BME 1132
Probability and Biostatistics

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Week-6

- Introduction to Probability
- Sample Space
- Events
- Set Theory
- Counting Techniques
    - Multiplication Rule
    - Permutations
    - Combinations
- Axioms of Probability

Sample Space

A sample space is often defined based on the objectives of the analysis, and is represented by the symbol S.

Flip a Coin

The possible outcomes when a coin is flipped, may be written

S = {H, T},

where *H* and *T* correspond to heads and tails, respectively.

Roll a Dice

We are interested in the number that shows on the top face, the sample space is

S1 = {1, 2, 3, 4, 5, 6}.

NOTE:
A sample space is **discrete** if it consists of a finite or countable infinite set of outcomes.
A sample space is **continuous** if it contains an interval (either finite or infinite) of real numbers.

Example: Camera Flash

Statement or Rule method

Consider an experiment that selects a cell phone camera and records the recycle time of a flash (the time taken to ready the camera for another flash).

It is convenient to define the sample space as simply the positive real line

$\boldsymbol{S = R^{+} = \{x \mid x > 0\}}$

If it is known that all recycle times are between 1.5 and 5 seconds, the sample space can be

$\boldsymbol{S = \{x \mid 1.5 < x < 5\}}$

If the objective of the analysis is to consider only whether the recycle time is low, medium, or high, the sample space can be taken to be the set of three outcomes

***S* = {*low*, *medium*, *high*}**

If the objective is only to evaluate whether or not a particular camera conforms to a minimum recycle time specification, the sample space can be simplified to a set of two outcomes

***S* = {*yes*, *no*}**

that indicates whether or not the camera conforms.

Event

For any given experiment, we may be interested in the occurrence of certain **events** rather than in the occurrence of a specific element in the sample space.

Example1:

We may be interested in the event A that the outcome when a die is tossed is divisible by 3.

$$A = \{3, 6\}$$

Example2:

Given the sample space

$S = \{t \mid t \geq 0\}$,

where t is the life in years of a certain electronic component, then the event A that the component fails before the end of the fifth year is the subset

$A = \{t \mid 0 \leq t < 5\}$.

Sample Space- Event- Probability

The **sample space** is the set of all possible outcomes. In referring to probabilities of events, an **event** is any set of outcomes of interest. The **probability** of an event is the relative frequency of this set of outcomes over an indefinitely large (or infinite) number of trials.

Probability- Examples

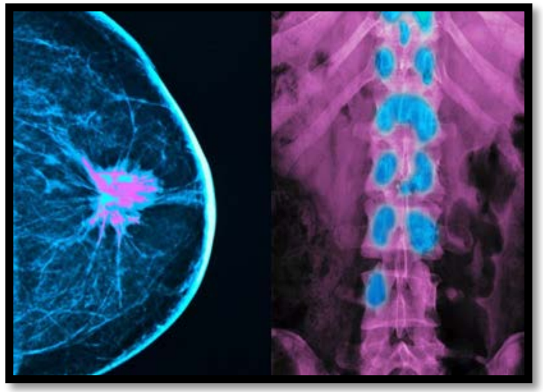
The **sample space** is the set of all possible outcomes. In referring to probabilities of events, an **event** is any set of outcomes of interest. The **probability** of an event is the relative frequency of this set of outcomes over an indefinitely large (or infinite) number of trials.

Pulmonary Disease The tuberculin skin test is a routine screening test used to detect tuberculosis. The results of this test can be categorized as either positive, negative, or uncertain.

If the probability of a positive test is .1, it means that if a large number of such tests were performed, about 10% would be positive. The actual percentage of positive tests will be increasingly close to .1 as the number of tests performed increases.

Cancer The probability of developing breast cancer over 40 years in 30-year-old women who have never had breast cancer is approximately 1/11.

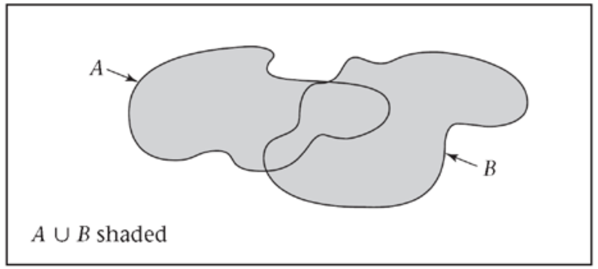
This probability means that over a large sample of 30-year-old women who have never had breast cancer, approximately 1 in 11 will develop the disease by age 0, with this proportion becoming increasingly close to 1 in 11 as the number of women sampled increases.

