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Example

$f(x) = e^{-2x} - \sin x - x + 2$

$x_{-1} = 0{,}5$

$x_0 = 1$

find, approximation root using secant

$$x_{(n+1)} = x_n - \frac{f(x_n)\,(x_{(n-1)} - x_n)}{f(x_{n-1}) - f(x_n)}$$

$n = 0$ start

1 iteration $\quad x_1 = x_0 - \dfrac{f(x_0)\,(x_{-1} - x_0)}{f(x_{-1}) - f(x_0)}$

$x_1 = 1{,}134275$

$x_2 = 1{,}170884$

$x_3 = 1.173486$

$x_4 = 1.173529$

$x_5 = \;?$

$x_6 = \;?$

$x_7 = \;?$

1. Find the second Taylor Polynominal $P_2(x)$ for function $f(x)=xe^x+x$ about $x_0=0$ and use $p_2(0.1)$ to $f(0.1)$. (hint: in the formula for the result, there will be 2 terms that are non-zero.)

A) $x+x^3$, 0.101
B) $2x+x^2$, 0.21(+)
C) $x+2x^2$, 0.12
D) $x+x^2$, 0.11
E) $2x+2x^2$, 0.22

ANS

$$f(x) = f(x_0) + f'(x_0)\cdot(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$f(x) = x e^x + x$

$f'(x) = e^x + x e^x + 1$

$f''(x) = e^x + e^x + x e^x$

$f(0) = 0$

$f'(0) = 2$

$P_2(x) = 0 + 2x + \dfrac{2 \cdot x^2}{2}$

$P_2(x) = 2x + x^2$

$P_2(0{,}1) = 2\cdot(0{,}1) + (0{,}1)^2$

$P_2(0{,}1) = 0{,}2 + 0{,}01$

$f'(0) = 2$

$f''(0) = 2$

$P_2(0,1) = 0,2 + 0,01$

$= 0.21$

2. Find the second Taylor Polynominal $P_3(x)$ for function $f(x)=\cos x-(x+1)^2$ about $x_0=0$ and use $p_3(0.1)$ to f(0.1). (hint: in the formula for the result, there will be 2 terms that are non-zero.)

A) $-2x-(3/2)x^2$, -0.215 ANS
B) $-x-3x^2$, -0.13
C) $-2x-2x^2$, -0.22
D) $-x-2x^2$, -0.12
E) $-3x-2x^2$, -0.32

$f(x) = f(x_0) + f'(x_0)\cdot(x-x_0) + \frac{f''(x-x_0)^2}{2!}$

$f(x) = \cos x - (x+1)^2$

$f'(x) = -\sin x - 2(x+1)$

$f''(x) = -\cos x - 2$

$f'''(x) = \sin x$

$f(0) = 0$

$f'(0) = -2$

$f''(0) = -3$

$P_3(x) = 0 + -2x + \frac{-3x^2}{2}$

$P_3(x) = -2x - \frac{3x^2}{2}$

$P(0,1) = -2(0,1) - \frac{3.(0,1)^2}{2}$

$= -0.215$

3. Function : f(x)=x-(sinx)/2 -3
   interval : [3 , 4]
   error :-
   starting point :-
   method : Bisection method
   Question : Find approximate Root (Use only 3 iteration)
   Hint : The result of the 1st iteration is 3.5. (x_1=3.5, x_3=?).
   ANS : 3.125

A) 3.125(+)
B) 3.625
C) 3.25
D) 3.375
E) 3.3125

$X_1 = \frac{3+4}{2} = 3.5$

$[3 , 3.5]$

C) 3.25
D) 3.375
E) 3.3125

[3 , 3.5]

$X_2 = \frac{3+3.5}{2} = 3.25$ ✓

[3, 3.25]

$X_3 = \frac{3+3.25}{2} = 3.125$ ✓ ANS

4. Function : $f(x)=e^x-x^2+3x-2$
   interval : [0.25,0.30]
   error :
   starting point :-
   method : Bisection method
   Question : Find approximate Root (Use only 3 iteration)
   Hint : The result of the 1st iteration is 0.275 . (x_1=0.275, x_3=?).

A) 0.25625(+)
B) 0.26875
C) 0.2875
D) 0.28125
E) 0.2625

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5. Function : $f(x)=e^{-x}-\sin x$
   interval : [0 , 1]
   error :-
   starting point :-
   method : Bisection method
   Question : Find approximate Root (Use only 3 iteration)
   Hint : The result of the 1st iteration is 0.5. (x_1=0.5, x_3=?).

A) 0.625(+)
B) 0.375
C) 0.875
D) 0.125
E) 0.75

$X_1 = \frac{0+1}{2} = 0.5$

[0.5 , 1]

B) 0.375
C) 0.875
D) 0.125
E) 0.75

2

[0,5, 1]

$X_2 = \frac{0,5+1}{2} = 0,75$

[0,5 , 0,75]

$X_3 = \frac{0,5+0,75}{2} = 0,625$

6. Function : $f(x)=e^{-x}-\cos(x)$
   interval :
   error : error 10^{-3}
   starting point : $x_0=1.25$
   method : Newton-Raphson
   Question : Find approximate Root
   Hint : Please do not make more than 3 iterations.

$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$

A) 1.2927(+)
B) 1.2835
C) 1.2786
D) 1.2694
E) 1.2548

START : 10 40

7. Function : $f(x)=-x^3-\cos x$
interval :
error : error 10^{-3}
starting point : $x_0=-1$
method : Newton-Raphson
Question : Find approximate Root
Hint : Please do not make more than 3 iterations.

A) -0.86547(+)
B) -0.86712
C) -0.86386

Homework

C) -0.86386
D) -0.86125
E) -0.86264

8. Function : $f(x)=-x^3-\cos x$
   interval :
   error :
   starting point : $x_{-1}=-1$ and $x_0=0$
   method : secant method
   Question : Find approximate Root (Use 6 iteration).
   Hint : Questions containing more than 3 iterations will not be asked in the exam.

A) -0.8665
B) -0.8723
C) -0.8598
D) -0.8832
E) -0.8145

$$x_{n+1} = x_n - \frac{f(x_n).(x_{n-1}-x_n)}{f(x_{n-1})-f(x_n)}$$

n=0

$$X_1 = X_0 - \frac{f(x_0)(X_{-1}-X_n)}{f(x_{-1})-f(x_0)}$$

$X_1 = -0.68507$
$X_2 = -1.25208$
$X_3 = -0.86721$
$X_4 = -0.86478$

9. Function : $f(x)=\ln x-\cos x$
   interval :
   error : error $=10^{-3}$
   starting point : $x_{-1}=1$ and $x_0=1.2$
   method : secant method
   Question : Find approximate Root (Use only 2 iteration)
   Hint : The result of the 1st iteration is 1.2999. ($x_1=1.2999$, $x_2=?$).

A) 1.3029(+)
B) 1.3158
C) 1.3227
D) 1.3381
E) 1.3429

Ragula falsi (6 iteration

$f(x)=x^3-2x^2-5$ [2,3]

$$c = \frac{af(b)-bf(a)}{}$$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

	x	f(x)
	2	-5
a	3	4
	2,555556	-1,37174
b	2,66905	-0,2338
	2,687326	-0,03632
	2,69014	-0,00556