

Which of the following is the general solution of  $y' + 4y = 0$  at  $x = 0$  using the power series?

- A)  $y = a_0(1 + 2x + 3x^2 + 4x^3 + \dots)$
- B)  $y = a_0(1 - 4x + 2x^2 + 8x^3 + \dots)$
- C)  $y = a_0(1 - 4x + 8x^2 - \frac{32}{3}x^3 + \dots)$
- D)  $y = a_0(1 - 4x + \frac{1}{4}x^2 - \frac{3}{2}x^3 + \dots)$
- E)  $y = a_0(1 + 4x - 3x^2 - 6x^3 + \dots)$

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$a_1 = -4a_0$$

$$y = a_0 + a_1 x + a_2 x^2 + \dots - 4a_0$$

$$a_2 = -\frac{4a_1}{2} = -2(-4a_0) = 8a_0$$

$$\sum_{n=1}^{\infty} n a_n x^{n-1} + L \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} L a_n x^n = 0 \Rightarrow \sum_{n=0}^{\infty} [(n+1) a_{n+1} + L a_n] x^n = 0$$

$$a_{n+1} = -\frac{L a_n}{n+1}, \quad n \geq 0$$

Which of the following is the recurrence relation of  $y'' - 4y' = 0$  at  $x = 4$  using the power series?

- A)  $a_{n+1} = 4 \frac{a_{n-1}}{n} \quad (n \geq 1)$
- B)  $a_{n+1} = 4 \frac{a_n}{(n+1)} \quad (n \geq 1)$
- C)  $a_{n+2} = 4 \frac{a_n}{(n+2)} \quad (n \geq 2)$
- D)  $a_{n+2} = 4 \frac{a_{n+1}}{n} \quad (n \geq 1)$
- E)  $a_{n+1} = \frac{a_n}{(n+2)} \quad (n \geq 1)$

$$y = \sum_{n=0}^{\infty} a_n (x-4)^n$$

$$y' = \sum_{n=1}^{\infty} n a_n (x-4)^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n (x-4)^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n (x-4)^{n-2} - L \sum_{n=1}^{\infty} n a_n (x-4)^{n-1} = 0$$

$$\sum_{n=1}^{\infty} (n+1) n a_{n+1} (x-4)^{n-1} - \sum_{n=1}^{\infty} L n a_n (x-4)^{n-1} = 0$$

$$(n+1) a_{n+1} = L n a_n \Rightarrow a_{n+2} = \frac{L a_n}{n+2}, \quad n \geq 1$$

$$a_{n+2} = \frac{L a_n}{(n+2)}$$

$n \geq 1$

The differential equation  $x^3y''' - 2x^2y'' - 4xy' + 8y = \overset{\curvearrowleft}{x^2} + \ln x$  is transformed into a differential equation with constant coefficients using a suitable transformation. Which of the following is the new differential equation?

$$x = e^t, \quad \ln x = t \quad D = \frac{d}{dt}$$

$$y' = e^{-t} Dy$$

$$y'' = e^{-2t} D(D-1)y$$

$$y''' = e^{-3t} D(D-1)(D-2)y$$

A)  $\frac{d^3y}{dt^3} - 5\frac{d^2y}{dt^2} + \underline{8\frac{dy}{dt}} = e^t + 2t$

~~B)  $\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} - 8y = e^t + 2t$~~

C)  $\frac{d^3y}{dt^3} - 5\frac{d^2y}{dt^2} + \underline{8y} = e^{2t} + t$

~~D)  $\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} - 8\frac{dy}{dt} = e^{2t} + t$~~

~~E)  $\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 18y = e^{2t} - t$~~

$$\left[ D(D-1)(D-2) - 2D(D-1) - LD + 8 \right] y = e^{2t} + t$$

$$\underbrace{D^3 - 3D^2 + 2D - 2D^2 + 2D - 1}_{(D^3 - 5D^2 + 8)} D + 8 y$$

If the two independent solutions of a fourth-order, homogeneous (non-second sided), linear differential equation are  $1$  and  $x^2 e^{-x}$ , then which of the following is this differential equation?

