PROB

29 Mart 2021 Pazartesi 06:57

START : 9:15

SOME IMPORTANT TERMS &CONCEPTS:

RANDOM EXPERIMENTS:

Experiments of any type where the outcome cannot be predicted are called random experiments.

SAMPLE SPACE:

A set of all possible outcomes from an experiment is called a sample space.

Eg: Consider a random experiment E of throwing 2 coins at a time. The possible outcomes are HH, TT, HT, TH.

These 4 outcomes constitute a sample space denoted by, S ={ HH, TT, HT, TH}.

TRAIL & EVENT:

Consider an experiment of throwing a coin.
When tossing a coin, we may get a head(H) or tail(T).
Here tossing of a coin is a trail and getting a hand or tail is an event.
In otherwords, "Every non-empty subset of A of the sample space S is called an event".

NULL EVENT:

An event having no sample point is called a null event and is denoted by Ø.

MUTUALLY EXCLUSIVE EVENTS:

Two events are said to be mutually exclusive when the occurrence of one affects the occurrence of the other.
In otherwords, if A & B are mutually exclusive events and if A happens then B will not happen and viceversa.
Eg: In tossing a coin the events head or tail are mutually exclusive, since both tail & head cannot appear in the same time.

EQUALLY LIKELY EVENTS:

Two events are said to be equally likely if one of them cannot be expected in the preference to the other.
Eg: In throwing a coin, the events head & tail have equal chances of occurrence.

INDEPENDENT & DEPENDENT EVENTS:

Two events are said to be independent when the actual happening of one does not influence in any way the happening of the other.

Events which are not independent are called dependent events.
Eg: If we draw a card in a pack of well shuffled cards and again draw a card from the rest of pack of cards (containing 51 cards) then the second draw is dependent on the first.

But if on the other hand, we draw a second card from the pack by replacing the first card drawn, the second draw is known as independent of the first.

An independent variable is exactly what it sounds like.

It is a variable that stands alone and isn't changed by the other variables you are trying to measure.

For example, someone's age might be an independent variable.
Other factors (such as what they eat, how much they go to school, how much television they watch) aren't going to change a person's age.

In fact, when you are looking for some kind of relationship between variables you are trying to see if the independent variable causes some kind of change in the other variables, or dependent variables.

a dependent variable is exactly what it sounds like.

It is something that depends on other factors.

For example, a test score could be a dependent variable because it could change depending on several factors such as how much you studied, how much sleep you got the night before you took the test, or even how hungry you were when you took it.

Usually when you are looking for a relationship between two things you are trying to find out what makes the dependent variable change the way it does.

Many people have trouble remembering which is the independent variable and which is the dependent variable.

An easy way to remember is to insert the names of the two variables you are using in this sentence in they way that makes the most sense.
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Experiment example

You are studying the impact of a new medication on the blood pressure of patients with hypertension.
To test whether the medication is effective, you divide your patients into two groups.

One group takes the medication, while the other group takes a sugar pill placebo.

- Your independent variable is the treatment that you vary between groups: which type of pill the patient receives.
- Your dependent variable is the outcome that you measure: the blood pressure of the patients.

independent

You are buying boxes of cookies at a bakery. Each box of cookies costs $4.

depending variable
independing variable

Which of the following statements are true?

Choose all answers that apply:

- (A) The dependent variable is the number of boxes of cookies you buy.
- (B) The independent variable is the number of boxes of cookies you buy.
- (C) The dependent variable is the amount of money you spend on the cookies.
- (D) The independent variable is the amount of money you spend on the cookies.

Does the amount of money you spend depend on the number of boxes of cookies you buy, or does the number of boxes of cookies you buy depend on the amount of money you spend?

The amount of money you spend depends on the number of boxes of cookies you buy. Therefore, the amount of money you spend is the dependent variable, and the number of boxes of cookies you buy is the independent variable.

The correct answers are:

- The independent variable is the number of boxes of cookies you buy.
- The dependent variable is the amount of money you spend on the cookies.

Probability is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome**. The **sample space** is the set of all possible outcomes. An **event** is an outcome or set of outcomes.

	Rolling a number cube	Spinning a spinner
Experiment or Situation		
Sample Space	{1, 2, 3, 4, 5, 6}	{red, blue, green, yellow}

Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.

Equally likely outcomes have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Theoretical Probability

For equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}.$$

Finding Theoretical Probability

A A CD has 5 upbeat dance songs and 7 slow ballads. What is the probability that a randomly selected song is an upbeat dance song?
There are 12 possible outcomes and 5 favorable outcomes.

$P(\text{upbeat dance song}) = \frac{5}{12} \approx 41.7\%$

$5+7=12$

$P(\text{upbeat dance song}) = \frac{5}{12} \approx 41.7\%$

Ex

A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability that the sum is 10?
There are 36 possible outcomes.

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

36 outcomes

6 + 4
4 – 6
5 – 5 → 3

$P(\text{sum is } 10) = \frac{\text{number of outcomes with sum of 10}}{36}$

$P(\text{sum is } 10) = \frac{3}{36} = \frac{1}{12}$

3 outcomes with a sum of 10: (4, 6) (5, 5), and (6, 4)

$\frac{3}{36} = \frac{1}{12}$

Experimental Probability

experimental probability = $\frac{\text{number of times the event occurs}}{\text{number of trials}}$

Events are **independent events** if the occurrence of one event does not affect the probability of the other.

If a coin is tossed twice, its landing heads up on the first toss and landing heads up on the second toss are independent events. The outcome of one toss does not affect the probability of heads on the other toss. To find the probability of tossing heads twice, multiply the individual probabilities, $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$.

Probability of Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

$P(A \text{ and } B) = P(A) \cdot P(B)$

Ex

Finding the Probability of Independent Events

Find each probability.

A spinning 4 and then 4 again on the spinner

Spinning a 4 once does not affect the probability of spinning a 4 again, so the events are independent.

