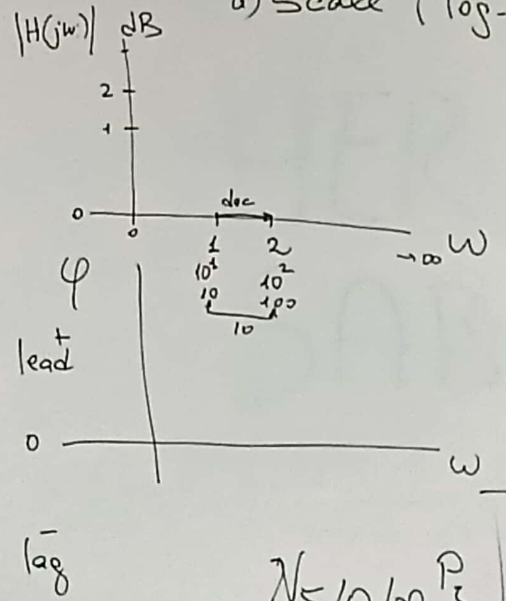
1) Bode Plot

a) Scale (log-log: dB - dec)



1924 WM Martin

J AIEE

Transmission Unit

$$\frac{P_2}{P_1} = 10^{aN} = 10^{\frac{N}{10}}$$

$$N = 10 \log \frac{P_2}{P_1}$$

$$\boxed{\log \frac{P_2}{P_1} = \frac{N}{10} \cancel{\log 10}}$$

Miles of Standard Cable

MSC

88 ohms
0.054 F

$\sqrt{RC}\sqrt{f}$, $a\sqrt{f}$

Power - Amplitude - Joule's Law

quadratic

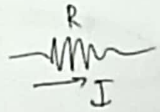
I, V, T,

Field Quantities

Root Square Power

RSP

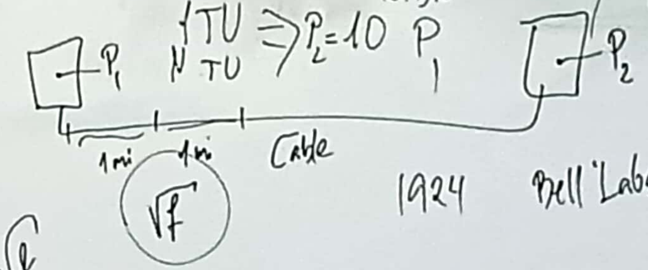
Graham Bell



$P = RI^2$

$P = V.I = RI.I = RI^2$

1 TU
N TU $\Rightarrow P_2 = 10^{(0.1)N} P_1$



1924 Bell Labs

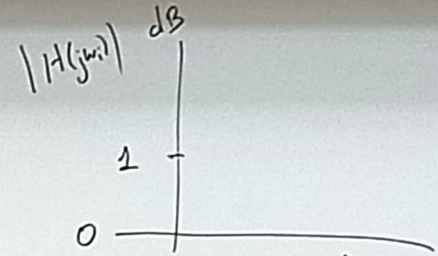
(dB) $N = 10 \log_{10} \frac{P_2}{P_1}$

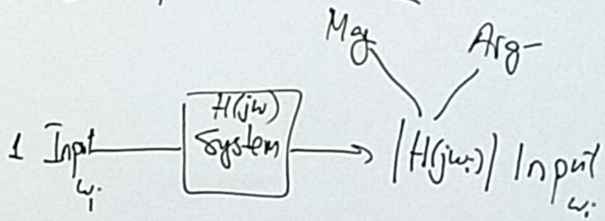
1924
$\downarrow$
1928

$\frac{P_2}{P_1} = 10^{\frac{1}{10}} = 1.28$

$\text{Bell} = \log_{10} \frac{P_2}{P_1}$

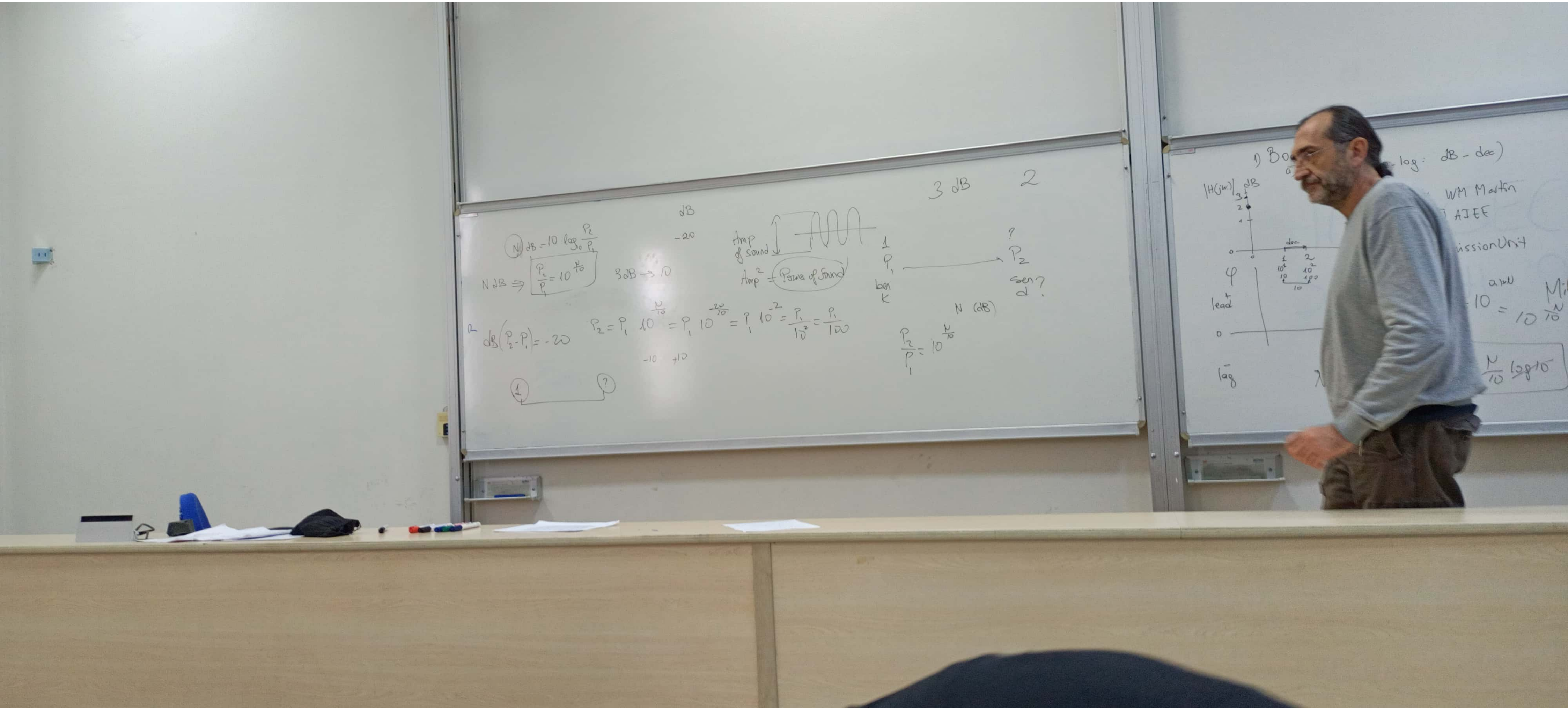
$$10 \log_{10} \frac{I_2^2 R}{I_1^2 R} = 10 \log_{10} \left(\frac{I_2}{I}\right)^2 = 20 \log_{10} \frac{I_2}{I_1}$$