$$r(r+1)^3 = 0$$

- A)  $y^{(4)} + 3y''' + 3y'' + y' = 0$
- B)  $y^{(4)} - 3y'' - 2y' = 0$
- C)  $y^{(4)} + 3y''' - 2y' = 0$
- D)  $y^{(4)} - 3y''' + 3y'' - y' = 0$
- E)  $y^{(4)} + y''' - 2y' = 0$

$$(r^4 + r^3 + 3r^2 + r) = 0$$

$$y^{(4)} + 3y''' + 3y'' + y' = 0$$

$$y = c_1 y_1 + \dots + c_L y_L$$

$$\downarrow$$

$$\rightarrow x^2 e^{-x}$$

$$y = e^{rx}$$

$$\begin{array}{l} r_1 = 0 \\ r_2 = -1 \\ r_3 = -1 \\ r_4 = -1 \end{array}$$

Which of the following is the general solution of  $y^{(6)} + 4y^{(4)} + 3y'' = 0$ ?

a)  $y = c_1 + c_2 x + c_3 x \cos x + c_4 \sin x + c_5 x \cos \sqrt{3}x + c_6 \sin \sqrt{3}x$

b)  $y = c_1 + c_2 x + c_3 \cos x + c_4 x \sin x + c_5 \cos \sqrt{3}x + c_6 x \sin \sqrt{3}x$

c)  $y = c_1 + c_2 x + c_3 \cos x + c_4 \sin x + c_5 \cos \sqrt{3}x + c_6 \sin \sqrt{3}x$

d)  $y = c_1 + c_2 x + c_3 \cos 2x + c_4 \sin 2x + c_5 \cos \sqrt{3}x + c_6 \sin \sqrt{3}x$

e)  $y = c_1 + c_2 x + c_3 x \cos x + c_4 x \sin x + c_5 \cos \sqrt{3}x + c_6 \sin \sqrt{3}x$

$$r^6 + 4r^4 + 3r^2 = 0 \Rightarrow r^2(r^4 + 4r^2 + 3) = 0 \quad r_1 = r_2 = 0$$

$$(r^2 + 1)(r^2 + 3) = 0 \quad r_{3,4} = \pm i \quad r_{5,6} = \pm \sqrt{3}i$$

$$y_h = c_1 + c_2 x + c_3 \cos x + c_4 \sin x + c_5 \cos \sqrt{3}x + c_6 \sin \sqrt{3}x$$

The solutions of the homogeneous (non-second handed) part of a differential equation with constant coefficients are given as  $\sin x$ ,  $\cos x$  and 1. If the right side of this differential equation is  $f(x) = 5x + e^{-x} - 2 \sin x$ , then which of the following gives the particular solution form of this equation?

- A)  $y_o = (ax + b) + Ae^{-x} + x(C \sin x + D \cos x)$
- B)  $y_o = (ax + b)x + Ae^{-x} + x(C \sin x + D \cos x)$
- C)  $y_o = (ax + b)x + Axe^x + C \sin x + D \cos x$
- D)  $y_o = (ax + b)x^2 + Ae^x + x(C \sin x + D \cos x)$
- E)  $y_o = (ax + b)x + Ae^{-x} + x^2(C \sin x + D \cos x)$

$$y_h \Rightarrow r_1 = 0 \\ r_2, 3 = -1$$

$$y_o = (ax + b)x + k e^{-x} + (A \cos x + B \sin x)x$$

Which of the following is the solution form of the right-sided (non-homogeneous) of the differential equation  $y''' - 2y'' + 2y' = e^x \cos x + x \sin 3x$ ?

- A)  $y_o = xe^x(Asinx + Bcosx) + (Cx + D)\sin 3x + (Ex + F)\cos 3x$
- B)  $y_o = e^x(Asinx + Bcosx) + (Cx + D)\sin 3x + (Ex + F)\cos 3x$
- C)  $y_o = x^2 e^x(Asinx + Bcosx) + x[(Cx + D)\sin 3x + (Ex + F)\cos 3x]$
- D)  $y_o = xe^x(Asinx + Bcosx) + x[(Cx + D)\sin 3x + (Ex + F)\cos 3x]$
- E)  $y_o = e^x(Asinx + Bcosx) + x[(Cx + D)\sin 3x + (Ex + F)\cos 3x]$

$$r^3 - 2r^2 + 2r = 0 \Rightarrow r(r^2 - 2r + 2) = 0 \Rightarrow r_1 = 0$$

$$r_{2,3} = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2}$$

$$r_{2,3} = 1 \pm i$$

$$y_p = e^x [A \cos x + B \sin x] + [C \cos x + D \sin x]x$$

Which of the following is the solution of the initial value problem  $y''' - y'' - 6y' = 0$ ,

