the heart As A Pump

blood commes in the left ventrical with pressure of left atrial into LV
the pressure in the Left vintrical is the input pressure
" " " " Aiorta is the output pressure

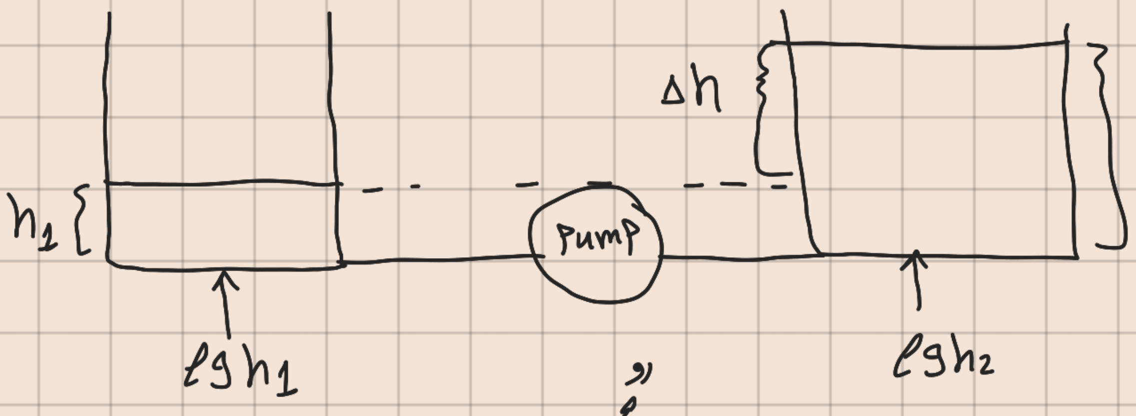
$$\Delta P = P_{out} - P_{input}$$
$$\Delta P = P_{AO} - P_{LA}$$

$$P = \rho g h$$

$$\Delta P = \rho g (h_o - h_i) = \rho g \Delta h$$

↳ Pump head (Pressn head)

the unit of Pressure => [Pa] or [mm Hg]



$$\Delta P = \rho g h_2 - \rho g h_1 = \rho g (h_2 - h_1) = \rho g \Delta h$$

where: Δh = Pump head

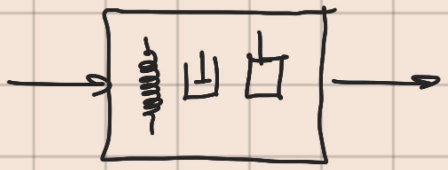
in the hydrolic system if you consider the mechanical system.

	MS	ES		
independen verable	x distance	q charge	KE	$m \rightleftharpoons L$
Dependent "				
effort "	f	V voltage	PE	$K \rightleftharpoons \frac{1}{C}$

Motion $\qquad v$ (valicty) $\qquad I$ (current) $\qquad$ HE $\qquad b \rightleftharpoons Re$

f

input



x

$\dot{x} = v$

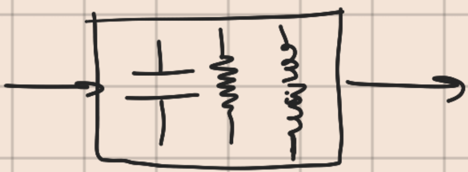
$\ddot{x} = a$

$\underbrace{(x, v, a)}_{\text{output}}$

- so the effort virable the input of the system
- the motion virable the output → independent virable

how we can model: constant virabiles lik $\underbrace{\text{sprine damper mass}}$,
in mechanical system

in electrical system we have capacitance which corespond to sprine and resistore which corespond to damper and the mass which is indectores



so mass is the analog of → indector $\quad m \rightleftharpoons L$

the stifince K is " " " → resistor capacitance (L)

$K \rightleftharpoons L$

the damper (b) is analog of → resistance $\quad b \rightleftharpoons Re$

Know we will switch to hydranic system.

MS	ES	HS
X {m}	q {coul}	
F	V	
$v = \dot{x}$ {m/s}	$I = \dot{q}$ {coul/sec}	
Fs: force in sprine $K \to f_s = Kx$	$C: V_c = \frac{1}{C} q$ the volt through the capacitance	
Force in the damper $b \to F_d = b\dot{x}$	$b: V_R = Re\, I$ ohm equation	
" " " mass $m \to F_m = m\ddot{x}$	$m: V_L = L I$	
	$\dot{q} = I \quad q = CV$	

HS

Volume V

Pressure P

motion virable $Q \; [\frac{L}{m}]$
↳ Fllow

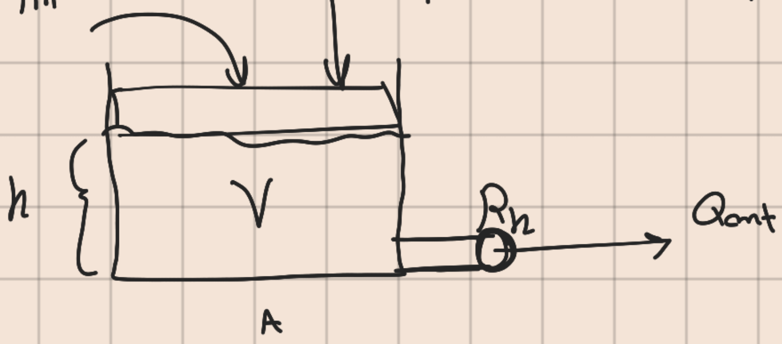
$$C = \frac{A}{\rho g}$$

$$P = R_h Q$$

For know we will ignore the mass it is camplectied

we are modeling the hydrolic system

P_{in} , P_{ext} → the inputes



the fluide from the input is going to discharge from this open Q_{out}

- now we will try to express the dynamic of hydromical system

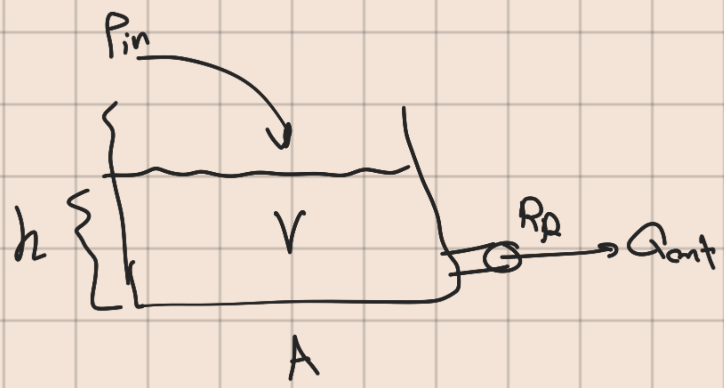
$$b\dot{x} + K x = f_{ext} \longrightarrow MS$$