$$\frac{\log |H(j\omega_i)|}{1}$$

$$\log|\cdot| - \cancel{\log 1}$$


3 dB
2
dB
Amp of sound
Power of sound
WM Martin
lead
log

3 dB 2

dB

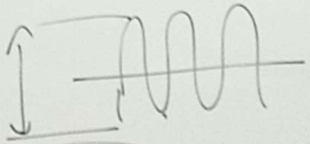
-20

Ⓝ $dB = 10 \log_{10} \frac{P_2}{P_1}$

$N\,dB \Rightarrow \boxed{\frac{P_2}{P_1} = 10^{\frac{N}{10}}}$

3 dB → 10

Amp of sound



$Amp^2 = $ Power of Sound

1
P_1
ben
K

? P_2 sen? d?

N (dB)

$\frac{P_2}{P_1} = 10^{\frac{N}{10}}$

$dB(P_2 - P_1) = -20$

$P_2 = P_1\, 10^{\frac{N}{10}} = P_1\, 10^{\frac{-20}{10}} = P_1\, 10^{-2} = \frac{P_1}{10^2} = \frac{P_1}{100}$

-10 +10

1 ?

Power Quantity : Power (Squared / Quadratic)

Root Power Quantity : Field Quantity V, I; Pressure, Force/Torque

dB	RPQ (V, I, P) $\left(\frac{V_2}{V_1}\right)^2$ $20 \log_{10} \frac{V_2}{V_1}$	Power $\left(\frac{V_2^2}{V_1^2}\right) = \frac{P_2}{P_1}$ $10 \log_{10} \frac{P_2}{P_1}$
−20	0.1	0.01
−10	0.3162	0.1
−6	0.501	0.25
−3	0.708	0.501
1	1.122	$1.258 = 10^{\frac{1}{10}}$

3	1.413	2
6	2	4
10	3.162	10
20	10	100

Every 3dB increase in sound represents doubling in sound intensit[y] (pow[er])

$|H(j\omega)|$ dB, 20 l[og]

10^1

Power Quantity : Power (Squared / Quadratic)

Root Power Quantity Field Quantity V, I, Pressure, Force/Torque

Every 3dB increase in sound represents doubling in sound intensity (power)

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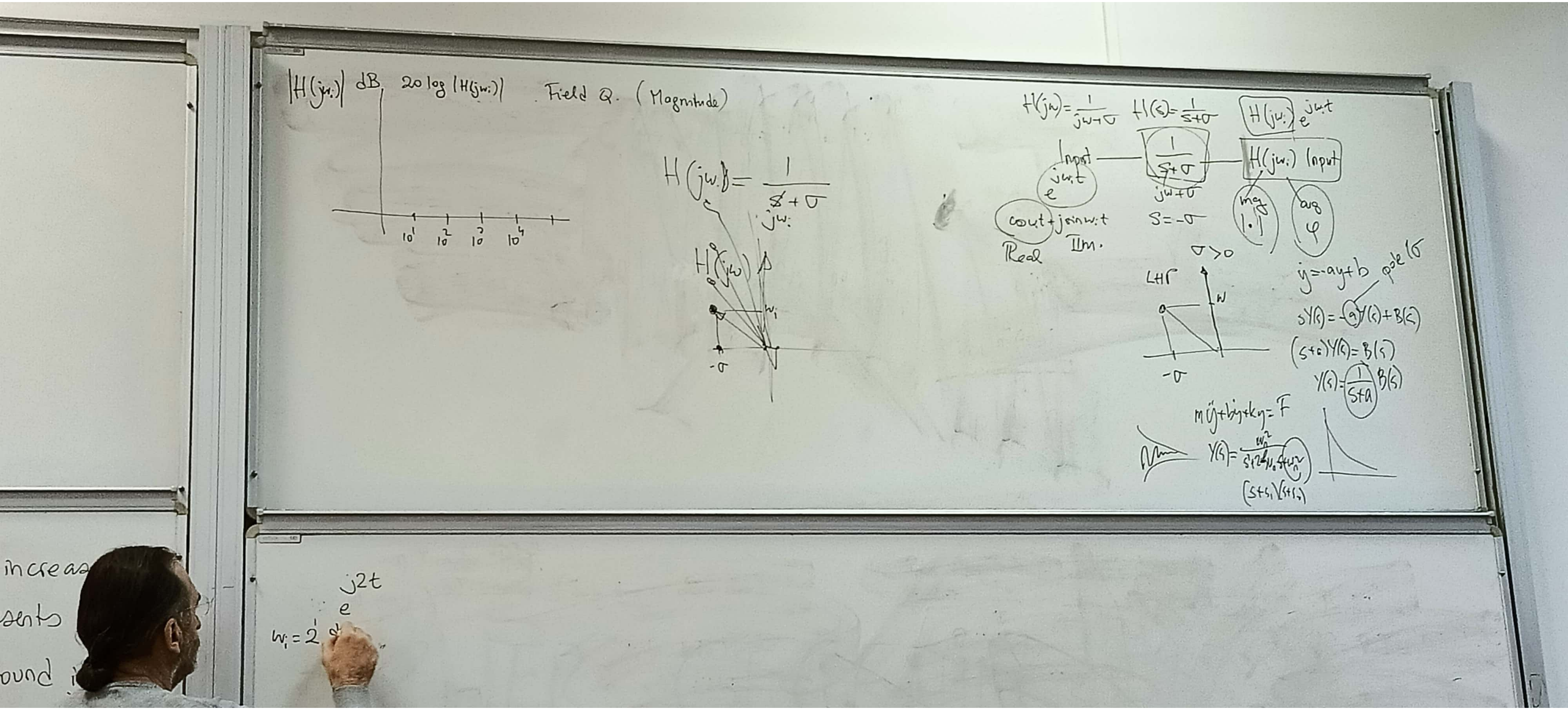
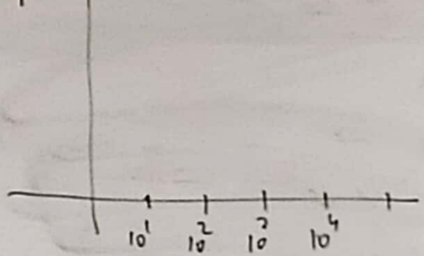
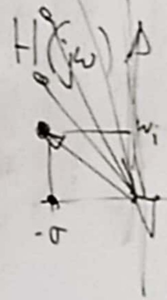

