BME 1132
Probability and Biostatistics

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Week-7

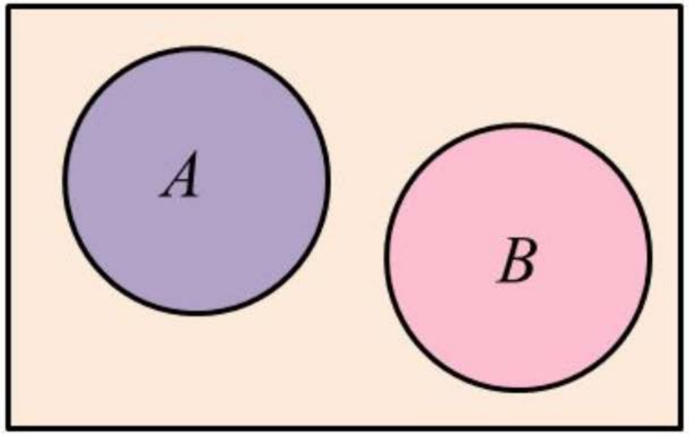
- Mutually Exclusive & Collectively Exhaustive
- Sample Space & Events Examples
- Introduction to Discrete and Continuous Probability
- Axioms of Probability
- Probability Calculation
- Summary

Sample Space

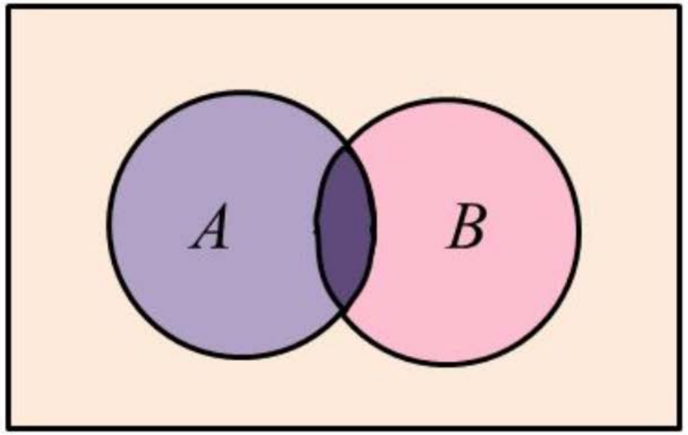
Set of possible outcome, **S** or **Ω**

Set must be;

- Mutually exclusive
- Collectively exhaustive
- At the right granularity



A and *B* are mutually exclusive



A and *B* are not mutually exclusive

Mutually Exclusive Events

Two events A and B are **mutually exclusive** if they cannot both happen at the same time.

Hypertension Let A be the event that a person has normotensive diastolic blood pressure (DBP) readings (DBP < 90), and let B be the event that a person has borderline DBP readings ($90 \leq$ DBP < 95). Suppose that $Pr(A) = 0.7$, and $Pr(B) = 0.1$.
Let Z be the event that a person has a DBP < 95. Then

$$Pr(Z) = Pr(A) + Pr(B) = 0.8$$

because the events A and B cannot occur at the same time.