$$R\dot{q} + \frac{1}{C} q = V_{in} \longrightarrow ES$$

First lets assume that there is not external pressure $P_{ext} = 0$

we just have an input flow and output flow and the volume accumlated in the risirver

حوضه أو مكان، ـــ يَ



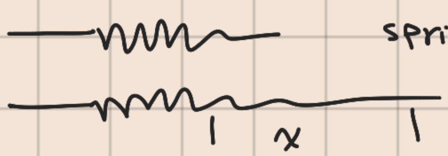
So when we see accumlate first thing come to mind integrate

accumlate ⟶ integrate

what does it integrate ⟶ it is integrate Volume

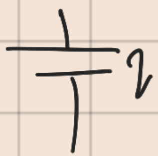
Ex:

MS ⟶



sprin what does it integral distance

ES ⟶



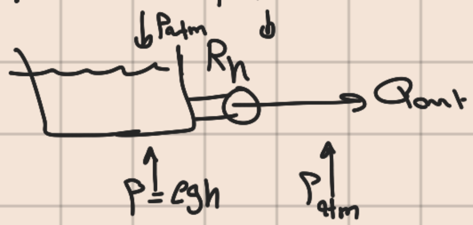
capacitonce integrate chorgeres

So firstiful

1- using ohm's equation

$$\underbrace{\Delta P = R_h Q}$$

⟶ we can apply this equation to the pipe

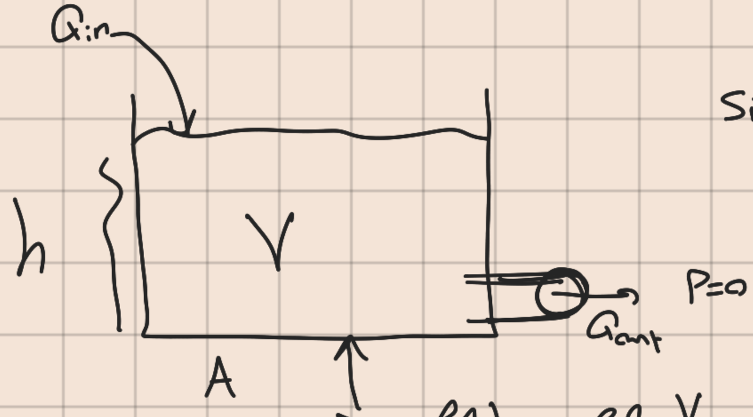


$$\Delta P = P_{in} - P_{out} = \rho g h_1 + P_{atm} - \rho g h_2 - P_{atm}$$

$$P = \rho g h + P_{atm}$$

$$\Delta P = \rho g \Delta h$$

so because of that we can ignore the atomasphire pressure when we calculate the changing in the pressure



Since : $V = A h \Rightarrow h = \frac{V}{A}$

$$P_{in} = \rho g h = \rho g \frac{V}{A} = \underbrace{\frac{\rho g}{A}}_{\text{inverse of the capacitonce}} V$$

$$P = \frac{1}{C} V$$

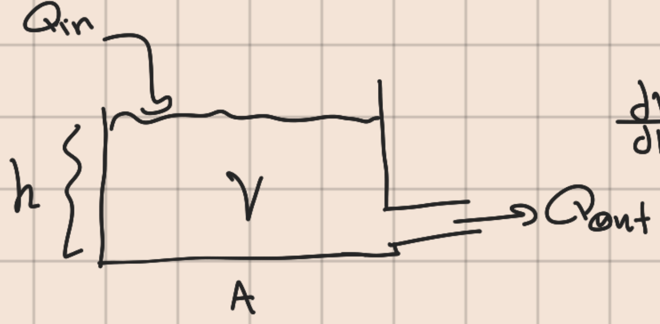
$$V = \frac{1}{C_e} q \qquad q = C_e V$$

$$\frac{1}{c} = \frac{\rho g}{A}$$

$$\Delta P = R_h Q_{out}$$

$$\boxed{\frac{\rho g}{A} V = R_h Q_{out}}$$ → ① first equation which comes from ohm's releationship

2- apply KCL



$$\frac{dV}{dt} = \dot{V}$$

$$Q_{in} - Q_{out} = \dot{V}$$ Continuity equation Hydromic KCL equation

$$\rho g \frac{V}{A} = R_h Q_{out}$$ since; $Q_{out} = Q_{in} - \dot{V}$

$$\Rightarrow \rho g \frac{V}{A} = R_h (Q_{in} - \dot{V})$$ → we have now differential eq. in terms V

dividing every think with R

$$\frac{\rho g}{R_h} \frac{V}{A} = \frac{R_h}{R_h}(Q_{in} - \dot{V})$$ since $\frac{\rho g}{A} = \frac{1}{c}$

$$\frac{1}{R_h C_h} V = Q_{in} - \dot{V} \Rightarrow \dot{V} = -\frac{1}{RC} V + Q_{in}$$

as we know the most generatic form for $-\frac{1}{RC}$ is →

$$\dot{y} = -ay + b$$ a: Pole the system $= \frac{1}{\tau} = \frac{1}{RC}$ (τ: time constant)