$$y(0) = 5, y'(0) = 0, y''(0) = 30 ?$$

A)  $y(x) = 3e^{2x} + 2e^{-3x}$

B)  $y(x) = 3e^{-2x} + 2e^{3x}$

C)  $y(x) = 2e^{-2x} + 3e^{3x}$

D)  $y(x) = 3e^{-2x} - 2e^{3x}$

E)  $y(x) = -2e^{-2x} + 3e^{3x}$

$$y_g = y_h$$

$$r_1 = 0$$

$$r^3 - r^2 - 6r = 0$$

$$r_2 = 3$$

$$r(r^2 - r - 6) = 0$$

$$r_3 = -2$$

$$-3 + 2$$

$$y = c_1 + c_2 e^{3x} + c_3 e^{-2x}$$

$$y(0) = 5 \Rightarrow [c_1 + c_2 + c_3 = 5]$$

$$y' = 3c_2 e^{3x} - 2c_3 e^{-2x}$$

$$y'' = 9c_2 e^{3x} + 4c_3 e^{-2x}$$

$$y'(0) = [3c_2 - 2c_3 = 0]$$

$$y''(0) = [9c_2 + 4c_3 = 30]$$

$$c_2 = 2$$

$$c_3 = 3$$

$$c_1 = 0$$

$$y = 2e^{3x} + 3e^{-2x}$$

Which of the following is the general solution of  $3y^2 y'' - 6y(y')^2 = 0$ ?

\* missing

A)  $y = c_1 x^2 + c_2 x$

$$y' = p, y'' = p \frac{dp}{dy}$$

B)  $y = \frac{1}{c_1 x^2 - c_2}$

$$3y^2 p \frac{dp}{dy} - 6y p^2 = 0$$

C)  $y = -\frac{1}{c_1 x + c_2}$

$$\cancel{3y^2 p} \left( y \frac{dp}{dy} - 2p \right) = 0$$

D)  $y = (c_1 x + c_2)$

1)  $p = 0 \Rightarrow y = c$  Nothing B.S.

E)  $y = c_1 x^2 + c_2$

2)  $y \frac{dp}{dy} = 2p$

$$\frac{dp}{p} = \frac{2dy}{y}$$

$$\ln p = 2 \ln y + \ln c_1 \Rightarrow p = y^2 \cdot c_1$$

$$\frac{dy}{dx} = c_1 y^2 \Rightarrow \int \frac{dy}{y^2} = \int c_1 dx$$

$$-\frac{1}{y} = c_1 x + c_2$$

$$y = -\frac{1}{c_1 x + c_2}$$

$x, y', y'' \Rightarrow y$  missing

The differential equation  $xy'' = (y')^2 \ln x - y'$ , ( $x > 0$ ) is transformed into a linear differential equation using a suitable transformation. Then which of the following is the new differential equation?

$$y' = p, \quad y'' = \frac{dp}{dx} = p'$$

A)  $z' - e^x z = 1$

B)  $z' + \frac{1}{x} z = e^x$

C)  $z' + \ln x z = x$

D)  $z' + \frac{1}{x} z = \frac{\ln x}{x} z^2$

E)  $z' - \frac{1}{x} z = -\frac{\ln x}{x}$

$$\begin{aligned} xp' &= p^2 \ln x - p \\ p' + \frac{p}{x} &= p^2 \frac{\ln x}{x} \end{aligned}$$

$p \rightarrow \text{dep}$   
 $x \rightarrow \text{ind.}$

true answer is E, the question is about linear D.E. not bernoulli

When the general solution of  $y'' + 4y = \tan 2x$  is written as  $y = \overbrace{U(x) \cos 2x} + \overbrace{V(x) \sin 2x}$ ,

then which of the following is  $\underline{V}'$ ?

$$r^2 + b = 0 \Rightarrow r_{1,2} = \pm 2i$$

A)  $V' = \frac{-1}{2} \cos 2x$

B)  $V' = \frac{1}{2} \sin 2x$

C)  $V' = \frac{-1}{4} \cos 2x$

D)  $V' = \frac{-1}{2} \sin 2x$

E)  $V' = 2 \sin 2x$

$$y_h = \underline{c_1 \cos 2x} + \underline{c_2 \sin 2x}$$

$$\begin{aligned} &\cancel{U'} \cos 2x + V' \sin 2x = 0 \\ &\cancel{-2V'} \sin 2x + 2V' \cos 2x = \tan 2x \\ &+ \underline{V' [2 \sin^2 2x + 2 \cos^2 2x]} = \underline{(\tan 2x) \cos 2x} \\ &V' = \frac{\sin 2x}{2} \end{aligned}$$

Which of the following system is used for the solution of the differential equation

6y'' - y' - 12y = Arcsinx ?

