MVA_WorldHappinessAnalysis_week6_PCA.R

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```r
#Soukhyada Vaidya
#Assignment: World Happiness Analysis
#Loading the data
worldh <- read.csv("C:/Users/Soukhyada/Desktop/MVA/WH_2017.csv")

#Loading packages required for the analysis
library(plyr)
library(plotly)
```

```
## Warning: package 'plotly' was built under R version 3.5.2
```

```
## Loading required package: ggplot2
```

```
## Warning: package 'ggplot2' was built under R version 3.5.2
```

```
##
## Attaching package: 'plotly'
```

```
## The following object is masked from 'package:ggplot2':
##
##     last_plot
```

```
## The following objects are masked from 'package:plyr':
##
##     arrange, mutate, rename, summarise
```

```
## The following object is masked from 'package:stats':
##
##     filter
```

```
## The following object is masked from 'package:graphics':
##
##     layout
```

```r
library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 3.5.2
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:plyr':
##
##     arrange, count, desc, failwith, id, mutate, rename, summarise,
##     summarize
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------------------------------- tidyverse 1
.2.1 --
```

```
## v tibble  2.0.1      v purrr   0.2.5
## v tidyr   0.8.2      v stringr 1.4.0
## v readr   1.3.1      v forcats 0.3.0
```

```
## Warning: package 'tibble' was built under R version 3.5.2
```

```
## Warning: package 'tidyr' was built under R version 3.5.2
```

```
## Warning: package 'readr' was built under R version 3.5.2
```

```
## Warning: package 'stringr' was built under R version 3.5.2
```

```
## Warning: package 'forcats' was built under R version 3.5.2
```

```
## -- Conflicts ------------------------------------------------------------------ tidyverse_conflic
ts() --
## x dplyr::arrange()   masks plotly::arrange(), plyr::arrange()
## x purrr::compact()   masks plyr::compact()
## x dplyr::count()     masks plyr::count()
## x dplyr::failwith()  masks plyr::failwith()
## x dplyr::filter()    masks plotly::filter(), stats::filter()
## x dplyr::id()        masks plyr::id()
## x dplyr::lag()       masks stats::lag()
## x dplyr::mutate()    masks plotly::mutate(), plyr::mutate()
## x dplyr::rename()    masks plotly::rename(), plyr::rename()
## x dplyr::summarise() masks plotly::summarise(), plyr::summarise()
## x dplyr::summarize() masks plyr::summarize()
```

```r
library(lubridate)
```

```
## Warning: package 'lubridate' was built under R version 3.5.2
```

```
##
## Attaching package: 'lubridate'
```

```
## The following object is masked from 'package:plyr':
##
##     here
```

```
## The following object is masked from 'package:base':
##
##     date
```

```r
library(caTools)
library(ggplot2)
library(ggthemes)
```

```
## Warning: package 'ggthemes' was built under R version 3.5.2
```

```r
library(reshape2)
```

```
##
## Attaching package: 'reshape2'
```

```
## The following object is masked from 'package:tidyr':
##
##     smiths
```

```r
library(data.table)
```

```
##
## Attaching package: 'data.table'
```

```
## The following objects are masked from 'package:reshape2':
##
##     dcast, melt
```

```
## The following objects are masked from 'package:lubridate':
##
##     hour, isoweek, mday, minute, month, quarter, second, wday,
##     week, yday, year
```

```
## The following object is masked from 'package:purrr':
##
##     transpose
```

```
## The following objects are masked from 'package:dplyr':
##
##     between, first, last
```

```r
library(tidyr)
library(corrgram)
```

```
## Warning: package 'corrgram' was built under R version 3.5.2
```

```
##
## Attaching package: 'corrgram'
```

```
## The following object is masked from 'package:plyr':
##
##     baseball
```

```r
library(corrplot)
```

```
## Warning: package 'corrplot' was built under R version 3.5.2
```

```
## corrplot 0.84 loaded
```

```r
library(formattable)
```

```
## Warning: package 'formattable' was built under R version 3.5.2
```

```
##
## Attaching package: 'formattable'
```

```
## The following object is masked from 'package:plotly':
##
##     style
```

```r
library(cowplot)
```

```
## Warning: package 'cowplot' was built under R version 3.5.2
```

```
##
## Attaching package: 'cowplot'
```

```
## The following object is masked from 'package:ggthemes':
##
##      theme_map
```

```
## The following object is masked from 'package:ggplot2':
##
##      ggsave
```

```r
library(ggpubr)
```

```
## Warning: package 'ggpubr' was built under R version 3.5.2
```

```
## Loading required package: magrittr
```

```
##
## Attaching package: 'magrittr'
```

```
## The following object is masked from 'package:purrr':
##
##      set_names
```

```
## The following object is masked from 'package:tidyr':
##
##      extract
```

```
##
## Attaching package: 'ggpubr'
```

```
## The following object is masked from 'package:cowplot':
##
##      get_legend
```

```
## The following object is masked from 'package:plyr':
##
##      mutate
```

```r
library(plot3D)
```

```
## Warning: package 'plot3D' was built under R version 3.5.2
```

```r
library(latexpdf)
```

```
## Warning: package 'latexpdf' was built under R version 3.5.2
```

```r
library(car)
```

```
## Warning: package 'car' was built under R version 3.5.2
```

```
## Loading required package: carData
```

```
##
## Attaching package: 'car'
```

```
## The following object is masked from 'package:purrr':
##
##      some
```

```
## The following object is masked from 'package:dplyr':
##
##     recode
```

```r
library(FactoMineR)
```

```
## Warning: package 'FactoMineR' was built under R version 3.5.2
```

```r
library(factoextra)
```

```
## Warning: package 'factoextra' was built under R version 3.5.2
```

```
## Welcome! Related Books: `Practical Guide To Cluster Analysis in R` at https://goo.gl/13EFCZ
```

```r
library(corrplot)
library(mice)
```

```
## Warning: package 'mice' was built under R version 3.5.2
```

```
## Loading required package: lattice
```

```
## Warning: package 'lattice' was built under R version 3.5.2
```

```
##
## Attaching package: 'lattice'
```

```
## The following object is masked from 'package:corrgram':
##
##     panel.fill
```

```
##
## Attaching package: 'mice'
```

```
## The following object is masked from 'package:tidyr':
##
##     complete
```

```
## The following objects are masked from 'package:base':
##
##     cbind, rbind
```

```r
#View the data
View(worldh)
#Displays the first few rows of the dataset
head(worldh)
```

```
##       Country Happiness.Rank Happiness.Score Whisker.high Whisker.low
## 1      Norway             1           7.537     7.594445    7.479556
## 2     Denmark             2           7.522     7.581728    7.462272
## 3     Iceland             3           7.504     7.622030    7.385970
## 4 Switzerland             4           7.494     7.561772    7.426227
## 5     Finland             5           7.469     7.527542    7.410458
## 6 Netherlands             6           7.377     7.427426    7.326574
##   Economy..GDP.per.Capita.   Family Health..Life.Expectancy.    Freedom
## 1                 1.616463 1.533524                0.7966665 0.6354226
## 2                 1.482383 1.551122                0.7925655 0.6260067
## 3                 1.480633 1.610574                0.8335521 0.6271626
## 4                 1.564980 1.516912                0.8581313 0.6200706
## 5                 1.443572 1.540247                0.8091577 0.6179509
## 6                 1.503945 1.428939                0.8106961 0.5853845
##   Generosity Trust..Government.Corruption. Dystopia.Residual
## 1  0.3620122                     0.3159638          2.277027
## 2  0.3552805                     0.4007701          2.313707
## 3  0.4755402                     0.1535266          2.322715
## 4  0.2905493                     0.3670073          2.276716
## 5  0.2454828                     0.3826115          2.430182
## 6  0.4704898                     0.2826618          2.294804
```

```
#Display the structure of the attributes
str(worldh)
```

```
## 'data.frame':    155 obs. of  12 variables:
##  $ Country                     : Factor w/ 155 levels "Afghanistan",..: 105 38 58 133 45 99 26 100 132 7
...
##  $ Happiness.Rank              : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ Happiness.Score             : num  7.54 7.52 7.5 7.49 7.47 ...
##  $ Whisker.high                : num  7.59 7.58 7.62 7.56 7.53 ...
##  $ Whisker.low                 : num  7.48 7.46 7.39 7.43 7.41 ...
##  $ Economy..GDP.per.Capita.    : num  1.62 1.48 1.48 1.56 1.44 ...
##  $ Family                      : num  1.53 1.55 1.61 1.52 1.54 ...
##  $ Health..Life.Expectancy.    : num  0.797 0.793 0.834 0.858 0.809 ...
##  $ Freedom                     : num  0.635 0.626 0.627 0.62 0.618 ...
##  $ Generosity                  : num  0.362 0.355 0.476 0.291 0.245 ...
##  $ Trust..Government.Corruption.: num  0.316 0.401 0.154 0.367 0.383 ...
##  $ Dystopia.Residual           : num  2.28 2.31 2.32 2.28 2.43 ...
```

```r
# Adding another column name "Continent"
worldh$Continent <- NA

# Deleting unnecessary columns (Whisker.high and Whisker.low)
worldh <- worldh[, -c(4,5)]

# Changing the name of columns
colnames (worldh) <- c("Country", "Happiness.Rank", "Happiness.Score",
                       "Economy", "Family", "Life.Expectancy", "Freedom", "Generosity",
                       "Trust", "Dystopia.Residual", "Continent")

# Adding the values for Continent name in the data.

worldh$Continent[which(worldh$Country %in% c("Israel", "United Arab Emirates", "Singapore", "Thailand", "Tai
wan Province of China",
                                             "Qatar", "Saudi Arabia", "Kuwait", "Bahrain", "Malaysia", "Uzb
ekistan", "Japan",
                                             "South Korea", "Turkmenistan", "Kazakhstan", "Turkey", "Hong K
ong S.A.R., China", "Philippines",
                                             "Jordan", "China", "Pakistan", "Indonesia", "Azerbaijan", "Leb
anon", "Vietnam",
                                             "Tajikistan", "Bhutan", "Kyrgyzstan", "Nepal", "Mongolia", "Pa
lestinian Territories",
                                             "Iran", "Bangladesh", "Myanmar", "Iraq", "Sri Lanka", "Armenia
", "India", "Georgia",
                                             "Cambodia", "Afghanistan", "Yemen", "Syria"))] <- "Asia"
worldh$Continent[which(worldh$Country %in% c("Norway", "Denmark", "Iceland", "Switzerland", "Finland",
                                             "Netherlands", "Sweden", "Austria", "Ireland", "Germany",
                                             "Belgium", "Luxembourg", "United Kingdom", "Czech Republic",
                                             "Malta", "France", "Spain", "Slovakia", "Poland", "Italy",
                                             "Russia", "Lithuania", "Latvia", "Moldova", "Romania",
                                             "Slovenia", "North Cyprus", "Cyprus", "Estonia", "Belarus",
                                             "Serbia", "Hungary", "Croatia", "Kosovo", "Montenegro",
                                             "Greece", "Portugal", "Bosnia and Herzegovina", "Macedonia",
                                             "Bulgaria", "Albania", "Ukraine"))] <- "Europe"
worldh$Continent[which(worldh$Country %in% c("Canada", "Costa Rica", "United States", "Mexico",
                                             "Panama","Trinidad and Tobago", "El Salvador", "Belize", "Guat
emala",
                                             "Jamaica", "Nicaragua", "Dominican Republic", "Honduras",
                                             "Haiti"))] <- "North America"
worldh$Continent[which(worldh$Country %in% c("Chile", "Brazil", "Argentina", "Uruguay",
                                             "Colombia", "Ecuador", "Bolivia", "Peru",
                                             "Paraguay", "Venezuela"))] <- "South America"
worldh$Continent[which(worldh$Country %in% c("New Zealand", "Australia"))] <- "Australia"
worldh$Continent[which(is.na(worldh$Continent))] <- "Africa"

# Moving the Continent column at the second position.

worldh <- worldh %>% select(Country,Continent, everything())

str(worldh)
```

```
## 'data.frame':    155 obs. of  11 variables:
##  $ Country          : Factor w/ 155 levels "Afghanistan",..: 105 38 58 133 45 99 26 100 132 7 ...
##  $ Continent        : chr  "Europe" "Europe" "Europe" "Europe" ...
##  $ Happiness.Rank   : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ Happiness.Score  : num  7.54 7.52 7.5 7.49 7.47 ...
##  $ Economy          : num  1.62 1.48 1.48 1.56 1.44 ...
##  $ Family           : num  1.53 1.55 1.61 1.52 1.54 ...
##  $ Life.Expectancy  : num  0.797 0.793 0.834 0.858 0.809 ...
##  $ Freedom          : num  0.635 0.626 0.627 0.62 0.618 ...
##  $ Generosity       : num  0.362 0.355 0.476 0.291 0.245 ...
##  $ Trust            : num  0.316 0.401 0.154 0.367 0.383 ...
##  $ Dystopia.Residual: num  2.28 2.31 2.32 2.28 2.43 ...
```

```
#Converting the Continent values into factorial.
worldh$Continent <- as.factor(worldh$Continent)

# Finding the correlation between numerical columns
Num.cols <- sapply(worldh, is.numeric)
Cor.data <- cor(worldh[, Num.cols])
corrplot(Cor.data, method = 'color')
```
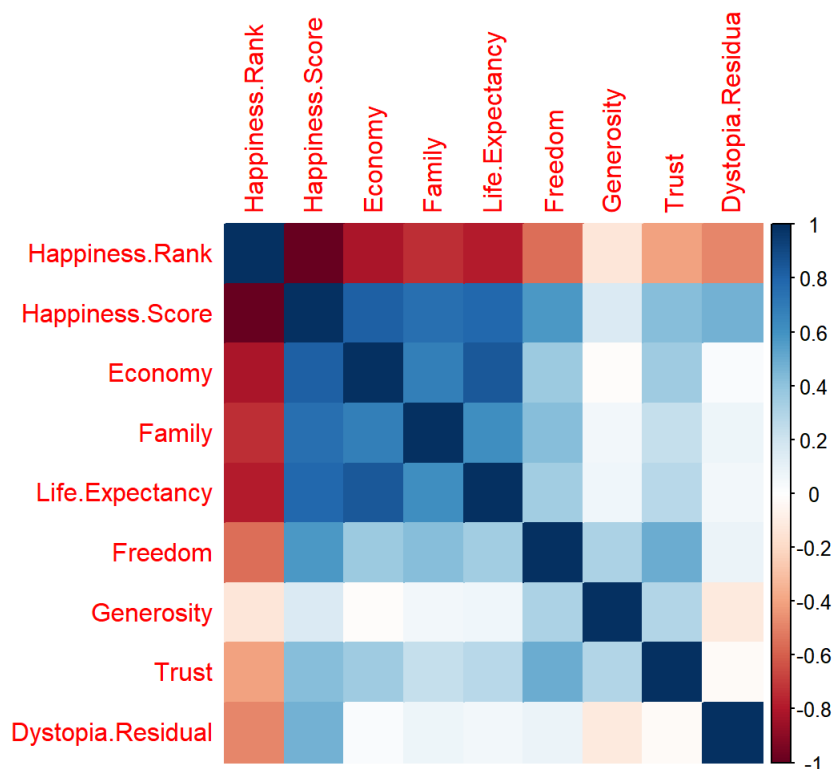


```
#Analysis: We can see there is an inverse correlation between "Happiness Rank" and all the other numerical v
ariables. In other words, the lower the happiness rank, the higher the happiness score, and the higher the o
ther seven factors that contribute to happiness. So let's remove the happiness rank, and see the correlation
again.

# Create a correlation plot
newdatacor = cor(worldh[c(3:10)])
corrplot(newdatacor, method = "number")
```
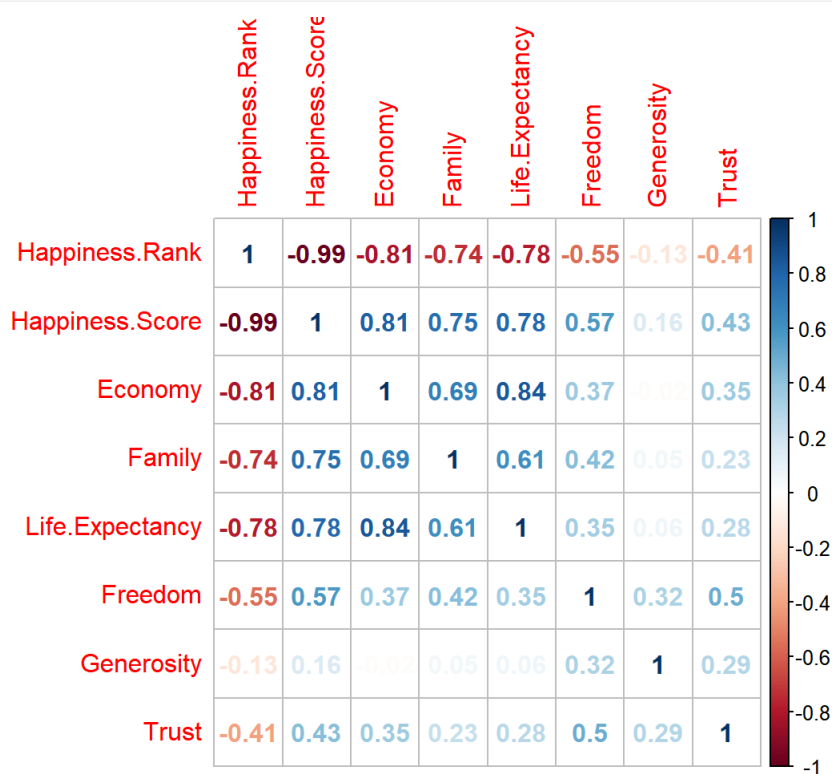
```
#Analysis: In the above cor plot, Economy, life expectancy, and family play the most significant role in con
tributing to happiness.
#Trust and generosity have the lowest impact on the happiness score.