So this is RC circuit with time constant

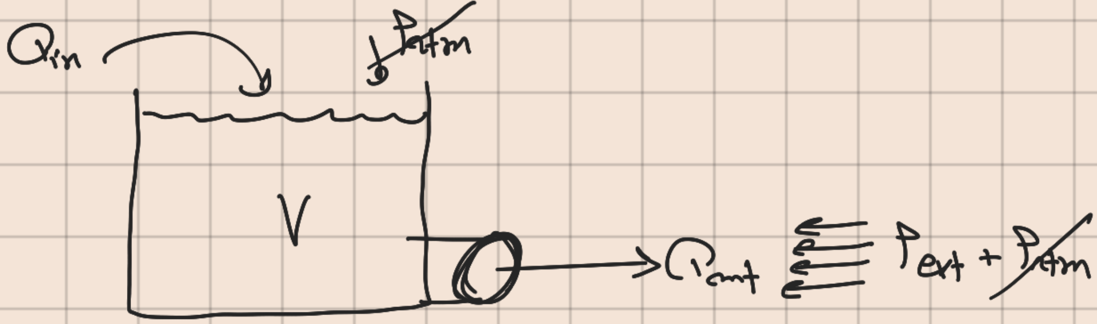
$$\tau = RC$$

$$\dot{V} = -\frac{1}{RC} V + Q_{in}$$

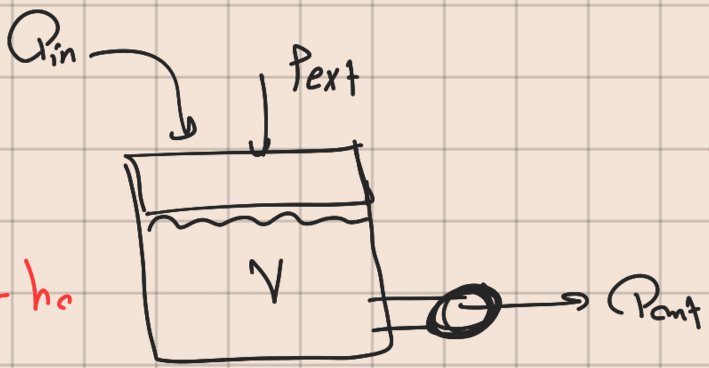
If now I want to add the pressure it exerted on it P_{ext} so it will be in terms Q $\Rightarrow$ since $P = RQ$

$$\frac{P}{R} = Q$$

$$\Rightarrow \quad \dot{V} = -\frac{1}{RC} V + Q_{in} + \frac{1}{R} P_{ext}$$



if you measure P over P_{atm} $\Rightarrow$ called as gauge pressure



$V_0 \leftarrow h_0$

$$\dot{V} = \overbrace{-\frac{1}{RC} V}^{\text{homogenous part}} + \underbrace{\left(Q_{in} + \frac{1}{R} P_{ext}\right)}_{\text{non homogenous part}}$$

you have to start from initial high h_0 initial volume V_0

$$\boxed{V(t) = \left(V_0 - \frac{b}{a}\right) e^{-\frac{t}{RC}} + \frac{b}{a}} \rightarrow \text{general solution to resirvore value}$$

$$\dot{y} = -ay + b$$
$$y(t) = \left(y_0 - \frac{b}{a}\right) e^{-\frac{t}{a}} + \frac{b}{a}$$

so $\Rightarrow$

$$\left.\begin{aligned} b &= Q_{in} - \frac{1}{R} P_{ext} \\ a &= \frac{1}{RC} \end{aligned}\right\} \rightarrow \frac{b}{a} = \frac{Q_{in} - \frac{1}{R} P_{ext}}{1/RC}$$

$$\frac{b}{a} = RCQ_{in} - CP_{ext}$$

Howmork: => you are going use matlab

Creat reservife (A) → the are we intersted

$C = \frac{A}{eg}$

determine Q_{in}

Calculate Q_{out}

" $\dot{q}$

R_h → calculate the resistance of the ...

model cordic viscular system

each pump in the heart fial in Valume

the flow that comes in from the LA to LV

the Volume in distolic (relaxtion) go up

and in the systol phase

the volume go down

if $Q_{in} > Q_{out}$ the Volume will increase

$Q_{in} - Q_{out} = q$

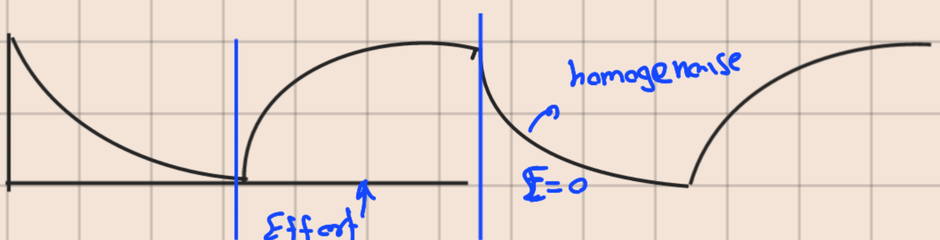
in some point when blood go to fiel the LV $Q_{out} = 0$

if $Q_{out} < Q_{in}$ the Volume will be decrease

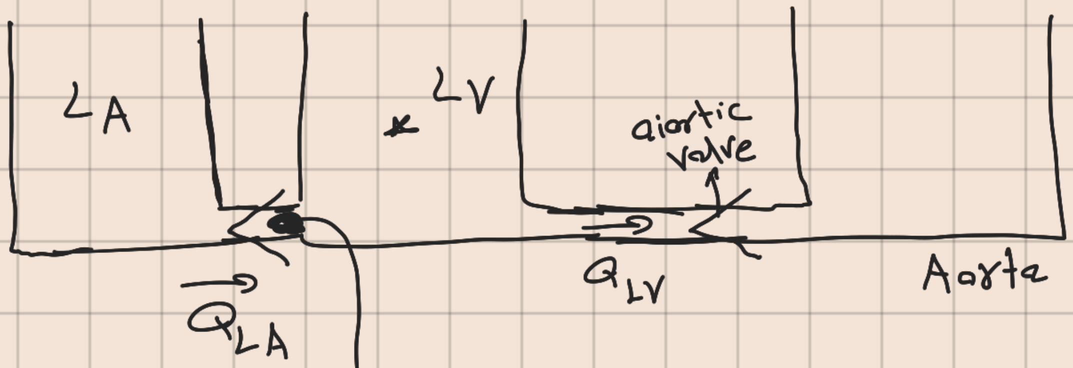
and $Q_{in} - Q_{out} = q \Rightarrow q < 0$