$P(4 \text{ and then } 4) = P(4) \cdot P(4)$

$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$

3 of the 8 equal sectors are labeled 4.

first second

$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{16}$

independent

$P(4 \text{ and then } 4) = P(4) \cdot P(4)$

$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$ *3 of the 8 equal sectors are labeled 4.*

Independent

B spinning red, then green, and then red on the spinner

The result of any spin does not affect the probability of any other outcome.

$P(\text{red, then green, and then red}) = P(\text{red}) \cdot P(\text{green}) \cdot P(\text{red})$

$= \frac{1}{4} \cdot \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{128}$ *2 of the 8 equal sectors are red; 3 are green.*

$\frac{\cancel{2}\,1}{\cancel{8}\,4} \cdot \frac{3}{8} \cdot \frac{\cancel{2}\,1}{\cancel{8}\,4}$

$= \frac{3}{128}$

Event are dependent events if the occurence of one event affect the probability of the other

To find the probability of dependent events, you can use **conditional probability** $P(B \mid A)$, the probability of event B, given that event A has occurred.

$P(B \mid A)$

Probability of Dependent Events

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$, where $P(B \mid A)$ is the probability of B, given that A has occurred.

$\frac{2}{36} = \frac{1}{18}$

Finding the Probability of Dependent Events

Two number cubes are rolled—one red and one blue. Explain why the events are dependent. Then find the indicated probability.

A The red cube shows a 1, and the sum is less than 4.

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

$\frac{6}{36} = \frac{1}{6}$

Step 1 Explain why the events are dependent.

$P(\text{red } 1) = \frac{6}{36} = \frac{1}{6}$ *Of 36 outcomes, 6 have a red 1.*

$P(\text{red } 1) = \frac{1}{6}$

$P(\text{sum} < 4 \mid \text{red } 1) = \frac{2}{6} = \frac{1}{3}$ *Of 6 outcomes with a red 1, 2 have a sum less than 4.*

$P(\text{sum} < 4 \mid \text{red } 1) = \frac{2}{6}$

$P(\text{sum} < 4 \mid \text{red } 1) = \frac{1}{3}$

The events "the red cube shows a 1" and "the sum is less than 4" are dependent because $P(\text{sum} < 4)$ is different when it is known that a red 1 has occurred.

Step 2 Find the probability.

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

$P(\text{red 1 and sum} < 4) = P(\text{red } 1) \cdot P(\text{sum} < 4 \mid \text{red } 1)$

$= \frac{1}{6} \cdot \frac{\cancel{2}\,1}{3} = \frac{1}{18}$

$P(\text{red } 1 \text{ and sum} < 4) = \underbrace{P(\text{red } 1)}_{\frac{1}{6}} \cdot \underbrace{P(\text{sum} < 4 \mid \text{red } 1)}_{\frac{1}{3}}$

$= \frac{1}{18}$

Explain why the events are dependent. Then find the indicated probability.

B The blue cube shows a multiple of 3, and the sum is 8.

The events are dependent because $P(\text{sum is } 8)$ is different when the blue cube shows a multiple of 3.

$$P(\text{blue multiple of } 3) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \frac{2}{12} = \frac{1}{6}$$

$$P(\text{blue multiple of 3 and sum is 8}) = P(\text{blue multiple of } 3) \cdot P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \left(\frac{1}{3}\right)\left(\frac{1}{6}\right) = \frac{1}{18}$$

Of 6 outcomes for blue, 2 have a multiple of 3. Of 12 outcomes that have a blue multiple of 3, 2 have a sum 8.

$= \frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

$P(\text{blue mult of } 3) = \frac{12}{36} = \frac{1}{3}$

$P(\text{sum is } 8 \mid \text{blue mult of } 3) = \frac{2}{12} = \frac{1}{6}$

cond

Using a Table to Find Conditional Probability

Largest Texas Counties' Votes for President 2004 (thousands)

County	Bush	Kerry	Other
Harris	581	472	5
Dallas	345	336	4
Tarrant	349	207	3
Bexar	260	210	3
Travis	148	197	5

Total vote: 559

$P(\text{Bush} \mid \text{Tarrant}) = \frac{349}{559}$

The table shows the approximate distribution of votes in Texas' five largest counties in the 2004 presidential election. Find each probability.

A that a voter from Tarrant County voted for George Bush

$$P(\text{Bush} \mid \text{Tarrant}) = \frac{349}{559} \approx 0.624$$

Use the Tarrant row. Of 559,000 Tarrant voters, 349,000 voted for Bush.

B that a voter voted for John Kerry and was from Dallas County

$$P(\text{Dallas} \mid \text{Kerry}) = \frac{336}{1422}$$

Of 1,422,000 who voted for Kerry, 336,000 were from Dallas County.

$$P(\text{Kerry and Dallas} \mid \text{Kerry}) = \frac{1422}{3125} \cdot \frac{336}{1422} \approx 0.108$$

There were 3,125,000 total voters.

$P(\text{Dallas} \mid \text{Kerry}) = \frac{336}{472 + 336 + 207 + 210 + 197}$

In many cases involving random selection, events are independent when there is replacement and dependent when there is not replacement.

EXAMPLE 4 **Determining Whether Events Are Independent or Dependent**

Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the probability.

independent

A selecting two aces when the first card is replaced

Replacing the first card means that the occurrence of the first selection will not affect the probability of the second selection, so the events are independent.

$$P(\text{ace} \mid \text{ace on first draw}) = P(\text{ace}) \cdot P(\text{ace})$$
$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

4 of the 52 cards are aces.

$\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$

B selecting a face card and then a 7 when the first card is not replaced

Not replacing the first card means that there will be fewer cards to choose from, affecting the probability of the second selection, so the events are dependent.

$$P(\text{face card}) \cdot P(7 \mid \text{first card was a face card})$$
$$= \frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221}$$

There are 12 face cards, four 7's and 51 cards available for the second selection.

$\frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221}$

dependent

Remember!

A standard card deck contains 4 suits of 13 cards each. The face cards are the jacks, queens, and kings.

Compound Events

A **simple event** is an event that describes a single outcome. A **compound event** is an event made up of two or more simple events. **Mutually exclusive events** are events that cannot both occur in the same trial of an experiment. Rolling a 1 and rolling a 2 on the same roll of a number cube are mutually exclusive events.

