BME4120
Biomedical Image Processing

Lecture 6

Frequency domain filters

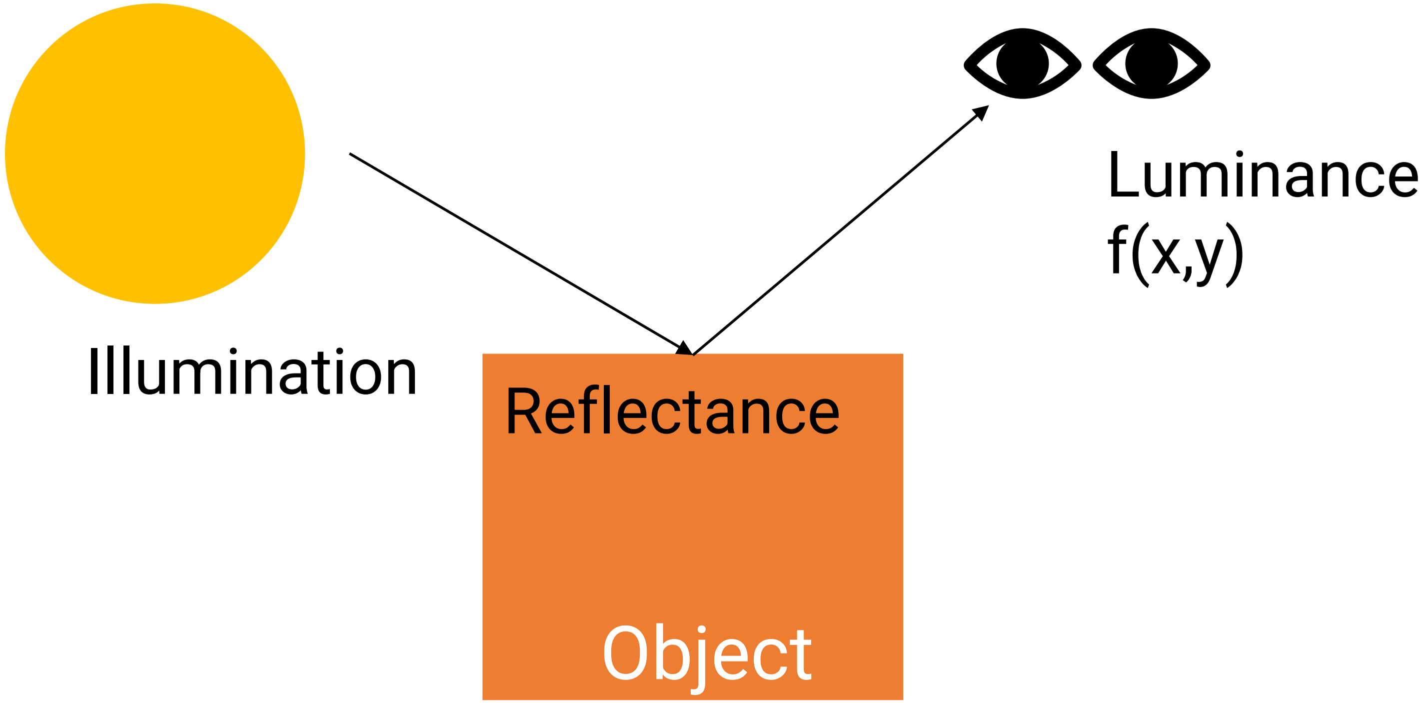
Frequency domain filters are ***mainly*** divided into four different types

- ❑Smoothing domain filters or low-pass filters
- ❑Sharpening domain filters or high-pass filters
- ❑**Homomorphic filters**
- ❑Color image enhancement methods or techniques

Illumination-Reflectance Model

An image can be modelled as the product of an illumination function and the reflectance at every point

$$f(x, y) = i(x, y)r(x, y)$$



Illumination-Reflectance Model

An image can be modelled as the product of an illumination function and the reflectance at every point

$$f(x,y) = i(x,y)r(x,y)$$

This model can be used to improve the quality of an image that was taken under poor illumination conditions (i.e., non-uniform distribution of intensity values or grayscale values in the image).

Illumination-Reflectance Model

The illumination is the primary contributor to the dynamic range, and it varies slowly in space.

Reflectance component represents the details of the object edges and varies rapidly in space.

→ Can associate ***low frequencies*** of FT of an image with ***illumination*** and ***high frequencies*** with ***reflectance***.

Illumination-Reflectance Model

Basic idea behind homomorphic filter: ***Separate*** illumination and reflectance and apply ***two different transfer functions*** for image enhancement.