Since $q = \dot{V} > 0$

$q = \dot{V} < 0$ and we know that expression is first order system and we know such that system the Volume increase exponentialy and decrease exponentialy



when it increase it is you apply external Effort and the external effort or force is applied is from the muscle and the force that the muscle creat we can represented as Pisten acting on the Top vinterical

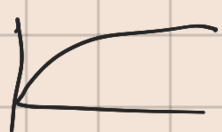


in here we have unitrary valve in one direction and that mean it flow from the LA and go in if the Pressure in LA is higher in the pressure here LV but until it reach to the pressure in LV higher then the Pressure in LA and then the Valve will be closed and there is no blood to flow we can represent this situation as diode

electrical equvilent

and the same think in the aiortic Valve when the pressure in Aorta higher then the pressure in the LV then the valve aiortic will close to prevent the flowing of the blood.

So during the pressure higher in LA and blood flowing into the LV the pressure in the Aorta in this time is higher the LV and the aiortic valve is closed and there no flowing blood from LV → Aorta and during this Pireod the volum in LV increases



the equation that represent this case is $\Rightarrow$

$$V = \left(V_0 - \frac{b}{q}\right) e^{-\frac{t}{RC}} + \frac{b}{q}$$

sice; $Q = \frac{V}{t} \Rightarrow V = Qt$

$P = \frac{eg}{A} V$

$tQ = V$ we can represent time by RC

$$\frac{b}{q} = RCQ_{in} + \frac{A}{eg}P$$

during diastolic

during systol $Q_{in} = 0$ and $P = P_{ext}$

$$\frac{b}{q} = RCQ_{in} + \frac{A}{eg}P_{ext} \rightarrow \text{during systole}$$

during distolic

so in to phases there is force | will explanle

force acting through the pressure it will press on the ventrical here the Effort = Pext

and this for the flow coming in and increase in the volume so in here

Effort = RQ

so our equation will have two form during distolic and systol

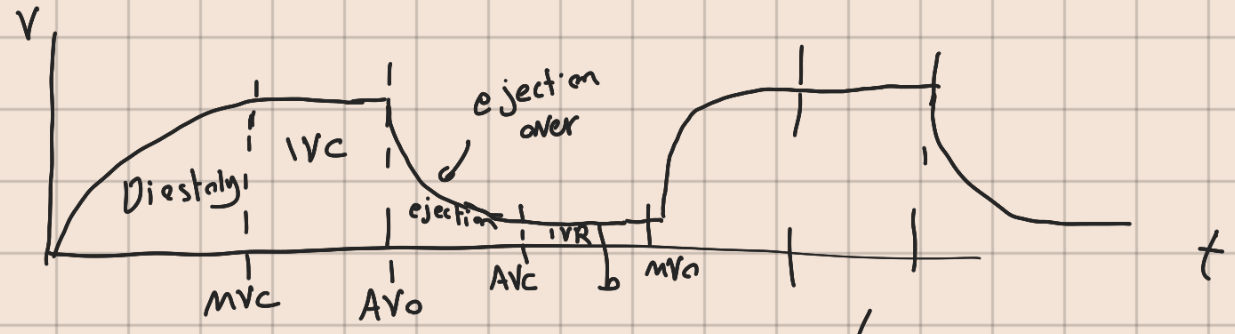
$$V = \begin{cases} RQ_{in} & \text{distolic} \\ CP & \text{systolic} \end{cases}$$

