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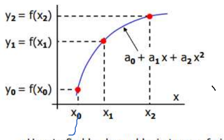
Polynomial Interpolation

• Quadratic Interpolation:



• Given: (x_0, y_0) , (x_1, y_1) and (x_2, y_2)

• A parabola passes from these three points.

• Similar to the linear case, the equation of this parabola can be written as

$$\boxed{f_2(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)}$$

Quadratic interpolation formula

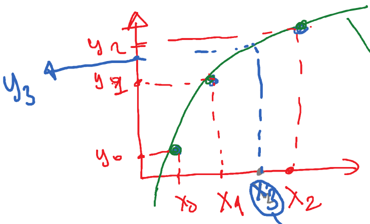
• How to find b_0, b_1 and b_2 in terms of given quantities?

• at $x=x_0$ $\quad f_2(x) = f(x_0) = b_0 \quad \rightarrow \quad b_0 = f(x_0)$

• at $x=x_1$ $\quad f_2(x) = f(x_1) = b_0 + b_1 x_1 \quad \rightarrow \quad b_1 = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$

• at $x=x_2$ $\quad f_2(x) = f(x_2) = b_0 + b_1(x_2-x_0) + b_2(x_2-x_0)(x_2-x_1) \quad \rightarrow \quad b_2 = \dfrac{\dfrac{f(x_2)-f(x_1)}{x_2-x_1} - \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$

In order to use the quadratic interpolation method, the cooordinates of three ~~different~~ points must be known.



x_0, y_0 known
x_1, y_1 known
x_2, y_2 known
x_3, y_3 unknown

$\boxed{y = ax^2 + bx + c}$

method

(1) Write the coefficients in their appropriate place in the equation

$$\left.\begin{aligned} a x_0^2 + b x_0 + c &= y_0 \\ a x_1^2 + b x_1 + c &= y_1 \\ a x_2^2 + b x_2 + c &= y_2 \end{aligned}\right\}$$

(2) method. A special equation is used

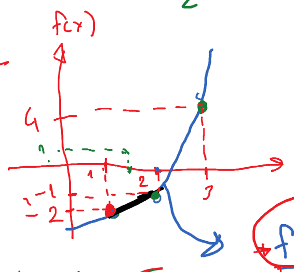
$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$b_2 = \frac{\dfrac{f(x_2)-f(x_1)}{x_2-x_1} - \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$$

Exa



$\boxed{x = 1.5}$ $\quad \boxed{f(x) = ?}$

$$f(x) = a x^2 + b x + c$$

(1) Method. (1, -2) (2, -1) (3, +4)

$$-2 = a + b + c$$
$$-1 = 4a + 2b + c$$

(1) Method $(1,-2)$, $(2,-1)$, $(3,+4)$

$-2 = a + b + c$

$-1 = 4a + 2b + c$

$4 = 9a + 3b + c$

$c = -2 - a - b$

$-1 = 4a + 2b + -2 - a - b$

$4 = 9a + 3b + -2 - a - b$

$(3a + b = 1)\cdot -2$

$8a + 2b = 6$

$2a = 4$

$a = 2$

$b = -5$

$c = 1$

$c = -2 - 2 - -5$

$f(x) = ax^2 + bx + c$

$\boxed{f(x) = 2x^2 - 5x + 1}$

$x = 1.5$ $\quad f(x) = ?$

$f(1.5) = 2\cdot(1.5)^2 - 5(1.5) + 1$

$f(1.5) = -2$

2 method $(1, -2)$, $(2, -1)$ $(3, 4)$ $(1.5, ?)$

$x_0 \; y_0 \; (f(x_0))$, $x_1 \; y_1 \; (f(x_1))$, $x_2 \; y_2 \; f(x_2)$, unknown

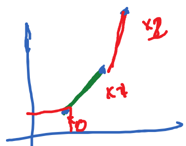
$x_1 \quad f(x_1)$

$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$

$b_0 = f(x_0)$

$b_1 = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} \longrightarrow \dfrac{-1 - -2}{2 - 1} = 1$