1 or 2

$\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Mutually Exclusive Events

WORDS	ALGEBRA	EXAMPLE
The probability of two mutually exclusive events *A* or *B* occurring is the sum of their individual probabilities.	For two mutually exclusive events *A* and *B*, $P(A \cup B) = P(A) + P(B)$.	When a number cube is rolled, $P(\text{less than } 3) =$ $P(1 \text{ or } 2) =$ $P(1) + P(2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

Inclusive events are events that have one or more outcomes in common. When you roll a number cube, the outcomes "rolling an even number" and "rolling a prime number" are not mutually exclusive. The number 2 is both prime and even, so the events are inclusive.

Remember!
Recall that the intersection symbol ∩ means "and."

There are 3 ways to roll an even number, $\{2, 4, 6\}$.
There are 3 ways to roll a prime number, $\{2, 3, 5\}$.

The outcome "2" is counted twice when outcomes are added $(3 + 3)$. The actual number of ways to roll an even number or a prime is $3 + 3 - 1 = 5$. The concept of subtracting the outcomes that are counted twice leads to the following probability formula.

Inclusive Events

WORDS	The probability of two inclusive events *A* or *B* occurring is the sum of their individual probabilities minus the probability of *both* occurring.
ALGEBRA	For two inclusive events *A* and *B*, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
EXAMPLE	When you roll a number cube, $P(\text{even number or prime}) =$ $P(\text{even or prime}) = P(\text{even}) + P(\text{prime}) - P(\text{even and prime})$ $= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$.

Finding Probabilities of Inclusive Events

Find each probability on a die.

A rolling a 5 or an odd number

$P(5 \text{ or odd}) = P(5) + P(\text{odd}) - P(5 \text{ and odd})$

$= \frac{1}{6} + \frac{3}{6} - \frac{1}{6}$ *5 is also an odd number.*

$= \frac{1}{2}$

B rolling at least one 4 when rolling 2 dice

$P(4 \text{ or } 4) = P(4) + P(4) - P(4 \text{ and } 4)$

$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36}$ *There is 1 outcome in 36 where both dice show 4.*

$= \frac{11}{36}$

1 odd
2
3 odd
4
5 — 5 odd
6

$P(5 \text{ or odd}) = P(5) + P(\text{odd}) - P(5 \text{ and odd})$

$= \frac{1}{6} \cdot \frac{3}{6} - \frac{1}{6} = \frac{1}{2}$

$= \frac{1}{6} \cdot \frac{3}{6} - \frac{1}{6} = \frac{1}{2}$

$P(4 \text{ or } 4) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36}$

$= \frac{11}{36}$

Health Application

Of 3510 drivers surveyed, 1950 were male and 103 were color-blind. Only 6 of the color-blind drivers were female. What is the probability that a driver was male or was color-blind?

Step 1 Use a Venn diagram.

Label as much information as you know. Being male and being color-blind are inclusive events.

Step 2 Find the number in the overlapping region.

Subtract 6 from 103. This is the number of color-blind males, 97.

Step 3 Find the probability.

$= P(\text{male} \cup \text{color-blind}) =$

$= P(\text{male}) + P(\text{color-blind}) - P(\text{male} \cap \text{color-blind})$

$= \frac{1950}{3510} + \frac{103}{3510} - \frac{97}{3510} = \frac{1956}{3510} \approx 0.557$

The probability that a driver was male or was color-blind is about 55.7%.

B R

START : 10 35

DEFINITION 3.1 The **sample space** is the set of all possible outcomes. In referring to probabilities of events, an **event** is any set of outcomes of interest. The **probability** of an event is the relative frequency of this set of outcomes over an indefinitely large (or infinite) number of trials.

Ex.
Pulmonary Disease The tuberculin skin test is a routine screening test used to detect tuberculosis.
The results of this test can be categorized as either positive, negative, or uncertain.

If the probability of a positive test is .1, it means that if a large number of such tests were performed, about 10% would be positive.

The actual percentage of positive tests will be increasingly close to .1 as the number of tests performed increases.

EQUATION 3.1

(1) The probability of an event E, denoted by $Pr(E)$, always satisfies $0 \le Pr(E) \le 1$.

(2) If outcomes A and B are two events that cannot both happen at the same time, then $Pr(A \text{ or } B \text{ occurs}) = Pr(A) + Pr(B)$.

EXAMPLE 3.6 **Hypertension** Let A be the event that a person has normotensive diastolic blood-pressure (DBP) readings (DBP < 90), and let B be the event that a person has borderline DBP readings ($90 \le \text{DBP} < 95$). Suppose that $Pr(A) = .7$, and $Pr(B) = .1$. Let Z be the event that a person has a DBP < 95. Then

$$Pr(Z) = Pr(A) + Pr(B) = .8$$

because the events A and B cannot occur at the same time.

DEFINITION 3.2 Two events A and B are **mutually exclusive** if they cannot both happen at the same time.

EXAMPLE 3.7 **Hypertension** Let X be DBP, C be the event $X \ge 90$, and D be the event $75 \le X \le 100$. Events C and D are *not* mutually exclusive, because they both occur when $90 \le X \le 100$.

DEFINITION 3.3 The symbol { } is used as shorthand for the phrase "the event."

DEFINITION 3.4 $A \cup B$ is the event that either A or B occurs, or they both occur.

Figure 3.1 diagrammatically depicts $A \cup B$ both for the case in which A and B are and are not mutually exclusive.

EXAMPLE 3.8 **Hypertension** Let events A and B be defined as in Example 3.6: $A = \{X < 90\}$, $B = \{90 \le X < 95\}$, where X = DBP. Then $A \cup B = \{X < 95\}$.

EXAMPLE 3.9 **Hypertension** Let events C and D be defined as in Example 3.7:

$$C = \{X \ge 90\} \quad D = \{75 \le X \le 100\}$$

Then $C \cup D = \{X \ge 75\}$

FIGURE 3.1 **Diagrammatic representation of $A \cup B$: (a) A, B mutually exclusive; (b) A, B not mutually exclusive**

(a)

(b)

DEFINITION 3.5 $A \cap B$ is the event that both A and B occur simultaneously. $A \cap B$ is depicted diagrammatically in Figure 3.2.