and the volume during two

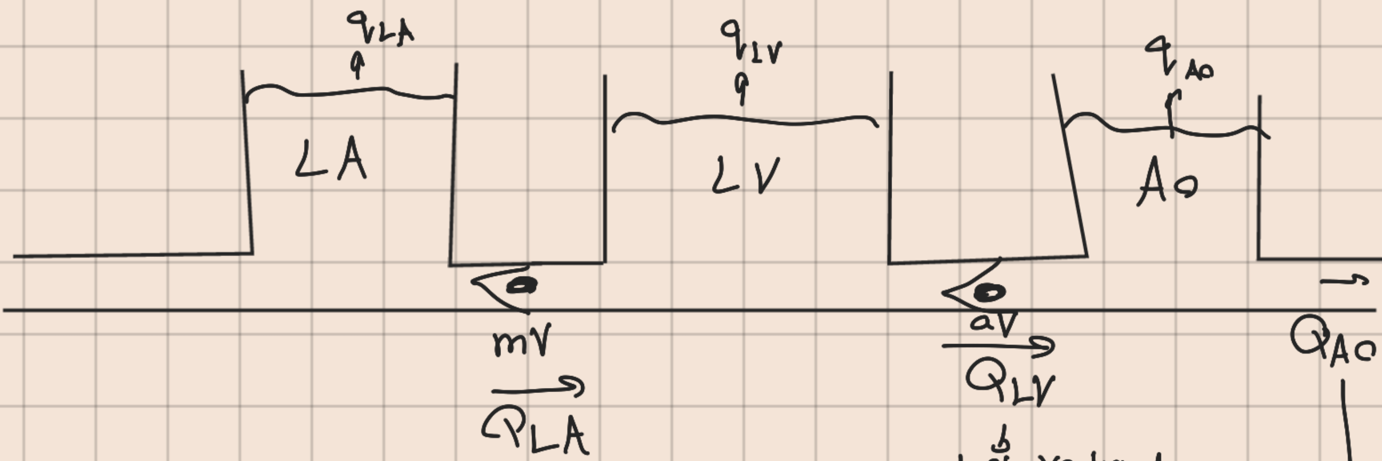
will not chang then $\frac{b}{q}$ will

$\frac{b}{q} = 0$

represented in graph



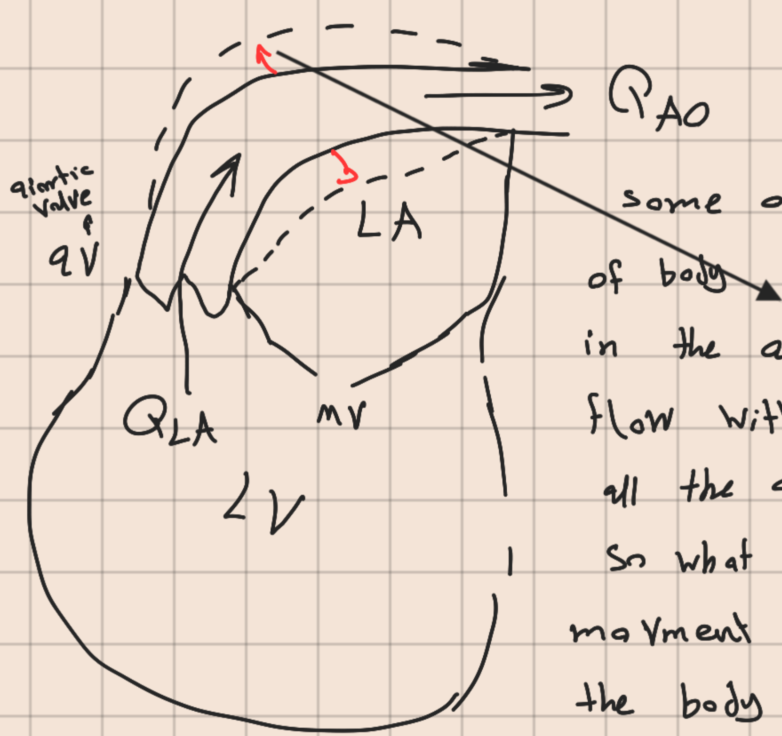
Homework



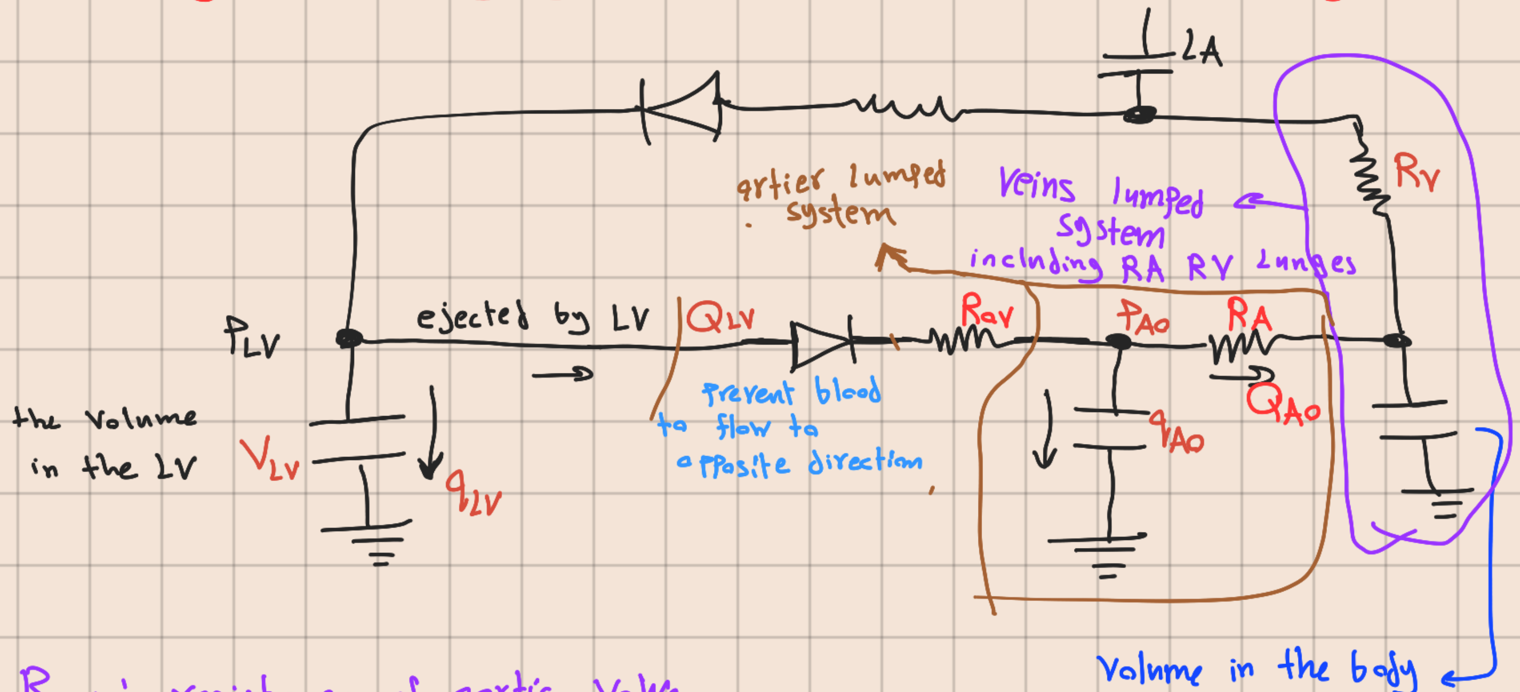
left ventricular blood during ejection going to aorta and rising the volume blood and then flowing to the body

during ejection in LV the aorto inside the iorta increasese

q_{Ao} ↑



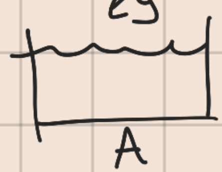
some of it goes to the rest of body and some of stay in the aiorta inlarge the flow with ventricular contraction all the artiers in the body so what we see up and down movment of aiorta inside the body the way we are going to model this as electrical system

