1. Relationship Between Derivative and Slope
2. Intermediate Value Theorem
3. Root Finding with Intermediate Value Theorem
4. Mean Value Therom
5. Rolle's Theorem

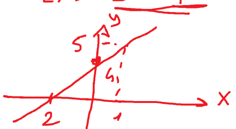
START: 09.15

Slope : gradient

The, derivative of a function $f(x)$ is the function $f'(x)$ which gives the gradient of the tangent to graph $y=f(x)$ at each value of x.

Example

$f(x) = 2x+4$

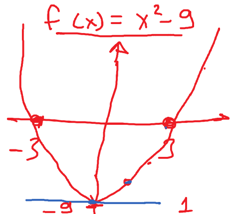


$f'(x) = 2$

Slope is constant

Example

$f(x) = x^2 - 9$



$f'(x) = 2x$

at $x=1$ slope 2

at $x=2$ slope 4

at $x=0$ slop 0

at $x=-1$ slope -2

⋮

① if the derivative of the function is always positive the function is constantly increasing.

② if the derivative of the function is always negative the function is constantly decreasing.

Intermediate Value Theorem

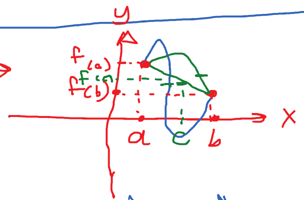
If f is a continuous function on a closed interval $[a,b]$ if y_0 is any value between $f(a)$ and $f(b)$ that $y_0 = f(c)$ for some c in $[a,b]$

$f(x)$ is a continuous

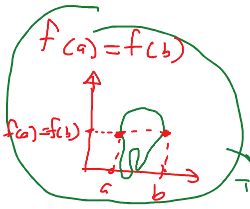
$f(a) \neq f(b)$

may be $f(a) > f(b)$

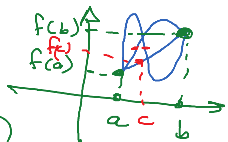
may be $f(b) > f(a)$



$f(a) > f(c) > f(b)$



There is not intermediate value



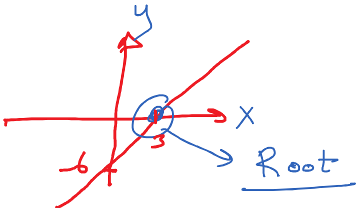
$f(b) > f(c) > f(a)$

A Consequence for Root finding with intermediate Value Therom.

Ex $f(x) = 2x-6$

0

$x = 3$



Root

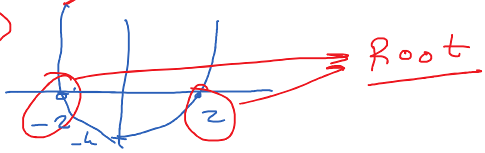
Ex $f(x) = x^2 - 4$

$0 = x^2 - 4$

$0 = (x-2)(x+2)$

$x_1 = 2$

$x_2 = -2$



Root

Root Finding with I.V.T.

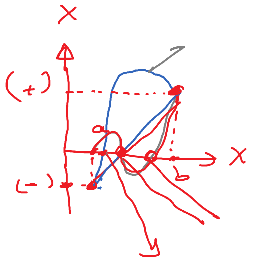
$f(x)$ is a continuous function on a closed interval $[a, b]$

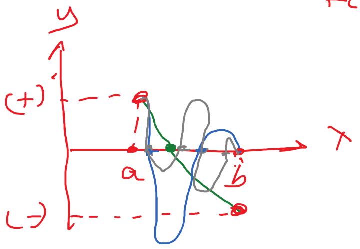
(*) if $f(a) \cdot f(b) < 0$

the function has at least one root

if $f(a) \cdot f(b) < 0 \Rightarrow$ may be $f(a) = -$, $f(b) = +$

may be $f(a) = +$, $f(b) = -$





Example

Show that there is a root of the equation

$x^3 - x - 1 = 0$ between 1 and 2. $\boxed{f(x) = x^3 - x - 1}$

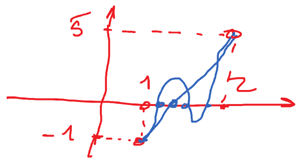
Step 1. Check if the function is continuous.

