SYLLABUS IS TENTATIVE
IT WILL BE UPDATED on a WEEKLY BASIS
ACCORDING to PROGRESS in CLASS

Course Code	**BME2122**
Course Title	**Signals and Systems**
Instructor	**Kamuran A. Kadıpaşaoğlu** **kamuran@yildiz.edu.tr** http://avesis.yildiz.edu.tr/kamuran/
Graduate Assistants	**Doğan Onur Arısoy** **Nihat Akkan** **ytubme2122@gmail.com**
Textbooks	**McClellan JH, Schaffer RW and Yoder MA** Signal Processing First Prentice Hall, US River, NJ, 2003 **Roberts JM** Signals and Systems: Analysis using Transform Methods and MATLAB 2nd ed., McGraw-Hill, New York, NY, 2012
Recommended Lectures	**Freeman Dennis** **Signals and Systems,** MIT OpenCourseWare, 2013
Recommended Study	Lecture Notes: Introduction to Engineering Fourier Analysis Signals and Systems

Important Info for Spring Semester 2021

Date/Time	Tue 14:00-16:50
Classroom	B-019 Amphi
Our First/Last Class	March 9/June 8
Midterm Exam (Week 8)	April 26-30
Final Exams	June 14-26
Last Day for Entry of Grades	June 28
Re-sit Exams	July 2-9
Last Day for Entry of Grades	July 12

Welcome to Class everyone.

Before starting this semester, it may be wise to be advised about the course organization so as to maximize your classroom performance and get the most fun out of the experience.

To begin, please follow the steps below.

1. Go to http://avesis.yildiz.edu.tr/kamuran/
2. Click on **Duyuru ve Dokümanlar**
3. Find the file "**Rules and Guidelines**"

    a. Download & save and print it
    b. Read, understand, and even memorize its contents.
    c. Pay particular attention to **Homework Policy** and **Grading Policy**

4. On the same page, find the file "**YourCourseCode_Syllabus_Semester_Year_#"**.

    a. The file will be updated on a weekly basis.
    b. Make sure you read the assignment(s) **BEFORE** coming to each class, **STARTING THIS WEEK**!
    c. Download and save any **Textbook**(s) that is (are) posted.

5. It is highly recommended that you

    a. Read the assigned chapter
    b. Solve the example and homework problems
    c. Prepare your questions for me

    BEFORE COMING TO CLASS.

6. There will be a homework assignment every week to be returned via e-mail to your assistant on Sunday night, no later than 24:00 hrs.
7. You will be asked to write your own syllabus for Midterm and Final exams. Details are on the next page.
8. You will be asked to make a weekly list of those among you, who can and wish to attend the face-to-face class (Max 17) and mail it to me at least a day before class. You will be responsible of rotating the list on a weekly basis if there are more than 17 on the list. (HES Code – No Risk – required)

I am looking forward to an educational and productive semester.
Good luck to all. KAK

EXAMS

1. This year, we will continue with last years' strategy for the Midterm and Final Exams which, I believe, provides a better measure of your overall grasp of the subject in the absence of face-to-face evaluation methods. You can find the topic and guidelines of your Final Exam HERE
2. The electronic version of your answers is due on the day scheduled for your exam, no later than 24:00 hrs. to be submitted via e-mail to your Teaching Assistant. NO excuse for tardiness.
3. If you start working on your Final Exam today, you will have approximately 14 weeks to finish it, which is almost 7 times as long as I would need to finish an equivalent load. So, I recommend you get to work ASAP and do NOT procrastinate.
4. This means, start writing your syllabus:
   a. Reflect on the philosophy and the overall message of each lecture,
   b. Try to understand why you are being thought this piece of information and how it will fit in the grander scheme of your formation as an engineer,
   c. Think of interesting examples that best illustrate the concept and its application to real life problems,
   d. Work the solution of the problem in a clear and understandable way, and
   e. Write them down using the Equation Toolbox in Word right after each class.
   f. You will be graded on your creativity and on your thoroughness.
5. As always, you are to write your answers in report format, following the standard rules.
6. Honesty is the first rule of professionalism. You should not recourse to copying and cheating out of self-respect, if anything else. Fear, more than anything else, the label of dishonesty and shun it at all cost. Once branded with it, it sticks for life.
7. There will be ABSOLUTELY NO tolerance for PLAGIARISM from any outside source, including but not limited to my class notes and syllabus, any inter-student (past or present) collaboration or some third-party tutor.
8. CRYING AFTER THE FACT WILL NOT SOLVE YOUR PROBLEM.
9. Follow updates on Homework from the link below
   https://docs.google.com/spreadsheets/d/1D3UrbIzS5P_kIrTA1u1FLyj_eGKoAC_Cnckp-MWFBoM/edit?usp=sharing

WK	DATE	HR	SUBJECT
1	Mar 9	1.	**Introduction to Class Rules, Homework Policy and Grading.**

2. System Definition, Signal Definition

Signal-System Equivalence

System Input/Output

Sensor Recording

3. Representation of Signals & Systems

a. Schematic

b. Block Diagram

c. Force Balance

d. Differential Equation and Initial Condition

e. Solution of differential equation

f. Parametric Representation

- Transient Phase (τ, Time Constant)
- Steady-State Phase (x_{ss} @ t_{ss})

g. Graphic Representation

h. Example: First Order Mechanical System_Spring and Damper

4. Introduction to Next Week's Topic

Continuous and Discrete Time Systems

Human time (Continuous) vs. Computer Time (Discrete)

HOMEWORK 1:

1. **Study**
   - Freeman Lectures: 1 and 2
   - KAK Lecture Notes: Introduction to Engineering-Chapter 3 and 4
     St1st Order Systems, Examples, Exercises
2. **Your Syllabus**
   - Write the Cover Letter addressed to the Department Chair:
     - ✓ Your first thoughts about the general philosophy of the course
     - ✓ Why is this course necessary for BME
     - ✓ What are the Learning Points
   - Remember, you will have plenty of time to update your Cover Letter as we move along into the semester and you learn more about the material.
   - Write the section about the first lecture, with original examples and solutions.
3. **List of Attendees:** Nominate candidates for Class Representative, vote, elect your Representative and have him/her send me the List (17 max) for next week (All attendees must get a HES Code that reads "No Risk" on mobile).