#Plotting ScatterPLot
plot_ly(data = worldh,
        x=~Economy, y=~Happiness.Score, type = "scatter",
        text = ~paste("Country:", Country)) %>%
  layout(title = "Happiness and GDP",
         xaxis = list(title = "GDP per Capita"),
         yaxis = list(title = "Happiness Score"))
```

```
## No scatter mode specifed:
##    Setting the mode to markers
##    Read more about this attribute -> https://plot.ly/r/reference/#scatter-mode
```

```
#Analysis: This interactive scatterplot shows that there is a strong positive correlation between GDP and Ha
ppiness.

#Let's do multiple Regression
dat <- worldh[c("Happiness.Score","Economy","Generosity")]
head(dat)
```

```
##   Happiness.Score  Economy Generosity
## 1          7.537 1.616463  0.3620122
## 2          7.522 1.482383  0.3552805
## 3          7.504 1.480633  0.4755402
## 4          7.494 1.564980  0.2905493
## 5          7.469 1.443572  0.2454828
## 6          7.377 1.503945  0.4704898
```

```
plot(dat)
```
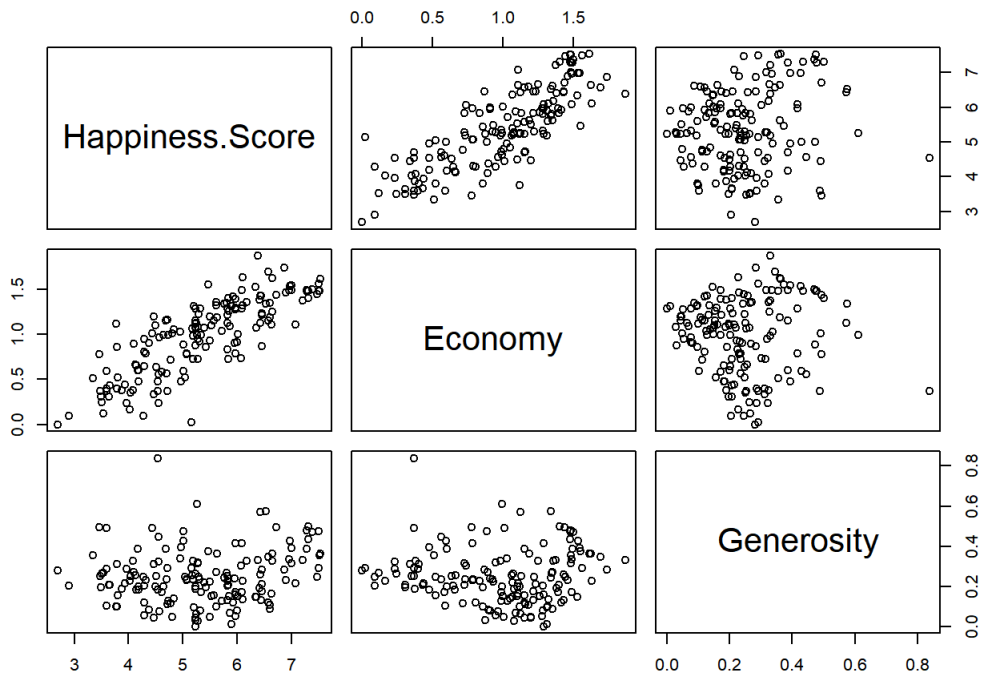
```
#It seems like there is a positive correlation between economy and happiness score but this is not true betw
een happiness score
#and generosity.

#3D plot of same
scatter3D(dat$Generosity, dat$Economy, dat$Happiness.Score, phi = 0, bty = "g",
          pch = 20, cex = 2, ticktype = "detailed",
          main = "Happiness data", xlab = "Generosity",
          ylab ="Economy", zlab = "Happiness.Score")
```

Happiness data
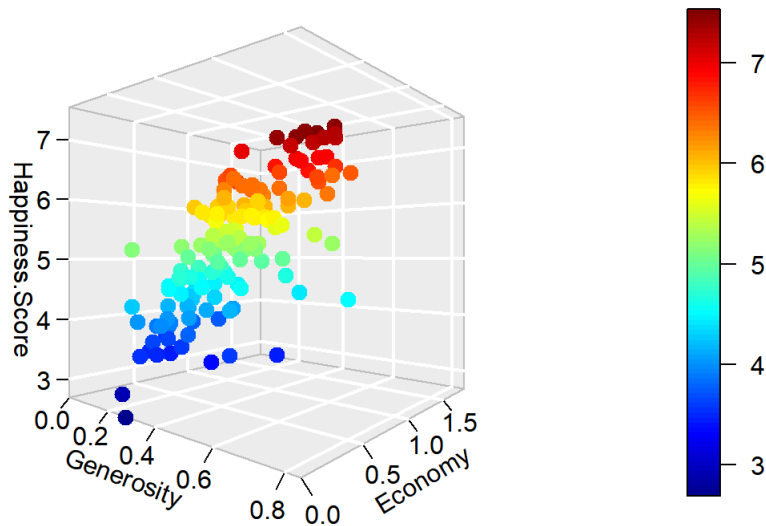
```
#From the scatter plot we cannot determine that combination of high economy and generosity leads to greater
happiness score.
#This is something we have to conclude after analyzing the effect of these 2 taken together.


# Checking the outliers in the dataset using the boxplot.
names(worldh)[4] <- "Happiness_Score"

ggplot(worldh, aes(x=Continent, y= Happiness_Score, colour = Continent)) +

  geom_boxplot() +

  theme(axis.text.x = element_text(angle = 60, hjust = 1)) +

  labs(title = "Happiness Score Boxplot",

       x = "Continent",

       y = "Happiness Score")
```
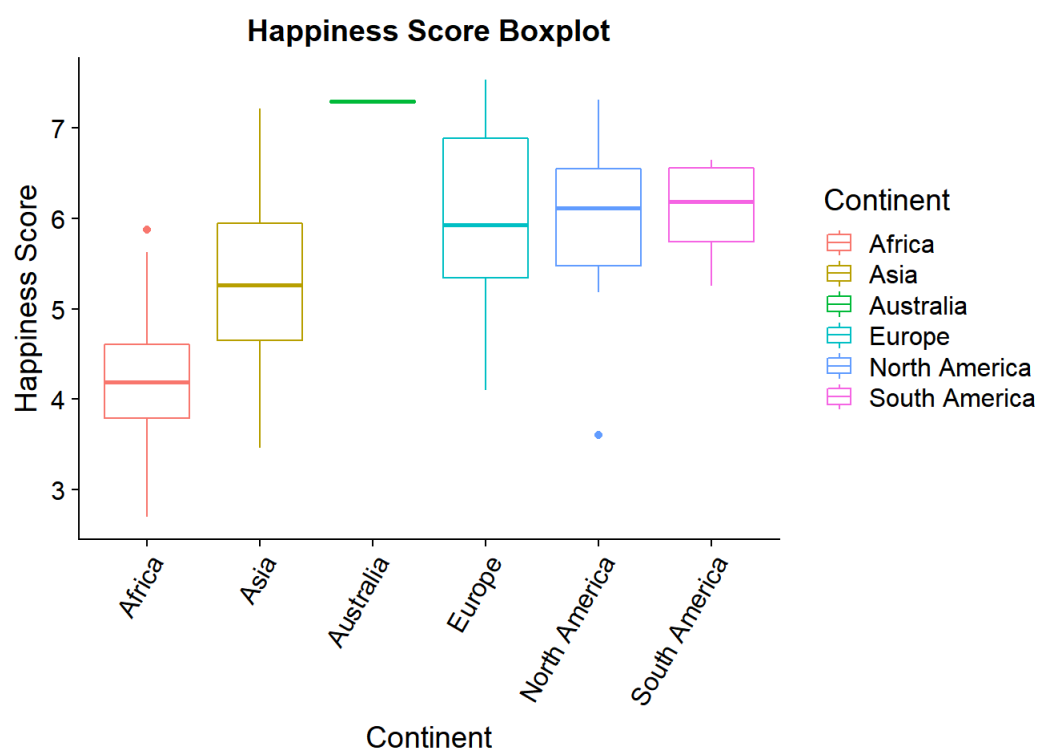


```
##Checking for normality using shaprio test

qqPlot(worldh$Economy)
```
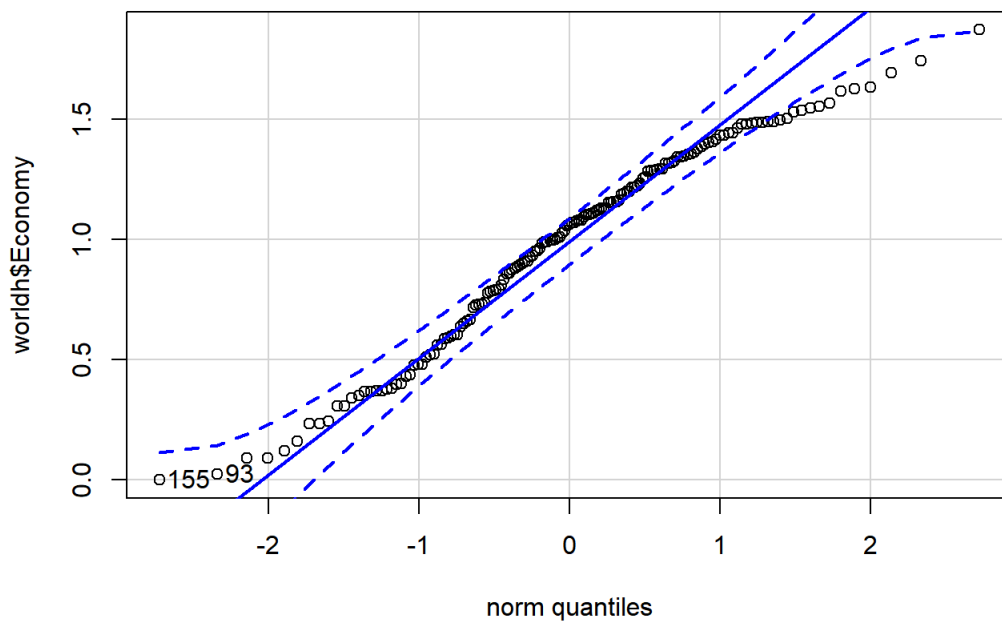
```
## [1] 155   93
```

```
shapiro.test(worldh$Economy)
```

```
##
##   Shapiro-Wilk normality test
##
## data:   worldh$Economy
## W = 0.96977, p-value = 0.00175
```

```
#p-value is greater than 0.05 implying that the data is not significantly different from normal distribution

qqPlot(worldh$Family)
```
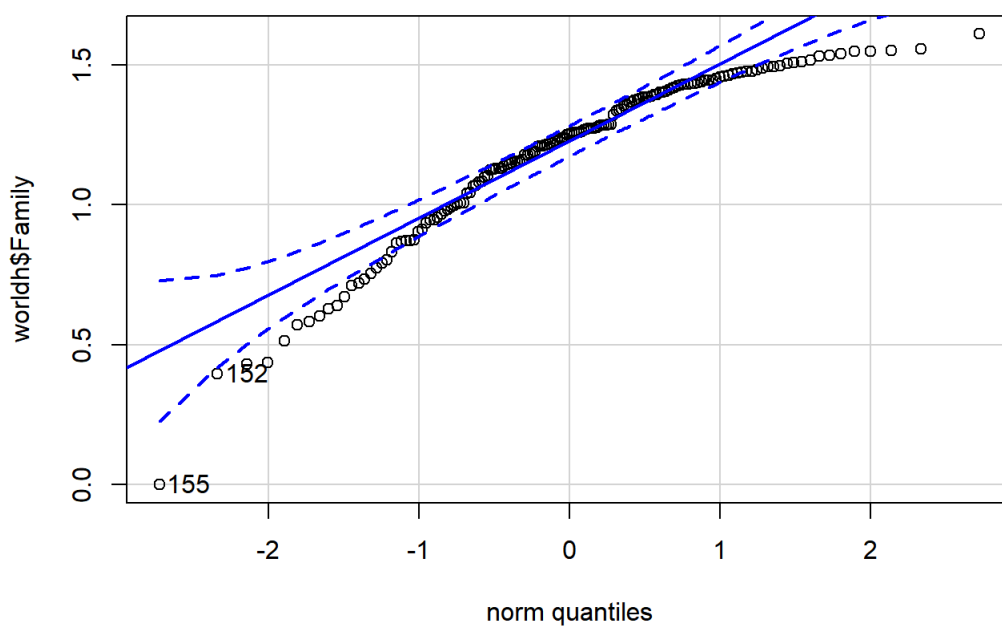
```
## [1] 155 152
```

```
shapiro.test(worldh$Family)
```

```
## 
##  Shapiro-Wilk normality test
## 
## data:  worldh$Family
## W = 0.91152, p-value = 4.186e-08
```

```
qqPlot(worldh$Life.Expectancy)
```
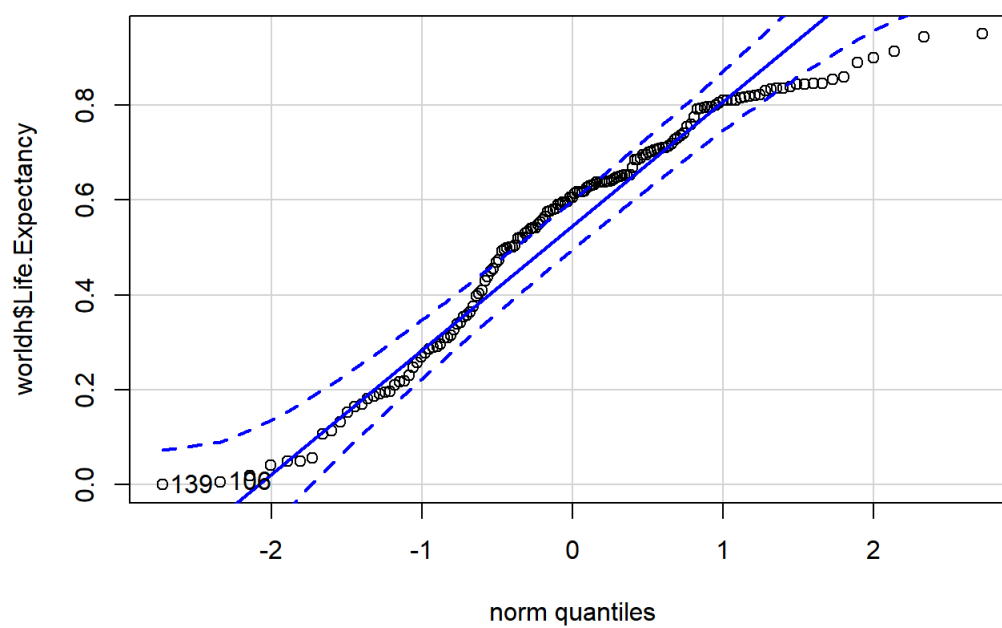


```
## [1] 139 106
```

```
shapiro.test(worldh$Life.Expectancy)
```

```
## 
##  Shapiro-Wilk normality test
## 
## data:  worldh$Life.Expectancy
## W = 0.94602, p-value = 1.135e-05
```

```
qqPlot(worldh$Freedom)
```
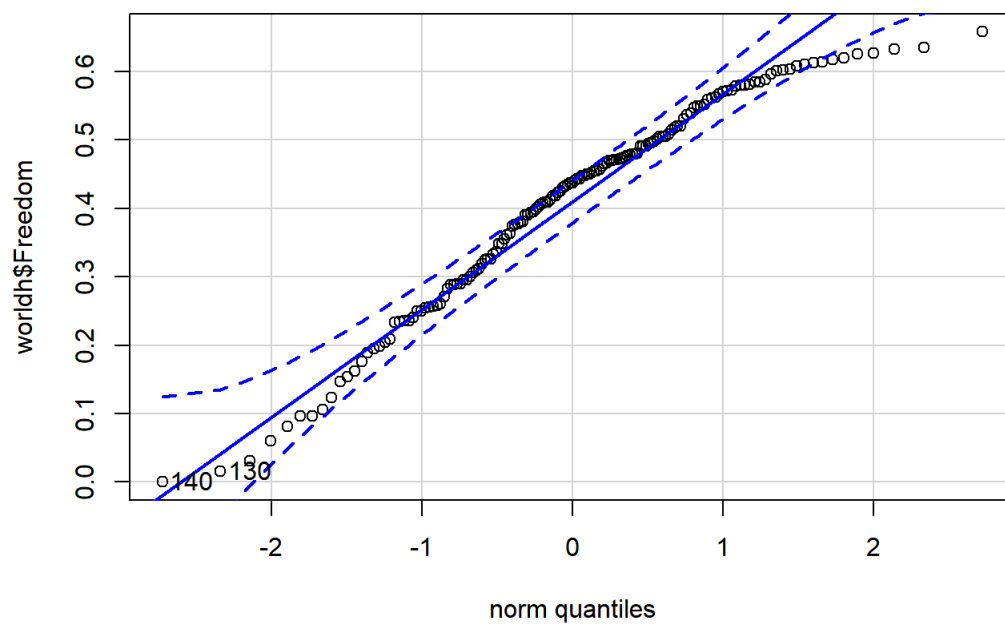
```
## [1] 140 130
```