*. There is no undefined.

*. Not a piecewise.

step 2. $f(1) = 1^3 - 1 - 1 = -1$ $\qquad f(-1) \cdot f(5) < 0$

$f(2) = 2^3 - 2 - 1 = 5$ $\qquad (-)\ (+)$



Example (prove that, a function has $\boxed{\text{only one}}$ root)

Show that there is only one root of the

$f(x) = x^4 - 3x + 1$ between 1 and 2

Step 1. Check continuous

step 2. $f(1) = -1$ $\qquad f(2) = 11$

at least one root

step 3. $f'(x) = 4x^3 - 3$

$1 < x < 2$

$1^3 < x^3 < 2^3$

$4 < 4x^3 < 32$

$4 - 3 < 4x^3 - 3 < 32 - 3$

$1 < 4x^3 - 3 < 29$

always positive

function has only one root

Comparison Exam

$f(x) = x^4 - 3x + 1$

$[1,2]$

Does it have roots?

is it only one?

Step 1. check Con.

Step 2. $f(1) = -1$

$f(2) = 11$

The function has at least one root

$f'(x) = 4x^3 - 3$

$1 < x < 2$

$1 < x^3 < 8$

$4 < 4x^3 < 32$

$1 < 4x^3 - 3 < 29$

positive

$f(x) = x^4 - 5x + 1$

$[1,2]$

Does it have roots.

is it only one

Step 1. check Cont..

Step 2. $f(1) = -3$

$f(2) = 7$

has at least one root

$f'(x) = 4x^3 - 5$

$1 < x < 2$

$1 < x^3 < 8$

$4 < 4x^3 < 32$

$-1 < 4x^3 - 5 < 27$

sometimes positive

negative.

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Mean Value Theorem.

The mean value theorem explain that.

if $f_{(x)}$ is continuous over the closed interval $[a,b]$

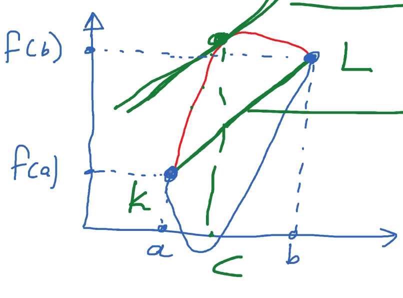
and differentiable over the open interval (a,b)

then,

there is a point $c \in (a,b)$ such that the tangent line

to the graph of $f(x)$ at c is parallel to the

secant line connecting.



tangent line

Secant line

Slope (Secant line) $\dfrac{f(b) - f(a)}{b - a}$

$f'(c)$.

Example: find the value that satisfies the mean value theorem in $[1,3]$ closed intervals of the function $f(x) = x^2 + 6$

Step 1. Check if the function is continuous.