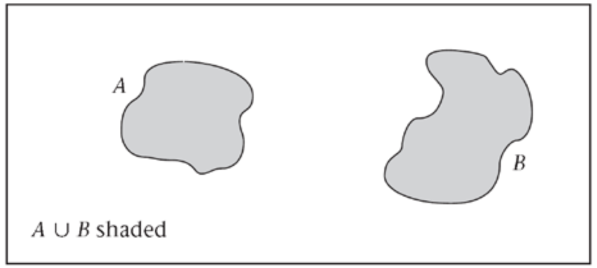


Some Useful Probabilistic Notation

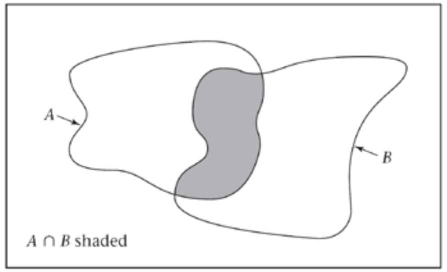
- The symbol { } is used as shorthand for the phrase *"the event"*.

- $A \cup B$ is the event that either A or B occurs, or they both occur.

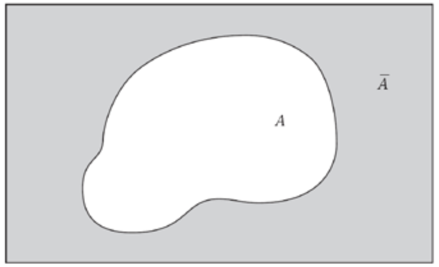




- $A \cap B$ is the event that both A and B occur simultaneously.



- $\bar{A}$ is the event that A does not occur. It is called the complement of A. Notice that $P(\bar{A}) = 1 - P(A)$, because A occurs only when A does not occur.



Example

Union, Intersection and Complement

As camera flash example, camera recycle times might use the sample space $S = R^{+}$, the set of positive real numbers.

Let,

$$E_1 = \{x \mid 10 \le x < 12\} \quad \text{and} \quad E_2 = \{x \mid 11 < x < 15\}$$

$$E_1 \cup E_2 = \{x \mid 10 \le x < 15\}$$

$$E_1 \cap E_2 = \{x \mid 11 < x < 12\}$$

$$E_1' = \{x \mid x < 10 \quad \text{or} \quad 12 \le x\}$$

$$E_1' \cap E_2 = \{x \mid 12 \le x < 15\}$$

NOTE:

Two events, denoted as E_1 and E_2, such that

$$E_1 \cap E_2 = \emptyset$$

are said to be **mutually exclusive**.