Illumination-Reflectance Model

An image can be modelled as the product of an illumination function and the reflectance at every point

$$f(x, y) = i(x, y)r(x, y)$$

Problem: FT of a product of two functions is not separable:

$$\mathcal{F}[f(x, y)] \neq \mathcal{F}[i(x, y)]\mathcal{F}[r(x, y)]$$

Solution:

$$\begin{aligned} z(x, y) &= \ln f(x, y) \\ &= \ln i(x, y) + \ln r(x, y) \end{aligned}$$

$$\mathcal{F}[z(x,y)] = \underbrace{\mathcal{F}[\ln f(x,y)]}_{\text{FT}} = \underbrace{\mathcal{F}[\ln i(x,y)]}_{\text{FT}} + \underbrace{\mathcal{F}[\ln r(x,y)]}_{\text{FT}}$$

$$Z(u, v) = F_i(u, v) + F_r(u, v)$$

Now the filter

We can filter Z(u,v) using a filter transfer function H(u,v):
(i.e., we must design filters separately for the illumination and reflectance components)

$$S(u,v) = H(u,v)Z(u,v)$$

FT of the result

Apply Inverse FT to the filtered image

In spatial domain, apply the inverse FT to the filtered image:

$$
\begin{aligned}
s(x,y) &= \mathcal{F}^{-1}[S(u,v)] \\
&= \mathcal{F}^{-1}[H(u,v)F_i(u,v) + H(u,v)F_r(u,v)] \\
&= \mathcal{F}^{-1}[H(u,v)F_i(u,v)] + \mathcal{F}^{-1}[H(u,v)F_r(u,v)] \\
&= i'(x,y) + r'(x,y)
\end{aligned}
$$

Enhanced Image

Lastly, because z(x, y) was formed by taking the natural logarithm of the input image, we reverse the process by taking the exponential (anti-log) of the filtered result to form the output image:

$$
\begin{aligned}
g(x, y) &= e^{s(x, y)} \\
&= e^{i'(x, y)} e^{r'(x, y)} \\
&= i_0(x, y)\, r_0(x, y)
\end{aligned}
$$

Illumination component of the output image

Reflectance component of the output image

Homomorphic Filtering

$$f(x,y) \rightarrow \boxed{\ln} \rightarrow \boxed{\mathrm{DFT}} \rightarrow \boxed{H(u,v)} \rightarrow \boxed{(\mathrm{DFT})^{-1}} \rightarrow \boxed{\exp} \rightarrow g(x,y)$$

Homomorphic Filtering Algorithm

The algorithm for homomorphic filtering is developed by considering the following factors:

Problem: If the illumination radiating to an object is non-uniform.
Objective: Simultaneously compress the gray level range and enhance the contrast, and the effect of non-uniform illumination is eliminated.
Methods: The illumination component of an image is characterized by slow spatial variations, while the reflectance components vary abruptly at the junctions of dissimilar objects. These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination and the high frequencies with reflectance.

Homomorphic Filtering Algorithm

Algorithm: Homomorphic filtering approach (HFA)
Input: Original image f(x, y)
Output: Enhanced image g(x, y)

Method:

begin
1. Acquire the input image using a scanner or digital camera and convert it into grayscale image f(x, y).
2. Resize and reformat the input image, if required.
3. Obtain z(x, y) as a logarithm of f(x, y).
4. Apply Fourier transform to z(x, y) and obtain Z(u, v).
5. Determine H(u, v), which must compress the dynamic range of i(x, y), and enhance the contrast of r(x, y) component.
6. Apply convolution between H(u, v) and Z(u, v) and obtain S(u, v).
7. Apply inverse Fourier transform of S(u,v) to obtain s(x,y)
8. Compute the inverse logarithm of s(x, y).
9. Obtain g(x, y) as an enhanced image.

end

Homomorphic Filtering Algorithm

- Can achieve a very good control over the illumination and reflectance components of an image using a homomorphic filter.

- This may require a filter function H(u, v) to be specified, affecting the low and high-frequency components of the Fourier transform.