Step 2. Calculate mean value

$f(1) = 7$

$f(3) = 15$

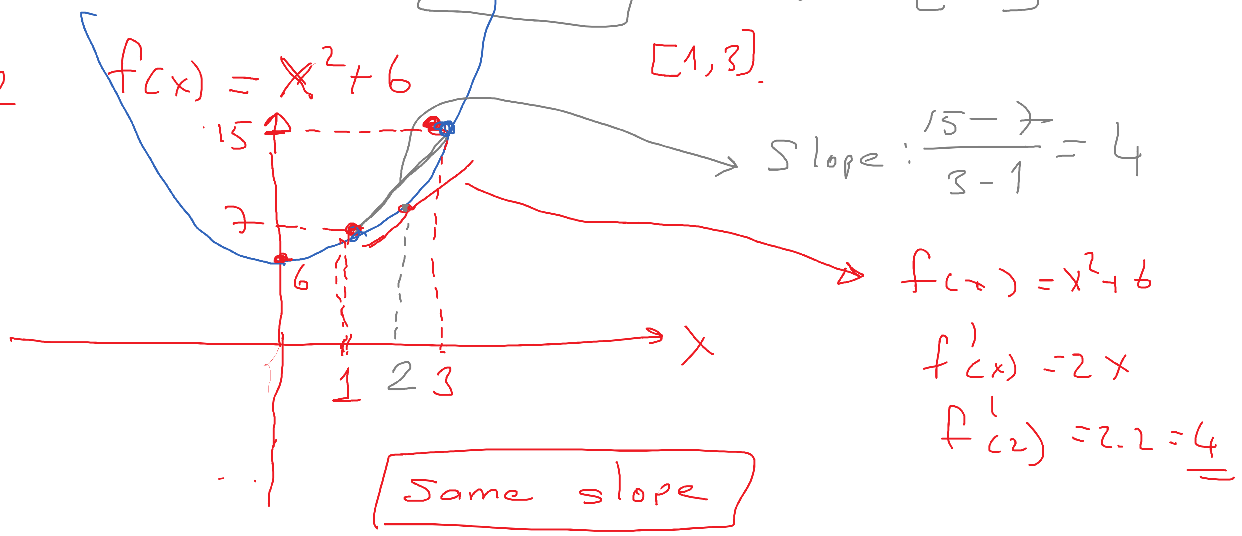
$$\text{Slope (mean value)} = \frac{f(3) - f(1)}{3 - 1} = \frac{15-7}{2} = 4$$

Step 3. find $f'(x) = 2X$

Step 4. Equalize at c point $f'(c) = 2c$

$2c = 4$ $\boxed{c = 2}$ $2 \in [1, 3]$

Graph $f(x) = X^2 + 6$ $[1, 3]$



Example: Determine all numbers c which satisfy the conclusions of the mean value theorem for

$$f(x) = X^3 + 2X + 4 \qquad [-1, 1]$$

Step 1. Check Count...

