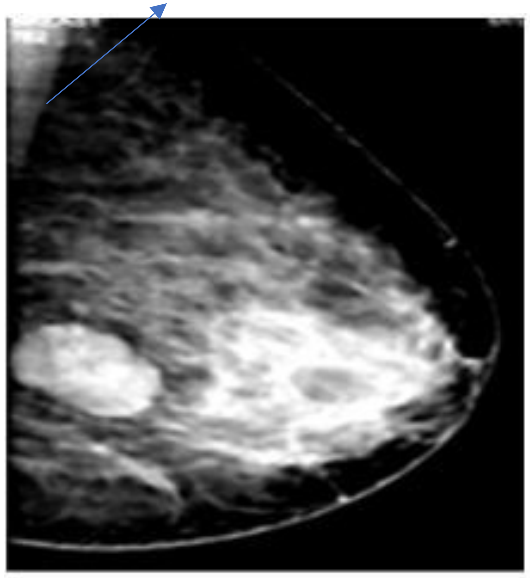
offsets

Summary of Shape Features

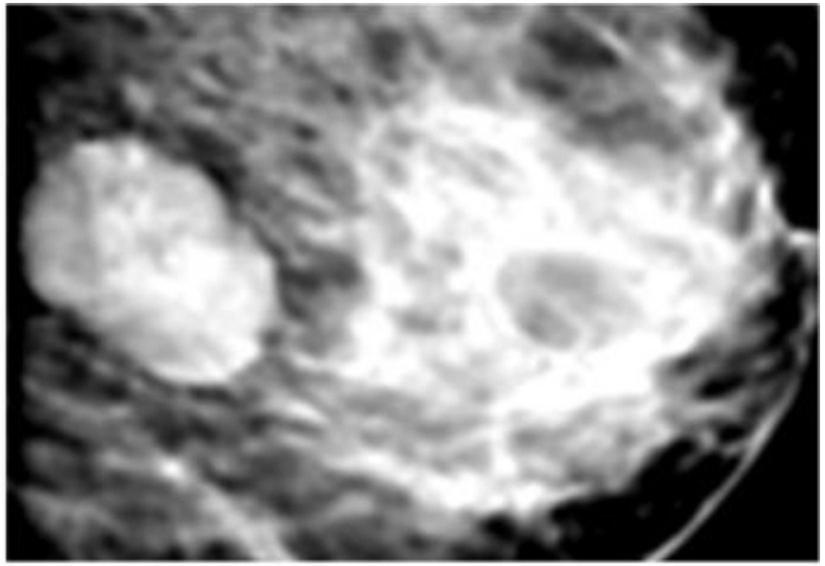
We must select a number of features as per the requirement and application type.

The selection of features is made to classify the cells.

May require magnifying the cells or tissues so that a proper interpretation is given.

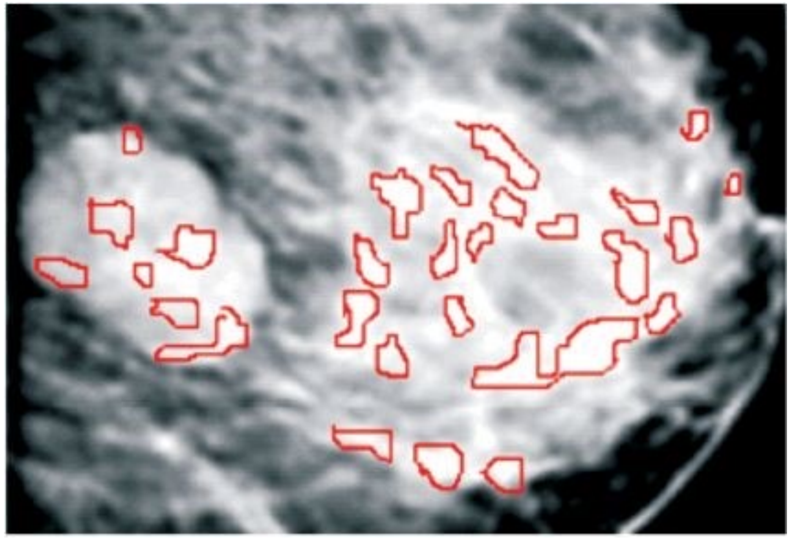


(a)



(b)

A breast image and its result in which the cells are magnified.



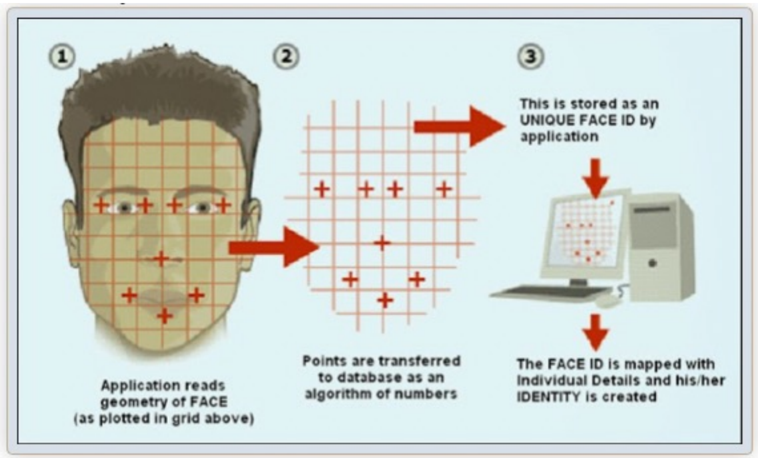
Segmented cells of different shapes such as visibly round shapes and irregular elongated shape. We can separate the cells or tissue into two different classes and count their population.

Principle Feature Analysis

The principal feature analysis (PFA) is similar to the principal component analysis (PCA) which is used as a standard technique in the statistical pattern recognition applications.

PCA/PFA is employed in signal processing for dimensionality reduction and feature extraction processes.

In the PFA, linear representation of the original objects or shapes using the minimum number of components which are the main components among the set of features, will be used as a method for feature extraction with the minimum mean square error.



https://www.matlabclass.com/2013/12/face-recognition-with-principal_12.html