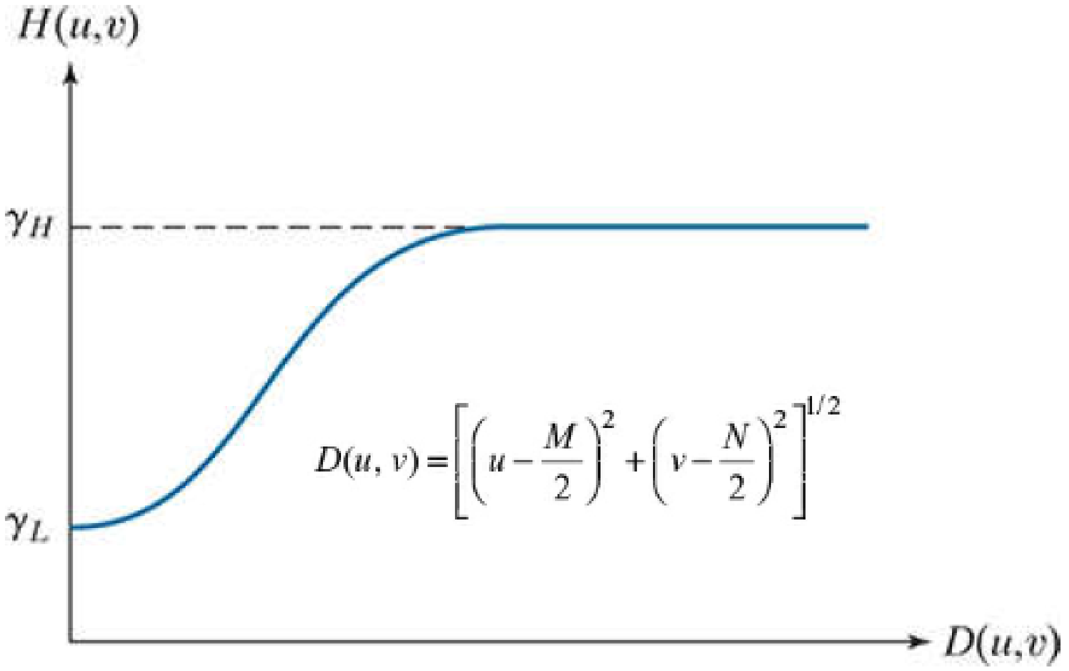


FIGURE 4.59

Radial cross section of a homomorphic filter transfer function.

Image: Gonzalez & Woods, Digital Image Processing (2018)

Homomorphic Filtering: Transfer Function

- For $\gamma_L < 1$ and $\gamma_H \geq 1$; H(u,v) will attenuate the contribution made by the low frequencies (illumination) and amplify the contribution made by high frequencies (reflectance).
- The net result is simultaneous dynamic range compression and contrast enhancement