|H(jwi)| dB, 20 log |H(jwi)| Field Q. (Magnitude)
H(jwi) = 1/(s+σ)
jwi
10^1 10^2 10^3 10^4
H(jw)
-σ
wi
H(jw) = 1/(jw+σ)
H(s) = 1/(s+σ)
H(jwi) e^{jwit}
Input
e^{jwit}
1/(s+σ)
H(jwi) Input
jw+σ
S = -σ
mag |.|
arg φ
cos + j sin wit
Real
Im.
σ > 0
LHP
w
-σ
y' = -ay + b
pole σ
sY(s) = -aY(s) + B(s)
(s+a)Y(s) = B(s)
Y(s) = 1/(s+a) B(s)
my'' + by' + ky = F
Y(s) = wn^2/(s^2 + 2ζwn s + wn^2)
(s+s1)(s+s2)
e^{j2t}
wi = 2
increas
sents
ound i

$|H(j\omega_i)|$ dB, $20\log|H(j\omega_i)|$ Field Q. (Magnitude)



$$H(j\omega_i) = \frac{1}{s+\sigma}$$

$j\omega_i$



$H(j\omega) = \frac{1}{j\omega+\sigma}$ $H(s) = \frac{1}{s+\sigma}$ $\boxed{H(j\omega_i)}\, e^{j\omega_i t}$

Input — $\boxed{\frac{1}{s+\sigma}}$ — $\boxed{H(j\omega_i)\ \text{Input}}$

$e^{j\omega_i t}$ $j\omega+\sigma$

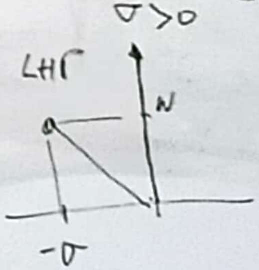
$s = -\sigma$

mag $|\cdot|$ arg φ

$\cos\omega_i t + j\sin\omega_i t$

Real Im.

$\sigma > 0$



$\dot{y} = -ay + b$ pole 1σ

$sY(s) = -aY(s) + B(s)$

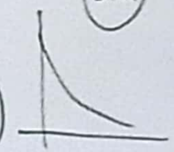
$(s+a)Y(s) = B(s)$

$Y(s) = \frac{1}{s+a} B(s)$

$m\ddot{y} + b\dot{y} + ky = F$

$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$(s+s_1)(s+s_2)$

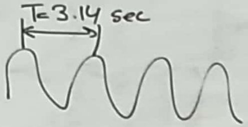


e^{j2t}

$w_i = 2 \ \frac{rad}{sec}$ —— $\boxed{H(s) = \frac{1}{s+2}}$ —— $\frac{?}{\lambda} e^{j2t}$

eigenvalue

$\dot{y} = -2y$



$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$H(s) \longrightarrow H(j\omega_i) = \frac{1}{j\omega_i + 2} = \frac{1}{j2+2}$$

phasor

$1.1 \angle \varphi$

$$H(j\omega_i) = H(j2) = \frac{1}{2\sqrt{2}} \angle -\frac{\pi}{4}$$

$$= \frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$$\frac{1}{a+jb} \cdot \frac{a-jb}{a-jb} = \frac{a-jb}{a^2+b^2} = \frac{a}{a^2+b^2} - j\frac{b}{a^2+b^2}$$

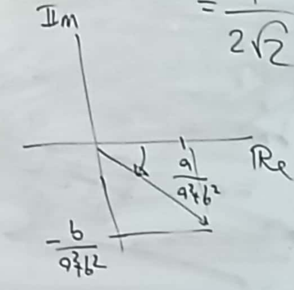
$$|\cdot| = \sqrt{\frac{a^2}{(a^2+b^2)^2} + \frac{b^2}{(a^2+b^2)^2}} = \sqrt{\frac{a^2+b^2}{(a^2+b^2)^2}} = \sqrt{\frac{1}{a^2+b^2}} = \sqrt{\frac{1}{4+4}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$\angle = \tan^{-1} \frac{\frac{-b}{a^2+b^2}}{\frac{a}{a^2+b^2}} = \tan^{-1} \frac{-b}{a}$$

$$= \tan^{-1} \frac{-2}{2} = \tan^{-1}(-1)$$

$$\varphi = -\frac{\pi}{4}$$



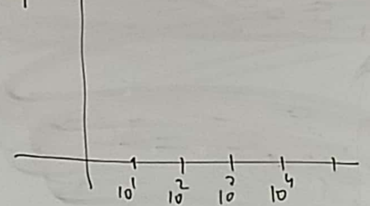
$(s+s_1)(s+s_2)$

e

tensity

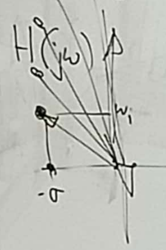
power)

$|H(j\omega_i)|$ dB, $20\log|H(j\omega_i)|$ Field Q. (Magnitude)



$$H(j\omega) = \frac{1}{\cancel{s}+\sigma}$$

$j\omega_i$



$H(j\omega) = \frac{1}{j\omega+\sigma}$ $H(s) = \frac{1}{s+\sigma}$ $\boxed{H(j\omega_i)}\, e^{j\omega_i t}$

Input —— $\boxed{\frac{1}{s+\sigma}}$ —— $\boxed{H(j\omega_i)\text{ Input}}$

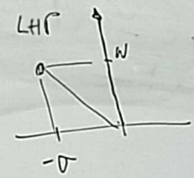
$e^{j\omega_i t}$ $j\omega+\sigma$

$\cos\omega_i t + j\sin\omega_i t$ $s=-\sigma$ mag $|\cdot|$ arg φ

Real Im.