~~X~~)  $c'_1 e^{\frac{3}{2}x} + c'_2 e^{-\frac{4}{3}x} = 0 \text{ ve } \frac{3}{2}c'_1 e^{\frac{3}{2}x} - \frac{4}{3}c'_2 e^{-\frac{4}{3}x} = \text{Arcsinx}$

B)  $c'_1 e^{\frac{3}{2}x} + c'_2 e^{-\frac{4}{3}x} = 0 \text{ ve } \frac{3}{2}c'_1 e^{\frac{3}{2}x} - \frac{4}{3}c'_2 e^{-\frac{4}{3}x} = \frac{1}{6} \text{Arcsinx}$

~~X~~)  $c'_1 e^{\frac{3}{2}x} + c'_2 e^{-\frac{4}{3}x} = 0 \text{ ve } \frac{3}{2}c'_1 e^{\frac{3}{2}x} + \frac{4}{3}c'_2 e^{-\frac{4}{3}x} = \text{Arcsinx}$

~~X~~)  $c'_1 e^{\frac{3}{2}x} + c'_2 e^{-\frac{4}{3}x} = 0 \text{ ve } \frac{3}{12}c'_1 e^{\frac{3}{2}x} - \frac{4}{18}c'_2 e^{-\frac{4}{3}x} = \frac{1}{6} \text{Arcsinx}$

~~X~~)  $\frac{3}{2}c'_1 e^{\frac{3}{2}x} - \frac{4}{3}c'_2 e^{-\frac{4}{3}x} = 0 \text{ ve } \frac{9}{4}c'_1 e^{\frac{3}{2}x} + \frac{16}{9}c'_2 e^{-\frac{4}{3}x} = \text{Arcsinx}$

$$6r^2 - r - 12 = 0$$

$$\begin{matrix} \downarrow & \downarrow \\ 3 & 4 \\ 2 & -3 \end{matrix}$$

$$(3r+4)(2r-3) = 0$$

$$r_1 = -\frac{4}{3}, \quad r_2 = \frac{3}{2}$$

$$y_h = c_1 e^{-\frac{4}{3}x} + c_2 e^{\frac{3}{2}x}$$

$$c_1 e^{-\frac{4}{3}x} + c_2 e^{\frac{3}{2}x} = 0$$

$$-\frac{4}{3}c_1 e^{-\frac{4}{3}x} + \frac{3}{2}c_2 e^{\frac{3}{2}x} = \frac{\text{arc sin}}{6}$$

If the general solution of  $y'' + ay' + by = 0$  is  $y = e^{-x}(c_1 \sin 2x + c_2 \cos 2x)$ , then what is the sum  $a + b$ ?

A) 2

B) 5

$$\overline{r^2 + ar + b = 0}$$

C) 7

D) 8

$$r_{1,2} = -1 \pm 2i$$

$$r_1 + r_2 = -a$$

$$\rightarrow (r-r_1)(r-r_2) = 0$$

$$r_1 r_2 = b$$

$$\rightarrow r^2 - (r_1+r_2)r + r_1 r_2 = 0$$

$$r_1 + r_2 = -1 + 2i - 1 - 2i = -2 = -a \Rightarrow a = 2$$

$$(-1+2i)(-1-2i) = 1 + 2i - 2i - 4i^2 = 5 = b \Rightarrow b = 5$$

## Out of the lesson subject

The differential equation  $y'' + 3y' + 2y = xe^{-x}$  is given. Then, which of the following is the reduced form of this differential equation?

- A)  $u'' - u' = x$    B)  $u'' + u = x$    C)  $u'' + u' = x$    D)  $u'' - u = x$   
 E)  $u'' + 2u' = x$

Which of the following is the coefficient of  $x^3$  which appears in the power series  $\sum_{n=0}^{\infty} a_n x^n$  of the differential equation  $x \frac{d^2y}{dx^2} + y = 0$ ?