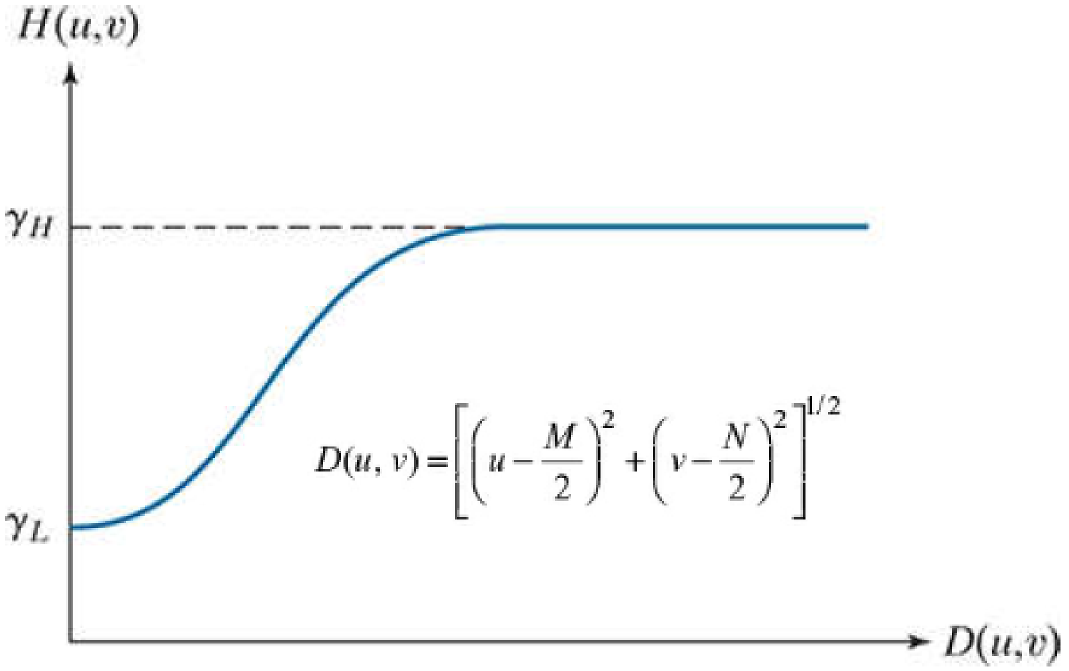


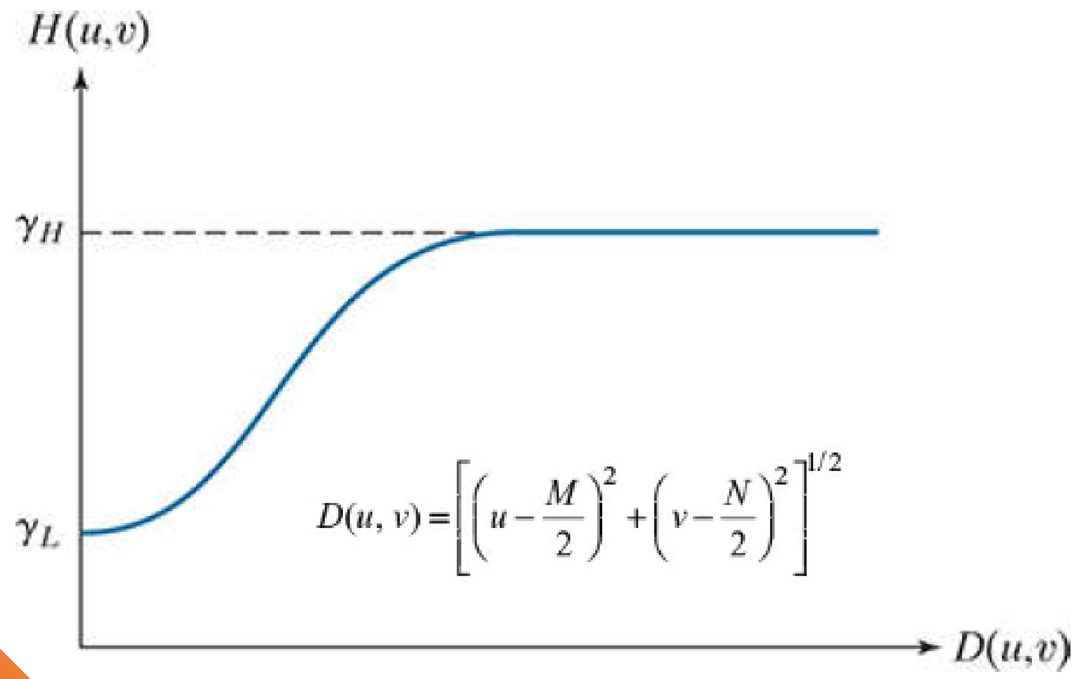
FIGURE 4.59

Radial cross section of a homomorphic filter transfer function.

Image: Gonzalez & Woods, Digital Image Processing (2018)

Homomorphic Filtering Transfer Function: Approximation

- The shape of the H(u,v) can be approximated using a highpass filter transfer function. E.g., using a slightly modified form of the GHPF function yields the homomorphic function



$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-cD^2(u,v/D_0^2)}\right] + \gamma_L$$

constant c controls the sharpness of the slope of the function

Image: Gonzalez & Woods, Digital Image Processing (2018)

Homomorphic Filtering: Example

Original image (a) is slightly blurred and many of its low-intensity features are obscured by the high intensity of the "hot spots" dominating the dynamic range of the display.

Enhanced image (b) has a much sharper slope and the transition between low and high frequencies much closer to the origin.

Full body PET scan of size 1162x746 pixels

Image: Gonzalez & Woods, Digital Image Processing (2018)

Homomorphic Filtering: Example

The effects of the dominant illumination components (the hot spots) were reduced.

Lower intensities to become more visible → dynamic range expanded

High frequencies are enhanced, and the reflectance components of the image (edge information) were sharpened.

Full body PET scan of size 1162x746 pixels

Image: Gonzalez & Woods, Digital Image Processing (2018)

Feature Extraction and Statistical Measurement

- ❑Selection of Features
- ❑Shape Related Features
- ❑Fourier Descriptors
- ❑Texture Features
- ❑Example: Breast Tissue Detection
- ❑Analysis of Tissue Structure

Feature Extraction and Statistical Methods

Enhanced images can be evaluated visually.
→ Visual inspection, however, is not enough for comparing the images with numerous changes.

Some features or parameters must be extracted from the images to compare them statistically.
→ E.g., it is very difficult to understand and interpret the mammography images due to small differences in the image intensity and densities of different tissues present in the images.

Feature extraction of statistical parameters is a very important process for the overall system performance in the pattern recognition and other image processing applications.

Feature Extraction and Statistical Methods

Statistical parameters provide a basic knowledge of using probabilistic and statistical methods for image analysis.

Various spatial statistical models are used to describe the patterns and their correlation, shape and structure of objects.