Dependent & Independent Multiplication Law of Probability

Two events A and B are called **independent** events if

$$Pr(A \cap B) = Pr(A) \times Pr(B)$$

Two events A, B are **dependent** if

$$Pr(A \cap B) \neq Pr(A) \times Pr(B)$$

Multiplication Law of Probability

If $A_1, \ldots, A_k$ are mutually independent events,

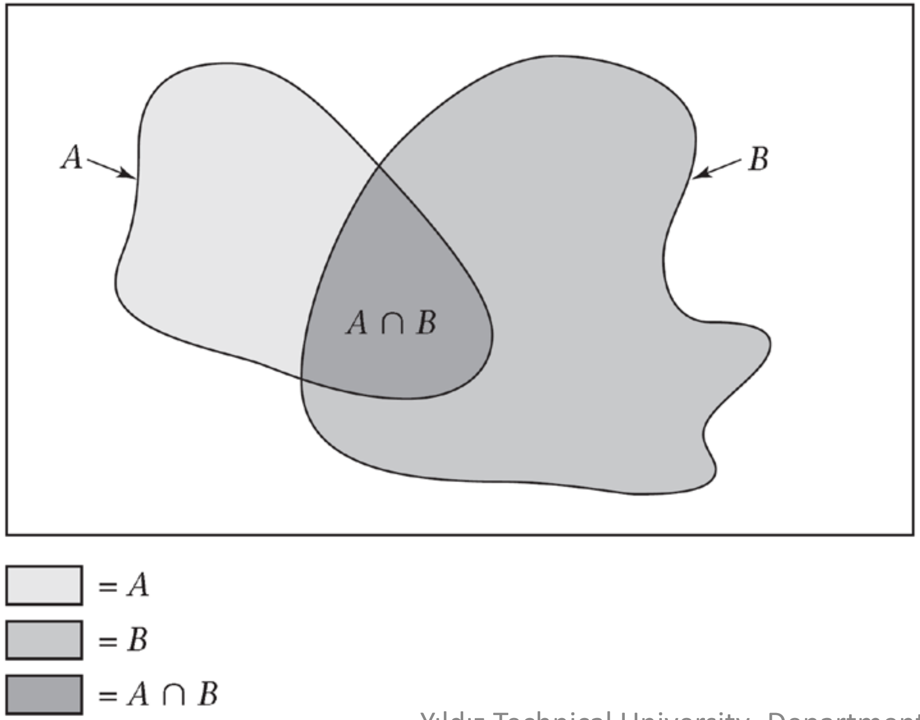
then $Pr(A_1 \cap A_2 \cap \ldots \cap A_k) = Pr(A_1) \times Pr(A_2) \times \ldots \times Pr(A_k)$

Addition Law of Probability

Addition Law of Probability

If A and B are any events,

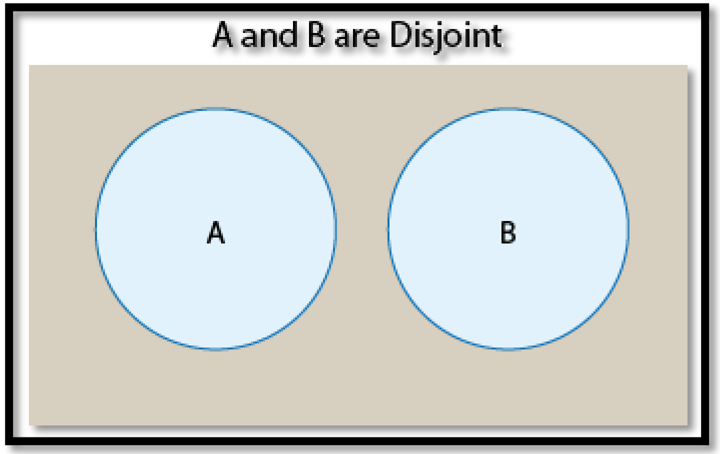
then $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$



Addition Rule

If A and B are **mutually exclusive** events,

$$P(A \cup B) = P(A) + P(B)$$



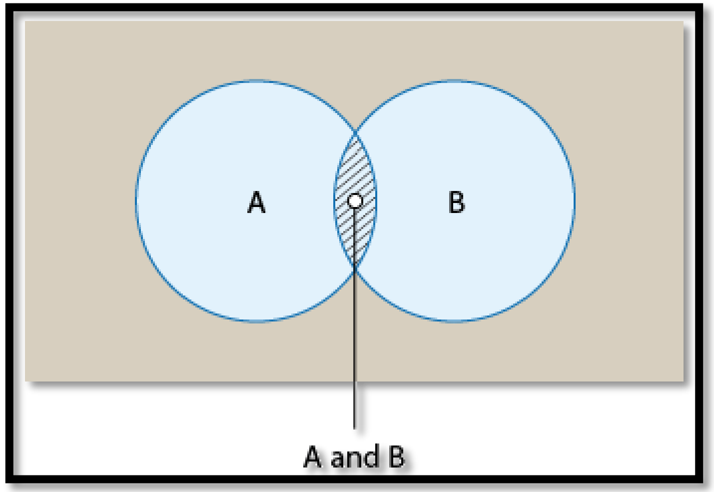
Three or more events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$
$$-P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

