

# Thermodynamics of Equilibrium

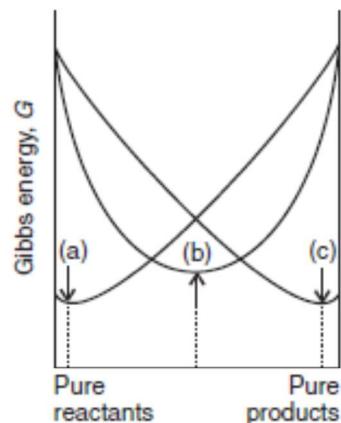
- Chemical thermodynamics is used to predict whether a mixture of reactants has a spontaneous tendency to change into products, to predict the composition of the reaction mixture at equilibrium, and to predict how that composition will be modified by changing the conditions.
- In biology, life is the avoidance of equilibrium, and the attainment of equilibrium is death, but knowing whether equilibrium lies in favor of reactants or products under certain conditions is a good indication of the feasibility of a biochemical reaction.
- Indeed, the material we cover in this chapter is of crucial importance for understanding the mechanisms of oxygen transport in blood, metabolism, and all the processes going on inside organisms.
- However, normally *thermodynamics is silent about the rates of reaction*.
- All it can do is to identify whether a particular reaction mixture has a tendency to form products; it cannot say whether that tendency will ever be realized.

# The Reaction Gibbs Energy

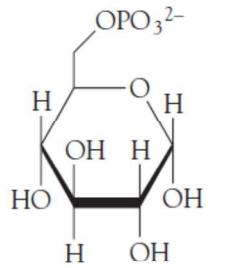
- The thermodynamic criterion for spontaneous change at constant temperature and pressure is  $\Delta G < 0$ .
- Thus, at constant temperature and pressure, a reaction mixture tends to adjust its composition until its Gibbs energy is a minimum.

The variation of Gibbs energy of a reaction mixture with progress of the reaction, pure reactants on the left and pure products on the right.

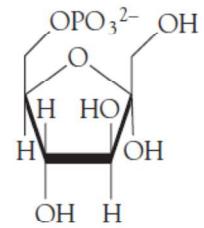
- (a) This reaction “does not go”: the minimum in the Gibbs energy occurs very close to the reactants.
- (b) This reaction reaches equilibrium with approximately equal amounts of reactants and products present in the mixture.
- (c) This reaction goes almost to completion, as the minimum in Gibbs energy lies very close to pure products.



- To explore metabolic processes, we need a measure of the driving power of a chemical reaction, and to understand the chemical composition of cells, we need to know what those compositions would be if the reactions taking place in them had reached equilibrium.
- Let's consider the isomerism of glucose-6-phosphate (G6P) to fructose-6-phosphate (F6P), which is an early step in the anaerobic breakdown of glucose:



1 Glucose-6-phosphate



2 Fructose-6-phosphate

- Suppose that in a short interval while the reaction is in progress, the amount of G6P changes infinitesimally by  $-dn$ .
- As a result of this change in amount, the contribution of G6P to the total Gibbs energy of the system changes by  $-\mu_{G6P}dn$ , where  $\mu_{G6P}$  is the chemical potential (the partial molar Gibbs energy) of G6P in the reaction mixture.
- In the same interval, the amount of F6P changes by  $+dn$ , so its contribution to the total Gibbs energy changes by  $+\mu_{F6P}dn$ , where  $\mu_{F6P}$  is the chemical potential of F6P.
- The change in Gibbs energy of the system is

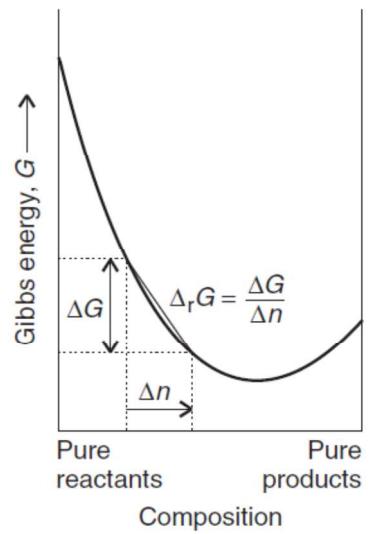
$$dG = \mu_{F6P}dn - \mu_{G6P}dn$$

- On dividing through by  $dn$ , we obtain the **reaction Gibbs energy**,  $\Delta_rG$ :

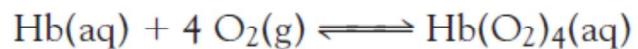
$$\frac{dG}{dn} = \mu_{F6P} - \mu_{G6P} = \Delta_rG$$

- There are two ways to interpret  $\Delta_r G$ :
- First, it is the difference of the chemical potentials of the products and reactants *at the composition of the reaction mixture*.
- Second, we can think of  $\Delta_r G$  as the derivative of  $G$  with respect to  $n$ , or the slope of the graph of  $G$  plotted against the changing composition of the system.