$b_2 = \dfrac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} = 2$



$f(x) = -2 + 1(x - 1) + 2(x - 1)(x - 2)$

$f(x) = -2 + x - 1 + 2x^2 \ldots$

$b_0 = f(x_0) = -2$

$b_1 = \dfrac{-1 - -2}{2 - 1} = 1$

$b_2 = \dfrac{\dfrac{4 - -1}{3 - 2} - \dfrac{-1 - -2}{2 - 1}}{3 - 1} = 2$

$f(x) = -2 + 1(x - 1) + 2(x - 1)(x - 2)$

$f(x) = -2 + x - 1 + 2x^2 - 4x - 2x + 4$

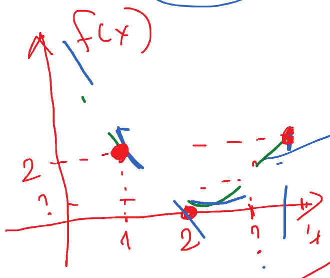
$\boxed{f(x) = 2x^2 - 5x + 1}$

$$f(1.5) = 2(1.5)^2 - 5(1.5) + 1$$

$$f(1.5) = -2$$

Ex $(1, 2)\ (2, 0)\ (4, 2)$

find $f(3) = ?$



$$f(x) = ax^2 + bx + c$$

① method $(1, 2)\ (2, 0)\ (4, 2)$

$$2 = a + b + c$$
$$0 = 4a + 2b + c$$
$$2 = 16a + 4b + c$$

$$c = 2 - a - b$$

$$\boxed{\begin{aligned} 0 &= 4a + 2b + 2 - a - b \\ 2 &= 16a + 4b + 2 - a - b \end{aligned}}$$

$$a = 1$$
$$b = -5$$
$$c = 6$$

$$f(x) = ax^2 + bx + c$$
$$= x^2 - 5x + 6$$

$$f(3) = 3^2 - 5.3 + 6 = \underline{\underline{0}}$$

2. method $\underset{x_0\ f(x_0)}{(1, 2)}$ $\underset{x_1\ f(x_1)}{(2, 0)}$ $\underset{x_2\ f(x_2)}{(4, 2)}$

$$\boxed{f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)}$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{}$$

$$b_2 = \frac{\frac{f(x_2)-f(x_1)}{x_2-x_1} - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$$

$$b_0 = f(1) = 2,$$

$$b_1 = \frac{0-2}{2-1} = -2$$

$$b_2 = \frac{\frac{2-0}{4-2} - \frac{0-2}{2-1}}{4-1} = 1$$

$$f(x) = 2 + (-2)(x-1) + 1(x-1)(x-2)$$

$$f(x) = 2 - 2x + 2 + x^2 - 3x + 2$$

$$\boxed{f(x) = x^2 - 5x + 6}$$

$$f(3) = 3^2 - 5.3 + 6 = 0$$

Example

x	3	4	5	6
$f(x) = \log(x)$	0.477	0.602	?	0.778

find $f(5)$ using quadratic interpolation method.

2. method

$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$\left(\overset{x_0}{3}, \overset{f(x_0)}{0.477}\right), \left(\overset{x_1}{4}, \overset{f(x_1)}{0.602}\right), \left(\overset{x_2}{6}, \overset{f(x_2)}{0.778}\right)$$

$$b_0 = 0.477$$

$$b_1 = \frac{0.602 - 0.477}{4 - 3} = 0.125$$

$$b_2 = \frac{\dfrac{0.778 - 0.602}{6 - 4} - \dfrac{0.602 - 0.477}{4 - 3}}{6 - 3} =$$

$$b_2 = \frac{0.088 - 0.125}{3} = -0.0123$$

$$\boxed{f(x) = 0.477 + 0.125(\overset{5}{x} - 3) - 0.0123(\overset{5}{x} - 3)(\overset{5}{x} - 4)}$$

$$\boxed{f(x) = -0.0123x^2 + 0.2051x - 0.0456}$$

$$f(5) = 0.477 + 0.250 - 0.0246$$

$$f(5) = 0.7024$$