WK	DATE	HR	SUBJECT
2	Mar 16		

1. **System Representations**
   - **a.** Analogy
   - **b.** From Differential Equation to Block Diagram
   - **c.** System as Integrator
   - **d.** Negative Feedback System and Examples
   - **e.** Open-loop and Closed Loop Representations
2. **Laplace Domain**
   - **a.** Linearity: Superposition and Scalability
   - **b.** Laplace Transform as Linear Operator
   - **c.** Advantages of Laplace Transform (LT)
     - i. Converts Calculus into Algebra
     - ii. Decouples System from its Output
     - iii. Turns Convolution into Multiplication
     - iv. Shows Initial Condition (bx_0) as Input (Force) before $t=0$
     - v. Shows Constant Input as Integrated Input (F/s)
   - **d.** Laplace Transform of Various Functions
   - **e.** Solution of 1st Order Differential Equation by LT
3. **Inputs**
   - **a.** Impulse (Delta Dirac), Step, Ramp
     - i. Delta Dirac Explained
     - **ii.** Usefulness of Delta Dirac
   - **b.** Laplace Transform of Impulse
   - **c.** Ramp Input Examples

HOMEWORK 2:

1. **Listen**
   Freeman Lectures 3 and 6.
   Solve HW Problems
2. **Study**
   KAK Lecture Notes: 1st Order Systems, Examples, Exercises
   2nd Order Systems
3. **Syllabus:**
   Lecture 2

WK	DATE	HR	SUBJECT
3	Mar 23		

1. **General**

Signals: Laplace Transform of Various Inputs

Systems: Canonical Form of Block Diagram

Open-loop Transfer Function (OLTF)

Closed-loop Transfer Function (CLTF)

Pole of a System: Root of the CLTF denominator

Laplace Domain

Complex vector j

Orthonormal Basis (i and j)

Real and Imaginary Axes (Stairway to Heaven)

Vectorial Representation of the Complex Number **s**

Cartesian Coordinates of the Complex Number **s**

Left Half Plane and Stability

2. **Second Order Systems: No damping**

Mass-Spring System (Homogeneous Solution)

Parametrization: Natural Frequency

Solution Strategy: Laplace Transform

Oscillatory Response, No Decay: Eternal Sinusoid

Complex Conjugate Pair

3. **Second Order Systems: Damping**

Mass-Spring-Damper System (Homogeneous Solution)

Parametrization:

Natural and Damped Frequencies

Damping Coefficient (Zeta)

Solution Strategy: Educated Guess

Euler's Identity

Complex Exponentials and Sinusoids

Trigonometric Identities

Damped Oscillatory Response

Integration Constants: Amplitude and Phase

4. Homework:

Syllabus: Write 3rd week of your syllabus.

Repeat calculations for homogeneous system

Give example: Non-homogeneous solution for 2nd order system (Both Educated Guess and Laplace Transform)

Correct past week's syllabi

WK	DATE	HR	SUBJECT
4	Mar 30		

I. System Description-Continuous Time Operator

Force Balance

$$\sum F = 0 \;\Rightarrow\; F_{spring} + F_{damper} - F_{ext} = 0$$

Equation of Motion (Differential Equation)

$$kx + b\dot{x} = F_{ext}$$

$$\dot{x} = -\frac{k}{b}x + \frac{1}{b}F_{ext}$$

Pole (Time Constant) as Sole Descriptor of System Behavior

$$pole \triangleq p = -\frac{k}{b} = \frac{1}{\tau}$$

Block Diagram in Time and Laplace Domains

Transfer Function as Algebraic Expression in s (Frequency)

$$G(s) = \frac{1}{s+p}$$

System as Integrator (with Memory)

Accumulator Operator

$$G(A) = \frac{A}{1+pA}$$

Read: Class Notes_Chapter 1_DISCRETE TIME_
[Part I_Continuous Time Re-cap]

Read: [Lecture Slides Chapter 3]

II. Convolution Theorem Introduction

1. Time-Shift
2. Examples and Exercises

Study [Lecture Slides Chapter 2]

III. Convolution Theorem

1. Linearity and Time-invariance Properties
2. Delta Combing a Function
3. Integral as Area under the Function
4. Sum becoming Integral in the Limit
5. Example: Exponential convolved with cosine

Read: Class Notes_Chapter 1_DISCRETE TIME_
[Part II_ Continuous-time Convolution]

HOMEWORK 3:

1. **Convolve general exponential signal with general cosine signal**
   a. **By hand (Compare result with Laplace method)**
   b. **Using Matlab (conv command and your own code)**
2. **Listen to** [Freeman Lectures 2 and 5]

WK	DATE	HR	SUBJECT
5	Apr 6		**Read: Class Notes_Chapter 1_DISCRETE TIME_** Part II _ Continuous-time Convolution

I. Introduction to Discrete Time Concepts

1. Analog-to-Digital Conversion
2. First-order Hold
3. Sampling the Data ($t \rightarrow KT_s$)
4. From Differential Equation to Difference Equation

$\dot{x}(t) = -ax(t) + b(t)$ $\qquad$ *Continuous Time*

$x[(k+1)T_s] = -ax[kT_s] + b[kT_s]$ $\qquad$ *Discrete Time* 1
$x[k+1] = -ax[k] + b[k]$ $\qquad$ *Discrete Time* 2
$x_{k+1} = -ax_k + b_k$ $\qquad$ *Discrete Time* 3

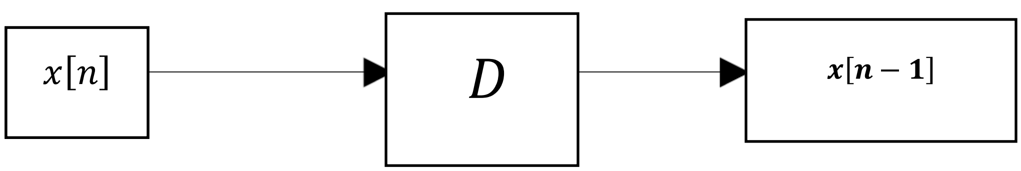
5. Forward Differentiation Rule

$\dot{x}(t) = -ax(t) + b(t)$ $\qquad$ *Continuous Time*
$x_{k+1} = (1 - aT_s)x_k + b_kT_s$ $\qquad$ *Discrete Time* 3