$$\Delta_r G = \frac{\Delta G}{\Delta n}$$



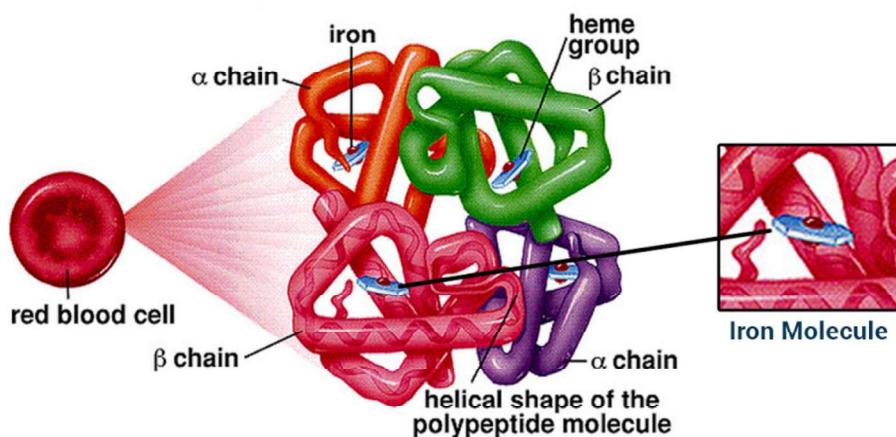
- Let's now consider the binding of oxygen to hemoglobin which provides a slightly more complicated example.



- Notice that both reactions that we analyze are specific examples of a general reaction of the form



- with arbitrary physical states.



- If the amount of Hb changes by  $-dn$ , then from the reaction stoichiometry we know that the change in the amount of O<sub>2</sub> will be  $-4dn$  and the change in the amount of Hb(O<sub>2</sub>)<sub>4</sub> will be  $+dn$ .
- Each change contributes to the change in the total Gibbs energy of the mixture, and the overall change will be:

$$\begin{aligned}\Delta G &= \mu_{\text{Hb}(\text{O}_2)_4} \times dn - \mu_{\text{Hb}} \times dn - \mu_{\text{O}_2} \times 4dn \\ &= (\mu_{\text{Hb}(\text{O}_2)_4} - \mu_{\text{Hb}} - 4\mu_{\text{O}_2})dn\end{aligned}$$

- where the  $\mu_j$  are the chemical potentials of the species in the reaction mixture.
- In this case, therefore, the reaction Gibbs energy is:

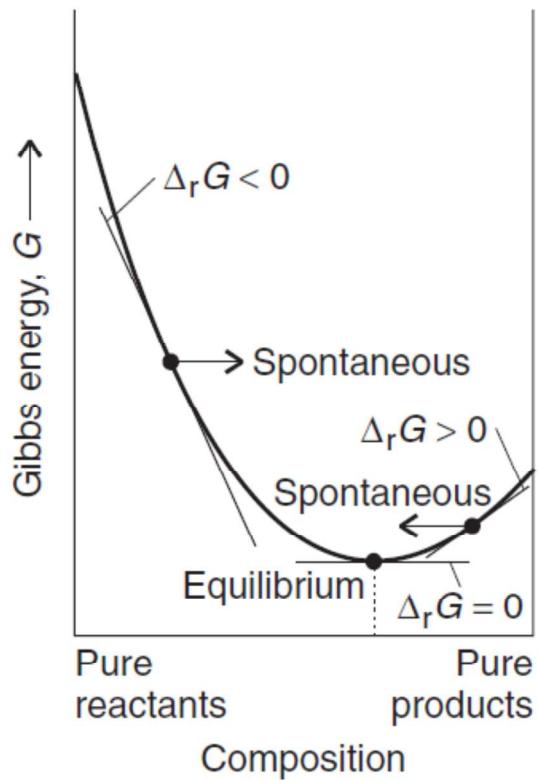
$$\Delta_r G = \frac{dG}{dn} = \mu_{\text{Hb}(\text{O}_2)_4} - (\mu_{\text{Hb}} + 4\mu_{\text{O}_2})$$

- Note that each chemical potential is multiplied by the corresponding stoichiometric coefficient and that reactants are subtracted from products.
- For the general reaction below:



$$\Delta_r G = (c\mu_C + d\mu_D) - (a\mu_A + b\mu_B)$$

- The chemical potential of a substance depends on the composition of the mixture in which it is present and is high when its concentration or partial pressure is high.
- Therefore,  $\Delta_r G$  changes as the composition changes.
- Remember that  $\Delta_r G$  is the *slope* of  $G$  plotted against composition.
- We see that  $\Delta_r G < 0$  and the slope of  $G$  is negative (down from left to right) when the mixture is rich in the reactants A and B because A and B are high.
- Conversely,  $\Delta_r G > 0$  and the slope of  $G$  is positive (up from left to right) when the mixture is rich in the products C and D because C and D are high.
- At compositions corresponding to  $\Delta_r G < 0$  the reaction tends to form more products; where  $\Delta_r G > 0$ , the reverse reaction is spontaneous, and the products tend to decompose into reactants.
- Where  $\Delta_r G = 0$  (at the minimum of the graph where the derivative is zero), the reaction has no tendency to form either products or reactants. In other words, the reaction is at equilibrium.
- That is, the criterion for chemical equilibrium at constant temperature and pressure is  $\Delta_r G = 0$ .



