12.04.2021

4 Nisan 2021 Pazar 19:12

# START: 09:15

1. Bisection Method (+)
2. False Position Method (+)
3. Newton-Raphson Method (+)
4. Secant Method (Today)

Need:
1. Notebook,
2. Calculator
3. Excel

① $f(x) = e^{-x} - x$

② $f(x) = x^3 - 5x + 1$

③ $\sqrt{5}$

Ex $f(x) = e^{-x} - x$ Newton-Raphson $x_0 = 0$

$$f'(x) = -e^{-x} - 1$$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

$$f(0) = 1$$

$$f'(0) = -2$$

n=0 $x_1 = x_0 - \frac{f(0)}{f'(0)} = 0 - \frac{1}{-2} = \frac{1}{2}$ $\boxed{x_1 = 0{,}5}$

n=1 $x_2 = x_1 - \frac{f(0{,}5)}{f'(0{,}5)} =$

❖ **Örnek :** **$f(x) = x^3 + 4x^2 - 3$** denklemini Newton-Raphson yöntemini kullanarak $x_0 = 0.7$ değerinden başlayarak, tol=5e-5 için çözünüz?

❶ $f(x) = x^3 + 4x^2 - 3 \Rightarrow f'(x) = 3x^2 + 8x$

❷ $x_0 = 0.7$ 'den başlayarak köke doğru yaklaşalım

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

❸ Durdurma Kriteri (Hata Sınırlaması) $|x_{i+1} - x_i| < \varepsilon_s$ yada iterasyon

$$x_1 = 0.7 - \frac{f(0.7)}{f'(0.7)} = 0.7 - \frac{(-0{,}6970)}{7{,}07} = 0{,}7986 \qquad |x_1 - x_0| = |0{,}7986 - 0.7| = 0.0986$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0{,}7986 - \frac{0{,}0602}{8{,}3019} = 0{,}7913 \qquad |x_2 - x_1| = |0{,}7913 - 0{,}7986| = 0.0073$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0{,}7913 - \frac{0{,}0003}{8{,}2092} = 0{,}7913 \qquad |x_3 - x_2| = |0{,}7913 - 0{,}7913| = 0.00004$$

3. Newton's equation $y^3 - 2y - 5 = 0$ has a root near $y = 2$. Starting with $y_0 = 2$, compute $y_1$, $y_2$, and $y_3$, the next three Newton-Raphson estimates for the root.

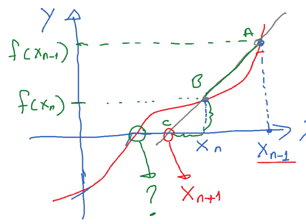
*Solution*: Let $f(y) = y^3 - 2y - 5$. Then $f'(y) = 3y^2 - 2$, and the Newton Method produces the recurrence

$$y_{n+1} = y_n - \frac{y_n^3 - 2y_n - 5}{3y_n^2 - 2} = \frac{2y_n^3 + 5}{3y_n^2 - 2}$$

(there was no good case for simplification here). Start with the estimate $y_0 = 2$. Then $y_1 = 21/10 = 2.1$. It follows that (to calculator accuracy) $y_2 = 2.094568121$ and $y_3 = 2.094551482$. These are almost the numbers that Newton obtained (see the notes). But Newton in effect used a rounded version of $y_2$, namely 2.0946.

## SECANT METHOD

2 starting point




$x_0 =$

$x_{-1} =$

(A-B) Slope $= \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$

||

(B-C) Slope $= \frac{f(x_n) - 0}{x_n - x_{(n+1)}}$

$$\frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n} \times \frac{f(x_n)}{x_n - x_{(n+1)}}$$

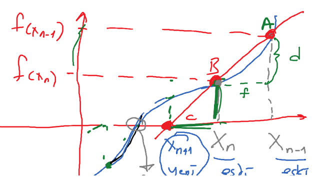
$$(x_n - x_{(n+1)})(f(x_{n-1}) - f(x_n)) = f(x_n)(x_{n-1} - x_n)$$

$$(x_n - x_{(n+1)}) = \frac{f(x_n)(x_{n-1} - x_n)}{f(x_{n-1}) - f(x_n)}$$

$$\boxed{x_{(n+1)} = x_n - \frac{f(x_n)(x_{n-1} - x_n)}{f(x_{n-1}) - f(x_n)}}$$

STOP Condition $|x_{n+1} - x_n| < \varepsilon$

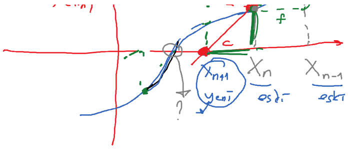
Example A real root of the equation $f(x) = x^3 - 5x + 1$ lies interval (0,1) perform four iteration of the secant method