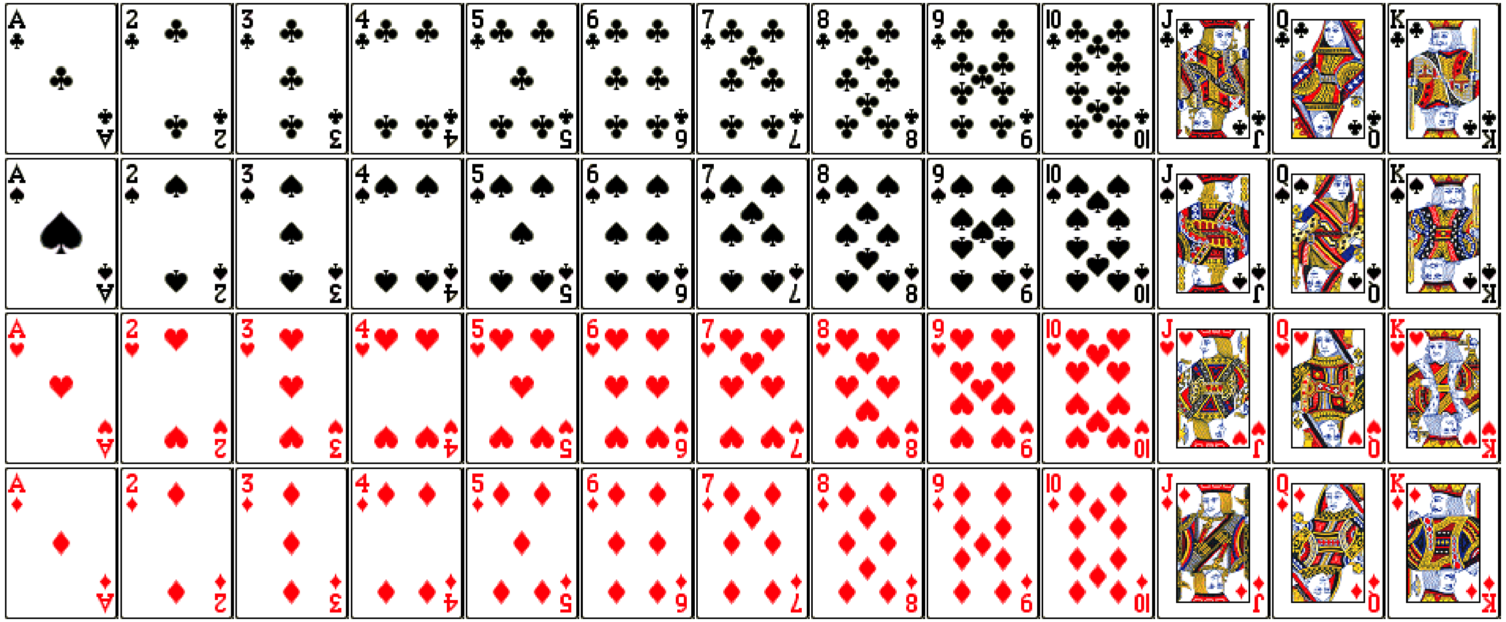
NOTE:
Two events, denoted as A and B, such that

$$A \cap B = \emptyset$$

are said to be **mutually exclusive**.

Not Mutually Exclusive Events

What is the probability that a card chosen at random from a standard deck of cards will be either a king or a heart?



Mutually Exclusive VS Independent

If two events A and B are independent a real-life example is the following. Consider a fair coin and a fair six-sided die. Let event A be obtaining heads, and event B be rolling a 6. Then we can reasonably assume that events A and B are independent, because the outcome of one does not affect the outcome of the other. The probability that both A and B occur is

P(A and B) = P(A)P(B) = (1/2)(1/6) = 1/12.

An example of a mutually exclusive event is the following. Consider a fair six-sided die as before, only in addition to the numbers 1 through 6 on each face, we have the property that the even-numbered faces are colored red, and the odd-numbered faces are colored green. Let event A be rolling a green face, and event B be rolling a 6. Then

P(B) = 1/6

P(A) = 1/2

as in our previous example. But it is obvious that events A and B cannot simultaneously occur, since rolling a 6 means the face is red, and rolling a green face means the number showing is odd. Therefore

P(A and B) = 0.