```
shapiro.test(worldh$Freedom)
```

```
## 
##  Shapiro-Wilk normality test
## 
## data:  worldh$Freedom
## W = 0.95945, p-value = 0.0001673
```

```
qqPlot(worldh$Generosity)
```



```
## [1] 114  81
```

```
shapiro.test(worldh$Generosity)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  worldh$Generosity
## W = 0.95783, p-value = 0.0001184
```

```
qqPlot(worldh$Trust)
```



```
## [1]  26 151
```

```
shapiro.test(worldh$Trust)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  worldh$Trust
## W = 0.83902, p-value = 9.204e-12
```

```
#Family,Life expectancy and trust variables are not normally distributed

####PCA################
act_col <- c(3, 5:10)

act_col
```
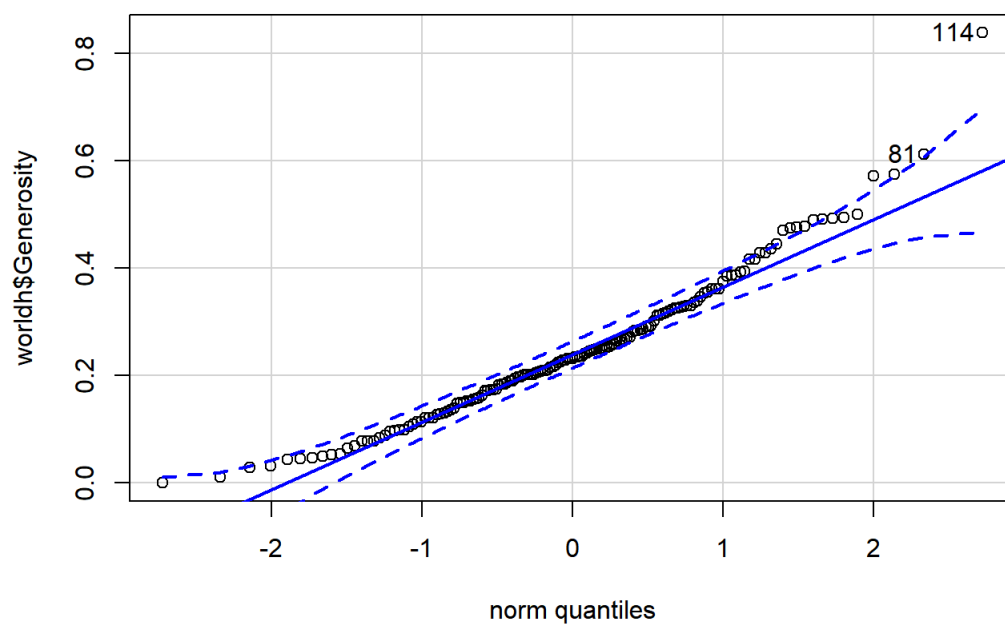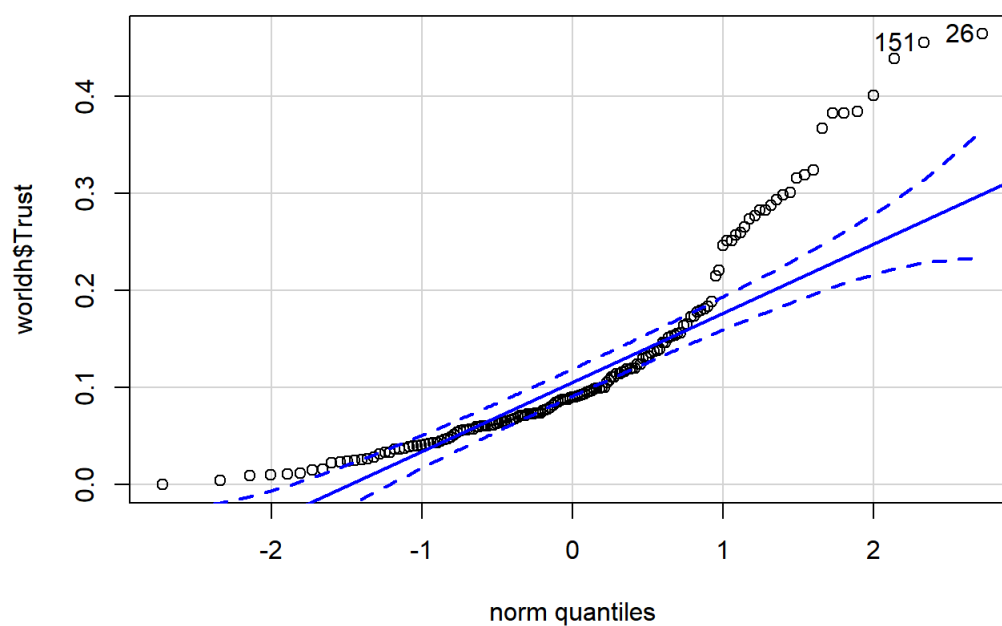
```
## [1]  3  5  6  7  8  9 10
```

```
happiness_new <- worldh[, act_col]

cor(happiness_new)
```

```
##                  Happiness.Rank      Economy      Family Life.Expectancy
## Happiness.Rank        1.0000000 -0.81324364 -0.73675268     -0.78071584
## Economy              -0.8132436  1.00000000  0.68829631      0.84307664
## Family               -0.7367527  0.68829631  1.00000000      0.61208006
## Life.Expectancy      -0.7807158  0.84307664  0.61208006      1.00000000
## Freedom              -0.5516078  0.36987339  0.42496576      0.34982679
## Generosity           -0.1326198 -0.01901125  0.05169263      0.06319149
## Trust                -0.4058423  0.35094410  0.23184139      0.27975198
##                       Freedom  Generosity      Trust
## Happiness.Rank     -0.5516078 -0.13261979 -0.4058423
## Economy             0.3698734 -0.01901125  0.3509441
## Family              0.4249658  0.05169263  0.2318414
## Life.Expectancy     0.3498268  0.06319149  0.2797520
## Freedom             1.0000000  0.31608271  0.4991828
## Generosity          0.3160827  1.00000000  0.2941595
## Trust               0.4991828  0.29415945  1.0000000
```

```
happiness_pca <- prcomp(happiness_new,scale=TRUE)

summary(happiness_pca)
```

```
## Importance of components:
##                           PC1    PC2     PC3     PC4     PC5     PC6
## Standard deviation     1.9426 1.1630 0.82011 0.73722 0.60113 0.40173
## Proportion of Variance 0.5391 0.1932 0.09608 0.07764 0.05162 0.02305
## Cumulative Proportion  0.5391 0.7324 0.82844 0.90608 0.95770 0.98076
##                            PC7
## Standard deviation     0.36702
## Proportion of Variance 0.01924
## Cumulative Proportion  1.00000
```

```
(eigen_happiness <- happiness_pca$sdev^2)
```

```
## [1] 3.7738458 1.3526400 0.6725747 0.5434929 0.3613563 0.1613830 0.1347073
```

```
eigen_happiness
```

```
## [1] 3.7738458 1.3526400 0.6725747 0.5434929 0.3613563 0.1613830 0.1347073
```

```
names(eigen_happiness) <- paste("PC",1:7,sep="")

sumlambdas <- sum(eigen_happiness)
sumlambdas
```

```
## [1] 7
```

```
propvar <- eigen_happiness/sumlambdas
propvar
```

```
##        PC1        PC2        PC3        PC4        PC5        PC6
## 0.53912084 0.19323429 0.09608209 0.07764184 0.05162233 0.02305471
##        PC7
## 0.01924390
```

```
cumvar_happiness <- cumsum(propvar)
cumvar_happiness
```

```
##       PC1       PC2       PC3       PC4       PC5       PC6       PC7
## 0.5391208 0.7323551 0.8284372 0.9060791 0.9577014 0.9807561 1.0000000
```

```r
matlambdas <- rbind(eigen_happiness,propvar,cumvar_happiness)
rownames(matlambdas) <- c("Eigenvalues","Prop. variance","Cum. prop. variance")

round(matlambdas,4)
```

```
##                          PC1    PC2    PC3    PC4    PC5    PC6    PC7
## Eigenvalues           3.7738 1.3526 0.6726 0.5435 0.3614 0.1614 0.1347
## Prop. variance        0.5391 0.1932 0.0961 0.0776 0.0516 0.0231 0.0192
## Cum. prop. variance   0.5391 0.7324 0.8284 0.9061 0.9577 0.9808 1.0000
```

```r
summary(happiness_pca)
```

```
## Importance of components:
##                           PC1    PC2     PC3     PC4     PC5     PC6
## Standard deviation     1.9426 1.1630 0.82011 0.73722 0.60113 0.40173
## Proportion of Variance 0.5391 0.1932 0.09608 0.07764 0.05162 0.02305
## Cumulative Proportion  0.5391 0.7324 0.82844 0.90608 0.95770 0.98076
##                            PC7
## Standard deviation     0.36702
## Proportion of Variance 0.01924
## Cumulative Proportion  1.00000
```

```r
happiness_pca$rotation
```

```
##                        PC1         PC2          PC3        PC4         PC5
## Happiness.Rank   0.4788810 -0.08240033  0.055747166 -0.0423546  0.05297915
## Economy         -0.4551507  0.25969334  0.004308571 -0.2549601 -0.10093319
## Family          -0.4133402  0.18852086 -0.186020546  0.3805632  0.72959861
## Life.Expectancy -0.4371899  0.23420596 -0.171512604 -0.3328363 -0.40772765
## Freedom         -0.3364051 -0.40606620  0.218916193  0.6702964 -0.43207360
## Generosity      -0.1052528 -0.67002041 -0.695293769 -0.2063681  0.05870829
## Trust           -0.2779915 -0.47070149  0.633636921 -0.4309386  0.31355300
##                        PC6         PC7
## Happiness.Rank  0.853440489  0.16677560
## Economy         0.117920263  0.79767244
## Family          0.259451316 -0.12061794
## Life.Expectancy 0.393633806 -0.54094969
## Freedom         0.181306473  0.07183781
## Generosity      0.005882616  0.10243048
## Trust           0.050645253 -0.11435316
```

```r
print(happiness_pca)
```

```
## Standard deviations (1, .., p=7):
## [1] 1.9426389 1.1630305 0.8201065 0.7372197 0.6011292 0.4017250 0.3670249
##
## Rotation (n x k) = (7 x 7):
##                        PC1         PC2          PC3        PC4         PC5
## Happiness.Rank   0.4788810 -0.08240033  0.055747166 -0.0423546  0.05297915
## Economy         -0.4551507  0.25969334  0.004308571 -0.2549601 -0.10093319
## Family          -0.4133402  0.18852086 -0.186020546  0.3805632  0.72959861
## Life.Expectancy -0.4371899  0.23420596 -0.171512604 -0.3328363 -0.40772765
## Freedom         -0.3364051 -0.40606620  0.218916193  0.6702964 -0.43207360
## Generosity      -0.1052528 -0.67002041 -0.695293769 -0.2063681  0.05870829
## Trust           -0.2779915 -0.47070149  0.633636921 -0.4309386  0.31355300
##                        PC6         PC7
## Happiness.Rank  0.853440489  0.16677560
## Economy         0.117920263  0.79767244
## Family          0.259451316 -0.12061794
## Life.Expectancy 0.393633806 -0.54094969
## Freedom         0.181306473  0.07183781
## Generosity      0.005882616  0.10243048
## Trust           0.050645253 -0.11435316
```

```r
happiness_pca$x
```

```
##             PC1         PC2         PC3          PC4          PC5
##  [1,] -3.57859076 -1.07900554  0.449013005 -0.1789550946  0.203059763
##  [2,] -3.64618244 -1.48980003  0.990024008 -0.4608502434  0.573909482
##  [3,] -3.21516819 -0.86940405 -1.236637827  0.4295796131 -0.057504412
##  [4,] -3.62968569 -0.90598652  1.082899829 -0.4344483627  0.241576488
##  [5,] -3.43368285 -0.85829325  1.429869160 -0.2687758954  0.449975887
##  [6,] -3.16035632 -1.46201066 -0.328555808 -0.5222806932  0.034869374
##  [7,] -3.28560398 -1.33882788 -0.131583784 -0.3238333967  0.059282840
##  [8,] -3.57677808 -2.13014748  0.105154667 -0.6567639162  0.592413042
##  [9,] -3.49956834 -1.54378394  0.741862247 -0.6603497428  0.328410764
## [10,] -3.36712682 -1.56090798 -0.298389383 -0.4704326429  0.206716019
## [11,] -1.89146538  0.54001301 -1.077606751 -0.3075777203 -0.263070983
## [12,] -1.84668885  0.35636402 -0.105505461  0.9083457772 -0.466672573
## [13,] -2.79237423 -0.36176314  0.041401371 -0.0658852016 -0.088381006
## [14,] -2.39916192 -0.20881363 -0.916697961 -0.1296829567 -0.184109722
## [15,] -3.26523471 -1.20263917 -0.100086896 -0.4336887413  0.433114686
## [16,] -2.90506434 -0.71664210  0.286038649 -0.3193487783  0.170536280
## [17,] -2.68665200 -0.02123863  0.628818619 -0.1796474566  0.060226074
## [18,] -3.37226839 -0.55115221  0.853494312 -0.5044411931  0.003137049
## [19,] -2.83869138 -1.30732108 -0.687932287 -0.7058434470  0.424782929
## [20,] -1.41918809  0.47802549 -1.037476067 -0.4497009465 -0.351629230
## [21,] -2.82447224 -1.31479390  0.716614012 -0.6150934463 -0.230138049
## [22,] -1.16451407  0.80030259  0.130516142  0.5166461105  0.251516346
## [23,] -1.53624301  1.65570184  0.031308277  0.8442186950 -0.452093769
## [24,] -1.33181556  1.27459580  0.107091847  0.9230011653 -0.222552066
## [25,] -1.00871753  0.94285709  0.522286962 -0.0762243420 -0.356430525
## [26,] -3.61831619 -1.42046514  1.369127960 -1.6878178770  0.192185459
## [27,] -2.60346850 -1.47688651 -1.691335457  0.0231127723 -0.134154767
## [28,] -1.89299703  0.18851224  0.635982360  0.6056855296 -0.189612638
## [29,] -0.65417688 -0.24797305 -0.393243481  0.9044988704 -0.320881822
## [30,] -1.47794544  0.55531392 -0.219829932  0.8289947492 -0.498043555
## [31,] -1.99212365  0.96902058  0.607000570 -0.1295288111 -0.237185195
## [32,] -1.55148595 -1.23419687 -2.276442976  0.7917838221 -0.286220885
## [33,] -1.43534318  1.07334010 -0.851359066 -0.2886707820 -0.122700987
## [34,] -1.83105589  1.41004449 -0.549688279  0.0881812476 -0.042525816
## [35,] -3.20075133 -1.56968139  1.610097003 -1.2013643237  0.129486212
## [36,] -0.79703459  0.96776524 -0.092367101  0.9897397046 -0.079930096
## [37,] -1.66551614  0.20129398  1.372031638 -0.5191461299  0.304637918
## [38,] -1.04702380  0.24183492 -1.102742908  0.9617409025 -0.232021779
## [39,] -1.80155406  0.02716588  0.651292246 -0.3460029121 -0.137015194
## [40,] -0.91643705  1.95923399 -0.577353743  0.1050966205  0.371143609
## [41,] -1.91561522 -0.03013221  1.202980084 -0.1176241826  0.013456985
## [42,] -0.93835238 -0.17505995 -1.395959938 -0.1647706719 -0.073508572
## [43,] -0.58530567 -0.33576072 -0.362186095  0.2275904509  0.028287419
## [44,] -0.84280002  0.54550115  0.470794033  0.2460946386 -0.151271947
## [45,] -0.17441884  0.98166526  0.628192537  0.5092685524 -0.358848890
## [46,] -1.32287800  0.99613248 -0.088869818  0.8807850976 -0.270599346
## [47,] -1.56548051 -1.96894618  0.025623351  1.0336480551  0.752459323
## [48,] -1.02931306  1.99551163 -0.849986193 -0.4690609301  0.110611821
## [49,] -0.54975375  1.89632080  0.179311508  0.7481764976  0.351493322
## [50,] -0.10770319 -0.38495000  0.179694864  0.8154683830 -0.598104104
## [51,] -2.00906491  1.01402221  0.594913430  0.0368696061 -0.338516257
## [52,] -0.30626081  2.67920504  0.003607533  0.1467905922  0.536153498
## [53,]  0.02588527  1.40016408  0.752302860 -0.8006129099  0.221427307
## [54,] -0.63166815  1.36162187 -0.174296089  0.0001178258  0.350049070
## [55,] -0.71913038  1.48977420 -0.623146362 -1.1600014202 -0.648640204
## [56,]  0.59692850  1.12916734 -0.851470784 -0.0079089301  0.248542490
## [57,] -0.36189391  1.27182198 -0.180236277  0.5317302292 -0.963852118
## [58,] -0.11082872 -0.11770014 -0.056839149  1.2389985644 -0.389824396
## [59,] -0.98489847 -0.45874536  0.709833059 -0.0473985221  1.304616725
## [60,] -0.91720786  0.56386753  0.064209941  0.2306271200  0.195240315
## [61,] -1.33555987  0.11545851 -0.031662720 -0.4928707486 -0.672457281
## [62,] -1.52801505  0.59772943 -0.585409784  0.8811346283 -0.560917982
## [63,] -0.19366186  0.99368154  0.110905915  0.6029463081 -0.495166180
## [64,] -0.61297600 -0.16425091 -1.063854732  0.3318621961 -0.599684897
## [65,] -0.67319291  0.90830926 -0.901750144 -0.6308319025 -0.842393336
## [66,] -1.37755476  0.82729656  0.939028050  0.2730222663  0.307097691
## [67,] -0.59575435  0.98895764  0.274504251 -0.3827137907  0.836595860
## [68,] -0.38878436  0.62892988  0.284154207  0.7346851340  0.142668841
## [69,] -0.23750484  1.72748114  0.561092208 -0.1217171325  0.319565230
## [70,] -0.57824668  0.30521353 -0.224895203  0.8816864072  0.468521131
## [71,] -2.26766775 -0.84938034  0.192306160 -1.3411506615 -0.284593363
## [72,] -0.15545867 -0.20919380  0.393974359  1.2567199316 -0.272985584
```