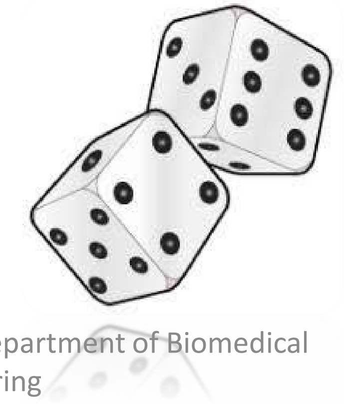
Addition Rule

If A and B are two events,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

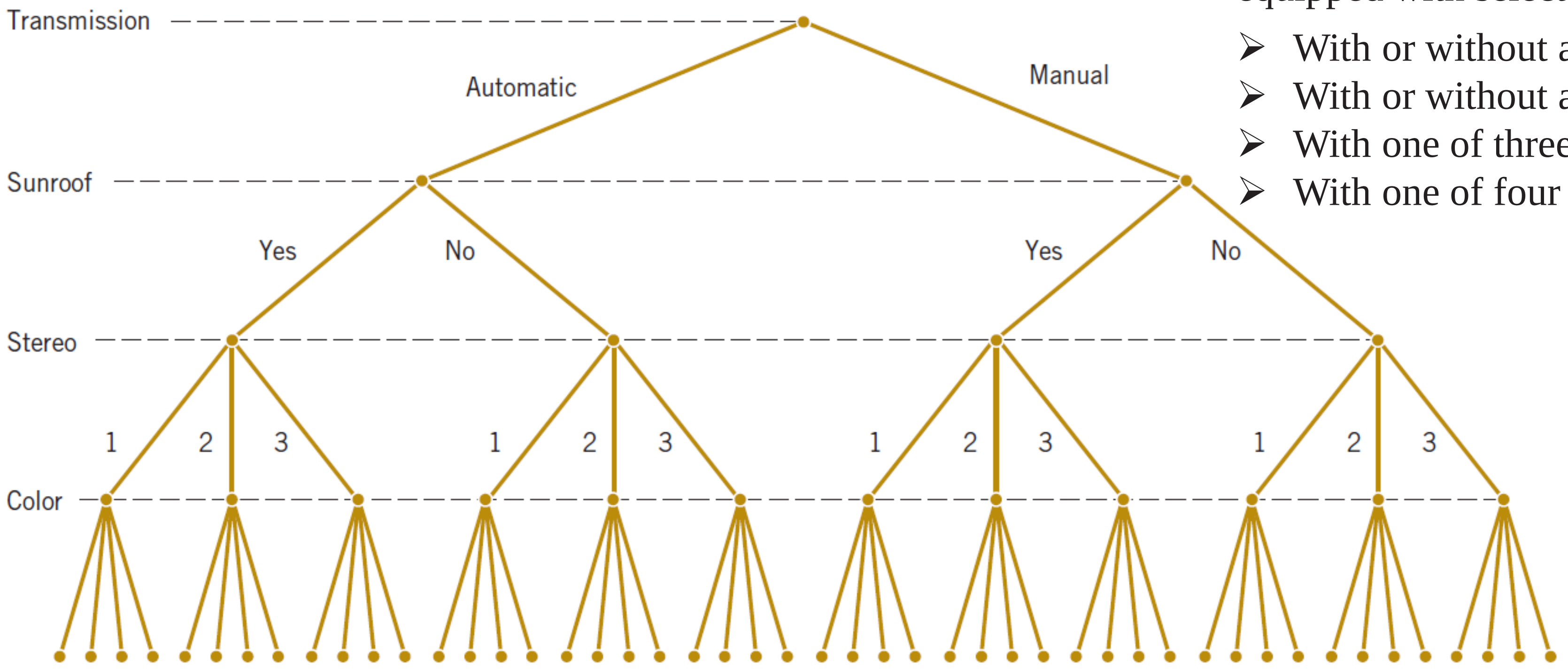


Example:
What is the probability of getting a total of 7 or 11 when a pair of fair dice is tossed?



Counting Techniques

This techniques are used to count of the numbers of outcomes in the sample space and various events in order to analyze the random experiments.



