Exercise 1

1. Which of the following propositional logic formulae are tautologies?
   a. $\big(p \wedge (p \Rightarrow q)\big) \Rightarrow q$
   b. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \vee \neg r)$
2. Which of the following propositional logic formulae are satisfiable?
   a. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \vee \neg r)$
   b. $p \Rightarrow (p \wedge q)$
   c. $(p \vee q) \wedge (\neg p \vee r) \wedge (\neg r \wedge \neg q)$
3.
   a. Write a specification for a function *FindMax(A)* that searches an array *A* of integers and returns the maximum value in the array.
   b. Write an implementation of the function *FindMax*, along with appropriate loop invariants that suffice to prove the function correct.
   c. Generate a set S of verification-conditions for the above example. This is a set of sentences S such that if every sentence in S is valid (i.e., a tautology), then the function is guaranteed to be correct.
   d. Check for the validity of the above verification-conditions using the SMT solver Z3.
4. Are the following rules valid for Hoare logic? Either sketch an informal proof or present a counterexample.
   a. $\dfrac{\vdash \{P_1\}\, S\, \{Q_1\}, \qquad \vdash \{P_2\}\, S\, \{Q_2\}}{\vdash \{P_1 \wedge P_2\}\ S\ \{Q_1 \wedge Q_2\}}$
   b. $\dfrac{\vdash \{P_1\}\, S\, \{Q_1\}, \qquad \vdash \{P_2\}\, S\, \{Q_2\}}{\vdash \{P_1 \vee P_2\}\ S\ \{Q_1 \vee Q_2\}}$