$\sigma > 0$

LHP



$\dot{y} = -ay + b$ pole σ

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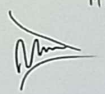
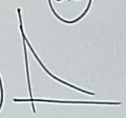


$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$(s+s_1)(s+s_2)$



$\dot{y} = -2y$

e^{j2t}

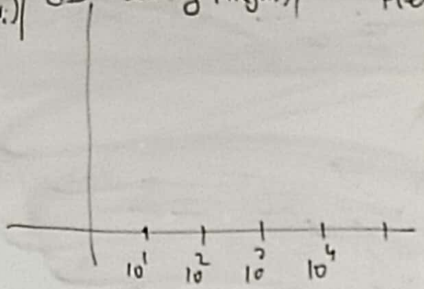
$\omega_i = 2$ ~~da~~ rad/sec —— $\boxed{H(s) = \frac{1}{s+2}}$ —— $\frac{?}{?}\, e^{j2t}$

$$\frac{1}{a+jb}\cdot\frac{a-jb}{a-jb} = \frac{a-jb}{a^2+b^2} = \frac{a}{a^2+b^2} - j\frac{b}{a^2+b^2}$$

$$|\cdot| = \sqrt{\frac{a^2}{(a^2+b^2)^2} + \frac{b^2}{(a^2+b^2)^2}} = \sqrt{\frac{a^2+b^2}{(a^2+b^2)^2}} = \sqrt{\frac{1}{a^2+b^2}} = \sqrt{\frac{1}{4+4}}$$

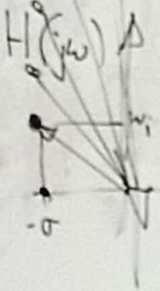
$-b$

$|H(j\omega_i)|$ dB, $20\log|H(j\omega_i)|$ Field Q. (Magnitude)



$$H(j\omega_i) = \frac{1}{s+\sigma}$$

$j\omega_i$



$H(j\omega) = \frac{1}{j\omega+\sigma}$ $H(s) = \frac{1}{s+\sigma}$ $\boxed{H(j\omega_i)}\, e^{j\omega_i t}$

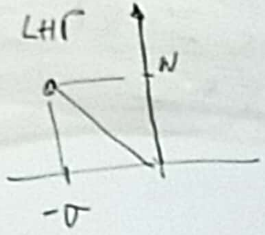
Input — $\boxed{\frac{1}{s+\sigma}}$ — $\boxed{H(j\omega_i)\ \text{Input}}$

$e^{j\omega_i t}$ $j\omega+\sigma$ mag $|\cdot|$ arg φ

cos + j sin $\omega_i t$ $s = -\sigma$

Real Im.

$\sigma > 0$



$\dot{y} = -ay + b$ pole

$sY(s) = -aY(s) + B(s)$

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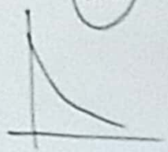
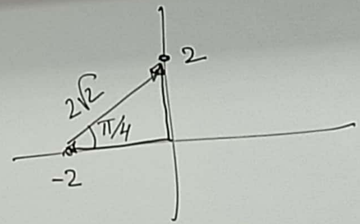
$(s+s_1)(s+s_2)$



	20	10
-6	0.501	0.25
-3	0.708	0.501
1	1.122	$1.258 = 10^{\frac{1}{10}}$

$$y(t) = \underbrace{\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}}_{\text{phasor / eigenvalue}} \underbrace{e^{j2t}}_{\text{input eigenvector}} = \underbrace{\frac{1}{2\sqrt{2}}}_{<1 \text{ attenuation}} e^{j(2t - \underbrace{\frac{\pi}{4}}_{\text{phase lag}})}$$



$\sigma = -2$

$\omega_1 = 2$

$$\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

Fundamental Rule of Algebra

Any n^{th} order system Can be represented by the product n 1^{st} order system

$$I(j\omega_i) \longrightarrow \boxed{G(s)} \longrightarrow G(j\omega_i)\, I(j\omega_i)$$

ratio of polynomials in s

$$G(s) = \frac{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_n s^0}{s^m + b_1 s^{m-1} + b_2 s^{m-3} + \cdots + b_n s^0}$$

$$\frac{\text{num}}{\text{denom.}} \text{ of TF} = \frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

z : zero

p : poles

$$= \frac{(M_{z_1} e^{j\varphi_{z_1}})(M_{z_2} e^{j\varphi_{z_2}}) \cdots M_{z_n} e^{j\varphi_{z_n}}}{M_{p_1} e^{j\varphi_{p_1}} \cdots M_{p_m} e^{j\varphi_{p_m}}} = \frac{M_{z_1} M_{z_2} \cdots M_{z_n}}{M_{p_1} M_{p_2} \cdots M_{p_m}} \; \frac{e^{j(\varphi_{z_1} + \varphi_{z_2} + \cdots + \varphi_{z_n})}}{e^{j(\varphi_{p_1} + \varphi_{p_2} + \cdots + \varphi_{p_m})}}$$

$$= \frac{\prod_{n=1}^{n} M_{z_n}}{\prod_{1}^{m} M_{p_m}} \, e^{j\left[\sum_{n=1}^{n} \varphi_{z_n} - \sum_{m=1}^{m} \varphi_{p_m}\right]}$$

Σ

-6	0.501	0.25	20	100	1000
-3	0.708	0.501			
1	1.122	$1.258 = 10^{\frac{1}{10}}$			

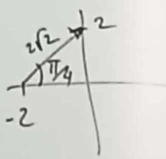
$$y(t) = \underbrace{\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}}_{\text{phasor/ eigenvalue}} \underbrace{e^{j2t}}_{\text{input eigenVector}} = \frac{1}{2\sqrt{2}} e^{j\left(2t - \frac{\pi}{4}\right)}$$

$\frac{\pi}{4}$: phase lag

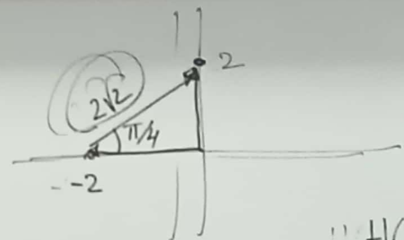
$\frac{1}{2\sqrt{2}} < 1$ attenuation

$$H(s) = \frac{1}{s+2}$$

e^{j2t}



$$H(j\omega_1) = \frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

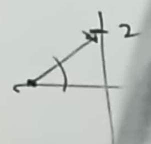


$\sigma = -2$

$\omega_1 = 2$

$$\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$H(s) = s-$

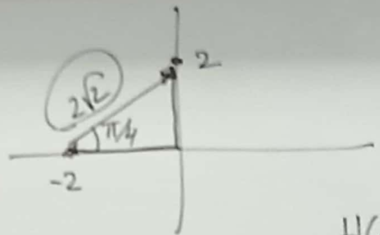


$H(j\omega_1) =$

-6	0.501	0.25	10	3.162	10
-3	0.708	0.501	20	10	100
1	1.122	$1.258 = 10^{\frac{1}{10}}$			

$$y(t) = \underbrace{\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}}_{\text{phasor / eigenvalue}} \underbrace{e^{j2t}}_{\text{input eigenVector}} = \underbrace{\frac{1}{2\sqrt{2}}}_{<1 \text{ attenuation}} \cdot e^{j\left(2t - \frac{\pi}{4}\right)}$$

phase lag



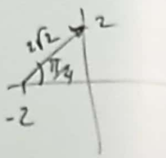
$$\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$\sigma = -2$

$\omega_1 = 2$

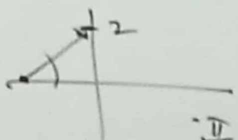
$$H(s) = \frac{1}{s+2}$$

e^{j2t}



$$H(j\omega) = \frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$$H(s) = s - z_1$$