# The variation of $\Delta rG$ with composition

- *The reactants and products in a biological cell are rarely at equilibrium, so we need to know how the reaction Gibbs energy depends on their concentrations.*
- Our starting point is the general expression for the composition dependence of the chemical potential that we derived previously:

$$\mu_J = \mu_J^\ominus + RT \ln a_J$$

- where  $a_J$  is the activity of the species J.

- When we are dealing with systems that may be treated as ideal, which will be the case throughout this class, we use the identifications given in the below table.
- For solutes in an ideal solution,  $a_J = [J]/c^\ominus$ , the molar concentration of J relative to the standard value  $c = 1 \text{ mol L}^{-1}$ .
- For perfect gases,  $a_J = p_J/p^\ominus$ , the partial pressure of J relative to the standard pressure  $p = 1 \text{ bar}$ .
- For pure solids and liquids,  $a_J = 1$ .

**Table 3.3** Activities and standard states\*

| Substance | Standard state                                | Activity, $a$   |
|-----------|---|-----------------|
| Solid     | Pure solid, 1 bar                             | 1               |
| Liquid    | Pure liquid, 1 bar                            | 1               |
| Gas       | Pure gas, 1 bar                               | $p/p^\ominus$   |
| Solute    | Molar concentration of $1 \text{ mol L}^{-1}$ | $[J]/c^\ominus$ |

$p^\ominus = 1 \text{ bar} (= 10^5 \text{ Pa})$ ,  $c^\ominus = 1 \text{ mol L}^{-1} (= 1 \text{ mol dm}^{-3})$ .

\*Activities are for perfect gases and ideal-dilute solutions; all activities are dimensionless.

- When we substitute below equation

$$\mu_J = \mu_J^\ominus + RT \ln a_J$$

- into

$$\Delta_r G = (c\mu_C + d\mu_D) - (a\mu_A + b\mu_B)$$

- we obtain:

$$\begin{aligned}\Delta_r G &= \{c(\mu_C^\ominus + RT \ln a_C) + d(\mu_D^\ominus + RT \ln a_D)\} \\ &\quad - \{a(\mu_A^\ominus + RT \ln a_A) + b(\mu_B^\ominus + RT \ln a_B)\} \\ &= \{(c\mu_C^\ominus + d\mu_D^\ominus) - (a\mu_A^\ominus + b\mu_B^\ominus)\} \\ &\quad + RT\{c \ln a_C + d \ln a_D - a \ln a_A - b \ln a_B\}\end{aligned}$$

- The first term on the right in the second equality is the **standard reaction Gibbs energy**,  $\Delta_r G^\ominus$ :

$$\Delta_r G^\ominus = \{c\mu_C^\ominus + d\mu_D^\ominus\} - \{a\mu_A^\ominus + b\mu_B^\ominus\}$$

- Because the standard states refer to the pure materials, the standard chemical potentials in this expression are the standard molar Gibbs energies of the (pure) species. Therefore:

$$\Delta_r G^\ominus = \{cG_m^\ominus(C) + dG_m^\ominus(D)\} - \{aG_m^\ominus(A) + bG_m^\ominus(B)\}$$

- At this stage, therefore, we know that

$$\Delta_r G = \Delta_r G^\ominus + RT\{c \ln a_C + d \ln a_D - a \ln a_A - b \ln a_B\}$$

- To make further progress:

$$\begin{aligned}
 c \ln a_C + d \ln a_D - a \ln a_A - b \ln a_B &= \ln a_C^c + \ln a_D^d - \ln a_A^a - \ln a_B^b \\
 &\quad \boxed{a \ln x = \ln x^a} \\
 &= \ln a_C^c a_D^d - \ln a_A^a a_B^b \\
 &\quad \boxed{\ln x + \ln y = \ln xy} \\
 &= \ln \frac{a_C^c a_D^d}{a_A^a a_B^b} \\
 &\quad \boxed{\ln x - \ln y = \ln x/y}
 \end{aligned}$$

- Therefore:

$$\Delta_r G = \Delta_r G^\ominus + RT \ln \frac{a_C^c a_D^d}{a_A^a a_B^b}$$

- To simplify the appearance of this expression still further, we introduce the (dimensionless) **reaction quotient**,  $Q$ , for the reaction:

$$Q = \frac{a_C^c a_D^d}{a_A^a a_B^b}$$

- We can now write the overall expression for the reaction Gibbs energy at any composition of the reaction mixture as:

$$\Delta_r G = \Delta_r G^\ominus + RT \ln Q$$

- Formulate the reaction quotients for the following reactions:
    - the isomerism of glucose-6-phosphate  $\text{G6P}(\text{aq}) \rightleftharpoons \text{F6P}(\text{aq})$
    - the binding of oxygen to hemoglobin  $\text{Hb}(\text{aq}) + 4 \text{O}_2(\text{g}) \rightleftharpoons \text{Hb}(\text{O}_2)_4(\text{aq})$
  - With  $[J]$  the numerical value of the molar concentration of  $J$  in moles per liter. So if  $[\text{F6P}] = 2.0 \text{ mmol L}^{-1}$ , corresponding to  $2.0 \times 10^{-3} \text{ mol L}^{-1}$ , we just write  $[\text{F6P}] = 2.0 \times 10^{-3}$  when using this expression.
- $$Q = \frac{a_{\text{F6P}}}{a_{\text{G6P}}} = \frac{[\text{F6P}]/c^\ominus}{[\text{G6P}]/c^\ominus} \quad Q = \frac{[\text{F6P}]}{[\text{G6P}]}$$
- $$Q = \frac{[\text{Hb}(\text{O}_2)_4]/c^\ominus}{([\text{Hb}]/c^\ominus)(p_{\text{O}_2}/p^\ominus)^4} \quad Q = \frac{[\text{Hb}(\text{O}_2)_4]}{[\text{Hb}]p_{\text{O}_2}^4}$$
- with  $p_J$  the numerical value of the partial pressure of  $J$  in bar (so if  $p_{\text{O}_2} = 2.0 \text{ bar}$ , we just write  $p_{\text{O}_2} = 2.0$  when using this expression).

- **Self exercise:** Write the reaction quotient for the esterification reaction  $\text{CH}_3\text{COOH} + \text{C}_2\text{H}_5\text{OH} \rightleftharpoons \text{CH}_3\text{COOC}_2\text{H}_5 + \text{H}_2\text{O}$ . (All four components are present in the reaction mixture as liquids: the mixture is not an aqueous solution.)

# Thermodynamics of Chemical Equilibrium

- *We need to be able to identify the equilibrium composition of a reaction so that we can discuss deviations from equilibrium systematically.*

# Thermodynamics of Chemical Equilibrium

- A **reversible reaction** can proceed in both the forward and backward directions.
- **Chemical equilibrium** is when the rate of the forward reaction equals the rate of the reverse reaction.
- All reactant and product concentrations are constant at equilibrium.



- At equilibrium, the reaction quotient has a certain value called the **equilibrium constant**,  $K$ , of the reaction:

$$K = \left( \frac{a_C^c a_D^d}{a_A^a a_B^b} \right)_{\text{equilibrium}}$$

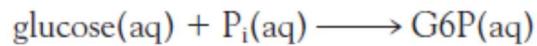
- Equilibrium constant,  $K$ , is a variable that describes a chemical reaction's tendency to proceed to completion, meaning all the reactants are converted to products.
- The relationship between the reaction Gibbs energy and the equilibrium constant is given in:

$$0 = \Delta_r G^\ominus + RT \ln K$$

$$\Delta_r G^\ominus = -RT \ln K$$

This is one of the most important equations in the whole of chemical thermodynamics. Its principal use is to predict the value of the equilibrium constant of any reaction from tables of thermodynamic data or by measuring the equilibrium constant of a reaction.

- Calculate the equilibrium constant of the first step in the glycolysis event given that the standard reaction Gibbs energy for the reaction is 14.0 kJ/mol at 37°C.



$$\begin{aligned}\ln K &= -\frac{\Delta_r G^\ominus}{RT} = -\frac{1.40 \times 10^4 \text{ J mol}^{-1}}{(8.314 \text{ } 47 \text{ J K}^{-1} \text{ mol}^{-1}) \times (310 \text{ K})} \\ &= -\frac{1.40 \times 10^4}{8.314 \text{ } 47 \times 310}\end{aligned}$$

$$K = e^{-\frac{1.40 \times 10^4}{8.314 \text{ } 47 \times 310}} = 4.4 \times 10^{-3}$$

- **Self-exercise:** Calculate the equilibrium constant of the reaction  $\text{N}_2(\text{g}) + 3 \text{H}_2(\text{g}) \leftrightarrow 2 \text{NH}_3(\text{g})$  at 25°C, given that  $\Delta rG = -32.90 \text{ kJ mol}^{-1}$ .

- According to below equation

$$\Delta_r G^\ominus = -RT \ln K$$

- $K > 1$  if  $\Delta_r G^\ominus < 0$ .  $K > 1$  implies that products are dominant at equilibrium, so we can conclude that a reaction is thermodynamically feasible if  $\Delta_r G^\ominus < 0$ .
- Conversely,  $K < 1$  if  $\Delta_r G^\ominus > 0$ , then we know that the reactants will be dominant in a reaction mixture at equilibrium if  $\Delta_r G^\ominus > 0$ . In other words, a reaction with  $\Delta_r G^\ominus > 0$  is not thermodynamically feasible.