```
##  [73,]  0.26648527  1.26041636 -0.788563334 -0.5989321997  0.289843681
##  [74,] -0.17409237  0.45745866  0.302882708  0.1632081830 -0.040863027
##  [75,]  0.05903833  2.29598688 -0.209045390 -0.5832185749  0.412109038
##  [76,] -0.33197617  0.37519103 -0.443981039  0.7498006646 -0.090676301
##  [77,]  0.33452834  0.93092983 -0.692061430 -0.9926451631 -0.682748842
##  [78,]  0.58314487  0.27905443 -0.962872710 -0.5307628055  0.152427759
##  [79,] -0.10269560  1.60241061  0.475974808  0.6817841380 -1.008908184
##  [80,]  1.65067783 -0.52569174 -0.157235263 -1.2057548562 -0.456481633
##  [81,] -0.06156536 -1.40892526 -2.512404729  0.2394920256  0.046607632
##  [82,]  0.29079981  2.17075839  0.002738907 -0.4736455003  0.946715989
##  [83,]  0.23374620  1.20623716 -0.385653862 -0.9171312566  0.385868835
##  [84,]  0.95528913  0.92988070  1.127073631 -0.0789082453 -1.299423528
##  [85,] -0.01294978  0.82009722  1.426935253 -0.1265528767  0.014506452
##  [86,] -0.61830356  0.15813968  0.317390932  0.9581800194  0.019936013
##  [87,]  0.32950064  2.90486958  0.113447802 -1.1721905755  0.168634474
##  [88,]  0.24559822  0.81460027 -0.880930177 -0.6020821659 -0.386340879
##  [89,] -0.73663365  1.55054859 -0.029805130  0.7699185416 -0.689563348
##  [90,]  0.74987016  0.76586740 -1.507612532 -0.9012619240 -0.308836785
##  [91,]  0.70128428  0.26860970 -0.323608229 -0.0055837434 -0.074890168
##  [92,]  0.21536946  0.60967600 -0.609945679 -0.2823611999 -0.067468059
##  [93,]  1.77607777 -2.84369936  1.672246024  0.6833648896 -0.233484007
##  [94,] -0.18812117 -0.20843391 -0.035343368  0.9733861324 -0.464525643
##  [95,]  1.58055347 -0.06025335 -0.183208747  1.2092278521  0.720584836
##  [96,]  0.40550577 -0.57273804  0.205847645  0.5808786866  0.271113368
##  [97,] -0.32807657 -1.66482410 -0.758504942  0.1740488023  0.512518641
##  [98,]  0.31763161 -0.78037975 -1.505172879  0.7683671383  0.322768775
##  [99,]  0.82251806 -0.98807412 -0.964680717  0.4676614686  0.021333727
## [100,] -0.05273460  0.19389174 -1.229961339  0.5241006045  0.580890276
## [101,]  0.62388282  0.34776390  0.508012972  1.4008505745  0.728460741
## [102,]  1.09942118  1.29185070  0.809408296 -0.6238623373 -0.752689428
## [103,]  1.12023428  1.01426108  0.288308146 -0.2669674840  0.266459312
## [104,]  1.00521750  0.77019321  0.549520524 -0.5853882128 -0.116687457
## [105,]  0.13334311  1.88281735 -0.420362235  0.1179282388  0.283636533
## [106,]  2.57469420 -0.84999467 -0.137953464  0.5892498983  0.718610429
## [107,]  2.04145380 -0.55883103  0.130249902  0.9102414959 -0.050471795
## [108,]  1.10977613 -0.09693228 -1.135804784 -1.5008363607 -1.078465629
## [109,]  0.86542303  0.56091616 -0.165689579 -0.4982266160 -1.451834146
## [110,]  1.35883007 -0.44017599  0.832847725  0.0578689887 -1.219182239
## [111,]  0.86081961  0.33814896  1.104860730  1.0453241874 -0.307614005
## [112,]  1.34738358 -1.47451269 -1.030803394  0.5448680342  0.028979708
## [113,]  2.08865072 -2.00573137  0.630966196  0.5921075370  0.177032308
## [114,]  0.55183758 -4.17156868 -2.293856130 -0.2421059723  0.444054020
## [115,]  1.35615544 -0.09187119  0.383802852  0.4500965647  0.423353296
## [116,]  1.59285208 -0.64501491  0.159500525  0.7634819166 -0.127968954
## [117,]  1.03354264  0.44766508  0.191230360 -0.7139680299 -0.153397200
## [118,]  1.10650532  1.33280785  0.827860567  0.1417263911  0.290635184
## [119,]  1.79986845 -1.40505687  0.322141350 -0.0791400991 -0.120796976
## [120,] -0.21459107 -1.33852695 -1.391473171  0.4529846546 -0.388206744
## [121,]  1.51521244  1.66387402  0.042250806 -0.6325094036 -0.300392207
## [122,]  1.43892688 -0.52584403  0.379371065  0.0620628581 -1.118821476
## [123,]  1.91327789  0.50785358  0.053139247 -0.7421947867  1.726155312
## [124,]  1.83486311  0.07159137  0.892826413  0.4519339686 -0.625017061
## [125,]  1.26881609  0.21659551  2.038510524 -1.6743249779 -1.076859518
## [126,]  2.64347168 -0.20711366 -0.357056245  0.5489262741  1.295403993
## [127,]  1.97238957 -0.04394473  0.337527277  0.6384363132  1.282533562
## [128,]  2.21146749 -0.92099559  0.941396371  0.7278300231  0.182928297
## [129,]  0.98345245 -1.61438997 -0.466739655  1.1365315716 -0.858873698
## [130,]  2.37518720  1.00413447 -0.215638919 -0.9729786728  1.654616555
## [131,]  2.06157488 -0.38843178 -0.188792818  0.5465048396 -0.560403533
## [132,]  1.22890977  1.12645653 -1.245347886 -0.6468514931  1.090057577
## [133,]  1.98155224 -1.02116536 -0.426608096  1.0168219553  0.365066781
## [134,]  2.30579180 -0.69970833  0.238917999  0.2382053163  0.661734024
## [135,]  2.37465573 -0.87923924  0.518214738  0.3430625446  0.423301845
## [136,]  2.81622514 -1.37160640  0.236038573  0.2479219275 -1.368156434
## [137,]  3.26673738  0.03810487 -0.045376385 -0.0789042527  0.963288203
## [138,]  2.38813295 -0.29412756  0.412581292  0.5524204241  0.655790520
## [139,]  2.28879123 -0.42997271  0.878680206  1.0719698515  1.135336469
## [140,]  3.02351471  1.35178692  0.332192021 -0.7621834317  1.699280417
## [141,]  3.65764812 -0.45726281 -0.429422385 -1.1794915517  0.017907334
## [142,]  0.83976623  0.36771883  0.960230241  0.9546267038  0.068322686
## [143,]  3.16815949 -0.85782974  0.648347013  0.1550807654 -1.375580499
## [144,]  2.83528246  0.14770720 -0.090942012 -0.4550997774  0.286495808
## [145,]  3.39869086 -1.20640549 -1.316955242 -1.9953016954  0.429055128
```

```
## [145,]   3.59869086 -1.20040549 -1.516955242 -1.9955016954  0.429055128
## [146,]   2.61049838  0.67610917  0.509489231 -0.0352743389  0.137672230
## [147,]   3.50623529 -0.71298832  0.119260545 -1.1451821612  0.148346569
## [148,]   3.11672979 -0.69073080 -0.221126791  0.4775553596  0.006091631
## [149,]   2.94328328 -1.07075480  0.340234851  0.1113525986  0.009353500
## [150,]   3.33086954 -0.89634966  0.838133192 -0.1659306678 -1.176304642
## [151,]   0.90890133 -2.93129847  2.696761035 -0.3454789097  0.531420637
## [152,]   2.71242548 -1.30599651 -0.934824563 -2.8828984344 -0.651856612
## [153,]   1.98262934 -0.92958240 -0.618511584  0.2775681081  0.075076935
## [154,]   4.24061565 -0.11655107  0.203365889 -1.0389509963  0.437649384
## [155,]   5.04420533 -1.54281984  0.449205665 -0.6897084890 -1.569703496
##                    PC6          PC7
##    [1,] -0.193278348  0.186116013
##    [2,] -0.172181065 -0.167388330
##    [3,] -0.148434455  0.084584687
##    [4,] -0.059863309 -0.152686457
##    [5,] -0.131868540 -0.329984556
##    [6,] -0.273250738  0.099234722
##    [7,] -0.142311156 -0.039944517
##    [8,] -0.059130911 -0.219963544
##    [9,] -0.060672088 -0.140365759
##   [10,] -0.045162961 -0.040299210
##   [11,] -0.537082655 -0.137569790
##   [12,] -0.474430284 -0.495376570
##   [13,] -0.167896822  0.013520788
##   [14,] -0.350806126  0.364060829
##   [15,]  0.013156479  0.084711002
##   [16,] -0.104216928  0.008229181
##   [17,] -0.114115888 -0.135163735
##   [18,]  0.128005879  0.326818335
##   [19,] -0.092388291  0.012645203
##   [20,] -0.551868609 -0.267821451
##   [21,] -0.196052533  0.527361820
##   [22,] -0.678138566 -0.263153265
##   [23,] -0.334650185 -0.057422210
##   [24,] -0.438445266 -0.246228590
##   [25,] -0.673123040 -0.353408820
##   [26,]  0.364472492 -0.070590320
##   [27,]  0.097452031  0.048672773
##   [28,] -0.174464087 -0.255861877
##   [29,] -0.794560353 -0.260264670
##   [30,] -0.276136008 -0.038219160
##   [31,] -0.008462204 -0.196264120
##   [32,] -0.285461377  0.214373048
##   [33,] -0.236382139  0.100714799
##   [34,]  0.117517903 -0.304710111
##   [35,]  0.168782065  0.923190698
##   [36,] -0.468588265 -0.137918913
##   [37,] -0.353711199  0.528005369
##   [38,] -0.454880796  0.797179815
##   [39,] -0.210909645  0.794175687
##   [40,] -0.264113185 -0.023098679
##   [41,] -0.053922441  0.382974407
##   [42,] -0.429221722  0.443728330
##   [43,] -0.423995421 -0.820826587
##   [44,] -0.270718033 -0.513527847
##   [45,] -0.575603297 -0.443294964
##   [46,]  0.054760571  0.075370909
##   [47,] -0.037456516 -0.411964425
##   [48,]  0.042234399 -0.152041873
##   [49,] -0.317218260  0.283495782
##   [50,] -0.664714591  0.117495964
##   [51,]  0.564339600 -0.306564081
##   [52,] -0.292703249  0.093755624
##   [53,] -0.582210864 -0.267733858
##   [54,] -0.168253394  0.088785754
##   [55,] -0.009844272 -0.101147476
##   [56,] -0.630939320 -0.662451468
##   [57,] -0.152649722  0.145044211
##   [58,] -0.367501250 -0.073342668
##   [59,] -0.154728084  0.191483215
##   [60,]  0.039953860  0.278527608
##   [61,]  0.338720944 -0.013120756
```

```
##  [62,]   0.590383391   0.122131642
##  [63,]  -0.106487132  -0.138329601
##  [64,]   0.018077536   0.346762516
##  [65,]   0.187413127   0.095062523
##  [66,]   0.473302424  -0.002775801
##  [67,]   0.089170435  -0.181086011
##  [68,]  -0.007236319   0.174643612
##  [69,]   0.015394869  -0.065462690
##  [70,]   0.228452326  -0.276232951
##  [71,]   0.932945889   0.068066874
##  [72,]  -0.030232900  -0.029535364
##  [73,]  -0.127612630  -0.137884220
##  [74,]   0.066194003  -0.193942698
##  [75,]   0.061235244   0.041635210
##  [76,]   0.301093229  -0.301690831
##  [77,]  -0.127683194   0.215326533
##  [78,]  -0.280845582   0.039237636
##  [79,]   0.354157646  -0.257612646
##  [80,]  -0.954540549   0.043669223
##  [81,]   0.043343159   0.545412232
##  [82,]   0.099433223  -0.158946467
##  [83,]   0.092818394  -0.108885105
##  [84,]  -0.240498389  -0.235426459
##  [85,]   0.137086945   0.162175842
##  [86,]   0.577179774   0.099462008
##  [87,]   0.303005372  -0.248459151
##  [88,]   0.279711377  -0.134052063
##  [89,]   0.916953061   0.082833148
##  [90,]   0.070157158  -0.157719551
##  [91,]   0.088550829  -0.467063578
##  [92,]   0.329947196   0.018053404
##  [93,]  -0.818051720  -0.626250051
##  [94,]   0.674596139  -0.471114794
##  [95,]  -0.595514360   0.889470731
##  [96,]   0.338744749  -0.783842741
##  [97,]   0.524972052   0.105934275
##  [98,]   0.482386426  -0.498876776
##  [99,]   0.190242444  -0.583944337
## [100,]   0.656908535   0.184009935
## [101,]   0.084266594   0.975985097
## [102,]   0.105128030  -0.040636462
## [103,]   0.184945634  -0.571351883
## [104,]   0.108465434   0.116800482
## [105,]   0.838879489  -0.058494468
## [106,]  -0.864297415   0.317525070
## [107,]  -0.490735969   0.450666220
## [108,]   0.109917634   0.551397073
## [109,]   0.466802080  -0.065687540
## [110,]   0.138098063  -0.427881179
## [111,]   0.299291233   0.565643543
## [112,]   0.050221774   0.160915722
## [113,]  -0.452224041  -0.115792885
## [114,]   0.382968464  -0.232439335
## [115,]   0.273880805  -0.547138793
## [116,]   0.011384763   0.306459823
## [117,]   0.346023696   0.495721485
## [118,]   0.317977481   0.864099704
## [119,]   0.001168142  -0.481115748
## [120,]   1.162528253   0.319529913
## [121,]   0.463609676  -0.236681338
## [122,]   0.288804793   0.252272601
## [123,]   0.021730864  -0.097740185
## [124,]   0.073556788   0.549716149
## [125,]   0.424162557  -0.193279842
## [126,]  -0.139181074  -0.721667143
## [127,]   0.103166213  -0.026038480
## [128,]  -0.199117047   0.704747772
## [129,]   0.745635607   0.092715666
## [130,]  -0.007590484  -0.039829069
## [131,]   0.178338082   0.435322992
## [132,]   0.833621680  -0.119180769
## [133,]   0.281298097  -0.002299740
## [134,]   0.095054122  -0.202298447
```