- A)  $\frac{2a_1}{5}$    B)  $\frac{a_1}{12}$    C)  $\frac{a_1 - a_0}{7}$    D)  $\frac{-a_0}{6}$    E)  $\frac{-a_0}{12}$

$$a_1 = ?$$

$$x \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} (n+1)n a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} [n(n+1) a_{n+1} + a_n] x^n + a_0 = 0$$

$$a_{n+1} = -\frac{a_n}{n(n+1)} \quad n \geq 1$$

↓

$$n=2 \Rightarrow a_3 = -\frac{a_2}{3 \cdot 2} = -\frac{1}{6} \left( -\frac{a_1}{1} \right) \Rightarrow \boxed{\frac{a_1}{12}}$$

$$n=1 \Rightarrow a_2 = -\frac{a_1}{2 \cdot 1}$$

Which of the following can NOT be the linearly independent solutions of the differential equation  $y'' = 0$ ?  $y = c_1 + c_2 x$

a)  $\{5, 3x\}$

b)  $\{5 + 2x, 3x\}$

c)  $\{2, 3x^2\}$

d)  $\{\underline{5-x}, \underline{3}\}$

e)  $\{5\underline{x-1}, \underline{2x}\}$

$$\begin{vmatrix} 5 & 3x \\ 0 & 3 \end{vmatrix} = 15 \neq 0$$

$$\begin{vmatrix} 2 & 3x^2 \\ 0 & 6x \end{vmatrix} = 12x \quad x \neq 0 \quad \checkmark$$

$$x = 0 \quad \times$$

$\times$  missing

The differential equation  $y^3 y'' = y^2 y' + 2y(y')^2$  is transformed into a first order differential equation using a suitable transformation. Then, which of the following is the new form of this differential equation?

A)  $yp' - p(y^2 - 2p) = 0$        B)  $p(y^2 \frac{dp}{dy} - y - 2p) = 0$

C)  $yp' - p(y^2 + 2py) = 0$       D)  $p(\frac{dp}{dy} - y^2 - 2p) = 0$

E)  $yp' + p(2y^2 - p) = 0$

$$y' = p, \quad y'' = p \frac{dp}{dy}$$

$$y^3 p \frac{dp}{dy} = y^2 p + 2y p^2 \Rightarrow p \underbrace{\left[ y^2 \frac{dp}{dy} - y - 2p \right]}_{=0} = 0$$

When the general solution of  $y''' + y' = \frac{1}{\sin^3 x}$  is searched as the form of

$$r^3 + r$$

$y = c_1(x) + c_2(x)\cos x + c_3(x)\sin x$  according to *Variation of Parameters Method*, then which of the following is the  $c_3(x)$  function?

- a)  $c_3(x) = \cot x + k$
- b)  $c_3(x) = \tan x + k$
- c)  $c_3(x) = \sec x + k$
- d)  $c_3(x) = \operatorname{cosec} x + k$
- e)  $c_3(x) = \cos x + k$

$$\begin{aligned} & \underline{c_1' + c_2' \cos x + c_3' \sin x = 0} \\ & \left. \begin{array}{l} \text{cosec } x \\ \sin x \end{array} \right\} \begin{array}{l} -c_2' \sin x + c_3' \cos x = 0 \\ -c_2' \cos x - c_3' \sin x = \frac{1}{\sin^3 x} \end{array} \end{aligned}$$

$$c_3' = \frac{-1}{\sin^2 x} \Rightarrow c_3 = k + \cot x$$

$$-\int \csc^2 x dx = \cot x$$

$x^2y'' + kxy' + y = \frac{1}{x} \ln x$ , ( $x > 0, k \in \mathbb{R}$ ) is transformed into a differential equation with constant coefficients. If the particular (nonhomogeneous) solution form of this new equation is  $y_p = (at + b)t^2 e^{-t}$ , then what is the value of  $k$ ?

$$x = e^t$$

- a) 0    b) 1    c) 2    d) 3    e) -1

$$D(D-1)y + k D^2 y + y = e^{-t} t$$

$$\underbrace{[D^2 + (k-1)D + 1]}_{r^2 + (k-1)r + 1 = 0} y = t e^{-t}$$

$$r^2 + (k-1)r + 1 = 0$$

$$y_p = (at + b)e^{-t}$$

$$t^2 \Rightarrow [r_1 = r_2 = -1]$$

$$r = -1 \Rightarrow 1 - (k-1) + 1 = 0$$

$$1 - k + 1 + 1 = 0$$

$$k = 3$$

$$y' = f(ax+by+c) \quad ax+by+c = u$$

The differential equation  $(1+y')\sin(x+y) = y'$  is transformed into a separable differential equation using a suitable transformation. Which of the following is the new differential equation?