- Below table summarizes the conditions under which  $\Delta_rG^\ominus < 0$  and  $K > 1$ .
- Because  $\Delta_rG^\ominus = \Delta_rH^\ominus - T\Delta_rS^\ominus$ , the standard reaction Gibbs energy is certainly negative if both  $\Delta_rH^\ominus < 0$  (an exothermic reaction) and  $\Delta_rS^\ominus > 0$  (a reaction system that becomes more disorderly, such as by forming a gas).
- The standard reaction Gibbs energy is also negative if the reaction is endothermic  $\Delta_rH^\ominus > 0$  and  $T\Delta_rS^\ominus$  is sufficiently large and positive.
- Note that for an endothermic reaction to have  $\Delta_rG^\ominus < 0$ , its standard reaction entropy *must* be positive.
- Moreover, the temperature must be high enough for  $T\Delta_rS^\ominus$  to be greater than  $\Delta_rH^\ominus$ .
- The switch of  $\Delta_rG^\ominus$  from positive to negative, corresponding to the switch from  $K > 1$  (the reaction “does not go”) to  $K < 1$  (the reaction “goes”), occurs at a temperature given by equating  $\Delta_rH^\ominus - T\Delta_rS^\ominus$  to 0, which gives

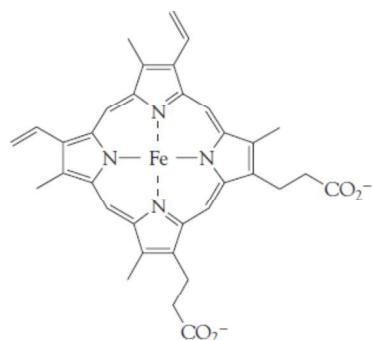
$$T = \frac{\Delta_rH^\ominus}{\Delta_rS^\ominus}$$

**Table 4.1** Thermodynamic criteria of spontaneity

- |   |   |
|---|---|
| 1. If the reaction is exothermic ( $\Delta_rH^\ominus < 0$ ) and $\Delta_rS^\ominus > 0$  | $\Delta_rG^\ominus < 0$ and $K > 1$ at all temperatures                                     |
| 2. If the reaction is exothermic ( $\Delta_rH^\ominus < 0$ ) and $\Delta_rS^\ominus < 0$  | $\Delta_rG^\ominus < 0$ and $K > 1$ provided that $T < \Delta_rH^\ominus/\Delta_rS^\ominus$ |
| 3. If the reaction is endothermic ( $\Delta_rH^\ominus > 0$ ) and $\Delta_rS^\ominus > 0$ | $\Delta_rG^\ominus < 0$ and $K > 1$ provided that $T > \Delta_rH^\ominus/\Delta_rS^\ominus$ |
| 4. If the reaction is endothermic ( $\Delta_rH^\ominus > 0$ ) and $\Delta_rS^\ominus < 0$ | $\Delta_rG^\ominus < 0$ and $K > 1$ at no temperature                                       |

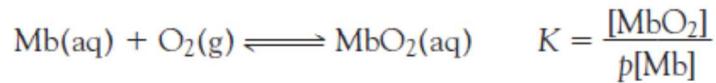
# Binding of oxygen to myoglobin and hemoglobin

- The protein myoglobin (Mb) stores O<sub>2</sub> in muscle, and the protein hemoglobin (Hb) transports O<sub>2</sub> in blood. These two proteins are related, for hemoglobin can be regarded, as a first approximation, as a tetramer of myoglobin.
- There are, in fact, slight differences in the peptide sequence of the myoglobin-like components of hemoglobin, but we can ignore them at this stage. In each protein, the O<sub>2</sub> molecule attaches to an iron ion in a heme group.



# Binding of oxygen to myoglobin and hemoglobin

- For our purposes here, we are concerned with the different equilibrium characteristics for the uptake of O<sub>2</sub> by myoglobin and hemoglobin.
- First, consider the equilibrium between Mb and O<sub>2</sub>:

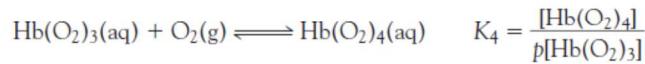
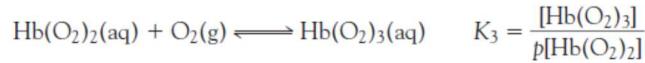
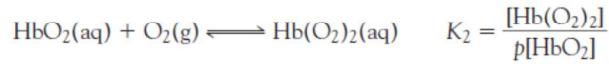
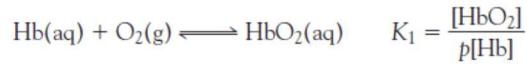


- where  $p$  is the numerical value of the partial pressure of O<sub>2</sub> gas in bar.
- It follows that the *fractional saturation*,  $s$ , the fraction of Mb molecules that are oxygenated, is

$$s = \frac{[\text{MbO}_2]}{[\text{Mb}]_{\text{total}}} = \frac{[\text{MbO}_2]}{[\text{Mb}] + [\text{MbO}_2]} = \frac{Kp}{1 + Kp}$$

# Binding of oxygen to myoglobin and hemoglobin

- Now consider the equilibrium between Hb and O<sub>2</sub>:



- To develop an expression for *s*, we express [Hb(O<sub>2</sub>)<sub>2</sub>] in terms of [HbO<sub>2</sub>] by using *K*<sub>2</sub>, then express [HbO<sub>2</sub>] in terms of [Hb] by using *K*<sub>1</sub>, and likewise for all the other concentrations of Hb(O<sub>2</sub>)<sub>3</sub> and Hb(O<sub>2</sub>)<sub>4</sub>. It follows that

$$\begin{aligned} [\text{HbO}_2] &= K_1 p[\text{Hb}] & [\text{Hb(O}_2)_2] &= K_1 K_2 p^2 [\text{Hb}] \\ [\text{Hb(O}_2)_3] &= K_1 K_2 K_3 p^3 [\text{Hb}] & [\text{Hb(O}_2)_4] &= K_1 K_2 K_3 K_4 p^4 [\text{Hb}] \end{aligned}$$

## Binding of oxygen to myoglobin and hemoglobin

- The total concentration of bound O<sub>2</sub> is

$$\begin{aligned} [\text{O}_2]_{\text{bound}} &= [\text{HbO}_2] + 2[\text{Hb}(\text{O}_2)_2] + 3[\text{Hb}(\text{O}_2)_3] + 4[\text{Hb}(\text{O}_2)_4] \\ &= (1 + 2K_2p + 3K_2K_3p^2 + 4K_2K_3K_4p^3)K_1p[\text{Hb}] \end{aligned}$$

- where we have used the fact that  $n$  O<sub>2</sub> molecules are bound in Hb(O<sub>2</sub>)<sub>n</sub>, so the concentration of bound O<sub>2</sub> in Hb(O<sub>2</sub>)<sub>2</sub> is 2[Hb(O<sub>2</sub>)<sub>2</sub>], and so on. The total concentration of hemoglobin is

$$[\text{Hb}]_{\text{total}} = (1 + K_1p + K_1K_2p^2 + K_1K_2K_3p^3 + K_1K_2K_3K_4p^4)[\text{Hb}]$$

- Because each Hb molecule has four sites at which O<sub>2</sub> can attach, the fractional saturation is

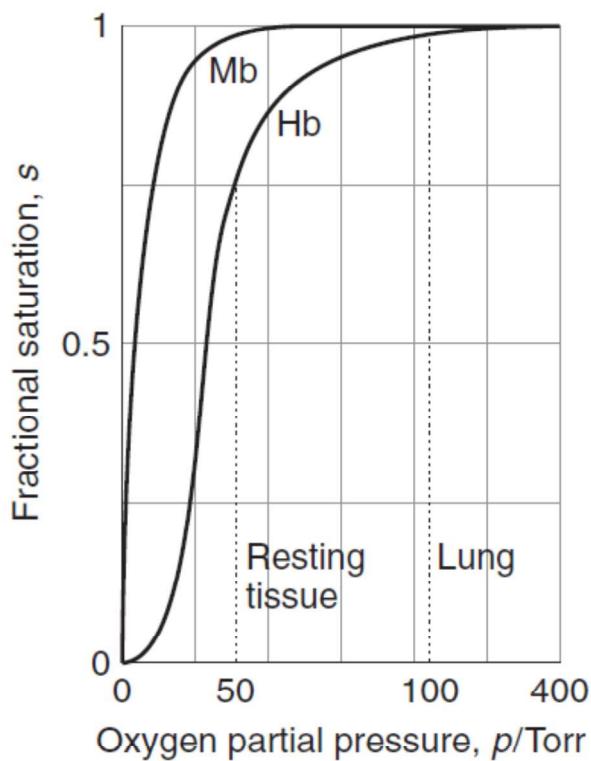
$$s = \frac{[\text{O}_2]_{\text{bound}}}{4[\text{Hb}]_{\text{total}}} = \frac{(1 + 2K_2p + 3K_2K_3p^2 + 4K_2K_3K_4p^3)K_1p}{4(1 + K_1p + K_1K_2p^2 + K_1K_2K_3p^3 + K_1K_2K_3K_4p^4)}$$

A reasonable fit of the experimental data can be obtained with  $K_1 = 0.01$ ,  $K_2 = 0.02$ ,  $K_3 = 0.04$ , and  $K_4 = 0.08$  when  $p$  is expressed in torr.

# Binding of oxygen to myoglobin and hemoglobin

- The binding of  $O_2$  to hemoglobin is an example of **cooperative binding**, in which the binding of a ligand (in this case  $O_2$ ) to a biopolymer (in this case Hb) becomes more favorable thermodynamically (that is, the equilibrium constant increases) as the number of bound ligands increases up to the maximum number of binding sites.
- Cooperative binding of  $O_2$  by hemoglobin is explained by an **allosteric effect**, in which an adjustment of the conformation of a molecule when one substrate binds affects the ease with which a subsequent substrate molecule binds.