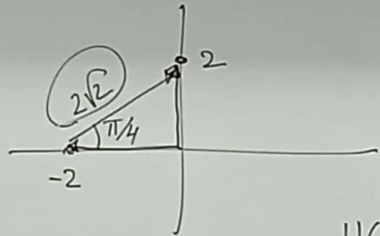
$$H(j\omega) = 2\sqrt{2}\, e^{j\frac{\pi}{4}}$$

$1 \qquad 1.122 \qquad 1.258 = 10^{\frac{1}{10}}$

$$y(t) = \underbrace{\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}}_{\text{phasor/ eigenvalue}} \underbrace{e^{j2t}}_{\text{input eigenvector}} = \frac{1}{2\sqrt{2}} e^{j\left(2t - \frac{\pi}{4}\right)}$$

$\frac{1}{2\sqrt{2}} < 1$ attenuation

phase lag



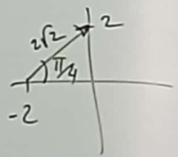
$$\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$\sigma = -2$

$\omega_1 = 2$

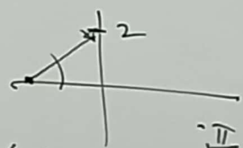
$$H(s) = \frac{1}{s+2}$$

e^{j2t}



$$H(j\omega_1) = \frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$$H(s) = s - z_1$$



$$H(j\omega_1) = 2\sqrt{2}\, e^{j\frac{\pi}{4}}$$

−6			20	10	100
−3	0.708	0.501			
1	1.122	$1.258 = 10^{\frac{1}{10}}$			

$$y(t) = \underbrace{\frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}}_{\text{phasor / eigenvalue}} \underbrace{e^{j2t}}_{\text{input eigenVector}} = \frac{1}{2\sqrt{2}} e^{j(2t - \frac{\pi}{4})}$$

$\frac{1}{2\sqrt{2}} < 1$ attenuation

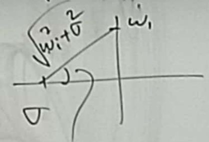
$-\frac{\pi}{4}$ phase lag

$H(s) = \frac{1}{s+\sigma} \to$ pole

pole : σ

input freq: ω_i

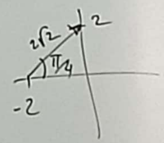
$$H(j\omega_i) = \frac{1}{\sqrt{\sigma^2+\omega_i^2}} e^{-j\tan^{-1}\frac{\omega_i}{\sigma}}$$



$\tan^{-1}\frac{\omega_i}{\sigma}$

$H(s) = \frac{1}{s+2}$

e^{j2t}



$$H(j\omega_i) = \frac{1}{2\sqrt{2}} e^{-j\frac{\pi}{4}}$$

$H(s) = s+\sigma$

ω_i input f.

$$H(j\omega_i) = \sqrt{\sigma^2+\omega_i^2}\, e^{j\tan^{-1}\frac{\omega_i}{\sigma}}$$

Fun…
Ru…

Any n…
Can be …
n 1^{st} …

pole: …

zero: $2\sqrt{…}$

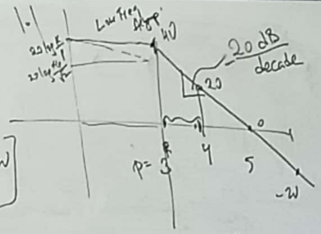
Single Pole system

$$|H(j\omega_i)| = \frac{K}{\sqrt{\omega_i^2 + p^2}} = \frac{K/p}{\sqrt{\frac{\omega_i^2}{p^2} + 1}} = \begin{cases} \frac{K}{p} & \omega_i \ll p \quad \text{VLF} \longrightarrow 20\log_{10} K/p \\ \frac{K/p}{\sqrt{2}} & \omega_i = p \longrightarrow 20\log \frac{K/p}{\sqrt{2}} \\ \frac{K/p}{\omega} & \omega_i \gg p \quad \text{VHF} \longrightarrow 20\left[\log K/p - \log \omega\right] \end{cases}$$

$$K/p\ \omega^{-1}$$

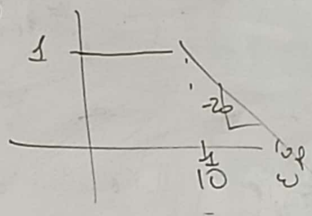
$$\log K/p\ \omega^{-1} = \log \frac{K}{p} - \log \omega$$

$$-20\ \frac{\text{dB}}{\text{decade}}$$



p=3 corner freq

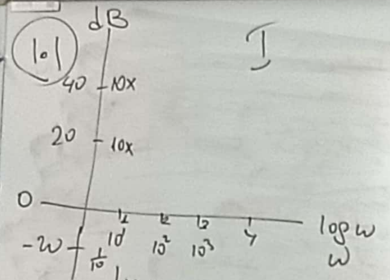
$$H(s) = \frac{100}{s+10}$$



$$\varphi(j\omega_i) = \tan^{-1} \frac{-\omega_i}{p}$$

I

$|\cdot|$



$N_{(dB)} = 20 \log \frac{|H_2|}{|H_1|} = 20 \log \frac{100}{1} = 40$

1 Input — [ ] — $H_2(j\omega_i)$ Input

$H_1(j\omega_i)$ 10

1

II $|\cdot|$ Fund. Rule of Alg.

$H(j\omega_i) \rightarrow \frac{(s-z)^n}{(s-p)^m}$

pole $\longrightarrow H(j\omega_i) = \frac{1}{\sqrt{\omega_i^2 + \sigma^2}}$ attenuation

ω_i

σ

$\angle \varphi(j\omega_i) = \tan^{-1} \frac{-\omega_i}{\sigma}$ lag

zero $\quad H(j\omega_i) = \sqrt{\omega_i^2 + \sigma^2}$ augment

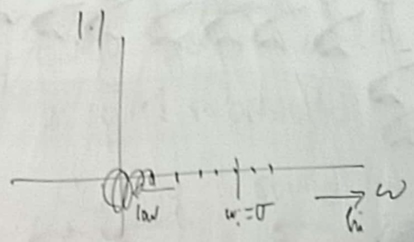
$\angle \varphi(j\omega_i) = \tan^{-1} \frac{\omega_i}{\sigma}$ lead