EXAMPLE 3.10 **Hypertension** Let events C and D be defined as in Example 3.7; that is,

$$C = \{X \geq 90\} \quad D = \{75 \leq X \leq 100\}$$

Then $C \cap D = \{90 \leq X \leq 100\}$

Notice that $A \cap B$ is not well defined for events A and B in Example 3.6 because both A and B cannot occur simultaneously. This is true for any mutually exclusive events.

FIGURE 3.2 **Diagrammatic representation of $A \cap B$**

Diagrammatic representation of $\overline{A}$

DEFINITION 3.6 $\overline{A}$ is the event that A does not occur. It is called the **complement** of A. Notice that $Pr(\overline{A}) = 1 - Pr(A)$, because $\overline{A}$ occurs only when A does not occur. Event $\overline{A}$ is diagrammed in Figure 3.3.

EXAMPLE 3.11 **Hypertension** Let events A and C be defined as in Examples 3.6 and 3.7; that is,

$$A = \{X < 90\} \quad C = \{X \geq 90\}$$

Then $C = \overline{A}$, because C can only occur when A does not occur. Notice that

$$Pr(C) = Pr(\overline{A}) = 1 - .7 = .3$$

Thus, if 70% of people have DBP < 90, then 30% of people must have DBP ≥ 90.

THE MULTIPLICATION LAW OF PROBABILITY

EXAMPLE 3.12 **Hypertension, Genetics** Suppose we are conducting a hypertension-screening program in the home. Consider all possible pairs of DBP measurements of the mother and father within a given family, assuming that the mother and father are not genetically related. This sample space consists of all pairs of numbers of the form (X, Y) where $X > 0$, $Y > 0$. Certain specific events might be of interest in this context. In particular, we might be interested in whether the mother or father is hypertensive, which is described, respectively, by events A = {mother's DBP ≥ 90}, B = {father's DBP ≥ 90}. These events are diagrammed in Figure 3.4.

Suppose we know that $Pr(A) = .1$, $Pr(B) = .2$. What can we say about $Pr(A \cap B)$ = Pr(mother's DBP ≥ 90 and father's DBP ≥ 90) = Pr(both mother and father are hypertensive)? We can say nothing unless we are willing to make certain assumptions.

DEFINITION 3.7 Two events A and B are called **independent** events if

$$Pr(A \cap B) = Pr(A) \times Pr(B)$$

FIGURE 3.4 **Possible diastolic blood-pressure measurements of the mother and father within a given family**

EXAMPLE 3.13 **Hypertension, Genetics** Compute the probability that both mother and father are hypertensive if the events in Example 3.12 are independent.

Solution: If A and B are independent events, then

$$Pr(A \cap B) = Pr(A) \times Pr(B) = .1(.2) = .02$$

One way to interpret this example is to assume that the hypertensive status of the mother does not depend at all on the hypertensive status of the father.

Thus, if these events are independent, then in 10% of all households where the father is hypertensive the mother is also hypertensive, and in 10% of all households where the father is not hypertensive the mother is hypertensive.

We would expect these two events to be independent if the primary determinants of elevated blood pressure were genetic.

However, if the primary determinants of elevated blood pressure were, to some extent, environmental, then we would expect the mother would be more likely to have elevated blood pressure (A true) if the father had elevated blood pressure (B true) than if the father did not have elevated blood pressure (B not true).

In this latter case the events would not be independent.

The implications of this lack of independence are discussed later in this chapter.

If two events are not independent, then they are said to be dependent.

DEFINITION 3.8 Two events A, B are **dependent** if

$$Pr(A \cap B) \neq Pr(A) \times Pr(B)$$

Example 3.14 is a classic example of dependent events.

EXAMPLE 3.14 **Hypertension, Genetics** Consider all possible DBP measurements from a mother and her first-born child. Let

$$A = \{\text{mother's DBP} \geq 90\} \quad B = \{\text{first-born child's DBP} \geq 80\}$$

Suppose $Pr(A \cap B) = .05$ $Pr(A) = .1$ $Pr(B) = .2$

Then $Pr(A \cap B) = .05 > Pr(A) \times Pr(B) = .02$

and the events A, B would be dependent.

This outcome would be expected because the mother and first-born child both share the same environment and are genetically related. In other words, the first-born child is more likely to have elevated blood pressure in households where the mother is hypertensive than in households where the mother is not hypertensive.

EXAMPLE 3.15 **Sexually Transmitted Disease** Suppose two doctors, A and B, test all patients coming into a clinic for syphilis. Let events A^+ = {doctor A makes a positive diagnosis} and B^+ = {doctor B makes a positive diagnosis}. Suppose doctor A diagnoses 10% of all patients as positive, doctor B diagnoses 17% of all patients as positive, and both doctors diagnose 8% of all patients as positive. Are the events A^+, B^+ independent?

Solution: We are given that

$$Pr(A^+) = .1 \quad Pr(B^+) = .17 \quad Pr(A^+ \cap B^+) = .08$$

Thus, $Pr(A^+ \cap B^+) = .08 > Pr(A^+) \times Pr(B^+) = .1(.17) = .017$

and the events are dependent. This result would be expected because there should be a similarity between how two doctors diagnose patients for syphilis.

Definition 3.7 can be generalized to the case of $k(>2)$ independent events. This is often called the *multiplication law of probability*.

EQUATION 3.2 **Multiplication Law of Probability**

If $A_1, \ldots, A_k$ are mutually independent events,

then $Pr(A_1 \cap A_2 \cap \ldots \cap A_k) = Pr(A_1) \times Pr(A_2) \times \ldots \times Pr(A_k)$

THE ADDITION LAW OF PROBABILITY

We have seen from the definition of probability that if A and B are mutually exclusive events, then $Pr(A \cup B) = Pr(A) + Pr(B)$. A more general formula for $Pr(A \cup B)$ can be developed when events A and B are not necessarily mutually exclusive. This formula, the *addition law of probability*, is stated as follows:

EQUATION 3.3 **Addition Law of Probability**

If A and B are any events,

then $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

This principle is diagrammed in Figure 3.5. Thus, to compute $Pr(A \cup B)$, add the probabilities of A and B separately and then subtract the overlap, which is $Pr(A \cap B)$.