$$y' [1 - \sin(x+y)] = \sin(x+y)$$

A)  $u' = \frac{1}{1-\sin u}$

$$y' = \frac{\sin(x+y)}{1 - \sin(x+y)} = f(x+y)$$

C)  $u' = \frac{1}{1-\cos u}$

$$\left. \begin{array}{l} x+y=u \\ 1+y'=u' \end{array} \right\} \quad u' = \frac{\sin u}{1 - \sin u} + 1$$

D)  $u' = \frac{1}{1+\cos u}$

$$u' = \frac{1}{1 - \sin u}$$

E)  $u' = \frac{1}{1-\sin^2 u}$

**SORU 3.**  $y'' + Ky' + 4y = x^2 e^x$      $y(0) = y'(0) = 0$     ~~başlangıç değer probleminin çözümü  $y = e^x(x^2 + 4x + 6) + 2(x - 3)e^{2x}$  ise K sayısı kaçtır?~~

~~Rsolution~~

?

a) 4

b) 2

c) ~~-4~~

d) -2

e) 0      CEVAP C

$$y = \underbrace{e^x(x^2 + 4x + 6)}_{y_p} + \underbrace{2(x - 3)e^{2x}}_{y_h}$$

$$r=2$$

$$r^2 + Kr + L = 0$$

$$r=2 \Rightarrow L + 4L + 2k = 0$$

$$k = -\frac{8}{2} = -4$$

y missing

linear d.e. missing a.

**SORU 4.**  $x^2y'' - (y')^2 + 3xy' = 0$  Diferansiyel denklemi için uygun değişken

dönüştürümü yaparak elde edilen lineer diferansiyel denklem aşağıdakilerden hangisidir?

suit.  
trans.

X)  $z' - 3\frac{z}{x} = -\frac{1}{x^2}$

$$y' = p, \quad y'' = \frac{dp}{dx}$$

X)  $2z' + zx = 4$

$$x^2 p' - p^2 + 3xp = 0$$

X)  $2z' + 4zx = 2$

$$p' - \frac{p^2}{x^2} + \frac{3p}{x} = 0$$

X)  $z' + z = x$

true answer is A, the question is about linear D.E. not bernoulli  
therefore, there is a true answer

error in the  
choice.

$$\boxed{z' + \frac{3z}{x} = \frac{z^2}{x^2}}$$

true answer

**SORU 6.** Cauchy-Euler ~~diferansiyel denklemi olduğu bilinen~~  $x^k y'' + xy' + y = kx+1$

~~diferansiyel denklemi, uygun bir değişken dönüşümü ile aşağıdakilerden hangi sabit katsayılı  
denklemeye dönüştür?~~ use a suitable trans.

a)  $\frac{d^2y}{dt^2} + y = 2e^t + 1$



$$\begin{matrix} 2 \\ \uparrow \\ k=2 \end{matrix}$$

b)  $\frac{d^2y}{dt^2} + \frac{dy}{dt} - y = 2t + 1$

$$x = e^t$$

c)  $\frac{d^2y}{dt^2} + y = 2e^t + 1$

d)  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - y = e^{2t} + e^t$

e)  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - y = t + 1$  (Cevap a)

$$D(D-1)y + Dy + y = 2e^t + 1$$

$$[D^2 - D + 1]y = 2e^t + 1 \Rightarrow y'' + y = 2e^t + 1$$

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**SORU 7.**  $y'' - 4y = \ln x^4$  ~~diferansiyel denklemi~~ genel çözümü  $y = f(x)e^{3x} + g(x)e^{-3x}$  şeklinde yazılıyor. Buna göre  $2f'(x)e^{3x} - g'(x)e^{-3x}$  aşağıdakilerden hangisidir?

a)  $\ln x$

b)  $2\ln x$

c) 0

d)  $\frac{2\ln x}{e^{3x}}$

e)  $4\ln x$

Cevap b)  $2\ln x$

$$2f'e^{3x} - g'e^{-3x}$$

$$f'e^{3x} + g'e^{-3x} = 0$$

$$+ 3f'e^{3x} - 3g'e^{-3x} = \ln x^4$$

$$\frac{4f'e^{3x} - 2g'e^{-3x}}{2} = \frac{\ln x^4}{2}$$

$$2f'e^{3x} - g'e^{-3x} = 2\ln x$$

or

$$= \ln x^2$$

singular sol.