Many systems use some features for detecting and classifying the abnormality as benign or malignant from the texture, statistical properties in spatial domain and wavelet-based fractal domain.

Selection of Features

Qualitative and quantitative features:
→They allow evaluating the image.

Some of the significant qualitative and quantitative parameters as well as some image features that can be used in the evaluation of performance of results of CAD systems.

- ❑ Signal-to-Noise Ratio (SNR)
- ❑ Peak Signal-to-Noise Ratio (PSNR)
- ❑ Mean Square Error (MSE)
- ❑ Root Mean Square Error (RMSE)
- ❑ Mean Absolute Error (MAE)
- ❑ Entropy

Signal-to-Noise Ratio (SNR)

Defined as the ratio of the signal power to the noise power.

The effect of noise can be quantified → quantitative assessment

Also gives quality assessment → since it is known that how much noise or degradation is mixed in the original image.

It can be expressed in decibel (dB).

A ratio higher than 1:1 indicates more signal than the noise.

The greater the S/N ratio, better is the signal strength or smaller is the noise level.

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}$$

$$(\mathrm{SNR})_{\mathrm{db}} = 10\log_{10}\left(\frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}\right)$$

Peak Signal-to-Noise Ratio (PSNR)

Defined as the ratio of the reference signal to the distortion signal in an image, given in decibels.

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{R^2}{\mathrm{MSE}}\right)$$

Used mostly for the comparison of image compression quality between the original image and the compressed image.

R is the maximum input image data type. For an eight-bit unsigned data type (grayscale image), R is 255; for double-precision floating-point data type, R is 1.

The greater the value of PSNR, better is the quality of the compressed or reconstructed image.

Mean Square Error (MSE)

It is calculated as a cumulative squared difference between the two images, that are generally an original image and a distorted image.

The computation of MSE is performed on pixel-by-pixel basis by summing up the squared differences of all the pixels and dividing by the total pixel count.

$$\mathrm{MSE}(P, Q) = \frac{1}{M} \sum_{i=1}^{M} (p_i - q_i)^2$$

(P and Q are the two images between which MSE is to be calculated. M is the number of pixels.)

MSE measures the average of the square of the error which is the amount by which the value differs.

It is an error metric that is used to compare the image compression quality.

Root Mean Square Error (RMSE)

A.k.a. root-mean-square deviation (RMSD).

It is used to measure of the difference between the predicted values and the actual values which are observed. This parameter is a good measure of accuracy.

$$\mathrm{RMSE} = \sqrt{\mathrm{MSE}}$$

Mean Absolute Error (MAE)

Defined as the average value of absolute errors which is calculated over the sample.

MAE is a linear score, measured as a mean value of all the individual differences which are weighted equally in the average

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n} | f_i - y_i |$$

f_i is the predicted value,
y_i is the true value
n is number of samples

Entropy

Entropy measures the amount of disorder or unpredictability of uncertain events.

This is defined as a measure of the uncertainty associated with a random variable and calculated as an expected outcome value.

Entropy is used to quantify the minimum descriptive complexity of a random variable

$$E(I) = -\sum_{k=0}^{L-1} p(k)\, log_2\left(p(k)\right)$$

(L= 256 for grey scale)

(p(k) normalized histogram counts)

Entropy serves as a measure of 'disorder'. As the entropy increases the events become less predictable.

Shape Related Features

The shape is an important visual feature that helps describe an image.

Mainly two types of shape: boundary-based shapes and region-based shapes

Include outer border line of a particular shape

Region-based shape representations may be of numerous types.

Properties of shape-based features

Some properties of shape-based features in an image:

- The features must not be affected with the change in location, rotation, translation and scaling. This property is called translation invariance.
- Affine transformations should be derived with the translation properties such as scaling, reflection, rotation and shear. The features extracted from the images remain invariant with the affine transforms.
- The features should remain similar with the same patterns in an image.
- Two different extracted features must be statistically independent of each other.
- Features must be reliable, identifiable and stable.
- The selected features should be robust against any variation due to noise present in the images, since some amount of noise is always there in the images.
- The shapes which are perceived similar should have same features.