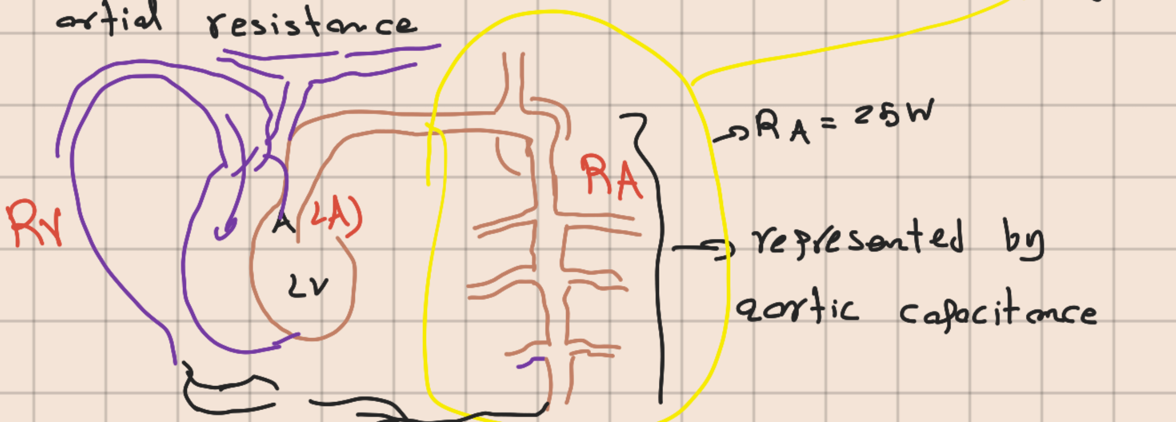


Rav : resistance of aortic valve

We are model the left ventrical the aorta lumped system and all these artial resistance

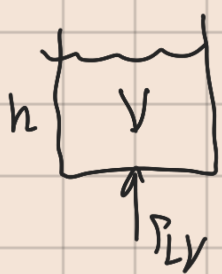
$C = \frac{A}{eg}$

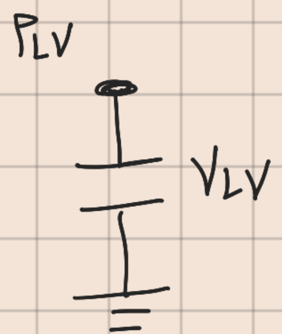




the capacity that acumlating valume on it

we hav 4 RC systems

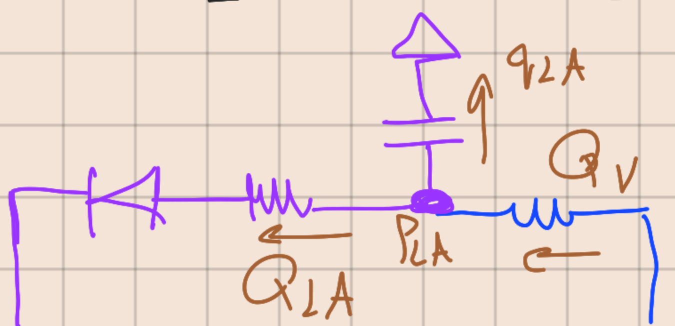


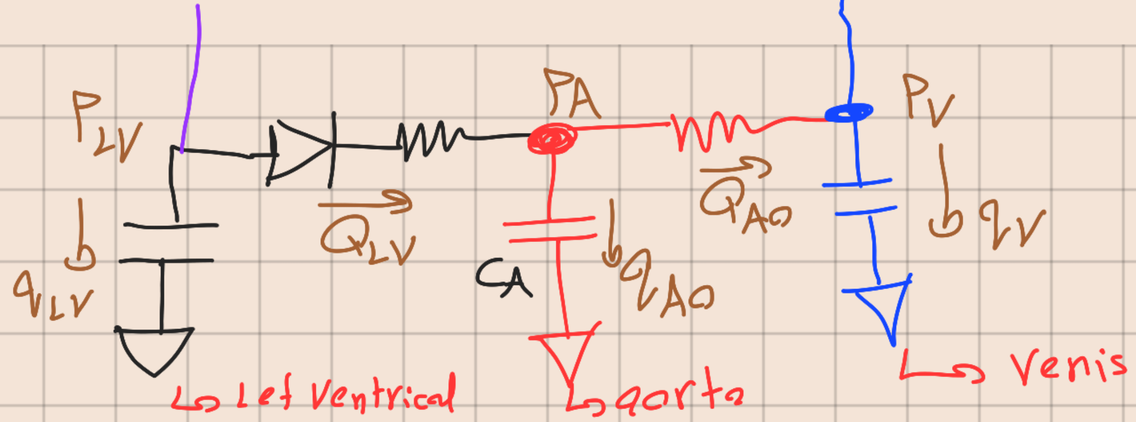


the volume creat a pressure in electrical system

$q = CV$ → voltage

in our system $V = CP \Rightarrow P = \frac{1}{C} V$





so for each system we have P, q, Q

node: $P\ q\ Q$ => unknown we have to get 3 equation for each node

We can apserve this equation by using

Ohm aort $\rightarrow R_A Q_{Ao} = P_{Ao} - P_V$

KCL $Q_{LV} = q_{Ao} + Q_{AO}$

Continuty

capacitance since $V = cP \rightarrow \dot{V} = c\dot{P}$ $\dot{V} = Q$

$Q = c\dot{P}$ $\rightarrow$ Q it is the volume that creat pressure

$Q = q_{AO}$ $\dot{P}_{Ao} = \frac{1}{c} q_{AO}$

Let's chos P as aunknown

$q_{Ao} = Q_{LV} - Q_{Ao}$ $Q_{Ao} = \frac{P_{AO} - P_V}{R_A}$

$\dot{P}_{Ao} = \frac{1}{c}(Q_{LV} - Q_{Ao})$

$\dot{P}_{Ao} = \frac{1}{c}\left\{ Q_{LV} - \left(\frac{P_{Ao}}{R_A} - \frac{P_V}{R_A} \right) \right.$