II. System Description-Discrete Time

1. **Delay Operator**, D**:** "Delay Box", to be used in DT Block Diagram:

$$Dx[n] = x[n-1]$$



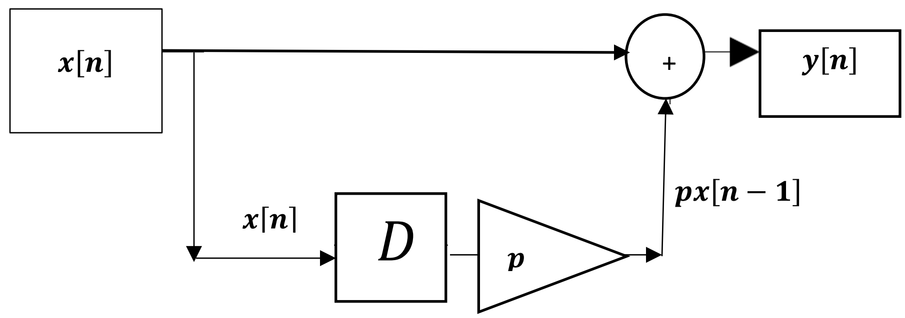
By definition, D holds the input for one period before letting it out. So, while

$x[n] \quad = x[0] \qquad for\ n = 0,$

$x[n-1] = x[0] \qquad for\ n = 1$

Consequently, we may think that the D box advances the time index, n, by one.

2. **Feed-forward System**



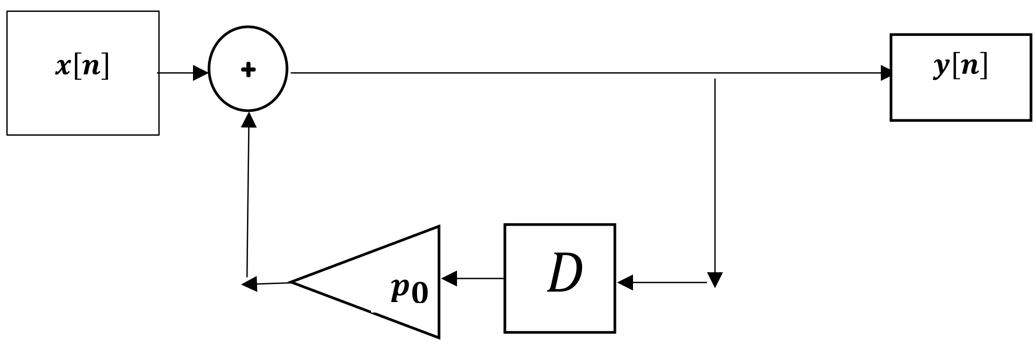
$$y[n] = x[n] + px[n-1]$$

if $\quad x[n] = \delta[n] = [1\ 0\ 0\ \ldots.]$ (unit impulse)

then $\quad y[n] = [1\ p\ 0\ 0\ \ldots.]$ (unit impulse response)

Forward system response fades away

3. **Feed-back System**



$$y[n] = py[n-1] + x[n]$$

if $\quad x[n] = \delta[n] = [1\ 0\ 0\ \ldots.]$ (unit impulse)

then $\quad y[n] = [1\ p\ \ p^2\ p^3\ \ldots p^n.]$ (unit impulse response)

Forward system response $\begin{cases} \text{decreases geometrically } & p < \pm 1 \\ \text{increases geometrically } & p < \pm 1 \end{cases}$

$x[n] = \alpha^n u[n] \quad$ and $\quad g[n] = u[n]$

4. **Recap: Discrete Time:** System Representations:
    1. Difference Equation

       $$y[n] = py[n-1] + x[n]$$

    2. Block Diagram
    3. Impulse Response (Infinite Series)

       $$y[n] = [p^0\ \ p^1\ \ p^2\ \ p^3 \dots] = p^n$$

    4. Operator Representation

       $$G(R) = \frac{1}{1-pR}$$

    5. Infinite Sum

       $$\frac{1}{1-pR} = p^0R^0 + p^1R^1 + +p^2R^2 + \cdots = \sum_{n=0}^{\infty} p^nR^n$$

5. **z-Transform**

$$z = \frac{1}{R}$$

$$G(z) = \frac{1}{1-pz^{-1}} = \sum_{n=0}^{\infty} y[n]z^{-n}$$

6. **HOMEWORK 4**

No HW this week (because Professor screwed up in Class)
Recommended (If you submit, you'll get a bonus):

a. Study Freeman Lectures 2 and **5 (very important)**
b. Recreate the table next page for
    i. various poles (smaller and bigger than 1; negative and positive)
    ii. various inputs (impulse and unit step)
    iii. plot the system response
    iv. compare to continuous time response

p_0	n	$x[n]$	p_0 y$[n-1]$	y$[n] = x[n] + p_0 y[n-1]$
1	-1	0	$1*0$	$0 = \quad 0 + \quad 1*0$
1	0	1	$1*0$	$1 = \quad 1 + \quad 1*0$
1	1	0	$1*1 = 1$	$1 = \quad 0 + \quad 1$
1	2	0	$1*1 = 1$	$1 = \quad 0 + \quad 1$
.				
.				
2	-1	0	$2*0$	$0 = \quad 0 + \quad 0$
2	0	1	$2*0$	$1 = \quad 1 + \quad 0$
2	1	0	$2*1$	$2 = \quad 0 + \quad 2$
2	2	0	$2*2 = 2^2$	$4 = \quad 0 + \quad 4$
2	3	0	$2*2^2 = 2^3$	$8 = \quad 0 + \quad 8$
2	4	0	$2*2^3 = 2^4$	$16 = \quad 0 + \quad 0$
.				.
2	1	0	$2*2^{n-1} = 2^n$	p_0^n

Therefore, the impulse sample response of the Canonical Form, is

$$\mathrm{y}[n] = \begin{cases} p_0^n & n \geq 0 \\ 0 & otherwise \end{cases}$$

By convention, the impulse sample response of a system is called h$[n]$

Notice that h$[n]$is NOT an infinite sum!