http://mathforum.org/library/drmath/view/69825.html

Sample Space: Discrete Example

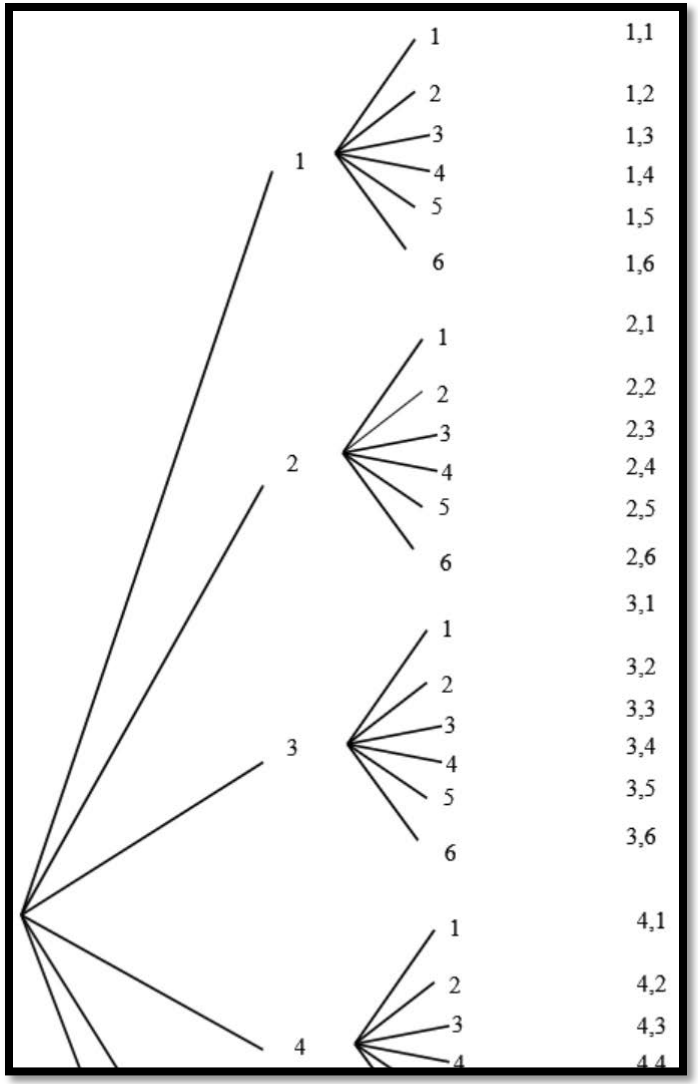
Roll two dice

$Y = second\ roll$

6						
5						
4						
3						
2						
1						
	1	2	3	4	5	6

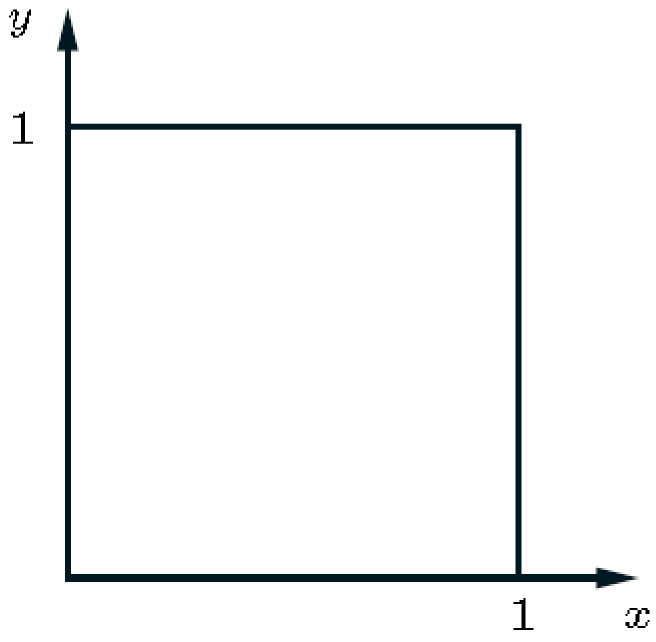
$X = first\ roll$

Tree diagram or
Sequential description



Sample Space: Continuous Example

(x, y) $such\ that = 0 \leq x, y \leq 1$



Axioms & Consequences

Axioms

Normalization: $P(\Omega) = 1$

Nonnegativity: $P(A) \geq 0$

Additivity: If $A \cap B = \emptyset, then\ P(A \cup B) = P(A) + P(B)$

Consequences

$\mathrm{P}(A) \leq 1$

$\mathrm{P}(A) + \mathrm{P}(A^c) = 1$

$\mathrm{P}(\emptyset) = 0$

$\mathrm{P}(A \cup B) \leq \mathrm{P}(A) + \mathrm{P}(B)$

$\mathrm{P}(A \cup B \cup C) = \mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C)$