Step 2. Calculate mean value $f(-1) = 1$ $f(1) = 7$

$$M.V = \frac{f(b) - f(a)}{b - a} = \frac{7-1}{1-(-1)} = 3$$

Step 3. find $f'(x) = 3X^2 + 2$

Step 4. Equalize : $f'(c) = 3c^2 + 2$

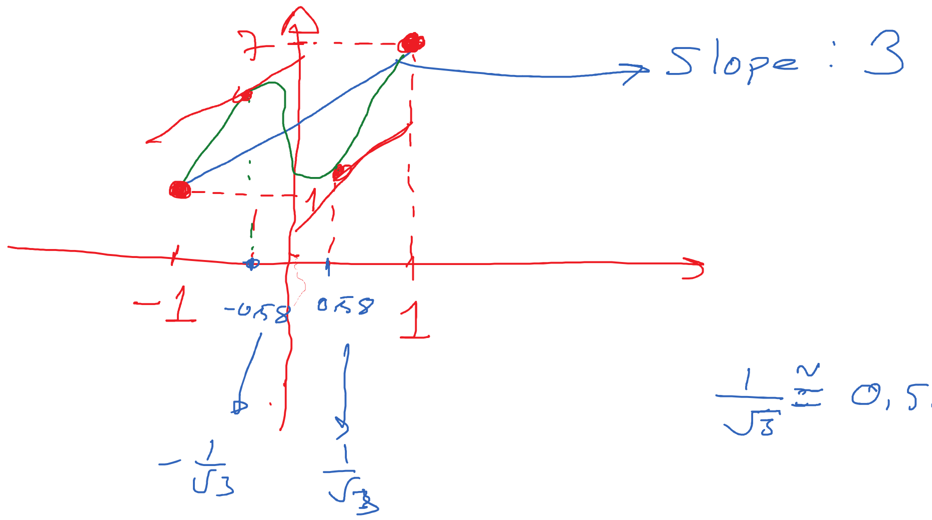
$3c^2 + 2 = 3$

$3c^2 = 1$

$c^2 = \frac{1}{3}$

$\boxed{c_1 = \frac{1}{\sqrt{3}}}$ $\boxed{c_2 = -\frac{1}{\sqrt{3}}}$

$f(x) = x^3 + 2x + 4$ $[-1, 1]$

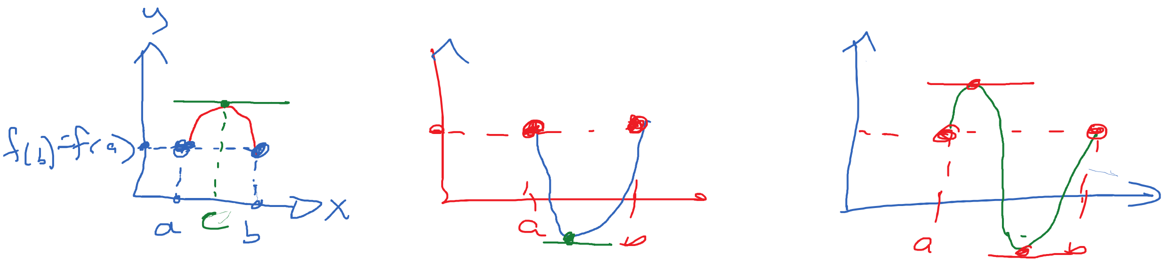


$\frac{1}{\sqrt{3}} \cong 0,58$

Rolle's Theorem

Let. f is a continuous function over the closed interval $[a,b]$ and differentiable over the open interval (a,b) such that $\boxed{f(a) = f(b)}$.

There then exists at least one $c \in (a,b)$ such that $f'(c) = 0$



$\boxed{f'(c) = 0}$

* f(x) is cont.. $[a,b]$
* f(x) is differantiable (a,b)
* if $f(a) = f(b)$
* There is at least on c point $(c \in (a,b))$

Example

For $f(x) = x^2$ on $[-2,2]$ verify that the function satisfies the criterra stated in Rolle's theorem and find all values $\underline{c}$ in the given interval where $f'(c) = 0$.