A list of shape-based parameters

- ❑ Distance
- ❑ Perimeter
- ❑ Convex Perimeter
- ❑ Major and Minor Axes
- ❑ Center of Gravity
- ❑ Axis of Inertia
- ❑ Aspect Ratio
- ❑ Eccentricity
- ❑ Circularity Ratio
- ❑ Rectangularity
- ❑ Convexity
- ❑ Solidity

Distance

1. Euclidean distance can be measured as

$$D_E = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

2. Magnitude distance is measured as

$$d_M = |x_1 - x_2| + |y_1 - y_2|$$

3. Maximum value distance can be calculated as

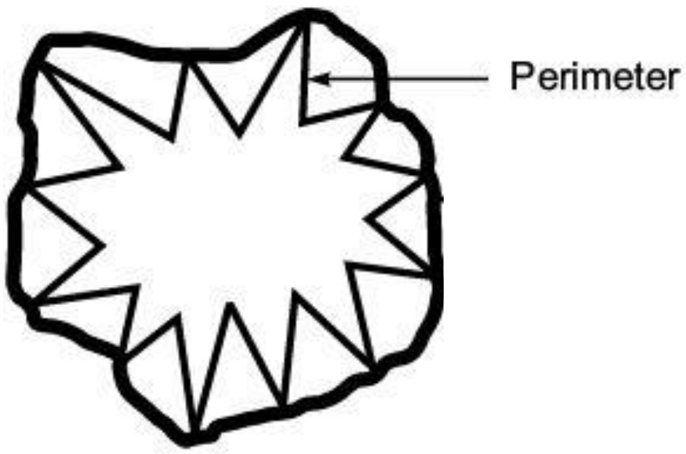
$$d_{\max} = \mathrm{MAX}\{|x_1 - x_2|, |y_1 - y_2|\}$$

Perimeter

The perimeter of a shape of an object or image is calculated as the number of pixel sides traversed around the boundary of that object starting at an arbitrary initial boundary pixel and returning to the initial pixel

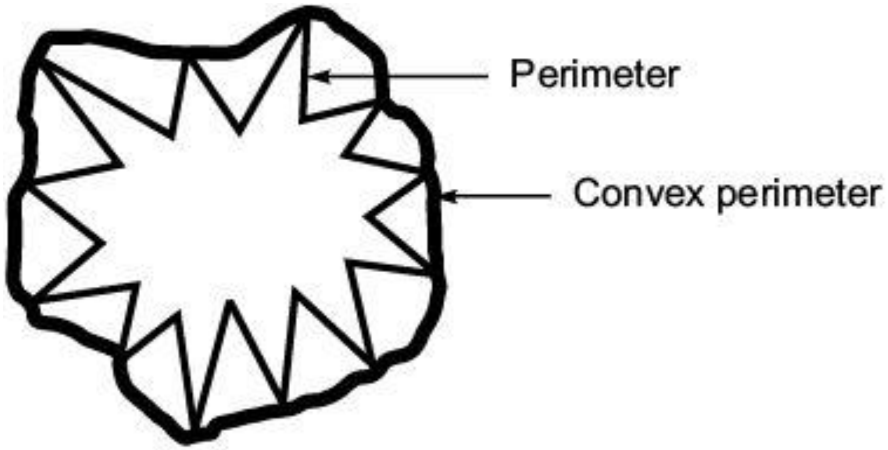
$$B_p = \sum_{i=1}^{N} d_i = \sum_{i=1}^{N-1} |x_i - x_{i+1}|$$

d_i is the pixel counts



Convex Perimeter

A convex is a set of points in some segments connecting each pair of its points.



Major and Minor Axes

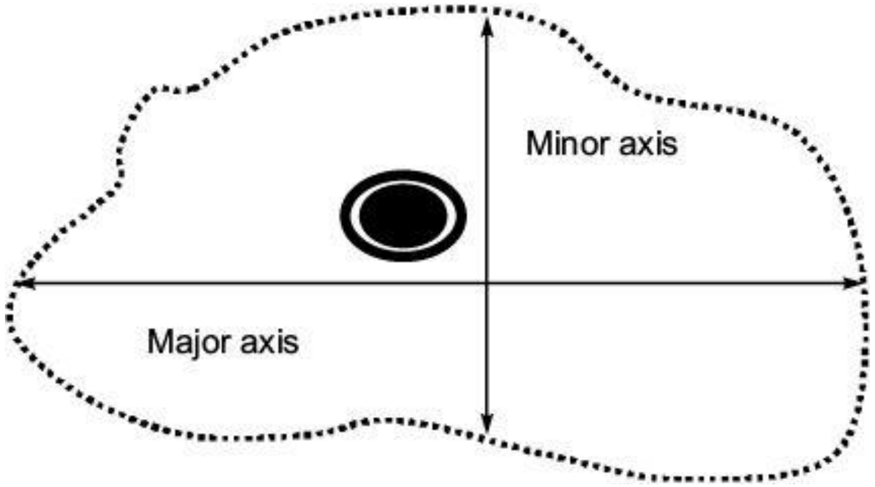
They provide information of an object such as elongation, eccentricity, elliptic variance, etc.