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

If $A \subset B$, then $\mathrm{P}(A) \leq \mathrm{P}(B)$

$\mathrm{P}(A \cup B \cup C) = \mathrm{P}(A) + \mathrm{P}(A^c \cap B) + \mathrm{P}(A^c \cap B^c \cap C)$

Probability Calculation: Discrete Example

Roll two dice

$Y = second\ roll$

6						
5						
4						
3						
2						
1						
	1	2	3	4	5	6

$X = first\ roll$

$P(Y = 1) = ?$

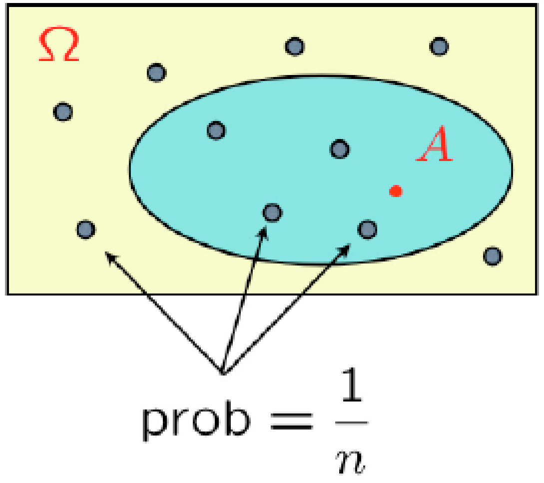
$Let\ Z = \min(X, Y)$

$P(Z = 6) = ?$

$P(Z = 4) = ?$

Note: *These are fair dice, so each outcome have probability* $1/36$.