Instead, it is a sequence, which means that successive terms are not added

$$\begin{array}{ll} \mathrm{n=} & 0,\quad 1,\quad 2,\quad 3,\quad 4, \ldots \ldots,\quad n \\ \mathrm{h}[n] = & p_0^0,\ p_0^1,\ p_0^2,\ p_0^3,\ p_0^4, \ldots \ldots, p_0^n \end{array}$$

Compare this sequence to the coefficients of the polynomial form of the system

$$H(R) = p_0^0 R^0 + p_0^1 R^1 + p_0^2 R^2 + p_0^3 R^3 + p_0^4 R^4 + \cdots + p_0^n R^n$$

Notice that the coefficients of R^n in H(R) match one-to-one to the values of the sequence h$[n]$ for the same value of n.

$$H(R) = \sum_{n=0}^{\infty} p_0^n R^n = \sum_{n=0}^{\infty} \mathrm{h}[n] R^n$$

WK	DATE	HR	SUBJECT
6	Apr 13		

I. **Recap: Discrete Time:** System Representations:

1. Difference Equation

$$y[n] = py[n-1] + x[n]$$

2. Block Diagram
3. Impulse Response (Infinite Series)

$$y[n] = [p^0 \;\; p^1 \;\; p^2 \;\; p^3 \ldots] = p^n$$

4. Operator Representation

$$G(R) = \frac{1}{1 - pR}$$

5. Infinite Sum

$$\frac{1}{1 - pR} = p^0R^0 + p^1R^1 + +p^2R^2 + \cdots = \sum_{n=0}^{\infty} p^nR^n$$

II. **z-Transform**

$$z = \frac{1}{R}$$

$$G(z) = \frac{1}{1 - pz^{-1}} = \sum_{n=0}^{\infty} y[n]z^{-n}$$

III. **z-Transform and Inverse z-Transform Examples**

1. **Manipulate signs of u[n]**
2. **Examples**
   a. **Inverse z-Transform Examples-1**
   b. **Inverse z-Transform Examples-2**
   c. **Inverse z-Transform Examples-3**

IV. **HOMEWORK**

1. **Study z-Transform (England) and complete Syllabus**
2. **Solve Examples in Freeman HW**
3. **Prepare for next week**
   Frequency Domain: Introduction

   Laplace Domain: $s = \sigma + j\omega$

   Frequency Domain: $\lim_{\sigma \to 0} s = j\omega$

   Period (T), Frequency (f), Angular Frequency (ω)
   Study: Class Notes-Frequency Domain

WK	DATE	HR	SUBJECT
7	Apr 20		

I. FUNDAMENTAL CONCEPTS

1. Period, Frequency, Angular Velocity

a. **Let a vector of norm $|\boldsymbol{r}|$ centered at the origin of a circle rotate at constant tangential speed, $\boldsymbol{v}$, (therefore at constant angular speed, $\boldsymbol{\omega}$) and tracing an arc s.**

b. **Project the motion of v on a longitudinal time axis.**

c. **Mark the following:**

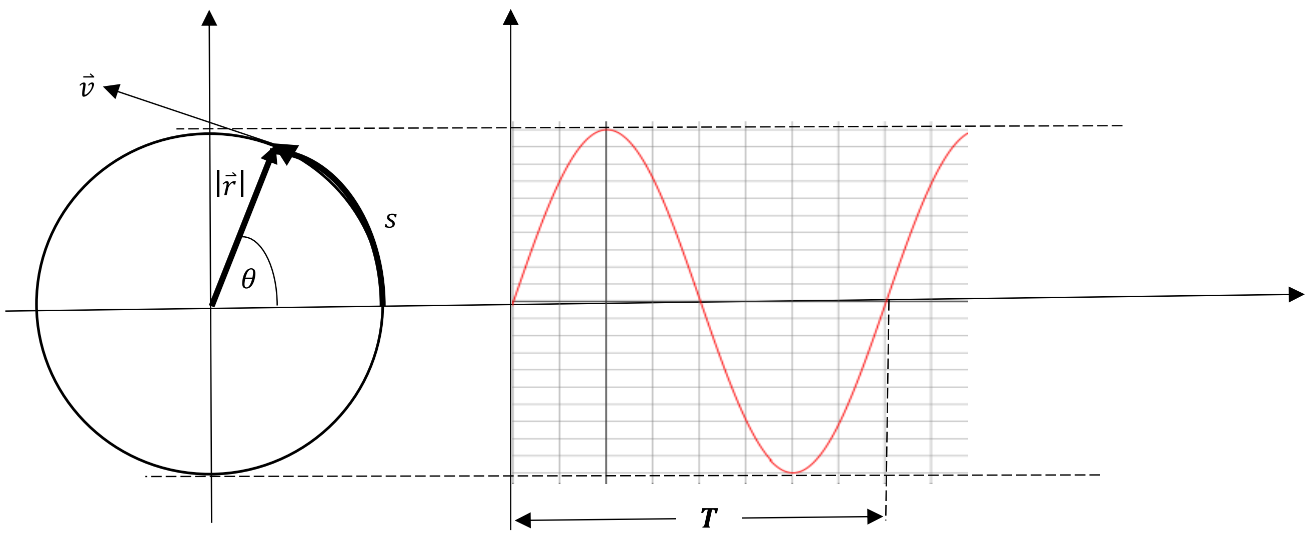
Arc length: $s = r\,\theta$

Linear velocity: $\dot{s} = r\dot{\theta}$

$v = r\omega$

Angle: $\theta = \omega t$

Period: $T = \dfrac{1}{f}$



in T seconds 1 Wave completed

⇒ in 1 second ? waves

$$= \frac{1\text{x}1\ waves}{T\ seconds}$$

$$\stackrel{\text{def}}{=} Frequency, f$$

2π radians in T seconds

? radians ⇒ 1 second

$$= \frac{1 \text{ x } 2\pi\ radians}{T\ seconds}$$

$$\Rightarrow \text{ angular velocity} \stackrel{\text{def}}{=} \omega = \frac{2\pi}{T} = 2\pi f$$

2. Matrix as System Representation

a. Ortho-normal Basis

- A basis of n vectors spans an n-dimensional Vector Space
- Therefore,
- Ortho: Dot product is zero, therefore vectors are mutually perpendicular
- Normal: Each vector's Euclidian Norm is unity
- An orthonormal basis also spans a vector space.

b. Matrix operates on a Vector

c. State-space Representation of a System

i. System states as integrator outputs
ii. Any n^{th} order system can be represented as a set of n 1^{st} order systems
iii. System Matrix and Input Matrix

d. EigenVectors and Eigenvalues (**Linear Algebra)**

$$[A]\vec{x} = \lambda\vec{x}$$

Matrix . Eigenvector = Eigenvalue . Eigenvector

Matrix does not **rotate** eigenvector,
Just changes its magnitude in proportion to **eigenvalue**

Example for the Syllabus:

i. Calculate the EV's and Ev's of a 2x2 matrix.
ii. Calculate the corresponding system parameters
iii. How do Ev's relate to system poles?