The major axis is made by connecting two points of an object where the object is more elongated and where the straight line drawn between these two points is the longest.

The major axis points are calculated by all possible combinations of perimeter pixels where the line is the longest.

$$\text{Major axis length} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Minor axis is drawn perpendicular to the major axis where this line has the maximum length. (after that, same formula)



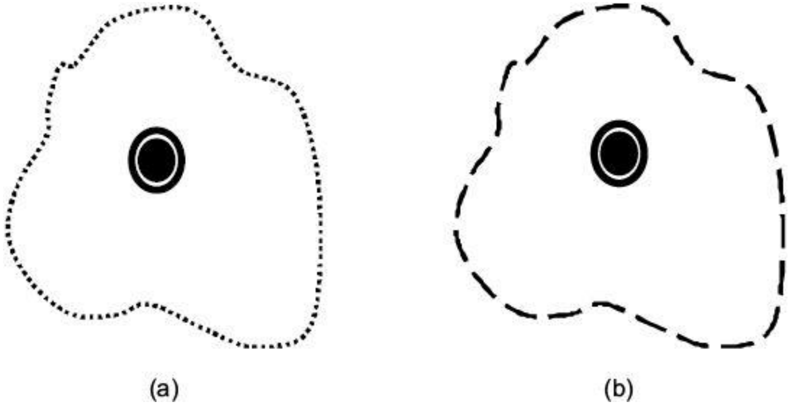
Center of Gravity

Defined as the point through which the resultant of force of gravity of a body acts.

The central points corresponding to volumes, surfaces and line segments are called centroids.

$$g_x = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$g_y = \frac{1}{N}\sum_{i=1}^{N} y_i$$



(a) Under uniform distribution
(b) Under non-uniform distribution

Axes of Inertia

Describes a single direction of line to maintain the orientation of a shape.

This axis is defined for the shape boundary which passes through the centroid of a shape.

$$\alpha = \frac{1}{2}\arctan\left(\frac{b}{a-c}\right), \qquad -\frac{\pi}{2} < a < \frac{\pi}{2}$$

$$\alpha = \sum_{i=0}^{N-1} x_i^2 \; b = 2\sum_{i=0}^{N-1} x_i y_i \text{ and } c = \sum_{i=0}^{N-1} y_i^2$$

Aspect Ratio

Aspect ratio is the ratio of the height to the width of a corresponding object or an image.

This ratio helps is distinguishing circular and square objects.

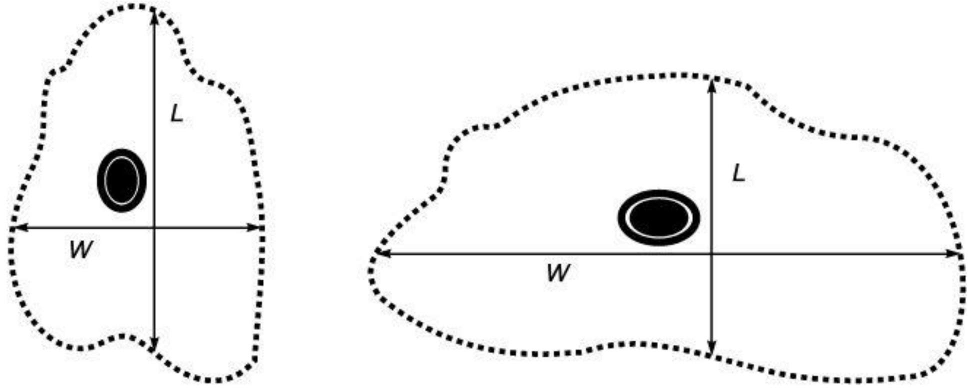
$$\text{Aspect ratio} = \frac{\text{Height}}{\text{Width}}$$

Eccentricity

Eccentricity is the ratio measured as the end-to-end distance of the major axis to that of the minor axis which can be calculated by the minimum bounding rectangle method.

$$E = 1 - \frac{\text{Width}}{\text{Length}}$$

(a) High value of eccentricity
(b) Low value of eccentricity



(a) (b)

E is in the range of (0, 1).

For a circle or square E = 0.
For shapes with large aspect ratios E ~ 1.

If the angle between the eccentricity line and the axis is less than 45°, then the shape will be ellipse.

For more than 45°, it will be hyperbola and the shape will be parabola for exact 45° of angle value.