$q_{LV} = \frac{P_{LV} - P_{Ao}}{R_{av}}$

$\dot{P}_{Ao} = \frac{1}{C_{Ao}} \left[\left(\frac{P_{LV}}{R_{av}} - \frac{P_{Ao}}{R_{av}} \right) - \left(\frac{P_{Ao}}{R_A} - \frac{P_V}{R_A} \right) \right]$

to simplify the equation let $\frac{1}{C_{Ao}} = e_{Ao}$ e: elostance steffinice

$R = \frac{1}{g}$

g = adiminth

$$\dot{P}_{Ao} = e_{Ao}\left[(g_a P_{LV} - g_a P_{AO}) - (g_A P_{Ao} - g_A P_V)\right]$$

$$\dot{P}_{Ao} = e_{Ao}\{(g_2 P_{LV} - (g_a + g_A) P_{Ao} + g_A P_V)\}$$

$$\dot{P}_{Ao} = \underbrace{e_A g_2 P_{LV}}_{b} - \underbrace{e_A (g_a + g_A)}_{-a} P_{Ao} + \underbrace{e_A g_A P_V}_{b} + \underset{P_{LA}}{0}$$

$$\dot{x} = -ax + b$$

if we do this equation for all node we get

$$\dot{P}_{LV} = a_1 P_{LV} + a_2 P_{Ao} + a_3 P_V + a_4 P_{LA}$$

$$\dot{P}_{Ao} = b_1 P_{LV} + b_2 P_{Ao} + b_3 P_V + b_4 P_{LA}$$

$$\dot{P}_V = c_1 P_{LV} + c_2 P_{Ao} + c_3 P_V + c_4 P_{LA}$$

$$\dot{P}_{LA} = d_1 P_{LV} + d_2 P_{Ao} + d_3 P_V + d_4 P_{LA}$$

so what we have is systemic of differential equation and each unknown express as linear sum of 4 unknown togeth this is matrix format

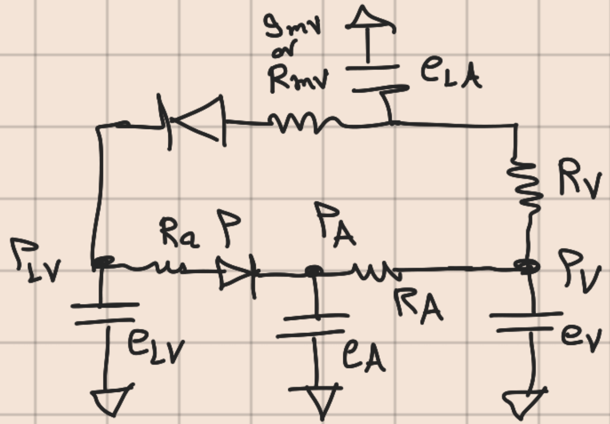
$$\begin{bmatrix} \dot{P}_{LV} \\ \dot{P}_{Ao} \\ \dot{P}_{V} \\ \dot{P}_{LA} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \begin{bmatrix} P_{LV} \\ P_{Ao} \\ P_{V} \\ P_{LA} \end{bmatrix}$$

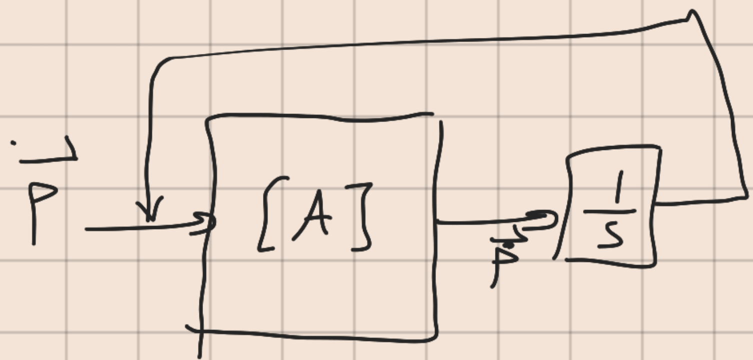
this is matrix format that can show us

$$\vec{\dot{P}} = [A] \vec{P}$$

so hole system made up by 4 unknown

and the constant one is the resistance and the other constant is the elastisty e





method called state-space analysis

second order case

$$F_m + F_d + F_K = F_{ext}$$

for balance

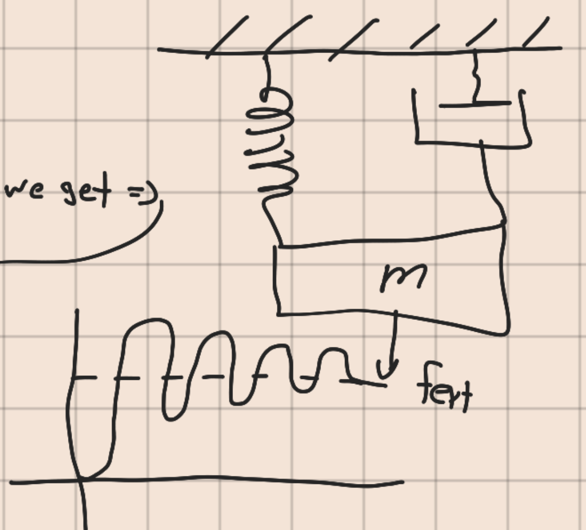
$$m\ddot{x} + b\dot{x} + Kx = F_{ext}$$

solving eq we get ⇒

$$x(t) = A e^{-\xi w_n t} \cos(w_d t + \theta)$$

$$w_n = \sqrt{\frac{K}{m}} \qquad \xi = \frac{b}{2 w_n m}$$

$$w_d = w_n \sqrt{1 - \xi^2}$$



to solve this equation in different method go to fundemental rule of Algebre

any n^{th} order equation is equivelent to the product of n 1^{st} order equation