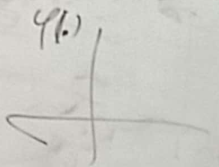
ω_i

σ

$|\cdot|$

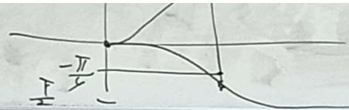
φ $\rightarrow$ log-log scale





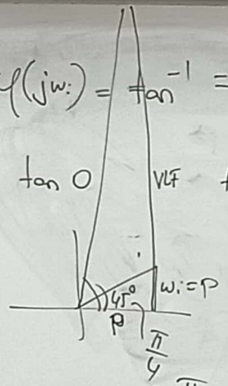
$\varphi(j\omega_i) = $

tan



$$\varphi(j\omega_i) = \tan^{-1} \frac{-\omega_i}{p}$$

$\tan 0$ VLF $\tan\varphi = 0$ $\varphi = 0$



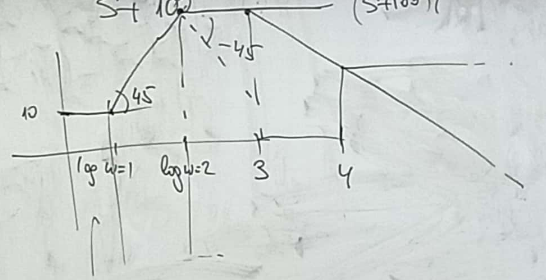
$\omega_i = p \longrightarrow \frac{\pi}{4}$

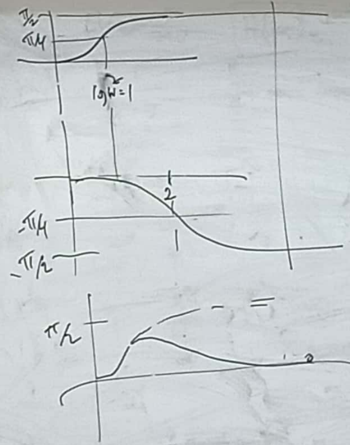
$\omega \gg p \longrightarrow \frac{\pi}{2}$

$H(\cdot)\ |\cdot| =$

$$H(s) = K\frac{s+10}{s+100}$$

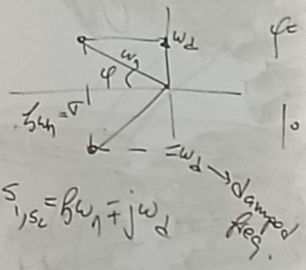
$\frac{(s+10)}{(s+100)(}$





$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \xrightarrow{s \to j\omega} G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n (j\omega) + \omega_n^2} = \frac{\omega_n^2 / \omega_n^2}{\underbrace{(\omega_n^2 - \omega^2)}_{Re} + j\underbrace{(2\zeta\omega_n\omega)}_{Im}}$$

evaluate G(s) @ input freq.



$$\varphi = \tan^{-1}\frac{\omega_d}{\zeta\omega_n} = \tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n\zeta} = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$|\cdot| = \sqrt{\omega_d^2 + \zeta^2\omega_n^2} = \sqrt{\omega_n^2(1-\zeta^2) + \zeta^2\omega_n^2} = \sqrt{\omega_n^2} = \omega_n$$

$$= \frac{\cancel{\omega_n^2}\ 1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right] + j\left[2\zeta\frac{\omega}{\omega_n}\right]}$$

$$s_{1}, s_2 = -\zeta\omega_n \mp j\omega_d$$

elegant

dactylic

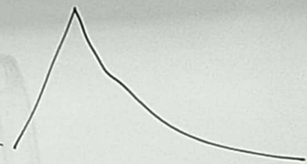
Socrates

Microbiology

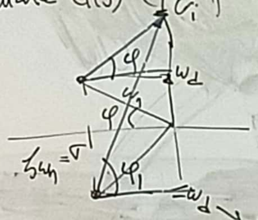
Microbiological

$H_1(j\omega)$

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\xrightarrow{s\to j\omega} G(j\omega)=\frac{\omega_n^2}{(j\omega)^2+2\zeta\omega_n(j\omega)+\omega_n^2}=\frac{\omega_n^2/\omega_n^2}{\underbrace{(\omega_n^2-\omega^2)}_{Re}+j\underbrace{(2\zeta\omega_n\omega)}_{Im}}$$



evaluate G(s) @ input freq.



$$\varphi=\tan^{-1}\frac{\omega_d}{\zeta\omega_n}=\tan^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n\zeta}=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$|\circ|=\sqrt{\omega_d^2+\zeta^2\omega_n^2}=\sqrt{\omega_n^2(1-\zeta^2)+\zeta^2\omega_n^2}=\sqrt{\omega_n^2}=\omega_n$$

$$=\frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]+j\left[2\zeta\frac{\omega}{\omega_n}\right]}$$

$s_1, s_2=\zeta\omega_n\mp j\omega_d$

$=\omega_d\to$ damped freq.

$$\frac{\prod_z}{\prod_p}e^{j\left(\frac{\varphi_1+\varphi_3}{z}-\frac{\varphi_1-\varphi_2}{p}\right)}$$

elegant

dactylic

Socrates

Microbiology

Microbiological

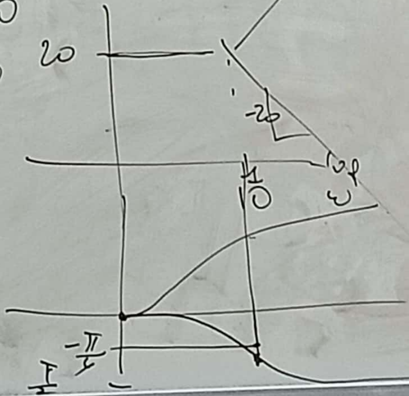
Single Pole system

$$|H(j\omega_i)| = \frac{K}{\sqrt{(\omega_i^2+p^2)\frac{p^2}{p^2}}} = \frac{K/p}{\sqrt{\frac{\omega_i^2}{p^2}+1}} =$$

$$H(s) = \frac{100}{s+10}$$

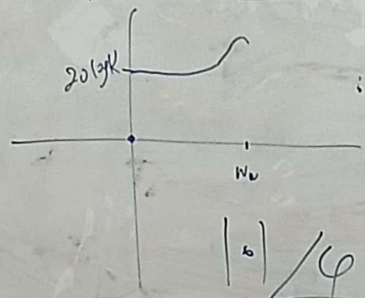
P / Z

$$\frac{K}{p} = \frac{100}{10} = 10$$



$$G(j\omega_i) = \frac{K}{\left[1-\left(\frac{\omega_i}{\omega_n}\right)^2\right] + j\left(2\zeta\frac{\omega_i}{\omega_n}\right)}$$

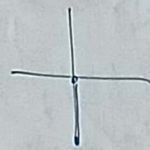
$|\cdot|$



$|\cdot| \angle\varphi$

$$20\log K - 20\log\frac{1}{1+j0} \qquad \omega_i \ll \omega_n$$

$$20\log\frac{K}{0+j2\zeta} \qquad \omega_i = \omega_n$$



$$20\log K - 20\log(2\zeta) \qquad \omega_i \gg \omega_n$$

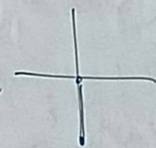
$$20\log\frac{K}{j2\zeta}\cdot\frac{j2\zeta}{j2\zeta} = 20\log\frac{j2K\zeta}{-4\zeta^2} = 20\log 2K\zeta - 20\log 4\zeta^2 = -40\log(4\zeta)$$