Example

An automobile manufacturer provides vehicles equipped with selected options. Each vehicle is ordered

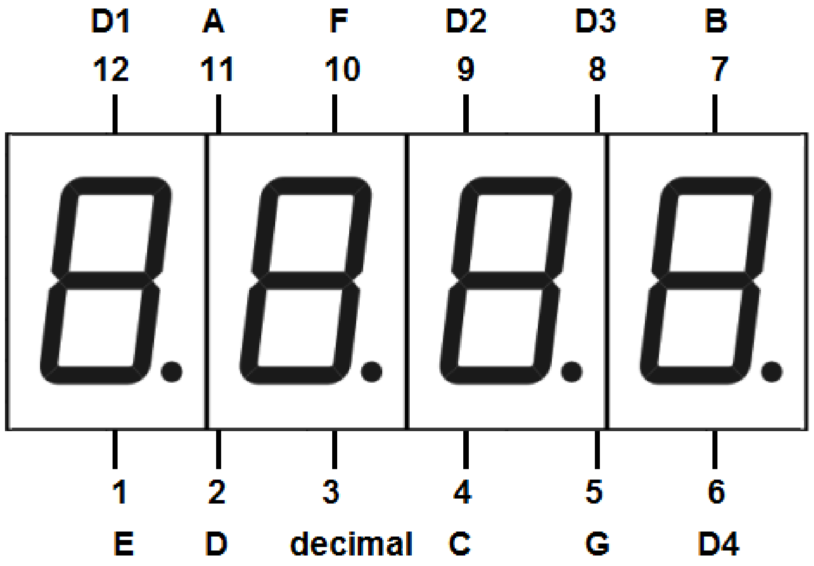
- With or without an automatic transmission
- With or without a sunroof
- With one of three choices of a stereo system
- With one of four exterior colors

Counting Techniques-

Multiplication Rule Examples

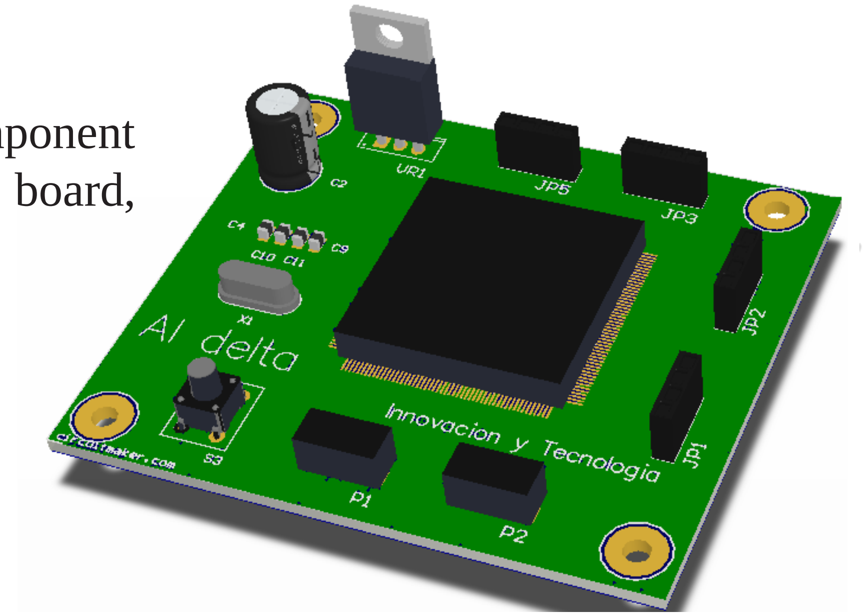
How many sample points are there in the sample space when a pair of fair dice is thrown once?

How many even four-digit numbers can be formed from the digits 0, 1, 2, 5, 6, and 9 if each digit can be used only once?



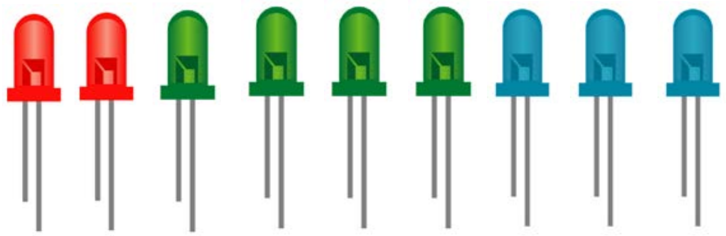
Counting Techniques-

Permutations- Examples



A printed circuit board has eight different locations in which a component can be placed. If **four different** components are to be placed on the board, how many different designs are possible?