```
## [135,]  0.158958570 -0.673166247
## [136,] -0.064028299 -0.270447092
## [137,] -0.424775457  0.382526225
## [138,]  0.181962451 -0.125137424
## [139,]  0.087634227  0.533086419
## [140,] -0.292788765  0.921759870
## [141,] -0.523352360  0.203246680
## [142,]  1.034660251  0.924362614
## [143,] -0.178776967  0.336050395
## [144,]  0.228037399 -0.595354299
## [145,] -0.307565614 -0.034378003
## [146,]  0.321276399  0.054379283
## [147,] -0.345268628  0.185689232
## [148,]  0.135651433 -0.441236917
## [149,]  0.112140708 -0.158756031
## [150,] -0.060479244 -0.028784797
## [151,]  0.997072472 -0.567340336
## [152,]  0.177626412  0.329349742
## [153,]  0.803875721  0.005725216
## [154,] -0.416996392 -0.419279115
## [155,] -0.968476275  0.168449208
```

happiness_new

```
##    Happiness.Rank   Economy    Family Life.Expectancy    Freedom
## 1               1 1.61646318 1.5335236    0.796666503 0.63542259
## 2               2 1.48238301 1.5511216    0.792565525 0.62600672
## 3               3 1.48063302 1.6105740    0.833552122 0.62716264
## 4               4 1.56497955 1.5169117    0.858131289 0.62007058
## 5               5 1.44357193 1.5402467    0.809157670 0.61795086
## 6               6 1.50394464 1.4289392    0.810696125 0.58538449
## 7               7 1.47920442 1.4813490    0.834557652 0.61110091
## 8               8 1.40570605 1.5481951    0.816759706 0.61406213
## 9               9 1.49438727 1.4781622    0.830875158 0.61292410
## 10             10 1.48441494 1.5100420    0.843886793 0.60160738
## 11             11 1.37538242 1.3762900    0.838404000 0.40598860
## 12             12 1.10970628 1.4164037    0.759509265 0.58013165
## 13             13 1.48709726 1.4599450    0.815328419 0.56776619
## 14             14 1.54625928 1.4199206    0.774286628 0.50574052
## 15             15 1.53570664 1.5582311    0.809782624 0.57311034
## 16             16 1.48792338 1.4725204    0.798950732 0.56251138
## 17             17 1.46378076 1.4623127    0.818091869 0.53977072
## 18             18 1.74194360 1.4575837    0.845089495 0.59662789
## 19             19 1.44163394 1.4964601    0.805335939 0.50819004
## 20             20 1.25278461 1.2840250    0.819479704 0.37689528
## 21             21 1.62634337 1.2664102    0.726798236 0.60834527
## 22             22 1.10735321 1.4313060    0.616552353 0.43745375
## 23             23 1.35268235 1.4338852    0.754444003 0.49094617
## 24             24 1.18529546 1.4404511    0.695137084 0.49451920
## 25             25 1.15318382 1.2108622    0.709978998 0.41273001
## 26             26 1.69227767 1.3538144    0.949492395 0.54984057
## 27             27 1.34327984 1.4884117    0.821944237 0.58876705
## 28             28 1.21755970 1.4122279    0.719216824 0.57939225
## 29             29 0.87200195 1.2555852    0.540239990 0.53131062
## 30             30 1.23374844 1.3731925    0.706156135 0.55002683
## 31             31 1.43092346 1.3877769    0.844465852 0.47022212
## 32             32 1.12786877 1.4257925    0.647239029 0.58020073
## 33             33 1.43362653 1.3845654    0.793984234 0.36146659
## 34             34 1.38439786 1.5320909    0.888960600 0.40878123
## 35             35 1.87076569 1.2742969    0.710098088 0.60413098
## 36             36 1.07062232 1.4021829    0.595027924 0.47748742
## 37             37 1.53062356 1.2866776    0.590148330 0.44975057
## 38             38 1.36135590 1.3802285    0.519983292 0.51863074
## 39             39 1.63295245 1.2596987    0.632105708 0.49633759
## 40             40 1.32539356 1.5050592    0.712732911 0.29581746
## 41             41 1.48841226 1.3231105    0.653133035 0.53674692
## 42             42 1.29121542 1.2846460    0.618784428 0.40226498
## 43             43 0.73729920 1.2872157    0.653095961 0.44755185
## 44             44 1.00082040 1.2861688    0.685636222 0.45519820
## 45             45 0.90978450 1.1821251    0.596018553 0.43245253
## 46             46 1.29178786 1.4457120    0.699475348 0.52034211
```

```
## 47            47 0.78644109 1.5489691    0.498272628 0.65824866
## 48            48 1.39506662 1.4449233    0.853144348 0.25645071
## 49            49 1.28177810 1.4692824    0.547349334 0.37378311
## 50            50 0.90797532 1.0814178    0.450191766 0.54750937
## 51            51 1.41691518 1.4363378    0.913475871 0.50562555
## 52            52 1.31458235 1.4735161    0.628949940 0.23423178
## 53            53 1.09186447 1.1462175    0.617584646 0.23333581
## 54            54 1.26074862 1.4047149    0.638566971 0.32570791
## 55            55 1.40167844 1.1282744    0.900214076 0.25792167
## 56            56 0.72887063 1.2518256    0.589465201 0.24072905
## 57            57 1.21768391 1.1500913    0.685158312 0.45700374
## 58            58 0.83375657 1.2276191    0.473630250 0.55873293
## 59            59 1.13077676 1.4931492    0.437726080 0.41827193
## 60            60 1.28455627 1.3843690    0.606041551 0.43745428
## 61            61 1.34691131 1.1863034    0.834647238 0.47120363
## 62            62 1.34120595 1.4525188    0.790828228 0.57257581
## 63            63 1.03522527 1.2187704    0.630166113 0.45000288
## 64            64 1.18939555 1.2095610    0.638007462 0.49124733
## 65            65 1.35593808 1.1313633    0.844714701 0.35511154
## 66            66 1.32087934 1.4766711    0.695168316 0.47913143
## 67            67 1.15655756 1.4449452    0.637714267 0.29540026
## 68            68 1.10180306 1.3575643    0.520169020 0.46573323
## 69            69 1.19827437 1.3377532    0.637605608 0.30074060
## 70            70 0.93253732 1.5072849    0.579250693 0.47350779
## 71            71 1.55167484 1.2627909    0.943062425 0.49096864
## 72            72 0.85769922 1.2539176    0.468009055 0.58521467
## 73            73 1.06931758 1.2581898    0.650784671 0.20871553
## 74            74 0.99101239 1.2390889    0.604590058 0.41842115
## 75            75 1.28601193 1.3431331    0.687763453 0.17586352
## 76            76 0.92557931 1.3682181    0.641022384 0.47430724
## 77            77 1.22255623 0.9679830    0.701288521 0.25577229
## 78            78 0.95148438 1.1378535    0.541452050 0.26028794
## 79            79 1.08116579 1.1608374    0.741415501 0.47278771
## 80            80 0.72688353 0.6726907    0.402047783 0.23521526
## 81            81 0.99553859 1.2744447    0.492345721 0.44332346
## 82            82 1.12843120 1.4313376    0.617144227 0.15399712
## 83            83 1.12112904 1.2383765    0.667464674 0.19498906
## 84            84 0.87811458 0.7748644    0.597710669 0.40815833
## 85            85 1.15360177 1.1524003    0.540775776 0.39815584
## 86            86 1.07937384 1.4024167    0.574873745 0.55258983
## 87            87 1.28948748 1.2394146    0.810198903 0.09573125
## 88            88 1.07498753 1.1296242    0.735081077 0.28851599
## 89            89 1.31517529 1.3670430    0.795843542 0.49846530
## 90            90 0.98240942 1.0693359    0.705186307 0.20440318
## 91            91 0.73057312 1.1439450    0.582569480 0.34807986
## 92            92 1.06457794 1.2078930    0.644948184 0.32590598
## 93            93 0.02264318 0.7211514    0.113989137 0.60212696
## 94            94 0.78854758 1.2774913    0.652168989 0.57105559
## 95            95 0.78375626 1.2157705    0.056915730 0.39495257
## 96            96 0.52471364 1.2714633    0.529235125 0.47156671
## 97            97 0.88541639 1.3401265    0.495879292 0.50153768
## 98            98 0.59622008 1.3942386    0.553457797 0.45494339
## 99            99 0.47982019 1.1792833    0.504130781 0.44030595
## 100          100 1.02723587 1.4930112    0.557783484 0.39414397
## 101          101 1.05469871 1.3847886    0.187080070 0.47924674
## 102          102 1.00726581 0.8683515    0.613212049 0.28968069
## 103          103 0.71624923 1.1556472    0.565666974 0.25471106
## 104          104 0.98970181 0.9974714    0.520187259 0.28211015
## 105          105 1.16145909 1.4343795    0.708217680 0.28923172
## 106          106 0.36842093 0.9841360    0.005564754 0.31869769
## 107          107 0.56430537 0.9460182    0.132892117 0.43038875
## 108          108 1.15687311 0.7115512    0.639333189 0.24932261
## 109          109 0.99619275 0.8036852    0.731159747 0.38149863
## 110          110 0.58668298 0.7351317    0.533241034 0.47835666
## 111          111 0.96443433 1.0984708    0.338611811 0.52030355
## 112          112 0.56047946 1.0679507    0.309988350 0.45276377
## 113          113 0.23430565 0.8707010    0.106654435 0.48079109
## 114          114 0.36711055 1.1232359    0.397522569 0.51449203
## 115          115 0.47930902 1.1796919    0.409362853 0.37792227
## 116          116 0.63640678 1.0031873    0.257835895 0.46160349
## 117          117 1.10271049 0.9786132    0.501180470 0.28855553
## 118          118 1.19821024 1.1556202    0.356578588 0.31232858
## 119          119 0.33923385 0.8646692    0.353409708 0.40884274
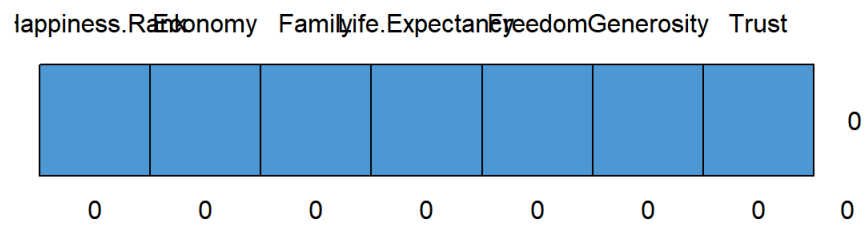```

```
## 120             120 1.00985014 1.2599764      0.625130832 0.56121325
## 121             121 0.90059674 1.0074837      0.637524426 0.19830327
## 122             122 0.79222125 0.7543726      0.455427617 0.46998700
## 123             123 0.64845729 1.2720308      0.285349280 0.09609804
## 124             124 0.80896425 0.8320444      0.289957434 0.43502587
## 125             125 0.95061266 0.5706149      0.649546981 0.30941004
## 126             126 0.09210235 1.2290235      0.191407025 0.23596135
## 127             127 0.47618049 1.2814734      0.169365674 0.30661374
## 128             128 0.60304892 0.9047800      0.048642170 0.44770619
## 129             129 0.60176510 1.0062383      0.429783404 0.63337582
## 130             130 0.65951669 1.2140086      0.290920824 0.01499586
## 131             131 0.66722482 0.8736647      0.295637727 0.42302629
## 132             132 0.89465195 1.3945376      0.575903952 0.12297478
## 133             133 0.38143072 1.1298277      0.217632607 0.44318596
## 134             134 0.35022771 1.0432800      0.215844259 0.32436785
## 135             135 0.16192533 0.9930250      0.268505007 0.36365870
## 136             136 0.23344204 0.5125688      0.315089583 0.46691465
## 137             137 0.43801299 0.9538559      0.041134715 0.16234203
## 138             138 0.37584653 1.0830959      0.196763754 0.33638421
## 139             139 0.52102125 1.1900952      0.000000000 0.39066130
## 140             140 0.85842818 1.1044120      0.049868666 0.00000000
## 141             141 0.40147722 0.5815433      0.180746779 0.10617952
## 142             142 1.12209415 1.2215550      0.341755509 0.50519633
## 143             143 0.43108541 0.4352998      0.209930211 0.42596278
## 144             144 0.30580869 0.9130204      0.375223309 0.18919677
## 145             145 0.36861026 0.6404498      0.277321130 0.03036986
## 146             146 0.59168345 0.9353822      0.310080916 0.24946372
## 147             147 0.39724863 0.6013231      0.163486004 0.14706244
## 148             148 0.11904179 0.8721179      0.229918197 0.33288118
## 149             149 0.24454993 0.7912447      0.194129139 0.34858751
## 150             150 0.30544472 0.4318825      0.247105569 0.38042614
## 151             151 0.36874589 0.9457070      0.326424807 0.58184385
## 152             152 0.77715313 0.3961026      0.500533342 0.08153944
## 153             153 0.51113588 1.0419898      0.364509284 0.39001778
## 154             154 0.09162257 0.6297936      0.151610792 0.05990075
## 155             155 0.00000000 0.0000000      0.018772686 0.27084205
##      Generosity        Trust
## 1    0.36201224 0.315963835
## 2    0.35528049 0.400770068
## 3    0.47554022 0.153526559
## 4    0.29054928 0.367007285
## 5    0.24548277 0.382611543
## 6    0.47048983 0.282661825
## 7    0.43553972 0.287371516
## 8    0.50000513 0.382816702
## 9    0.38539925 0.384398729
## 10   0.47769925 0.301183730
## 11   0.33008265 0.085242100
## 12   0.21461323 0.100106589
## 13   0.31647232 0.221060365
## 14   0.39257878 0.135638788
## 15   0.42785832 0.298388153
## 16   0.33626917 0.276731938
## 17   0.23150334 0.251343131
## 18   0.28318098 0.318834424
## 19   0.49277416 0.265428066
## 20   0.32666242 0.082287982
## 21   0.36094195 0.324489564
## 22   0.16234989 0.111092761
## 23   0.08810676 0.036872927
## 24   0.10945706 0.059739888
## 25   0.12099043 0.132774115
## 26   0.34596598 0.464307785
## 27   0.57473058 0.153066069
## 28   0.17509693 0.178061873
## 29   0.28348839 0.077223279
## 30   0.21055694 0.070983924
## 31   0.12976231 0.172502428
## 32   0.57212311 0.031612735
## 33   0.25836048 0.063829236
## 34   0.19013357 0.070914097
## 35   0.33047387 0.439299256
## 36   0.14901447 0.046668742
```

```
## 36   0.14901447 0.046668742
## 37   0.14761601 0.273432255
## 38   0.32529646 0.008964816
## 39   0.22828980 0.215159550
## 40   0.13654448 0.024210852
## 41   0.17266849 0.257042170
## 42   0.41660893 0.065600708
## 43   0.30167422 0.130687982
## 44   0.15011247 0.140134647
## 45   0.07825799 0.089980960
## 46   0.15846597 0.059307806
## 47   0.41598365 0.246528223
## 48   0.17278965 0.028028091
## 49   0.05226382 0.032962881
## 50   0.24001564 0.096581072
## 51   0.12057277 0.163760737
## 52   0.01016466 0.011865643
## 53   0.06943665 0.146096110
## 54   0.15307479 0.073842727
## 55   0.20667437 0.063282669
## 56   0.20877913 0.010091286
## 57   0.13351992 0.004387901
## 58   0.22556072 0.060477726
## 59   0.24992499 0.259270340
## 60   0.20196442 0.119282886
## 61   0.26684570 0.155353352
## 62   0.24264909 0.045128979
## 63   0.12681971 0.047049087
## 64   0.36093375 0.042181555
## 65   0.27125430 0.041237976
## 66   0.09889081 0.183248922
## 67   0.15513751 0.156313822
## 68   0.15207367 0.092610210
## 69   0.04669304 0.099671580
## 70   0.22415066 0.091065913
## 71   0.37446579 0.293933749
## 72   0.19351342 0.099331893
## 73   0.22012588 0.040903781
## 74   0.17217046 0.119803272
## 75   0.07840166 0.036636937
## 76   0.23381834 0.055267781
## 77   0.24800298 0.043103110
## 78   0.31993145 0.057471618
## 79   0.02880684 0.022794275
## 80   0.31544602 0.124348067
## 81   0.61170459 0.015317135
## 82   0.06501963 0.064491123
## 83   0.19791102 0.088174194
## 84   0.03220996 0.087763183
## 85   0.04526934 0.180987507
## 86   0.18696785 0.113945253
## 87   0.00000000 0.043289777
## 88   0.26445076 0.037513830
## 89   0.09510271 0.015869452
## 90   0.32886750 0.000000000
## 91   0.23618887 0.073345453
## 92   0.25376096 0.060277794
## 93   0.29163131 0.282410324
## 94   0.23496805 0.087633237
## 95   0.23094720 0.026121566
## 96   0.24899764 0.146377146
## 97   0.47405455 0.173380390
## 98   0.42858037 0.039439179
## 99   0.39409617 0.072975546
## 100  0.33846423 0.032902289
## 101  0.13936238 0.072509497
## 102  0.04969336 0.086723149
## 103  0.11417317 0.089282602
## 104  0.12863144 0.114381365
## 105  0.11317769 0.011051531
## 106  0.29304090 0.071095176
## 107  0.23629846 0.051306631
## 108  0.38724291 0.048761073
```

```
## 109 0.20131294 0.039864216
## 110 0.17225535 0.123717859
## 111 0.07713374 0.093146972
## 112 0.44486031 0.064641319
## 113 0.32222810 0.179436386
## 114 0.83807516 0.188816205
## 115 0.18346889 0.115460448
## 116 0.24958014 0.078213550
## 117 0.19963726 0.107215755
## 118 0.04378538 0.076046787
## 119 0.31265074 0.165455714
## 120 0.49086356 0.073653966
## 121 0.08348809 0.026674422
## 122 0.23153849 0.092226885
## 123 0.20187002 0.136957005
## 124 0.12085213 0.079618134
## 125 0.05400882 0.251666635
## 126 0.24645583 0.060241356
## 127 0.18335420 0.104970247
## 128 0.20123747 0.130061775
## 129 0.38592297 0.068105951
## 130 0.18231745 0.089847520
## 131 0.25692394 0.025336370
## 132 0.27006146 0.023029471
## 133 0.32576606 0.057069719
## 134 0.25086468 0.120328106
## 135 0.22867385 0.138572946
## 136 0.28717047 0.072711654
## 137 0.21611385 0.053581882
## 138 0.18914349 0.095375381
## 139 0.15749727 0.119094640
## 140 0.09792649 0.069720335
## 141 0.31187093 0.061157830
## 142 0.09934845 0.098583199
## 143 0.20794846 0.060929015
## 144 0.20873253 0.067231975
## 145 0.48920378 0.099872150
## 146 0.10412521 0.056767423
## 147 0.28567082 0.116793513
## 148 0.26654989 0.038948249
## 149 0.26481509 0.110937618
## 150 0.19689615 0.095665015
## 151 0.25275603 0.455220014
## 152 0.49366373 0.151347131
## 153 0.35425636 0.066035107
## 154 0.20443518 0.084147945
## 155 0.28087649 0.056565076
```

```
md.pattern(happiness_new)
```

```
##   /\     /\
## {  `---'  }
## {  O   O  }
## ==>  V <==  No need for mice. This data set is completely observed.
## \  \|/  /
##   `-----'
```