Discrete Uniform Law

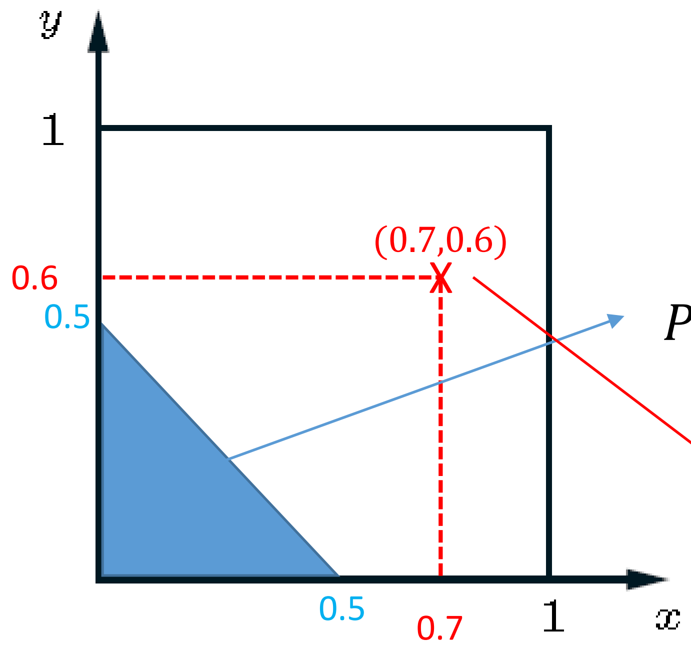


Assume Ω consists of n equally likely elements,
Assume A consists of k elements

$$P(A) = k\,\frac{1}{n}$$

Probability Calculation: Continuous Example

$(x, y)\ such\ that = 0 \leq x, y \leq 1$

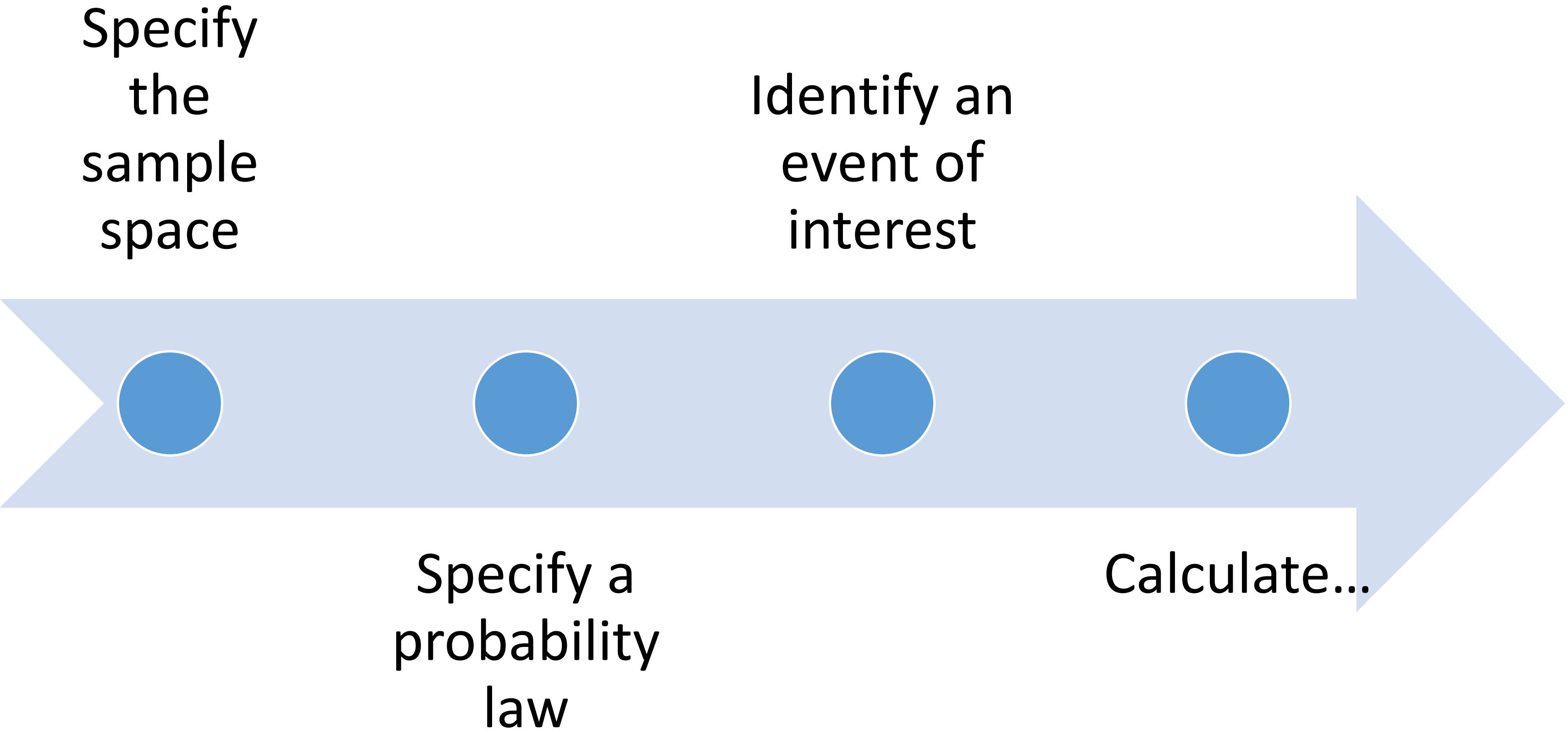