FIGURE 3.5 **Diagrammatic representation of the addition law of probability**

EXAMPLE 3.16 **Sexually Transmitted Disease** Consider the data in Example 3.15. Suppose a patient is referred for further lab tests if either doctor A or B makes a positive diagnosis. What is the probability that a patient will be referred for further lab tests?

Solution: The event that either doctor makes a positive diagnosis can be represented by $A^+ \cup B^+$. We know that

$$Pr(A^+) = .1 \qquad Pr(B^+) = .17 \qquad Pr(A^+ \cap B^+) = .08$$

Therefore, from the addition law of probability,

$$Pr(A^+ \cup B^+) = Pr(A^+) + Pr(B^+) - Pr(A^+ \cap B^+) = .1 + .17 - .08 = .19$$

Thus, 19% of all patients will be referred for further lab tests.

Special cases of the addition law are of interest.

First, if events A and B are mutually exclusive, then Pr(A ∩ B) = 0 and the addition law reduces to Pr(A ∪ B) = Pr(A) + Pr(B).

This property is given in Equation 3.1 for probabilities over any two mutually exclusive events.

Second, if events A and B are independent, then by definition Pr(A ∩ B) = Pr(A) × Pr(B) and Pr(A ∪ B) can be rewritten as Pr(A) + Pr (B) − Pr(A) × Pr(B).

This leads to the following important special case of the addition law.

Special cases of the addition law are of interest. First, if events A and B are *mutually exclusive*, then $Pr(A \cap B) = 0$ and the addition law reduces to $Pr(A \cup B) = Pr(A) + Pr(B)$. This property is given in Equation 3.1 for probabilities over any two mutually exclusive events. Second, if events A and B are *independent*, then by definition $Pr(A \cap B) = Pr(A) \times Pr(B)$ and $Pr(A \cup B)$ can be rewritten as $Pr(A) + Pr(B) - Pr(A) \times Pr(B)$. This leads to the following important special case of the addition law.

EQUATION 3.4 **Addition Law of Probability for Independent Events**

If two events A and B are independent, then

$$Pr(A \cup B) = Pr(A) + Pr(B) \times [1 - Pr(A)]$$

This special case of the addition law can be interpreted as follows:

The event A ∪ B can be separated into two mutually exclusive events: {A occurs} and {B occurs and A does not occur}.

Furthermore, because of the independence of A and B, the probability of the latter event can be written as Pr(B) × [1 — Pr(A)].

This probability is diagrammed in Figure 3.6.

This special case of the addition law can be interpreted as follows: The event $A \cup B$ can be separated into two mutually exclusive events: {A occurs} and {B occurs and A does not occur}. Furthermore, because of the independence of A and B, the probability of the latter event can be written as $Pr(B) \times [1 - Pr(A)]$. This probability is diagrammed in Figure 3.6.

FIGURE 3.6 **Diagrammatic representation of the addition law of probability for independent events**

EXAMPLE 3.17 **Hypertension** Look at Example 3.12, where

$$A = \{\text{mother's DBP} \geq 90\} \quad \text{and} \quad B = \{\text{father's DBP} \geq 90\}$$

$Pr(A) = .1$, $Pr(B) = .2$, and assume A and B are independent events. Suppose a "hypertensive household" is defined as one in which either the mother or the father is hypertensive, with hypertension defined for the mother and father, respectively, in terms of events A and B. What is the probability of a hypertensive household?

Solution: Pr(hypertensive household) is

$$Pr(A \cup B) = Pr(A) + Pr(B) \times [1 - Pr(A)] = .1 + .2(.9) = .28$$

Thus, 28% of all households will be hypertensive.

It is possible to extend the addition law to more than two events. In particular, if there are three events A, B, and C, then

$$Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B) - Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C)$$

This result can be generalized to an arbitrary number of events, although that is beyond the scope of this text (see [3]).

$P(\bar{A}) = P(A^c)$

THEOREMS ON PROBABILITY:

THEOREM 1: Probability of an impossible event is zero. i.e) $P(\emptyset) = 0$

THEOREM 2: Probability of the complementary event $\bar{A}$ of A is given by, $P(\bar{A}) = 1 - P(A)$.

THEOREM 3: For any two events A & B, $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

THEOREM 4: If A and B are two events such that $A \subset B$, then $P(B \cap \bar{A}) = P(B) - P(A)$.

THEOREM 5: If $B \subset A$, then $P(A) \geq P(B)$.

THEOREM 6: If $A \cap B = \emptyset$, then $P(A) \leq P(\bar{B})$.

Example set of 52 playing cards; 13 of each suit: clubs, diamonds, hearts, and spades

PROBLEMS:

1. If from a pack of cards a single card is drawn. What is the prob. that it is either a spade or a king?

$P(\text{spade} + \text{king}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Sol: $P(A) = P(\text{a spade card}) = \frac{13}{52}$

$P(B) = P(\text{a king card}) = \frac{4}{52}$

$P(A) + P(B) - P(A \cap B)$

$\frac{13}{52} + \frac{4}{52} - \boxed{\frac{13}{52} \cdot \frac{4}{52}}$

$= \frac{4}{13}$

$P(\text{either a spade or a king card}) = P(A \text{ or } B)$

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{4}{52} - \left[\frac{13}{52} \times \frac{4}{52}\right]$

$= \frac{4}{13}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) = \frac{3}{4} + \frac{2}{3} - \left(\frac{3}{4} \cdot \frac{2}{3}\right)$

2. A person is known to hit the target in 3 out of 4 shots, whereas another person is known to hit the target in 2 out of 3 shots. Find the probability of the targets being hit at all when they both person try.

$\frac{3}{} + \frac{2}{} - \frac{1}{} = \frac{11}{}$

they both person try.

