

		Surname		Type
Group Number		Name		A
List Number		e-mail		
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**ATTENTION:** There is normally only one correct answer for each question and each correct answer is worth the same point. Only the answers on your answer sheet will be graded. Please be sure that you have marked all of your answers on the answer sheet by using a pencil (not pen).

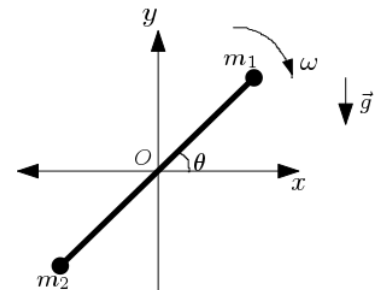
### Questions 1-3

The angular position of a rigid body rotating about a fixed-axis is given as  $\theta(t) = a + bt - ct^3$  with  $t$  is in seconds and  $\theta$  in radians,  $a$ ,  $b$ ,  $c$  are constants. At  $t = 0$  the object has angular speed  $2 \text{ rad/s}$  and at  $t = 1.5 \text{ s}$  it has angular acceleration of  $18 \text{ rad/s}^2$

- Which of the following is the constant  $b$  with its SI unit?  
(a)  $2 \text{ rad/s}$  (b)  $3 \text{ rad/s}^2$  (c)  $2 \text{ rad/s}^2$  (d)  $3 \text{ rad/s}$  (e)  $1.5 \text{ rad/s}$
- If the angular momentum of the object at  $t = 0$  is  $12 \text{ kgm}^2/\text{s}$ , what is the rotational inertia of the object relative to the given rotation axis?  
(a)  $7.5 \text{ kgm}^2$  (b)  $7 \text{ kgm}^2$  (c)  $6 \text{ kgm}^2$  (d)  $8 \text{ kgm}^2$  (e)  $5 \text{ kgm}^2$
- What is the torque on the object relative to the rotation axis at  $t = 1.5 \text{ s}$ ?  
(a)  $108 \text{ Nm}$  (b)  $72 \text{ Nm}$  (c)  $63 \text{ Nm}$  (d)  $54 \text{ Nm}$  (e)  $90 \text{ Nm}$

### Questions 4-6

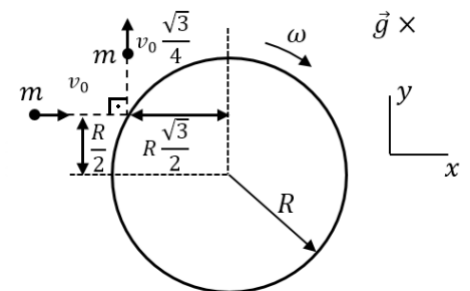
A uniform rigid rod of mass  $M$  and length  $\ell$  rotates in the vertical plane about a frictionless pivot passing through its center. Two point masses  $m_1$  and  $m_2$  are attached at the ends of the rod, as shown in the figure. (For a uniform rigid rod of mass  $M$  and length  $\ell$ ,  $I_{cm} = \frac{1}{12}M\ell^2$ .)