Happiness.Rank Economy Family Life.Expectancy Freedom Generosity Trust

0   0   0   0   0   0   0   0

```
##      Happiness.Rank Economy Family Life.Expectancy Freedom Generosity Trust
## 155              1       1      1               1       1          1     1
##                  0       0      0               0       0          0     0
##
## 155 0
##      0
```

```r
happy.pca <- PCA(happiness_new, graph = F)
eig.val <- get_eigenvalue(happy.pca)
eig.val
```

```
##       eigenvalue variance.percent cumulative.variance.percent
## Dim.1  3.7738458        53.912084                    53.91208
## Dim.2  1.3526400        19.323429                    73.23551
## Dim.3  0.6725747         9.608209                    82.84372
## Dim.4  0.5434929         7.764184                    90.60791
## Dim.5  0.3613563         5.162233                    95.77014
## Dim.6  0.1613830         2.305471                    98.07561
## Dim.7  0.1347073         1.924390                   100.00000
```

```r
fviz_eig(happy.pca, addlabels = TRUE, ylim = c(0, 60), linecolor = "purple", barfill = "orange", barcolor =
"orange")
```

Scree plot
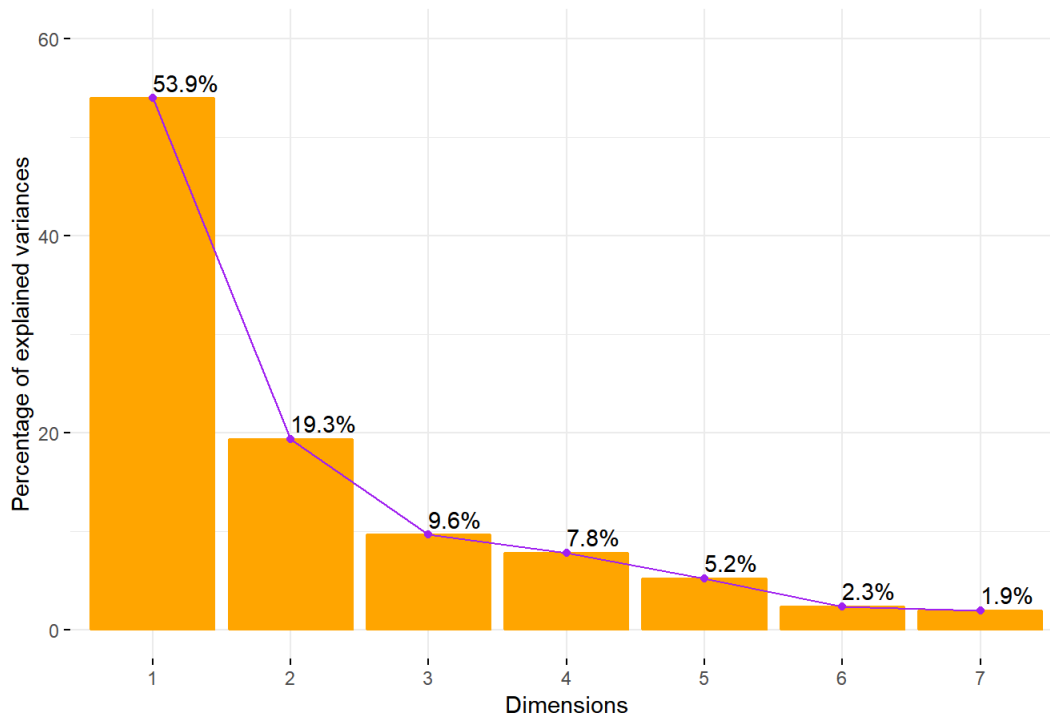

```
#Showing the variables
var <- get_pca_var(happy.pca)
fviz_pca_var(happy.pca, col.var = "darkblue")
```

Variables - PCA
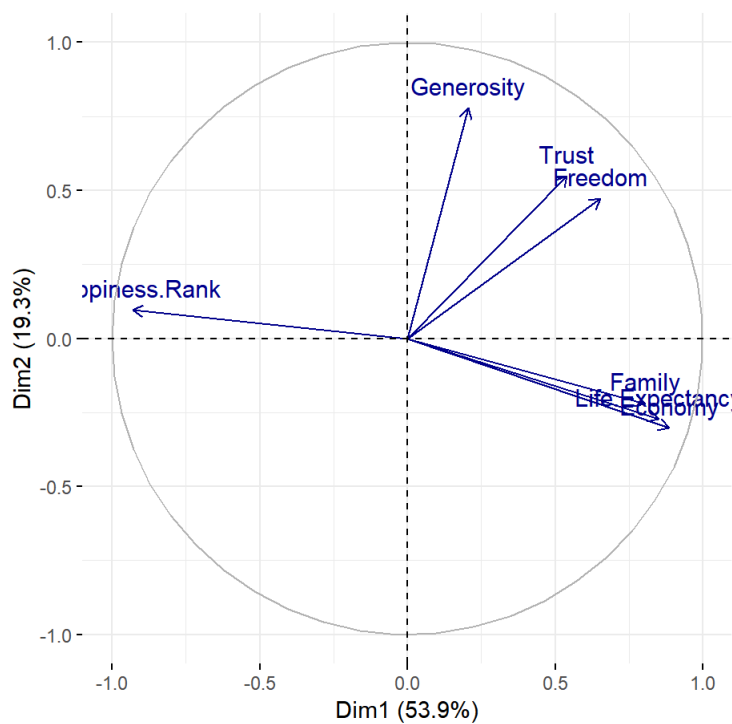

```
#Analysis:We see that for instance family, life expectancy and economy are highly correlated. Trust in the g
overnment and freedom are also correlated.
#We also see that life expectancy, etc are more correlated with the first dimension whereas freedom, generou
sity are more correlated with the second dimension.

#Here ,Cos2 shows the quality of representation
fviz_cos2(happy.pca, choice ="var", axes = 1:2, top = 10, color = "dark blue" )
```

Cos2 of variables to Dim-1-2
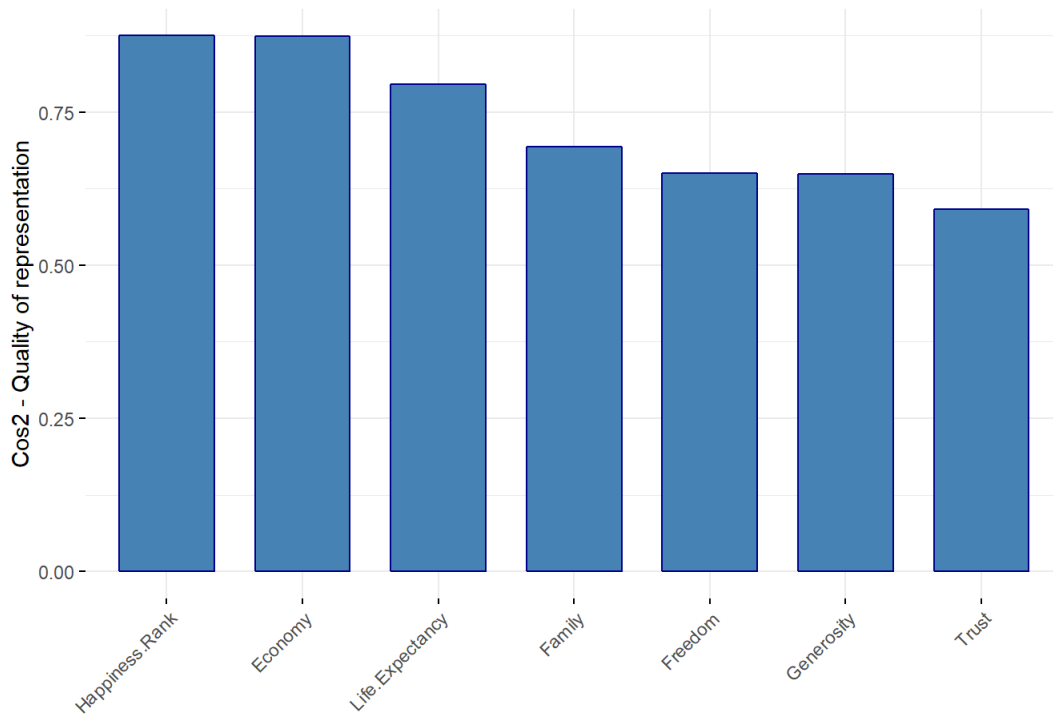

```
#Contribution of the variables
var$contrib
```

```
##                      Dim.1       Dim.2        Dim.3       Dim.4      Dim.5
## Happiness.Rank   22.932697   0.6789814  0.310774650   0.1793912   0.2806790
## Economy          20.716212   6.7440632  0.001856379   6.5004663   1.0187508
## Family           17.085010   3.5540115  3.460364349  14.4828341  53.2314125
## Life.Expectancy  19.113500   5.4852433  2.941657344  11.0780005  16.6241839
## Freedom          11.316840  16.4889758  4.792429952  44.9297245  18.6687593
## Generosity        1.107816  44.8927351 48.343342584   4.2587796   0.3446663
## Trust             7.727925  22.1559896 40.149574744  18.5708037   9.8315482
```

```
#Contribution of the top 5 variables
fviz_contrib(happy.pca, choice = "var", axes = 1, top = 5)
```