Step 1 : Check continuous

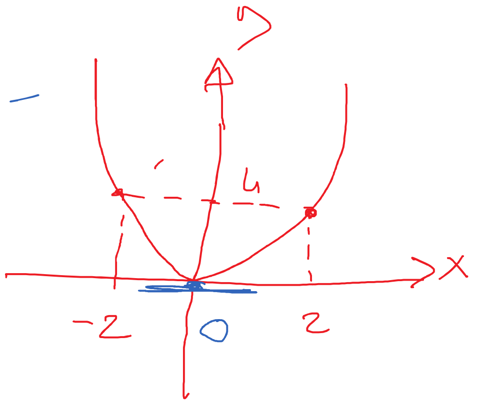
Step 2 : find $f(-2) = 4$, $f(2) = 4$

$f(-2) \stackrel{?}{=} f(2)$ yes

Step 3 . find $f'(x) = 2x$

Step 4 . Equalize $f'(c) = 2c = 0$

$\boxed{c = 0}$



Ex $f(x) = x^2 + 2x$ over $[-2, 0]$ Rolle's theorem

Step 1 . check

Step 2 . find $f(-2) = 0$, $f(0) = 0$

same

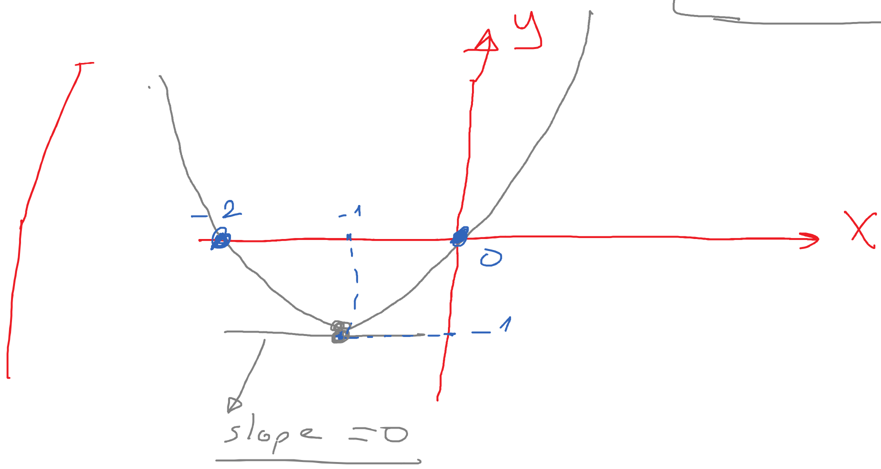
Step 3 . find $f'(x) = 2x + 2$

Step 4 . Equalize . $f'(c) = 2c + 2 = 0$

$\boxed{c = -1}$

always Rolle's theorem

$\boxed{f(x) = x^2 + 2x}$



Example $f(x) = x^3 - 4x$ $[-2, 2]$ Rolle's

Step 1 . check

Step 2 . find $f(-2) = 0$ $f(2) = 0$

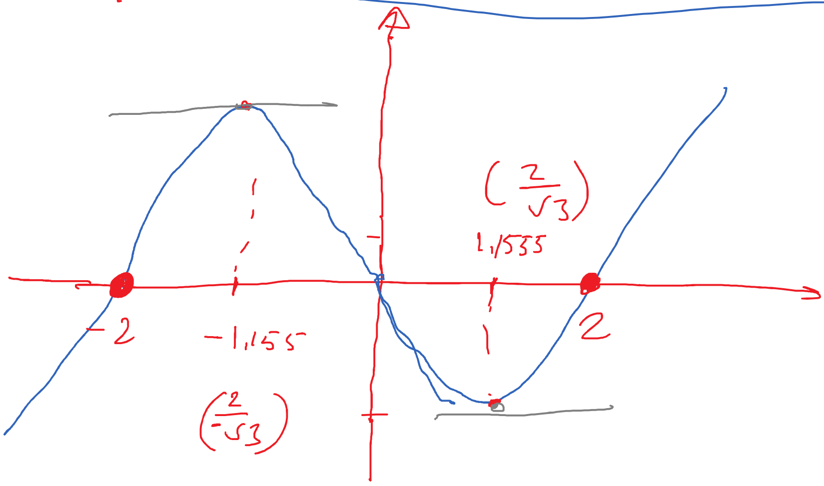
same

step 3. $f'(x)=3x^2-4$

step 4. Equalize $f'(c)=3c^2-4=0$ $c^2=\frac{4}{3}$

$c_1=\frac{2}{\sqrt{3}}$ $c_2=-\frac{2}{\sqrt{3}}$

$f(x)=x^3-4x$ $c_1\cong-1{,}155$ $c_2\cong1{,}155$



Homework and Possible question on the exam.

(6) Given the function $f(x)=x^2-6x+5$
Find all values of c in the open interval $(2,4)$
such that $f'(c)=0$. (Use Rolle's theorem)

(7) - - - - - - $f(x)=x^2+8x+14$ - - - -
- - — — — — - - $(-6,-2)$
— — - - $f'(c)=0$

(8) — — — - - $f(x)=-2x^2-8x+6$ - - -
- — — — - — $[1,3]$
— - - - $f'(c)=0$ — — - -

(9) Given the function, $f(x)=x^2-3x+5$ on interval
$[1,4]$. Find a point $\underline{\underline{c}}$ satisfying the condition
of mean value theorem

(10) Determine all numbers c which satisfy the conclusion of the mean value theorem for.

$f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$

(11) Show that there is a root of the equation $x^3 - x - 1 = 0$ between 1 and 2. (use Int. ~~theo.~~ Th)

(12) Show that there is only one root of the $x^4 - 4x + 2$ between 1 and 2

(13) Show that there is at least one root of the $x^5 - 2x^3 - 2 = 0$ between $x = 0$ and $x = 2$

Handwriting