$\frac{3}{4} + \frac{2}{3} - \frac{1}{2} = \frac{11}{12}$

Sol: The prob. that the first person hit the target = P (A) = $\frac{3}{4}$

The prob. that the second person hit the target = P (B) = $\frac{2}{3}$

The two events are not mutually exclusive, since both persons hit the same target.

$$P(A \text{ or } B) = P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{4} + \frac{2}{3} - \left[\frac{3}{4} \times \frac{2}{3}\right]$$

$$= \frac{11}{12}$$

$$\begin{array}{r} 0.21 \\ 0.14 \\ +\ 0.59 \\ \hline 0.94 \end{array}$$

$1 - 0.94 = 0.06$

1. If P (A) = 0.35, P (B) = 0.73, P (A ∩ B) = 0.14. Find P ($\bar{A} \cup \bar{B}$)

$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$

$= 1 - 0.14$

$= 0.86$

$$\begin{array}{r} 0.21 \\ 0.59 \\ 0.06 \\ + \\ \hline 0.86 \end{array}$$

Sol: Using Demargon's Law,

$$\bar{A} \cup \bar{B} = \overline{A \cup B}$$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cup B})$$

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$$

$$= 1 - 0.14 = 0.86$$

2. A bag contains 8 white and 10 black balls. Two balls are drawn in succession. What is the prob. that first is white and second is black.

Sol: Total no. of balls = 8 + 10 = 18

P (drawing one white ball from 8 balls) = $\frac{8}{18}$

P (drawing one black ball from 10 balls) = $\frac{10}{18}$

P (drawing first white & second black) = $\frac{8}{18} \times \frac{10}{18}$

$= \frac{20}{81}$

Two persons A & B appear in an interview for 2 vacancies for the same post. The probability of A's selection is $\frac{1}{7}$ and that of B's selection is $\frac{1}{5}$. What is the probability that, i) both of them will be selected, ii) none of them will be selected.

P (none of them) $= \frac{6}{7} \cdot \frac{4}{5} = \frac{24}{35}$

$\frac{1}{7} \cdot \frac{1}{5} = \frac{1}{35}$

Sol: P (A selected) = $\frac{1}{7}$

P (B selected) = $\frac{1}{5}$

P (A will not be selected) = $1 - \frac{1}{7} = \frac{6}{7}$

P (B will not be selected) = $1 - \frac{1}{5} = \frac{4}{5}$

P (A will not be selected) = $1 - \frac{1}{7} = \frac{6}{7}$

P (B will not be selected) = $1 - \frac{1}{5} = \frac{4}{5}$

i) P (Both of them will be selected) = P (A) × P (B)

$= \frac{1}{7} \times \frac{1}{5}$

$= \frac{1}{35}$

ii) P (none of them will be selected) = P (A) × P (B)

$= \frac{6}{7} \times \frac{4}{5}$

$= \frac{24}{35}$

4. A problem in mathematics is given to 3 students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the prob. that the problem will be solved?

Sol: P (A will not solve the problem) = $1 - \frac{1}{2} = \frac{1}{2}$

P (B will not solve the problem) = $1 - \frac{1}{3} = \frac{2}{3}$

P (C will not solve the problem) = $1 - \frac{1}{4} = \frac{3}{4}$

P (all three will not solve the problem) = $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{4}$

Prob

not solve

$1 - \frac{1}{4} = \frac{3}{4}$

∴ P (all the three will solve the problem) = $1 - \frac{1}{4} = \frac{3}{4}$

5. What is the chance of getting two sixes in two rolling of a single die?

Sol: P (getting a six in first rolling) $= \frac{1}{6}$

P (getting a six in second rolling) $= \frac{1}{6}$

Since two rolling are independent.

$\therefore$ P (getting two sixes in 2 rolls) $= \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

An article manufactured by a company consists of two parts A & B. In the process of manufacture of part A, 9 out of 100 are likely to be defective. Similarly, 5 0ut of 100 are likely to be defective in the manufacture of part B. Calculate the prob. that the assembled article will not be (assuming that the events of finding the part A non-defective and that of B are independent).

$P(A) = \frac{9}{100}$ $\quad$ P (A will not be def) $= 1 - \frac{9}{100} = \frac{91}{100}$

$P(B) = \frac{5}{100}$ $\quad$ P (B will not be def) $= 1 - \frac{5}{100} = \frac{95}{100}$

Sol: Prob. that part A will be defective $= \frac{9}{100}$ $\quad$ $\frac{91}{100} \cdot \frac{95}{100} = 0.8645$

$\therefore$ P (A will not be defective) $= 1 - \frac{9}{100}$

$= \frac{100 - 9}{100}$

$= \frac{91}{100}$

$\therefore$ P (the assembled article will not be defective) = P (A will not be defective) $\times$

P (B will not be defective)

$= \frac{91}{100} \times \frac{95}{100}$

$= 0.86$

7. From a bag containing 4 white and 6 black balls, two balls are drawn at random. If the balls are drawn one after the other without replacement, find the probability that

i) both balls are white.

ii) both balls are black.

iii) the first ball is white and the second ball is black.

iv) one ball is white and the other is black.

iii) the first ball is white and the second ball is black.

iv) one ball is white and the other is black.

$\frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

$\frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

$\frac{4}{10} \cdot \frac{6}{9} = \frac{24}{90} = \frac{4}{15}$

Sol: Total no. of balls = 4 + 6 = 10

i) P (first ball is white) $= \frac{4}{10}$

P (second ball is white) $= \frac{3}{9}$

$\therefore$ P (both balls are white) $= \frac{4}{10} \times \frac{3}{9}$

$= \frac{2}{15}$

ii) P (first ball is black) $= \frac{6}{10}$

P (second ball is black) $= \frac{5}{9}$

$\therefore$ P (both balls are black) $= \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

iii) P (first ball is white) $= \frac{4}{10}$

iv) one ball is white and the other is black:

$= \frac{4}{15}$

iv) a) P (first ball is white & second ball is black) $= \frac{4}{10} \times \frac{6}{9}$

$= \frac{24}{90}$

b) P (first ball is black & second ball is white) $= \frac{6}{10} \times \frac{4}{9}$

$= \frac{24}{90}$

Hence both events (a) & (b) are mutually exclusive.