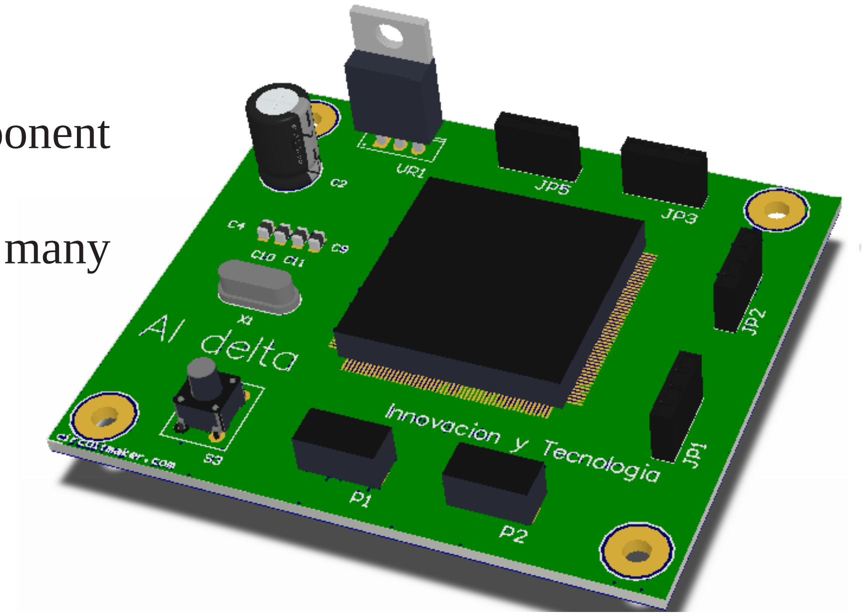
Nine LEDs are to be arranged on a string. Two of the LEDs are red, four are green, and three are blue.
In how many ways can the LEDs be arranged?

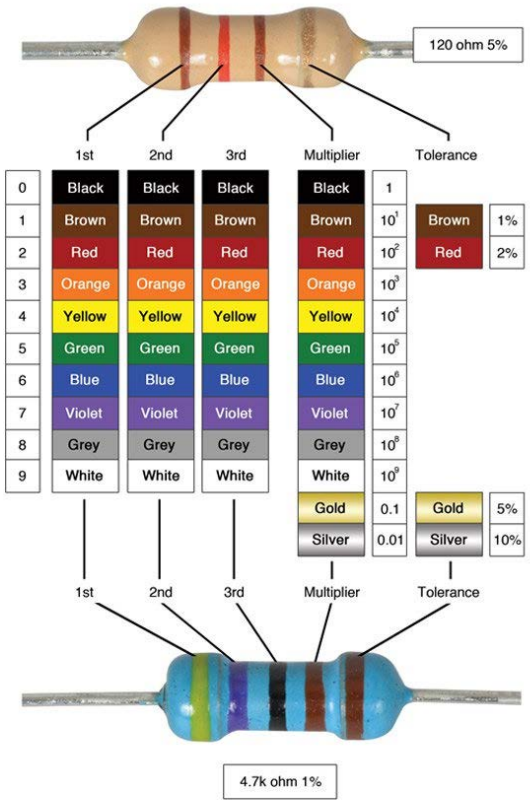


Counting Techniques-

Combinations- Examples

A printed circuit board has eight different locations in which a component can be placed.
If **five identical** components are to be placed on the board, how many different designs are possible?





A researcher will buy 5 resistors that have different tolerance limits of 120 Ω (10 resistors) and 4.7 kΩ (5 resistors).
How many ways are there that he can get 3 of 120 Ω and 2 of 4.7 kΩ?

Axioms of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

(1) $P(S)=1$

(2) $0 \le P(E) \le 1$

(3) For two events E_1 and E_2 with $E_1 \cap E_2 = \varnothing$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

These axioms imply the results $P(\varnothing) = 0$

$$P(E') = 1 - P(E)$$

If the event E_1 is contained in the event E_2 results;

$$P(E_1) \le P(E_2)$$

Questions?