- What is the angular momentum of the system relative to the center of mass of the rod, if the whole assembly is rotating about this point with an angular speed  $\omega$ ?  
(a)  $(M + m_1 + m_2) \frac{\omega \ell^2}{2}$   
(b)  $(\frac{M}{3} + m_1 + m_2) \frac{\omega \ell^2}{4}$  (c)  $(\frac{M}{12} + m_1 + m_2) \frac{\omega \ell^2}{2}$  (d)  $(\frac{M}{2} + m_1 + m_2) \frac{\omega \ell^2}{4}$  (e)  $(M + m_1 + m_2) \frac{\omega \ell^2}{4}$
- What is the magnitude of the angular acceleration of the system when the rod makes an angle  $\theta$  with the horizontal, assuming  $m_2 > m_1$ ?  
(a)  $\frac{2(m_2 - m_1)}{(\frac{M}{3} + m_1 + m_2)} \frac{g \cos \theta}{\ell}$  (b)  $\frac{2(m_2 - m_1)}{(\frac{M}{2} + m_1 + m_2)} \frac{g \cos \theta}{\ell}$   
(c)  $\frac{(2m_2 - m_1)}{(\frac{M}{2} + m_1 + m_2)} \frac{g \cos \theta}{\ell}$  (d)  $\frac{(2m_2 - m_1)}{(\frac{M}{3} + m_1 + m_2)} \frac{g \cos \theta}{\ell}$  (e)  $\frac{2(m_2 - m_1)}{(M + m_1 + m_2)} \frac{g \cos \theta}{\ell}$
- What is the kinetic energy of the system when its angular speed is  $\omega$ ?  
(a)  $\frac{1}{8} (\frac{M}{3} + m_1 + m_2) \omega^2 \ell^2$  (b)  $\frac{1}{6} (M + m_1 + m_2) \omega^2 \ell^2$   
(c)  $\frac{1}{2} (\frac{M}{6} + m_1 + m_2) \omega^2 \ell^2$  (d)  $\frac{1}{2} (\frac{M}{3} + m_1 + m_2) \omega^2 \ell^2$  (e)  $\frac{1}{2} (\frac{M}{12} + m_1 + m_2) \omega^2 \ell^2$

### Questions 7-10

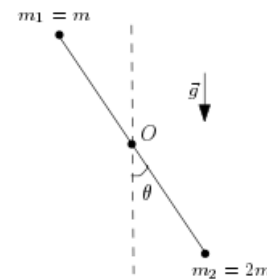
A disk of mass  $M$  and radius  $R$  is located on a frictionless table and pivoted at its center, and initially at rest. A point mass of  $m$  with an initial speed  $v_0$  hits and scatters from the disk as shown in the figure. (For a disk of mass  $M$  and radius  $R$ ,  $I_{cm} = \frac{1}{2}MR^2$ .)



- What are the conserved quantities in this collision?  
(a)  $\vec{L}$  relative to the center of mass of the disk (b)  $\vec{p}$  and  $\vec{L}$  relative to the every point in space (c)  $\vec{L}$  relative to the point of collision (d)  $\vec{p}$  and  $\vec{L}$  relative to the point of collision (e)  $\vec{p}$
- What is the magnitude of the angular speed of the disk just after the collision?  
(a)  $\frac{3mv_0}{4MR}$  (b)  $\frac{mv_0}{4MR}$  (c)  $\frac{mv_0}{2MR}$  (d)  $\frac{3mv_0}{5MR}$  (e)  $\frac{2mv_0}{5MR}$
- What is the impulse transferred to the mass  $m$  during the collision?  
(a)  $-mv_0\hat{j}/2$  (b)  $mv_0\hat{i}$  (c)  $mv_0(\hat{i} + \hat{j}/2)$  (d)  $-2mv_0(2\hat{i} - \hat{j})$  (e)  $-mv_0(\hat{i} - \sqrt{3}\hat{j}/4)$
- If the disk were not pivoted at the beginning, what would be the center of mass velocity,  $v_{cm}$ , of the disk just after the collision?  
(a)  $\frac{mv_0}{M}(\hat{i} - \hat{j})$  (b)  $\frac{mv_0}{2M}(\hat{i} - \hat{j})$  (c)  $\frac{mv_0}{3M}(2\hat{i} - \hat{j})$  (d)  $\frac{mv_0}{M}(\hat{i} - \sqrt{3}\hat{j}/4)$  (e)  $\frac{2mv_0}{M}(\hat{i} - \hat{j})$

## Questions 11-13

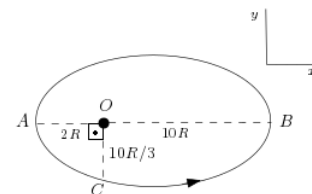
A uniform rod of mass  $M = 3m$  and length  $L$  is pinned to a wall at its center of mass  $O$  in the vertical plane. It is free to be able to rotate about this point. Two point masses  $m_1 = m$  and  $m_2 = 2m$  are attached to the ends of the rod, as shown in the figure.