Single Pole system

$$|H(j\omega_i)| = \frac{K}{\sqrt{(\omega_i^2 + p^2)\frac{p^2}{p^2}}} = \frac{K/p}{\sqrt{\frac{\omega_i^2}{p^2} + 1}} =$$

$$G(j\omega_i) = \frac{K}{\left[1 - \left(\frac{\omega_i}{\omega_n}\right)^2\right] + j\left(2\zeta\left(\frac{\omega_i}{\omega_n}\right)\right)}$$

$$20\log K - 20\log\frac{1}{1+j0} \qquad \omega_i \ll \omega_n$$



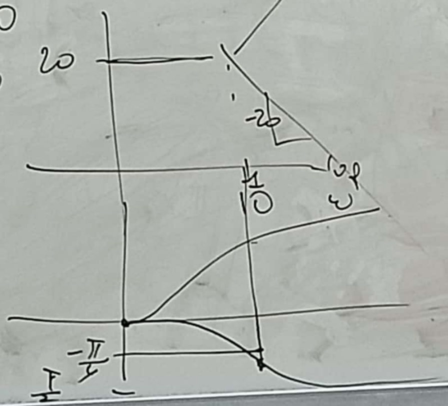
$$20\log\frac{K}{0+j2\zeta} \qquad \omega_i = \omega_n$$

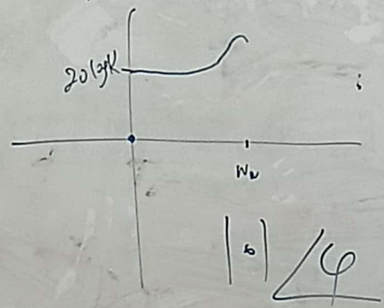
$$20\log K - 20\log(2\zeta) \qquad \omega_i \gg \omega_n$$

$$20\log\frac{K}{j2\zeta}\cdot\frac{j2\zeta}{j2\zeta} = 20\log\frac{j2K\zeta}{-4\zeta^2} = 20\log 2K\zeta - 20\log 4\zeta^2 = -40\log(4\zeta)$$

$$H(s) = \frac{100}{s+10} \qquad P/Z$$

$$\frac{K}{p} = \frac{100}{10} = 10$$





II |o| Fund. P.C

$H(j\omega_i) \rightarrow$

pole $\longrightarrow H(j\omega_i) = \frac{1}{\sqrt{\omega_i^2+\sigma^2}}$

$\angle \varphi(j\omega_i)\ \tan^{-1}\frac{-\omega}{\sigma}$

zero $H(j\omega_i) = \sqrt{\omega_i^2+}$

$\angle \varphi(j\omega_i) = \tan^{-1}$

$\frac{|H_2|}{|H_1|} = 20\log\frac{100}{1} = 40$

$H_2(j\omega_i)$ Input 10

$$\frac{K}{\left[1-\left(\frac{\omega_i}{\omega_n}\right)^2\right]+j\left(2\zeta\frac{\omega_i}{\omega_n}\right)}$$

$$\frac{1-\left(\frac{\omega_i}{\omega_n}\right)^2-j\left(2\zeta\frac{\omega_i}{\omega_n}\right)}{\left[1-\left(\frac{\omega_i}{\omega_n}\right)^2\right]^2+4\zeta^2\left(\frac{\omega_i}{\omega_n}\right)^2}$$

$\varphi(j\omega_i) = \tan^{-1}\frac{-\omega_i}{p}$

$\tan 0$ VLF $\tan\varphi = 0$ $\varphi = 0$

$\omega_i = p$

$\omega_i = p \longrightarrow \frac{\pi}{4}$

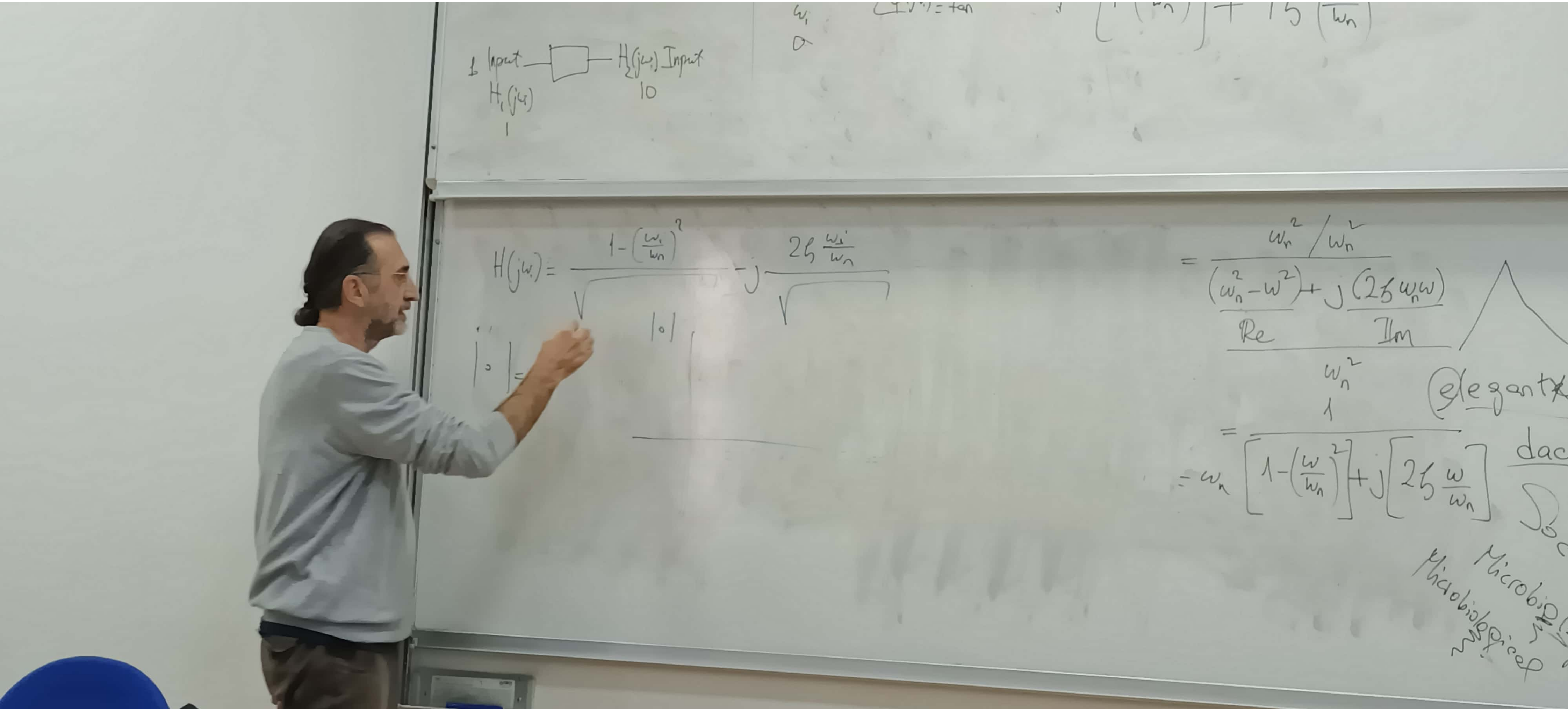
$\omega \gg p \longrightarrow \frac{\pi}{2}$