Contribution of variables to Dim-1
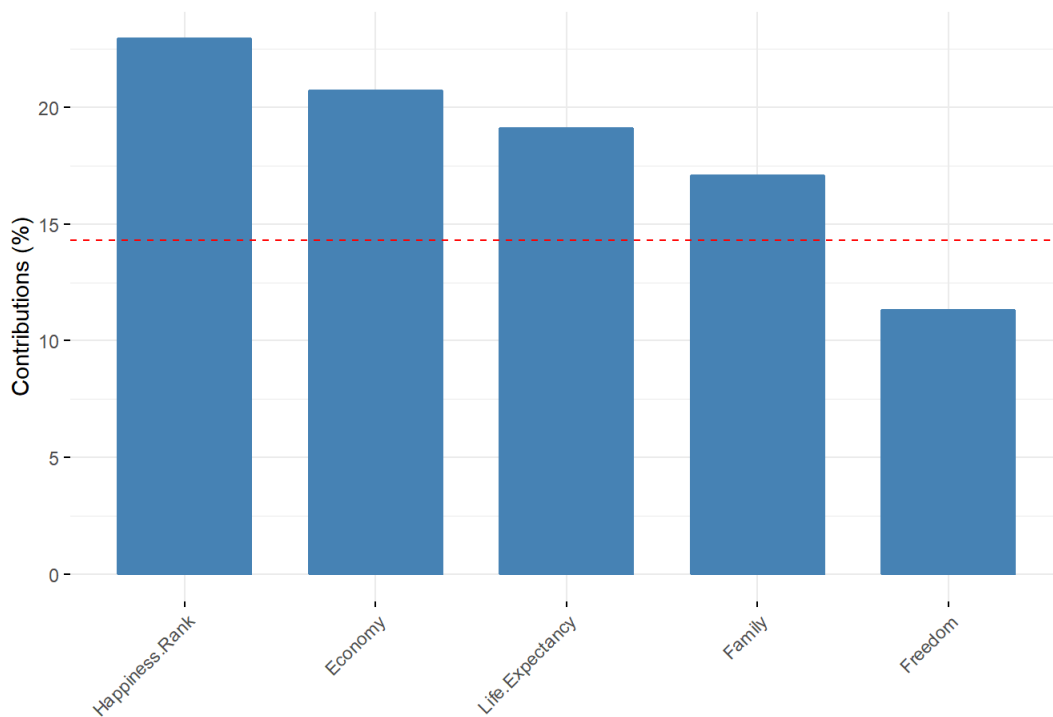
```
#PCA plot with "fviz_pca_ind"
ind <- get_pca_ind(happy.pca)
ind
```

```
## Principal Component Analysis Results for individuals
##   ===================================================
##   Name         Description
## 1 "$coord"    "Coordinates for the individuals"
## 2 "$cos2"     "Cos2 for the individuals"
## 3 "$contrib" "contributions of the individuals"
```

```
happy.pca$ind
```

```
## $coord
##           Dim.1        Dim.2        Dim.3         Dim.4        Dim.5
## 1     3.59019076  1.08250314 -0.450468480  0.1795351775  0.203717981
## 2     3.65800154  1.49462921 -0.993233171  0.4623440892  0.575769809
## 3     3.22559015  0.87222222  1.240646388 -0.4309720952 -0.057690812
## 4     3.64145132  0.90892328 -1.086410048  0.4358566269  0.242359558
## 5     3.44481314  0.86107541 -1.434504080  0.2696471323  0.451434484
## 6     3.17060061  1.46674977  0.329620821  0.5239736659  0.034982403
## 7     3.29625427  1.34316769  0.132010313  0.3248831025  0.059475006
## 8     3.58837220  2.13705235 -0.105495526  0.6588928161  0.594333348
## 9     3.51091218  1.54878811 -0.744266994  0.6624902662  0.329475308
## 10    3.37804136  1.56596767  0.299356612  0.4719575501  0.207386088
## 11    1.89759657 -0.54176346  1.081099812  0.3085747334 -0.263923728
## 12    1.85267489 -0.35751917  0.105847457 -0.9112901797 -0.468185291
## 13    2.80142571  0.36293579 -0.041535574  0.0660987685 -0.088667493
## 14    2.40693880  0.20949050  0.919669437  0.1301033239 -0.184706513
## 15    3.27581897  1.20653752  0.100411327  0.4350945432  0.434518627
## 16    2.91448110  0.71896509 -0.286965843  0.3203839472  0.171089074
## 17    2.69536078  0.02130748 -0.630856935  0.1802297838  0.060421297
## 18    3.38319960  0.55293877 -0.856260913  0.5060763390  0.003147218
## 19    2.84789300  1.31155876  0.690162219  0.7081314382  0.426159863
## 20    1.42378839 -0.47957501  1.040839045  0.4511586520 -0.352769036
## 21    2.83362777  1.31905581 -0.718936916  0.6170872714 -0.230884041
## 22    1.16828884 -0.80289677 -0.130939210 -0.5183208187  0.252331636
## 23    1.54122275 -1.66106879 -0.031409762 -0.8469552295 -0.453559231
## 24    1.33613264 -1.27872740 -0.107438985 -0.9259930732 -0.223273469
## 25    1.01198728 -0.94591337 -0.523979955  0.0764714232 -0.357585894
## 26    3.63004496  1.42506958 -1.373565987  1.6932889379  0.192808428
## 27    2.61190764  1.48167383  1.696817921 -0.0231876923 -0.134589630
## 28    1.89913318 -0.18912330 -0.638043896 -0.6076488590 -0.190227267
## 29    0.65629739  0.24877686  0.394518179 -0.9074308031 -0.321921960
## 30    1.48273620 -0.55711397  0.220542511 -0.8316819353 -0.499657963
## 31    1.99858112 -0.97216166 -0.608968162  0.1299486787 -0.237954031
## 32    1.55651509  1.23819752  2.283822065 -0.7943503891 -0.287148669
## 33    1.43999584 -1.07681933  0.854118746  0.2896065082 -0.123098722
## 34    1.83699125 -1.41461515  0.551470094 -0.0884670870 -0.042663663
## 35    3.21112656  1.57476951 -1.615316132  1.2052585457  0.129905942
## 36    0.79961818 -0.97090225  0.092666509 -0.9929479454 -0.080189189
## 37    1.67091491 -0.20194648 -1.376479078  0.5208289418  0.305625401
## 38    1.05041773 -0.24261883  1.106317449 -0.9648583853 -0.232773877
## 39    1.80739380 -0.02725394 -0.653403410  0.3471244804 -0.137459329
## 40    0.91940768 -1.96558485  0.579225235 -0.1054372912  0.372346671
## 41    1.92182469  0.03022989 -1.206879543  0.1180054614  0.013500606
## 42    0.94139405  0.17562741  1.400484941  0.1653047760 -0.073746850
## 43    0.58720294  0.33684909  0.363360121 -0.2283281853  0.028379113
## 44    0.84553195 -0.54726939 -0.472320112 -0.2468923543 -0.151762295
## 45    0.17498422 -0.98484733 -0.630228823 -0.5109193462 -0.360012099
## 46    1.32716611 -0.99936145  0.089157889 -0.8836401621 -0.271476494
## 47    1.57055502  1.97532852 -0.025706409 -1.0369986248  0.754898419
## 48    1.03264958 -2.00198008  0.852741423  0.4705813909  0.110970370
## 49    0.55153578 -1.90246773 -0.179892746 -0.7506017114  0.352632688
## 50    0.10805231  0.38619782 -0.180277345 -0.8181117234 -0.600042858
## 51    2.01557730 -1.01730916 -0.596841842 -0.0369891189 -0.339613557
## 52    0.30725355 -2.68788968 -0.003619227 -0.1472664138  0.537891440
## 53   -0.02596917 -1.40470271 -0.754741449  0.8032080963  0.222145063
## 54    0.63371570 -1.36603557  0.174861070 -0.0001182077  0.351183754
## 55    0.72146144 -1.49460330  0.625166290  1.1637615643 -0.650742770
```

```
## 56  -0.59886345 -1.13282753  0.854230826  0.0079345669  0.249348140
## 57   0.36306699 -1.27594459  0.180820513 -0.5334538325 -0.966976444
## 58   0.11118797  0.11808167  0.057023393 -1.2430147777 -0.391088012
## 59   0.98809102  0.46023239 -0.712133983  0.0475521644  1.308845639
## 60   0.92018099 -0.56569531 -0.064418078 -0.2313746977  0.195873186
## 61   1.33988909 -0.11583277  0.031765355  0.4944683889 -0.674637051
## 62   1.53296811 -0.59966697  0.587307389 -0.8839908259 -0.562736198
## 63   0.19428962 -0.99690256 -0.111265417 -0.6049007583 -0.496771261
## 64   0.61496296  0.16478333  1.067303217 -0.3329379273 -0.601628775
## 65   0.67537507 -0.91125355  0.904673167  0.6328767438 -0.845123953
## 66   1.38202010 -0.82997824 -0.942071909 -0.2739072679  0.308093148
## 67   0.59768548 -0.99216335 -0.275394056  0.3839543573  0.839307685
## 68   0.39004461 -0.63096855 -0.285075293 -0.7370666156  0.143131302
## 69   0.23827472 -1.73308077 -0.562910988  0.1221116786  0.320601101
## 70   0.58012107 -0.30620288  0.225624200 -0.8845443934  0.470039843
## 71   2.27501840  0.85213361 -0.192929520  1.3454980008 -0.285515873
## 72   0.15596259  0.20987190 -0.395251427 -1.2607935887 -0.273870467
## 73  -0.26734908 -1.26450200  0.791119462  0.6008736382  0.290783210
## 74   0.17465669 -0.45894151 -0.303864502 -0.1637372222 -0.040995484
## 75  -0.05922970 -2.30342932  0.209723011  0.5851090777  0.413444889
## 76   0.33305227 -0.37640721  0.445420204 -0.7522311431 -0.090970228
## 77  -0.33561271 -0.93394744  0.694304747  0.9958628220 -0.684961972
## 78  -0.58503514 -0.27995899  0.965993861  0.5324832729  0.152921854
## 79   0.10302849 -1.60760482 -0.477517680 -0.6839941410 -1.012178559
## 80  -1.65602850  0.52739577  0.157744941  1.2096633100 -0.457961318
## 81   0.06176493  1.41349229  2.520548688 -0.2402683389  0.046758710
## 82  -0.29174244 -2.17779490 -0.002747785  0.4751808220  0.949784769
## 83  -0.23450389 -1.21014718  0.386903959  0.9201041371  0.387119629
## 84  -0.95838569 -0.93289491 -1.130727040  0.0791640263 -1.303635609
## 85   0.01299175 -0.82275556 -1.431560663  0.1269630978  0.014553475
## 86   0.62030780 -0.15865229 -0.318419754 -0.9612859596  0.020000636
## 87  -0.33056872 -2.91428572 -0.113815543  1.1759902308  0.169181103
## 88  -0.24639433 -0.81724080  0.883785712  0.6040338150 -0.387593203
## 89   0.73902145 -1.55557469  0.029901743 -0.7724142322 -0.691798568
## 90  -0.75230087 -0.76834996  1.512499457  0.9041833642 -0.309837879
## 91  -0.70355749 -0.26948040  0.324657205  0.0056018431 -0.075132925
## 92  -0.21606758 -0.61165227  0.611922817  0.2832764736 -0.067686757
## 93  -1.78183493  2.85291721 -1.677666609 -0.6855800166 -0.234240846
## 94   0.18873097  0.20910954  0.035457933 -0.9765413632 -0.466031403
## 95  -1.58567683  0.06044866  0.183802618 -1.2131475637  0.722920611
## 96  -0.40682022  0.57459457 -0.206514900 -0.5827616046  0.271992182
## 97   0.32914003  1.67022062  0.760963635 -0.1746129815  0.514179970
## 98  -0.31866122  0.78290935  1.510051896 -0.7708578001  0.323815030
## 99  -0.82518425  0.99127696  0.967807729 -0.4691773930  0.021402880
## 100  0.05290554 -0.19452024  1.233948258 -0.5257994763  0.582773232
## 101 -0.62590514 -0.34889118 -0.509659696 -1.4053914310  0.730822047
## 102 -1.10298496 -1.29603823 -0.812031993  0.6258845868 -0.755129270
## 103 -1.12386552 -1.01754880 -0.289242696  0.2678328590  0.267323039
## 104 -1.00847591 -0.77268979 -0.551301795  0.5872857485 -0.117065699
## 105 -0.13377534 -1.88892050  0.421724839 -0.1183105031  0.284555941
## 106 -2.58304007  0.85274993  0.138400640 -0.5911599516  0.720939804
## 107 -2.04807118  0.56064248 -0.130672107 -0.9131920434 -0.050635399
## 108 -1.11337347  0.09724649  1.139486494  1.5057013211 -1.081961475
## 109 -0.86822829 -0.56273437  0.166226661  0.4998416173 -1.456540265
## 110 -1.36323472  0.44160282 -0.835547400 -0.0580565710 -1.223134218
## 111 -0.86360996 -0.33924507 -1.108442136 -1.0487126053 -0.308611136
## 112 -1.35175113  1.47929232  1.034144743 -0.5466342237  0.029073645
## 113 -2.09542108  2.01223295 -0.633011472 -0.5940268534  0.177606159
## 114 -0.55362636  4.18509082  2.301291664  0.2428907587  0.445493421
## 115 -1.36055142  0.09216899 -0.385046949 -0.4515555525  0.424725596
## 116 -1.59801531  0.64710572 -0.160017546 -0.7659567429 -0.128383765
## 117 -1.03689287 -0.44911619 -0.191850233  0.7162823570 -0.153894437
## 118 -1.11009206 -1.33712815 -0.830544077 -0.1421857971  0.291577277
## 119 -1.80570273  1.40961136 -0.323185571  0.0793966317 -0.121188539
## 120  0.21528667  1.34286579  1.395983630 -0.4544530041 -0.389465116
## 121 -1.52012400 -1.66926747 -0.042387762  0.6345596825 -0.301365928
## 122 -1.44359116  0.52754855 -0.380600796 -0.0622640349 -1.122448135
## 123 -1.91947978 -0.50949979 -0.053311498  0.7446006108  1.731750643
## 124 -1.84081082 -0.07182344 -0.895720510 -0.4533989124 -0.627043054
## 125 -1.27292896 -0.21729760 -2.045118355  1.6797523017 -1.080350157
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## 131 -2.06825748  0.38969089  0.189404790 -0.5482763348 -0.562220081
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## 134 -2.31326602  0.70197644 -0.239692452 -0.2389774587  0.663879034
## 135 -2.38235317  0.88208929 -0.519894531 -0.3441745817  0.424673978
## 136 -2.82535393  1.37605246 -0.236803692 -0.2487255663 -1.372591312
## 137 -3.27732651 -0.03822839  0.045523472  0.0791600208  0.966410701
## 138 -2.39587408  0.29508097 -0.413918674 -0.5542110947  0.657916265
## 139 -2.29621034  0.43136647 -0.881528448 -1.0754446414  1.139016662
## 140 -3.03331543 -1.35616873 -0.333268822  0.7646540489  1.704788634
## 141 -3.66950439  0.45874502  0.430814357  1.1833148731  0.017965380
## 142 -0.84248834 -0.36891079 -0.963342827 -0.9577211259  0.068544153
## 143 -3.17842908  0.86061040 -0.650448629 -0.1555834596 -1.380039443
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## 147 -3.51760075  0.71529947 -0.119647128  1.1488942687  0.148827434
## 148 -3.12683266  0.69296980  0.221843574 -0.4791033551  0.006111377
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## 150 -3.34166655  0.89925518 -0.840850001  0.1664685320 -1.180117633
## 151 -0.91184753  2.94080028 -2.705502585  0.3465987794  0.533143237
## 152 -2.72121780  1.31022990  0.937854796  2.8922433485 -0.653969605
## 153 -1.98905603  0.93259564  0.620516489 -0.2784678450  0.075320297
## 154 -4.25436160  0.11692887 -0.204025100  1.0423187556  0.439068024
## 155 -5.06055612  1.54782089 -0.450661765  0.6919441788 -1.574791689
##
## $cos2
##              Dim.1        Dim.2        Dim.3        Dim.4        Dim.5
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## 2     7.777504e-01 1.298433e-01 5.733963e-02 1.242462e-02 1.926860e-02
## 3     8.051190e-01 5.887027e-02 1.191071e-01 1.437276e-02 2.575461e-04
## 4     8.531622e-01 5.315413e-02 7.593992e-02 1.222278e-02 3.779226e-03
## 5     7.874624e-01 4.920179e-02 1.365533e-01 4.824920e-03 1.352346e-02
## 6     7.932040e-01 1.697516e-01 8.572966e-03 2.166311e-02 9.656085e-05
## 7     8.476659e-01 1.407485e-01 1.359562e-03 8.234506e-03 2.759639e-04
## 8     7.038552e-01 2.496426e-01 6.083541e-04 2.373110e-02 1.930850e-02
## 9     7.776893e-01 1.513388e-01 3.494811e-02 2.769014e-02 6.848760e-03
## 10    8.023333e-01 1.724211e-01 6.300900e-03 1.566141e-02 3.024016e-03
## 11    6.502810e-01 5.300449e-02 2.110692e-01 1.719547e-02 1.257912e-02
## 12    6.737322e-01 2.508924e-02 2.199124e-03 1.630055e-01 4.302535e-02
## 13    9.782810e-01 1.641969e-02 2.150531e-04 5.446182e-04 9.800184e-04
## 14    8.286471e-01 6.277232e-03 1.209771e-01 2.421120e-03 4.879822e-03
## 15    8.528629e-01 1.156967e-01 8.013179e-04 1.504550e-02 1.500570e-02
## 16    9.196463e-01 5.596467e-02 8.915768e-03 1.111322e-02 3.169151e-03
## 17    9.397153e-01 5.872547e-05 5.147831e-02 4.201609e-03 4.722180e-04
## 18    8.896981e-01 2.376520e-02 5.699013e-02 1.990763e-02 7.699119e-07
## 19    7.373968e-01 1.563974e-01 4.330682e-02 4.559127e-02 1.651198e-02
## 20    5.008789e-01 5.682703e-02 2.676757e-01 5.029218e-02 3.074842e-02
## 21    7.273753e-01 1.576156e-01 4.682249e-02 3.449579e-02 4.829040e-03
## 22    4.720265e-01 2.229387e-01 5.929327e-03 9.291017e-02 2.201960e-02
## 23    3.847005e-01 4.468555e-01 1.597796e-04 1.161750e-01 3.331654e-02
## 24    3.886254e-01 3.559492e-01 2.512788e-03 1.866584e-01 1.085191e-02
## 25    3.520654e-01 3.075926e-01 9.438491e-02 2.010350e-03 4.395760e-02
## 26    6.543511e-01 1.008461e-01 9.368841e-02 1.423800e-01 1.846030e-03
## 27    5.719749e-01 1.840630e-01 2.413970e-01 4.507922e-05 1.518744e-03
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## 45  1.320012e-02 4.181363e-01 1.712287e-01 1.125343e-01 5.587458e-02
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## 151 4.487506e-02 4.667579e-01 3.950540e-01 6.483577e-03 1.534081e-02