Ex: $ax^2 + bx + c = 0$

$(x - x_1)(x - x_2) = 0$

$x = x_1 \quad x = x_2$

we can do same think to

$\ddot{x} + 2\zeta w_n \dot{x} + w_n^2 x = 0$

$s^2 X(s) + 2\zeta w_n s X(s) + w_n^2 X(s) = 0 \qquad \frac{1}{X(s)}$

$s^2 + 2\zeta w_n s + w_n^2$

$(s - s_1)(s - s_2) = 0$

in laplace domian we turn differential equation to algebra equation

so how to solve this equation $m\ddot{x} + b\dot{x} + Kx = f_{ext}$ using fundemental algebre

$$\frac{m\ddot{x}}{m} + \frac{b\dot{x}}{m} + \frac{Kx}{m} = \frac{1}{m} f_{ext} \implies$$

$$\boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = \frac{1}{m} f_{ext}} \rightarrow$$

try to expressed as 2^{nd} order defferential equation product of 2 1^{st} ord eq.

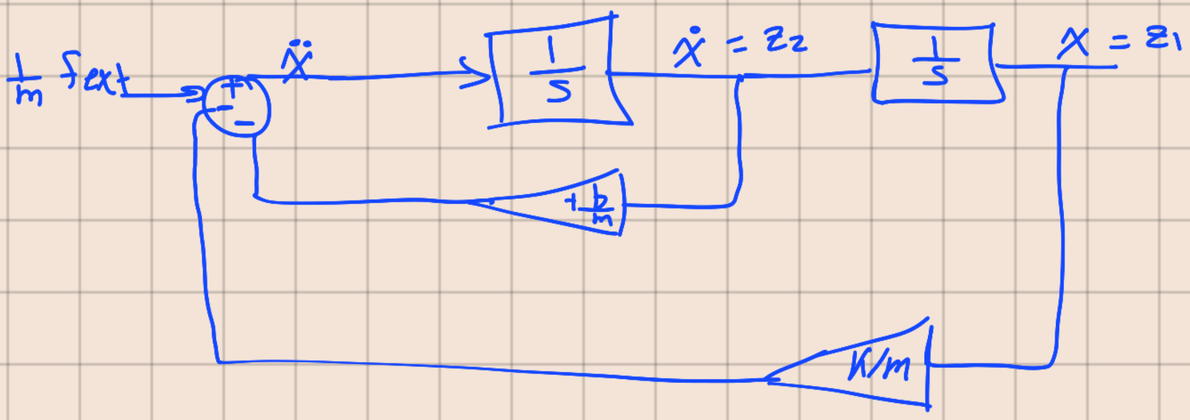
$\frac{b}{m} = 2\zeta w_n$

$\frac{K}{m} = w_n^2$

$$\ddot{x} + 2\zeta w_n \dot{x} + w_n^2 x = \frac{1}{m} f_{ext}$$

$$\ddot{x} = -\frac{K}{m}x - \frac{b}{m}\dot{x} + \frac{1}{m}f_{ext}$$

Change of Voribels $\longrightarrow$ $x \rightarrow z$



if $x = z_1 \Rightarrow \dot{x} = \dot{z}_1 \longrightarrow \ddot{x} = \ddot{z}_1$

$\dot{x} = z_2 \longrightarrow \ddot{x} = \dot{z}_2$

$$\ddot{x} = -\frac{K}{m}x - \frac{b}{m}\dot{x} + \frac{1}{m}f_{ext}$$

$$\dot{z}_2 = -\frac{K}{m}z_1 - \frac{b}{m}z_2 + \frac{1}{m}f_{ext}$$

$$\dot{z}_1 = z_2 \qquad \dot{\vec{p}} = [A]\vec{P}$$

$$\dot{z}_1 = 0z_1 + 1z_2 + 0f_{ext}$$

$$\dot{z}_2 = -\frac{K}{m}z_1 - \frac{b}{m}z_2 + \frac{1}{m}f_{ext}$$

$$\begin{bmatrix} \dot{z}_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f_{ext}$$

$$[\dot{\vec{z}}] = [A][z] + [B]f_{ext}]$$

$$\dot{z} = Az + Bu$$

non homogennse

$$\dot{y} = -ay + b$$

$$\dot{P} = [A]P + [B]u$$

---

in order to solve the system ohm's KCL capacitity

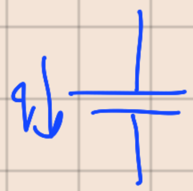
but for the Left Ventrical the capacity will gona have tricke

$$\text{volume} \leftarrow V = CP$$

$$\dot{V} = C\dot{P}$$

$$q = C\dot{P}$$

$$\dot{P} = \frac{1}{C} q$$



$$e = \frac{1}{C}$$

$$\dot{P} = eq$$

So in the left vintrical you have proplem that capectonc not constant for all other nodes the capactance is constant but for the left ventrical C will not be constant

$$\dot{P} = \underset{\downarrow}{e} q$$

$$e = \frac{1}{C}$$ → remember the elasty changes by the time

So you he $e_{LV}(t)$ for LV

$$V_{LV}(t) = C_{LV}(t)\, P_{LV}(t)$$

$$\dot{V}(t) = \dot{C}_{LV} P_{LV} + C_{LV}\dot{P}$$

so that will complecate your medirm exam

$$\dot{V} \Rightarrow q_{LV}$$

$$P_{LV} = \frac{1}{C_{LV}} V_{LV}$$

$$P_{LV} = e_{LV} V_{LV}$$

$$\dot{P}_{LV} = \dot{e}_{LV} V_{LV} + e_{LV}\dot{V}_{LV}$$

$$\dot{P}_{LV} = \frac{\dot{e}_{LV}}{e_{LV}} P_{LV} + e_{LV} q_{LV}$$

Since $e_{LV} V_{LV} = P_{LV}$

$$V_{LV} = \frac{P_{LV}}{e_{LV}}$$