**SORU 10.**  $y = xy' + \cos(y')$  ~~diferansiyel denklemi~~ tekil çözümü aşağıdakilerden hangisidir?

a)  $y = x \operatorname{Arcsin} x + \sqrt{1-x^2}$

b)  $y = x \operatorname{Arcsin} x - \sqrt{1-x^2}$

c)  $y = x \operatorname{Arcsin} x + \sqrt{1+x^2}$

d)  $y = x \operatorname{Arccos} x + \sqrt{1-x^2}$

e)  $y = x \operatorname{Arccos} x - \sqrt{1+x^2}$

$$y' = p$$

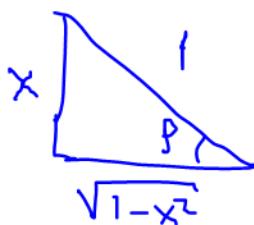
$$y = px + \cos p \quad \text{X}$$

$$y' = p'x + p - p \sin p$$

$$p' [x - \sin p] = 0$$

$x = \sin p$

$y = px + \cos p$



$y = x \operatorname{Arcsin} x + \sqrt{1-x^2}$

**SORU 11.**  $y'' + 2y' + y = te^{-t}$ ,  $y(0) = -1$ ,  $y'(0) = +2$  başlangıç değer problemlerine

Laplace dönüşümü uygulandığında elde edilecek ifade aşağıdakilerden hangisidir?

a)  $Y(s) = \frac{1}{(s+1)^4} + \frac{s}{(s+1)^2}$

b)  $Y(s) = \frac{1}{(s-1)^4} - \frac{s}{(s-1)^2}$

c)  $\textcircled{d} Y(s) = \frac{1}{(s+1)^4} - \frac{s}{(s+1)^2}$

d)  $Y(s) = \frac{1}{(s-1)^4} - \frac{s}{(s+1)^2}$

e)  $Y(s) = \frac{1}{(s+1)^4} - \frac{s}{(s-1)^2}$

$$\mathcal{L}\{te^{-t}\} = \mathcal{F}(s+1) = \frac{1}{(s+1)^2}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2} = \mathcal{F}(s)$$

$$5+2+2$$

2



$$s^2 Y(s) - \underbrace{sy(0)}_{-(s+1)^2} - \underbrace{y'(0)}_{-1} + 2 \left[ sY(s) - \underbrace{y(0)}_{-1} \right] + Y(s) = \frac{1}{(s+1)^2}$$

$$\underbrace{(s^2 + 2s + 1)}_{(s+1)^2} Y(s) = \frac{1}{(s+1)^2} - s$$

$$Y(s) = \frac{1}{(s+1)^2} - \frac{s}{(s+1)^2}$$

Lö2

**SORU 15.**  $f_1(t) = t^5 e^{-3t}$ ,  $f_2(t) = e^{2t} \sin^2(3t)$  fonksiyonlarının Laplace dönüşümüleri mevcut olduğuna göre  $L\left\{\frac{1}{5}f_1(t) + 2f_2(t)\right\}$  değeri aşağıdakilerden hangisidir?

- a)  $\frac{4!}{(s+3)^6} + \frac{1}{s-2} + \frac{2-s}{(s-2)^2+36}$   
b)  $\frac{5!}{(s-3)^6} - \frac{1}{s-2} + \frac{s-2}{(s-2)^2+6}$   
c)  $\frac{4!}{(s+2)^6} + \frac{1}{s+2} + \frac{2-s}{(s-2)^2+36}$   
d)  $\frac{5!}{(s+3)^6} + \frac{1}{s-2} - \frac{2-s}{(s-2)^2+36}$   
e)  $\frac{4!}{(s+3)^5} + \frac{1}{s-2} + \frac{2-s}{(s-2)^2+36}$

**SORU 16.**  $y = (-x + 4)y' + 2(y')^2$  Lagrange diferansiyel denkleminin çözümü sırasında ortaya çıkan Lineer diferansiyel denklem aşağıdakilerden hangisidir? Lineer !

a)  $\frac{dx}{dp} + \frac{1}{2p}x = \frac{2}{p} + 2$

~~b)  $\frac{dp}{dx} + \frac{1}{2x}p = \frac{x}{2}$~~

~~c)  $\frac{dx}{dp} + 2x = \frac{p}{2} - 2$~~

~~d)  $\frac{dx}{dp} - \frac{1}{2p}x = -\frac{p}{2} - 1$~~

~~e)  $\frac{dx}{dp} - \frac{1}{4p}x = \frac{2}{p} + 2$~~

$y' = p$

$y = (-x + 4)p + 2p^2$

$y' = -p + (-x + 4)p' + 2pp'$

$2p = \boxed{p} [Lp - x + 4]$

$\frac{dp}{dx}$

$\frac{dx}{dp} = \frac{Lp - x + 4}{2p}$

$\frac{dx}{dp} + \frac{x}{2p} = \frac{2p + 2}{p} = 2 + \frac{2}{p}$

**Çöz**

**SORU 20.**  $x \frac{d^2y}{dx^2} + y = 0$  diferansiyel denkleminin  $\sum_{n=0}^{\infty} a_n x^n$  şeklindeki kuvvet serisi çözümünde  $x^3$  teriminin katsayısı aşağıdakilerden hangisidir?

