Random variable

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A random variable is a function that assigns a real number to each outcome in the sample space of a random experiment.

Notation: A random variable is denoted by an uppercase letter such as X and Y .

After experiment is conducted, the measured value of the random variable is denoted by a lowercase letter such as x and y.

If S is a sample space with a probability measure and X is a real-valued function defined over the elements of S, then X is called a random variable.

In this course we shall always denote random variables by capital letters such as X, Y etc., and their values by the corresponding lowercase letters such as x and y, respectively.

Random variable is not a variable. Also, it is not random.
Thus someone named it inappropriately.

Example:
Suppose that a coin is tossed twice so that the sample space is Let X represent the number of heads.

What is the random variable for X.

H

0 1 2

$X = \{0, 1, 2\}$

Head → not (0)
→ only 1 time
→ 2 times

S = HH, HT, TH, TT

S = {HH,HT,TH,TT}.

Ex. Suppose that a coin is tossed three so that the sample space is
Let Y represent the number of heads. What is the random variable for X.

$X = \{0, 1, 2, 3\}$

Ex.
A dice is tossed 2 times. What is the random variable for 5.

$R.V = \{0, 1, 2 \}$

Discrete Probability Distributions (Probability Mass Function)

probability distributions is the distribution of probabilities belonging to random variables.

There are 3 step in probability and statistical systems related to this issue.

Step1. Experiment
Step2. Random variable
Step3. Probability Distributions (Probability Mass Function)

Let X be a discrete random variable,and suppose that the possible values that it can assume are given by x1,x2,x3,. . . ,arranged in some order.

Suppose also that these values are assumed with probabilities given by

$$P(X = x_k) = f(x_k) \qquad k = 1, 2, \ldots$$

It is convenient to introduce the probability function,also referred to as probability distribution,given by

$$P(X = x) = f(x)$$

For $x = x_k$, this reduces to (1) while for other values of x, $f(x) = 0$.
In general, $f(x)$ is a probability function if

1. $f(x) \geq 0$
2. $\sum_x f(x) = 1$

check

where the sum in 2 is taken over all possible values of x.

For $x = x_k$, this reduces to (1) while for other values of x, $f(x) = 0$.

In general, $f(x)$ is a probability function if

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2. $\sum_x f(x) = 1$ → check

where the sum in 2 is taken over all possible values of x.

Before getting into the details of the subject, let's solve a simple example.

HH
HT
TH
TT

A coin is tossed 2 times.

a) What is the probability that no heads appear. → $\frac{1}{4}$
b) What is the probability of going heads one time. → $\frac{2}{4} = \frac{1}{2}$
c) What is the probability of going heads 2 times. → $\frac{1}{4}$
ç) What is the random variable for head → $H = \{0, 1, 2\}$
d) What is the probability distributions head

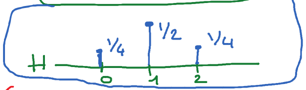
H	0	1	2
P(H)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$f(x) \geq 0$ (+)

$\frac{1}{4} + \frac{1}{2} + \frac{1}{4} \overset{?}{=} 1$

(2)

$\boxed{1 \overset{\checkmark}{=} 1}$



Ex. A coin is tossed 3 times. What is the probability distributions for head

Step 1.

HHH
HHT
HTH
THH
HTT
THT
TTH
TTT

Step 2. Random variable $X = \{0, 1, 2, 3\}$

Step 3. Proble. distr.

X	0	1	2	3
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

1/8, 3/8, 3/8, 1/8

Check please

1. $f(x) \geq 0$ ✓
2. $\sum_x f(x) = 1$ $\quad \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{8}{8} = 1$

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Ex

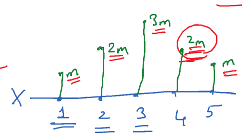
X	1	4	9
$f(x=x_i)$	d	3d	6d

This function is discrete probability distribution

what is d $\quad$ d = ?

