03.05.2021

3 Mayıs 2021 Pazartesi 09:06

START : 9 15

LINEAR SYSTEMS

$$\begin{aligned} x+3y+2z&=20 \\ x+2y-3z&=7 \\ 2x+5y&=16 \end{aligned}$$

$$\cancel{\begin{aligned} x^2+y&=10 \\ 2x-6y&=9 \end{aligned}}$$

NON-LINEAR S.

$$\cancel{\begin{aligned} x\cdot y&=4 \\ x+4y&=7 \end{aligned}}$$

NON-LINEAR

4 METHOD

1- Gauss Elimination Method.
2- Gauss-Jordan Elimination method.

Linear Algebra

3- Jacobi Iteration method
4- Gauss-Seidel Iteration method.

GAUSS ELIMINATION METHOD.

upper trianguler matrix

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

Ex

$$\begin{aligned} x+y+z&=6 \\ 2x+y-z&=1 \\ -x+2y+2z&=9 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -1 & 1 \\ -1 & 2 & 2 & 9 \end{array}\right] \xrightarrow[R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & -11 \\ 0 & 3 & 3 & 15 \end{array}\right]$$

Step 1

Step 2

make 0

0

$$3R_2+R_3 \to R_3$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & -11 \\ 0 & 0 & -6 & -18 \end{array}\right] \end{array}$$

$0.x + 0.y + (-6).z = -18$ $\boxed{z = 3}$

$0.x + (-1).y + (-3)z = -11$ $y = 2$

$x + y + z = 6$

$x + 2 + 3 = 6$ $\boxed{x = 1}$

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*. Gauss Elimination Method with Trivial Pivoting

If the diagonals are zero in the matrix system, it is the process of recovering the diagonals from zero.

Ex

$$\begin{aligned} y + z &= 2 \\ +y - z &= 1 \\ 2x - y - z &= 0 \end{aligned}$$

Apply the Gaussian Elimination method with trivial pivoting to solve the follow system of equations.

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 0 & 1 & 1 & 2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & -1 & 0 \end{array}\right] \end{array} \xrightarrow[R_1 \leftrightarrow R_2]{\text{Trivial pivoting}} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & -1 & -1 & 0 \end{array}\right]$$

0

x y z

$$\xrightarrow{-2R_1+R_3\to R_3}\left[\begin{array}{ccc|c}1&1&-1&1\\0&1&1&2\\0&-3&1&-2\end{array}\right]\xrightarrow{3R_2+R_3\to R_3}\begin{array}{ccc}x&y&z\\ & & \end{array}\left[\begin{array}{ccc|c}1&1&-1&1\\0&1&1&2\\0&0&4&4\end{array}\right]$$

$-3 \to 0$

$0x+0y+4z=4 \quad z=1$

$0x+1y+1z=2 \quad y=1$

$x+y-1=1 \quad x=1$

Gaussian Elimination Method with Partial Pivoting

We call (term in diagonals) pivot term

Gaussian Elimination with Partial Pivoting ensures that each step of forward elimination is performed with the pivoting element having the largest absolute value.

$$\begin{aligned} x+3y+2z&=5\\ 2x+4y-6z&=-4\\ x+5y+3z&=10\end{aligned}$$

Apply the Gaussian Elimination method with partial pivoting to solve the equation system

$$\left[\begin{array}{ccc|c}1&3&2&5\\2&4&-6&-4\\1&5&3&10\end{array}\right]\xrightarrow[R_1\leftrightarrow R_2]{\text{Partial pivoting}}\left[\begin{array}{ccc|c}2&4&-6&-4\\1&3&2&5\\1&5&3&10\end{array}\right]$$

ok

$$\xrightarrow[-\frac{1}{2}R_1+R_3\to R_3]{-\frac{1}{2}R_1+R_2\to R_2}\left[\begin{array}{ccc|c}2&4&-6&-4\\0&1&5&7\\0&3&6&12\end{array}\right]$$

ok

$$-\frac{1}{2}R_1 + R_3 \rightarrow R_3 \quad \left[\begin{array}{ccc|c} 0 & 3 & 6 & 12 \end{array}\right]$$

New partial pivoting

$$\xrightarrow[R_2 \leftrightarrow R_3]{} \left[\begin{array}{ccc|c} 2 & 4 & -6 & -4 \\ 0 & 3 & 6 & 12 \\ 0 & 2 & 5 & 7 \end{array}\right]$$

$2 \rightarrow 0$

$$\xrightarrow[]{-\frac{R_2}{3} + R_3 \rightarrow R_3} \left[\begin{array}{ccc|c} 2 & 4 & -6 & -4 \\ 0 & 3 & 6 & 12 \\ 0 & & & 3 \end{array}\right]$$