- The differing shapes of the saturation curves for myoglobin and hemoglobin have important consequences for the way  $O_2$  is made available in the body: in particular, the greater sharpness of the Hb saturation curve means that Hb can load  $O_2$  more fully in the lungs and unload it more fully in different regions of the organism.
- In the lungs, where  $p \approx 105$  Torr (14 kPa),  $s \approx 0.98$ , representing almost complete saturation. In resting muscular tissue,  $p$  is equivalent to about 38 Torr (5 kPa), corresponding to  $s \approx 0.75$ , implying that sufficient  $O_2$  is still available should a sudden surge of activity take place.
- If the local partial pressure falls to 22 Torr (3 kPa),  $s$  falls to about 0.1. Note that the steepest part of the curve falls in the range of typical tissue oxygen partial pressure.
- Myoglobin, on the other hand, begins to release  $O_2$  only when  $p$  has fallen below about 22 Torr, so it acts as a reserve to be drawn on only when the Hb oxygen has been used up.



# The standard reaction Gibbs Energy

- It is a useful indicator of the energy available from catabolism to drive anabolic processes.
- The standard reaction Gibbs energy,  $\Delta_rG^\ominus$ , is defined as the difference in standard molar Gibbs energies of the products and the reactants weighted by the stoichiometric coefficients,  $v$ , in the chemical equation:

$$\Delta_rG^\ominus = \sum vG_m^\ominus(\text{products}) - \sum vG_m^\ominus(\text{reactants})$$

- However, when the standard molar Gibbs energies are not known, the standard reaction enthalpy can be calculated from standard enthalpies of formation and the standard reaction entropy can be calculated from Third-Law entropies. Then two quantities are combined by using:

$$\Delta_rG^\ominus = \Delta_rH^\ominus - T\Delta_rS^\ominus$$

- Evaluate the standard reaction Gibbs energy at 25°C for the reaction  $\text{CO}_2(\text{g}) + \text{H}_2\text{O}(\text{l}) \rightarrow \text{H}_2\text{CO}_3(\text{aq})$  catalyzed by the enzyme carbonic anhydrase in red blood cells.

$$\Delta_r H^\ominus = \sum \nu \Delta_f H^\ominus(\text{products}) - \sum \nu \Delta_f H^\ominus(\text{reactants})$$

$$\Delta_r S^\ominus = \sum \nu S_m^\ominus(\text{products}) - \sum \nu S_m^\ominus(\text{reactants})$$

$$\begin{aligned}\Delta_r H^\ominus &= \Delta_f H^\ominus(\text{H}_2\text{CO}_3, \text{aq}) - \{\Delta_f H^\ominus(\text{CO}_2, \text{g}) + \Delta_f H^\ominus(\text{H}_2\text{O}, \text{l})\} \\ &= -699.65 \text{ kJ mol}^{-1} - \{(-110.53 \text{ kJ mol}^{-1}) + (-285.83 \text{ kJ mol}^{-1})\} \\ &= -303.29 \text{ kJ mol}^{-1}\end{aligned}$$

$$\begin{aligned}\Delta_r S^\ominus &= S_m^\ominus(\text{H}_2\text{CO}_3, \text{aq}) - \{S_m^\ominus(\text{CO}_2, \text{g}) + S_m^\ominus(\text{H}_2\text{O}, \text{l})\} \\ &= (187.4 \text{ J K}^{-1} \text{ mol}^{-1}) \\ &\quad - \{(213.74 \text{ J K}^{-1} \text{ mol}^{-1}) + (69.91 \text{ J K}^{-1} \text{ mol}^{-1})\} \\ &= -96.3 \text{ J K}^{-1} \text{ mol}^{-1} \blacksquare\end{aligned}$$

$$\begin{aligned}\Delta_r G^\ominus &= (-303.29 \text{ kJ mol}^{-1}) - (298.15 \text{ K}) \times (-9.63 \times 10^{-2} \text{ kJ K}^{-1} \text{ mol}^{-1}) \\ &= -274.6 \text{ kJ mol}^{-1}\end{aligned}$$

- The standard Gibbs energy of almost any reaction can also be calculated from standard Gibbs energies of formation using the formula:

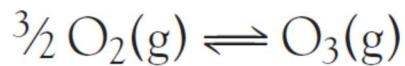
$$\Delta_f G^\ominus = \sum \nu \Delta_f G^\ominus(\text{products}) - \sum \nu \Delta_f G^\ominus(\text{reactants})$$

- Determine the standard reaction Gibbs energy for the complete oxidation of solid sucrose,  $C_{12}H_{22}O_{11}(s)$ , by oxygen gas to carbon dioxide gas and liquid water using the standard Gibbs energies of formation.