$d + 3d + 6d = 1 \qquad \boxed{d = \frac{1}{10}}$

Example



a) m = ? $\quad m + 2m + 3m + 2m + m = 1 \quad m = \frac{1}{9}$

b) $P(X=4) = ?$ $\quad 2 \cdot \frac{1}{9} = \frac{2}{9}$

c) $P(X > 3) = ?$ $\quad P(4) + P(5) = 2 \cdot \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$

d) $P(1 \leq x < 4) = P_{(X=1)} + P_{(X=2)} + P_{(X=3)}$

$\frac{1}{9} + 2 \cdot \frac{1}{9} + 3 \cdot \frac{1}{9} = \frac{6}{9}$

Example A bag contains 3 red and 4 white balls.

Two draws are made without replacement.

Find, discrete probability distribution for red balls.

Step 1

Step 2. Random Variable $X = \{0, 1, 2\}$

$E[X] = 0 \cdot \frac{2}{7} + 1 \cdot \frac{4}{7} + 2 \cdot \frac{1}{7}$

Step 1

Step 2 - Random Variable $X = \{0, 1, 2\}$

No red (0) $= \frac{4}{7} \cdot \frac{\cancel{8}\,2}{\cancel{6}\,2} = \frac{2}{7}$

only 1 red (1) $= rw + wr$

$\frac{3}{7} \cdot \frac{4}{6} + \frac{4}{7} \cdot \frac{3}{6} = \frac{4}{7}$

2 red (2) $= \overset{rr}{\frac{3}{7} \cdot \frac{2}{6}} = \frac{1}{7}$

Step 3

X	0	1	2
$P(X=x_n)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

$E[X] = 0 \cdot \frac{2}{7} + 1 \cdot \frac{4}{7} + 2 \cdot \frac{1}{7}$

$= 0 + \frac{4}{7} + \frac{2}{7}$

$= \frac{6}{7}$

$\frac{2}{7} + \frac{4}{7} + \frac{1}{7} \overset{?}{=} 1$

$1 = 1$

Expected Value for discrete probability distribution

$E[X]$ = expected value

$E[X] = \sum X \cdot P(X = x_n)$

Example

X	0	1	2	3
$P(X=x_n)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{4}$

$\frac{1}{12} + \frac{1}{6} + \frac{1}{2} + \frac{1}{4} \overset{?}{=} 1$

$E[X] = 0 \cdot \frac{1}{12} + 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4}$

$E[X] = \frac{23}{12}$

$\boxed{\text{Expected value} = \text{mean}}$

Variance for discrete probability distribution

$Var[X]$ or σ^2

$$\boxed{Var[X] = E[X^2] - (E[X])^2}$$

Ex

X	1	2	3
$P(X=x_n)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

Check

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{2} \overset{?}{=} 1$$

$Var[X] = ?$

$$E[X] = 1\cdot\frac{1}{4} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{2} = \frac{9}{4}$$

$$E[X^2] = 1^2\cdot\frac{1}{4} + 2^2\cdot\frac{1}{4} + 3^2\cdot\frac{1}{2} = \frac{23}{4}$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$= \frac{23}{4} - \left(\frac{9}{4}\right)^2 = \frac{11}{16} = \underline{0{,}6875}$$

$$\sqrt{0{,}6875}$$

Conditional probability for Discrete probability distribution

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Ex

X	1	2	3	4
$P(X=x_n)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$

a) $P(X=1 \mid X=4) = ?$ $\rightarrow 0$

b) $P(X=1 \mid X \leq 3) = ?$ $= \dfrac{P(X=1 \cap X \leq 3)}{P(X \leq 3)} = \dfrac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{6}+\frac{1}{4}} = \dfrac{1}{6}$

c) $P(X<2 \mid X \leq 2) = ?$ $= \dfrac{P(X<2 \cap X \leq 2)}{P(X \leq 2)} = \dfrac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{6}} = \dfrac{1}{3}$

X	1	2	3	4
$P(X=x_n)$	1	1	1	1

$P(X=x_n)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$