- A)  $\frac{2a_1}{5}$
- B)  $\frac{a_1}{6}$  doğru
- C)  $\frac{a_1 - a_0}{7}$
- D)  $\frac{-a_0}{6}$
- E)  $\frac{-a_0}{12}$

**Çöz**

**SORU 19.**  $y'' = 1 + (y')^2$  diferansiyel denkleminin genel çözümü aşağıdakilerden hangisidir?

- a)  $y = -\ln[\cos(x + c_1)] + c_2$
- b)  $y = \ln[\sin(x + c_1)] + c_2$
- c)  $y = -\ln[\sec(x + c_1)] + c_2$
- d)  $y = \ln(x + c_1) + c_2$
- e)  $y = \ln[x + c_1] + c_2 x$

**SORU 13.**  $u = u(x)$  ve  $v = v(x)$  olmak üzere  $\begin{cases} u' = 4u - v \\ v' = -4u + 4v \end{cases}$  sisteminin çözümü aşağıdakilerden hangisidir?

- a)  $u(x) = c_1 e^{2x} + c_2 e^{6x}, v(x) = \underline{2c_1} e^{2x} - \underline{2c_2} e^{6x}$
- b)  $u(x) = c_1 e^{-2x} + c_2 e^{6x}, v(x) = \underline{2c_1} e^{-2x} - \underline{2c_2} e^{6x}$
- c)  $u(x) = c_1 e^{2x} + c_2 e^{-6x}, v(x) = \underline{2c_1} e^{2x} - \underline{2c_2} e^{-6x}$
- d)  $u(x) = c_1 e^{-2x} + c_2 e^{-6x}, v(x) = 2c_1 e^{-2x} - 2c_2 e^{-6x}$
- e)  $u(x) = c_1 e^{2x} - 2c_2 e^{6x}, v(x) = c_1 e^{2x} + c_2 e^{6x}$

+D<sup>2</sup> 8D+12

- D<sup>2</sup> +8D -16+L

$$4/(D-L)u + v = 0$$

$$D^2u + (D-L)v = 0$$

$$\frac{L_v - (D^2 - 8D + 16)v = 0}{\Rightarrow (D^2 - 8D + 12)v = D}$$

$$v = c_1 e^{6x} + c_2 e^{2x}$$

$$-6 \quad -2$$

$$u = \frac{-(D-L)v}{L} = -\frac{1}{4} \left[ 6c_1 e^{6x} + 2c_2 e^{2x} - 4c_1 e^{-6x} - 4c_2 e^{-2x} \right]$$

$$u = -\frac{c_1}{2} e^{6x} + \frac{c_2}{2} e^{2x}$$

$$-\frac{c_1}{2} = k_1$$

$$\frac{c_2}{2} = k_2$$

**SORU 8.**  $y''' + 2y'' - 3y'' - 4y' + 4y = \tan x$  diferansiyel denkleminin lineer bağımsız  $D$ ne çözümlerinden biri  $\boxed{xe^{kx}}$  ise  $k$ nın alabileceği en büyük değer aşağıdakilerden hangisidir?  $\rightarrow$

- a-) 1
- b-) -1
- c-) 2
- d-) -2
- e-) 0

Cevap A

$$\frac{r^4 + 2r^3 - 3r^2 - 4r + 4}{r^4 - r^3} \left| \begin{array}{c} r-1 \\ r^3 \end{array} \right.$$

$$(r-1)(r^2 + 4r)$$

$$= (r-1)r^3 + (r^3 - r^2 - 4r + 4)$$

$$3r^2(r-1) - 4(r-1)$$

$$= (r-1)[r^3 + 3r^2 - 4]$$

$$\frac{r^3 + 3r^2 - 4}{r^2 - r} \left| \begin{array}{c} r-1 \\ r^2 + 4r \end{array} \right.$$