$\therefore$ P (one ball is white & the other is black) $= \frac{24}{90} + \frac{24}{90}$

$= \frac{8}{15}$

8. Find the probability in each of the below four cases, if the balls are drawn one after the other with replacement. A bag containing 4 white & 6 black balls, 2 balls are drawn at random.

   i) both balls are white.
   ii) both balls are black.
   iii) the first ball is white and the second ball is black.
   iv) one ball is white and the other is black.

Sol: Total no. of balls = 4 + 6 = 10

i) P (first ball is white) $= \frac{4}{10}$

P (second ball is white) $= \frac{4}{10}$

$\therefore$ P (both balls are white) $= \frac{4}{10} \times \frac{4}{10}$

$= \frac{4}{25}$

ii) P (first ball is black) $= \frac{6}{10}$

P (second ball is black) $= \frac{6}{10}$

$\therefore$ P (both balls are black) $= \frac{6}{10} \times \frac{6}{10}$

$= \frac{9}{25}$

iii) P (first ball is white) $= \frac{4}{10}$

P (second ball is black) $= \frac{6}{10}$

$\therefore$ P (first ball is white & second ball is black) $= \frac{4}{10} \times \frac{6}{10}$

$= \frac{6}{25}$

iv) P (first ball is white & second ball is black) $= \frac{4}{10} \times \frac{6}{10}$

$= \frac{6}{25}$

CONDITIONAL PROBABILITY:

The conditional probability of event A, when the event B has already happened is defined as,

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}, P(B) \neq 0 \text{ (OR) } P(A \cap B) = P(A/B).P(B)$$

If A & B are mutually exclusive events then,

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}, P(A) \neq 0$$

PROBLEMS:

1. A bag contains 3 red & 4 white balls. Two draws are made without replacement. What is the prob. that both the balls are red.

$\frac{3}{7} \cdot \frac{2}{6} = \frac{1}{7}$

Sol: P (drawing a red ball in the first draw) $= \frac{3}{7}$

i.e) P (A) $= \frac{3}{7}$

P (drawing a red ball in the first draw given that first ball drawn is red) $= \frac{2}{6}$

i.e) $P(B/A) = \frac{2}{6}$

$\frac{3}{7} \cdot \frac{2}{6} = \frac{1}{7}$

2. Find the prob. of drawing a queen and a king from a pack of cards in two consecutive draws, the cards drawn not being replaced.

Sol: P (drawing a queen card) $= \frac{4}{52}$

i.e) P (A) $= \frac{4}{52}$

$P(A) = \frac{4}{52}$

P (drawing a king after a queen has been drawn) $= \frac{4}{51}$

$P(B) = \frac{4}{51}$

i.e) $P(B/A) = \frac{4}{51}$

$\therefore P(A \cap B) = P(B/A) \times P(A)$

$= \frac{4}{51} \times \frac{4}{52}$

$= \frac{4}{663}$

$P(A).(P(B) = \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}$

$P(A) = \frac{44}{100}$

$P(B) = \frac{62}{100}$

$P(A \cap C) = 0$

$P(B \cap C) = 24$

$P(B) = \frac{62}{100}$ | $P(A \cap C) = 0$ $P(B \cap C) = \frac{24}{100}$

$P(C) = \frac{32}{100}$

$P(A/B)$

3. In a box there are 100 resistors having resistance and tolerance as shown in the following table. Let a resistor be selected from the box and assume each resistor has the same likelihood of being chosen. Define three events A as draw a 47Ω resistor, B as draw a resistor with 5% tolerance and C as draw a 100Ω resistor. Find $P(A/B), P(A/C), P(B/C)$.

Resistance Ω	5%	10%	Total
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{28}{100}}{\frac{62}{100}} = \frac{28}{62}$

$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{\frac{32}{100}} = 0$

$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{24}{100}}{\frac{32}{100}} = \frac{24}{32}$

Sol: $P(A) = P(47\Omega) = \frac{44}{100}$

$P(B) = P(5\%) = \frac{62}{100}$

$P(C) = P(100\Omega) = \frac{32}{100}$

The joint probabilities are,

$P(A \cap B) = P(47\Omega \cap 5\%)$

$= \frac{28}{100}$

$P(A \cap C) = P(47\Omega \cap 100\Omega)$

$= 0$

$P(B \cap C) = P(5\% \cap 100\Omega)$

$= \frac{24}{100}$

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{28/100}{62/100}$

$= \frac{28}{62}$

$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{32/100}$

$= 0$

$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{24/100}{32/100}$

$= \frac{24}{32}$

4. The Hindu newspaper publishes three columns entitled politics (A), books(B), cinema(C). Reading habits of a randomly selected reader with respect to three columns are,

Read Regularly	A	B	C	A ∩ B	A ∩ C	B ∩ C	A ∩ B ∩ C
Probability	0.14	0.23	0.37	0.08	0.09	0.13	0.05

Find P (A/B), P (A/B∪C), P (A/reads atleast one), P (A∪B /C).

Sol:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.08}{0.23}$$

$$= 0.348$$

$$P(A/B \cup C) = \frac{P[A \cap (B \cup C)]}{P(B \cup C)}$$

$$= \frac{0.04+0.05+0.03}{0.47}$$

$$= 0.255$$

P (A / reads atleast one) = P [A / (A∪ $B \cup C$)]

$$= \frac{P[A \cap (A \cup B \cup C)]}{P(A \cup B \cup C)}$$

$$= \frac{P(A)}{P(A \cup B \cup C)}$$

$$= \frac{0.14}{0.49}$$

$$= 0.286$$

$$P(A \cup B / C) = \frac{P[(A \cup B) \cap C]}{P(C)}$$

$$= \frac{0.04+0.05+0.08}{0.37}$$

$$= 0.459$$

PROBLEMS:

1. In tossing a coin, what is the prob. of getting a head.
   Sol: Total no. of events = {H, T}= 2
   Favourable event = {H}= 1
   Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$
   = 1/2
2. In throwing a die, the prob. of getting 2.
   Sol: Total no. of events = {1,2,3,4,5,6}= 6
   Favourable event = {2}= 1
   Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$
   = 1/6
3. Find the prob. of throwing 7 with two dice.
   Sol: Total no. of possible ways of throwing a dice twice = 36 ways
   Number of ways of getting 7 is, (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) = 6
   Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$
   = 6/36 = 1/6
4. A bag contains 6 red & 7 black balls. Find the prob. of drawing a red ball.
   Sol: Total no. of possible ways of getting 1 ball = 6 + 7

Number of ways of getting 1 red ball = 6

Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$ = 6/13

5. Find the prob. of a card drawn at random from an ordinary pack, is a diamond.

Sol: Total no. of possible ways of getting 1 card = 52

Number of ways of getting 1 diamond card is 6

Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$

= 13/52 = 14

6. From a pack of 52 cards, 1 card is drawn at random. Find the prob. of getting a queen.

Sol: A queen may be chosen in 4 ways.

Total no. of ways of selecting 1 card = 52

Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ caseS$ = 4/52 = 1/13

7. Find the prob. of throwing: (a) 4, (b) an odd number, (c) an even number with an ordinary die (six faced).

Sol: a) When throwing a die there is only one way of getting 4.

Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$ = 1/6

b) Number of ways of falling an odd number is 1, 3, 5 = 3

Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$ = 3/6 = 1/2

c) Number of ways of falling an even number is 2, 4, 6 = 3

Probability = $Number\ of\ favourable\ casesTotal\ number\ of\ exhaustive\ cases$ = 3/6 = 1/2

8. From a group of 3 Indians, 4 Pakistanis, and 5 Americans, a sub-committee of four people is selected by lots. Find the probability that the sub-committee will consist of

i) 2 Indians and 2 Pakistanis.

ii) 1 Indians, 1 Pakistanis and 2 Americans.

iii) 4 Americans.

Sol: Total no. of people = 3 + 4 + 5 = 12

∴ 4 people can be chosen from 12 people = 12C4 ways

= 12 ×11 ×10 ×91 ×2 ×3×4 = 495 ways

i) 2 Indians can be chosen from 3 Indians = 3C2 ways

2 Pakistanis can be chosen from 4 Pakistanis = 4C2 ways

∴ No. of favourable cases = 3C2× 4C2

∴ Prob. = 3C2× 4C2495 = 255

ii) 1 Indian can be chosen from 3 Indians = 3C1 ways

1 Pakistani can be chosen from 4 Pakistanis = 4C1 ways

2 Americans can be chosen from 5 Americans = 5C2 ways

Favourable events = 3C1× 4C1 × 5C2

∴ Prob. = 3C1× 4C2× 5C2 495 = 833

iii) 4 Americans can be chosen from 5 Americans = 5C4 ways

∴ Prob. = 5C4495 = 199

9. A bag contains 7 white, 6 red & 5 black balls. Two balls are drawn at random. Find the prob. that they both will be white.

Sol: Total no. of balls = 7 + 6 + 5 = 18

From there 18 balls, 2 balls can be drawn in 18C2 ways i.e) 18 × 171 ×2 = 153

2 white balls can be drawn from 7 white balls = 7C2 ways = 21

∴ Favourable cases = 21

P(drawing 2 white balls) = 21153 = 751

10. A bag contains 10 white, 6 red, 4 black & 7 blue balls. 5 balls are drawn at random. What is the prob. that 2 of them are red and one is black?

Sol: Total no. of balls = 10 + 6 + 4 + 7 =27

5 balls can be drawn from these 27 balls = 27C5 ways

= 27 × 26 ×25 × 24 ×231 ×2 ×3×4 ×5

= 80730 ways

Total no. of exhaustive events = 80730

2 red balls can be drawn from 6 red balls = 6C2 ways

= 6 × 51 ×2 = 15 ways

1 black balls can be drawn from 4 black balls = 4C1 ways = 4

∴ No. of favourable cases = 15 × 4 = 60

Probability = $Number\ of\ favourable\ cases/Total\ number\ of\ exhaustive\ cases$

= 6080730 = 68073

11. What is the prob. of having a king and a queen, when 2 cards are drawn from a pack of 52 cards?

Sol: 2 cards can be drawn from a pack of 52 cards = 52C2 ways

= 52 × 511 ×2 = 1326 ways

1 queen card can be drawn from 4 queen cards = 4C1 ways

1 king card can be drawn from 4 king cards = 4C1 ways

Favourable cases = 4 × 4 = 16 ways

P(drawing 1 queen & 1 king card) = $Number\ of\ favourable\ casesTotal\ number\ of\ exhaustive\ cases$

= 161326 = 8663

12. What is the prob. that out of 6 cards taken from a full pack, 3 will be black and 3 will be red?

Sol: A full pack contains 52cards. Out of 52 cards, 26 cards are red & 26

black cards .
6 cards can be chosen from 52 cards = $52C6$ ways
3 black cards can be chosen from 26 black cards = $26C3$ ways
3 red cards can be chosen from 26 red cards = $26C3$ ways
Favourable cases = $26C3 \times 26C3$
Probability = $Number\ of\ favourable\ casesTotal\ number\ of\ exhaustive\ cases$
= $26C3 \times 26C3\ 52C6$
13. Find the prob. that a hand at bridge will consist of 3 spades, 5 hearts, 2 diamonds & 3 clubs?
Sol: Total no. of balls = 3 + 5 + 2 + 3 = 13
From 52 cards, 13 cards are chosen in $52C13$ ways
In a pack of 52 cards, there are 13 cards of each type.
3 spades can be chosen from 13 spades = $13C3$ ways
5 hearts can be chosen from 13 hearts = $13C5$ ways
2 diamonds can be chosen from 13 diamonds = $13C2$ ways
3 clubs can be chosen from 13 clubs = $13C3$ ways
Hence the total no. of favourable cases are = $13C3 \times 13C5 \times 13C2 \times 13C3$
Probability = $Number\ of\ favourable\ casesTotal\ number\ of\ exhaustive\ cases$
= $13C3 \times 13C5 \times 13C2 \times 13C352C13$