## 152 3.910765e-01 9.066302e-02 4.645221e-02 4.417787e-01 2.258657e-02
## 153 6.655154e-01 1.463022e-01 6.476953e-02 1.304410e-02 9.543070e-04
## 154 9.147652e-01 6.910101e-04 2.103817e-03 5.490880e-02 9.743264e-03
## 155 7.968166e-01 7.454239e-02 6.319218e-03 1.489718e-02 7.716282e-02
##
## $contrib
##             Dim.1        Dim.2        Dim.3        Dim.4        Dim.5
## 1     2.203531e+00 0.5589132348 1.946510e-01 3.826252e-02 7.409543e-02
## 2     2.287557e+00 1.0654988855 9.463030e-01 2.537494e-01 5.918757e-01
## 3     1.778699e+00 0.3628610574 1.476467e+00 2.204818e-01 5.942183e-03
## 4     2.266904e+00 0.3940401888 1.132180e+00 2.255079e-01 1.048704e-01
## 5     2.028689e+00 0.3536457460 1.973928e+00 8.631098e-02 3.638498e-01
## 6     1.718570e+00 1.0261199287 1.042214e-01 3.259068e-01 2.184902e-03
## 7     1.857486e+00 0.8604914091 1.671643e-02 1.252935e-01 6.315412e-03
## 8     2.201299e+00 2.1782934594 1.067569e-02 5.153520e-01 6.306565e-01
## 9     2.107289e+00 1.1441160570 5.313557e-01 5.209949e-01 1.938110e-01
## 10    1.950806e+00 1.1696384814 8.596180e-02 2.644115e-01 7.678775e-02
## 11    6.155905e-01 0.1399927310 1.121139e+00 1.130304e-01 1.243625e-01
## 12    5.867898e-01 0.0609655847 1.074703e-02 9.857977e-01 3.913526e-01
## 13    1.341660e+00 0.0628269052 1.654887e-03 5.186342e-03 1.403659e-02
## 14    9.904082e-01 0.0209321926 8.113184e-01 2.009330e-02 6.091119e-02
## 15    1.834526e+00 0.6943328793 9.671486e-03 2.247200e-01 3.370928e-01
## 16    1.452134e+00 0.2465481090 7.899293e-02 1.218473e-01 5.226094e-02
## 17    1.241990e+00 0.0002165460 3.817592e-01 3.855916e-02 6.517976e-03
## 18    1.956768e+00 0.1458277402 7.032991e-01 3.040230e-01 1.768424e-05
## 19    1.386537e+00 0.8204678550 4.569095e-01 5.952538e-01 3.242484e-01
## 20    3.465573e-01 0.1096981142 1.039190e+00 2.416201e-01 2.221844e-01
## 21    1.372681e+00 0.8298744727 4.958032e-01 4.520304e-01 9.517448e-02
## 22    2.333375e-01 0.3074719407 1.644626e-02 3.189126e-01 1.136779e-01
## 23    4.060831e-01 1.3160164090 9.463616e-04 8.515210e-01 3.672829e-01
## 24    3.051991e-01 0.7799055486 1.107265e-02 1.017864e+00 8.900347e-02
## 25    1.750791e-01 0.4267649959 2.633644e-01 6.941811e-03 2.282935e-01
## 26    2.252725e+00 0.9686306456 1.809784e+00 3.403585e+00 6.637197e-02
## 27    1.166272e+00 1.0471075470 2.761839e+00 6.382480e-04 3.234118e-02
## 28    6.165879e-01 0.0170598871 3.905071e-01 4.383084e-01 6.460680e-02
## 29    7.363521e-02 0.0295192978 1.493007e-01 9.774656e-01 1.850265e-01
## 30    3.758477e-01 0.1480383263 4.665653e-02 8.210866e-01 4.457366e-01
```

```
## 31   6.828538e-01 0.4507783477 3.557271e-01 2.004556e-02 1.010925e-01
## 32   4.141816e-01 0.7312500753 5.003251e+00 7.490290e-01 1.472131e-01
## 33   3.544922e-01 0.5530593717 6.997845e-01 9.956142e-02 2.705452e-02
## 34   5.768971e-01 0.9544708703 2.917237e-01 9.290474e-03 3.249743e-03
## 35   1.762784e+00 1.1828238166 2.502896e+00 1.724388e+00 3.012942e-02
## 36   1.093074e-01 0.4496111587 8.237082e-03 1.170381e+00 1.148059e-02
## 37   4.773015e-01 0.0194517475 1.817469e+00 3.220065e-01 1.667676e-01
## 38   1.886289e-01 0.0280759900 1.174052e+00 1.105100e+00 9.673891e-02
## 39   5.584570e-01 0.0003542785 4.095346e-01 1.430359e-01 3.373503e-02
## 40   1.445108e-01 1.8427637403 3.218272e-01 1.319663e-02 2.475299e-01
## 41   6.314103e-01 0.0004358721 1.397191e+00 1.653022e-02 3.254162e-04
## 42   1.515050e-01 0.0147119640 1.881415e+00 3.243734e-02 9.710009e-03
## 43   5.894685e-02 0.0541198785 1.266492e-01 6.188615e-02 1.437905e-03
## 44   1.222205e-01 0.1428526772 2.139935e-01 7.235853e-02 4.112070e-02
## 45   5.234581e-03 0.4626194528 3.809994e-01 3.098697e-01 2.314019e-01
## 46   3.011166e-01 0.4763555721 7.625132e-03 9.268839e-01 1.315822e-01
## 47   4.216872e-01 1.8610786824 6.338852e-04 1.276529e+00 1.017442e+00
## 48   1.823014e-01 1.9116376258 6.975294e-01 2.628718e-01 2.198603e-02
## 49   5.200338e-02 1.7263174579 3.104239e-02 6.687958e-01 2.220127e-01
## 50   1.995962e-03 0.0711386636 3.117526e-02 7.945105e-01 6.428316e-01
## 51   6.945173e-01 0.4936191266 3.417011e-01 1.624136e-03 2.059220e-01
## 52   1.613905e-02 3.4459497996 1.256491e-05 2.574435e-02 5.165614e-01
## 53   1.152923e-04 0.9411414630 5.464170e-01 7.658269e-01 8.810611e-02
## 54   6.865514e-02 0.8900411428 2.933013e-02 1.658693e-08 2.201920e-01
## 55   8.898369e-02 1.0654619364 3.749030e-01 1.607691e+00 7.560515e-01
## 56   6.131119e-02 0.6120877093 6.999681e-01 7.473446e-05 1.110056e-01
## 57   2.253501e-02 0.7765147277 3.136340e-02 3.378066e-01 1.669416e+00
## 58   2.113488e-03 0.0066504501 3.119133e-03 1.834117e+00 2.730749e-01
## 59   1.669083e-01 0.1010276889 4.864646e-01 2.684201e-03 3.058508e+00
## 60   1.447540e-01 0.1526339771 3.980552e-03 6.354863e-02 6.849875e-02
## 61   3.069176e-01 0.0063995432 9.679106e-04 2.902361e-01 8.125930e-01
## 62   4.017449e-01 0.1715166662 3.308710e-01 9.276197e-01 5.653826e-01
## 63   6.453322e-03 0.4740143485 1.187540e-02 4.343529e-01 4.406012e-01
## 64   6.465201e-02 0.0129512769 1.092706e+00 1.315834e-01 6.462341e-01
## 65   7.797838e-02 0.3960632364 7.850751e-01 4.754586e-01 1.275186e+00
## 66   3.265223e-01 0.3285636243 8.513260e-01 8.905977e-02 1.694715e-01
## 67   6.107023e-02 0.4695181938 7.275066e-02 1.749982e-01 1.257694e+00
## 68   2.600834e-02 0.1898897386 7.795554e-02 6.448934e-01 3.657648e-02
## 69   9.705995e-03 1.4325957944 3.039536e-01 1.770063e-02 1.835113e-01
## 70   5.753358e-02 0.0447202906 4.883140e-02 9.287818e-01 3.944592e-01
## 71   8.848180e-01 0.3463390428 3.570468e-02 2.149020e+00 1.455436e-01
## 72   4.158388e-03 0.0210084818 1.498562e-01 1.886959e+00 1.339131e-01
## 73   1.221917e-02 0.7626497108 6.003603e-01 4.285887e-01 1.509633e-01
## 74   5.215003e-03 0.1004617520 8.857022e-02 3.182506e-02 3.000577e-03
## 75   5.997402e-04 2.5306719393 4.219101e-02 4.063948e-01 3.051884e-01
## 76   1.896308e-02 0.0675774708 1.903126e-01 6.717026e-01 1.477513e-02
## 77   1.925577e-02 0.4160360220 4.624109e-01 1.177263e+00 8.376558e-01
## 78   5.851242e-02 0.0373830639 8.951103e-01 3.365785e-01 4.175147e-02
## 79   1.814675e-03 1.2326638684 2.187292e-01 5.553659e-01 1.829140e+00
## 80   4.688347e-01 0.1326659141 2.386924e-02 1.737015e+00 3.744469e-01
## 81   6.521808e-04 0.9529562370 6.094218e+00 6.852792e-02 3.903532e-03
## 82   1.455068e-02 2.2621428223 7.242579e-06 2.680355e-01 1.610583e+00
## 83   9.401221e-03 0.6984936306 1.435933e-01 1.004959e+00 2.675612e-01
## 84   1.570235e-01 0.4150988364 1.226432e+00 7.439268e-03 3.034206e+00
## 85   2.885491e-05 0.3228700200 1.965836e+00 1.913504e-02 3.781517e-04
## 86   6.578072e-02 0.0120054592 9.725855e-02 1.096932e+00 7.142008e-04
## 87   1.868132e-02 4.0508888056 1.242599e-02 1.641655e+00 5.110182e-02
## 88   1.037876e-02 0.3185562601 7.492414e-01 4.331087e-01 2.682162e-01
## 89   9.336804e-02 1.1541647461 8.576712e-04 7.082311e-01 8.544606e-01
## 90   9.675363e-02 0.2815815629 2.194412e+00 9.704819e-01 1.713964e-01
## 91   8.462201e-02 0.0346370118 1.011062e-01 3.725086e-05 1.007844e-02
## 92   7.981111e-03 0.1784412456 3.591874e-01 9.525668e-02 8.179752e-03
## 93   5.427741e-01 3.8820793166 2.699847e+00 5.579442e-01 9.796207e-02
## 94   6.089344e-03 0.0208561325 1.206020e-03 1.132024e+00 3.877601e-01
## 95   4.298466e-01 0.0017428476 3.240643e-02 1.747035e+00 9.330693e-01
## 96   2.829366e-02 0.1574740898 4.091010e-02 4.031404e-01 1.320826e-01
## 97   1.852020e-02 1.3305578165 5.554636e-01 3.619324e-02 4.720228e-01
## 98   1.735971e-02 0.2923539881 2.187316e+00 7.053797e-01 1.872090e-01
## 99   1.164089e-01 0.4686796408 8.984749e-01 2.613055e-01 8.178566e-04
## 100  4.785049e-04 0.0180474418 1.460568e+00 3.281820e-01 6.063619e-01
## 101  6.697322e-02 0.0580585300 2.491658e-01 2.344601e+00 9.535775e-01
## 102  2.079808e-01 0.8011644900 6.325197e-01 4.650107e-01 1.018065e+00
## 103  2.159299e-01 0.4938517111 8.025140e-02 8.515343e-02 1.275867e-01
```

```
## 103 2.159299e-01 0.4956517111 8.025140e-02 8.515545e-02 1.275807e-01
## 104 1.738662e-01 0.2847714297 2.915457e-01 4.094241e-01 2.446764e-02
## 105 3.059400e-03 1.7018192276 1.706028e-01 1.661579e-02 1.445666e-01
## 106 1.140634e+00 0.3468402153 1.837401e-02 4.148436e-01 9.279631e-01
## 107 7.170909e-01 0.1499194786 1.637923e-02 9.899167e-01 4.577635e-03
## 108 2.119170e-01 0.0045105959 1.245507e+00 2.691238e+00 2.090048e+00
## 109 1.288701e-01 0.1510403360 2.650508e-02 2.965782e-01 3.787718e+00
## 110 3.177060e-01 0.0930143027 6.696841e-01 4.001080e-03 2.671043e+00
## 111 1.275027e-01 0.0548925189 1.178566e+00 1.305532e+00 1.700419e-01
## 112 3.123760e-01 1.0437441968 1.025866e+00 3.547056e-01 1.509147e-03
## 113 7.506315e-01 1.9312681489 3.843713e-01 4.188771e-01 5.631820e-02
## 114 5.239836e-02 8.3540263292 5.080087e+00 7.003198e-02 3.543360e-01
## 115 3.164565e-01 0.0040518725 1.422182e-01 2.420454e-01 3.220695e-01
## 116 4.365621e-01 0.1997268945 2.456196e-02 6.964387e-01 2.942747e-02
## 117 1.838027e-01 0.0962062933 3.530632e-02 6.090360e-01 4.228424e-02
## 118 2.106697e-01 0.8527704228 6.616879e-01 2.399865e-02 1.517889e-01
## 119 5.574124e-01 0.9477304823 1.001917e-01 7.483049e-03 2.622140e-02
## 120 7.923525e-03 0.8601046284 1.869341e+00 2.451616e-01 2.708132e-01
## 121 3.950410e-01 1.3290396195 1.723490e-03 4.779906e-01 1.621515e-01
## 122 3.562645e-01 0.1327427892 1.389528e-01 4.602025e-03 2.249393e+00
## 123 6.298704e-01 0.1238152348 2.726274e-03 6.581444e-01 5.354309e+00
## 124 5.792986e-01 0.0024604719 7.696138e-01 2.440256e-01 7.019843e-01
## 125 2.770081e-01 0.0225214283 4.012035e+00 3.349384e+00 2.083828e+00
## 126 1.202387e+00 0.0205927595 1.230870e-01 3.600091e-01 3.015464e+00
## 127 6.693920e-01 0.0009270660 1.099908e-01 4.869906e-01 2.955842e+00
## 128 8.415041e-01 0.4072041026 8.556257e-01 6.329149e-01 6.013197e-02
## 129 1.664186e-01 1.2511631522 2.103237e-01 1.543293e+00 1.325569e+00
## 130 9.707129e-01 0.4840394941 4.489449e-02 1.131077e+00 4.919698e+00
## 131 7.312963e-01 0.0724313475 3.441198e-02 3.568399e-01 5.643459e-01
## 132 2.598574e-01 0.6091523416 1.497339e+00 4.999132e-01 2.135220e+00
## 133 6.756257e-01 0.5005980931 1.757102e-01 1.235308e+00 2.394905e-01
## 134 9.148193e-01 0.2350342401 5.511077e-02 6.779354e-02 7.868838e-01
## 135 9.702786e-01 0.3711172702 2.592736e-01 1.406151e-01 3.219912e-01
## 136 1.364677e+00 0.9031420188 5.379040e-02 7.343706e-02 3.363684e+00
## 137 1.836215e+00 0.0006970406 1.987918e-03 7.438515e-03 1.667463e+00
## 138 9.813233e-01 0.0415306262 1.643455e-01 3.646069e-01 7.728122e-01
## 139 9.013791e-01 0.0887521394 7.454190e-01 1.372937e+00 2.316290e+00
## 140 1.572966e+00 0.8772300913 1.065411e-01 6.940718e-01 5.188882e+00
## 141 2.301966e+00 0.1003757490 1.780361e-01 1.662169e+00 5.762428e-04
## 142 1.213422e-01 0.0649125672 8.902040e-01 1.088811e+00 8.388292e-03
## 143 1.727067e+00 0.3532638885 4.058390e-01 2.873434e-02 3.400288e+00
## 144 1.383209e+00 0.0104737060 7.984871e-03 2.474564e-01 1.474960e-01
## 145 1.987552e+00 0.6986885973 1.674483e+00 4.756665e+00 3.308033e-01
## 146 1.172579e+00 0.2194475287 2.506160e-01 1.486631e-03 3.405935e-02
## 147 2.115326e+00 0.2440404828 1.373195e-02 1.566876e+00 3.954565e-02
## 148 1.671450e+00 0.2290417648 4.720864e-02 2.724789e-01 6.668235e-05
## 149 1.490594e+00 0.5503983261 1.117626e-01 1.481447e-02 1.572143e-04
## 150 1.909020e+00 0.3857020654 6.782110e-01 3.289567e-02 2.486470e+00
## 151 1.421440e-01 4.1249351728 7.021400e+00 1.426029e-01 5.074818e-01
## 152 1.265933e+00 0.8188061175 8.437213e-01 9.929882e+00 7.635682e-01
## 153 6.763604e-01 0.4148325501 3.693469e-01 9.205016e-02 1.012877e-02
## 154 3.094233e+00 0.0065212314 3.992960e-02 1.289661e+00 3.441885e-01
## 155 4.378049e+00 1.1426874900 1.948181e-01 5.683509e-01 4.427708e+00
##
## $dist
##          1         2         3         4         5         6         7
## 3.7961035 4.1478569 3.5948364 3.9423821 3.8819573 3.5599926 3.5802087
##          8         9        10        11        12        13        14
## 4.2771633 3.9812266 3.7712694 2.3531703 2.2571248 2.8323526 2.6441144
##         15        16        17        18        19        20        21
## 3.5471560 3.0391409 2.7804734 3.5867928 3.3164468 2.0117734 3.3224888
##         22        23        24        25        26        27        28
## 1.7004624 2.4848721 2.1433050 1.7055458 4.4875220 3.4535786 2.1334301
##         29        30        31        32        33        34        35
## 1.5096120 1.8913199 2.3286515 3.1644011 2.0317777 2.4076524 4.2138451
##         36        37        38        39        40        41        42
## 1.6802230 2.3449152 2.0540685 2.1244585 2.2945241 2.3055451 1.8151559
##         43        44        45        46        47        48        49
## 1.2256896 1.2886774 1.5230353 1.9055964 2.8612398 2.4617638 2.1967844
##         50        51        52        53        54        55        56
## 1.2966464 2.4465352 2.7794356 1.9108396 1.5677954 2.2211214 1.8100399
##         57        58        59        60        61        62        63
## 1.7483685 1.3670924 1.8632259 1.1586555 1.6201893 2.1256675 1.2989620
```

```
##        64        65        66        67        68        69        70
## 1.4624049 1.8066994 1.9702201 1.5199751 1.1072619 1.8706845 1.2703908
##        71        72        73        74        75        76        77
## 2.9515503 1.3751355 1.6665657 0.6357865 2.4232389 1.0990845 1.7294346
##        78        79        80        81        82        83        84
## 1.3197147 2.1233045 2.3743331 2.9523132 2.4476885 1.6389632 2.2106725
##        85        86        87        88        89        90        91
## 1.6697843 1.3345677 3.1908318 1.4565673 2.2125182 2.0946386 0.9519184
##        92        93        94        95        96        97        98
## 0.9949776 3.9650746 1.3902687 2.3873386 1.2983809 2.0150897 2.0441449
##        99       100       101       102       103       104       105
## 1.7889742 1.6271956 2.0611503 2.1284793 1.6995077 1.5172962 2.1378658
##       106       107       108       109       110       111       112
## 3.0234079 2.4102446 2.5106647 1.9217531 2.1105581 1.9227011 2.3266386
##       113       114       115       116       117       118       119
## 3.0731606 4.8556037 1.6640172 1.9224553 1.4893784 2.1606797 2.3676957
##       120       121       122       123       124       125       126
## 2.3705754 2.4218230 1.9796189 2.7404955 2.2592936 3.1713533 3.1210155
##       127       128       129       130       131       132       133
## 2.4717214 2.7883193 2.5350895 3.2326160 2.3032299 2.5862987 2.5375579
##       134       135       136       137       138       139       140
## 2.5395970 2.7393995 3.4577211 3.4660498 2.6053223 2.9972400 3.9476201
##       141       142       143       144       145       146       147
## 3.9470395 2.1525118 3.6524912 2.9700961 4.3732562 2.7762392 3.7942431
##       148       149       150       151       152       153       154
## 3.2788041 3.1686328 3.7559724 4.3044701 4.3514342 2.4381924 4.4481515
##       155
## 5.6691645
```

```
#Plotting the graph
fviz_pca_ind (happy.pca, pointsize = "cos2", pointshape = 22, fill = "blue", repel = TRUE)
```



```
#Method to show only the 50 countries best represented.
plot(happy.pca,  select = "cos2 50", cex=1,  col.ind = "darkblue", title = "50 countries with highest cos2",
cex.main=2, col.main= "darkblue")
```

50 countries with highest cos2