$3z = 3 \quad \boxed{z = 1}$

$3y + 6z = 12 \quad \boxed{y = 2}$

$2x + 4y - 6z = -4$

$\boxed{x = -3}$

Jacobi Iteration Method

$\boxed{\text{START: } 10^{\underline{20}}}$

Jacobi

Writing the system of linear equation

$Ax = b$ format

coefficent matrix

unknown matrix

result matrix

Example

$5x + 2y + z = 12$

$$5x + 2y + z = 12$$
$$2x + 3y + 4z = 20$$
$$-4x + 5y + 3z = 15$$

Step 1 Write $Ax = b$ format

$$\begin{bmatrix} 5 & 2 & 1 \\ 2 & 3 & 4 \\ -4 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 15 \end{bmatrix} \qquad R_2 \Leftrightarrow R_3$$

Step 2 Write the largest absolute number in each column in the diagonal.

$$\begin{bmatrix} 5 & 2 & 1 \\ -4 & 5 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 20 \end{bmatrix}$$

Step 3 Find x from 1. equation
Find y from 2. equation
Find z from 3. equation

$$5x + 2y + z = 12 \qquad x = \frac{12 - 2y - z}{5}$$

$$-4x + 5y + 3z = 15 \qquad y = \frac{15 + 4x - 3z}{5}$$

$$2x + 3y + 4z = 20 \qquad z = \frac{20 - 2x - 3y}{4}$$

Step 4 (Iteration)

The starting point is given to you

by the person asking the question.

If the problem is yourself, you must determine the starting point.

But generally the (0,0,0) point is considered the starting point.

$$x = \frac{12-2y-z}{5} \qquad y = \frac{15+4x-3z}{5} \qquad z = \frac{20-2x-3y}{4}$$

$\boxed{\begin{aligned} x&=0 \\ y&=0 \\ z&=0 \end{aligned}}$ starting point

1.	2. iterat	3. it	4	5	6
x = 2.4	x = 0,2	x = 1,32	x = 0.82	1,07	0,96
y = 3	y = 1,92	y = 2,23	y = 1.98	2.05	2,00
z = 5	z = 1,55	z = 3.46	z = 2.66	3.11	2.92

Example: Use the Jacobi method to approximate the solution of the following system of linear equations. (Starting point 0,0,0)

$$2x - y - 7z = 3$$
$$5x - 2y + 3z = -1$$
$$-3x + 9y + z = 2$$

$$\begin{bmatrix} 2 & -1 & -7 \\ 5 & -2 & +3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \qquad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 5 & -2 & +3 \\ -3 & +9 & +1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 5 & -2 & +3 \\ 2 & -1 & -7 \\ -3 & +9 & +1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$\begin{bmatrix} 5 & -2 & +3 \\ -3 & 9 & +1 \\ 2 & -1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$5x - 2y + 3z = -1$$
$$-3x + 9y + z = 2$$
$$2x - y - 7z = 3$$

$$x = \frac{-1 + 2y - 3z}{5}$$
$$y = \frac{2 + 3x - z}{9}$$
$$z = \frac{3 - 2x + y}{-7}$$

$x = 0$
$y = 0$
$z = 0$

	1 it	2 it	3	4	5	6	7
X	-0,2	0.146	0,192	0.181	0,185	0.186	0.186
Y	0.222	0.203	0,328	0.332	0,329	0,331	0,331
Z	-0,43	-0,517	-0.416	-0.421	-0.4224	-0,423	-0,423

3 different ways

Example

(1) Analytical methods

Example

$$x - 5y = -4$$
$$7x - y = 6$$

① Analytical methods

$x = 1$
$y = 1$

② Jacobi (False)

$x = 5y - 4$
$y = 7x - 6$

starting point (0,0)

	1.ite	2.it	3.it	4.it	5
x	-4	-34	-174	-1246	-6124
y	-6	-34	-244	-1246	-8534

$$x - 5y = -4$$
$$7x - y = 6$$

$$\begin{bmatrix} 1 & -5 \\ 7 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

③ Jacobi (True)

$$x - 5y = -4$$
$$7x - y = 6$$

$$\begin{bmatrix} 1 & -5 \\ 7 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -1 \end{bmatrix}\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -1 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

$$7x - y = 6 \longrightarrow x = \frac{6+y}{7}$$

$$x - 5y = -4 \qquad y = \frac{4+x}{5}$$

Starting point $(0,0)$

	1.it	2.it	3.it	4.it	5.it
X	0.857	0.97	0,99		
y	0.8	0,97	0,99		

Homework

27.

$$\left.\begin{aligned} 3x - y &= -4 \\ 2x + 5y &= 2 \end{aligned}\right) \text{Jacobi}$$

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$$\left.\begin{aligned} 4x+2y-z&=-1\\ x+3z&=-4\\ 3x-5y+z&=3 \end{aligned}\right\}$$ Jakobi

29 28 Homework using Gaussın elemitıon

10.05.2021 Ders Yok

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