$$P(\{(x,y) | x + y \leq 1/2\}) = \frac{0.5.0.5}{2} = \frac{1}{8}$$

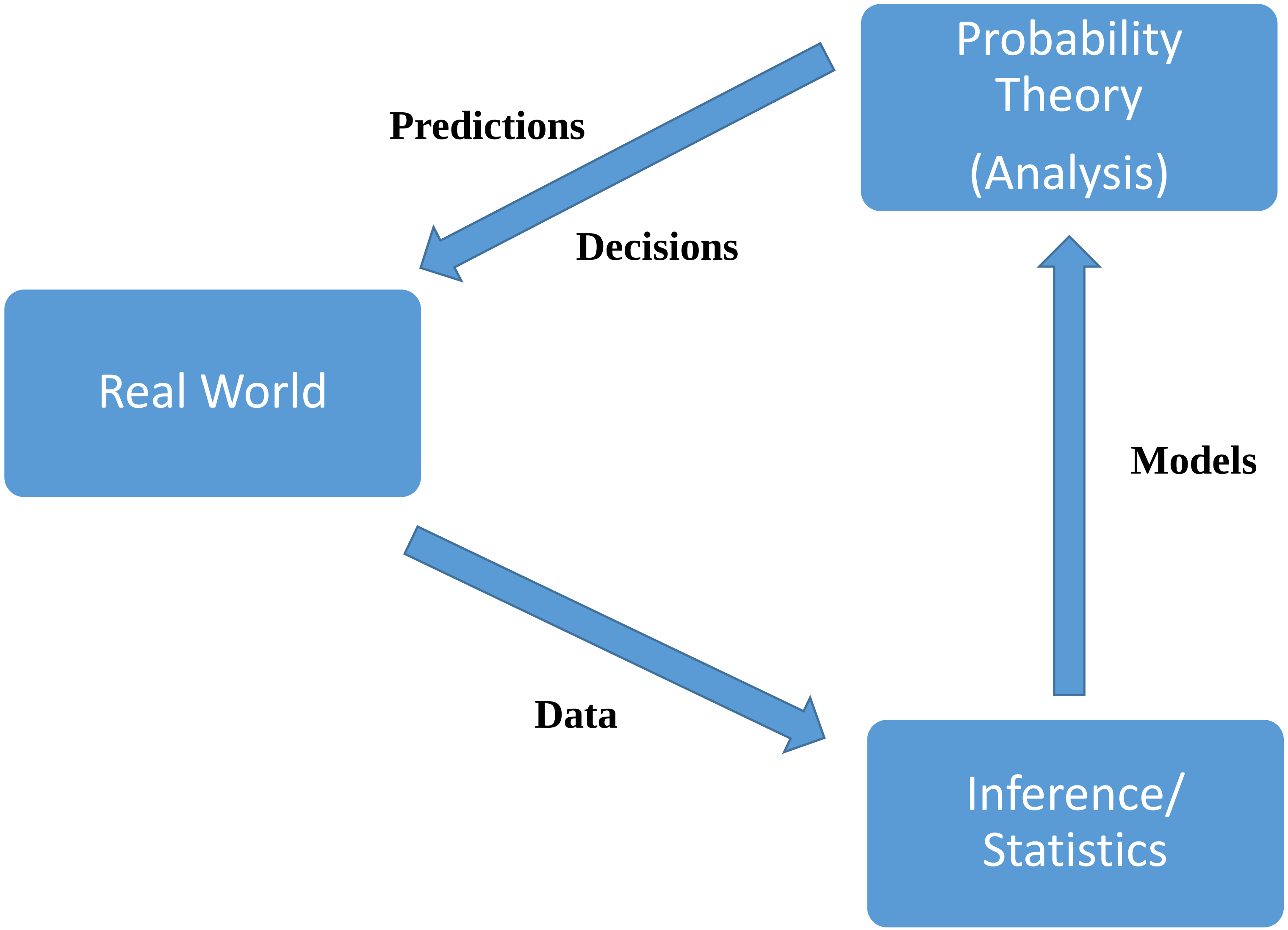
$$P(\{0.7, 0.6\}) = 0$$

<u>Uniform probability law:</u> $Probability = Area$

Probability Calculation Steps



Questions?


Probability Theory
(Analysis)
Predictions
Decisions
Real World
Models
Data
Inference/ Statistics