11. What is the rotational inertia of the system about the point  $O$ ?  
 (a)  $2mL^2/3$  (b)  $2mL^2/5$  (c)  $3mL^2/2$  (d)  $mL^2$  (e)  $3mL^2/4$
12. Which of the following is the period of the system for small oscillations?  
 (a)  $2\pi\sqrt{\frac{3L}{g}}$  (b)  $2\pi\sqrt{\frac{2L}{g}}$  (c)  $2\pi\sqrt{\frac{2L}{3g}}$  (d)  $2\pi\sqrt{\frac{3L}{2g}}$  (e)  $2\pi\sqrt{\frac{3L}{4g}}$
13. If the system starts oscillation from an initial angle  $\theta_{max}$ , which of the following is the required time to reach  $\theta_{max}/2$ , in terms of the period  $T$  of the small oscillations?  
 (a)  $T/8$  (b)  $T/5$  (c)  $T/12$  (d)  $T/10$  (e)  $T/6$

## Questions 14-18

A planet of mass  $m$  is moving on an elliptical path about a star of mass  $M$  ( $m \ll M$ ) at point  $O$ , as shown in the figure. The point  $A$  is the closest and the point  $B$  is the farthest point of the planet from the star. The distance between the planet and the star is  $2R$  when the planet is at point  $A$ , and  $10R$  when it is at point  $B$ .



14. What is the length of the semimajor axis (half of long axis of the ellipse) of the elliptical orbit?  
 (a)  $9R$  (b)  $8R$  (c)  $12R$  (d)  $6R$  (e)  $10R$
15. What is the total mechanical energy of the system?  
 (a)  $-\frac{GMm}{8R}$  (b)  $-\frac{GMm}{6R}$  (c)  $-\frac{GMm}{10R}$  (d)  $-\frac{GMm}{9R}$  (e)  $-\frac{GMm}{12R}$
16. What is the speed of the planet at point  $C$ ?  
 (a)  $\sqrt{\frac{3GM}{4R}}$  (b)  $\sqrt{\frac{5GM}{21R}}$  (c)  $\sqrt{\frac{7GM}{9R}}$  (d)  $\sqrt{\frac{14GM}{27R}}$  (e)  $\sqrt{\frac{13GM}{30R}}$
17. What is the acceleration vector of the planet at point  $C$ ?  
 (a)  $\frac{3GM}{10R^2}\hat{i}$  (b)  $\frac{9GM}{10R^2}\hat{i}$  (c)  $-\frac{3GM}{10R^2}\hat{j}$  (d)  $\frac{7GM}{100R^2}\hat{j}$  (e)  $\frac{9GM}{100R^2}\hat{j}$
18. What is the time to reach from  $A$  to  $B$  on this elliptical orbit?  
 (a)  $8\pi\sqrt{\frac{4R^3}{GM}}$  (b)  $6\pi\sqrt{\frac{3R^3}{GM}}$  (c)  $8\pi\sqrt{\frac{6R^3}{5GM}}$  (d)  $12\pi\sqrt{\frac{6R^3}{5GM}}$  (e)  $6\pi\sqrt{\frac{6R^3}{GM}}$

## Questions 19-20

A spring-mass system is composed of a mass  $m = 200\text{ g}$  and a massless spring of force constant  $k$  obeying Hooke's Law, and the whole system is located on a horizontal frictionless table. The mass  $m$  makes oscillations about the equilibrium position  $x = 0$  according to the relation  $x(t) = (15\text{ cm})\sin 2\pi t$ . (You can take  $\pi = 3$ .)

19. What is the force constant  $k$  of the spring?  
 (a)  $36/5\text{ N/m}$  (b)  $36\text{ N/m}$  (c)  $54\text{ N/m}$  (d)  $72/5\text{ N/m}$  (e)  $54/4\text{ N/m}$
20. What is the total mechanical energy of the system?  
 (a)  $9/50\text{ J}$  (b)  $81/1000\text{ J}$  (c)  $8/25\text{ J}$  (d)  $81/130\text{ J}$  (e)  $2/25\text{ J}$