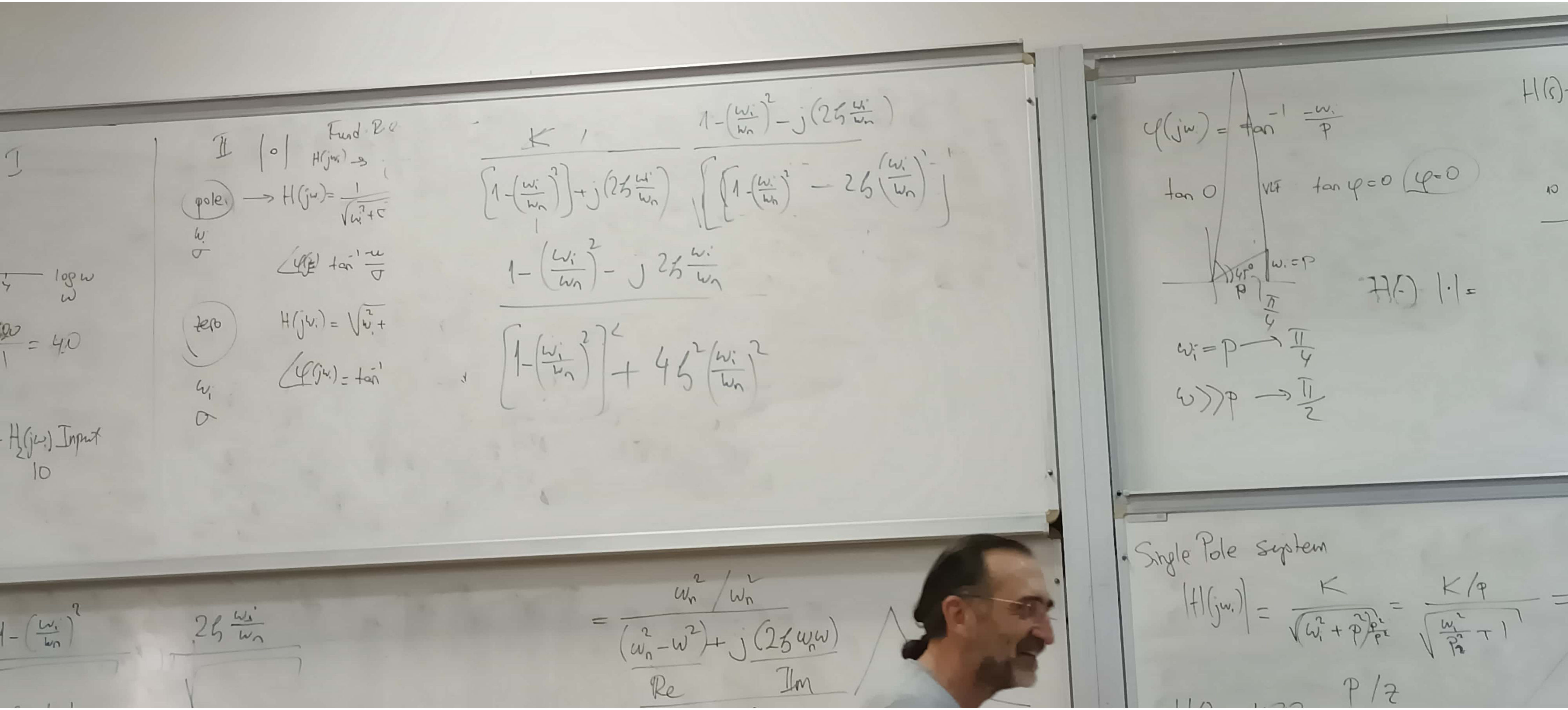
Single Pole system

$$|H|(j\omega_i) = \frac{K}{\sqrt{\omega_i^2+p^2}} = \frac{K/}{\sqrt{\frac{\omega_i^2}{p^2}}}$$

$$\frac{1-\left(\frac{\omega_i}{\omega_n}\right)^2}{} - j\frac{2\zeta\frac{\omega_i}{\omega_n}}{}$$

$$= \frac{\omega_n^2/\omega_n^2}{(\omega_n^2-\omega^2)+j(2\zeta\omega_n\omega)}$$


Input
H1(jw)
1
H2(jw) Input
10
H(jw) =
Re
Im
elegant
Microbiological


Single Pole system
pole
zero
log ω
Input
ωᵢ = p
ω ≫ p
Re
Im
P / Z

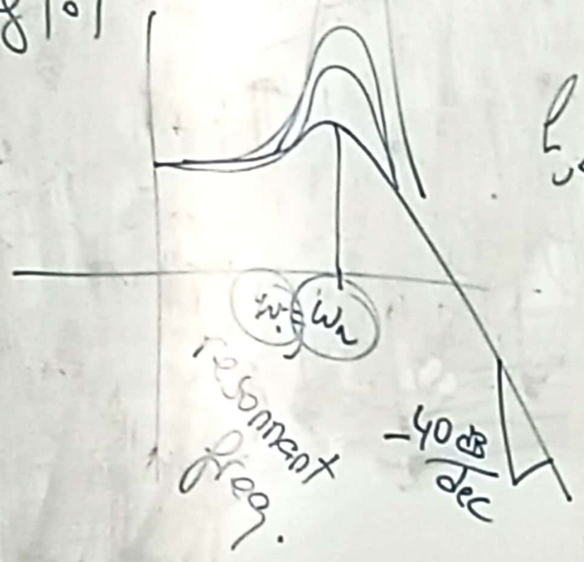
$$H(j\omega_i) \doteq \frac{1-\left(\frac{\omega_i}{\omega_n}\right)^2}{\sqrt{\quad}} - j\,\frac{2\zeta\frac{\omega_i}{\omega_n}}{\sqrt{\quad}} = \qquad = \frac{\omega_n^2 / \omega_n^2}{\underbrace{(\omega_n^2-\omega^2)}_{Re} + j\,(\ldots}$$

$$= \frac{\omega_n^2}{\ldots}$$

$$= \frac{1}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right.\ldots}$$

$$= \omega_n$$

$$|\circ| =$$



$\zeta << 1$