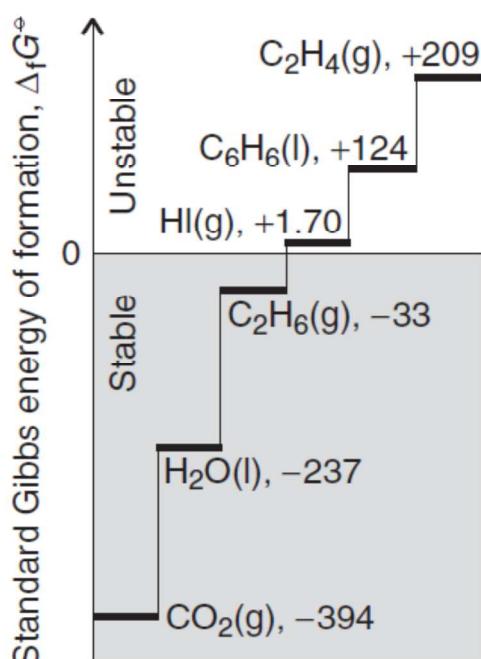
$$\begin{aligned}
 \Delta_r G^\ominus &= \{12\Delta_f G^\ominus(CO_2, g) + 11\Delta_f G^\ominus(H_2O, l)\} \\
 &\quad - \{\Delta_f G^\ominus(C_{12}H_{22}O_{11}, s) + 12\Delta_f G^\ominus(O_2, g)\} \\
 &= \{12 \times (-394 \text{ kJ mol}^{-1}) + 11 \times (-237 \text{ kJ mol}^{-1})\} \\
 &\quad - \{-1543 \text{ kJ mol}^{-1} + 0\} \\
 &= -5.79 \times 10^3 \text{ kJ mol}^{-1} \blacksquare
 \end{aligned}$$

- Standard Gibbs energies of formation of compounds have their own significance:
- A compound with  $\Delta_f G^\theta > 0$  is **thermodynamically unstable** with respect to its elements or that it is **endergonic**.
- Thus, the endergonic substance ozone, for which  $\Delta_f G^\theta = 163 \text{ kJ/mol}$ , has a spontaneous tendency to decompose into oxygen under standard conditions at 25°C.



- However, although ozone is thermodynamically unstable, it can survive if the reactions that convert it into oxygen are slow. That is the case in the upper atmosphere, and the  $\text{O}_3$  molecules in the ozone layer survive for long periods.
- Benzene ( $\Delta_f G^\theta = 124 \text{ kJ mol}^{-1}$ ) is also thermodynamically unstable with respect to its elements ( $K = 1.8 \times 10^{-22}$ ).
- However, the fact that bottles of benzene are everyday laboratory commodities also reminds us of the point made at the start of the second law class, that ***spontaneity is a thermodynamic tendency that might not be realized at a significant rate in practice.***

- Another useful point that can be made about standard Gibbs energies of formation is that there is no point in searching for *direct* syntheses of a thermodynamically unstable compound from its elements, because the reaction does not occur in the required direction: the *reverse* reaction, decomposition, is spontaneous.
- Endergonic compounds must be synthesized by alternative routes or under conditions for which their Gibbs energy of formation is negative and they lie beneath thermodynamic sea level.
- Compounds with  $\Delta_f G^\ominus < 0$  (corresponding to  $K > 1$  for their formation reactions) are said to be **thermodynamically stable** with respect to their elements or **exergonic**.
- Exergonic compounds lie below the thermodynamic sea level of the elements (under standard conditions). An example is the exergonic compound ethane, with  $\Delta_f G^\ominus = -33 \text{ kJ mol}^{-1}$ : the negative sign shows that the formation of ethane gas is spontaneous in the sense that  $K > 1$  (in fact,  $K = 7.1 \times 10^5$  at  $25^\circ\text{C}$ ).



# The response of equilibria to conditions

- **Le Chatelier's principle:**
  - When a system at equilibrium is subjected to a disturbance, the composition of the system adjusts so as to tend to minimize the effect of the disturbance.
- We need to keep in mind that some changes in conditions affect the value of  $\Delta rG^\theta$  and therefore of  $K$  (temperature is the only instance), whereas others change the consequences of  $K$  having a particular fixed value without changing the value of  $K$  (the pressure, for instance).

# The effect of catalyst

- Catalysts provide an alternative, faster route from reactants to products. Although the new route from reactants to products is faster, the initial reactants and the final products are the same.
- The quantity  $\Delta rG^\theta$  is defined as the difference of the standard molar Gibbs energies of the reactants and products, so it is independent of the path linking the two.
- It follows that an alternative pathway between reactants and products leaves  $\Delta rG^\theta$  and therefore  $K$  unchanged.
- That is, *the presence of a catalyst does not change the equilibrium constant of a reaction.*
- *Enzymes are biological versions of catalysts and are so ubiquitous that we need to know how their action affects chemical equilibria.*

# The effect of temperature

- According to Le Chatelier's principle, we can expect a reaction to respond to a lowering of temperature by releasing heat and to respond to an increase of temperature by absorbing heat. That is:
- When the temperature is raised, the equilibrium composition of an exothermic reaction will tend to shift toward reactants; the equilibrium composition of an endothermic reaction will tend to shift toward products.
- Equilibrium constant K changes as temperature is changed:
  - *The equilibrium constant of an exothermic reaction decreases with an increase in temperature.*
  - *The equilibrium constant of an endothermic reaction increases with temperature.*