3. Moving from Laplace to Frequency Domain

$$s = \sigma + j\omega$$

$$\lim_{\sigma \to 0} H(s) = \lim_{s \to j\omega} H(s) = H(j\omega)$$

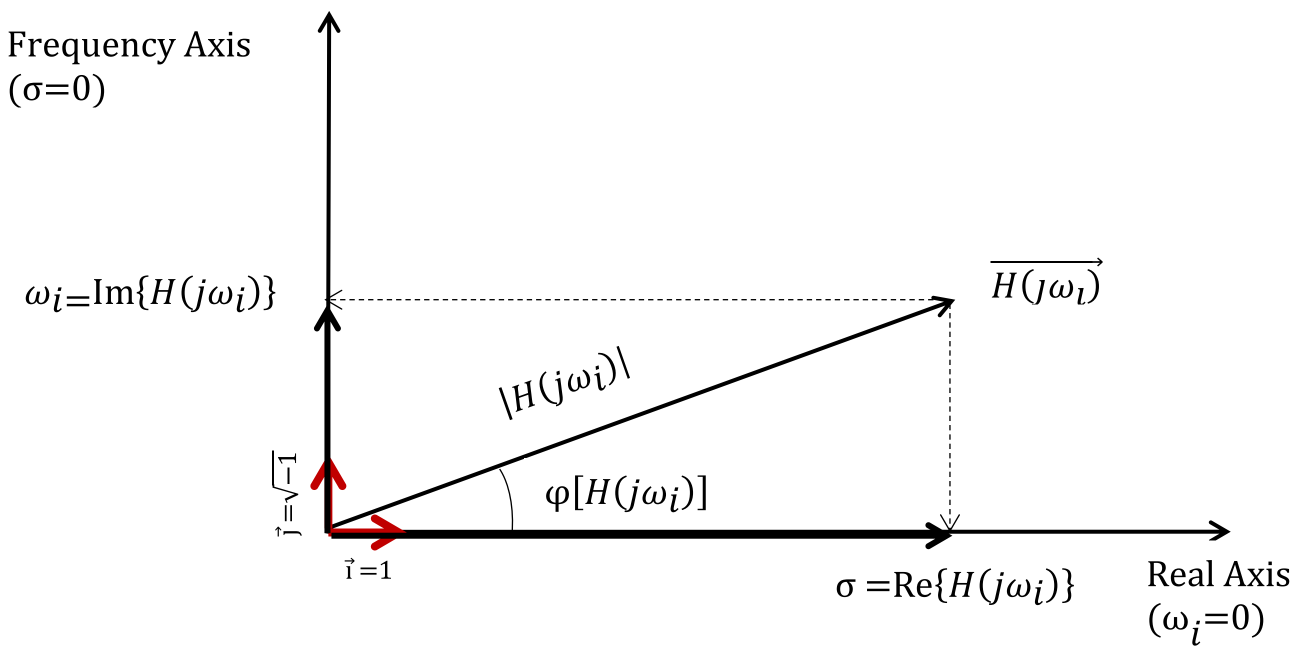
4. **Phasor**

a. A stationary vector in the Complex Plane $\Rightarrow$ No time

b. Exponential representation
$\boldsymbol{H(j\omega_i) = e^{j\omega_i}}$

c. Cartesian Representation

$$e^{j\omega} = cos(\omega) + j\ sin(\omega)$$



d. Polar representation

$$H(j\omega) = |H(j\omega)|e^{j\varphi\{H(j\omega)\}}$$

where

$$|H(j\omega)| \stackrel{\text{def}}{=} \quad Magnitude$$

$$\varphi\{H(j\omega)\} \stackrel{\text{def}}{=} \quad Phase$$

e. **Complex Exponential (Euler's Formula)**

$$\mathcal{L}\{e^{j\omega}\} = \frac{1}{s-j\omega} = \frac{1}{s-j\omega}\,\frac{s+j\omega}{s+j\omega}$$

$$= \frac{s+j\omega}{s^2+\omega^2} = \frac{s}{s^2+\omega^2} + j\frac{\omega}{s^2+\omega^2}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2} + j\frac{\omega}{s^2+\omega^2}\right\}$$

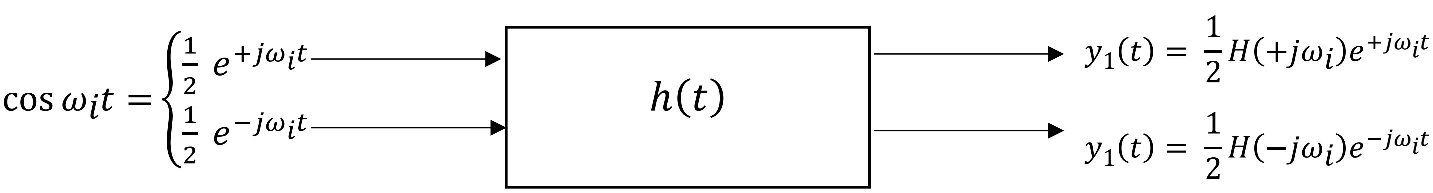
$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+\omega^2}\right\} + j\mathcal{L}^{-1}\left\{\frac{\omega}{s^2+\omega^2}\right\}$$

$$e^{j\omega t} = cos(\omega t) + j\ sin(\omega t)$$

II. APPLICATION to FREQUENCY DOMAIN ANALYSIS

1. System (LTI) does not **change the frequency** of complex exponential

   Just changes its magnitude and phase in proportion to **phasor**



2. **Signals and Systems**

$$[System]\,.\,\overrightarrow{Signal} \quad = \quad \lambda \quad . \quad \overrightarrow{Signal}$$

$$\overrightarrow{Signal}\,.\,[System] \quad = \quad \lambda \quad . \quad \overrightarrow{Signal} \qquad \text{(Commutation)}$$

$$e^{\pm j\omega_i t} \ .\ H(j\omega) \quad = \quad H(\pm j\omega_i)\ .\ e^{\pm j\omega_i t}$$

III. BODE PLOTS-I

1. The eigenvalue (phasor) of the system is evaluated at the frequency (ω_i) of the input signal.
2. In a Bode Plot, the response of the system (the magnitude and phase of the phasor) is plotted for all possible values of the input signal frequency, $0 < \omega_i < \infty$
3. Log scales are used on the frequency (horizontal) and magnitude (vertical) axes to squeeze more data, to linearize curves and to express magnitude as Decibels $[20 \log H(+j\omega_i)]$, which is a unit of signal power.