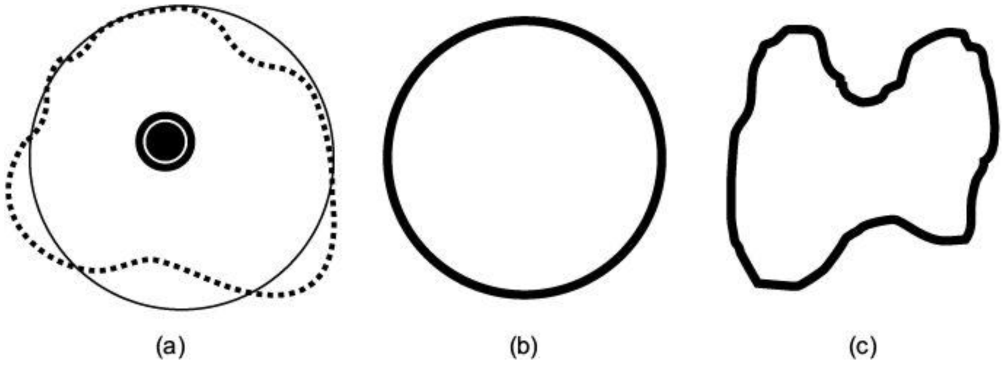
Circularity Ratio

It is used to measure compactness of a shape. It is calculated for any two dimensional shape with a given set of its area and perimeter values.

Circularity ratio represents the analogy of a shape to a circle.

$$C = \frac{A_s}{A_c}$$

A_s is the area of a shape and A_c is the area of a circle having the same perimeter as the given shape



(a) Circle variance; (b) High roundness resulting circle and (c) Lower roundness with varying shape than circle.

Rectangularity

Defined as the ratio of the area of a region to the area of its minimum bounding rectangle.

A minimum bounding rectangle must contain all the points.

Rectangularity indicates the normalized discrepancies between the areas of the rectangle and the region.

$$\text{Rectangularity} = \frac{A_s}{A_r}$$

A_s is the area of a shape and A_r is the area of the minimum bounding rectangle

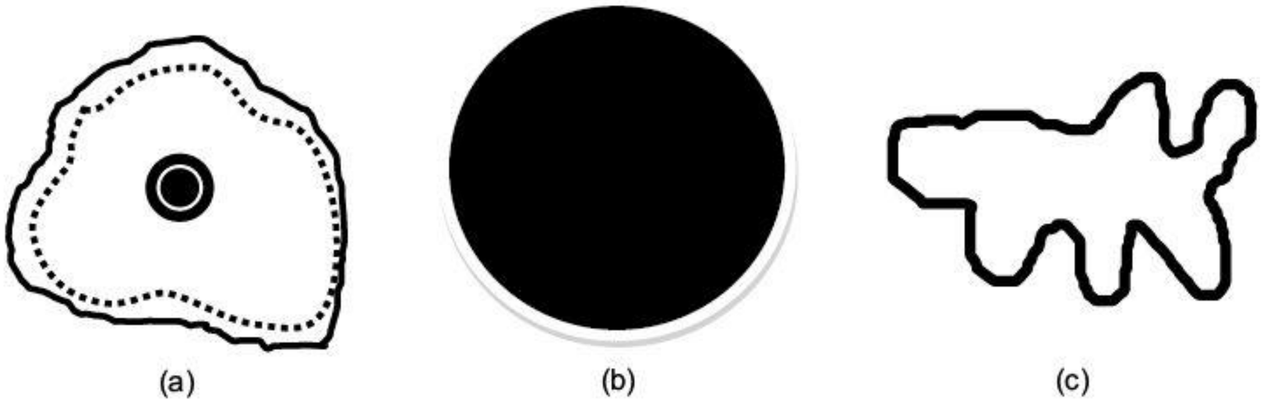
Convexity

Convexity of an object or image is the ratio of perimeter of the convex object to the original perimeter of the object.

This parameter is used to measure the difference in objects or images from the corresponding convex object

$$\text{Convexity} = \frac{\text{Convex perimeter}}{\text{Perimeter}}$$

(a) Convex region; (b) High convexity region and (c) Low convexity region.



Solidity

Defined as the ratio of the total area to the convex area of the same object which describes the extent to which the shape is convex or concave.

$$\text{Solidity} = \frac{\text{Total area}}{\text{Convex area}}$$

Solidity of a convex shape is always one and it is lower for an object with rough perimeter or having holes in it.

Shape Representation

An efficient object representation or shape representation can be achieved by using boundary-based shape descriptors.

Shape signatures are a set of one-dimensional functions that form these descriptors.

The degree of similarity between the objects or images helps to identify objects. The descriptors capture the perceptual features of the shape such as complex coordinates, centroid distance function, tangent angle or turning angles, curvature function, area function, triangle representation and chord length function, etc.

Next lecture we will discuss these.