A−B ⇒ C

B−C ⇒ D

(A−B) Slope $= \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$




(A–B)
$$\text{Slope} = \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$$

(B–C)
$$\text{slope} = \frac{f(x_n)}{x_n - x_{n+1}}$$

$$\frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n} = \frac{f(x_n)}{x_n - x_{n+1}}$$

Example $\quad f(x) = x^3 - 5x + 1 \quad$ (0,1)

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)\,(x_{n-1} - x_n)}{f(x_{n-1}) - f(x_n)}}$$

$\boxed{x_{(-1)} = 0, \quad x_{(0)} = 1}$

$n=1$

$n=0$

$$x_1 = x_0 - \frac{f(x_0)\,(x_{-1} - x_0)}{f(x_{-1}) - f(x_0)} \qquad x_{-1} = 0, \quad x_{(0)} = 1$$

$$x_1 = 1 - \frac{f(1)\,(0-1)}{f(0) - f(1)} \qquad f(0) = 1, \quad f(1) = -3$$

$f(x) = x^3 - 5x + 1$

$$x_1 = 1 - \frac{-3\,(-1)}{1 - -3} = 1 - \frac{3}{4} = 0{,}25$$

old point $\quad x_0 = 1 \quad x_1 = 0{,}25 \quad x_2 = ?$

$n=1$

$$x_2 = x_1 - \frac{f(x_1)\cdot(x_0 - x_1)}{f(x_0) - f(x_1)} \qquad f(1) = \qquad f(0{,}25) =$$

$$= 0{,}25 - \frac{f(0{,}25)\,(1 - 0{,}25)}{f(1) - f(0{,}25)}$$

$$x_2 = 0{,}186440678$$

| x | fx | |
|---|---|---|
| 0 | 1 | |
| 1 | -3 | 0,25 |
| 0,25 | -0,234375 | 0,186440678 |
| 0,186440678 | 0,074277312 | 0,201736256 |
| 0,201736256 | -0,000471116 | 0,201639853 |
| 0,201639853 | -8,64229E-07 | 0,201639676 |
| 0,201639676 | 1,03527E-11 | 0,201639676 |

old point $\quad x_1 = 0{,}25 \quad x_2 = 0{,}186440678 \quad x_3 = ?$

new point

$\boxed{n=2}$

$$x_3 = x_2 - \frac{f(x_2)\cdot(x_1 - x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = 0{,}186440678 - \frac{0{,}074277312\,(0{,}25 - 0{,}186440678)}{(-0{,}234375) - (0{,}074277312)}$$

$$x_3 = 0{,}201736256$$

Newton–R $\qquad\qquad$ Secant

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad\qquad x_{n+1} = x_n - \frac{f(x_n)\,(x_{n-1} - x_n)}{f(x_{n-1}) - f(x_n)}$$

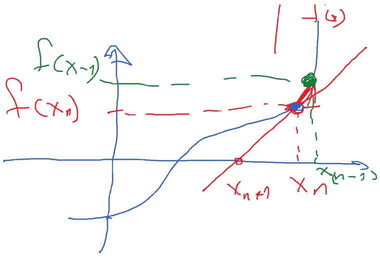
$\text{slope} = f'(x)$

$f(x_{n-1})$ $\qquad f(x)$

$$\text{slope} = \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$$

$$\text{slope} = f'(x)$$

$$\text{slope} = \frac{f(x_n)}{x_n - x_{(n+1)}}$$




$$\boxed{\text{slope}} = \frac{f(x_{n-1}) - f(x_n)}{x_{(n-1)} - x_n}$$

türev

$$x_{n+1} = x_n - \frac{f(x_n)}{\dfrac{f(x_{n-1}) - f(x_n)}{x_{(n-1)} - x_n}} = x_n - \frac{f(x_n)(x_{n-1} - x_n)}{f(x_{n-1}) - f(x_n)}$$

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Find an approximation to $\sqrt{5}$ correct to four decimal places using the secant method.

$$\sqrt{5} = ?$$

$$\boxed{x = \sqrt{5}}$$

$$x^2 = 5$$

$$\boxed{x^2 - 5 = 0}$$ function

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