IV. HOMEWORK 5:

1. Study Dennis Freeman Lectures 9 and 11:
   Frequency Response and Bode Plot
2. **Study: Class Notes-Frequency Domain**
3. **SYLLABUS: Text and Examples**
   a. **z-Transform**
   b. **Inverse z-Transform**
   c. **Eigenvectors and eigenvalues of a second order sytem matrix**
   d. **Frequency Domain Introduction**

WK	DATE	HR	SUBJECT
8	Apr 28		MIDTERM WEEK No Class

WK	DATE	HR	SUBJECT
9	May 4	Bode Plot of First-order System (single pole)	

I. Recap:

1. Transfer Function as Ratio of Polynomials

a. **Root**: Scalar that drives a polynomial to zero

b. **Zero**: Root of the TF numerator

c. **Pole**: Root of the TF denominator

d. **Fundamental Theorem of Algebra**

Any nth order polynomial can be written as the product of n 1st order polynomials

$$G(s) = \frac{q(s)}{d(s)} = \frac{a_0 s^0 + a_1 s^1 + \cdots + a_n s^n}{b_0 s^0 + b_1 s^1 + \cdots + b_m s^m} = \frac{\sum_0^n a_n s^n}{\sum_0^m b_m s^m}$$

$$G(s) = K\,\frac{(s - z_1)(s - z_2)\ldots(s - z_n)}{(s - p_1)(s - p_2)\ldots(s - p_m)} = K\,\frac{\prod_1^n (s - z_n)}{\prod_1^m (s - p_m)}$$

Note that when

$$\begin{array}{lll} s = z_1, z_2, \ldots, z_n, & \mathrm{num(s)} = 0 & \Rightarrow \quad G(s) = 0 \\ s = p_1, p_2, \ldots, p_m, & \mathrm{denom(s)} = 0 & \Rightarrow \quad G(s) = \infty \end{array}$$

e. Phasor (Eigenvalue) as Ratio of Polynomials

i. Complex exponentials are EigenVectors of LTI systems

$$U(s) = e^{s_i t} \longrightarrow \boxed{G(s)} \longrightarrow Y(s) = G(s_i)e^{s_i t}$$

$$U(j\omega_i) = e^{j\omega_i t} \longrightarrow \boxed{G(j\omega)} \longrightarrow Y(j\omega) = G(j\omega_i)e^{j\omega_i t}$$

$$U(s) = e^{s_i t} \longrightarrow \boxed{G(s) = K_1 \frac{\prod_1^n (s - z_n)}{\prod_1^m (s - p_m)}} \longrightarrow Y(s) = K_2 \frac{\prod_1^n (s_i - z_n)}{\prod_1^m (s_i - p_m)} e^{s_i t}$$

ii. $\boldsymbol{s_i} = \omega_i$ is the frequency of the **input signal**

iii. **z_n and p_m** are the zeros and poles of the **system**

iv. Here, $G(s_i) = K_2 \frac{\prod_1^n (s_i - z_n)}{\prod_1^m (s_i - p_m)} = G(j\omega_i) = K_2 \frac{\prod_1^n (j\omega_i - z_n)}{\prod_1^m (j\omega_i - p_m)}$ is the Eigenvalue of the output signal.

v. The Eigenvalue is
- The system TF, $G(s)$, evaluated at s_i
- A Phasor, therefore
  - it is a vector and
  - it can be expressed in
    - Cartesian coordinates
    - Polar coordinates
    - Complex exponetial (Euler)

f. Consider the example of the simplest phasor: **Single zero**

$U(s) = e^{s_i t} \longrightarrow \boxed{G(s) = s - z} \longrightarrow Y(s) = (s_i - z)e^{s_i t}$

Here, $G(s_i) = s_i - z = G(j\omega_i) = j\omega_i - z$ is the Eigenvalue of the output signal.

If we express the system Eigenvalue as a phasor

$G(s_i) = -z + j\omega_i$ Cartesian coordinates

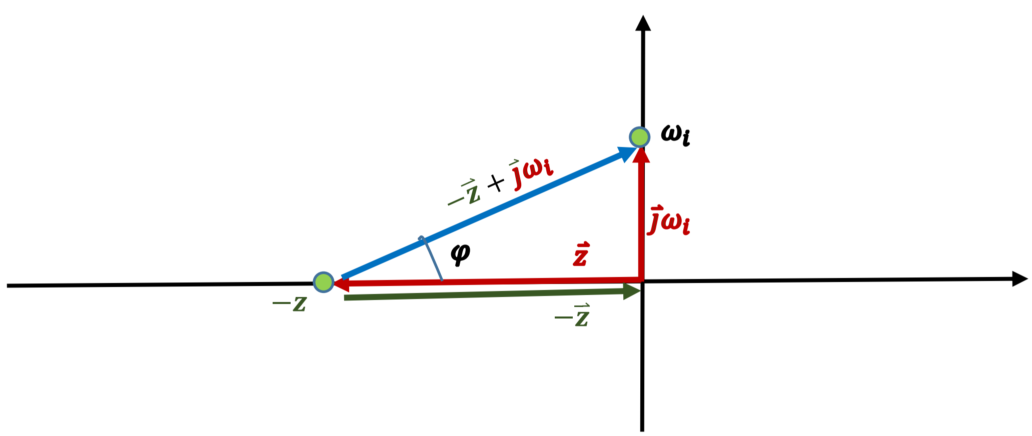
$G(s_i) = |G(s_i)|\ \varphi(s_i)$ Polar coordinates

$$= \sqrt{z^2 + \omega_i^2}\ \tan^{-1}\frac{\omega_i}{z}$$

$G(s_i) = |G(s_i)|e^{j\varphi}$ Euler Notation

$$= \sqrt{z^2 + \omega_i^2}\ e^{j\varphi}$$

If we graphically show the system Eigenvalue on the Complex Plane



g. Therefore, the magnitude (length) and argument (phase angle) of the Eigenvalue Phasor is a function of both the system pole, p, and the input signal frequency, ω_i

h. Therefore, the output of the system will be of the form (using the Euler form of the Phasor above)

$$Y(j\omega) = G(j\omega_i)e^{j\omega_i t}$$
$$= \sqrt{z^2 + \omega_i^2}\; e^{j\varphi}e^{j\omega_i t}$$
$$= \sqrt{z^2 + \omega_i^2}\; e^{j(\omega_i t+\varphi)}$$

f. Therefore,

i. The frequency of the output signal will remain the same as that of the input signal, $\boldsymbol{\omega_i}$

ii. The amplitude of the output signal will be modified by a factor of $\sqrt{z^2 + \omega_i^2}$

iii. The output signal will be out-of-phase with the input signal by a phase angle, φ

II. Complex Calculus to express TF of a System or Phasor

a. Single zero with forward gain K

$$H(j\omega) = K(j\omega + z)$$

$$|H(j\omega)| = K\sqrt{\omega^2 + z^2} \quad \text{(from Phythagorian Theorem)}$$

$$and\ \ \varphi\{H(j\omega)\} = tan^{-1}\left(\frac{\omega}{z}\right) \quad \text{(from trionometry)}$$

$$H(j\omega) = K\sqrt{\omega^2 + z^2}\; e^{jtan^{-1}\left(\frac{\omega}{p}\right)}$$

b. Single pole with forward gain K

$$H(j\omega) = \frac{K}{j\omega + p} = K\left[\frac{p}{\omega^2 + p^2} - j\frac{\omega}{\omega^2 + p^2}\right]$$

$$|H(j\omega)| = \frac{K}{\sqrt{\omega^2 + p^2}} \;\; and \;\; \varphi\{H(j\omega)\} = tan^{-1}\left(\frac{-\omega}{p}\right)$$

$$H(j\omega) = \frac{K}{\sqrt{\omega^2 + p^2}} e^{j tan^{-1}\left(\frac{-\omega}{p}\right)}$$

III. Bode Plots-II

1. **Evaluate System Phasor, $\boldsymbol{H(j\omega)}$, at input Frequency, ω_i**

c. Representations
(Example: Single pole, p, with forward gain, K)

$$H(j\omega_i) = K\,\frac{1}{j\omega_i + p} = K\,\frac{1}{p + j\omega_i} \quad (pole\ representation)$$

$$H(j\omega_i) = K\frac{1}{j\omega_i + \frac{1}{\tau}} = K\tau\,\frac{1}{1 + j\tau\omega_i} \quad \begin{pmatrix} time - constant \\ representation \end{pmatrix}$$

$$H(j\omega_i) = K\frac{\frac{1}{p}}{j\frac{\omega_i}{p} + 1} = \frac{K}{p}\,\frac{1}{1 + j\frac{\omega_i}{p}} \quad \begin{pmatrix} frequency\ ratio \\ representation \end{pmatrix}$$

d. Real and Imaginary Parts

$$H(j\omega_i) = K\left[\frac{p}{p^2 + \omega_i^2} - j\frac{\omega_i}{p^2 + \omega_i^2}\right]$$

$$H(j\omega_i) = K\tau\left[\frac{1}{1 + (\omega_i\tau)^2} - j\frac{\omega_i\tau}{1 + (\omega_i\tau)^2}\right]$$

$$H(j\omega_i) = \frac{K}{p}\left[\frac{1}{1 + \left(\frac{\omega_i}{p}\right)^2} - j\frac{\frac{\omega_i}{p}}{1 + \left(\frac{\omega_i}{p}\right)^2}\right]$$

e. Magnitude and Phase

$$H(j\omega_i) = \frac{K}{\sqrt{{\omega_i}^2 + p^2}} e^{jtan^{-1}\left(\frac{-\omega_i}{p}\right)}$$

$$H(j\omega_i) = K\tau \frac{1}{\sqrt{1 + (\omega_i\tau)^2}} e^{jtan^{-1}\left(\frac{-\omega_i}{p}\right)}$$

$$H(j\omega_i) = \frac{K}{p} \frac{1}{\sqrt{1 + \left(\frac{\omega_i}{p}\right)^2}} e^{jtan^{-1}\left(\frac{-\omega_i}{p}\right)}$$

IV. Bode in Matlab (System with forward gain and single pole)

```
>> close all; clear all; clc;
>>K=10; p=5;                        %Gain and Pole
>>num=K; denom=[1 5];               %Numerator and Denominator
>>H=tf(num, denom);                 %Transfer Function
>>bode(H), grid;                    %Draw Magnitude and Phase
```

V. HOMEWORK 5:

1. Study Dennis Freeman Lectures 9 and 11:
   Frequency Response and Bode Plot
2. **Study: Class Notes-Frequency Domain**
3. **SYLLABUS: Text and Examples**

WK	DATE	HR	SUBJECT
10	May 11		**Canceled by Presidential Decree!!!**

WK	DATE	HR	SUBJECT
11	May 18		**Bode Plots-III: Second-order System**

I. Transfer Function of an Electrical System

$$L\ddot{q} + R\dot{q} + \tfrac{1}{C}q = V_{in} = \tfrac{1}{C}q_{in}$$

$$s^2LQ(s) + RQ(s) + \tfrac{1}{C}Q(s) = \tfrac{1}{C}Q_{in}(s)$$

$$Q(s) = \frac{\frac{1}{C}}{s^2L + sR + \frac{1}{C}}\,Q_{in}(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}\,Q_{in}(s)$$

$$H(s) = \frac{Q(s)}{Q_{in}(s)} = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$where \qquad \omega_n \stackrel{\text{def}}{=} \frac{1}{LC} \qquad and \qquad \varsigma \stackrel{\text{def}}{=} \frac{R}{2L\omega_n}$$

$$H(j\omega_i) = \frac{\omega_n^2}{(j\omega_i)^2 + 2\varsigma\omega_n(j\omega_i) + \omega_n^2} = \frac{\omega_n^2}{(\omega_n^2 - \omega_i^2) + j(2\varsigma\omega_n\omega_i)}$$

$$H(j\omega_i) = \frac{\omega_n^2}{(\omega_n^2 - \omega_i^2) + j(2\varsigma\omega_n\omega_i)} = \frac{\frac{\omega_n^2}{\omega_n^2}}{\frac{(\omega_n^2 - \omega_i^2)}{\omega_n^2} + \frac{j(2\varsigma\omega_n\omega_i)}{\omega_n^2}}$$

$$H(j\omega_i) = \frac{1}{\left(1 - \frac{\omega_i^2}{\omega_n^2}\right) + j\left(2\varsigma\frac{\omega_i}{\omega_n}\right)} = \frac{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] - j2\varsigma\left(\frac{\omega_i}{\omega_n}\right)}{\sqrt{\left[1 - \left(\frac{\omega_i}{\omega_n}\right)^2\right]^2 + (2\varsigma)^2\left(\frac{\omega_i}{\omega_n}\right)^2}}$$

$$|H(j\omega_i)| = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_i}{\omega_n}\right)^2\right]^2 + (2\varsigma)^2\left(\frac{\omega_i}{\omega_n}\right)^2}}\, e^{j\tan^{-1}\frac{-2\varsigma\left(\frac{\omega_i}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}}$$

II. Plotting the Magnitude and Phase on Log-scale

Bode can be constructed by plotting

- $0 < \omega_i < \infty$ on the horizontal axis (abscissa)
- $20\,log|\boldsymbol{H}(\boldsymbol{j}\boldsymbol{\omega}_i)|$ and $\boldsymbol{\varphi}[\boldsymbol{H}(\boldsymbol{j}\boldsymbol{\omega}_i)]$
  on two separate vertical axes (ordinates).

III. First-order System with Single Pole

$$20\,log|\boldsymbol{H}(\boldsymbol{j}\boldsymbol{\omega}_i)| = \begin{cases} 20\,log\,K & \omega_i \ll \boldsymbol{p} \quad (\omega_i \to 0) \\ 20\,logK - 3 & \omega_i = \mathbf{p} \\ 20\,logK - 20\log\omega_i & \omega_i \gg \boldsymbol{p} \quad (\omega_i \to \infty) \end{cases}$$

$$\boldsymbol{\varphi}[\boldsymbol{H}(\boldsymbol{j}\boldsymbol{\omega}_i)] = \begin{cases} 0 & \omega_i \ll \boldsymbol{p} \quad (\omega_i \to 0) \\ -\dfrac{\pi}{4} & \omega_i = \mathbf{p} \\ -\dfrac{\pi}{2} & \omega_i \gg \boldsymbol{p} \quad (\omega_i \to \infty) \end{cases}$$

IV. Second-order System with Distinct Poles ($\boldsymbol{p}_1\boldsymbol{p}_2 = \boldsymbol{\omega}_n^2$)

$$20\,log|\boldsymbol{H}(\boldsymbol{j}\boldsymbol{\omega}_i)| = \begin{cases} 20\,log\,K & \omega_i \ll \omega_n \quad (\omega_i \to 0) \\ 20\,logK - 6 & \omega_i = \omega_n \\ 20\,logK - 40\log\omega_i & \omega_i \gg \omega_n \quad (\omega_i \to \infty) \end{cases}$$

$$\boldsymbol{\varphi}[\boldsymbol{H}(\boldsymbol{j}\boldsymbol{\omega}_i)] = \begin{cases} 0 & \omega_i \ll \omega_n \quad (\omega_i \to 0) \\ -\dfrac{\pi}{2} & \omega_i = \omega_n \\ -\pi & \omega_i \gg \omega_n \quad (\omega_i \to \infty) \end{cases}$$

V. Bode in Matlab (System with forward gain and single pole)

```
>> close all; clear all; clc;
>>K=10; p=5;                     %Gain and Pole
>>num=K; denom = [1 5];          %Numerator and Denominator
>>H=tf(num, denom);              %Transfer Function
>>bode(H), grid;                 %Draw Magnitude and Phase
```

VI. HOMEWORK 6: Listen:

Brian Douglas

[Fourier Transform 1](), [Fourier Transform 2, Laplace Transform]()

Dennis Freeman Lectures

Lecture 14: [Fourier Representations]()
Lecture 15: [Fourier Series]()
Lecture 16: [Fourier Transform]()

WK	DATE	HR	SUBJECT
12	May 25		

I. **FOURIER ANALYSIS-I**

Class Notes: Fourier Analysis_I

1. **Input Signal Types**
   Impulse, Step, Ramp
   Bounded exponential
   Eternal Sinusoid: Use Euler's Rule
      Decomposition into Complex Exponentials
   Arbitrary Input (Periodic Signal): Use Fourier Series:
      Decomposition into weighted Complex Exponentials

2. **Algebraic and Geometric Vector Representations**
   Dot (Inner Product)
   Vector-Function Analogy
3. **Complex Fourier Series**
4. **Example:** Square Wave

II. **HOMEWORK 6:**

1. **Syllabus**
2. **Watch and Study: Dennis Freeman Lectures**
   Lecture 17: Discrete Time (DT) Frequency Representations
   Lecture 18: Relations among Fourier Representations
   Lecture 19: Applications of Fourier Transforms

WK	DATE	HR	SUBJECT
13	Jun 1		

I. FOURIER ANALYSIS-I

I. Recap: Complex Fourier Coefficients
Class Notes: Fourier Analysis_I

II. Real Fourier Coefficients
Class Notes: Fourier Analysis-II
Example: Square Wave

III. Continuous-Time Fourier Transform.
Class Notes: Fourier Analysis-III

a. Continuous-Time Aperiodic Signals.
b. Continuous-Time Fourier Transform.
c. Properties of Continuous-Time Fourier Transform

II. HOMEWORK
I. Syllabus
II. Properties of FS and FT
III. Generate:
Square Wave
Triangle Wave
Saw-tooth Wave
1. Analytically in Word using Equation Toolbox,
2. Matlab: Write your own Code
IV. Watch and Study: Dennis Freeman Lectures
Lecture 17: Discrete Time (DT) Frequency Representations
Lecture 18: Relations among Fourier Representations
Lecture 19: Applications of Fourier Transforms

WK	DATE	HR	SUBJECT
14	Jun 8		**The Discrete Time Fourier Transform for Discrete-Time Signals.** Discrete-Time Aperiodic Signals. Discrete-Time Fourier Transform. Properties of Discrete-Time Fourier Transform Analogies HOMEWORK 7: Syllabus Include all of the below Problems Indicated in red letters in Fourier Analysis IV:DTFT Exercises in Freeman Lectures/Homework Study: Class Notes: Fourier Analysis IV: DTFT Freeman Lecture 17: Discrete Time (DT) Frequency Representations Freeman Lecture 18: Relations among Fourier Representations

WK	DATE	HR	SUBJECT
15	TBD		
		I.	DFT and FFT Study: Fourier Analysis V: DFT Study: Fourier Analysis VI: FFT Study Richard Rudke, DSP Lecture 11: FFT
		II.	SAMPLING THEOREM Study: Fourier Analysis VII: Sampling Theorem Adapted from: Dennis Freeman Lecture 21
		III.	RELATIONSHIP btw FOURIER SERIES and TRANSFORMS Fourier Analysis VIII: Relationships Fourier Analysis X: Applications Frequency Analysis: Filters